STRUCTURAL IVE FOR DYNAMIC TREATMENT EFFECTS: SPANKING EFFECT ON BEHAVIOR

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- Finding effects of sequential treatments on response y measured at the end of a trial when some treatments are affected by interim responses; e.g., effects of spanking on behavior when spanking is adjusted depending on interim behaviors.

- A headway, '*G estimation (or G computation algorithm)* ', has been made in 1980's by Robins in (bio-) statistics generalizing the usual static treatment effect analysis under 'selection on observables'. But hard to implement.

- A much simpler econometric alternative to G estimation is proposed—*IVE's for a linear structural model.* Also relation between this approach and *Granger causality* is explored.

- Our approach and G estimation are applied to find the effects of spanking on behavior. Mild spanking at early years reduces a child's behavior problems later.

1 Failure of Dynamic Panel Data Model

Two Period Model. Suppose

$$(x_0, y_0),\ (d_1, \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}),\ (d_2, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix})$$

where x_0 and y_0 are the baseline covariate and response, and treatment d_t at period t temporally precedes (x_t', y_t), $t = 1, 2$.

Total Effect. Interested in the *total* effect of the treatment *profile* $d \equiv (d_1, d_2)'$ on the last response y_2. Graphically, omitting y_0, x_0, x_1, x_2,

Total Effects

$$\begin{array}{ccccc}
d_2 & \longrightarrow & \longrightarrow & \longrightarrow & y_2 \\
 & \nwarrow & & \nearrow & \uparrow \\
 & & y_1 & & \uparrow \\
 & \nearrow & & & \uparrow \\
d_1 & \longrightarrow & \longrightarrow & \longrightarrow & \uparrow
\end{array}$$

d_2 has only a direct effect on y_2, but d_1 has direct *and* indirect (through y_1) effects.

Main Dilemma. If y_1 is controlled, then the indirect effect of d_1 on y_2 is not identified. If y_1 is not controlled, then the effect of d_2 on y_2 gets distorted as y_1 becomes a 'common factor' (confounder) for d_2 and y_2: even if there is no effect of d_2 on y_2, a spurious effect may be found due to not controlling y_1.

y_1 controlled:
Direct Effects Only

$$\begin{array}{ccccc}
d_2 & \longrightarrow & \longrightarrow & \longrightarrow & y_2 \\
 & & & & \uparrow \\
 & & & & \uparrow \\
 & & & & \uparrow \\
d_1 & \longrightarrow & \longrightarrow & \longrightarrow & \uparrow
\end{array}$$

y_1 not controlled:
Confounded Effect

$$\begin{array}{ccccc}
d_2 & & & & y_2 \\
 & \nwarrow & & \nearrow & \uparrow \\
 & & y_1 & & \uparrow \\
 & \nearrow & & & \uparrow \\
d_1 & \longrightarrow & \longrightarrow & \longrightarrow & \uparrow
\end{array}$$

Non-Lagged-Response Confounder. The total (direct+indirect) effect composition with a confounder can occur also with a covariate $w_1 \neq y_1$. For instance, y_2 can be death and w_1 is a 'death-predictor'.

$$
\begin{array}{ccccc}
d_2 & \longrightarrow & \longrightarrow & \longrightarrow & y_2 \\
 & \nwarrow & & \nearrow & \uparrow \\
 & & w_1 & & \uparrow \\
 & \nearrow & & & \uparrow \\
d_1 & \longrightarrow & \longrightarrow & \longrightarrow & \uparrow
\end{array}
$$

Inadequate Dynamic Model. A typical 'first-lag' dynamic model

$$y_{i2} = \beta_1 + \beta_y y_{i1} + \beta_{d1} d_{i1} + \beta_{d2} d_{i2} + x'_{x2} x_{i2} + v_{i2}$$

is misleading, because the *indirect effect of* d_1 *on* y_2 *through* y_1 *is missed* by controlling y_1. Intuitively, if the effect of d_1 on y_1 is γ_y, then the indirect effect of d_1 on y_2 through y_1 is $\beta_y \gamma_y$. This dynamic model identifies only the direct effects β_{d1} and β_{d2} of d_1 and d_2 on y_2. The desired *total effect of the treatment profile is* $\beta_{d1} + \beta_y \gamma_y$ *(from* d_1*) plus* β_{d2} *(from* d_2*).*

Total Mean Marginal Effect. A common goal across different approaches is to find $E(y_2^{jk} - y_2^{00})$ where

$$
\begin{array}{rcl}
y_2^{jk} & : & \text{potential response when } d_1 = j \text{ and } d_2 = k \\
(y_1^j & : & \text{potential response when } d_1 = j)
\end{array}
$$

With d_1 and d_2 observed, $y_1 = y_1^{d_1}$ and $y_2 = y_2^{d_1 d_2}$.

2 IVE's for Linear Structural Models

Two-Step IVE for First-Lag Model. Consider

$$\begin{aligned} y_{i1} &= \gamma_1 + \gamma_y y_{i0} + \gamma_d d_{i1} + \gamma_x' x_{i1} + v_{i1}, \\ y_{i2} &= \beta_1 + \beta_y y_{i1} + \beta_{d1} d_{i1} + \beta_{d2} d_{i2} + \beta_{x2}' x_{i2} + v_{i2}. \end{aligned} \qquad \text{(First-lag)}$$

Estimate the two equations separately with IVE to find

$$\begin{aligned} \textit{direct and indirect effects of } d_1 \text{ on } y_2 &: \quad \beta_{d1} + \beta_y \gamma_d, \\ \textit{direct effect of } d_2 \text{ on } y_2 &: \quad \beta_{d2}. \end{aligned}$$

This is a *two-step IVE.* The source for the instruments in the y_1 equation is x_0, and the source for the instruments in the y_2 equation is x_0 and x_1—owing to the exclusion restriction of lagged covariates.

One-Step IVE under "Stationarity". Suppose

$$\textit{equal contemporaneous effects}: \ \gamma_d = \beta_{d2}$$

that the effect of d_1 on y_1 is the same as the effect of d_2 on y_2. This is a stationarity-type assumption, under which

$$d_1 \text{ effect is } \beta_{d1} + \beta_y \beta_{d2} \quad \text{and} \quad d_2 \text{ effect is } \beta_{d2}.$$

These are identified with the y_2-equation only. Useful if not enough instruments for the y_1 equation.

One-Step IVE for Last-Lag Model. Instead of doing IVE twice or only once under $\gamma_d = \beta_{d2}$, another alternative is substituting the y_1-equation into y_1 to get

$$y_2 = (\beta_1 + \beta_y \gamma_1) + \beta_y \gamma_y y_0 \ + (\beta_{d1} + \beta_y \gamma_d) d_1 + \beta_{d2} d_2 \ + \beta_y \gamma_x' x_1 + \beta_{x2}' x_2 \ + (\beta_y v_1 + v_2). \qquad \text{(Last-lag)}$$

Unusual in that the last-lag response y_0 is included. Apply IVE only once to find the total effect of d_1 and d_2 as the sum of the coefficients of d_1 and d_2. This last-lag IVE is simpler, but there are two disadvantages: decomposition of the total effect of d_1 cannot be done, and there is in general less instrument source because x_1 and x_2 are included in the right-hand side and the error term consists of v_1 and v_2.

Mean Effect from Linear-Model Parameters. Equation Last-lag is in fact derived from its counter-factual version—this makes our approach 'structural':

$$y_2^{jk} = (\beta_1 + \beta_y\gamma_1) + \beta_y\gamma_y y_0 \ + (\beta_{d1} + \beta_y\gamma_d)j + \beta_{d2}k \ + \beta_y\gamma_x' x_1 + \beta_{x2}' x_2 \ + (\beta_y v_1 + v_2).$$

This connects the structural linear-model parameters to $E(y_2^{jk} - y_2^{00})$:

$$\begin{aligned} y_2^{jk} - y_2^{00} &= (\beta_{d1} + \beta_y\gamma_d)j + \beta_{d2}k \\ &\Longrightarrow \quad E(y_2^{jk} - y_2^{00}) = (\beta_{d1} + \beta_y\gamma_d) + \beta_{d2} \quad \text{when } j = k = 1. \end{aligned}$$

Nonlinear Effects. Even if spanking is beneficial, too much spanking is likely to be harmful: nonlinear effects. Suppose that the effects of d_1 and d_2 are quadratic:

$$\begin{aligned} y_1 &= \gamma_1 + \gamma_d d_1 + \gamma_{dq} d_1^2 + \gamma_y y_0 + \gamma_x' x_1 + v_1, \\ y_2 &= \beta_1 + \beta_{d1} d_1 + \beta_{d1q} d_1^2 + \beta_{d2} d_2 + \beta_{d2q} d_2^2 + \beta_y y_1 + \beta_x' x_2 + v_2. \end{aligned}$$

With the first derivatives, the three key effects are

$$\begin{aligned} \text{direct and indirect effects of } d_1 &= j: \quad \beta_{d1} + 2\beta_{d1q} j, \quad \beta_y(\gamma_d + 2\gamma_{dq} j) \\ \text{direct effect of } d_2 &= k: \quad \beta_{d2} + 2\beta_{d2q} k. \end{aligned}$$

Estimable with a two-step IVE. Alternatively, estimate the last-lag model with a single IVE:

$$\begin{aligned} y_2 = \ & (\beta_1 + \beta_y\gamma_1) \ + (\beta_{d1} + \beta_y\gamma_d) d_1 + (\beta_{d1q} + \beta_y\gamma_{dq}) d_1^2 \ + \beta_{d2} d_2 + \beta_{d2q} d_2^2 \\ & + \beta_y\gamma_y y_0 \ + \beta_y\gamma_x' x_1 + \beta_x' x_2 \ + (\beta_y v_1 + v_2). \end{aligned}$$

An extension to three-periods is also available.

3 Comparison to Granger Causality

Granger Causality. Granger non-causality of d_t on y_t is often tested by $H_0 : \beta_{d1} = \beta_{d2} = 0$ in

$$y_2 = \beta_1 + \beta_{y1} y_1 + \beta_{y0} y_0 + \beta_{d1} d_1 + \beta_{d2} d_2 + \beta'_{x2} x_2 + \beta'_{x1} x_1 + \beta'_{x0} x_0 + v_2 \quad \text{(Granger-Cause)}$$

where *all lagged* d and y appear on the right-hand side. But *this is only for the direct effect of* d, because y_1 and y_0 are included in the model.

Resolution under Stationarity. The deficiency disappears under the stationarity assumption of equal contemporaneous effects $\gamma_d = \beta_{d2}$, because the indirect effect $\beta_y \gamma_d$ becomes zero when the two direct effects β_{d1} and β_{d2} are zero. This solution, however, works only for *the test* of non-causality. For the effect magnitude, the equation 'Granger-Cause' still misses the indirect effect.

Differently from the preceding equations, both y_1 and y_0 appear in the equation 'Granger-Cause'. But this is not a distinguishing character of the Granger causality, because both may be included in the preceding equations as well. It is the lack of awareness that the confounding by y_1 affecting both d_2 and y_2 is avoided by controlling for y_1, which then unfortunately misses the indirect effect of d_1 on y_2 through y_1.

4 G estimation by Robins

Define

$$X_2 \equiv (x_0', x_1', x_2')',$$

and '$a \amalg b|c$' as the conditional independence of a and b given c.

G estimation. Assume '*no unobserved confounder' (NUC)* or 'selection-on-observables (y_0, X_2)':

$$\text{NUC (a): } y_2^{jk} \amalg d_1 \ |(y_0, X_2) \qquad \text{NUC (b): } y_2^{jk} \amalg d_2 \ |(d_1, y_1, y_0, X_2).$$

G-estimation under NUC is

$$E(y_2^{jk}|y_0, X_2) = \int E(y_2|d_1 = j, d_2 = k, y_1, y_0, X_2) f(y_1|d_1 = j, y_0, X_2) \partial y_1. \tag{G-Estimation}$$

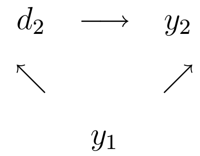
The right-hand side is identified, and so is the conditional effect $E(y_2^{jk}|y_0, X_2)$. Then

$$E(y_2^{jk} - y_2^{00}) = \int \{E(y_2^{jk}|y_0, X_2) - E(y_2^{00}|y_0, X_2)\} \partial F(y_0, X_2).$$

How It Works. The G estimation works because the right-hand side is

$$\begin{aligned}
& \int E(y_2^{jk}|d_1 = j, d_2 = k, \ y_1, y_0, X_2) f(y_1|d_1 = j, y_0, X_2) \partial y_1 \\
= & \int E(y_2^{jk}|d_1 = j, \ y_1, y_0, X_2) f(y_1|d_1 = j, y_0, X_2) \partial y_1 \qquad \text{(due to NUC (b))} \\
= & \ E(y_2^{jk}|d_1 = j, y_0, X_2) \qquad \text{(for } y_1 \text{ is integrated out)} \\
= & \ E(y_2^{jk}|y_0, X_2) \qquad \text{(due to NUC (a))}.
\end{aligned}$$

One-Period Case. Without the first period, the dynamic effect diagram becomes



This is the static 'common factor' model with y_1 as a confounder. NUC becomes $y_2^k \amalg d_2|(y_1, X_2)$, which is the usual selection-on-observables (y_2, X_2') for the one-shot treatment d_2. The G estimation becomes

$$\int E(y_2^k|d_2 = k, y_1, X_2) f(y_1|X_2) \partial y_1 = \int E(y_2^k|y_1, X_2) f(y_1|X_2) \partial y_1 = E(y_2^k|X_2).$$

But as X_2 gets integrated out eventually for the total (marginal) effect, instead of G estimation, we can just use

$$\int E(y_2^k|d_2 = k, y_1, X_2) \partial F(y_1, X_2) = \int E(y_2^k|y_1, X_2) \partial F(y_1, X_2) = E(y_2^k).$$

When a linear structural model holds, one can estimate the dynamic treatment effect using IVE, and the same effect gets estimated with G estimation. Lee (2005) shows this in a simpler setting without covariates.

Binary-Response G estimation. With a binary y, the G estimation becomes

$$\begin{aligned} E(y_2^{jk}|y_0, X_2) &= P(y_2 = 1|d_1 = j, d_2 = k, y_1 = 0, y_0, X_2) \cdot P(y_1 = 0|d_1 = j, y_0, X_2) \\ +P(y_2 &= 1|d_1 = j, d_2 = k, y_1 = 1, y_0, X_2) P(y_1 = 1|d_1 = j, y_0, X_2) \text{(G-Binary)} \end{aligned}$$

Apply probit to y_2 on d_1, d_2, y_1, y_0, X_2 to obtain the two probit probabilities for $y_2 = 1$:

$$\Phi(\psi_1 + \psi_{d1} d_1 + \psi_{d2} d_2 + \psi_{y1} y_1 + \psi_{y0} y_0 + \psi_x' X_2).$$

Also apply probit to y_1 on d_1, y_0, X_2 to get the probit probabilities for $y_1 = 0, 1$:

$$\Phi(\eta_1 + \eta_{d1} d_1 + \eta_{y0} y_0 + \eta_x' X_2).$$

Substituting these into G-Binary will do. This binary version used for our data.

5 Structural Nested Model

Instead of G estimation, there are other estimation methods available for dynamic causal inference (Robins, 1998,1999). A simple version of 'Structural Nested Model' used for our data; see, e.g., Witteman et al. (1998) for an epidemiological application.

Relating Treated Response to Untreated Response. Suppose

$$y_2^{00} = y_2^{jk} \frac{\exp(\psi_o j) + \exp(\psi_o k)}{2} \iff y_2^{jk} = y_2^{00} \frac{2}{\exp(\psi_o j) + \exp(\psi_o k)}.$$

The treatments multiplicatively alter the no-treatment response y_2^{00}. The mean effect $E(y_2^{jk} - y_2^{00})$ is

$$E[y_2^{00} \cdot \{\frac{2}{\exp(\psi_o j) + \exp(\psi_o k)} - 1\}].$$

Structural Nested Model. If $y_2^{jk} \amalg d_2 | observables$ as in NUC(b), then the above display implies $y_2^{00} \amalg d_2 | observables$ as well. Defining

$$S_i(\psi) \equiv y_{i2} \frac{\exp(\psi d_{i1}) + \exp(\psi d_{i2})}{2},$$

we get $S_i(\psi_o) = y_{i2}^{00}$. Transforming the treatments into binary, the true value of θ in the following logit should be zero if $\psi = \psi_o$:

$$P(d_2 = 1 | y_1, y_0, d_1, X_2) = \frac{\exp\{\beta_2'(y_1, y_0, d_1, X_2') + \theta S(\psi)\}}{1 + \exp\{\beta_2'(y_1, y_0, d_1, X_2') + \theta S(\psi)\}}.$$

Getting CI's. Depending on ψ, we get different t-ratios $t_N(\psi)$ for θ. Following the duality between a test and the confidence interval (CI), a 95% CI for ψ is $\{\psi : |t_N(\psi)| < 1.96\}$. The middle point of the CI, or ψ for $\hat{\theta} = 0$ may be used as a point estimator $\hat{\psi}$ of ψ.

(Dis-) Advantages. The main disadvantage of this simple approach is the same-effect restriction for d_1 and d_2 and the arbitrary functional form assumption linking all counter-factuals y_2^{jk} to y_2^{00}, but the main advantage—computational ease—is simply incomparable with other dynamic causal effect estimators.

Relaxing the Same Effect. The same-effect assumption can be relaxed: adopt, instead of $S(\psi)$,

$$S_2(\psi_0, \psi_1) \equiv y_2 \frac{\exp(\psi_0 d_1) + \exp(\psi_1 d_2)}{2} \quad \text{and}$$

$$P(d_2 = 1|y_1, y_0, d_1, X_2) = \frac{\exp\{\beta_2'(y_1, y_0, d_1, X_2') + \theta_1 S_2(\psi_0, \psi_1) + \theta_2 S_2(\psi_0, \psi_1)^2\}}{1 + \exp\{\beta_2'(y_1, y_0, d_1, X_2') + \theta_1 S_2(\psi_0, \psi_1) + \theta_2 S_2(\psi_0, \psi_1)^2\}}$$

We set $\psi_1 = c\psi_0$ in our empirical analysis to estimate ψ_0 from each fixed level of c. As c changes around one, the estimate for ψ_0 will change, showing how robust the results are to the same-effect assumption.

6 Data Description

Data. The NLSY79 child sample contains information on children born to the women respondents of the NLSY79, surveyed three times over 1986-1998. Information when children are 2-3, 4-5, and 6-7 years old ($N \simeq 1000$). Since severe spanking is likely to harm children and since most children are spanked modestly, we focus on the effects of moderate spanking.

BPI (bad behavior). For children 4 years old and above, behavior is measured by the Behavior Problems Index (BPI). *A higher BPI represents more behavior problems.* BPI has mean 105.3 and SD 14.7 around age 6-7, and mean 104.8 with SD 14.8 around age 4-5. Two binary variables were also constructed for BPI (1 if a child's BPI is higher than the mean and 0 otherwise).

MSD (good behavior). Since there is no BPI for age below 4, we use Motors and Social Development Scale (MSD) which measures development in motor, cognitive, communication, and social skills. Differently from BPI, *a higher MSD means better development.* MSD has mean 102.7 with SD 14.1.

Spanking Question. The survey asks "About how many times, if any, have you had to spank your child *in the past week*?" Over 90% were spanked at least once before age 5. As children grew, spanking dropped: 87% spanked their toddlers at least once, but only 68% spanked their five year olds. The reported spanking may not be the regular spanking frequency. The binary variable for 'ever spanking' might be more reliable.

7 Empirical Results

Moderate Spanking Good. Let y_2 and y_1 be BPI at age 6-7 and 4-5; d_2 and d_1 are the spanking frequencies at age 4-5 and 2-3 (or their binary versions). We have

$$\text{effect of } d_1 : \textit{direct} + \textit{indirect} : \widehat{\beta}_{d1} + \widehat{\beta}_y \widehat{\gamma}_d = -4.03 + 0.52 \times (-4.07) = -6.15$$

which is 42% of SD(BPI). The bootstrap bias-corrected 95% CI is $(-59.3,\ 6.5)$. The total effect of $d = (d_1, d_2)'$ and its 95% CI are

$$-6.15\ (d_1 \text{ effect})\ +\ 1.42\ (d_2 \text{ effect}) = -4.73 \quad \text{and} \quad (-37.7,\ 13.6).$$

While d_1 has an intended effect, d_2 does not.

Severe Spanking Bad. When models quadratic in d_1 and d_2 are used,

$$\text{effect of } d_1 = -7.33 + 3.46 d_1 \qquad \text{and} \qquad \text{effect of } d_2 = -1.6 + 1.56 d_2.$$

Moderate spanking reduces BPI as the negative 'intercepts' indicate, but too much spanking is harmful as the positive 'slopes' show. The harmful effect is greater at earlier ages.

G estimation with Binary Response. With binary y and G-estimation, we get

$$\text{total effect} : E(y_2^{11}) - E(y_2^{00}) = 0.047;\ \ 95\% \text{ CI } (-0.4, 0.48).$$

This can be decomposed into two parts: the effect of spanking at age 4-5 (conditional on spanking at age 2-3) and the effect of spanking at age 2-3 (conditional on no spanking at age 4-5), which are, respectively,

$$\begin{aligned} E(y_2^{11}) - E(y_2^{10}) &= 0.16;\ \ 95\% \text{ CI } (-0.14, 0.30) \\ E(y_2^{10}) - E(y_2^{00}) &= -0.12;\ \ 95\% \text{ CI } (-0.64, 0.35). \end{aligned}$$

Moderate spanking at age 2-3 followed by no spanking at age 4-5 reduces behavior problems at age 6-7 relative to no spanking at all, whereas continued spanking at age

2-3 and 4-5 tends to increase them relative to spanking only at age 2-3. This opposite pattern was noted also in the above IVE.

Structural Nested Model. $\widehat{\psi_0} = 0.04$ corresponding to 4.3 points BPI reduction effect (about 30% of one SD). This is similar to the IVE finding. When the same-effect assumption was relaxed with $\widehat{\psi_1} = c\widehat{\psi_0}$, where $\widehat{\psi_0}$ is for the spanking effect at age 2-3 and $\widehat{\psi_1}$ is for the spanking effect at age 4-5, and c is a positive number, $\widehat{\psi_0}$ varied from 0.20 to 0.01 as c changes from 1/4 to 4. This corresponds to BPI reduction of 12.64 to 2.11 at age 6-7.

Granger Causality. With all lagged responses controlled, still the lagged spankings were significant, rejecting the Granger non-causality. As noted already, the coefficients of d_1 and d_2 show only the direct effects. The coefficient of spanking at age 2-3 was always negative and significant across most model specifications; the coefficient of spanking at age 4-5 was also negative, but often insignificant.

8 Conclusions

Main Contribution. When a treatment is repeated over time and the final response is measured at the end, we showed how to estimate the total treatment effects with IVE applied to linear structural models. Early treatments are allowed to have an immediate (direct) effect as well as a lingering (indirect) effect through interim responses. Also, interim treatments are allowed to be affected by interim responses. Our IVE approach identifies the same total-effect of the entire treatment 'profile' as 'G estimation' in (bio-) statistics does.

Indirect Effect and Granger Causality. Regarding controlling interim (i.e., lagged) responses, there is a dilemma: if not controlled, they become a confounder, because the treatment and control groups differ systematically in the interim responses; if controlled, the indirect effects are missed. The latter happens in the usual Granger causality model where all interim responses are controlled, missing all indirect effects. Nonetheless, when the hypothesis of no causality is not rejected, the Granger non-causality inference is valid under a stationarity assumption.

Empirical Finding. The IVE approach and two versions of G estimation were applied to the spanking effect on child behavior. Moderate spanking seems to work, and spanking at age 2-3 has a stronger effect on reducing behavior problems at age 6-7 than spanking at age 4-5 does. Spanking at age 2-3 reduces Behavior Problems Index at age 6-7 by 42% of one SD. In comparison, the effects of spanking at age 4-5 are small and ambiguous in sign. These results seem at odds with findings in the psychology literature where no proper dynamic causal framework has been used.

Our approach takes 'one empirically feasible step' from the usual Granger causality toward the full causal analysis allowing for feedbacks from interim responses.

Appendix: Three-Period Extension

The treatment profile is $d = (d_1, d_2, d_3)'$ and the observation sequence is

$$(x_0, y_0),\ (d_1, \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}),\ (d_2, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}),\ (d_3, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}).$$

The desired effect is $E(y_3^{jkl} - y_3^{000})$. The direct and indirect effects are:

$$\begin{array}{ccccccc}
 & & & & \longleftarrow & \longleftarrow & \\
 & & & \swarrow & & & \nwarrow \\
 & & y_2 & \longrightarrow & d_3 & & \uparrow \\
 & \nearrow & & \searrow & \downarrow & & \uparrow \\
d_2 & \longrightarrow & \longrightarrow & \longrightarrow & y_3 & & \uparrow \\
 & \nwarrow & & & \uparrow & \nearrow & \\
 & & y_1 & \longleftarrow & d_1 & &
\end{array}$$

The linear contemporaneous-covariate models are

$$\begin{aligned}
y_1^j &= \gamma_{11} + \gamma_{1d1} j + \gamma_{1y} y_0 + \gamma'_{1x} x_1 + v_1, \\
y_2^{jk} &= \gamma_{21} + \gamma_{2d1} j + \gamma_{2d2} k + \gamma_{2y} y_1^j + \gamma'_{2x} x_2 + v_2, \\
y_3^{jkl} &= \beta_1 + \beta_{d1} j + \beta_{d2} k + \beta_{d3} l + \beta_y y_2^{jk} + \beta'_x x_3 + v_3.
\end{aligned}$$

The y_3^{jkl} RF with both y_{i2}^{jk} and y_{i1}^j removed is

$$\begin{aligned}
y_3^{jkl} = {} & \{\beta_1 + \beta_y(\gamma_{21} + \gamma_{2y}\gamma_{11})\} + \{\beta_{d1} + \beta_y(\gamma_{2d1} + \gamma_{2y}\gamma_{1d1})\} j + (\beta_{d2} + \beta_y \gamma_{2d2}) k + \beta_{d3} l \\
& + \beta_y \gamma_{2y} \gamma_{1y} y_0 + \beta_y \gamma_{2y} \gamma'_{1x} x_1 + \beta_y \gamma'_{2x} x_2 + \beta'_x x_3 + (\beta_y \gamma_{2y} v_1 + \beta_y v_2 + v_3).
\end{aligned}$$

This shows five effects to be identified:

$$\begin{aligned}
\text{direct and indirect (through } y_1, y_2\text{) effects of } d_1 &: \ \beta_{d1}, \ \ \beta_y(\gamma_{2d1} + \gamma_{2y}\gamma_{1d1}) \\
\text{direct and indirect (through } y_2\text{) effects of } d_2 &: \ \beta_{d2}, \ \ \beta_y \gamma_{2d2} \\
\text{direct effect of } d_3 &: \ \beta_{d3}.
\end{aligned}$$

The first-lag model IVE for these effects are

- Step 1: estimate γ_{1d1} in the y_1 equation with regressors (d_1, y_0, x_1); x_0 provides the instrument source for d_1 and y_0.
- Step 2: estimate γ_{2d1}, γ_{2d2}, and γ_{2y} in the y_2 equation with regressors (d_1, d_2, y_1, x_2); x_0 and x_1 are the instrument source for d_1, d_2, and y_1.
- Step 3: estimate β_{d1}, β_{d2}, β_{d3}, and β_y in the y_3 equation with regressors $(d_1, d_2, d_3, y_2, x_3)$; x_0, x_1, and x_2 are the instrument source for d_1, d_2, d_3, and y_2.

Imposing the equal contemporaneous effect assumption

$$\gamma_{1d1} = \gamma_{2d2}$$

that the effect of d_1 on y_1 is the same as the effect of d_2 on y_2, there is no need to estimate the y_1 equation. Further impose

$$\beta_{d3} = \gamma_{1d1} = \gamma_{2d2}, \quad \gamma_{2y} = \beta_y, \quad \gamma_{2d1} = \beta_{d2}.$$

Under these, estimate only the y_3 equation, and

$$d_1 \text{ effect } \beta_{d1} + \beta_y(\beta_{d2} + \beta_y \beta_{d3}), \quad d_2 \text{ effect } \beta_{d2} + \beta_y \beta_{d3}, \;\; d_3 \text{ effect } \beta_{d3}.$$

The Granger non-causality test becomes equivalent to our approach under the strengthened stationarity-type assumption, because all indirect effects are zero when $\beta_{d1} = \beta_{d2} = \beta_{d3} = 0$.

Turning to the last-lag model IVE, consider the observed version of the above y_3^{jkl} RF with y_{i2}^{jk} and y_{i1}^{j} removed; only y_0 is left as a lagged response on the right-hand side. The observed version has regressors $(d_1, d_2, d_3, y_0, x_1, x_2, x_3)$. The instrument source for d_1, d_2, d_3, y_0 is x_0. This last-lag model IVE is a single step method as in the two-period case.