EVOLUTIONARY EFFICIENCY AND HAPPINESS

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ABSTRACT. We model happiness as a measurement tool used to rank alternative actions. The quality of the measurement is enhanced by a happiness function that adapts to the available opportunities, a property favored by evolution. The optimal function is based on a time-varying reference point—or performance benchmark—that is updated in a statistically optimal way. Habits and peer comparisons arise as special cases of this process. This also results in a volatile level of happiness that continuously reverts to its long-term mean. Throughout, we draw a parallel with a problem of optimal incentives, which allows us to apply statistical insights from agency theory to the study of happiness.

1. Introduction

For long, utility was assumed to depend only on the absolute levels of our material outcomes. However, a large body of research now argues that utility, whether defined in terms of decision-making or hedonic experience, is sharply dependent on the difference between these outcomes and a time-varying reference point—examples include Markowitz [1952], Stigler and Becker [1977], Frank [1985], Constantinides [1990], Easterlin [1995], Clark and Oswald [1996], and Frederick and Loewenstein [1999]. Two pervasive phenomena in these lines are habituation (e.g., becoming accustomed to an expensive life-style or a physical handicap) and peer comparisons (e.g., caring about relative wages or consumption levels); both of which can be described by a reference point that is determined, respectively, by past outcomes and by the outcomes of peers. In fact, these phenomena appear to be innate: they are present in young children, and have been documented in every known human culture—Brown [1999]. Suggesting in turn that they served an evolutionary role in the descent of our species.

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In this paper, we are interested in the hedonic aspects of utility, or happiness. At a biological level, we view happiness as a decision-making device that translates potential choices into a ranking criteria—e.g., Damasio [1994], Robson [2001a]. Accordingly, we adopt an evolutionary approach to uncover its rationale. Here we propose that habituation and peer comparisons can arise as special cases of the same general phenomenon, namely, a reference point that is updated over time in a statistically meaningful way. In addition, our approach will encompass two well-known psychological phenomena: a reference point that is affected by the individual’s expectations, and a happiness level that although volatile, continuously reverts to its long-term mean. We argue that these phenomena can be derived from the same underlying biological role.

The endogenous variable in our model is the individual’s happiness function. We assume that this function is implicitly designed by nature. Following the general framework of Binmore [1994], Robson [2001a], Samuelson [2004], and Samuelson and Swinkels [2004], we focus directly on the limiting outcome of evolution. This outcome is described as the solution to a metaphorical principal-agent problem, where the principal corresponds to the evolutionary process controlling the innate characteristics of the individual, or agent, who in turn serves the purpose of genetic replication. Crucially, when we speak of the evolutionary end-point, we refer to the ancestral hunter-gather environment and the suitable adaptations developed back then. In the modern world, in contrast, the rate of environmental change has dwarfed the rate of evolutionary adaptation, resulting in a level of maladaptation in many respects. Our approach will be relevant to the extent that the adaptations developed in the ancestral environment are still present in our innate characteristics today.

The happiness function, in particular, will serve as a measurement tool used to compare alternative choices. Following Frederick and Loewenstein [1999], and Robson [2001b], our central assumption will be an exogenous limit over the precision of the agent’s measurement, from which a role for adaptation is derived. For example, these authors argue that an adaptive utility is analogous to an eye that adjusts to the luminosity of the environment in order to increase its accuracy, or a voltmeter.
that delivers a more precise measurement when calibrated to the specific problem at hand.\footnote{Frederick and Loewenstein [1999] also argue that hedonic adaptation can serve a protective role against extreme emotional states. In section 6, we suggest a way in which these two approaches can be combined.}

Here we take the analogy one step further. We consider an abstract choice setting where the agent must compare alternative inputs $x$ towards the production of a random output $y$ (consider, for example, a hunter-gatherer who is searching for fruit). Associated to each level of $y$, there will be a real-valued hedonic utility, or happiness, $V(y)$. The agent will measure the impact of $x$ by means of the conditional expectation $E[V(y) \mid x]$, where $V(y)$ serves as a “lens” that can adjust to the environment. The quality of the measurement will be restricted by two constraints. The first is an upper and lower bound on the happiness function $V(y)$, which we interpret as a physical limit on the emotions that the body can produce. The second constraint describes a limit on the agent’s perception sensitivity: we assume that two alternatives $x_1$ and $x_2$ cannot be distinguished by the agent whenever the difference between $E[V(y) \mid x_1]$ and $E[V(y) \mid x_2]$ is smaller than some minimum threshold. When combined, these constraints will provide the basis for an adaptive $V(y)$.

The fact that the agent’s choice is over inputs, while happiness depends on the level of output, will lead to a parallel between the evolutionary problem and a standard problem of optimal incentives under moral hazard—where $V(y)$ corresponds to a performance reward for the agent. In both cases, the principal who designs $V(y)$ will seek to maximize the signal value of this function (due to measurement limits in the former case, and due to a cost of effort in the latter). This parallel will be central to our approach, since it will allow us to import a number of statistical insights from incentive theory to the study of happiness.

The optimal happiness functions we derive correspond to an extreme version of the $S$-shaped value functions in prospect theory (Kahneman and Tversky [1979]), with a slope that is fully concentrated around an endogenous reference point. The position of this reference point is determined by the underlying technological opportunities. Moreover, it will optimally adjust over time so that it constantly matches the agent’s potential. In order to illustrate this adaptive mechanism, we consider an environment where the optimal reference point corresponds to the conditional
expectation of output based on all available information. In particular, this expectation will exploit information contained in past levels of output and in the output of peers, from which habituation and peer comparisons are derived. Consequently, the specific functional form for these habits and peer comparisons will reflect their underlying informative role.

A distinctive result is a generalized form of habituation. From a standard class of output technologies, we derive a reference point that is positively related to past levels of output as well as contemporaneous peer output (describing standard negative externalities), but negatively related to past values of peer output (which departs from usual formulations). This represents an individual who becomes habituated not only to her own success, but also to the success of her peers. For example, the negative effect of a permanent increase in peer output will fade away over time.

We begin our analysis with a static model where the basic evolutionary problem is presented. We then extend this model to a dynamic setup where adaptation is discussed.

2. The Static Model

Consider a representative agent (i.e., a hunter-gatherer) who faces an abstract one-shot project. To fix ideas, suppose this project amounts to an opportunity to collect fruit. The agent first observes the current state of nature \( s \), which describes the physical configuration of the world, such as the presence of fruit and dangers in specific locations. Next, she selects a course of action \( x \in X \), which represents the strategy adopted, such as traveling in a certain direction or climbing a particular tree. The combination of \( x \) and \( s \) randomly determines a level of output \( y \in \mathbb{R} \) – the amount of fruit collected. Denote the conditional probability distribution of output by \( f(y \mid x, s) \), a function known by the agent.

Beyond this example, output \( y \) is meant to summarize the achievement of proximate evolutionary goals. Namely, those tangible goals that favored the ultimate evolutionary goal of genetic replication during the ancestral environment – examples presumably included wealth, health, and sex, as well as the well-being of friends and kin. Accordingly, the decision variable \( x \) represents the actions taken in pursuit of these goals.

We are interested in the case where the agent’s decision is guided by emotional rewards that are based on the realized level of output – as opposed to rewards that are based directly on \( x \). This means that the agent will be allocated full autonomy
over the choice of $x$, rather than being prescribed a specific behavior. The idea is that because of an informational advantage that stems from her observation of $s$ (representing a complex environment), the agent has comparative advantages when it comes to selecting the appropriate means of production—Binmore [1994, p.151] and Robson [2001a] follow a similar approach.\(^2\) Emotional rewards will take the form of a one-dimensional level of happiness $V(y)$ (a summary of emotions), experienced once $y$ is realized. We assume that the agent can freely dispose of $y$, which allows us to focus without loss on non-decreasing happiness functions.

The agent will measure the impact of alternative choices $x$ via the expected value of happiness $E[V \mid x, s] \equiv \int V(y)f(y \mid x, s)dy$. Here the happiness function serves as a “lens” that transforms a functional space of probability distributions $f$ into a single dimension, expected happiness, from which a decision criteria is obtained.\(^3\) Observe that this setup allows for a distinction between hedonic utility and decision utility. The former refers to the emotional experience, $V(y)$, whereas the latter refers to the standard notion of decision-theoretic utility (a ranking of alternative choices), given here by the state-contingent utility function $u(x, s) \equiv E[V \mid x, s]$.

The happiness function will be implicitly designed by an evolutionary process, which we call the “principal.” When designing this function, the metaphorical objective for the principal is to promote the production of $y$, which is simply another way to say that in a population of individuals endowed with a diversity of happiness functions, those producing higher levels of $y$ will have a reproductive advantage. For concreteness, we assume that the principal seeks to maximize the expected value of $y$—which leads to the same results as maximizing the expected value of any other increasing function of $y$.\(^4\)

Rather than studying the evolutionary trial-and-error dynamics, we are interested in describing the limiting outcome once sufficient experimentation and selection have taken place, while holding the environment fixed. We represent this limiting outcome by means of an optimization problem where the principal directly selects a happiness function that maximizes her objective. (Recall that, in general, an evolutionary process where genetic traits are passed on to offspring with small random

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\(^2\)Samuelson and Swinkels [2004] study a model where part of this autonomy is subtracted in order to compensate for cognitive biases.

\(^3\)See Damasio [1994] for a neurological foundation of emotions as a decision-making device, and Robson [2001a] for an evolutionary foundation of expected utility.

\(^4\)The technologies we consider below fit the assumptions under which Robson [1996] shows that expected-value criteria are optimal.
variations might converge to a local maximum that is not globally optimal. However, for the technologies considered below, the global optimum will coincide with a local maximum that is unique.)

2.1. **Measurement Imperfections.** Our theory is based on two constraints that, when combined, will limit the precision of the agent’s ranking of $x$. The first is a physical bound that limits the highest and lowest value that $V$ can take: normalizing these bounds to 0 and 1, we assume that $V(y) \in [0, 1]$ for all $y$. In all the applications below, these bounds will bind, implying that the principal would benefit from an organism capable of more extreme emotions. In practice, however, expanding these bounds will presumably have an evolutionary cost as well—for example, additional energy would need to be devoted towards building and maintaining a larger emotional apparatus. As a result, the principal will have an incentive to use whatever bounds are available in the most efficient possible way. This is the problem we focus on.

The second constraint imposes a limit on the agent’s ability to measure, or perceive, small differences in her objective $E[V \mid x, s]$. We represent this imperfection using a reduced form that allows for a simple analysis. Given any pair of choices $x_1$ and $x_2$, we assume that there is a minimum threshold $\varepsilon > 0$ such that whenever $|E[V \mid x_1, s] - E[V \mid x_2, s]| \leq \varepsilon$, these two choices cannot be ranked. Accordingly, all choices that deliver an expected payoff within $\varepsilon$ distance of the optimized value $\max_x E[V \mid x, s]$ will be part of the same indifference set, denoted the “satisficing” set. We assume that the agent’s choice is randomly drawn from this set. For our purposes, it suffices to assume that her draw is monotonic in the sense that the probability assigned to any subset of the satisficing set is inversely proportional to the size of the latter.\(^5\)

This assumption imposes a coarseness in the agent’s measurement analogous to a computer that must round any small difference to zero because it uses only finitely many digits, or an eye that cannot rank the luminosity of two sources when they are sufficiently similar. In both these examples, because of the imperfection, adapting to the problem at hand will improve the measurement quality: an eye uses a pupil

\(^5\)Along similar lines, Simon [1959, p.261] argues that when the utilities of two alternatives are only slightly different, the subject is likely to vacillate in his choice. As an empirical precedent, he reports an experiment where subjects are asked to rank two unequal weights: when these weights approach each other, the frequency of a correct answer approaches $1/2$. A related type of imperfect optimization is used for $\varepsilon$-equilibria in games—Radner [1980].
that opens in the dark in order to maximize the relative differences in luminosity, while a computer uses a decimal point that floats. The function $V$ will play a very similar role.

As shown below, the principal would benefit from a smaller $\varepsilon$. However, reducing $\varepsilon$ is likely to require additional energy as well—e.g., in the form of repeated measurements, or more intensive optimization techniques. In this case, the principal will also benefit from a happiness function that best aids an imperfect machinery, whatever the size of the underlying imperfection happens to be. We approach this problem by first solving for the optimal happiness function for any given small $\varepsilon$, and then characterizing the limit as $\varepsilon \to 0$. This limit will serve as an analytically convenient representation of an environment where small imperfections remain.

2.2. Output Technology. We focus on output technologies of the form

$$y = E[y \mid x, s] + z,$$

where $z$ is an exogenous shock drawn from a continuous density function that has full support, and is strictly monotonic on either side of its mean—such as a normal. The shock is realized after $x$ is selected. We assume that $E[y \mid x, s]$ is continuous in $x$, while $X$ (the choice space) is a compact subset of $\mathbb{R}^N$, which guarantee that the choice space is sufficiently rich, and an optimal action always exists.

Under these technologies, the conditional densities $f(y \mid x, s)$ are single-peaked at their mean $E[y \mid x, s]$, and are single-crossing in $x$: for all $x_1 \neq x_2$, $f(y \mid x_1, s)$ and $f(y \mid x_2, s)$ intersect for only one value of $y$ (in this case, between their two means). These densities are also ordered across $x$ according to first-order stochastic dominance. These are the key distributional properties we employ.

2.3. A One-Dimensional Formulation. The agent’s choice problem can be simplified to an equivalent problem where her choice is one-dimensional. To do this, we define a real-valued index $\varphi(x, s)$ as follows:

$$\varphi(x, s) \equiv \frac{E[y \mid x, s] - \min_x E[y \mid x, s]}{\max_x E[y \mid x, s] - \min_x E[y \mid x, s]}.$$

Notice that, for any given $s$, $\varphi(x, s)$ ranges from zero to one. We refer to $\varphi$ as the “efficiency” of the agent’s decision—for example, $\varphi(x, s) = 1$ means that, given $s$, the agent selected the optimal action. Using this index, we can express $E[y \mid x, s]$ as a function of $\varphi$ and $s$ alone: $E[y \mid x, s] = E[y \mid \varphi(x, s), s] \equiv \varphi \max_x E[y \mid x, s] +$
(1 − ϕ) \min_x E[y \mid x, s]. Accordingly, output becomes
\begin{equation}
y = E[y \mid \varphi, s] + z, \tag{1}
\end{equation}
where \(E[y \mid \varphi, s]\) is increasing and continuous in \(\varphi\).

This formulation will allow us to express the agent’s problem, without loss of generality, as one where she directly selects the level of \(\varphi\), subject to \(\varphi \in [0, 1]\), while \(x\) is sent to the background. Note that this simplification is for analytical purposes only: the existence of an underlying complex problem –where the agent actually compares values of \(x\), not values of \(\varphi\)– remains essential for the interpretation of the model.

We begin our analysis with the simplest case where \(E[y \mid \varphi, s]\) is independent of the state \(s\), which means that both \(\max_x E[y \mid x, s]\) and \(\min_x E[y \mid x, s]\) are independent of \(s\). In this case, while \(s\) might affect the value of each particular choice \(x\), it does not change the agent’s overall output potential. As a result, the conditional density of \(y\) simplifies to \(f(y \mid \varphi)\). We return to the general case in the dynamic model below.

2.4. The Optimal Happiness Function. Expressed in terms of \(\varphi \in [0, 1]\), the agent’s objective function is given by
\[E[V \mid \varphi] \equiv \int V(y)f(y \mid \varphi)dy.\]
Accordingly, her satisficing set corresponds to the set of choices \(\varphi\) that deliver an expected happiness within \(\varepsilon\) distance of \(\max \varphi E[V \mid \varphi]\). Notice that under the technologies above, the densities \(f(y \mid \varphi)\) have full support and are ordered across \(\varphi\) according to first-order stochastic dominance. As a result, for any \(V\) that is not constant (and non-decreasing), \(E[V \mid \varphi]\) is increasing in \(\varphi\), and therefore maximized at \(\varphi = 1\). Thus, the satisficing set, now given by \(\{\varphi : E[V \mid \varphi] \geq E[V \mid 1] - \varepsilon\}\), becomes the interval \([\varphi_{\min}(V, \varepsilon), 1]\), where the lower boundary \(\varphi_{\min}(V, \varepsilon)\) is uniquely determined by the equality \(E[V \mid \varphi_{\min}(V, \varepsilon)] = E[V \mid 1] - \varepsilon\). This boundary represents the lowest efficiency level \(\varphi\) that can arise in equilibrium.

From the principal’s standpoint, the impact of \(V\) is fully summarized by the value of \(\varphi_{\min}(V, \varepsilon)\), with a larger value being strictly preferred. Her problem can therefore be expressed as
\begin{equation}
\max_{\varphi} \varphi_{\min}(V, \varepsilon) \tag{I}
s.t. V(y) \in [0, 1] \text{ for all } y,
\end{equation}
which corresponds to minimizing the set of inefficient choices \(\varphi < 1\) that the agent confuses with \(\varphi = 1\). Let \(\varphi^*\) denote the optimized value for this problem –which for
any $\varepsilon > 0$, is smaller than 1. The following Lemma will allow us to solve for the optimal $V$ using a dual approach:\footnote{Existence of an optimal $V$ follows from a standard argument.}

**Lemma 1.** Suppose $V^*$ is a solution to problem I (namely, $\varphi_{\text{min}}(V^*, \varepsilon) = \varphi^*$). Then, $V^*$ must also solve

\[
(II) \quad \max_V E[V \mid 1] - E[V \mid \varphi^*]
\]

s.t. $V(y) \in [0, 1]$ for all $y$.

**Proof.** Suppose not. Then there must exist a $V \neq V^*$ (satisfying the constraint) such that $E[V \mid 1] - E[V \mid \varphi^*] > E[V^* \mid 1] - E[V^* \mid \varphi^*] \equiv \varepsilon$. But this implies that $\varphi_{\text{min}}(V, \varepsilon) > \varphi^*$, a contradiction. \hfill \blacksquare

In other words, in order for $V^*$ to be optimal, there cannot exist an alternative $V$ that leads to a difference between $E[V \mid 1]$ and $E[V \mid \varphi_{\text{min}}(V^*, \varepsilon)]$ larger than $\varepsilon$, since this would deliver a boundary $\varphi_{\text{min}}(V, \varepsilon)$ larger than $\varphi_{\text{min}}(V^*, \varepsilon)$.

**Proposition 1.** Problem I is solved by a one-step happiness function $V^*$ such that

\[
V^*(y) = \begin{cases} 
1 & \text{for all } y \geq \hat{y}, \\
0 & \text{for all } y < \hat{y},
\end{cases}
\]

where the threshold $\hat{y}$ is uniquely determined by the equality $f(\hat{y} \mid 1) = f(\hat{y} \mid \varphi^*)$. Moreover, this solution is unique up to a measure-zero subset.

**Proof.** The objective in problem II is equal to $\int V(y) [f(y \mid 1) - f(y \mid \varphi^*)] dy$. This integral is maximized by setting $V(y) = 1$ for every $y$ such that $f(y \mid 1) \geq f(y \mid \varphi^*)$, and $V(y) = 0$ for every $y$ such that $f(y \mid 1) < f(y \mid \varphi^*)$. Moreover, from the single-crossing of $f$, we have $f(y \mid 1) > f(y \mid \varphi^*)$ for all $y > \hat{y}$, and $f(y \mid 1) < f(y \mid \varphi^*)$ for all $y < \hat{y}$. Finally, since $f(y \mid 1) \neq f(y \mid \varphi^*)$ almost everywhere, this solution is unique up to a zero-measure subset, and therefore solves problem I as well. \hfill \blacksquare

This result can be illustrated graphically. The upper panel of Figure 1 graphs the conditional density $f(y \mid \varphi)$. The bold curve represents $f(y \mid 1)$, the most desirable function for the principal, while the dashed curve represents $f(y \mid \varphi^*)$—where $\varphi^*$, by definition, will always belong to the satisfying set. The dual objective is to maximize the difference in expected happiness under these two alternatives, which is the only way to exclude every $\varphi < \varphi^*$ from the satisfying set. As depicted in the lower panel, this is achieved by a $V$ that maximally rewards all values of $y$ for which
Figure 1. The Optimal $V$

$f(y \mid 1) > f(y \mid \varphi^*)$, and vice versa: under any other $V$, the two distributions would appear more similar for the agent. The threshold $\hat{y}$ lies where the two densities intersect. Under the technologies in (1), this occurs between the peaks $E[y \mid \varphi^*]$ and $E[y \mid 1]$.

A statistical parallel can be drawn with a problem of optimal incentives—e.g., Holmstrom [1979], Levin [2003]. Interpret $V$ as a performance bonus, $\varphi$ as a costly effort variable, and the bounds for $V$ as a two-sided limited-liability constraint. Under this interpretation, the one-step bonus above will maximally punish the agent following a deviation to $\varphi^*$. Accordingly, we can view this bonus as implicitly testing the null "$\varphi = 1$" against the alternative "$\varphi = \varphi^*$." In this case, the null will be rejected whenever the likelihood ratio $\frac{f(y \mid \varphi^*)}{f(y \mid 1)}$ exceeds one.

2.5. The Limit When $\varepsilon \to 0$. As $\varepsilon$ converges to zero, the lower bound of the satisficing set $\varphi^*$ converges to one. Consequently, as suggested by Figure 1, the happiness threshold $\hat{y}$—which lies between $E[y \mid \varphi^*]$ and $E[y \mid 1]$—converges to $E[y \mid 1]$. Proposition 2 follows as a result:
Proposition 2. Given $\varepsilon$, let $V^*(\varepsilon)$ denote the optimal one-step function characterized by Proposition 1. Then, in the limit as $\varepsilon$ converges to zero, $V^*(\varepsilon)$ converges point-wise to the one-step function

$$V(y) = \begin{cases} 
1 & \text{for all } y \geq E[y \mid 1], \\
0 & \text{for all } y < E[y \mid 1]. 
\end{cases}$$

Up to a measure-zero subset, this limiting function uniquely maximizes the derivative $\frac{\partial}{\partial \varphi} E[V \mid \varphi]_{\varphi=1} \equiv \int V(y) f_{\varphi}(y \mid \varphi)|_{\varphi=1} dy$, which represents the limiting version of the objective in the dual problem II. Maximizing this derivative guarantees that marginal deviations away from $\varphi = 1$ have a maximal impact over the agent’s objective, thus improving her ability to discriminate. The analogy in optimal incentives is a first-order approach where all incentive power is focused over small effort deviations (e.g., Rogerson [1985], Levin [2003]).

These extreme rewards arise because the agent, by construction, is risk-neutral with respect to happiness, and happiness is costless to the principal within the bounds. Smoother happiness functions would arise if instead of imposing the $[0, 1]$ bounds, we assumed that there is a neutral level of happiness that is physiologically optimal, and that deviations from this level are increasingly costly. The Appendix studies this case. There, we present a parameterization that delivers a family of smooth S-shaped curves with differing slopes around the reference point (similar to those used by Kahneman and Tversky [1979]). In the limit as these slopes converge to infinity, the curves converge to the one-step functions above. The analytical advantage of the limiting functions is that they are fully characterized by the position of $\hat{y}$.

3. The Dynamic Model

We now extend the model to a dynamic setup where the agent lives for multiple periods $t = 1, 2, \ldots$. We equate every period with one separate project—the simplest possible case. At the beginning of period $t$, the agent observes a state $s_t$ and selects an action $x_t \in X$. Output is then given by $y_t = E[y_t \mid x_t, s_t] + z_t$, which we assume satisfies the same properties as above, with $z_t$ i.i.d. across time. As before, we use the one-dimensional representation in (1), where $y_t = E[y_t \mid \varphi_t, s_t] + z_t$, and the agent directly selects the efficiency level $\varphi_t \in [0, 1]$. After $y_t$ is realized, the agent experiences a happiness level $V_t \in [0, 1]$.

In contrast to the special case studied in the static model, we now allow the extreme values $\max_{x_t} E[y_t \mid x_t, s_t]$ and $\min_{x_t} E[y_t \mid x_t, s_t]$ to vary with $s_t$, so that
$E[y_t | \varphi_t, s_t]$ also varies with $s_t$. In particular, we assume that these extreme values now depend on a subset of $s_t$, denoted $\Omega_t$. Accordingly, output becomes

$$\text{(2)} \quad y_t = E[y_t | \varphi_t, \Omega_t] + z_t.$$  

The new variable $\Omega_t$ may represent, for example, current weather conditions. We assume that $\Omega_t$ can be encoded, together with $y_t$, into the happiness function (which means that $\Omega_t$ must be relatively simple). As a result, happiness becomes $V_t(y_t, \Omega_t)$. This dependence on $\Omega_t$ implies that happiness can now adapt to changes in output potential.

The agent’s objective for period $t$ is to maximize the expected value of $V_t$, as opposed to some expected discounted value of future happiness. In other words, everything the agent cares about, present and future, is reflected in present emotions. This model will capture forward-looking behavior by interpreting a given project as being forward looking itself, and rewarded by current happiness. Consider, for example, a hunter-gatherer who eats in excess of current needs in order to accumulate fat, or helps a friend, precisely because it makes her feel happy today (a modern counterpart might be an individual who invests in her retirement funds for precisely the same reason). Below, we also discuss the case where the agent internalizes future happiness above and beyond $V_t$.

Since the agent faces a separate project every period, any such period is identical to the static model above—save for the presence of $\Omega_t$. Moreover, $\Omega_t$ simply enters as a technological constant in all the previous analysis. As a result, from Proposition 2, the optimal limiting function $V_t$ (as $\varepsilon \to 0$) will be a one-step function with threshold $\hat{y}_t = E[y_t | \varphi_t, \Omega_t]_{|\varphi_t=1}$ —namely, the peak of the density $f(y_t | \varphi_t, \Omega_t)_{|\varphi_t=1}$. In other words, the impact of $\Omega_t$ will occur via a threshold $\hat{y}_t$ that is updated in a statistically optimal way.

3.1. Habituation. We proceed with two simple examples where $\hat{y}_t$ incorporates a habit due to an $\Omega_t$ that is correlated across time. Possible causes for this correlation include environmental shocks and an intrinsic talent that persists over time.

**Example 1: A Markovian Habit.** Suppose output is given by $y_t = \varphi_t + \theta_t$, where $\theta_t$ is a random shock that follows the Markovian process $\theta_t = \theta_{t-1} + z_t$. Equivalently, output can be expressed as $y_t = \varphi_t + \theta_{t-1} + z_t$. This technology satisfies equation (2) with $\Omega_t = \theta_{t-1}$, which is correlated across time. Notice that $\theta_{t-1}$ can be inferred from the lagged equality $y_{t-1} = \varphi_{t-1} + \theta_{t-1}$. As a result, output becomes
In equilibrium, once \( \phi_t = \phi_{t-1} = 1 \), this equation reduces to \( y_t = y_{t-1} + z_t \), from which it follows that \( y_{t-1} \) (the best predictor of \( y_t \)) becomes the optimal reference point:

\[
\hat{y}_t = E[y_t | \phi_t, \Omega_t] | \phi_{t-1} = 1 = y_{t-1}.
\]

In this case, the agent will experience a high level of happiness if and only if her current output exceeds what she achieved one period ago. This result follows from an optimal statistical inference. In order to best guide the agent, the principal will employ her most accurate source of information regarding \( \phi_t \) (analogous to an optimal incentive scheme). From the equality \( y_t - y_{t-1} = \phi_t - \phi_{t-1} + z_t \), we learn that the most accurate source is the difference \( y_t - y_{t-1} \). As a result, this difference becomes the carrier of happiness. In contrast, if the reference point did not adapt, as \( \theta_t \) drifts to extreme values, all decisions \( \phi_t \) would appear increasingly good or increasingly bad, and thus increasingly similar.

On the other hand, observe that a reduction in \( \phi_t \) will affect \( y_t \) in exactly the same way as a low realization of \( z_t \), implying that these two variables cannot be distinguished by \( V_t \). As a consequence, the principal must punish the agent following low realizations of \( z_t \), and vice versa: happiness is inevitably affected by chance. In fact, in equilibrium, the sole carrier of happiness becomes the random shock \( z_t \) (which equals \( y_t - \hat{y}_t \)). This implies that the expected value of happiness will be the same for every period, regardless of past levels of output: the effects of the shocks are always short-lived. These features are shared by all the examples that follow. In many languages, the word “happiness” is closely linked to “fortune” and “luck.” For the ancient Greeks, happiness (eudaimonia) was ultimately determined by the will of the gods.\(^7\)

**Example 2: Auto-Regressive Habits.** Suppose output is given by \( y_t = \phi_t + \theta_t \), and \( \theta_t \) follows the auto-regressive process \( \theta_t = \sum \alpha_s \theta_{t-s} + z_t \) for arbitrary constants \( \alpha_s \), and \( s \geq 1 \). In this case, \( \Omega_t \) is the vector \((\theta_{t-1}, \theta_{t-2}, \ldots)\). Following similar steps to those in Example 1, output becomes \( y_t = \phi_t + \sum \alpha_s (y_{t-s} - \phi_{t-s}) + z_t \). In equilibrium, this equation reduces to \( y_t = \sum \alpha_s y_{t-s} + (1 - \sum \alpha_s) + z_t \), from which we obtain

\[
\hat{y}_t = E[y_t | \varphi_t, \Omega_t] | \varphi_{t-1} = 1 = \sum \alpha_s y_{t-s} + (1 - \sum \alpha_s).
\]

\(^7\)“When viewed through mortal eyes, the world’s happenings—and so our happiness—could only appear random, a function of chance” —McMahon [2004, p.7].
The reference point is now a weighted average between past levels of output and the equilibrium efficiency level $\varphi_t = 1$ (e.g., allowing habituation to occur at a slower rate than in Example 1). The specific weights guarantee that the carrier of happiness $y_t - \hat{y}_t$ employs only the new information contained in $y_t$.

3.2. Habits and Forward-Looking Behavior. Notice that the presence of habits does not deter the agent from selecting $\varphi_t = 1$. A possible interpretation is that the agent is simply unaware of these habits, and therefore does not take them into account. This interpretation is consistent with a common finding in the psychology literature that individuals tend to underestimate the degree to which they will adapt to changing circumstances –Gilbert et al. [1998], Loewenstein and Schkade [1999].

This opens the possibility that a rational agent, who recognizes the existence of these habits (and manages to internalize their effect), might benefit from a deviation. Whether such a profitable deviation exists will depend on how the reference point $\hat{y}_t$ is determined outside equilibrium. So far, this issue has not been discussed.

Consider the technologies of Example 2. In general, there will be several alternative formulations for the reference point, all of which are equivalent in equilibrium. In one extreme, $\hat{y}_t$ may correspond to an exogenous function of past levels of output, namely, $\hat{y}_t = \sum \alpha_s y_{t-s} + (1 - \sum \alpha_s)$. In the other extreme, $\hat{y}_t$ may equal the best predictor of $y_t$ conditional on $\varphi_t = 1$ and all past information, namely, $\hat{y}_t = E[y_t | \varphi_t, \Omega_t]_{\varphi_t=1} = \sum \alpha_s \theta_{t-s} + 1$ (where the values of $\theta_{t-s}$ are inferred from the technological equalities $\theta_{t-s} = y_{t-s} - \varphi_{t-s}$).

When $\varphi_{t-s} = 1$ for all $s$, both formulations coincide. The difference arises outside equilibrium:

In the former case, a reduction in $\varphi_t$ will reduce future reference points by reducing $y_t$, thus increasing the expected value of future happiness. As a result, the deviation might indeed be beneficial. In the latter case, in contrast, the agent understands that a reduction in $\varphi_t$ will not affect her future reference points because these only depend on the underlying technological shock $\theta_t$, and not on the particular value of $y_t$. This case describes an agent who cannot change his future output expectations.

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8See also Frey and Stutzer [2004] for a study of biased predictions, and Burnham and Phelan [2001] for a witty account.

9In this case, the agent uses her best cognitive abilities to form an expectation, only to then compare her actual success against this self-imposed benchmark. Such a procedure would add flexibility to the reference point, which can be evolutionarily advantageous in an environment where the parameters of the technological process change over time.

For example, if the agent maximizes a geometrically-discounted sum of future happiness levels at rate $\beta$, a marginal deviation away from $\varphi_t = 1$ will be beneficial if and only if $\sum \beta^s \alpha_s > 1$. 

by merely reducing \( \varphi_t \). Therefore, a deviation will never be beneficial. As a result, this agent can be described as either maximizing a present discounted value of future happiness levels, or equivalently, as maximizing current happiness alone.

The above distinction will also be relevant for policy. In the former case, for instance, a tax policy that leads to an increasing income profile over the life cycle might increase long-term happiness. In the latter case, in contrast, expectations may fully adjust to the policy, therefore eliminating the desired effect.

4. Multiple Agents

In order to derive peer effects, we extend the model to include multiple agents \( i \). Actions for period \( t \), denoted \( \varphi^i_t \), are selected simultaneously, and they randomly determine an output level \( y^i_t \) for each agent. Let \( \bar{y}_t \) denote the average output across agents, and let \( w^i_t \equiv y^i_t - \bar{y}_t \) denote relative output. The new assumption is that the agents will experience common productivity shocks (e.g., due to a shared environment), implying that peer output becomes valuable when assessing individual performance.\(^\text{11} \) Dropping the \( i \) superscript, we focus on technologies such that

\[
 w_t = E[w_t | \varphi_t, \Omega_t] + z_t,
\]

which we assume satisfies our previous assumptions with \( w_t \) in the place of \( y_t \). In addition, we assume that \( z_t \) is independent across agents and that the population average for these shocks is zero –i.e., an exact law of large numbers applies.

Although the principal cares only about \( y_t \), happiness is also allowed to depend on \( \bar{y}_t \) (as well as \( \Omega_t \)). From (3), the conditional density \( f(y_t | \varphi_t, \bar{y}_t, \Omega_t) \) depends on \( y_t \) and \( \bar{y}_t \) only through \( w_t \). As a result, happiness can be expressed without loss as \( V_t(w_t, \Omega_t) \). It follows that this model is identical to the model with a single agent, with \( w_t \) replacing \( y_t \). Consequently, from Proposition 2, the optimal \( V_t \) is a one-step function with \( V_t(w_t, \Omega_t) = 1 \) for all \( w_t \geq \widehat{w}_t \), and \( V_t(w_t, \Omega_t) = 0 \) otherwise –where \( \widehat{w}_t = E[w_t | \varphi_t, \Omega_t]|_{\varphi_t=1} \).

\(^\text{11}\)If the principal directly benefited from relative output \( w^i_t \), peer effects would immediately arise –see, for example, Cole et al. [1992] for the potential benefits conveyed by \( w^i_t \). Here we show how these effects can extend beyond any direct advantage of achieving a high \( w^i_t \). In a complementary approach, Samuelson [2004] derives peer effects that lead to an imitation of consumption levels (even when only absolute consumption is relevant), which may be desirable in an environment where the optimal level of consumption is not fully known by the agent.
Example 3: Static Peer Comparisons. Suppose output for each agent is given by \( y_t = \varphi_t + \Gamma_t + z_t \). The term \( \Gamma_t \) represents an aggregate shock that is shared by all agents, whereas \( z_t \) is the idiosyncratic shock from (3). Both \( \Gamma_t \) and \( z_t \) are realized after \( \varphi_t \) is selected. No restrictions over the distribution of \( \Gamma_t \) are imposed. In this case, \( \Omega_t \) will be redundant. In equilibrium, by averaging across agents we obtain \( \overline{y}_t = 1 + \Gamma_t \). Therefore, \( y_t - \overline{y}_t = w_t = (\varphi_t - 1) + z_t \), which satisfies (3). The optimal reference point for \( w_t \) becomes \( \hat{w}_t = E[w_t | \varphi_t] | \varphi_t = 1 = 0 \). Thus, the reference point for \( y_t \) is given by \( \hat{y}_t = \overline{y}_t \).

The carrier of happiness now becomes the agent’s relative income \( y_t - \overline{y}_t \). The reason why \( \overline{y}_t \) enters the happiness function is because it filters out the aggregate shock \( \Gamma_t \), and thus increases the statistical power of the measurement device. The resulting happiness function is analogous to a relative performance scheme inside a firm. By tightening the connection between effort and reward, its effect is to magnify the cost of withdrawing effort – e.g., Lazear and Rosen [1981], Green and Stokey [1983].

A distinctive implication of the model arises when habits and peer comparisons are combined. We begin with a simple example that combines the technologies from Examples 1 and 3:

Example 4: A Markovian Habit and Dynamic Peer Comparisons. Suppose output for each agent is given by \( y_t = \varphi_t + \Gamma_t + \theta_t \), where \( \Gamma_t \) is an arbitrary aggregate shock, and \( \theta_t \) follows the Markovian process \( \theta_t = \theta_{t-1} + z_t \). The difference with Example 1 is the presence of \( \Gamma_t \), and the difference with Example 3 is the persistence of \( z_t \). Using this technology, we can write \( y_t - y_{t-1} = (\varphi_t - \varphi_{t-1}) + (\Gamma_t - \Gamma_{t-1}) + z_t \). Moreover, in equilibrium, \( \overline{y}_t - \overline{y}_{t-1} = \Gamma_t - \Gamma_{t-1} \). Combining these expressions, we obtain \( w_t = (\varphi_t - \varphi_{t-1}) + w_{t-1} + z_t \), which satisfies (3) (with \( \Omega_t = w_{t-1} = \theta_{t-1} - \overline{\theta}_{t-1} \)). As a result, the reference point in terms of \( w_t \) becomes \( \hat{w}_t = w_{t-1} \). Accordingly, the reference point in terms of \( y_t \) becomes

\[
\hat{y}_t = y_{t-1} + \overline{y}_t - \overline{y}_{t-1}.
\]

This reference point is not the mere sum of \( y_{t-1} \) and \( \overline{y}_t \). Such a reference point would imply that the carrier of happiness is the difference between the increase in output \( \Delta y_t = y_t - y_{t-1} \) and the average peer output \( \overline{y}_t \). This would lead to a
comparision between an innovation and an absolute level. Rather, the carrier of happiness is the difference in differences

\[ \Delta y_t - \Delta \bar{y}_t, \]

where the role of \( \bar{y}_{t-1} \) is to filter out the lagged aggregate shock \( \Gamma_{t-1} \).

Thus, while \( y_{t-1} \) and \( \bar{y}_t \) reduce current happiness, \( y_t \) has the opposite effect. We interpret this as a generalized process of habituation that extends to the output of peers. Consider, for example, a sudden and permanent increase in \( y_t \), while holding \( y_t \) constant. This increase will initially shift the reference point to the right, with a likely decrease in happiness. But after one period, \( y_t \) will enter the reference point with a negative sign, shifting it back to its original level. As a result, the agent will have successfully coped. Equivalently, this agent has become habituated to her new lower social position \( w_t \), even though her own income has not changed.\(^{12}\)

We conclude with a result that encompasses all the examples above:

**Proposition 3.** Suppose output for each agent is given by \( y_t = \varphi_t + \Gamma_t + \theta_t \), where \( \Gamma_t \) is an arbitrary aggregate shock, and \( \theta_t \) follows the auto-regression \( \theta_t = \sum \alpha_s \theta_{t-s} + z_t \) for arbitrary constants \( \alpha_s \), and \( z_t \) i.i.d. Then, the optimal reference point for period \( t \) is given by

\[ \hat{y}_t = \sum \alpha_s y_{t-s} + \bar{y}_t - \sum \alpha_s \bar{y}_{t-s}. \]

**Proof.** Using the above technology, we can write

\[ y_t - \sum \alpha_s y_{t-s} = \varphi_t - \sum \alpha_s \varphi_{t-s} + \Gamma_t - \sum \alpha_s \Gamma_{t-s} + z_t. \]

Therefore, in equilibrium, \( \bar{y}_t - \sum \alpha_s \bar{y}_{t-s} = (1 - \sum \alpha_s) + \Gamma_t - \sum \alpha_s \Gamma_{t-s} \). Combining these two expressions, we obtain

\[ w_t = \sum \alpha_s w_{t-s} + (\varphi_t - 1) - \sum \alpha_s (\varphi_{t-s} - 1) + z_t, \]

which satisfies (3). The result follows from setting \( \varphi_t = \varphi_{t-s} = 1 \) (so that \( \hat{w}_t = \sum \alpha_s w_{t-s} \)), and rearranging terms. \( \blacksquare \)

The carrier of happiness is now the generalized difference in differences

\[ y_t - \sum \alpha_s y_{t-s} - \left[ \bar{y}_t - \sum \alpha_s \bar{y}_{t-s} \right]. \]

\(^{12}\)This may potentially describe an agent who copes with peer success by changing her reference group. To the best of our knowledge, a satisfactory formal model of reference-group formation is yet to be developed.
The term $\sum \alpha_s y_{t-s}$ corresponds to a conventional habit, whereas the presence of $\sum \alpha_s \overline{y}_{t-s}$ again results in habituation to peers. Regardless of the properties of the aggregate shocks (including any intertemporal correlations), the same coefficients enter both forms of habituation. The reason is that $y_{t-s}$ is impacted by the aggregate shock $\Gamma_{t-s}$, which is redundant when assessing $\varphi_t$. Subtracting $\overline{y}_{t-s}$ from $y_{t-s}$ filters out this shock. The implication is that lagged output and lagged peer output have the opposite effect over happiness. Consider, for example, an individual with a stable level of wealth who compares himself with a neighbor who is currently wealthier. The above formulation allows him to feel better when, for as long as he can remember, this neighbor has always been wealthier—as opposed to the case where their relative fortunes have been recently reversed.

4.1. Income and Happiness Surveys. A reference point that depends negatively on the lagged income of peers might also be useful when describing the happiness surveys. To illustrate this, suppose we restrict to linear reference points of the form

$$\tilde{y}_t = \sum \alpha_s y_{t-s} + \lambda \overline{y}_t + \sum \beta_s \overline{y}_{t-s},$$

where $y_t$ denotes income, perhaps measured in logs, and where the sum of the right-hand coefficients does not exceed one—a condition required if a general increase in income is to have a non-negative impact over happiness. Such a model would explain habituation to own income via positive coefficients $\alpha_s$ (for example, full habituation to permanent changes in $y_t$ would require $\sum \alpha_s = 1$), and it would explain a concern for relative income via a positive $\lambda$.

A number of authors have argued that beyond the point where basic needs have been covered, an individual’s absolute level of income has a minimal impact over her reported happiness. Rather, the bulk of its impact comes from the relative social position it conveys—e.g., Easterlin [1995], Oswald [1997], Frank [2004]. In the above model, this would correspond to a $\lambda$ close to 1—so that simultaneous increases in $y_t$ and $\overline{y}_t$ mostly cancel each other out.\footnote{Easterlin [1995], for example, implicitly favors a $\lambda = 1$. This extreme version is necessary to eliminate any time-series association between income and happiness when the growth rate for income varies with time.}

But when combined with the initial restriction that $\sum \alpha_s + \lambda + \sum \beta_s \leq 1$, notice that a $\lambda$ that is close to 1 may coexist with significant habits for own income only when the term $\sum \beta_s$ happens to be negative—as suggested by the Proposition above.
5. Conclusions

We have modeled happiness as a measurement instrument that guides the agent’s decisions. The quality of the measurement is enhanced when the happiness function adapts to the current decision environment –adaptation is thus favored by nature. In the model, this adaptation occurs through an output benchmark, or reference point, that integrates all information that can predict the agent’s performance. Whenever output is correlated across time and across agents, habits and peer comparisons arise as special cases.

Our goal has been to argue that happiness, as best observed by current empirical methods, contains the signs of statistical inference. In particular, we have suggested a statistical parallel between happiness and an optimal incentive scheme that seeks to promote effort, allowing us to import the insights of agency theory to the study of our innate psychological features. This approach rationalizes, for example, why serendipity has an impact over happiness –and why this impact is short-lived.

The examples of habits and peer comparisons that we considered are far from exhaustive. Possible extensions could address the issue of habituation patterns that differ systematically according to the type of good involved, as well as the endogenous formation of reference groups. In both cases, statistical principles may prove to play a role.

6. Appendix: Costly Happiness

Here we replace the assumption that happiness is bounded. Instead, we assume that the hedonic machinery of the agent is such that there exists a specific level of happiness $V_0$ that is most desirable from a physiological point of view, and deviations away from $V_0$ are increasingly costly. For example, $V_0$ might consume the least amount of calories when experienced. Alternatively, this might be a happiness level that maximizes the health of the agent.$^{14}$

In particular, we assume that whenever $V(y)$ departs from $V_0$, it causes an indirect evolutionary cost $C(V(y), V_0) \in \mathbb{R}$. Assuming risk-neutrality, and an additive structure for this cost, the problem for the principal becomes

$$\max_{V} \frac{\partial}{\partial \varphi} E[V \mid \varphi] \bigg|_{\varphi=1} - E[C \mid V],$$

$^{14}$An analogy employed in the psychology literature is that happiness is like blood sugar –e.g., Wilson et al. [2002]. See also Sapolsky [1999] for a discussion of the health costs of negative emotions.
where the first term is the limiting version of the objective in \( II \), and the second term is the expected value of \( C \) in equilibrium. (Notice that the assumption of a bounded happiness would correspond to the case where \( C \) is zero within \([0, 1]\), and infinity outside this range.) The principal must now trade off the signal value of a departure from \( V_0 \) against its physiological cost.

We illustrate this trade-off for the case where \( C(V(y), V_0) \) takes the simple form 
\[
\frac{1}{n} |V(y) - V_0|^n,
\]
and output is given by \( y = \varphi + z \), with \( z \) distributed as a standard normal. This specification allows for a solution in closed form that is parameterized by the degree of curvature \( n \) in the cost function. The first-order conditions for \( V(y) \) become

\[
(V(y) - V_0)^{n-1} = \frac{f_\varphi(y | 1)}{f(y | 1)} = y - \hat{y}, \quad \text{for } V(y) \geq V_0, \quad \text{and}
\]

\[
(V_0 - V(y))^{n-1} = -\frac{f_\varphi(y | 1)}{f(y | 1)} = \hat{y} - y, \quad \text{for } V(y) < V_0,
\]

where \( \hat{y} = E[V | 1] \).\(^{15}\)

\(^{15}\)The equality \( \frac{f_\varphi(y | 1)}{f(y | 1)} = y - \hat{y} \) is derived from the normal density

\[
f(y | \varphi) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(y - \varphi)^2\right).
\]
Figure 2 (drawn to scale) plots \( V(y) - V_0 \) as a function of \( y - \widehat{y} \), the carrier of happiness. When \( n = 2 \), the optimal happiness function is a 45° line. Whenever \( n > 2 \), the optimal function becomes S-shaped: concave to the right of the reference point, and convex to the left. Moreover, as \( n \) becomes large, the optimal function converges to a one-step function with \( V(y) - V_0 = 1 \) to the right of \( \widehat{y} \), and \( V(y) - V_0 = -1 \) to the left. After a normalization of units, this limiting case coincides with the functions derived in Proposition 2.

References


