A Theory of International Currency and Seigniorage Competition*

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July 2005

Abstract

This paper explicitly considers strategic interaction between governments to study currency competition and its effects on the circulation of currencies and welfare in a two-country, two-currency search theoretic model. Each government uses seigniorage to provide public goods. Agents consume private goods, and the public goods of their own country. We have several findings. The negative impact of a country's inflationary policy on the realm of circulation of its currency imposes an inflation discipline: the more open a country is, the stronger is the discipline. The worldwide circulation of a currency increases seigniorage and welfare and decreases the inflation rate of the issuing country compared to autarky. The other country, since the tax base is reduced due to the use of foreign currency, raises its inflation rate. However, there is a limit on the rate beyond which it cannot maintain the circulation of national money. Under strategic interaction between governments in selecting equilibrium, the larger country would try to lower the inflation rate to make its currency circulate abroad, while the other country may also lower the inflation rate to sustain its national currency as the sole medium of exchange.

*We thank Kazuya Kamiya, Takashi Shimizu, Victor Rios-Rull, V.V. Chari and Shouyong Shi for valuable comments.

1 Introduction

Money, either minted or printed, has long been used to provide the economy with a means of payment and to generate revenues for the governments to finance public spending. These two functions of money issuance are interrelated with each other. If the government is a sole issuer of currency, it is easier for it to collect seigniorage at the expense of providing a "stable" means of payment than otherwise. In the fifteenth century, for example, the Yuan dynasty enjoyed the monopoly power of issuing paper money in China, paying little attention to the control of inflation, until its economic and military power declined. On the other hand, if multiple states issue monies, competition for wider circulation imposes an inflation discipline. In the seventeenth century, the Spanish Monarchy pursued a policy of "price discrimination" among its own Castillian currencies: it faced competition from other states minting large-denomination coins, forcing it not to seek an additional short-term revenue, while petty coinage was a local monopoly, allowing the Monarchy to collect a good amount of seigniorage (Motomura, 1994). More recently, the United States, the country that would seem best able to impose a seigniorage tax on foreigners,[1] has a relatively stable monetary policy.[2] These observations, especially the U.S. case, have puzzled some economists who argued that an inflationary bias is inevitable in an environment where the tax burden falls partially on foreigners.[3] Intuitively speaking, their models ignored the aspect of currency competition to attract their "users". While an inflation tax on a currency is an efficient means to collect revenues in the short-run, it may diminish its value and realm of circulation in the long-run. To better understand the above observations, therefore, we need a model with endogenous determination of the realms of circulation of currencies.

This paper studies currency competition between governments and its effects on the circu-

[1] Jefferson (1998) computes seigniorage estimates in US and found that in the early 1990s the seigniorage contributed by the rest of the world has even surpassed the domestic contribution. This could also be viewed as a cost for not providing stable national currency in emerging and transition economies.

[2] According to Fisher (1982), in the United States seigniorage averaged about .5 percent of GNP and only 2 or 3 percent of total revenue collected between 1960 and 1978. Seigniorage tax accounted for about 15 percent of total revenue between 1973 and 1978 in Italy, of which the currency is not a 'vehicle' currency.

[3] For example, Canzoneri (1989) uses a two-country model with cash-in-advance constraints to show that a government will opt for an inflation bias if the tax burden falls partially on foreigners. Cooper and Kempf (2003) uses a two-country overlapping generations model to show gains from a monetary union due to reduced transaction costs and low inflation rates.

lation of currencies and welfare levels in a two-country, two-currency search theoretic model due to Matsuyama et al. (1993).[4] Each country consists of infinitely-lived private agents and a government. A representative agent obtains utility from private good, and the public good of his own country. Each government prints fiat money, taxes on money holdings, and uses seigniorage to purchase private goods and provide public goods. Agents interact with home and foreign agents with different frequencies, reflecting the relative country size and the degree of international economic integration. Agents choose which money to hold to conduct trade. In so doing, they take into account the relative frequency of trade, which may differ across currencies, and the risk of confiscation (a proxy for inflation) that each currency is subject to.[5]

We first study the effects of inflation taxes on the circulation of currencies. If the degree of economic integration is sufficiently low, there exists an equilibrium where the two national currencies circulate only locally. We call this situation autarky. The higher the degree of economic integration becomes, the more likely is one of the currencies to circulate internationally. In particular, the larger country is more likely to have its currency circulate internationally than the smaller country. We find that the higher the inflation rate on a given currency is, the less likely is it to circulate locally and internationally. More specifically, the greater the foreign inflation tax is relative to home inflation tax, the more attractive home currency becomes relative to foreign currency, and therefore, the higher incentive agents have to use home currency. A sufficiently high inflation tax eliminates its chance of domestic circulation as well as worldwide circulation. Therefore, the negative impact of a country's inflationary policy on the circulation of its currency imposes an inflation discipline. This is one of the issues that cannot be analyzed in a framework with no endogenous emergence of an international currency.

We then consider a policy game. We first study a situation in which all the agents and

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[4]There are preceding works using search-theoretic models to study the issues of international currency. Zhou (1997) considers preference shocks to induce currency exchange in a framework similar to Matsuyama et al. (1993). Wright and Trejos (2001) considers a search model with divisible goods to study the determination of exchange rate. Curtis and Waller (2003) shows how currency restrictions and government transactions policy affect the values of fiat currencies in a two-country model. Ravikumar and Wallace (2002) shows that a uniform currency can eliminate inferior equilibria associated with distinct currencies.

[5]Previous studies on how trade frictions and government policy influence the circulation and value of a medium of exchange include Li (1995), Aiyagari and Wallace (1997) and Li and Wright (1998). In Li (1995) the government taxing fiat money holding increases the risk (cost) of holding money, which we adopt here as the proxy for inflation.

the governments believe a particular equilibrium to prevail, and the two governments choose tax rates simultaneously, measuring the payoff of each government by the utility of its own representative agent. For the country that issues the international currency, two opposing forces affect the optimal inflation rate: the enlarged tax base, and the tax burden that falls partially on foreigners. If the former effect dominates the latter, we observe a *lower* inflation rate on a currency circulating abroad than under autarky. The country with the local currency, on the contrary, has an incentive to raise the inflation rate to collect seigniorage, because the tax base shrinks due to the use of foreign currency. However, the possibility of abandoning the use of home currency provides a force to curb the inflation tendency. The force is stronger as the degree of "openness" facing the country is higher, since this increases the gains of using foreign currency.[6]

As for the welfare issue, a country that successfully has its currency to circulate abroad will enjoy higher welfare than under autarky: both the amount of public good provided and private consumption are higher, since it can collect seigniorage from foreigners, and the trade opportunities expand. The other country, however, may not benefit from the circulation of foreign currency. The resulting change in welfare depends on the positive effect of an increase in trade opportunity and the negative effect of losing the tax base. If the degree of "openness" facing the country is sufficiently small, using foreign currency is not beneficial because the seigniorage is partially taken away, while there is little benefit from trade.

We also consider the situation where both governments choose inflation tax rates, understanding the possibility that their choices affect which type of equilibrium to prevail. Although the outcome depends on the details of the policy game, we are still able to make some predictions on the equilibrium selection and provide economic intuition. One of the key findings is that the larger country may try to make its currency circulate internationally by lowering the inflation tax. Knowing this, the smaller country has an incentive to prevent its tax base from diminishing by lowering its tax rate accordingly. As a result, it would raise less revenue than when there is no such strategic interaction. In return, it can ensure the existence of the equilibrium it prefers.

[6]Romer (1993) finds negative correlation between openness and inflation and argues that the absence of precommitment in monetary policy leading to excessive inflation is the underlying mechanism. Here we provide another mechanism: the negative impact of a country's inflationary policy on the realm of circulation of its currency imposes an inflation discipline, and the higher the degree of openness is, the stronger is the discipline.

One interesting question is the following: will a government raise the inflation rate *after* it has successfully made its national currency circulate abroad? We find that this time-inconsistency problem is not likely to arise if the "degree of openness" is sufficiently high, since in this case, the government can make the currency attractive enough to foreigners without lowering too much the inflation rate.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 discusses the existence and properties of various types of equilibria under inflation tax policy. In section 4, we study currency competition between governments and its effects on welfare. Section 5 discusses strategic selection of equilibrium. Section 6 concludes with suggestions for possible modifications and extensions.

2 The Basic Model

The Environment

Time is discrete and the horizon is infinite. There is a $[0,1]$ continuum of infinitely-lived agents with unit mass. The agents are divided into two regions, Home and Foreign. Let $n \in (0,1)$ be the size of Home population. There are k ($k \geq 3$) types of indivisible goods. Within each economy, there are equal proportions of k types of agents, who specialize in consumption, production and storage. A type i agent derives utility only from consuming type i good and can produce only good $i+1$ (mod k). Agent i can only store his production good costlessly up to one unit; he can neither produce nor store other types of goods. Hence, there is no double coincidence of wants. Let $u > 0$ be the instantaneous utility from consuming an agent's consumption good and δ his discount rate.

There are two distinguishable fiat money, Home currency and Foreign currency. Each currency is indivisible. An agent can store only one unit of good or one unit of currency at a time. Let m_h (m_f) denote the fraction of Home agents holding the Home (Foreign) currency. The inventory distribution of Home agents can be summarized by $X = (1-m_h-m_f, m_h, m_f)$. Likewise, let m_h^* (m_f^*) denote the fraction of Foreign agents holding the Home (Foreign) currency. The inventory distribution of Foreign agents can be summarized by $X^* = (1-m_h^*-m_f^*, m_h^*, m_f^*)$. Let m and $m^* \in (0,1)$ denote the supply of the Home currency per Home agent and that of

Foreign currency per Foreign agent, respectively. Then,

$$nm = nm_h + (1-n)m_h^*, \ (1-n)m^* = nm_f + (1-n)m_f^*.$$

Agents are matched randomly in pairs, but not in a uniform fashion. Let $\beta \in (0,1)$. A Home agent meets another Home agent with probability n, and meets a Foreign agent with probability $\beta(1-n)$. A Foreign agent meets a Home and another Foreign agent with probability βn and $(1-n)$, respectively. Thus, agents who live in different countries meet less frequently than a pair of agents who live in the same country. Note that the above description implies the probability of meeting a trade partner also depends on the size of country. We can interpret β as the degree of economic integration or a measure of the trading frictions in international trade. An increase in β reduces international trade frictions, because higher β makes it easier to meet with foreign citizens. Similarly, a higher n not only makes it easier for the Home agents to meet with their fellow citizens but also makes it easier for Foreign to trade with Home agents. That is, a higher n increases the economies of scale in transactions with Home agents. However, an increase in n reduces the relative size of Foreign country and so increases the local trade frictions in Foreign country.

Trade entails a one-for-one swap of inventories, and takes place if and only if both agents agree to trade. The trade partner's type and inventory are observable, trade histories are not. Agents are unable to commit to future actions, and proposed transfers cannot be enforced. Thus, people trade when there is a single coincidence of wants, and all trades involve the use of a tangible medium of exchange.

The role of government in the provision of public goods

In each country there is a government whose role is to print fiat money, tax money holdings and provide public goods from the private goods that it purchases. An agent who holds Home (Foreign) currency is subject to a probability τ_h (τ_f) that his money would be confiscated by the government of Home (Foreign) country. The rate τ_h (τ_f) can be interpreted as a tax rate that a government imposes on money holdings in order to provide public goods. We can also interpret τ_h (τ_f) as inflationary tax.

When a Home currency holder and a commodity holder are matched and about to trade, a Home government agent steps in with probability τ_h, confiscating money from the money holder and purchasing the commodity from the commodity holder. The same arrangement is made for Foreign government. In this series of moves, the money holder loses what he had without obtaining his consumption good and goes back to the status of holding commodity, the commodity holder becomes money holder just like when he trades with the private agent, and the government obtains the commodity.

The government transforms the private goods it purchases into public goods from which every private agent in the country enjoys the utility of $\phi(G)$ where G is the total quantity of private goods purchased by the government in a unit of time. We assume $\phi(0) = 0$, $\phi'(G) \to \infty$ as $G \to 0, \phi'(G) > 0$ and $\phi''(G) < 0$. Public goods are nonstorable (e.g., army service).[7]

Strategies and equilibria

An agent chooses trade strategies to maximize his expected discounted utility, taking as given others' strategies and the distribution of inventories. We restrict our attention to pure strategies which only depend on his nationality and the objects he and his trading partner have in inventory. Thus, the Home agent's trade strategy can be described as

$$s_{ab} = \begin{cases} 1 \text{ if he trades object } a \text{ for } b \\ 0 \text{ otherwise,} \end{cases}$$

where $a, b = g, h$, or f, and $a \neq b$. Similarly, the Foreign agent's trade strategy is given by $s^*_{ab} = 0$ or 1. We consider only time-independent strategies. Given that the physical environment is stationary and the planning horizon is infinite, we can therefore confine our attention to steady-state equilibrium.

Let V_g, V_h and V_f denote the expected discounted utility to a Home agent holding his production good, the Home currency, and Foreign currency, respectively. Let P_{ab} (P^*_{ab}) denote the transition probability with which a Home (Foreign) agent switches his inventory from object a to object b. Then, the Bellman's equations are

$$V_g = [(1 - P_{gh} - P_{gf})V_g + P_{gh}V_h + P_{gf}V_f]/(1 + \delta), \quad (1)$$

$$V_h = [\tau_h P_{hg}V_g + (1 - \tau_h)P_{hg}(u + V_g) + (1 - P_{hg} - P_{hf})V_h + P_{hf}V_f]/(1 + \delta), \quad (2)$$

$$V_f = [\tau_f P_{fg}V_g + (1 - \tau_f)P_{fg}(u + V_g) + P_{fh}V_h + (1 - P_{fg} - P_{fh})V_f]/(1 + \delta). \quad (3)$$

[7] We may assume that Home government and Foreign government have different efficiency in providing public goods; e.g., the quantity of public goods G is a fraction γ of total consumption goods purchased by the government, and both countries may have different γ's. We may also assume that Home and Foreign agents have different preferences for public goods.

Note that the first terms in the RHS of equality in (2) and (3) imply that, if an agent's currency is confiscated by the issuing government (with probability τ_h and τ_f that his money is confiscated by Home and Foreign government, respectively), his value becomes that of holding production good. The value functions and strategies must satisfy the following incentive compatibility constraints:

$$
\begin{aligned}
s_{gb} &= 1 \text{ iff } V_g < V_b \ (b = h \text{ or } f), \\
s_{ag} &= 1 \text{ iff } V_a < u + V_g \ (a = h \text{ or } f), \\
s_{ab} &= 1 \text{ iff } V_a < V_b \ (a, b = h \text{ or } f).
\end{aligned}
$$

For example, $V_g > V_f$ is the sufficient and necessary condition for a Home agent not to trade his production good for Foreign currency.

We restrict our attention to the equilibrium where agents always accept their local currency; i.e., Home currency is accepted by the Home agents and Foreign currency is accepted by the Foreign agents. We are left four types of equilibria – no international currency, Foreign currency is the only international currency, Home currency is the only international currency and both currencies circulate in both countries. We characterize the existence conditions in terms of β and n, the extent of international and local trade frictions, as well as tax rates τ_h and τ_f.

First of all, in any of these equilibria, we have $P_{fh} = P_{hf} = P^*_{fh} = P^*_{hf} = 0$. Given the tie-breaking rule, no two agents in the same country exchange Home currency and Foreign currency; indeed, for currency exchange to occur between two, say, Home agents, we need $s_{hf} = s_{fh} = 1$, which implies $V_f > V_h$ and $V_h > V_f$: a contradiction. Therefore, the only possibility for currency exchange is between agents from different countries. Due to the nature of equilibrium, this may happen only when both currencies circulate worldwide. In this case, we need to have, say, $V_h > V_f$ and $V^*_f > V^*_h$ (the opposite case has a similar consequence). If $\tau_h = \tau_f$ holds, then the two currencies are perfect substitutes, and therefore, $V_h = V_f$ and $V^*_f = V^*_h$, which is a contradiction. But, if, say, τ_h becomes smaller (resp. greater) than τ_f, then Home currency is more (resp. less) attractive for both Home and Foreign agents than Foreign currency. Thus, both Home and Foreign agents have the same incentive concerning the acceptance of currency, and therefore, there is no room for currency exchange.

Before conducting equilibrium analysis, let us calculate the value functions from (1), (2) and

(3):

$$V_g = [(\delta + P_{fg})(1-\tau_h)P_{gh}P_{hg} + (\delta + P_{hg})(1-\tau_f)P_{gf}P_{fg}]\, u/P, \tag{4}$$

$$V_h = [(1-\tau_h)[(\delta + P_{gh})(\delta + P_{fg}) + \delta P_{gf}] + (1-\tau_f)P_{gf}P_{fg})]\, P_{hg}u/P, \tag{5}$$

$$V_f = [(1-\tau_f)[(\delta + P_{gf})(\delta + P_{hg}) + \delta P_{gh}] + (1-\tau_h)P_{gh}P_{hg})]\, P_{fg}u/P, \tag{6}$$

where

$$P = \delta\, [(\delta + P_{gh} + P_{hg})(\delta + P_{fg}) + P_{gf}(\delta + P_{hg})]\,.$$

Using the above value functions, we are able to state some general results.

Proposition 2.1. *In a steady-state equilibrium,*

1. $u + V_g > V_g, V_h, V_f$.

2. $\max\{V_h, V_f\} > V_g$.

3. $V_h > (<) V_g$ *iff* $(1-\tau_h)P_{hg}(\delta + P_{fg} + P_{gf}) > (<)(1-\tau_f)P_{gf}P_{fg}$.

4. $V_f > (<) V_g$ *iff* $(1-\tau_f)P_{fg}(\delta + P_{hg} + P_{gh}) > (<)(1-\tau_h)P_{gh}P_{hg}$.

The same relations hold for a Foreign agent, with relevant variables starred.

3 Equilibria

3.1 Equilibrium with two local currencies: Equilibrium A

In this equilibrium a Home agent trades his production good for the Home currency, the Home currency for his consumption good, but does not accept Foreign currency ($u + V_g > V_h > V_g \geq V_f$). A Foreign agent trades his production good for the Foreign currency, the Foreign currency for his consumption good, but does not accept the Home currency ($u + V_g^* > V_f^* > V_g^* \geq V_h^*$). There is no international currency and no international trade in this equilibrium. The inventory distributions are given by $X = (1-m, m, 0)$ and $X^* = (1-m^*, m^*, 0)$. The transition probabilities in this equilibrium for a Home agent are:

$$\begin{aligned} &P_{gh} = nm/k, && P_{hg} = n(1-m)/k \\ &P_{fg} = \beta(1-n)(1-m^*)/k, && P_{gf} = P_{hf} = P_{fh} = 0. \end{aligned} \tag{7}$$

Note that P_{gh} incorporates the opportunity to sell goods to acquire money from private agents and Home government with probabilities $nm(1-\tau_h)/k$ and $nm\tau_h/k$, respectively. If a Home agent ever holds Foreign currency, then given others' strategies the chance that he can acquire consumption goods is from trading with Foreign sellers, of which probability is $(1-\tau_f)P_{fg} = \beta(1-n)(1-\tau_f)(1-m^*)/k$. Similarly, the transition probabilities for a Foreign agent are:

$$\begin{aligned} P^*_{gf} &= (1-n)m^*/k, \quad P^*_{fg} = (1-n)(1-m^*)/k \\ P^*_{hg} &= \beta n(1-m)/k, \quad P^*_{gh} = P^*_{fh} = P^*_{hf} = 0. \end{aligned} \tag{8}$$

To find the existence conditions for Equilibrium A, we verify the incentive constraints $u+V_g > V_h > V_g > V_f$ and $u+V^*_g > V^*_f > V^*_g > V^*_h$. From Proposition 2.1, $V_g \geq V_f$ and $V^*_g \geq V^*_h$ imply other inequalities. We have $V_g \geq V_f$ (Home agents do not accept Foreign currency) iff $\beta \leq \beta_A$, where

$$\beta_A \equiv \frac{m(1-m)n^2}{(1-n)(1-m^*)(k\delta+n)} \frac{1-\tau_h}{1-\tau_f}.$$

Likewise, Foreign agents do not accept Home currency, or $V^*_g \geq V^*_h$, iff $\beta \leq \beta^*_A$, where

$$\beta^*_A \equiv \frac{m^*(1-m^*)(1-n)^2}{n(1-m)(k\delta+1-n)} \frac{1-\tau_f}{1-\tau_h}.$$

In the sequel, we focus on the case where agents are sufficiently patient relative to matching frequency, i.e., we study the limiting situation where δ goes to zero. Taking the limit, we obtain

$$\lim_{\delta\to 0} \beta_A = \frac{m(1-m)n}{(1-n)(1-m^*)} \frac{1-\tau_h}{1-\tau_f} \tag{9}$$

and

$$\lim_{\delta\to 0} \beta^*_A = \frac{m^*(1-m^*)(1-n)}{n(1-m)} \frac{1-\tau_f}{1-\tau_h}. \tag{10}$$

Given parameter values of m, m^*, k, τ_h, and τ_f, $\beta \leq \beta_A$, $\beta \leq \beta^*_A$ give the existence conditions of equilibrium A on (n,β) space, shown in Figure 1.[8] The region of existence of Equilibrium A on (n,β)-space depends on the ratio $(1-\tau_h)/(1-\tau_f)$. The less $(1-\tau_h)/(1-\tau_f)$ is, the less is β_A and the greater is β^*_A. In other words, as τ_h increases and/or τ_f decreases, the locus $\beta=\beta_A$ shifts downward, while the locus $\beta=\beta^*_A$ shifts upward (see Figure 1). If we interpret (τ_h,τ_f) as a proxy for the rate of inflation, then this change is intuitive. The less $(1-\tau_h)/(1-\tau_f)$ is,

[8] The parameters are $m = m^* = .3, k = 10, u = 1$.

the less attractive Home currency becomes relative to Foreign currency, and therefore, the less (resp. more) incentive agents have to use Home currency (resp. Foreign currency).

The downward shift of β_A implies that under an inflationary policy, staying autarchy is not the best response unless the population size of the country is big enough to offset the negative impacts due to the risk of confiscating currency. Thus, for a given pair of (n, β), if a country adopts too high an inflation tax rate, it may destroy the equilibrium with two currency areas.

Given a policy pair (τ_h, τ_f), if the degree of economic integration is sufficiently small, national currency circulates only locally; there is no international currency and no international trade. Other things being equal, this equilibrium is less likely to survive if the country size is more uneven. If n is sufficiently large, the trade with Home agents is so easy that Foreign agents would have incentives to use Home currency. For a given n, Equilibrium A does not exist when β is sufficiently high, either. The higher the degree of economic integration becomes, the easier trade with foreigners, and the higher the incentive to accept foreign currency becomes.

3.2 Equilibria with one local currency and one international currency: Equilibria F and H

We discuss the existence conditions for Equilibrium F, where Home currency is accepted only in Home country, while Foreign currency circulates in both Home and Foreign country as an international medium of exchange. Equilibrium H is the mirror image of Equilibrium F and can be characterized in a similar manner.

Equilibrium F requires $u + V_g > V_h, V_f > V_g$ and $u + V_g^* > V_f^* > V_g^* \geq V_h^*$. When agents follow these strategies, $m_h = m$ and so $X = (1 - m - m_f, m, m_f)$ and $X^* = (1 - m_f^*, 0, m_f^*)$. The steady state requires that the ratios of commodity holders to the Foreign currency holders in the two countries be equalized, i.e.,

$$\frac{m_f^*}{1 - m_f^*} = \frac{m_f}{1 - m - m_f}.$$

From the steady state condition, $m_f = (1 - m)m_f^*$. Therefore we can rewrite the inventory distributions in terms of m_f^* as $X = ((1-m)(1-m_f^*), m, (1-m)m_f^*)$ and $X^* = (1-m_f^*, 0, m_f^*)$. The total supply of Foreign currency must equal the total amount circulates in both countries

$$(1 - n)m^* = n(1 - m)m_f^* + (1 - n)m_f^* = (1 - nm)m_f^*. \tag{11}$$

The transition probabilities for a Home agent are

$$
\begin{aligned}
P_{gh} &= nm/k, \\
P_{gf} &= Bm_f^*/k, \\
P_{hg} &= n(1-m)(1-m_f^*)/k, \\
P_{fg} &= B(1-m_f^*)/k, \\
P_{hf} &= P_{fh} = 0,
\end{aligned} \tag{12}
$$

where $B = n(1-m) + \beta(1-n)$, and for a Foreign agent

$$
\begin{aligned}
P_{gf}^* &= B^* m_f^*/k, \\
P_{hg}^* &= \beta n(1-m)(1-m_f^*)/k, \\
P_{fg}^* &= B^*(1-m_f^*)/k, \\
P_{gh}^* &= P_{fh}^* = P_{hf}^* = 0,
\end{aligned} \tag{13}
$$

where $B^* = \beta n(1-m) + (1-n)$, and m_f^* satisfies (11).

From Proposition 2.1, it suffices to check that Home agents accept Home currency ($V_g < V_h$), and that Foreign agents do not accept Home currency ($V_g^* \geq V_h^*$).

First, substituting (12) into the third and forth claims of Proposition 2.1, and taking the limit of δ going to zero, we have $V_h > V_g$ iff

$$
\beta \leq \beta_F \equiv \frac{n(1-m)}{1-n} \left[\frac{1-\tau_h}{1-\tau_f} \frac{1}{m_f^*} - 1 \right], \tag{14}
$$

and $V_g^* \geq V_h^*$ iff

$$
\beta \leq \beta_F^* \equiv \frac{1-n}{n(1-m)} \left[\frac{1-\tau_h}{1-\tau_f} \frac{1}{m_f^*} - 1 \right]^{-1}. \tag{15}
$$

Equilibrium F exists if and only if the two incentive constraints hold, given (12), (13) and (11). We depict the equilibrium region defined by (14) and (15) on the space of (n, β) in Figure 2. Given other parameters, an increase in τ_h leads to a decrease in β_F, while an increase in τ_f leads to an increase in β_F. Likewise, an increase in τ_h leads to an increase in β_F^*, while an increase in τ_f leads to a decrease in β_F^*. Hence, the higher the Home inflation tax is (or similarly, the lower Foreign inflation tax is), the less likely Home and Foreign agents are to use Home currency.

3.3 Equilibrium with two international currencies: Equilibrium U

In this equilibrium, both currencies circulate side by side, i.e., they are both universally accepted: $u+V_g > V_h, V_f > V_g$, and $u+V_g^* > V_f^*, V_h^* > V_g^*$. When agents follow these strategies, $X = X^*$, and $m_h = m_h^* = nm$, and $m_f = m_f^* = (1-n)m^*$. The transition probabilities are

$$\begin{aligned}
P_{gh} &= nm[n+\beta(1-n)]/k,\\
P_{gf} &= [n+\beta(1-n)](1-n)m^*/k,\\
P_{hg} &= P_{fg} = [n+\beta(1-n)][1-nm-(1-n)m^*]/k,\\
P_{gh}^* &= nm[\beta n+(1-n)]/k,\\
P_{gf}^* &= [\beta n+(1-n)](1-n)m^*/k,\\
P_{hg}^* &= P_{fg}^* = [\beta n+(1-n)][1-nm-(1-n)m^*]/k,\\
P_{hf} &= P_{fh} = P_{fh}^* = P_{hf}^* = 0.
\end{aligned} \tag{16}$$

Given any $\tau_h > 0$ and $\tau_f > 0$, $V_h > V_g$ ($\Leftrightarrow V_h^* > V_g^*$) holds iff

$$\frac{1-\tau_h}{1-\tau_f} > \frac{(1-n)m^*}{1-nm}, \tag{17}$$

and $V_f > V_g$ ($\Leftrightarrow V_f^* > V_g^*$) holds iff

$$\frac{1-\tau_f}{1-\tau_h} > \frac{nm}{1-(1-n)m^*}. \tag{18}$$

Combining (17) and (18), we ensure that the existence of equilibrium U iff

$$\frac{1-n}{2-n} < \frac{1-\tau_h}{1-\tau_f} < \frac{1+n}{2n}.$$

If the tax rate of, say, Home currency is sufficiently high in comparison with that of Foreign currency, then agents start rejecting Home currency, and the more Foreign currency balance we have, the lower this threshold is since each agent can have Foreign currency relatively quickly after he rejects Home currency.

This result is in contrast to Matsuyama *et al* (1993) in which the unified currency equilibrium exists for all parameter values. The reason for this difference is that currencies are no longer perfect substitutes even in this equilibrium if the tax rates are different. Indeed, if $\tau_h = \tau_f$ holds, then the two currencies become perfect substitutes, and such an equilibrium exists under all parameter values.

4 Policies and Welfare

The following two sections discuss currency competition between governments and its effects on welfare and the determination of currency regimes.

The welfare of Home country (resp. Foreign country), denoted by W (resp. W^*), consists of the long-run expected (average) value of each agent in Home (resp. Foreign) country from private transactions and the payoff stream of the representative Home (resp. Foreign) agent obtained from public goods. To be concrete, we use the following specifications:

$$\begin{aligned} W &\equiv \delta[(1-m_h-m_f)V_g + m_h V_h + m_f V_f] + n\phi(G) \\ &= [m_h(1-\tau_h)P_{hg} + m_f(1-\tau_f)P_{fg}]u + n\phi((m_h P_{hg} + m_h^* P_{hg}^*)\tau_h), \end{aligned} \tag{19}$$

$$\begin{aligned} W^* &\equiv \delta[(1-m_h^*-m_f^*)V_g^* + m_h^* V_h^* + m_f^* V_f^*] + (1-n)\phi(G^*) \\ &= [m_h^*(1-\tau_h)P_{hg}^* + m_f^*(1-\tau_f)P_{fg}^*]u + (1-n)\phi((m_f P_{fg} + m_f^* P_{fg}^*)\tau_f), \end{aligned} \tag{20}$$

where $G = (m_h P_{hg} + \beta m_h^* P_{hg}^*)\tau_h$ and $G^* = (\beta m_f P_{fg} + m_f^* P_{fg}^*)\tau_f$ are the total amounts of public goods, measured by private goods, in each period provided by Home and Foreign governments, respectively. Using these values as the payoffs of the respective governments, we analyze a situation where the two countries use the tax rates and, in some case, money balances as policy instruments. We first confine our attention to each type of equilibrium, and then consider a regime change from one type of equilibrium to another, e.g., equilibrium A to F.

In the sequel, we sometimes use

$$\phi(G) = \alpha \ln G \tag{21}$$

to obtain a closed form solution.

4.1 Equilibrium A

Consider an interior solution to the policy game where all the agents as well as governments believe Equilibrium A to prevail. Substituting transition probabilities (7) into (19), and differentiating it with respect to m, we obtain

$$\frac{\partial W}{\partial m} = \frac{1}{k}\left[(1-\tau_h)nu + n\phi'(\cdot)\tau_h\right](1-2m) = 0. \tag{22}$$

Therefore, the optimal money balance is $m^A = 1/2$. Similarly, we have $m^{*A} = 1/2$ for Foreign money balance where the superscript "A" stands for Equilibrium A. Differentiating (19) with respect to τ_h, we obtain

$$\frac{\partial W}{\partial \tau_h} = \frac{m^A(1-m^A)n}{k}\left[-u + n\phi'(\cdot)\right] = 0,$$

or

$$n\phi'(m^A(1-m^A)n\tau_h^A/k) = u. \tag{23}$$

Similarly, we have

$$(1-n)\phi'(m^{*A}(1-m^{*A})(1-n)\tau_f^A/k) = u. \tag{24}$$

If we use the specification (21), then (23) is rewritten as:

$$\tau_h^A = \frac{k\alpha}{m(1-m)u}.$$

In a similar manner, the optimal tax rate for Foreign government is given by:

$$\tau_f^A = \frac{k\alpha}{m^*(1-m^*)u}.$$

Substituting $m^A = m^{*A} = 1/2$ into the above solutions, we finally obtain

$$\tau_h^A = \frac{4k\alpha}{u} \tag{25}$$

$$\tau_f^A = \frac{4k\alpha}{u}. \tag{26}$$

Note that this solution exists if and only if $4k\alpha < u$, which we assume hereafter.

4.2 Equilibrium F

We conduct an analysis similar to the previous subsection, albeit more complicated than that. We assume that the governments believe Equilibrium F to prevail. Also, to simplify the illustration in this subsection, we assume that $n < 1/2$ holds.

First of all, if we differentiate W^* with respect to m_f^* after substituting (13) into (20), we obtain

$$\frac{\partial W^*}{\partial m_f^*} = \left[(1-\tau_f)B^*/k + (1-n)\phi'(\cdot)(\beta(1-m)B + B^*)\tau_f/k\right](1-2m_f^*) = 0,$$

which implies $m_f^* = 1/2$. Therefore, the optimal money balance for Foreign country is given by $m_f^* = 1/2$. On the other hand, the optimal balance of Home currency is not independent of other parameters. In the sequel, we let $m_f^* = 1/2$ and $m = \bar{m}$ as given and examine the policy game where τ_h and τ_f are chosen simultaneously.[9]

Foreign country's problem is straightforward, which is to choose τ_f to maximize W^*. Differentiating W^* with respect to τ_f, we obtain

$$\frac{\partial W^*}{\partial \tau_f^F} = \frac{m_f^*(1-m_f^*)}{k}\left[-B^*u + (\beta(1-\bar{m})B + B^*)(1-n)\phi'(\cdot)\right] = 0,$$

or

$$(1-n)\phi'((\beta(1-\bar{m})B + B^*)m_f^*(1-m_f^*)\tau_f^F/k) = \frac{B^*}{\beta(1-\bar{m})B + B^*}u. \tag{27}$$

Using (21), we have

$$\tau_f^F = \frac{1}{\theta^*(1-\bar{m})+1}\frac{4k\alpha}{u} = \frac{1}{\theta^*(1-\bar{m})+1}\tau_f^A < \tau_f^A, \tag{28}$$

where

$$\theta^* = \frac{\beta n}{1-n}$$

is the degree of "openness" of Foreign country.

This implies that Foreign country has a lower inflation rate when its currency becomes an international currency than otherwise. Note that we have two opposing forces. If we look into the arguments of ϕ' of both (24) and (27) at $m^{*A} = m_f^{*F} = 1/2$, we notice that $(1-n) < \beta(1-\bar{m})B + B^*$. This inequality implies that the tax base for Foreign currency is larger in Equilibrium F than in Equilibrium A, which gives the government an incentive to reduce the inflation rate. On the other hand, the right hand side of (24) is greater than that of (27). This corresponds to the extent to which Foreign government can raise revenue from Home agents,

[9] We may consider a two stage game where m_f^* and m are chosen first and τ_h and τ_f are chosen secondly. One can think of $\bar{m}$ as a solution to such a problem, though we do not explicitly solve for $\bar{m}$.

which gives it an incentive to raise the inflation rate.[10] Under the current specification, however, the effect of an increased tax base dominates that of collecting seigniorage from Home agents.

Home country is faced with the constraint that its currency has to be accepted by Home agents, i.e., $\beta \leq \beta_F$. Thus, its problem is given by

$$\max_{\tau_h \geq 0} W \quad \text{s.t. } \beta \leq \beta_F, \quad \text{given } m = \bar{m}, \tag{29}$$

where β_F and W come from (14) and (19) together with (12). Solving this problem in the standard fashion, we obtain

$$\tau_h^F = \begin{cases} \dfrac{2}{\bar{m}(1-\bar{m})}\dfrac{k\alpha}{u} & \text{if } \theta \leq \bar{\theta}, \\ 1 - \dfrac{1-\tau_f^F}{2}\left[1 + \dfrac{\theta}{1-\bar{m}}\right] & \text{if } \bar{\theta} < \theta < \Theta, \end{cases} \tag{30}$$

where

$$\theta \equiv \frac{\beta(1-n)}{n}$$

is the degree of "openness" of Home country, and

$$\bar{\theta} = \frac{2}{1-\tau_f^F}\left[(1-\bar{m}) - \frac{2k\alpha}{\bar{m}u}\right] - (1-\bar{m}),$$

$$\Theta = \frac{1+\tau_f^F}{1-\tau_f^F}(1-\bar{m}).$$

If the degree of "openness" is not too high, or $\theta < \bar{\theta}$, Home country can freely choose its tax rate, or to be precise, $\beta = \beta_F$ is not binding. In this case, since $\bar{m}(1-\bar{m}) \leq 1/4$ holds, we have

$$\tau_h^F \geq \frac{8k\alpha}{u} = 2\tau_f^A.$$

In other words, the country with local currency has an incentive to raise its tax rate to collect seigniorage due to the internationalization of Foreign currency. If the degree of integration proceeds further, or $\theta \in (\bar{\theta}, \Theta)$, then $\beta = \beta_F$ becomes binding: an inflation discipline is needed in order to keep Home currency in circulation. Beyond Θ, equilibrium F no longer exists since even if Home government sets $\tau_h = 0$, Home agents have no incentive to accept Home currency.[11]

[10] The right hand side of (27) represents the relative utility sacrifice from private consumption of foreign agents due to the inflation tax. This ratio is less than 1 in equilibrium F because the tax burden falls partially on Home agents, and this creates incentive to adopt a higher tax rate.

[11] Note that while $\bar{\theta} < \Theta$ and $\Theta > 0$ always hold, $\bar{\theta}$ can be negative. If this is the case, (30) is reduced to $\tau_h^F = 1 - (1-\tau_f^F)[1+\theta/(1-\bar{m})]/2$ for $\theta < \Theta$.

4.3 Equilibrium U

The analysis of this equilibrium is easier than that of equilibrium F. Indeed, it is verified that at the optimum, we have

$$\tau_h^U = \frac{k\alpha}{m[n+\beta(1-n)][1-nm-(1-n)m^*]u}, \tag{31}$$

$$\tau_f^U = \frac{k\alpha}{m^*[\beta n+(1-n)][1-nm-(1-n)m^*]u}, \tag{32}$$

provided that (31) (resp. (32)) satisfies (17) (resp. (18)); for if not, Home (resp. Foreign) currency would not be accepted by anyone. Therefore, if (17) is violated, it is Home government that lowers the inflation rate to meet the constraint, i.e.,

$$\tau_h^U = 1-(1-\tau_f^U)\frac{(1-n)m^*}{1-nm}, \tag{33}$$

where τ_f^U is given by (32). Similarly, if (18) is violated, then we have

$$\tau_f^U = 1-(1-\tau_h^U)\frac{nm}{1-(1-n)m^*}, \tag{34}$$

where τ_h^U is given by (31).

In order to compare them with the corresponding rates in equilibria A and F, we let $m = m^* = 1/2$.[12] Then it is verified that $\tau_h^U > \tau_h^A$ and $\tau_f^U > \tau_f^A$ hold.[13] Both countries have incentives to increase the tax rates to collect seigniorage from the other country. One can also verify that $\partial\tau_h^U/\partial\theta < 0$ and $\partial\tau_f^U/\partial\theta^* < 0$, i.e., as the degree of "openness" increases, optimal tax rate under equilibrium U would be lower. If $n < 1/2$, then we have $\tau_f^U < \tau_h^U$, i.e., the government of the larger country imposes a lower inflation rate than that of the smaller country.

[12] It is verified that while $W + W^*$ is maximized at $m = m^* = 1/2$, $\partial W/\partial m > 0$ at $m = m^* = 1/2$. We hold that m and m^* are not policy variables, but historically determined ones.

[13] Equations (31) and (32) are equivalent to

$$\tau_h^U = \frac{1}{n+\beta(1-n)}\frac{4k\alpha}{u} > \frac{4k\alpha}{u} = \tau_h^A,$$

$$\tau_f^U = \frac{1}{\beta n+(1-n)}\frac{4k\alpha}{u} > \frac{4k\alpha}{u} = \tau_f^A.$$

Also, we verify that (33) and (34) are greater than $4k\alpha/u$ where we make use of $4k\alpha/u < 1$:

$$\begin{aligned}\tau_h^U - \tau_h^A &= \frac{1}{2-n} + \frac{1}{\beta n+(1-n)}\frac{1-n}{2-n}\frac{4k\alpha}{u} - \frac{4k\alpha}{u} \\ &\geq \frac{4k\alpha}{2u}[\beta n(3-n)+n(1-n)] > 0.\end{aligned}$$

4.4 Welfare comparisons

This subsection compares equilibria A, F, and U in terms of welfare. Let us compare equilibria A and F first. To begin with, it is obvious from (20) that W^* is larger in equilibrium F than in equilibrium A. This is fairly intuitive since both the trade opportunity and the tax base are larger in the former than in the latter.

On the other hand, the direction of change in W is unclear since we have the positive effect of an increase in trade opportunity and the negative effect of losing the tax base. These effects change as the "openness" of Home country changes.

Suppose that θ, or β, is close to zero. We evaluate W and W^* at $m^A = m_f^{*F} = 1/2$ and $m^F = \bar{m}, \theta = 0$ and substitute the optimal τ's into the expressions to obtain

$$
\begin{aligned}
W^A|_{\beta=0} &= \left[\frac{nu}{4k} - n\alpha\right] + n\alpha \ln \frac{n\alpha}{u}, \\
W^F|_{\beta=0} &= \left[\frac{1-\bar{m}^2}{4k} nu - \left\{1 + (1-\bar{m})^2\right\} n\alpha\right] + n\alpha \ln \frac{n\alpha}{u}, \\
W^U|_{\beta=0} &= \left[\frac{nu}{4k} - 2n\alpha\right] + n\alpha \ln \frac{\alpha}{u},
\end{aligned}
$$

$$
\begin{aligned}
W^{*A}|_{\beta=0} &= \left[\frac{(1-n)u}{4k} - (1-n)\alpha\right] + (1-n)\alpha \ln \frac{(1-n)\alpha}{u}, \\
W^{*F}|_{\beta=0} &= \left[\frac{(1-n)u}{4k} - (1-n)\alpha\right] + (1-n)\alpha \ln \left[\left\{n(1-\bar{m})^2 + (1-n)\right\} \frac{\alpha}{u}\right], \\
W^{*U}|_{\beta=0} &= \left[\frac{(1-n)u}{4k} - 2(1-n)\alpha\right] + (1-n)\alpha \ln \frac{\alpha}{u}.
\end{aligned}
$$

One can show that

$$
W^F - W^A|_{\beta=0} = -n\overline{m}^2 u/4k - \alpha n(1-\overline{m})^2 < 0.
$$

Thus, if the "openness" is sufficiently low, then equilibrium A is preferred to equilibrium F by Home country. This is because the seigniorage is partially taken away by Foreign government, while there is little benefit from trade.

If the "openness" of Home country increases, this may not be the case. To see this, we evaluate W^A and W^F at $\beta = \beta_A \equiv n/2(1-n)$. Its sign is ambiguous, but we have numerical examples of both cases, $W^F - W^A > 0$ and $W^F - W^A < 0$, as shown in Table 1.

	$\alpha = 1, k = 10, u = 1000, \theta = .5$				$\alpha = 1, k = 6, u = 100, \theta = .5$			
	$\bar{m} = 0.2$				$\bar{m} = 0.4$			
n	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
$\tau_f^A(\tau_h^A)$	0.04	0.04	0.04	0.04	0.24	0.24	0.24	0.24
τ_h^F	0.125	0.125	0.125	0.125	0.3025	0.2993	0.2918	0.2775
τ_f^F	0.0398	0.0390	0.0373	0.034	0.2391	0.2356	0.2275	0.2118
τ_h^U	0.2667	0.1333	0.0889	0.0667	0.6519	0.6848	0.5333	0.4000
τ_f^U	0.0442	0.0485	0.0523	0.0546	0.2650	0.2909	0.3140	0.3273
W^A	1.4790	3.0966	4.7665	6.4704	-0.3741	-0.6096	-0.7927	-0.9419
W^F	2.2755	2.1951	2.1696	2.1821	-0.3923	-0.6448	-0.8420	-0.9991
W^U	2.7206	5.718	8.8400	12.073	-0.2828	-0.3681	-0.3381	-0.1726
W^{*A}	15.288	13.495	11.715	9.949	-1.3895	-1.3293	-1.2566	-1.1696
W^{*F}	15.405	14.020	13.060	12.723	-1.372	-1.251	-1.057	-0.763
W^{*U}	14.832	13.606	12.896	12.938	-1.6040	-1.6604	-1.4151	-0.9582

Table 1: equilibria A, F, U if $\beta = \beta_A$

Next, we study equilibrium U in comparison with other equilibria. We have

$$\begin{aligned} W^U - W^A|_{\beta=0} &= -\alpha n(1+\ln n), \\ W^U - W^F|_{\beta=0} &= \alpha n\overline{m}(-2+\overline{m}) + n\overline{m}^2 u - \alpha n \ln n, \\ W^{*U} - W^{*A}|_{\beta=0} &= -\alpha(1-n)[1+\ln(1-n)], \\ W^{*U} - W^{*F}|_{\beta=0} &= -\alpha(1-n)[1+\ln(1-n\overline{m}(2-\overline{m}))]. \end{aligned}$$

If β is close to zero, there is no gain from trade and so the signs of $W^U - W^A$ and $W^{*U} - W^{*A}$ depend upon the relative country size: $W^U - W^A|_{\beta=0} > 0$ iff $n < 1/e$, and $W^{*U} - W^{*A}|_{\beta=0} > 0$ iff $n > 1-1/e$. In other words, the smaller the country size is, the more likely it is to gain by the global circulation of both currencies. The reason is simple: if the country size is small, it can obtain huge seigniorage from abroad provided that it succeeds in circulating its own currency, the difficulty of which is, of course, a different question.

If the country size is about the same, or $1/e < n < 1-1/e$, then both countries lose due to the change from equilibrium A to equilibrium U. The situation exhibits the one similar to the prisoner's dilemma, i.e., $W^U - W^A|_{\beta=0} < 0$ and $W^{*U} - W^{*A}|_{\beta=0} < 0$.

The sign of $W^U - W^F|_{\beta=0}$ is ambiguous but one can show that it is positive as long as u is sufficiently large. Since we know $W^{*F} - W^{*A}|_{\beta=0} > 0$ and $W^{*U} - W^{*F}|_{\beta=0} > 0$ iff $n < (1-1/e)/(2\bar{m} - \bar{m}^2)$, in the neighborhood of $\beta = 0$, we have

$$\begin{aligned} W^U &> W^A > W^F, \\ W^{*F} &> W^{*A} > W^{*U}, \end{aligned}$$

if $n < 1/e$,

$$\begin{aligned} W^A &> W^U > W^F, \\ W^{*F} &> W^{*A} > W^{*U}, \end{aligned}$$

if $1/e < n < 1-1/e$,

$$\begin{aligned} W^A &> W^U > W^F, \\ W^{*F} &> W^{*U} > W^{*A}, \end{aligned}$$

if $1-1/e < n < (1-1/e)/(2\bar{m} - \bar{m}^2)$ and

$$\begin{aligned} W^A &> W^U > W^F, \\ W^{*U} &> W^{*F} > W^{*A}, \end{aligned}$$

if $n > (1 - 1/e)/(2\bar{m} - \bar{m}^2)$.

If β is relatively large, we do not have such a clear relationship since we now have another effect, gains from trade.[14] In order to compare across equilibria, we assume that $\beta = \beta_A$, which is the greatest β for which equilibrium A exists. In this case, we have some numerical examples shown in Table 1.

5 Strategic selection of equilibrium and time inconsistency

In the previous sections, we confine our attention to the situations the governments believe a certain equilibrium to prevail and try to meet the constraint it faces to sustain the equilibrium. This section goes one step further, albeit not technically rigorous, and considers a situation in which the governments choose the tax rates, understanding the possibility that their choices affect the type of equilibrium to prevail. Unlike other sections, this section is illustrative rather than analytical.

We focus on the equilibrium selection between equilibria A and F. For this purpose, assume $n < 1/2$, and that $\beta \leq \beta_A$ holds under $\tau_h = \tau_f = 4k\alpha/u$. Assume further that $m = m^* = 1/2$ if equilibrium A prevails, and that $m^*_f = 1/2$ and $m = \bar{m}$ if equilibrium F prevails. We finally assume that equilibrium A initially prevails.[15]

We assume that once Home agents start accepting Foreign currency, this process continues until equilibrium F prevails with the money balances as assumed, and the governments care only about the final (stationary) outcome.

Let us fix $\tau_h = 4k\alpha/u$ for the moment and consider the incentive of Foreign government. In order to have equilibrium F, Foreign government lowers τ_f to make Foreign currency attractive to Home agents. This happens if $\beta > \beta_A$ occurs under $(\tau_h, \tau_f) = (4k\alpha/u, \tau_f)$. From (9), the threshold value of τ_f, denoted by $\bar{\tau}_f$, is given by

$$\bar{\tau}_f = 1 - \frac{1 - 4k\alpha/u}{2\theta}. \tag{35}$$

There are two questions that are of particular interest. The first is whether or not Foreign

[14] To be precise, this is not the standard gains from trade due to comparative advantages; rather, it is gains from trade opportunities.

[15] Note that which equilibrium prevails initially is crucial in the present analysis since the process from, say, equilibrium A to equilibrium F is irreversible.

government raises the tax rate from $\bar{\tau}_f$ after equilibrium F prevails, i.e., whether the time inconsistency problem arises or not. We can examine it by comparing $\bar{\tau}_f$ with τ_f^F as given in the previous section. Subtracting (28) from (35), we obtain

$$\bar{\tau}_f - \tau_f^F = 1 - \frac{1}{2\theta} + \left[\frac{1}{2\theta} - \frac{1}{\theta^*(1-\bar{m})+1}\right]\frac{4k\alpha}{u}.$$

By definition, $\theta = \theta^* = 0$ at $\beta = 0$, and $\theta = 1/2$ at $\beta = \beta_A$ under $\tau_h = \tau_f$. Therefore, we have $\bar{\tau}_f < \tau_f^F$ if θ is close to zero since $4k\alpha/u < 1$, and $\bar{\tau}_f > \tau_f^F$ if θ is close to a half. As the degree of "openness" facing Home country is higher, there is larger gains from accepting Foreign currency, and this offsets partially the negative effect due to a higher τ_f and thus, allows for a higher threshold $\bar{\tau}_f$. In other words, the time inconsistency problem is less likely to arise if the degree of "openness" is high. This also implies that, given other parameters, the larger Foreign country is, the more likely it is the case that by choosing the optimal inflation rate it can ensure the existence of its preferred equilibrium without facing the time inconsistency problem.

The second question is whether or not Home government has an incentive to prevent equilibrium F from prevailing by lowering its tax rate as well. To begin with, Home government has to set the rate as low as

$$\bar{\tau}_h = 1 - 2\theta(1 - \tau_f)$$

for this purpose. As one may see, it depends upon Foreign government's decision.

In order to analyze this situation, we need to specify a scenario or a game. We consider two suggestive, but not necessarily most plausible, scenarios.[16] The first scenario is as follows:

Step 1. Home government chooses τ_h. After observing it, Foreign government chooses τ_f.

Step 2-a. If $\beta \leq \beta_A$ holds under (τ_h, τ_f) determined in Step 1, then equilibrium A prevails under $(\tau_h^A, \tau_f^A) = (4k\alpha/u, 4k\alpha/u)$ and $m = m^* = 1/2$.

Step 2-b. If $\beta > \beta_A$ holds under (τ_h, τ_f) determined in Step 1, then equilibrium F prevails under (τ_h^F, τ_f^F), $m_f^* = 1/2$, and $m = \bar{m}$.

[16] We have chosen these scenarios not because they are most realistic, but because they are more tractable than some other (more realistic) scenarios. For example, one may wonder why sequential moves are introduced in Step 1. If we modify it to a simultaneous move game, then in the first scenario, we typically have multiple equilibria, and in the second, we sometimes have no pure strategy equilibrium.

In the first scenario, the question is reduced to whether $W^F > W^A$ or not since Foreign government always prefers equilibrium F to A, i.e., $W^{*F} > W^{*A}$, and therefore, the comparison that we made in the previous subsections directly applies. If $W^F > W^A$ holds, then Home government may intentionally raise the tax rate beyond τ_h^A in order to allow Foreign government to choose a sufficiently low tax rate to switch to equilibrium F.[17] From (9) we know that, given τ_h the threshold value of τ_f is $\bar{\tau}_f = 1 - (1 - \tau_h)/2\theta$. Note that $\bar{\tau}_f > 0$ iff $n < 2\beta/(2\beta + 1 - \tau_h)$. That is, given τ_h, the larger Foreign country is, the more likely equilibrium F is to prevail.

The second scenario is the one in which Step 2-a is replaced by the following:

Step 2-a'. If $\beta \leq \beta_A$ holds under (τ_h, τ_f) determined in Step 1, then equilibrium A prevails under (τ_h, τ_f) and $m = m^* = 1/2$.

In the second scenario, Home government may have to pay an extra cost to maintain equilibrium A. Since Foreign government has an incentive to lower its tax rate as low as zero if doing so leads to equilibrium F, Home government has to set $\bar{\tau}_h$ at $1 - 2\theta$ if it wishes to prevent the regime change. The higher the degree of "openness" is, the larger is the gains from accepting Foreign currency, and therefore, the higher is the cost for Home government to maintain equilibrium A.

Some numerical examples are shown in Table 2. In this table, cases (1)-(4) induce the same equilibrium, F, in both scenarios. Home government prefers equilibrium F to A. In cases (1) and (2), however, Foreign government cannot attain equilibrium F if Home government chooses τ_h^A. Therefore, Home government sets its rate sufficiently high so that Foreign government can induce equilibrium F by choosing a sufficiently low rate.

These scenarios sometimes induce different results. See cases (5)-(8). They exhibit $W^F < W^A$. Therefore, in the first scenario, Home government chooses a sufficiently low inflation rate to prevent the change. In the second scenario, however, Home government has to commit to a low tax rate to prevent equilibrium F, incurring an extra cost to keep the tax rate that would be non-optimal had there been no concern for equilibrium selection. Consequently, Home government

[17] To be precise, we need to consider the possibility that equilibrium U would prevail. We assume that, when Foreign government lowers the tax rate, it's more likely that equilibrium F, rather than equilibrium U, would prevail. The justification of this assumption is that under a sufficiently low τ_f, given that Home agents accept Foreign currency, and that other Foreign agents do not accept Home currency, no Foreign agent would have an incentive to deviate to accept Home currency.

	$\alpha = 1, n = .2, \bar{m} = .4$							
	$k = 10, u = 1000$				$k = 6, u = 100$			
case	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
θ	0.4	0.45	0.48	0.49	0.4	0.45	0.48	0.49
$\tau_f^A(\tau_h^A)$	0.04	0.04	0.04	0.04	0.24	0.24	0.24	0.24
$\bar{\tau}_h$	0.2	0.1	0.04	0.02	0.2	0.1	0.04	0.02
τ_h^F	0.0833	0.0833	0.0833	0.0833	0.3637	0.3315	0.3122	0.3057
τ_f^F	0.0398	0.0390	0.0373	0.0340	0.2391	0.2356	0.2275	0.2118
W^A	3.0966	3.0966	3.0966	3.0966	-0.6096	-0.6096	-0.6096	-0.6096
$\bar{W}^A$	3.0966	3.0966	3.0966	3.0579	-0.6127	-0.6680	-0.8013	-0.9232
W^F	3.3783	3.5227	3.6093	3.6381	-0.6703	-0.6566	-0.6493	-0.6470
Scenario 1	Eqm F	Eqm F	Eqm F	Eqm F	Eqm A	Eqm A	Eqm A	Eqm A
Scenario 2	Eqm F	Eqm F	Eqm F	Eqm F	Eqm $\bar{A}$	Eqm F	Eqm F	Eqm F

Table 2: Strategic equilibrium selection

may no longer wish to maintain equilibrium A. In case (5), it chooses $\bar{\tau}_h$ since $\bar{W}^A > W^F$ holds. However, in cases (6)-(8), since $\bar{W}^A < W^F$ holds, Home government does not choose $\bar{\tau}_h$ but some rate higher than that to allow Foreign government to implement equilibrium F.

6 Conclusion

The issues on currency competition have been discussed in many previous studies, yet there has been few work modelling it in an environment with endogenous determination of the realms of circulation of currencies and strategic interaction between money issuers. Studies that are silent on the issue of endogenous emergence of an international medium of exchange cannot answer the question as to how inflation taxes affect the realms of circulation of currencies and how this effect serves an inflation discipline.

We have developed a framework to study currency competition between governments and its effects on the circulation of currencies and welfare levels in a two-country, two-currency search theoretic model. This model provides us with some additional insights. For example, the negative impact of a country's inflationary policy on the realm of circulation of its currency imposes an inflation discipline on the government: the more open a country is, the stronger is the discipline. This result offers another account for the empirical evidence that the degree of "openness" is negatively correlated with inflation rates (Romer, 1993).

We also find that, at least under the current specification, the issuing country of an international currency has an incentive to choose a lower inflation rate than in autarky. The other country, since the tax base is reduced due to the use of foreign currency, chooses a higher inflation rate. However, there is a limit of the inflation rate beyond which it cannot sustain the circulation of its national currency. If the governments act strategically in selecting equilibrium, the larger country would try to lower the inflation rate to make its currency circulate internationally. The other country, knowing this, may lower the inflation rate to maintain its national currency as the sole medium of exchange to prevent the tax base from diminishing. This phenomenon would not have arisen without the strategic interaction between the governments.

Another implication is on the cost and benefit of having two international currencies. Our model suggests that if the two countries are of similar size, they both lose by shifting from autarky to the equilibrium with universally circulating currencies. This result is in contrast to those in the previous studies with two-country two-currency search-theoretic models, which argue that a unified currency regime is universally preferred. The difference lies in the fact that the current model takes into account a negative effect caused by competition on seigniorage collection.

One can study many other issues in the present framework, with some modifications and extensions. First, we may be able to address issues on trade as well as monetary issues in a unified framework. In the present model, the incentive to trade with foreigners is simply created by expanded trade opportunities. If we consider international trade based on the comparative advantage, some results may still be carried over. For example, if the gains from trade are not too large, then trade liberalization policy may decrease the welfare of the country that starts using foreign currency.

Next, the equilibrium with two local currencies entails no international trade, which is not the case in reality. Zhou (1997) introduces preference shocks to Matsuyama et al. (1993) to induce currency exchange between agents so that they engage in international trade, while both currencies remain local. Another possibility is to introduce a currency exchange market. One way is to endogenize the matching process so that they can go to the market whenever they wish to exchange their money. One can also consider profit-maximizing financial intermediaries or central banks to exchange currencies with other agents.

A more challenging modification is to relax the assumption on indivisibility of fiat currencies and restriction on inventory holding to discuss issues on inflation and exchange rates.[18] To do so, one must overcome the problem of tracking inventory, which is a difficult task even in models with a single currency (see, for example, Green and Zhou 1998 and Kamiya and Shimizu 2004).[19]

Finally, introducing more than two currencies in the present model may help us to address issues on currency zoning. For example, some countries such as Turkey that has been using dollar face a new alternative of Euro, and it is not clear which currency they end up using. It is interesting to know whether or not the circulation of two international currencies increases welfare, and its implications on the policies of the governments whose currencies circulate only locally and of the governments issuing international currencies.

[18]The autarkic equilibrium in this model may disappear if both goods and money are divisible, and if the marginal cost of production at zero output level is zero. We thank Shouyong Shi for pointing out this possibility.

[19]Head and Shi (2003) gets around this problem by using a continuum-of-household-members model with two currencies and two countries to study the effects of inflation on exchange rates. However, it does not consider various currency regimes, interaction between governments, and relationship between regimes and governments' behavior.

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