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Abstract

Human tasks are often multidimensional. Holmstrom and Milgrom (1991) concluded that "high-powered" incentives cannot work unless all dimensions of these tasks are observable in the firm. However, as this study shows, if the firm can observe the price vector of its products in the market, distinguish each dimension of the price vector, and connect the information with signals from workers in the firm, then the use of multitask "high-powered" incentives becomes feasible. Product differentiation with committed quality satisfies those conditions, which has been practiced by Japanese, but not by Western, manufacturing for a century.

Key words: multitask incentives, high-powered incentives, hedonic prices, contract theory, Japanese manufacturing.
JEL: L22, D23, N65.

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I. Introduction: hedonic prices and effort vectors

A. "Low-powered" incentives in Western manufacturing and theory

Stock prices are often thought to be good signals of executives' performance, given that profits are their "products." Then, the price of products could similarly be used as a signal to evaluate the performance of workers. This paper analyzes conditions under which "high-powered" incentives,[1] connected with information about the product price, could be efficient.

Most tasks are multidimensional especially where multiple dimensions of product quality are critical. In that case it could be beneficial to control relevant dimensions of workers' task to keep the optimal quality vector.

However, a common style of business in American manufacturing has been the mass production of goods by unskilled workers. At this extreme, the benefit from multidimensional incentives is relatively small because the premium derived from better quality is small. On the other hand, Europe has a tradition of manufacturing luxury goods by skilled workers. In this case, a multidimensional incentive becomes less manageable because it is harder to evaluate the artistically high quality in separate and standardized terms. At both extremes, multidimensional incentives for workers do not seem to work well. Production of standardized or luxury goods with "low-powered" incentives for workers is an equilibrium in Western manufacturing.

This fits the argument forwarded by Ronald Coase and elaborated by Oliver Williamson. Coase thought a firm replaces the price mechanism with a planned coordination of inputs to avoid the cost of pricing.[2] Here a firm is supposed to be intrinsically reluctant to price inputs, or equivalently, provide incentives. Williamson presented this idea using a clear concept: "low-powered" incentives are generally used in the firm while "high-powered" incentives, which explicitly price inputs, exist in the market.[3] They believed the main role of modern firms lies in minimizing transaction costs rather than providing incentives with workers, and this views coincided with practices in the Western manufacturing in the last century.

B. Linking "hedonic prices" with incentives

However, as mentioned by Alchian and Demsetz (1972) and Rosen (1988), the centralized pricing of tasks through a better-organized flow of information within the firm could be more efficient than decentralized pricing in the market, if it is too costly to observe the market price of each task because of jointed production, although the signals related to the performance of each task are observable in the firm. This in-house pricing that composes the internal labor market can be used for career concerns as long-term evaluation ,combined with "low-powered" incentives for short-term evaluation, as shown in Williamson, Wachter and Harris (1975), and formalized as the "career concerns" model by Holmstrom (1999) and Dewatripont, Jewitt and

[1] "High powered" incentives reflect a payment explicitly based on observed performance and "low-powered" on a payment based on the opportunity cost of labor. Hence a "low-powered" incentive can be interpreted as a compensation scheme that only satisfies the participation constraint. Williamson (1985), pp. 131-162.

[2] Coase (1937), p .391.

[3] Williamson (1985), pp. 131-162.

Tirole (1999). But in theory the centralized pricing of tasks could also work for short-term evaluation, *i.e.*, provide "high-powered" incentives. If a firm can infer implicit relative prices of each dimension of tasks evaluated in the market, then the firm can adjust its "high-powered" incentive as an in-house pricing to maximize its profit. Observed multidimensional prices called "hedonic prices" could thus be useful instruments to give "high-powered" incentives.

As Rosen (1974) pointed out, "hedonic prices" of differentiated products are more observable than those of generic goods. But at the same time, the mystical value of luxuries might be difficult to decompose into several dimensions of a hedonic price. In contrast, hedonic prices of differentiated goods consisting of several decomposable factors of quality in the middle-range market seem easier to identify. Then it is feasible for firms in the middle range market to connect the information about the hedonic prices of their products with the multidimensional tasks required in production to provide explicit multitask "high-powered" with workers and to optimally control their multidimensional efforts. Hence the production for middle range market with the "high-powered" incentives for workers is another possible equilibrium, with other equilibria of "low-powered incentives" with standardized or luxury goods seen in the Western manufacturing.

Japanese manufacturing linked hedonic prices to incentives and moved to this equilibrium from the beginning of its modernization since the middle 1880s, and the earliest and the most important example was the modernization of silk reeling from the middle 1880s to the 1900s.[4] The industry was one of the most important driving force of the Japanese economy due to its huge exports to the US before World War II, multidimensional wage schemes developed by the early 1900s. Interestingly this was the time when firms tried to differentiate their products by establishing their own brands in the New York market, which was the largest in the world. While the quality of raw silk consisted of various aspects, Japanese manufacturers focused on a few critical and well-observed dimensions of quality, committed to them under their brand names, and provided workers with incentives to focus on these dimensions of quality. Japanese raw silk acquired a vast share in the US market, although the high-end of the market was continuously held by Italian raw silk.

C. Multiple signals of multiple tasks

This study is motivated by Holmstrom and Milgrom (1991). An insight of them was in their focus on the optimization of the "direction" of vector-valued effort rather than just its "scalar."[5] Most work done by human beings are multidimensional while work done by engines are measured by a single scalar, such as horse-power or torque. For instance, individual workers often have to perform along at least two dimensions: increasing productivity and retaining quality. As easily imagined, it is more difficult to make individuals work both fast and carefully, rather than

[4] The silk reeling industry is an industry that produces raw silk threads from cocoons. Raw silk is used as a material for luxury clothes. For descriptive research on the silk reeling industry of Japan, see Nakabayashi (2005, forthcoming).

[5] Hence, a "direction" mentions exactly the "direction" of a vector in $\mathbb{R}^m$ where $m \geq 2$, rather than just $+$ or $-$ in $\mathbb{R}$.

just work fast or just carefully. Indeed the result of Holmstrom and Milgrom (1991) emphasizes it is difficult for multidimensional “high-powered” incentives to work in a firm.

However, as implied by Holmstrom and Milgrom (1991), it could become easier to use if some signals of each dimension of effort vector become observable in the firm. One important signal that the firm could observe is the price vector of its products; the hedonic price. As shown later, implementing “high-powered” incentives become feasible if the firm can observe the hedonic price of its products. Intuitively, *The more signals, the better it is for the firm to provide incentives*, and this intuition is easily evoked by the “sufficient statistic” result in Holmstrom (1979). To be utilized for multidimensional incentives, however, multiple signals have to satisfy another condition: each dimension of each signal must be distinguishable to the principal. While those conditions will be analyzed in detail in the next section, predictions from the analysis are summarized as follows:

> *If a firm knows the multidimensional price vector in the product market, distinguishes each dimension of the price vector, and establishes a production organization where the information is preserved and utilized to control production, then the firm can optimize the effort vectors of its workers by introduction of multidimensional incentives.*

Or, we can summarize main features of the conditions for multitask incentives to work on the models in Holmstrom and Milgrom (1991) and in this study as follows:

Multitask incentives based on one signal (Holmstrom and Milgrom (1991))
: *Each dimension of the vector-valued signal must be observable, but each dimension of the error vector of the vector-valued signal can be dependent to each other.*

Multitask incentives based on more than one signals (this study)
: *Some dimensions of some vector-valued signals can be unobservable, but each dimension of the error vector of all vector-valued signals must be independent to each other*.

Hence, this study inquires conditions under which the constraints on multitask “high-powered” incentives are relaxed in a sense, but are strengthened in another sense. If a firm can observe the price vector of its products, the dependance on the signal in the firm could be reduced. However, for the price vector to be useful information to give incentives, it is necessary each dimension of the price vector can be distinguished. This condition seems to be satisfied in the middle range market of the products, but neither in the low-end nor in the high-end market.

After Holmstrom and Milgrom (1991) presented their theoretical prediction about the difficulty of applying multitask incentives within a firm, the conditions under which their use becomes feasible have never been considered.[6] This paper tries to close this gap in the literature.

[6] An empirical study by Margaret Slade, which applies the Holmstrom and Milgrom (1991) model, showed how their model exactly fits transactions in the market, not within a firm, of the Western world (Slade (1996)). In the Holmstrom and Milgrom (1991) model, the risk aversion of the agent plays a critical role, and so another theoretical prediction by them was that multitask incentives could be optimal for executives who bear more risk than employees. Applications of their model on compensations for executives were presented by Feltham and Xie (1994) and Preyra and Pink (2001).

In the section **II**, some theoretical predictions are deduced from the model under hedonic prices. While Holmstrom and Milgrom (1991) analyzed the function of incentives within an organization, this paper is also interested in the stream of information that flows from the market into an organization. Inquired first will be the condition where more than two signals can be utilized to provide incentives, then considers a case where two signals – observed performance in the firm and observed price vector of the products in the market – are utilized.

Section **III** estimates the compensation scheme in a silk reeling factory, and depicts its changes from the 1890s to 1910s. It will be shown that a 4-dimensional wage scheme was established in the early 1900s when the firm established its own brand name to differentiate its products.

Section **IV** evaluates how each dimension of the workers' effort was optimized.

Section **V** sums up the results and discusses further related topics.

II. Theoretical prediction

A. The standard model of multitask incentives

The model from Holmstrom and Milgrom (1991) is first outlined here. Consider a multidimensional task. An agent has a particular amount of attention as her/his endowment, and allots her/his attention to each dimension, which composes an effort vector $\boldsymbol{t}^{\mathrm{T}} = (t_1, t_2, ..., t_m)$.

Suppose that there are two dimensions t_1 and t_2 in the task, and that the principal observes signals x_1 and x_2 of t_1 and t_2, and relates these signals to the wage. If the two dimensions are substitutes and one of them is unobservable, then the agent tries to get a higher wage by increasing her/his effort for the observable one and decreasing effort for the unobservable one. In this case, it is necessary to use "low-powered" instead of "high-powered" incentives in order to have workers pay attention to both of t_1 and t_2. We can summarize the discussion on "low-powered" incentives in Holmstrom and Milgrom (1991) as follows: Suppose that inputs to each dimension of the task are substitutes. Then, unless every dimension of the task is sufficiently observable, multitask "high-powered" incentives are generally difficult to apply, and "low-powered" incentives are more efficient.[7]

B. Hedonic prices in the market and multitasking in the firm

Since Court (1939), the quality of products has been thought to be intrinsically multidimensional.[8] Suppose that consumers in the market have a multidimensional utility function that is concave with respect to every term of the product quality, and they assign an l-list of quality magnitude $(q_1, q_2..., q_l)$ to amount Q of a product they purchase; ***i.e.***, suppose that the market has a hedonic price function $p(\boldsymbol{q}')$. Hence, the hedonic price $p(\boldsymbol{q}')$ is a mapping from a vector $\boldsymbol{q}'$, whose coordinates specify the quality magnitude of the product to a specific amount of

[7] Holmstrom and Milgrom (1991), pp. 34-35.

[8] Also see Court (1941a, 1941b), Griliches (1961), Lancaster (1966), Baumol (1967), Rosen (1974), and Epple (1987).

money p. By marginally changing the relevant quality q_i, where $i = 1, ..., l$, the firm observes marginal changes of price, thus can approximate the shape of hedonic price function $p(\boldsymbol{q'})$ in the neighborhood of the amount Q of the product.

Then, the firm can infer the $(l + 1)$-dimensional price vector of the product $\boldsymbol{p}(\boldsymbol{q})^{\mathrm{T}} = \boldsymbol{p}(p_1(q_1), ..., p_l(q_l), p_{l+1}(q_{l+1}))$, where q_{l+1} specifies the quantity, *i.e.*, $q_{l+1} = Q$.

C. The model of multitask incentives with multiple signals

Now consider a case where a risk-neutral principal utilizes n signals of a risk-averse agent's effort vector $\boldsymbol{t}$. Put $m = l + 1$, which is the number of dimension of the price vector of product, and suppose,

$\boldsymbol{t} \gg 0$: k-dimensional effort vector generated by the agent.

$C(\boldsymbol{t})$: private cost function of the agent, which is strictly convex.

$B(\boldsymbol{t})$: gross benefit to the principal, realized by the agent's effort.

$\boldsymbol{\mu}_j(\boldsymbol{t}) : \mathbb{R}^k \to \mathbb{R}^m_+$ where $j = 1, ..., n$: m-dimensional outcome realized by the k-dimensional effort vector $\boldsymbol{t}$.

$\boldsymbol{x}_j(\boldsymbol{t}) = \boldsymbol{\mu}_j(\boldsymbol{t}) + \boldsymbol{\epsilon}_j$: m-dimensional signal vector of m-dimensional outcome vector $\boldsymbol{\mu}_j(\boldsymbol{t})$, observed by the principal. $\boldsymbol{\epsilon}_j$ stands for the measurement error vector of the outcome vector $\boldsymbol{\mu}_j$. $\boldsymbol{\epsilon}_1, ..., \boldsymbol{\epsilon}_n$ are independently distributed with $\boldsymbol{\epsilon}_j \sim N(0, \boldsymbol{\Sigma}_j)$.

$u\left(w - C(\boldsymbol{t})\right) = -e^{-r[w - C(\boldsymbol{t})]}$: Constant Absolute Risk-Averse utility function of the agent.[9]

$\boldsymbol{\alpha}$: m-dimensional incentive vector.

β: transfer of total surplus for allocation.

Suppose $k = m$, and the i-th coordinate of $\boldsymbol{\mu}_j$, μ_{ji}, is a one-to-one mapping of the i-th coordinate t_i of $\boldsymbol{t}$. Then the principal can use the signal $\boldsymbol{x}_j$, $j = 1, ..., n$, to construct the compensation scheme , which extends the model of Holmstrom and Milgrom (1991) for multiple signals, such that

$$\text{(1)} \quad w = \boldsymbol{\alpha}^{\mathrm{T}} \left[\boldsymbol{\Gamma}_1 \boldsymbol{x}_1(\boldsymbol{t}) + \boldsymbol{\Gamma}_2 \boldsymbol{x}_2(\boldsymbol{t}) +, ..., + \boldsymbol{\Gamma}_n \boldsymbol{x}_n(\boldsymbol{t})\right] + \beta = \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \left[\boldsymbol{\mu}_h(\boldsymbol{t}) + \boldsymbol{\epsilon}_h\right]\right] + \beta,$$

where $\boldsymbol{\Gamma}_j$ is a $k \times k$ matrix. Then,

$$u\left(\mathrm{CE}\right) = \mathrm{E}\left[u\left(w - C(\boldsymbol{t})\right)\right] = -e^{-r\left[\boldsymbol{\alpha}^{\mathrm{T}} \sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\mu}_h(\boldsymbol{t}) + \beta - C(\boldsymbol{t}) - \frac{r}{2} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h\right] \boldsymbol{\alpha}\right]},$$

so that the agent's Certainty Equivalent is

[9]Holmstrom and Milgrom (1987), p. 307.

$$\mathrm{CE} = \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\mu}_h(\boldsymbol{t}) \right] + \beta - C(\boldsymbol{t}) - \frac{r}{2} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h \right] \boldsymbol{\alpha},$$

and the Total Surplus is

$$\mathrm{TS} = B(\boldsymbol{t}) - C(\boldsymbol{t}) - \frac{r}{2} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h \right] \boldsymbol{\alpha}.$$

Then the contract $(\boldsymbol{t}, \boldsymbol{\alpha}, \beta, \boldsymbol{\Gamma}_j)$, $j = 1, ..., n$, must solve

$$\max_{\boldsymbol{\Gamma_j},\ \boldsymbol{\alpha}} B(\boldsymbol{t}) - C(\boldsymbol{t}) - \frac{r}{2} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h \right] \boldsymbol{\alpha}, \quad j = 1, ..., n, \tag{2}$$

$$\textit{subject to } \boldsymbol{t} = \operatorname*{\mathbf{argmax}}_{\boldsymbol{t'}} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\mu}_h(\boldsymbol{t'}) \right] + \beta - C(\boldsymbol{t'}). \qquad \text{(IC)} \tag{3}$$

The principal must choose the optimal weight of signals $\boldsymbol{\Gamma}_j^*$ such that the variance of wage $\boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h \right] \boldsymbol{\alpha}$ is minimized. Here we have to note there is a condition to be satisfied by each covariance matrix $\boldsymbol{\Sigma}_j$, for the uniqueness of this problem's solution.

Proposition 1.
For $j = 1, ..., n$, suppose that each dimension of the measurement error $\boldsymbol{\epsilon}_j$ of the signal $\boldsymbol{x}_j$ is nonzero so that the variance of each dimension of $\boldsymbol{\epsilon}_j$ is positive. Then, the optimal contract which solves (2) is unique only if all dimensions of $\boldsymbol{\epsilon}_j$ are independent of each other, so that the covariance matrix $\boldsymbol{\Sigma}_j$ of $\boldsymbol{\epsilon}_j$ is diagonal.

Proof. See the Appendix.

Proposition 1 says each dimension of the signals must be clearly distinguishable from others by the principal. If some dimensions of $\boldsymbol{\epsilon}_j$ are not independent, then the signal $\boldsymbol{x}_j$ cannot be utilized as a signal of the same dimensions of the effort vector $\boldsymbol{t}$ with other signals $\boldsymbol{x}_1, ..., \boldsymbol{x}_{j-1}, \boldsymbol{x}_{j+1}, ..., \boldsymbol{x}_n$ in the compensation scheme (1). If the principal nevertheless wants to use such $\boldsymbol{x}_j$, he should use it as a signal of another aspect of the agent's effort.

Therefore suppose for the reminder of this paper that each dimension of the measurement error $\boldsymbol{\epsilon}_j$ is independent to each other so that $\boldsymbol{\Sigma}_j$ is diagonal, hence suppose that optimal $\boldsymbol{\Gamma}_j$ is a diagonal matrix whose diagonal entries are $\gamma_{j\ ii}$, for $j = 1, ..., n$ and $i = 1, ...k$. Also suppose that $0 < \gamma_{j\ ii}$, $\sum_{h=1}^{n} \gamma_{h\ ii} = 1$, for $j = 1, ..., n$, $i = 1, ..., k$, and normalize outcomes such that $\boldsymbol{\mu}_1(\boldsymbol{t}) = \boldsymbol{\mu}_2(\boldsymbol{t}) =, ..., = \boldsymbol{\mu}_n(\boldsymbol{t}) = \boldsymbol{t}$. Then, since $\sum_{h=1}^{n} \boldsymbol{\Gamma}_h = \boldsymbol{I}$, (2) and (3) are reduced to

$$\max_{\boldsymbol{\Gamma_j},\ \boldsymbol{\alpha}} B(\boldsymbol{t}) - C(\boldsymbol{t}) - \frac{r}{2} \boldsymbol{\alpha}^{\mathrm{T}} \left[\sum_{h=1}^{n} \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_h \boldsymbol{\Gamma}_h \right] \boldsymbol{\alpha}, \quad j = 1, ..., n, \tag{4}$$

$$(5) \qquad subject\ to\ \boldsymbol{t} = \underset{\boldsymbol{t'}}{\mathbf{argmax}}\ \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{t'} + \beta - C(\boldsymbol{t'}).$$

Denoting C_{ij} as the Hessian of $C(\boldsymbol{t})$ with respect to $\boldsymbol{t}$, the first-order condition of (5) gives

$$(6) \qquad \boldsymbol{\alpha} = \left[\frac{\partial C(\boldsymbol{t})}{\partial \boldsymbol{t}}\right]^{\mathrm{T}}, \quad \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{t}} = [C_{ij}]\,, \quad \frac{\partial \boldsymbol{t}}{\partial \boldsymbol{\alpha}} = [C_{ij}]^{-1}\,.$$

The first order condition of (4) with respect to $\boldsymbol{\Gamma}_j$ is

$$(7) \qquad \boldsymbol{\Gamma}_j = \boldsymbol{\Sigma}_j^{-1}\left[\sum_{g=1}^{n}\boldsymbol{\Sigma}_g^{-1}\right]^{-1},$$

which minimizes the risk to the risk-averse agent, *i.e.*, the variance of wage. [10]

The first-order condition of (3) with respect to $\boldsymbol{\alpha}$ is

$$(8) \qquad \boldsymbol{\alpha} = \left[\frac{\partial B(\boldsymbol{t})}{\partial \boldsymbol{t}}\left[\boldsymbol{I} + r\left[\sum_{h=1}^{n}\boldsymbol{\Gamma}_h\boldsymbol{\Sigma}_h\boldsymbol{\Gamma}_h\right][C_{ij}]\right]^{-1}\right]^{\mathrm{T}}.$$

Combining (7) with (8) gives

$$(9) \qquad \boldsymbol{\alpha} = \left[\frac{\partial B(\boldsymbol{t})}{\partial \boldsymbol{t}}\left[\boldsymbol{I} + r\left[\sum_{h=1}^{n}\left[\boldsymbol{\Sigma}_h^{-1}\left[\sum_{g=1}^{n}\boldsymbol{\Sigma}_g^{-1}\right]^{-2}\right]\right][C_{ij}]\right]^{-1}\right]^{\mathrm{T}}$$

The following is a description of this procedure by the principal: The principal

1. observes the signals $\boldsymbol{x}_j(\boldsymbol{t})$, for $j = 1, 2, ..., n$,
2. assigns the information weight matrix $\boldsymbol{\Gamma}_j$ to $\boldsymbol{x}_j(\boldsymbol{t})$, such that the variance of the compensation, which is the risk to the agent, is minimized,

[10] Put $\sigma_{j,ii}^2, j = 1, ..., n, i = 1, ..., k$, as the diagonal entry on the i-th row of the covariance matrix $\boldsymbol{\Sigma}_j$ of the error vector $\boldsymbol{\epsilon}_j$ of signal $\boldsymbol{x}_j$. Then, the row vector $(\gamma_{1,ii}, ..., \gamma_{j,ii}, ..., \gamma_{n,ii})$ consisting of diagonal entries on i-th rows of $\boldsymbol{\Gamma}_j, j = 1, ..., n$, is given as the "minimum variance portfolio"

$$(\gamma_{1,ii}^*, ..., \gamma_{n,ii}^*) = \begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\sigma_{1,ii}^2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_{n,ii}^2\end{pmatrix}^{-1} \Bigg/ \begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\sigma_{1,ii}^2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_{n,ii}^2\end{pmatrix}^{-1}\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}.$$

Then, $\boldsymbol{\Gamma}_j^*$ whose diagonal entries are $\gamma_{j,11}^*, ..., \gamma_{j,kk}^*$ above is equal to (7).

3. and then chooses an incentive vector $\boldsymbol{\alpha}$ such that the total surplus should be maximized.

Now suppose $n = 2$, that is, suppose that the firm uses two signals of the effort vector $\boldsymbol{t}$ of the agent; the quality and quantity of the product generated in the firm $\boldsymbol{\mu}_1(\boldsymbol{t}) \equiv \boldsymbol{\mu}(\boldsymbol{t})$, and the price vector of the product in the market $\boldsymbol{\mu}_2(\boldsymbol{t}) \equiv \boldsymbol{p}(\boldsymbol{t})$. Let $\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{\mu}(\boldsymbol{t}) + \boldsymbol{\epsilon}$ be the performance of the worker observed with some noise in the firm , and let $\tilde{\boldsymbol{p}}(\boldsymbol{t}) = \boldsymbol{p}(\boldsymbol{t}) + \boldsymbol{\varepsilon}$ be the observed price vector with some noise in the market. The compensation scheme is the given by $w = \boldsymbol{\alpha}^{\mathrm{T}}\left[\boldsymbol{\Gamma}_{\boldsymbol{\mu}}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{\Gamma}_{\boldsymbol{p}}\tilde{\boldsymbol{p}}(\boldsymbol{t})\right] + \beta$. Normalize $\boldsymbol{\mu}(\boldsymbol{t})$ and $\boldsymbol{p}(\boldsymbol{t})$ such that $\boldsymbol{\mu}(\boldsymbol{t}) = \boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{t}$. In addition, consider the case where $k = 2$, such that

$$\boldsymbol{t} = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}, \ \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \ \boldsymbol{\Sigma}_{\boldsymbol{\mu}} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}, \ \boldsymbol{\Sigma}_{\boldsymbol{p}} = \begin{pmatrix} \varsigma_1^2 & 0 \\ 0 & \varsigma_2^2 \end{pmatrix},$$

where t_1 and t_2 respectively stand for the quality and the quantity of the product.

Then, from (8), we have the optimal $\boldsymbol{\alpha}$ such that,

(10)

$$\boldsymbol{\alpha} = \begin{pmatrix} (\sigma_1^2+\varsigma_1^2)\left[(\sigma_2^2+\varsigma_2^2+r\sigma_2^2\varsigma_2^2C_{22})\,B_1 - r\sigma_2^2\varsigma_2^2B_2C_{12}\right] \\ (\sigma_2^2+\varsigma_2^2)\left[(\sigma_1^2+\varsigma_1^2+r\sigma_1^2\varsigma_1^2C_{11})\,B_2 - r\sigma_1^2\varsigma_1^2B_1C_{12}\right] \end{pmatrix} \times$$
$$\left[(\sigma_1^2+\varsigma_1^2)(\sigma_2^2+\varsigma_2^2)\left(1+\frac{r\sigma_1^2\varsigma_1^2C_{11}}{\sigma_1^2+\varsigma_1^2}+\frac{r\sigma_2^2\varsigma_2^2C_{22}}{\sigma_2^2+\varsigma_2^2}\right)+r^2\sigma_1^2\varsigma_1^2\sigma_2^2\varsigma_2^2\left(C_{11}C_{22}-C_{12}^2\right)\right]^{-1}.$$

An interpretation of the result can be summarized as **Proposition 2** and **Proposition 3** below.

Proposition 2.
(a) *If any dimension of the performance vector $\boldsymbol{\mu}(\boldsymbol{t})$ of workers in the firm or the price vector $\boldsymbol{p}(\boldsymbol{t})$ of the product in the market is neither perfectly unobservable, nor perfectly observable (i.e., $0 < \sigma_i^2 < +\infty$ and $0 < \varsigma_i^2 < +\infty$, for $i = 1, 2$), then it is optimal to use information about both $\boldsymbol{\mu}(\boldsymbol{t})$ and $\boldsymbol{p}(\boldsymbol{t})$ for each dimension of the effort vector $\boldsymbol{t}$ in order to provide an incentive.*
(b) *For the i-th dimension of $\boldsymbol{t}$, if $\boldsymbol{\mu}(\boldsymbol{t})$ or $\boldsymbol{p}(\boldsymbol{t})$ becomes perfectly unobservable ($\sigma_i^2 \to +\infty$ or $\varsigma_i^2 \to +\infty$), then it is optimal to ignore the signal from the unobservable one and use information only about the observable one to provide an incentive. For the i-th dimension of $\boldsymbol{t}$, if $\boldsymbol{\mu}(\boldsymbol{t})$ or $\boldsymbol{p}(\boldsymbol{t})$ becomes perfectly observable ($\sigma_i^2 \to 0$ or $\varsigma_i^2 \to 0$), then it is optimal to use information only about the perfectly observable one.*

Proof. See Appendix.

Consider the conditional joint distribution for a given effort t_i, $f_x(x_i|t_i)f_{\tilde{p}}(\tilde{p}_i|x_i,t_i) = f(x_i,\tilde{p}_i|t_i)$. Suppose $0 < \sigma_i^2 < +\infty$, and let t_i^h and t_i^l denote high and low achievement of dimension i of $\boldsymbol{t}$, respectively. Then the likelihood ratio $f(x_i,\tilde{p}_i|t_i^h)/f(x_i,\tilde{p}_i|t_i^l)$ depends on $\tilde{p}_i$ if and only if $0 < \varsigma_i^2 < +\infty$. Hence **Proposition 2** shows that the "sufficient statistic" result of Holmstrom (1979) holds also for this mechanism.

Proposition 3.
Suppose $0 < \sigma_i^2 < +\infty$ and $0 < \varsigma_i^2 < +\infty$. If the i-th dimension of the performance vector $\boldsymbol{\mu}(\boldsymbol{t})$ in the firm or the price vector $\boldsymbol{p}(\boldsymbol{t})$ in the market becomes less observable $(\sigma_i^2$ or ς_i^2 increases$)$, then it is optimal to weaken the incentive for that dimension.

Proof. See Appendix.

As **Proposition 3** indicates, when the i-th dimension of $\boldsymbol{\mu}(\boldsymbol{t})$ becomes less observable, the firm can keep the incentive $\boldsymbol{\alpha}$, but must re-weight i-th dimension of $\boldsymbol{p}(\boldsymbol{t})$ by adjusting γ_{ii} accordingly, instead of giving up the whole incentive. This shows that the conditions for the multitask "high-powered" incentive to work can be relaxed if the firm uses information about $\boldsymbol{p}(\boldsymbol{t})$ as well as $\boldsymbol{\mu}(\boldsymbol{t})$.

As shown by Holmstrom and Milgrom (1991), as long as the firm uses only information within the organization, the conditions for the multitask "high-powered" incentives to work are strict. However, if each dimension of the price vector $\boldsymbol{p}(\boldsymbol{t})$ is observed and distinguishable from each other as **Proposition 1** requires, and if the firm can preserve the stream of information about the price vector, then a multidimensional incentive becomes easier to use, as **Proposition 2** and **Proposition 3** indicate.

Related to this point, another remark is also necessary.

Remark.
Since $\boldsymbol{p}(\boldsymbol{t})$ shows the aggregate performance of all workers of the firm while $\boldsymbol{x}(\boldsymbol{t})$ is individually generated, information about $\boldsymbol{p}(\boldsymbol{t})$ can be utilized to provide an incentive only for aggregate performance, not for individual ones.

Now that the benefit from the market-oriented multidimensional incentives looks rather straightforward, the next question is under what conditions the price vector can be observed and each dimension of it can be distinguishable by the firm. The best-case scenario is a monopoly, where the firm can easily observe the marginal change of each coordinate of $\boldsymbol{p}(\boldsymbol{t})$ by marginally increasing each coordinate of $\boldsymbol{q}(\boldsymbol{t})$, without any noise from the pricing of other sellers' products. The worst-case scenario is a firm that just sells a generic product at the same price as other sellers in the market do, where the firm is never able to observe a marginal price increase from their efforts to enhance a dimension of quality.

However, if information about quality of a product is not perfectly observed by buyers at the time when they buy it in the market, then some firms will try to establish their own brands that guarantee some quality in order to receive a quality premium from buyers.[11] Consider the establishment of a brand a little more carefully. In practice, there are two kinds of brands. One is a brand of luxury goods such as the European fashion brands. The quality of these products is hard to evaluate at the time of purchase, so consumers choose a product by relying on the established image of its brand. The other type is typically a brand of electric appliances or automobiles for the middle-range market. The quality of these products can be seen in catalogues or as samples, and firms whose catalogue specs and samples are credible find it

[11] Klein, Crawford and Alchian (1978), p. 306. Klein and Leffler (1981).

optimal to establish their brands to reflect this. The quality of Japanese electric appliances and cars can be seen in catalogues and as samples at shops, and their catalogues and samples are believed to almost exactly show the real quality of these products so that the reputation of their brands are kept.

The condition of **Proposition 1** might not be satisfied in the first type of brands; luxury brands. However, the second type of brands of differentiated products in the middle-range market could satisfy the condition of **Proposition 1**. Hence, if the second type of brand is established, then the price vector is observable to the supplier when consumers differentiate the multidimensional quality of the product and then buy it. Even if the market for such brands is competitive among suppliers that have established their brands so that suppliers are price takers, each supplier can observe the price vector of her/his products.

Then the firm may try to use the information about the price vector of its brand to control incentives within its production organizations, which indeed happened in Japanese manufacturing a century ago. Before studying the details, let's summarize the theoretical prediction.

Prediction 1.
If a firm can observe the price vector of its products by differentiating it through the establishment of a brand name, and if the firm can construct a production organization where the information is preserved and utilized to control production, then the firm can introduce multitask incentives and optimize the effort vector of workers (from **Proposition 1** *and* **Proposition 2***).*

Prediction 2.
If a firm can observe the price vector by establishing its brand so that it can use the information about the price vector as well as the outcome observed in the firm to provide incentives, then these incentives will be given for the aggregate performance of all workers as well as individual performance, since the price vector is a signal of aggregate performance (from **Remark***).*

Prediction 3.
Consider a product whose brand is established in the market. If the relative observability of a dimension of quality changes in the market, then the multitask incentive must change accordingly(from **Proposition 3***).*

III. The wage scheme in early 20th century Japan

A. Establishment of the brand

The modern silk reeling industry in Japan grew in the middle 1880s and increased exports to the U.S. dramatically. The Japanese share of the U.S. market had reached 50 percent by the end of the 1880s, 70 percent by 1910, and 80 percent in the 1920s, overwhelming both the Italian and the Chinese silk reeling industries. This was the first case where a competitive export industry led Japan's economic development, which has been repeated by various manufacturing industries since then.[12]

When raw silk was traded in the market, it was priced according to several factors of quality. However, it had been almost impossible for silk reeling manufacturers to acquire information about this price vector in the New York market until the early 1880s. Prior to the 1880s, the Western trading companies put their trademarks, or "private chops," on the raw silk after they inspected and re-packed it before exporting it to Europe and the U.S. Thus, information about the price and quality premium came to belong to the Western trading companies that owned the "private chops."

In the middle 1880s, however, leading manufacturers organized cooperatives for joint inspection and shipment, and they put their trademarks, or "original chops,"[13] on their products. The New York raw silk market was a spot market where raw silk was traded by sample. Thus the establishment of a brand meant that its samples were recognized as credible, and the producer of the brand was able to observe the price vector of her/his products. By establishing producers' brands, the quality premium and the information about price thus belonged to those manufacturers' cooperatives, not to Western trading companies. It meant, however, that the most important information about the price and effort vectors was still unknown to each member manufacturer of a cooperative. Only the cooperatives' headquarters, which conducted the joint inspection of products, recorded the performance of workers, and guaranteed the quality of the brands to the market, possessed the information necessary to evaluate the performance of workers.

Hence, major manufacturers withdrew from cooperatives, built large factories that included inspection processes from the late 1890s to the early 1900s, and established their own brands. This allowed respective silk reeling manufacturers to finally grasp the stream of information about the price vector in the market. Then multitask incentives were introduced by those manufacturers. Usually, *Labor productivity*, *Material productivity*,[14] *Evenness of threads*[15] and *Luster of threads* of raw silk were monitored to provide incentives.

B. Determination of the wage

Now let us inquire about the real process of optimizing the effort vector. One example here is that of the Kasahara Factory, in Suwa County, Nagano Prefecture, which was the center of the industry in Japan. In the silk reeling industry in Suwa County, all workers were young and female, all of them lived in dormitories of the firms they worked for, and all their living expenses were paid by the firms. They were not unionized in the relevant period, so that obstacles to the introduction of new management practices were small. Employment contracts were usually one-year and turnover rate in a factory between two consecutive years was generally high.[16]

[12] Nakabayashi (2003), pp. 1-59.

[13] Trademarks of trading companies were called "private chops" and those of silk reeling manufacturers were called "original chops." Duran (1913), pp. 105-106.

[14] *Material productivity* was the amount of a product over a unit of material (cocoon), which revealed the performance of economizing on the raw material: cocoons.

[15] *Evenness of threads* was the most important factor of quality in the U.S. market, where power looms for mass production prevailed.

[16] Thus the career concern was not relevant.

Also The technology of reeling raw silk did not require workers to literally cooperate. In general wages were determined by the ex post relative evaluation by which the effects of common exogenous shocks were excluded; hence the incentives of risk-averse workers were enhanced.[17] Workers received by lump-sum payments at the end of year, in addition to the living expenses.[18] The Kasahara Factory followed these common practices.

Until the early 1900s, the Kasahara Factory belonged to a cooperative for joint inspection and shipment under the cooperative's brand. Those days, only *Labor productivity* and *Material productivity* had been systematically recorded in the Kasahara Factory to determine wages, not dimensions related to the quality of the products. Raw silk that did not satisfy a specific level of quality was excluded from shipment, but the results of inspections were not used for determining wages. The Kasahara Factory stopped cooperative inspections and shipments in 1903, and began to inspect raw silk independently and ship it under its own brand in 1904.

Since 1904, in order to determine wages, $1)$ *Labor productivity*, $2)$ *Material productivity*, $3)$ *Evenness of threads*, and $4)$ *Luster of threads* were systematically recorded at the Kasahara Factory, where *Luster* and *Evenness* were critical dimensions of quality. These dimensions were recorded for all produced raw silk everyday during the inspection process before shipment. During the year under the contracts, workers' performances were recorded every day, relative performances were calculated every half month, and workers' efforts followed the overseers' guidance based on the recent two-week performance. [19] Then wages were paid at lump sum according to the relative performance of each worker at the end of the year. This practice spread throughout Suwa County in the 1900s, a case of which was the Kasahara Factory.

Table 1 shows the wage distribution that does not include the living expense paid by the firm. Large variance in the table indicate a feature of the "high-powered" incentive of this wage system.

C. Construction of the wage scheme

In the Kasahara Factory, the wage scheme had been two dimensional until the early 1900s, as shown in Table 2. [20]

[17]Holmstrom (1982a).

[18]Hunter (2003), pp. 144-189. Nakabayashi (2003), pp. 241-255.

[19]Therefore the practice satisfied the condition in which a linear compensation scheme can be assumed (Holmstrom and Milgrom (1987), pp. 316-322).

[20]The coefficients α_3 and α_4 of x_3 and x_4 are unstable in some years on Table 2. It is supposedly from two reasons. One is that t_3: *Evenness of threads* and t_4: *Luster of threads*, both of which were for cleaner threads, presumably interacted with each other. Indeed, the coefficient of the interaction term of a standardized regression through the origin with year dummies *Y19XX* is significant as follows:

$$\begin{aligned}
\underset{p \text{ value for } t}{w^{\textit{1904-1913}}} =& \underset{0.000}{1.387} x_1^{\textit{1904-1913}} + \underset{0.000}{2.506} x_2^{\textit{1904-1913}} + \underset{0.001}{0.018} x_3^{\textit{1904-1913}} + \underset{0.000}{0.041} x_4^{\textit{1904-1913}} + \underset{0.000}{0.025} (x_3 x_4)^{\textit{1904-1913}} \\
& - \underset{0.000}{1.085} \textit{Y1904} - \underset{0.000}{1.116} \textit{Y1905} - \underset{0.000}{1.026} \textit{Y1906} - \underset{0.000}{1.301} \textit{Y1907} - \underset{0.000}{0.708} \textit{Y1908} \\
& - \underset{0.000}{0.798} \textit{Y1909} - \underset{0.000}{0.809} \textit{Y1910} - \underset{0.000}{0.809} \textit{Y1911} - \underset{0.000}{0.460} \textit{Y1912} - \underset{0.000}{0.780} \textit{Y1913}
\end{aligned}$$

p value for F : 0.000. The number of samples: $2,235$.

Scheme 1897
w: *wage*. x_1: *Labor productivity*. x_2: *Material productivity*.

(11) $$w = \alpha_1 x_1 + \alpha_2 x_2 + \beta, \;\; \alpha_1, \alpha_2 > 0.$$

However, since 1904, the wage scheme has had four dimensions as follows:

Scheme 1904
w: *wage*. x_1: *Labor productivity*. x_2: *Material productivity*. x_3: *Evenness of threads*. x_4: *Luster of threads*.

(12) $$w = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \beta, \;\; \alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0.$$

The regressions of (12) are in Table 2. In 1904, the Kasahara Factory incorporated an inspection process into its own factory and began to sell its raw silk under its own brand. With this organizational change, the Kasahara Factory could acquire the signal $\boldsymbol{x}(\boldsymbol{t})$ of the effort vector $\boldsymbol{t}$ that included quality dimensions, recorded in the daily inspections. Furthermore, it was able to acquire information about the price vector $\tilde{\boldsymbol{p}}(\boldsymbol{t})$ of its own products, *i.e.*, another signal of $\boldsymbol{t}$. **Prediction 1** suggests at this point the Kasahara Factory could introduce an explicitly multidimensional "high-powered" incentive scheme, and it was exactly what happened.

The aggregate performance, given the price in the market, can be approximated by the return on sales, i.e., profits over sales. Indeed a regression of the real wage to the return on sales through 1904-1913 shows a significant relation.[21] This indicates wages, at least to some degree, depended on the overall performance of the firm. In other words, that portion of the wage was determined as a reward for the aggregate performance of the whole factory, a result **Prediction 2** suggests.

Since it was not necessary for workers to literally cooperate, a reward for aggregate performance was just for aggregated effort, not cooperative activity.[22] However, a contemporary

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The other reason was a discrepancy between *ex ante* incentive and *ex post* performance. If all of workers were on the path of incentive mechanism and performed exactly as guided, then their performance should have concentrated on the optimal point so that their performance should be the same, hence a regression of observed *ex post* differences of their performance to the wage should not have fit. For instance, the sign of α_4 in 1910 is opposite in Table 2, but it was not because workers did not follow the incentive for t_4, rather because they followed it very well as shown on Table 3-c (discussed in the next section).

The wage scheme in Table 1 estimates the final result of calculating a wage. The actual practice leading to the result was a little more complicated. See Nakabayashi (2003), pp. 256-268, and Nakabayashi (2005).

[21] A standardized regression is as follows:
RW: real wage. ROS: return on sales of the Kasahara Factory.

$$\underset{p\text{ value for }t}{RW}{}^{\textit{1904-1913}} = \underset{0.000}{1.067}x_1^{\textit{1904-1913}} - \underset{0.000}{0.108}x_2^{\textit{1904-1913}} + \underset{0.000}{0.052}x_3^{\textit{1904-1913}} + \underset{0.000}{0.024}x_4^{\textit{1904-1913}} + \underset{0.000}{0.025}ROS^{\textit{1904-1913}}.$$

p value for F : 0.000. The sample numbers:$2,235$.

[22] Hence it was different from the team production in Itoh (1992),Che and Yoo (2001).

observer in a local newspaper pointed out that rewards for aggregate performance were also useful to keep up workers' morale.[23]

Next, we will check whether the effort vector was really optimized during the process.

IV. Optimization of the effort vector

A. Substitute dimensions

In the silk reeling industry, *Material productivity* t_2, *Evenness of threads* t_3, and *Luster of threads* t_4, all of which need careful processing, were obviously substitutes for *Labor productivity* t_1. If the relative price of labor and material changes, and/or the relative "price" of each dimension of the quality changes, the effort vector that maximizes profit needs to change accordingly.

B. Controlling the direction of the effort vector

The firm's target was some optimal effort vector $\boldsymbol{t}^{*\mathrm{T}} = (t_1^*, t_2^*, t_3^*, t_4^*) = t_1^*(1, t_2^*/t_1^*, t_3^*/t_1^*, t_4^*/t_1^*)$. Neither *Material productivity* t_2 , *Evenness of threads* t_3, nor *Luster of threads* t_4 needed to be enhanced infinitely. Rather, given the relative price to clear the product market, the labor market, and the raw material market, optimal levels of the quality of product and the material productivity were decided such that profit was maximized.

Therefore, if workers' ability was sufficiently high, their attention had to be allotted to each of *Material productivity* t_2, *Evenness of threads* t_3, and *Luster of threads* t_4 such that the optimal levels of them were satisfied, and the rest amount of workers' attention, if they had, had to be allotted to *Labor productivity* t_1, at the optimum. However, if some workers' ability was not sufficient so that their attention was not enough to satisfy the optimal levels of the t_2, t_3, and t_4, the firm had to instruct them to enhance all of the t_1, t_2, t_3, and t_4.

Thus, according to the distribution of ability of workers, the firm was supposed to instruct less able workers to enhance all dimensions t_1, t_2, t_3, and t_4 with some weights, and instruct abler workers to fix dimensions t_2, t_3, and t_4 at the optimal level and to enhance t_1.

Hence, on the (t_1, t_i) plane $(i = 2, 3, 4)$, plots of performance (t_1, t_i) of workers were supposed to follow an increasing and bounded-above function $g(t_1)$ if the effort vector $\boldsymbol{t}$ was optimized. Then, the image of the signal $\boldsymbol{x}(\boldsymbol{t})$ on the (x_1, x_i) plane $(i = 2, 3, 4)$ was also supposed to converge an increasing and bounded-above function according to the optimization of the effort vector $\boldsymbol{t}$.

Motivated by a Gaussian kernel regression of *Labor productivity* x_1 and *Material productivity* x_2 (Figure 1), we take a first approximation of such a function with

(13) $$x_i = \eta_1/x_1 + \eta_2, \;\; 0 < x_1; \eta_1 < 0; i = 2, 3, 4.$$

[23] Haizanbo, "Kawagishimura no ichinich (5)" (One day in the Kawagishi Village (5)), *Shinano Mainichi Shimbun* (Shinano daily), Nagano, July 28, 1903.

C. The optimized effort vector and information from the market

The image of $\boldsymbol{x}(\boldsymbol{t})$ in the *Labor productivity* x_1- *Material productivity* x_2 plane had not been optimized at all in 1897 or 1901 (Table 3-*a*), even though *Material productivity* x_2 had been recorded (Table 2). The inspection process had been carried out by the headquarters of the cooperative before 1904, not the Kasahara Factory, so that even information about *Material productivity* had probably not been handed to Kasahara on a daily basis. Hence the Kasahara Factory could neither monitor nor instruct the effort vector $\boldsymbol{t}$ of workers on daily basis. Consequently, workers increased *Labor productivity* x_1 by decreasing *Material productivity* x_2 below the optimal level. It was optimized after 1904 (Table 3-*a*).

The process of optimization can also be seen in the *Labor productivity* x_1 - *Evenness of threads* x_3 plane since 1904 (Table 3-*b*).

For *Labor productivity* t_1, *Material productivity*t_2, and *Evenness of threads* t_3, the effort vector $\boldsymbol{t}$ has been well controlled since 1904, when the Kasahara Factory started independent inspection and made its original brand. On the other hand, the image of $\boldsymbol{t}$ on the *Labor productivity* t_1 - *Luster of threads* t_4 plane was optimized as late as 1908 (Table 3-*c*).

Interestingly, some changes occurred in the US market at the beginning of the 1908 season. The Silk Association of America, the industrial body consisting of silk manufacturers and merchants, suggested a method of classification for Japanese brands of raw silk, using a standard brand as a measure. Using this standardized measure allowed buyers to more easily differentiate the quality of raw silk sold by different brands.[24] At the same time it enabled manufacturers to better observe the price vector from the purchasing behavior of buyers in the New York market. Indeed, *Luster of threads* became more effective for the determination of wages in 1908-1913 than it had been in 1904-1907. [25]

As **Prediction 3** claims, faced with a changing market, the firm observed the *Luster* dimension $\tilde{p}_4$ of $\tilde{\boldsymbol{p}}(\boldsymbol{t})$ more clearly, and enhanced the incentive α_4 for dimension x_4 of the observed performance $\boldsymbol{x}(\boldsymbol{t})$, both of which were signals of t_4 of the effort vector $\boldsymbol{t}$. Then, the image of $\boldsymbol{x}(\boldsymbol{t})$ on the (x_1, x_4) plane converged to the optimal curve (Table 3-*c*, Figure 2), which reflects the optimization of $\boldsymbol{t}$ on the (t_1, t_4) plane.

The incentive scheme worked on the basis of information from the product market, as well as information in the firm, as **Prediction 1** mentions. Moral hazard by workers had been a serious

[24] "Classifications of raw silks," *The American Silk Journal*, New York, vol.27, no.7, 1908, p. 23.

[25] Standardized regressions through the origin with year dummies *Y19XX* are below.

$$\underset{p\text{ value for }t}{w^{\textit{1904-1907}}} = \underset{0.000}{1.334}x_1^{\textit{1904-1907}} + \underset{0.009}{1.115}x_2^{\textit{1904-1907}} + \underset{0.001}{0.078}x_3^{\textit{1904-1907}} + \underset{0.012}{0.040}x_4^{\textit{1904-1907}} + \underset{0.147}{0.021}(x_3x_4)^{\textit{1904-1907}}$$
$$- \underset{0.001}{0.677}\textit{Y1904} - \underset{0.001}{0.706}\textit{Y1905} - \underset{0.000}{0.721}\textit{Y1906} - \underset{0.000}{0.953}\textit{Y1907}.$$

p value for F : 0.000. The number of samples: 768.

$$\underset{p\text{ value for }t}{w^{\textit{1908-1913}}} = \underset{0.000}{1.371}x_1^{\textit{1908-1913}} + \underset{0.000}{5.139}x_2^{\textit{1908-1913}} + \underset{0.004}{0.018}x_3^{\textit{1908-1913}} + \underset{0.000}{0.021}x_4^{\textit{1908-1913}} + \underset{0.000}{0.028}(x_3x_4)^{\textit{1908-1913}}$$
$$- \underset{0.000}{2.162}\textit{Y1908} - \underset{0.000}{2.487}\textit{Y1909} - \underset{0.000}{2.481}\textit{Y1910} - \underset{0.000}{2.488}\textit{Y1911} - \underset{0.000}{1.322}\textit{Y1912} - \underset{0.000}{2.424}\textit{Y1913}.$$

p value for F : 0.000. The number of samples: $1,467$.
The coefficient of x_4 was more significant in 1908-1913.

problem until the early 1900s, but after 1904 this problem had been almost completely solved. After 1904, the Kasahara Factory established its own brand as an instrument to capture the stream of information about the price vector of its products, utilized it to control the incentives of workers, and explicitly optimized the multidimensional effort vector of workers.

V. Discussion: a viewpoint of comparative analysis

A. A tradition of multitask incentives

As predicted by the model, the establishment of a brand that enabled firms to observe the price vectors of their products made the introduction of "high-powered" multitask incentives for workers possible.

In the Japanese silk reeling industry, manufacturers established their own brands to acquire quality premiums that had belonged to trading companies. The establishment of brands accompanied the construction of an organization to inspect the quality. Within this organization, the multidimensional performance of workers had begun to be monitored and recorded. As a result of these organizational changes, silk reeling manufacturers acquired information about the price vector in the market and about performance of their workers in their factories on a daily basis. By taking advantage of this condition, they were able to optimize the effort vectors of workers by connecting information about the price vector with that of the effort vector. The information stream of the price vector from the product market was efficiently utilized for controlling incentives in the firm.

B. Segmented quality of Japanese cars

This parable of a historical experiment contains some implication to understand the contemporary difference between Japanese and Western manufacturing.

For two decades, Japanese cars have had a good reputation because of their quality, and, in 2006, *Consumer Reports*[26] finally ranked "Japanese models as its top choice in all 10 vehicle categories." However, what is this good quality? The annual car-buying guide of *Consumer Reports* "is based on tests of more than 200 models, covering performance, comfort, safety and fuel economy, among other factors."[27] Thus the kind of market reports usually evaluates cars along several categories, such as engine troubles within a few years after purchase, or driver satisfaction in handling, and so forth. Then those reports classify cars by summing up the points given to respective categories or terms. Therefore, they implicitly assume that consumers' evaluations, or utility functions, are additively separable across standardized categories. In other words, the fact that Japanese cars achieve high quality ratings means the additively separable dimension of their quality components are high. Japanese car manufacturers likely optimize the quality components of their cars subject to the multidimensional price that reflect the additively separable benefits their customers receive from the quality. On the other hand, the benefits

[26] *Consumer Reports New Car Buying Guide 2006*, Consumer Reports Books, June 12, 2006.
[27] *Financial Times*, March 2, 2006, p. 15.

of owning a *Corvette* or *Porsche* seem to be hard to separate and calculate in their separated components, and in such a high-end market, Japanese cars have been relatively unsuccessful. The Japanese auto industry is typically good at hedonic approaches to quality control. This is exactly a point directly related to **Prediction 1**. Japanese manufacturing has focused on the middle range of the market, where each dimension of the quality can be easily distinguished from the others, as **Prediction 1** requires.

Another particular feature of Japanese manufacturing is that multidimensional evaluations for wage and promotion schemes are imposed on blue-collar workers as well as white-collar workers to keep product quality high.[28] Combined with career concerns, these compensation schemes consist of multitask incentives.

Given those casual observations with the analysis of this research, the multitask "high-powered" incentives on shop floors seem to work better, if the quality control is conducted in well-defined multidimensional terms.

That story also induces us to return to the understanding of the "borders of firms" implied by Alchian and Demsetz (1972). A firm tries to exclude intermediary players and incorporate transactions if it can acquire a quality premium with smaller transaction costs. At the same time, the firm can increase total surplus if it can preserve the information stream from the product market to optimize the effort vectors of workers in the production organization. Hence, the borders of the firm can also be decided by the benefits from quality and incentive controls, as well as the transaction cost, as seen in Japanese manufacturing more than one century ago.

C. For a comparative analysis

It has been shown that a multitask "high-powered" incentives can be efficient under some conditions. If so, why are similar organizations rare in the West? In the US, Ford style of the "contractually fixed wage" became dominant for blue-collar workers in the auto industry in the 1920-30s, ironically when they discovered the concept of "hedonic prices" (Court (1939)). European auto industries had followed suit by the 1970s. A reason why the fixed wage was taken there were with some combination of the new technology for mass production and the new management affiliated with it. However, another point was that incentive wages had to face union conflicts, and a fixed wage has been a part of agreements between firms and unions in Western manufacturing industries until now. In the auto industry, for instance, while many Japanese practices were introduced in the 1990s, the industry's standard pay structure based on an agreement between the Big Three and the UAW remained, and the Japanese style of an individualized and "capability-based" wage system was rejected.[29]

The inertia of American industrial relations can be traced back to the very beginning of industrialization in the US.[30] Hence an important factor is probably the historical path of the industrialization. The modern textile industry was a newly implemented industry in 19th cen-

[28] Aoki (1988), pp. 49-98.

[29] Jürgens, Malsch and Dohse (2003), pp. 215-280. Wheeler (1993), pp. 71-75. Weinstein and Kochan (1995), pp. 12-15. Adler, Kochan, MacDuffie, Pil and Rubinstein (1997), pp. 68-69. Sloane and Witney (2004), pp. 280-282. For an overview on the industrial relations in Japan, see Nakamura and Nitta (1995).

[30] Kochan and Katz (2000), pp. 17-48.

tury Japan, and young female workers did not have a history of guilds. However, in the Western world, especially male workers had a tradition of guild-like unions and sometimes they were against firms that tried to control production.[31] This seems to be a reason why multitask incentives are not offered to blue-collar workers in the West but they are in Japan.

This difference could also be part of explanation of why Western manufacturing is generally competitive only in the production for the low-end or the high-end market, but not for the middle-range market. Production of generic or luxury goods does not require or suit multitask incentives. On the other hand, consumers of middle-range products can be recognized across standardized terms, hence multitask incentives work well for production of middle-range differentiated products.

Appendix: proofs of the propositions

Proposition 1.

Proof.

$$\frac{\partial \boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{\Gamma}_j \boldsymbol{\Sigma}_j \boldsymbol{\Gamma}_j \boldsymbol{\alpha}}{\partial \boldsymbol{\Gamma}_j} = \boldsymbol{\Sigma}_j^{\mathrm{T}} \boldsymbol{\Gamma}_j^{\mathrm{T}} \boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} + \boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{\Gamma}_j^{\mathrm{T}} \boldsymbol{\Sigma}_j^{\mathrm{T}}.$$

Since the rank of $\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}}$ is 1, solution $\boldsymbol{\Gamma}_j^*$ of $\boldsymbol{\Sigma}_j^{\mathrm{T}} \boldsymbol{\Gamma}_j^{\mathrm{T}} \boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} + \boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{\Gamma}_j^{\mathrm{T}} \boldsymbol{\Sigma}_j^{\mathrm{T}} = 0$ is unique only if each row of $\boldsymbol{\Gamma}_j$ contains only one nonzero entry. Such a $\boldsymbol{\Gamma}_j^*$ minimizes $\boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{\Gamma}_j \boldsymbol{\Sigma}_j \boldsymbol{\Gamma}_j \boldsymbol{\alpha}$ only if each row of $\boldsymbol{\Sigma}_j$ contains only one nonzero entry. Since each dimension of $\boldsymbol{\epsilon}_j$ is positive, it means $\boldsymbol{\Sigma}_j$ must be diagonal, *i.e.*, each dimension of $\boldsymbol{\epsilon}_j$ must be independent to each other. □

Proposition 2.

Proof. (*a*) By (7), if $0 < \sigma_i^2 < +\infty$, and if $0 < \varsigma_i^2 < +\infty$, then $0 < \gamma_{ii} < 1$, so that information both about the performance observed in the firm x_i and the price observed in the market $\tilde{p}_i$ is utilized to determine the wage w.
(*b*) For the observed performance in the firm $\boldsymbol{x}(\boldsymbol{t})$, as $\sigma_i^2 \to +\infty$, $\gamma_{ii} \to 0$, where information only about the price $\tilde{p}_i$ is utilized. As $\sigma_i^2 \to 0$, $\gamma_{ii} \to 1$, where information only about the performance in the firm x_i is utilized. For the price vector $\boldsymbol{p}(\boldsymbol{t})$, the proof is analogous. □

Proposition 3.

Proof. By (10),

$$\alpha_1 = \frac{\left[\left(\sigma_2^2 + \varsigma_2^2 + r\sigma_2^2\varsigma_2^2 C_{22}\right) B_1 - r\sigma_2^2\varsigma_2^2 B_2 C_{12}\right]}{\left(\sigma_2^2 + \varsigma_2^2\right)\left[1 + r\dfrac{\sigma_1^2\varsigma_1^2}{\sigma_1^2 + \varsigma_1^2} C_{11}\right] + r\sigma_2^2\varsigma_2^2 C_{22} + r^2\sigma_2^2\varsigma_2^2 \dfrac{\sigma_1^2\varsigma_1^2}{\sigma_1^2 + \varsigma_1^2}\left(C_{11}C_{22} - C_{12}^2\right)}$$

Since $C(\boldsymbol{t})$ is strictly convex $(C_{11}C_{22} - C_{12}^2 > 0)$ and $\sigma_1^2\varsigma_1^2/(\sigma_2^2 + \varsigma_1^2)$ is strictly increasing in σ_1^2 and in ς_1^2, α_1 decreases if σ_1^2 and/or ς_1^2 increases.

□

[31] Clark (1984).

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Historical document

"Kasaharagumi shiryo (Documents of Kasahara group)," held by Okaya Sanshi Hakubutsukan (The Silk Museum of Okaya), Okaya, Nagano Prefecture, Japan.

Table 1 Wage in the Kasahara Fusakichi Factory, Suwa County, Nagano Prefecture.

year	Number of sample	Distribution average	maximum	minimum	median	variance	kurtosis	skewness
		sen per workday	sen per workday	sen per workday	sen per workday			
1897	138	18.067	32.042	7.413	17.997	29.547	0.003	0.362
1901	163	20.898	44.098	0.638	21.999	61.410	0.377	-0.034
1904	191	22.521	53.983	6.667	23.104	74.095	0.389	0.367
1905	199	20.040	46.415	3.625	20.652	60.918	0.088	0.268
1906	150	23.196	49.549	1.735	23.994	92.764	-0.341	0.034
1907	228	23.191	52.140	0.736	22.852	92.683	0.499	0.254
1908	251	22.931	59.780	1.774	24.001	99.890	0.178	0.309
1909	351	22.682	58.235	1.264	22.999	112.989	-0.229	0.405
1910	351	26.083	60.229	1.364	25.000	130.243	-0.258	0.247
1911	343	23.450	66.998	0.000	22.998	146.768	-0.001	0.405
1912	88	23.779	47.996	2.478	25.494	130.575	-1.062	-0.023
1913	83	23.209	60.000	3.080	20.000	159.526	0.011	0.794

source : Fusakichi Kasahara, "Seishi keisan bo" (Book for evaluation of silk reeling). Kasaharagumishiryo (Documents of Kasahara goroup).

notes: "Wage" does not contain supplemental payment, which amounts to 5-10% of wage. 1 sen (0.01 yen) was approximately 0.5 cent of U.S. dollar.
Number of sample is small in 1906, 1912, and 1913, because some books have been lost for those years. However, there is not any bias in distribution of performance depending on each book, so that this loss does not affect the rsult of estimation.

Table 2 Wage and observed performance

w: Wage. $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$: incentive vector. $\boldsymbol{x} = (x_1, x_2, x_3, x_4)$: signal of effort vector $\boldsymbol{t}$ in the firm, where x_1: *Labor productivity*. x_2: *Material productivity*. x_3: *Evenness of threads*. x_4: *Luster of threads*. β: transfer of surplus.

$w = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \beta.$

year	α_1	α_2	α_3	α_4	β	standard error	R^2	p value for F
1897	0.777	0.209	–	–	0.000	0.031	0.681	0.000
p value for t	0.000	0.000			(0.000)			
1901	0.867	0.191	–	–	0.000	0.038	0.766	0.000
p value for t	0.000	0.000			(0.000)			
1904	0.838	0.190	0.042	0.093	0.000	0.038	0.809	0.000
p value for t	0.000	0.000	0.209	0.005	(0.000)			
1905	0.836	0.099	0.116	0.144	0.000	0.031	0.841	0.000
p value for t	0.000	0.001	0.000	0.000	(0.000)			
1906	0.799	0.150	0.052	0.164	0.000	0.044	0.803	0.000
p value for t	0.000	0.000	0.173	0.000	(0.000)			
1907	0.848	0.019	0.010	0.155	0.000	0.047	0.762	0.000
p value for t	0.000	0.637	0.788	0.000	(0.208)			
1908	0.794	0.143	0.114	-0.022	0.000	0.048	0.777	0.000
p value for t	0.000	0.000	0.001	0.492	(0.000)			
1909	0.860	0.073	0.173	0.016	0.000	0.037	0.883	0.000
p value for t	0.000	0.001	0.000	0.436	(0.000)			
1910	0.826	0.191	0.088	-0.046	0.000	0.049	0.821	0.000
p value for t	0.000	0.000	0.001	0.075	(0.000)			
1911	0.826	0.192	0.084	0.035	0.000	0.047	0.850	0.000
p value for t	0.000	0.000	0.000	0.129	(0.000)			
1912	0.754	0.176	-0.007	0.033	0.000	0.058	0.762	0.000
p value for t	0.000	0.006	0.912	0.619	(0.002)			
1913	0.780	0.202	0.030	0.079	0.000	0.051	0.846	0.000
p value for t	0.000	0.000	0.572	0.160	(0.000)			

source: Fusakichi Kasahara, "Seishi keisan bo."

note: Coeffecients are the results of a standadized linear regression so that the transfer β is normalized as 0. The p value for t of β is from an unstandardized regression. The number of samples is the same as Table 1.

Table 3-*a* Optimization of effort vector ***t*** (*Labor productivity-Material productivity* plane)

x_1: *Labor productivity* . x_2: *Material productivity* .

$t_2 = \eta_1/x_1 + \eta_2$.

year	η_1	η_2	standard error	R^2	p value for F
1897	-0.116	0.000	0.404	0.013	0.177
p value for t	0.177	(0.000)			
1901	0.035	0.000	0.467	0.001	0.661
p value for t	0.661	(0.000)			
1904	-0.212	0.000	0.446	0.045	0.003
p value for t	0.003	(0.000)			
1905	-0.299	0.000	0.403	0.089	0.000
p value for t	0.000	(0.000)			
1906	-0.539	0.000	0.365	0.290	0.000
p value for t	0.000	(0.000)			
1907	-0.713	0.000	0.700	0.509	0.000
p value for t	0.000	(0.000)			
1908	-0.471	0.000	0.261	0.221	0.000
p value for t	0.000	(0.000)			
1909	-0.468	0.000	0.257	0.219	0.000
p value for t	0.000	(0.000)			
1910	-0.386	0.000	0.224	0.149	0.000
p value for t	0.000	(0.000)			
1911	-0.359	0.000	0.270	0.129	0.000
p value for t	0.000	(0.000)			
1912	-0.563	0.000	0.249	0.317	0.000
p value for t	0.000	(0.000)			
1913	-0.338	0.000	0.400	0.114	0.002
p value for t	0.002	(0.000)			

source : Fusakichi Kasahara, "Seishi keisan bo."

note : Coefficients are the results of standardized linear regression while the p value for t of η_2 is from an unstandardized regression. The number of samples is the same as Table 1.

Table 3-*b* Optimization of effort vector $\boldsymbol{t}$ (*Labor productivity-Evenness of threads* plane)

x_1: *Labor productivity*. x_3: *Evenness of threads*.

$x_3 = \eta_1/x_1 + \eta_2$.

Year	η_1	η_2	standard error	R^2	p value for F
1904	-0.156	0.000	0.000	0.024	0.032
p value for t	0.032	(0.000)			
1905	-0.383	0.000	0.000	0.147	0.000
p value for t	0.000	(0.000)			
1906	-0.270	0.000	0.000	0.073	0.001
p value for t	0.001	(0.000)			
1907	-0.589	0.000	0.000	0.347	0.000
p value for t	0.000	(0.000)			
1908	-0.150	0.000	0.000	0.023	0.017
p value for t	0.017	(0.000)			
1909	-0.216	0.000	0.000	0.047	0.000
p value for t	0.000	(0.000)			
1910	-0.266	0.000	0.000	0.071	0.000
p value for t	0.000	(0.000)			
1911	-0.132	0.000	0.000	0.017	0.015
p value for t	0.015	(0.000)			
1912	-0.094	0.000	0.000	0.009	0.384
p value for t	0.384	(0.014)			
1913	-0.071	0.000	0.000	0.005	0.523
p value for t	0.523	(0.207)			

source: Fusakichi Kasahara, "Seishi keisan bo."

note: Coefficients are the results of standardized linear regression while the p value for t of η_2 is from an unstandardized regression. The number of samples is the same as Table 1.

Table 3-*c* Optimization of effort vector ***t*** (*Labor productivity-Luster of threads* plane)

x_1: *Labor productivity* . x_4: *Luster of threads* .

$t_4 = \eta_1 / x_1 + \eta_2$.

year	η_1	η_2	standard error	R^2	p value for F
1904	-0.003	0.000	0.012	0.000	0.970
p value for t	0.970	(0.103)			
1905	0.017	0.000	0.015	0.000	0.816
p value for t	0.816	(0.000)			
1906	0.035	0.000	0.008	0.001	0.675
p value for t	0.675	(0.027)			
1907	0.027	0.000	0.011	0.001	0.682
p value for t	0.682	(0.267)			
1908	-0.154	0.000	0.014	0.024	0.015
p value for t	0.015	(0.174)			
1909	-0.106	0.000	0.014	0.011	0.047
p value for t	0.047	(0.001)			
1910	-0.339	0.000	0.012	0.115	0.000
p value for t	0.000	(0.000)			
1911	-0.214	0.000	0.014	0.046	0.000
p value for t	0.000	(0.852)			
1912	-0.431	0.000	0.013	0.186	0.000
p value for t	0.000	(0.029)			
1913	-0.492	0.000	0.010	0.242	0.000
p value for t	0.000	(0.000)			

source : Fusakichi Kasahara, "Seishi keisan bo."

note : Coefficients are the results of standardized linear regression while the p value for t of η_2 is from an unstandardized regression. The number of sumples is the same as Table 1.

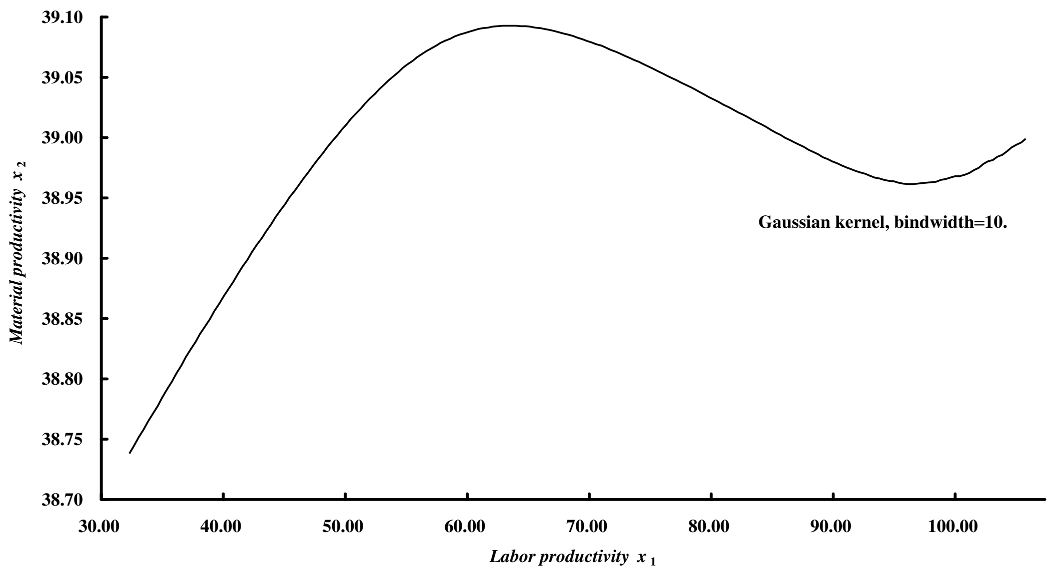

Figure 1: Gaussian kernel regression (*Labor productivity-Material productivity*): Kasahara Factory, 1904.
Source: "Seishi keisan bo".
Note: *Material productivity*: produced raw silk per 4 *shou* (7,216 liters) of cocoon. *Labor productivity*: *momme*(3.75 grams)*s* of raw silk per workday.

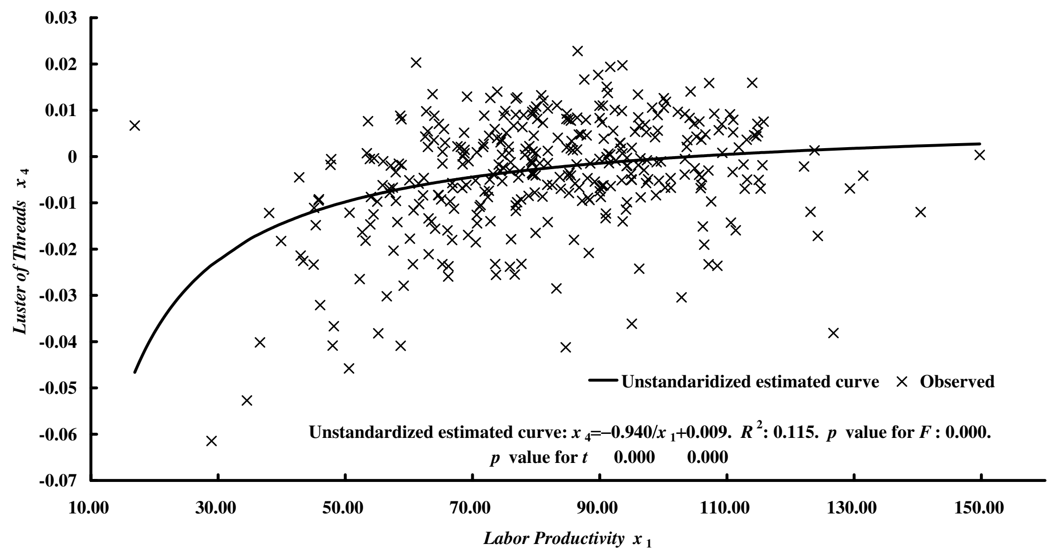

Figure 2: Optimization of effort vector $\boldsymbol{t}$ (*Labor productivity-Luster of threads*): Kasahara Factory, 1910.
Source: "Seishi keisan bo".
Note: *Luster of threads*: points of Luster per 1 *momme* (3.75 grams) of raw silk. *Labor productivity*: *mommes* of raw silk per workday.