Informative Advertising with Spillover Effects*

Hiroyuki Nakata†

December 4, 2006

Abstract

This paper studies the effects of advertising in a Bertrand duopoly model with informative advertising, which is an extension of Grossman and Shapiro (1984). It introduces spillover advertising effects on top of the direct advertising effects. Also, the model involves R&D activities, which make the model dynamical. It is shown that the spillover effects would make the environment more monopolistic: higher prices and profits with lower R&D expenditures.

1 Introduction

There is no doubt that advertising is an important economic activity, which we inevitably face on a daily basis. Naturally, it has been attracting substantial attention of economists. Economists have identified three main roles of advertising: the persuasive view, the informative view, and the complementary view.[1] This paper focuses on the informative role of advertising.

Informative advertising provides an interesting implication regarding the value of information. As the well celebrated classical result by Blackwell (1953) shows, more information is always good in the context of a static single decision maker problem. However, as the example by Hirshleifer (1971) indicates, more information may make the agents worse off in a more generic case. Sulganik and Zilcha (1997) generalises the example of Hirshleifer (1971) and argues that such an apparently pathological case arises when the additional information changes the opportunity sets of the agents. Now the informative view of advertising postulates that advertising does change the opportunity sets the agents *recognise*. Hence, it is not trivial whether information is beneficial or not to the welfare of the economy. Indeed, the existing literature suggests mixed results concerning the welfare effect of advertising.

---

*Preliminary. Comments are welcome.

†Department of Accounting, Finance and Management, University of Essex, Wivenhoe Park, Colchester CO4 3SQ, United Kingdom; E-mail: `hnakata@stanfordalumni.org` or `hnakata@essex.ac.uk`.

[1] Bagwell (2005) provides an extensive literature review.

Butters (1977) initiated the line of research on informative advertising in oligopoly models.[2] It is a model with a homogeneous good, in which each firm chooses its price to advertise and the number of (price) advertisements to send out. It shows that the equilibrium advertising level is socially optimal, while price has no welfare implications.[3] Some extensions of Butters (1977), however, show that equilibrium advertising is insufficient (e.g. Stegeman [1991] and Stahl [1994]). Grossman and Shapiro (1984) extends Butters (1977) to allow for differentiated products. As opposed to Butters (1977), it shows that the equilibrium advertising may be insufficient or excessive. Also, it provides a surprising result: the equilibrium profit is larger when advertising is more costly. Moreover, some other works show that firms may keep some consumers uninformed even if the advertising cost is zero (e.g. Ferschtman and Muller [1993] and Boyer and Moreaux [1999]).

Now, as is often argued, the internet is becoming an important channel through which consumers collect information and purchase products or services. It is therefore not surprising to see that more and more firms advertise and offer their products or services via the internet. Easier and cheaper communication via the internet would make advertising more effective in improving the recognition of products or services. However, it may cause larger spillover advertising effects simultaneously, because a consumer who has seen an advertisement of a product would have an easy access to information about its competing products when communication is easy and cheap. While the direct advertising effect would be advantageous for the advertising firm, the spillover effect would be disadvantageous for it. Since the spillover effect is an externality that exist in both directions, the effect on the economy is not trivial. Cellini and Lambertini (2003) examines such an effect in a Cournot oligopoly, and finds that the equilibrium advertising is higher when firms form a cartel than when they act non-cooperatively.

Our model also examines the direct and spillover advertising effects by extending Grossman and Shapiro (1984), which is a Bertrand oligopoly. On top of examining the direct and spillover advertising effects on prices, this paper analyses such effects on R&D activities. Usually R&D activity is rather expensive, and thus, the firms would avoid spending on R&D if they do not face any competition. In our model, the two advertising effects influence the competitive environment, and thus, we can expect some impacts on R&D, too. Note that the addition of R&D makes the model dynamical.

However, unlike some existing works that make advertising fully dynamical, this paper does not (e.g. Nerlove and Arrow [1962], Friedman [1983], Doraszelski and Markovich [2005]). We focus on the situation in which two established firms competing with each other by introducing new products in each period, while the new products will remain unrecognised unless the consumers receive advertisements then. Hence, the impact of advertising on recognition is static in nature.

[2]Other classical works on informative advertising include Nelson (1974) and Lynk (1981).

[3]Welfare is measured with respect to post-advertisement preferences. See Dixit and Norman (1978). Also, we shall discuss this issue later in this paper.

The rest of the paper proceeds as follows. Section 2 introduces the model and the definition of the equilibrium. Then, section 3 examines the comparative statics of the equilibrium, and analyses the welfare effects of advertising. Section 4 discusses the implications and limits of the results, and section 5 concludes the paper.

2 The Model

In this section, we first explain the structure of the model, and then, define its equilibrium. Then, we define the symmetric equilibrium and stationary equilibrium, and show that they are equivalent in our model. Also, the uniqueness of such an equilibrium is also shown.

2.1 The Structure of the Model

The model in this paper essentially follows that of Grossman and Shapiro (1984), although our model is dynamical and introduces spillover effects and R&D activities. To make the model analytically tractable, we focus on the case where there are only two firms. Namely, we follow the model in Tirole (1988, Section 7.3.2) in which firms are located on a line rather than on a circle in Grossman and Shapiro (1984).

More specifically, consider an economy in which there are two firms competing in the same industry. Each firm has only one product in every period, and is infinitely lived. On the other hand, there is a continuum of consumers who are located on the unit interval $[0, 1]$ in each period, while they are indexed by $h \in [0, 1]$.

We assume that each consumer consumes at most only one unit of the product of the industry in every period t ($t = 0, 1, 2, ...$). Consumer h's subjective (monetary) evaluation of firm 1's product is $v_t^1 - \eta h$ and that of firm 2's product is $v_t^2 - \eta(1-h)$, where η (≥ 0) is a parameter that measures diversity in tastes among the consumers, while v_t^k is the maximum subjective evaluation of firm k's product in period t.[4]

The maximum subjective evaluation of firm k's product is determined as follows:

$$v_{t+1}^k = v_t^k + f(z_t^k), \tag{1}$$

where z_t^k (≥ 0) denotes firm k's R&D expenditure in period t, while we assume $v_0^k = v_0$ ($k = 1, 2$), $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$. Hence, firm k's R&D expenditure alters consumers' subjective evaluations of firm k's product in the next period, while the R&D activities exhibit 'diminishing returns to scale'.

The consumer surplus of consumer h in period t is $v_t^1 - \eta h - p_t^1$ if he buys firm 1's product at price p_t^1, while it is $v_t^2 - \eta(1-h) - p_t^2$ if he buys firm 2's

[4]This effectively means that firm 1 is located at 0 and firm 2 at 1 on the unit interval $[0, 1]$.

product at price p_t^2.[5] However, we assume that consumers do not *a priori* recognise the products available in each period t unless the firms advertise their products then. Namely, consumers are passive about the products and/or brands. This assumption follows Butters (1977) and Grossman and Shapiro (1984). Nevertheless, we assume that the recognition of a firm's product may be affected by the advertisement of the other firm. We call such effects the *spillover effects*. Moreover, we assume that the firms cannot target specific consumers (i.e. no target advertising), but can only affect the overall recognition.

Upon these observations, we assume that firm k's product's recognition in period t takes the following functional form: For $k = 1, 2$,

$$\phi_t^k = \phi^k(q_t^1, q_t^2) = \min\left\{\sqrt{aq_t^k + bq_t^{(k)}}, 1\right\}, \quad \text{for all } t, \tag{2}$$

where ϕ_t^k denotes the proportion of agents who recognise firm k's product in period t, q_t^k denotes firm k's advertisement expenditure in period t, and $q_t^{(k)}$ denotes the other firm's advertisement expenditure in period t, while a and b are parameters with $a \geq b \geq 0$ and $a > 0$. Note that when $b > 0$, there is a spillover effect. Also, we omit the subscript t whenever it is explicitly or implicitly understood.

To describe the demand for the products, we need to consider two different situations. The first one is when a consumer recognises only one firm's product. In this case, the consumer buys the firm's product as long as his consumer surplus from the product is positive. The other is the case in which the consumer recognises the products of both firms. In this case, the consumer buys the product of the firm that yields a larger consumer surplus.

Following Grossman and Shapiro (1984), it is straightforward to derive the demand for firm k's product as follows:

$$\begin{aligned} x_t^k &= x^k\left(\mathbf{p}_t, \phi^k(\mathbf{q}_t), \phi^{(k)}(\mathbf{q}_t), v_t^k, v_t^{(k)}\right) \\ &= \frac{\phi^1(\mathbf{q}_t)\phi^2(\mathbf{q}_t)(v_t^k - v_t^{(k)} + p_t^{(k)} - p_t^k)}{\eta} + \frac{\phi^k(\mathbf{q}_t)(2 - \phi^{(k)}(\mathbf{q}_t))}{2}, \end{aligned}$$

where $\mathbf{p}_t = (p_t^1, p_t^2)$ and $\mathbf{q}_t = (q_t^1, q_t^2)$.

For the production side, following Grossman and Shapiro (1984) we assume that the cost functions of the firms are identical and take the following form:

$$C(x_t^k) = cx_t^k + \zeta,$$

where c is a parameter that measures the variable cost, ζ is the fixed cost, and x_t^k is the number of units of firm k's product sold in period t. On top of production costs, each firm spends on advertisement as well as on R&D. Thus, the profit of firm k in period t is defined as follows:

$$\pi_t^k = (p_t^k - c) \cdot x_t^k - \zeta - q_t^k - z_t^k.$$

[5] The concept of consumer surplus here is not very clear cut since consumers do not recognise the products a priori. Consumer surplus here is conditional on the knowledge of the product, and thus, it can be understood as an ex post concept. We discuss this point more in details in Section 4.

We assume that the firms are profit maximisers, and that, each firm chooses the price of its product p_t^k as well as the advertising expenditure q_t^k and the R&D expenditure z_t^k in each period. Furthermore, we assume that each firm knows the structure of the model described here, while there is no informational asymmetry as far as the description of this reduced form model is concerned.

Firm k solves the following profit maximisation problem:

$$\begin{aligned} \max_{(p_t^k, q_t^k, z_t^k)_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \delta^t \left[(p_t^k - c) \cdot x^k \left(\mathbf{p}_t, \phi^k(\mathbf{q}_t), \phi^{(k)}(\mathbf{q}_t), v_t^k, v_t^{(k)} \right) - \zeta - q_t^k - z_t^k \right] \\ \text{s.t.} \quad & v_{t+1}^k = v_t^k + f(z_t^k), \\ & \left(p_t^{(k)*}, q_t^{(k)*}, z_t^{(k)*} \right)_{t=0}^{\infty} \text{ given,} \end{aligned}$$

where $\delta \in (0,1)$ is the discount factor. This formulation assumes that each firm possesses the correct (common) belief, and that it is common knowledge. Each firm therefore effectively has a perfect foresight about the entire future sequence of the opponent's actions $(p_t^{(k)}, q_t^{(k)}, z_t^{(k)})_{t=0}^{\infty}$.

The first order conditions of firm k's optimisation problem are the following (for interior solutions such that $\phi_t^k \in (0,1)$):

$$0 = \frac{\phi_t^1 \phi_t^2}{\eta} \cdot (\Delta v_t^k - \Delta p_t^k - p_t^k + c) + \frac{\phi_t^k (2 - \phi_t^{(k)})}{2} \tag{3}$$

$$1 = (p_t^k - c) \left[\frac{a(\phi_t^{(k)})^2 + b(\phi^k)^2}{2\phi_t^k \phi_t^{(k)}} \cdot \frac{\Delta v_t^k - \Delta p_t^k}{\eta} + \frac{a(2 - \phi_t^{(k)})}{4\phi_t^k} - \frac{b\phi_t^k}{4\phi_t^{(k)}} \right] \tag{4}$$

$$1 = \delta(p_{t+1}^k - c) \cdot \frac{\phi_{t+1}^1 \phi_{t+1}^2}{\eta} \cdot f'(z_t^k) \tag{5}$$

for all t, where $\Delta v_t^k := v_t^k - v_t^{(k)}$ and $\Delta p_t^k := p_t^k - p_t^{(k)}$.

Note that the choice of (p_t^k, q_t^k) is essentially static and is without dynamical concerns. On the other hand, the choice of z_t^k has dynamical consequences, where v_t^k is really a state variable. As it is clear from equation (5) that the choice of z_t^k is determined by applying backward induction: taking $(p_{t+1}^k, \phi_{t+1}^1, \phi_{t+1}^2)$ as given to derive the optimal z_t^k. Obviously, the equilibrium concept resulting from optimising behaviours of the firms is Nash equilibrium.[6]

2.2 The Equilibrium

In what follows, first we provide the definition of equilibrium, followed by those of stationary equilibrium and symmetric equilibrium. Then, we show the equivalence between symmetric equilibrium and stationary equilibrium

[6]The definition here is that of an open-loop equilibrium. However, there is no distinction between an open-loop equilibrium and a closed-loop equilibrium in the symmetric case in our model, since the state variables (v_t^1, v_t^2) do not affect the control variables. Namely, an open loop equilibrium is time consistent, and is subgame perfect.

in our model. Also, we show the uniqueness of symmetric or stationary equilibrium.

Depending on the parameters (a, b, η), there may well be corner solutions such that $\phi_t^k = 1$. However, this is not an interesting case, and thus, we rule out such equilibria, and focus on the equilibria such that $\phi_t^k \in (0, 1)$ for all k, t. Hence, we define the interior equilibrium of the model as follows, while defining $\mathbf{z}_t = (z_t^1, z_t^2)$.

Definition: *An interior equilibrium of the model is a sequence* $(\mathbf{p}_t, \mathbf{q}_t, \mathbf{z}_t)_{t=0}^{\infty}$ *that satisfies conditions (3)—(5) and* $\phi_t^k \in (0, 1)$ *for all* k, t. □

Next, we define the stationary equilibrium as follows.

Definition: *A stationary equilibrium is an interior equilibrium in which*

$$(\mathbf{p}_t^*, \mathbf{q}_t^*, \mathbf{z}_t^*) = (\mathbf{p}^*, \mathbf{q}^*, \mathbf{z}^*)$$

holds for all t. □

Thus, a stationary equilibrium is an interior equilibrium in which all control variables are time-invariant. Note however that the subjective evaluations of the products are not necessarily and typically not time-invariant, because $v_{t+1}^k = v_t^k + f(z_t^k) > v_t^k$ if $z_t^k > 0$. Also, we define the symmetric equilibrium as follows.

Definition: *A symmetric equilibrium is an interior equilibrium in which*

$$(p_t^{1*}, q_t^{1*}, z_t^{1*}) = (p_t^{2*}, q_t^{2*}, z_t^{2*})$$

holds for all t. □

Note that the definition of stationary equilibrium does not require v_t^k to be constant over time. Hence, even in a stationary equilibrium, the consumer surplus may continue to rise over time.

With the definitions above, we are ready to state the following proposition.

Proposition 1: *A stationary equilibrium prevails if and only if it is a symmetric equilibrium.*

(Proof)
First, we prove the necessity. We prove it by contradiction. Suppose a stationary equilibrium prevails, but it is not symmetric. Then, either $z^1 > z^2$ or $z^1 < z^2$ holds. We only focus on the case $z^1 > z^2$, since the same logic applies to the other case. $z^1 > z^2$ implies that $f(z^1) > f(z^2)$, since $f'(\cdot) > 0$ by assumption. It follows that

$$\begin{aligned} v_{t+1}^1 - v_{t+1}^2 &= v_t^1 - v_t^2 + f(z^1) - f(z^2) \\ &> v_t^1 - v_t^2. \end{aligned}$$

However, this is a contradiction with (3) unless either $\phi^1 = 0$ or $\phi^2 = 0$ holds, which cannot be compatible with (4). Hence, $z^1 = z^2$ must hold.

It follows from (5) that $z^1 = z^2$ implies $p^1 = p^2$. This in turn implies that (3) yields

$$\frac{\phi^1(2-\phi^2)}{2} = \frac{\phi^2(2-\phi^1)}{2}.$$

Hence, $\phi^1 = \phi^2$ and also $q^1 = q^2$. This completes the proof of necessity.

Next we prove the sufficiency. Observe that for symmetric equilibria the first order conditions (3)—(5) can be rewritten as follows:

$$p_t^* - c = \frac{2-\phi_t^*}{\phi_t^*} \cdot \eta, \tag{6}$$

$$p_t^* - c = \frac{4\phi_t^*}{2a-(a+b)\phi_t^*}, \tag{7}$$

$$1 = \delta(p_{t+1}^* - c) \cdot \frac{(\phi_{t+1}^*)^2}{\eta} \cdot f'(z_t^*). \tag{8}$$

(6) and (7) imply that the following quadratic equation holds:

$$(a+b-4)\,(\phi_t^*)^2 - 2(2a+b)\phi_t^* + 4a = 0.$$

Hence, when $a+b < 4$, there is only one positive root, which implies $\phi_t^* = \phi^*$ for all t if such a positive root is not greater than 1. On the other hand, when $a+b > 4$, the signs of the two roots are both positive. In this case, the quadratic function $(a+b-4)\,(\phi_t^*)^2 - 2(2a+b)\phi_t^* + 4a$ is at the minimum when

$$\phi_t^* = \hat{\phi} = \frac{2a+b}{a+b-4}.$$

Observe that $\hat{\phi} > 1$ if and only if $a > -4$. Since $a > 0$ by assumption, $\hat{\phi} > 1$ holds. Thus, there is only one root that is between 0 and 1. Hence, there is at most only one positive root, and thus, $\phi_t^* = \phi^*$ for all t. It follows that $q_t^* = q^*$ for all t, while both (6) and (7) imply that $p_t^* = p^*$, which in turn implies from (8) that $z_t^* = z^*$ for all t. This completes the proof. ■

The proof for the sufficiency of proposition 1 yields the following proposition:

Proposition 2: *Given the parameters, there exists at most one symmetric/stationary equilibrium.* ■

Propositions 1 and 2 provide some justifications to focus on symmetric/stationary equilibrium. The parametric set-up is symmetrical between the two firms; thus, it is natural to focus on symmetric equilibrium. Proposition 1 then shows that a stationary equilibrium is equivalent to a symmetric equilibrium, and thus, we may conclude that it is natural to focus on stationary equilibrium. Moreover, proposition 2 shows the uniqueness of a symmetric/stationary equilibrium. Hence, any comparative statics results can be understood to be generic as long as we focus on symmetric/stationary equilibrium.

Hence, from now on, we focus only on symmetric/stationary equilibria: $p_t^1 = p_t^2 = p^*$, $q_t^1 = q_t^2 = q^*$ and $z_t^1 = z_t^2 = z^*$ hold for all t. Consequently, we rewrite the first order conditions above as follows:

$$\begin{aligned} p^* - c &= \frac{(2-\phi^*)\eta}{\phi^*}, && (9)\\ p^* - c &= \frac{4\phi^*}{2a-(a+b)\phi^*}, && (10)\\ 1 &= \delta(p^*-c)\cdot\frac{(\phi^*)^2}{\eta}\cdot f'(z^*). && (11) \end{aligned}$$

It is easy to show that the second order conditions hold.

$$\begin{aligned} \frac{\partial^2\pi^k}{\partial(p^k)^2} &= -\frac{2(\phi^*)^2}{\eta} < 0,\\ \frac{\partial^2\pi^k}{\partial(q^k)^2} &= (p^*-c)\cdot\frac{1}{8(\phi^*)^2}\cdot\left[(a-b)^2-\frac{2a^2}{\phi^*}\right] < 0,\\ \frac{\partial^2\pi^k}{\partial(p^k)^2}\cdot\frac{\partial^2\pi^k}{\partial(q^k)^2}-\left(\frac{\partial^2\pi^k}{\partial q^k\partial p^k}\right)^2 &= \frac{a^2(\phi^*-2)^2+2ab\phi^*(2-\phi^*)+b^2((\phi^*)^2-2\phi^*-2)}{8(\phi^*)^2}\\ &\geq \frac{1}{8(\phi^*)^2}\cdot\left[2ab\phi^*(2-\phi^*)+2((\phi^*)^2-3\phi^*+1)b^2\right]\\ &\geq \frac{1}{8(\phi^*)^2}\cdot\left[2ab\phi^*(1-\phi^*)+2(1-\phi^*)^2b^2\right]\\ &> 0,\\ \delta(p^*-c)\cdot\frac{(\phi^*)^2}{\eta}\cdot f''(z^*) &< 0. \end{aligned}$$

It is clear that $f''(z^*) < 0$ must hold, which we assumed before. In other words, although the assumption may have appeared ad hoc since it is concerning the subjective monetary evaluation of the consumers, it is in fact essential. Moreover, the profit function in each period is bounded as long as price p^* is bounded, which is indeed the case here. Hence, the objective function over the infinite periods is also bounded, and thus, equations (9)–(11) do characterise the solution.

3 Comparative Statics and Welfare Analysis

In this section, we examine the comparative statics of the endogenous variables as well as welfare implications. In so doing, we consider two cases: the static case and the dynamical case. The static case is the case in which there are no R&D activities. This makes the situation static, because the only dynamical element of the model is the R&D activities. In this case, consumers' subjective evaluations of the products are constant over time. The other case, the dynamical case, is thus the case in which there are R&D activities. In this case, the subjective evaluations of the products change over time.

Nevertheless, we sustain our focus on stationary equilibrium, and thus, all (equilibrium values of) control variables are time-invariant. It follows that the results for control variables in the static case carry through in the dynamical case, too. However, the results for profits must be considered separately, since profits are affected by R&D expenditures, which appear only in the dynamical case.

3.1 The Static Case

In what follows, we examine the effects of the parameters on the endogenous variables in the static case, in which R&D expenditures are absent, and thus, the maximum subjective evaluation of the products is constant at v_0. From the first order conditions (9) and (10), it is routine to show the following:

$$\begin{pmatrix} \frac{\partial p^*}{\partial a} \\ \frac{\partial q^*}{\partial a} \\ \frac{\partial p^*}{\partial b} \\ \frac{\partial q^*}{\partial b} \\ \frac{\partial p^*}{\partial \eta} \\ \frac{\partial q^*}{\partial \eta} \end{pmatrix} = \frac{1}{D^*} \begin{pmatrix} \frac{2(2-\phi^*)\eta}{G^* q^*} \\ \frac{\eta}{2\phi^*(a+b)} - \frac{4\eta\phi^*[a(1-\phi^*)+b(2-\phi^*)]}{G^*(a+b)} \\ -\frac{2\phi^*\eta}{G^*} \\ -\frac{\eta}{2\phi(a+b)} + \frac{4\phi^*[a+(a+b)\phi^*]}{G^*(a+b)} \\ \frac{2a(\phi^*-2)}{G^* q^*} \\ \frac{\phi^*-2}{2\phi^*} \end{pmatrix},$$

where

$$\begin{aligned} D^* &= -\frac{4a\phi^*}{G^* q^*} - \frac{\eta}{2q^*\phi^*} < 0, \\ G^* &= [2a - (a+b)\phi^*]^2 > 0. \end{aligned}$$

Hence, it is clear that $\partial p^*/\partial a < 0$, $\partial p^*/\partial b > 0$, $\partial p^*/\partial \eta > 0$ and $\partial q^*/\partial \eta > 0$. Although the sign of $\partial q^*/\partial b$ appears to be indeterminate, this is not the case. We know from (9) that

$$\frac{\partial p^*}{\partial a} = -\frac{2\eta}{(\phi^*)^2} \cdot \frac{\partial \phi^*}{\partial a}, \tag{12}$$

$$\frac{\partial p^*}{\partial b} = -\frac{2\eta}{(\phi^*)^2} \cdot \frac{\partial \phi^*}{\partial b}. \tag{13}$$

Thus, $\partial \phi^*/\partial a > 0$ and $\partial \phi^*/\partial b < 0$. Observe that (2) implies that

$$\begin{aligned} \frac{\partial \phi^*}{\partial b} &= \left.\frac{\partial \phi^*}{\partial b}\right|_* + \left(\frac{\partial \phi^*}{\partial q^k} + \frac{\partial \phi^*}{\partial q^{(k)}}\right) \cdot \frac{\partial q^*}{\partial b} \\ &= \frac{q^*}{2\phi^*} + \frac{a+b}{2\phi^*} \cdot \frac{\partial q^*}{\partial b} \\ &< 0, \end{aligned}$$

where $\left.\frac{\partial \phi^*}{\partial b}\right|_*$ denotes the direct effect of b on ϕ^*. It follows that $\partial q^*/\partial b < 0$. However, the sign of $\partial q^*/\partial a$ is indeterminate, because

$$\begin{aligned} \frac{\partial \phi^*}{\partial a} &= \left.\frac{\partial \phi^*}{\partial a}\right|_* + \left(\frac{\partial \phi^*}{\partial q^k} + \frac{\partial \phi^*}{\partial q^{(k)}}\right) \cdot \frac{\partial q^*}{\partial a} \\ &= \frac{q^*}{2\phi^*} + \frac{a+b}{2\phi^*} \cdot \frac{\partial q^*}{\partial a} \\ &> 0. \end{aligned}$$

Also, from (10), we obtain

$$\frac{\partial p^*}{\partial \eta} = \frac{8a}{[2a - (a+b)\phi^*]^2} \cdot \frac{\partial \phi^*}{\partial \eta}.$$

We know that $\partial p^*/\partial \eta > 0$. Hence, $\partial \phi^*/\partial \eta > 0$ holds.

Moreover, by construction, the equilibrium product sales is

$$x^* = \frac{\phi^* \left(2 - (\phi^*)^2\right)}{2}.$$

It follows that

$$\begin{aligned}
\frac{\partial x^*}{\partial a} &= (1 - \phi^*) \cdot \frac{\partial \phi^*}{\partial a} > 0, \\
\frac{\partial x^*}{\partial b} &= (1 - \phi^*) \cdot \frac{\partial \phi^*}{\partial b} < 0, \\
\frac{\partial x^*}{\partial \eta} &= (1 - \phi^*) \cdot \frac{\partial \phi^*}{\partial \eta} > 0.
\end{aligned}$$

The following proposition summarises the comparative statics results regarding advertising, pricing and product recognition.

Proposition 3:
(a) *The equilibrium price level is decreasing in the degree of direct advertising effect, increasing both in the degree of the spillover advertising effect and in the degree of diversity in tastes, i.e.* $\frac{\partial p^*}{\partial a} < 0$, $\frac{\partial p^*}{\partial b} > 0$ *and* $\frac{\partial p^*}{\partial \eta} > 0$.

(b) *The equilibrium advertising level is decreasing in the degree of the spillover advertising effect, and increasing in the degree of diversity in tastes, i.e.* $\frac{\partial q^*}{\partial b} < 0$ *and* $\frac{\partial q^*}{\partial \eta} > 0$.

(c) *The equilibrium product recognition is increasing in the degree of the direct advertising effect, decreasing in the degree of the spillover advertising effect, and increasing in the degree of diversity in tastes, i.e.* $\frac{\partial \phi^*}{\partial a} > 0$, $\frac{\partial \phi^*}{\partial b} < 0$, *and* $\frac{\partial \phi^*}{\partial \eta} > 0$.

(d) *The equilibrium product sales is increasing in the degree of the direct advertising effect, decreasing in the degree of the spillover advertising effect, and increasing in the degree of diversity in tastes, i.e.* $\frac{\partial x^*}{\partial a} > 0$, $\frac{\partial x^*}{\partial b} < 0$, *and* $\frac{\partial x^*}{\partial \eta} > 0$.

Before examining the case in which z_t^k is controllable, we examine the effects on profits in the static case, i.e. v_t^k is fixed. In this case, the following holds:

Proposition 4: *In the static case in which* $v_t^1 = v_t^2 = v_0$ *for all* t*, the equilibrium profit level is decreasing in the degree of the direct advertising effect, increasing both in the degree of the spillover advertising effect and in the degree of diversity in tastes, i.e.* $\frac{\partial \pi^*}{\partial a} < 0$, $\frac{\partial \pi^*}{\partial b} > 0$ *and* $\frac{\partial \pi^*}{\partial \eta} > 0$.

(Proof) See appendix. ■

parameters	p^*	q^*	ϕ^*	x^*	π^*
a	$-$	?	$+$	$+$	$-$
b	$+$	$-$	$-$	$-$	$+$
η	$+$	$+$	$+$	$+$	$+$

Table 1: Comparative Statics: Static Case

Table 1 summarises the results of Propositions 3 and 4. p^*, q^*, ϕ^* and x^* are unaffected by the maximum subjective evaluation of the products v_t, and thus, the effects on them are the same whether or not we are analysing the static case or the dynamical case. However, effects on profits are different. Profits in the static case do not involve R&D expenditures z_t, while they do involve z_t in the dynamical case. Thus, the effects on profits are affected by the effects on R&D expenditures in the dynamical case, and consequently, the results may well be different between the two cases.

The results with respect to a are consistent with the findings of Grossman and Shapiro (1984). When the direct advertising effect is larger (i.e. larger a), the equilibrium advertising expenditure may or may not increase. Nevertheless, in this case, the equilibrium recognition level of the product becomes higher, the equilibrium number of units sold increases, and the equilibrium price and profit fall. Hence, it can be understood that the environment of the industry becomes more competitive when the parameter a is larger.

In contrast, the results with respect to b suggest that larger spillover effects (i.e. larger b) make the economy more monopolistic—higher prices and larger profits. Observe that larger spillover effects drive down the equilibrium advertising expenditures that are small enough so that the product recognition remains at a low level. By recognising that an increase in its advertising expenditure would improve both the recognition of its own product and that of the competitor's product, each firm restricts its advertising expenditure so that it can enjoy a more monopolistic environment.

When the degree of diversity in tastes is higher (i.e. larger η), all endogenous variables increase. To understand why, notice that each firm has more consumers who value its own product far more than its competitor's and vice versa when η is large. Hence, the price elasticities of the demand for the products become low when η is large, and thus, the firms can charge higher prices. Moreover, the damage caused by the spillover effects will be smaller. Hence, firms spend more on advertising when η is large, and consequently, the recognition of the products and the sales increase.

3.2 The Dynamical Case

In what follows, we examine the comparative statics regarding R&D expenditures and profits, which involve dynamical considerations. Because we are focusing on stationary equilibria (or equivalently symmetric equilibria), comparative statics become simple: the analysis becomes as though it is static.

To examine the effects on R&D decisions, we use (9) and (11) to derive the following:

$$1 = (2 - \phi^*)\,\phi^* f'(z^*).$$

It follows that

$$\begin{aligned}
\frac{\partial z^*}{\partial a} &= -\frac{2(1-\phi^*)}{(2-\phi^*)\phi^*} \cdot \frac{f'(z^*)}{f''(z^*)} \cdot \frac{\partial \phi^*}{\partial a} > 0, \\
\frac{\partial z^*}{\partial b} &= -\frac{2(1-\phi^*)}{(2-\phi^*)\phi^*} \cdot \frac{f'(z^*)}{f''(z^*)} \cdot \frac{\partial \phi^*}{\partial b} < 0, \\
\frac{\partial z^*}{\partial \eta} &= -\frac{2(1-\phi^*)}{(2-\phi^*)\phi^*} \cdot \frac{f'(z^*)}{f''(z^*)} \cdot \frac{\partial \phi^*}{\partial \eta} > 0.
\end{aligned}$$

The following proposition summarises the above results.

Proposition 5: *The equilibrium R&D expenditure is increasing in the degree of the direct advertising effect, decreasing in the degree of the spillover advertising effect, and increasing in the degree of diversity in tastes, i.e.* $\frac{\partial z^*}{\partial a} > 0$, $\frac{\partial z^*}{\partial b} < 0$, *and* $\frac{\partial z^*}{\partial \eta} > 0$.

Proposition 5 suggests that the direct advertising effect (i.e. a) enhances R&D activities, while the spillover advertising effect (i.e. b) has a negative effect on them. The difference reflects the fact that the two advertising effects have opposing impacts on the recognition of the products: the direct effect has a positive impact and the spillover effect has a negative one. As explained earlier, wider recognition of the products means that more consumers choose a product rather than merely deciding whether or not to take the one (and the only one) they recognise. Hence, the direct effect makes the environment more competitive for the firms, while the spillover effect makes it more monopolistic. Firms make R&D expenditures to sustain a competitive edge, and they do more so when the environment is more competitive. Hence, a larger direct effect causes a fiercer R&D race between the firms, while a larger spillover effect eases the race.

We can therefore conclude the following: When the direct advertising effects are larger (i.e. larger a) the welfare of the consumers improves through lower equilibrium prices and more rapid product improvements (through more active R&D activities). In contrast, the spillover advertising effects (i.e. b) have an adverse impacts on the consumer welfare: Higher equilibrium prices and slower product improvements.

Next, we examine the effects on profits. Observe that the following equations hold:

$$\begin{aligned}
\frac{\partial \pi^*}{\partial a} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial a} + \left(\frac{\partial \pi^*}{\partial \phi^k} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \right) \cdot \frac{\partial \phi^*}{\partial a} - \frac{\partial q^*}{\partial a} - \frac{\partial z^*}{\partial a}, \\
\frac{\partial \pi^*}{\partial b} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial b} + \left(\frac{\partial \pi^*}{\partial \phi^k} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \right) \cdot \frac{\partial \phi^*}{\partial b} - \frac{\partial q^*}{\partial b} - \frac{\partial z^*}{\partial b}, \\
\frac{\partial \pi^*}{\partial \eta} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial \eta} + \left(\frac{\partial \pi^*}{\partial \phi^k} \cdot \frac{\partial \phi^{k*}}{\partial q^{(k)}} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \cdot \frac{\partial \phi^{(k)*}}{\partial q^{(k)}} \right) \cdot \frac{\partial q^*}{\partial \eta} - \frac{\partial z^*}{\partial \eta}.
\end{aligned}$$

All equations have an extra term that involves partial derivative of z^* on top of the equations found in the proof for proposition 4. It is then straightforward that

$$\frac{\partial \pi^*}{\partial a} < 0, \quad \frac{\partial \pi^*}{\partial b} > 0,$$

while the sign of $\frac{\partial \pi^*}{\partial \eta}$ is indeterminate.

Before providing a result regarding $\frac{\partial \pi^*}{\partial \eta}$, let us introduce the following definition:

$$\alpha^* := -\frac{f''(z^*)}{f'(z^*)},$$

which is analogous to the coefficient of absolute risk aversion. With this definition, we claim the following proposition:

Proposition 6: *The equilibrium profit level is decreasing in the degree of the direct advertising effect and increasing in the degree of the spillover advertising effect, i.e.* $\frac{\partial \pi^*}{\partial a} < 0$ *and* $\frac{\partial \pi^*}{\partial b} > 0$*. Also, there exists a threshold* $\hat{\alpha}$ *such that*

$$\frac{\partial \pi^*}{\partial \eta} \gtreqless 0 \quad \textit{if and only if} \quad \alpha^* \gtreqless \hat{\alpha}.$$

(Proof) See above for $\partial \pi^* / \partial a$ and $\partial \pi^* / \partial b$. We know from Proposition 4 that

$$\frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial \eta} + \left(\frac{\partial \pi^*}{\partial \phi^k} \cdot \frac{\partial \phi^{k*}}{\partial q^{(k)}} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \cdot \frac{\partial \phi^{(k)*}}{\partial q^{(k)}} \right) \cdot \frac{\partial q^*}{\partial \eta} > 0,$$

while we know from Proposition 5 that $\partial z^* / \partial \eta > 0$. It follows that the sign of $\partial \pi^* / \partial \eta$ depends on the relative magnitude of $\partial z^* / \partial \eta$. Hence, we can define $\hat{\alpha}$ as is done in the claim. ■

parameters	p^*	q^*	ϕ^*	x^*	z^*	π^*
a	$-$	?	$+$	$+$	$+$	$-$
b	$+$	$-$	$-$	$-$	$-$	$+$
η	$+$	$+$	$+$	$+$	$+$	?

Table 2: Comparative Statics: Dynamical Case

Table 2 summarises the results. The direct advertising effect makes the environment more competitive by enhancing wider recognition of the products, which in turn lowers the prices and causes larger R&D expenditures. Thus, it reduces the equilibrium profit levels of the firms. On the other hand, the spillover effect has the exact opposite effects. The sign of $\partial \pi^* / \partial \eta$ depends on the magnitude of α^*, i.e. it is positive when α^* is small, while it is negative when α^* is large. In other words, when $f(\cdot)$ is close to (affine) linear, the equilibrium profits would be decreasing in the degree of diversity of tastes, while the profits would be increasing in it when $f(\cdot)$ shows a strong 'diminishing returns to scale' in R&D activities.

3.3 Welfare Analysis

In what follows, we examine the welfare implications of advertising. To do so, we first define the first best of the static case to examine the welfare effects for such a case. Then, we examine the welfare effects for the dynamical case by extending the analysis for the static case.

The total surplus of the economy is the sum of the aggregate consumer surplus and the profits of the firms. The aggregate consumer surplus can be understood as the sum of surplus for two separate groups of consumers; consumers who are purchasing the potentially more preferable product, and consumers who are purchasing the potentially less preferable product because of the lack of recognition. The average consumer surplus of the first group is $v - 0.25\eta - p$ while that of the second group is $v - 0.75\eta - p$. It is easy to check that fraction ϕ of the total population falls into the first group, while fraction $\phi(1-\phi)$ of the total population falls into the second group. Hence, the aggregate consumer surplus CS (in each period) is

$$\begin{aligned} CS &= \phi(v - 0.25\eta - p) + \phi(1-\phi)(v - 0.75\eta - p) \\ &= \phi(2-\phi)v - 0.25\eta\phi(4-3\phi) - \phi(2-\phi)p. \end{aligned}$$

On the other hand, in the static case, the aggregate profit of the firms π is

$$\begin{aligned} \pi &= \phi(2-\phi)(p-c) - 2\xi - 2q \\ &= \phi(2-\phi)(p-c) - 2\xi - \frac{2(\phi)^2}{a+b}, \end{aligned}$$

where the last line follows from $\phi = \sqrt{(a+b)q}$. It follows that the total surplus of the economy is

$$CS + \pi = \phi(2-\phi)(v-c) - 0.25\eta\phi(4-3\phi) - 2\xi - \frac{2(\phi)^2}{a+b}.$$

Note that the price p plays no role here, and ϕ is the only endogenous variable. In other words, it is sufficient to see the impacts on ϕ to examine the welfare effects.

Consequently, the first best of the economy is given by the following optimisation problem.

$$\max_{\phi} \phi(2-\phi)(v-c) - 0.25\eta\phi(4-3\phi) - 2\xi - \frac{2(\phi)^2}{a+b}.$$

It is then straightforward to obtain the first best recognition level ϕ^{FB} as follows.

$$\phi^{FB} = \frac{2(v-c) - \eta}{2(v-c) + \frac{4}{a+b} - 1.5\eta}. \tag{14}$$

It follows that

$$\frac{\partial \phi^{FB}}{\partial a} > 0, \quad \frac{\partial \phi^{FB}}{\partial b} > 0.$$

Recall that in the decentralised equilibrium $\partial\phi^*/\partial a > 0$ and $\partial\phi^*/\partial b < 0$ hold. Hence, the spillover advertising effects have opposing directions for the first best and for the decentralised equilibrium. In particular, it is easy to show that $\phi^{FB} > \phi^*$ holds, when the maximum subjective evaluation v is large. Hence, the following proposition holds.

Proposition 7: *The spillover advertising effects make the advertising insufficient when the maximum subjective evaluation v is large in the static case.*

On the other hand, it is not very clear if the direct advertising effects cause an insufficient level of advertising even if v is large, since it has positive effects on the recognition level in both cases.

The discrepancy regarding the spillover advertising effects arises because they are treated symmetrically with the direct advertising effects in the first best, while they are regarded differently in the decentralised equilibrium. In fact, the spillover effect to the opponent is a negative externality, whilst the spillover effect from the opponent is a positive externality. In the decentralised equilibrium, the firms fail to internalise these externalities, and consequently, they advertise insufficiently when v is sufficiently large.

In the dynamical case, the maximum subjective evaluations v_t^k grows over time. Hence, unless we introduce depreciation for v_t^k, the total surplus will be completely dominated by the aggregate consumer surplus, which is nonsensical. In fact, full recognition is the first best (i.e. $\phi^{FB} = 1$) if there is no depreciation. To make the analysis meaningful, we need to introduce depreciation so that there is a steady state for v_t^k. With an introduction of depreciation, the condition (14) remains valid in the dynamical case, although we need to examine the R&D expenditures, too.

Regarding the R&D expenditures, the following condition characterises the first best.

$$\delta\phi^{FB}\left(2 - \phi^{FB}\right) f'(z^{FB}) = 1.$$

Hence, z^{FB} increases if and only if ϕ^{FB} increases. It follows that the spillover advertising effects have a positive effect on the R&D expenditures in the first best. However, we know that they have a negative effect on the R&D expenditures in the decentralised equilibrium. Hence, we claim the following.

Proposition 8: *When the steady state level of v_t^k is large, the spillover advertising effects make the R&D expenditures insufficient. Also, the spillover effects have a negative effect on the total surplus, when the steady state level of v_t^k is large.*

On the other hand, the direct advertising effects have a positive impact on the R&D expenditures for both the first case and the decentralised equilibrium, and thus, the conclusion is indeterminate.

4 Discussion

In this section, we discuss the implications and limits of our results. One particular result, which is apparently counter-intuitive, is the negative impact of the spillover advertising effect on the recognition of the products, i.e. $\partial\phi^*/\partial b < 0$. It may appear that the spillover effect should imply a wider recognition of the products, since that should indeed be the case had the advertising expenditures remained the same. However, a decrease in the advertising expenditure caused by the spillover effect is so great that the recognition of the products actually falls. Note that the spillover effect improves the recognition of the opponent firm's product, which means that the environment the firm faces becomes more competitive since there will be more customers who choose between the products of the two firms rather than deciding whether or not to buy the only one they recognise. Hence, each firm has incentives to restrict its advertising expenditure so that the environment remains more monopolistic.

To understand this prediction of the model better, let us consider the consumer digital camera market as a concrete example. Digital camera manufacturers such as Canon, Nikon or Sony advertise their products via various media, explaining that their products have acquired better features such as higher pixels. When consumers are faced with the advertisement (say by Canon), they only recognise the advertised product (say Canon) per se initially. However, when they go to a shop (a conventional one or an online one), they usually have a chance to compare similar products of the competitors (say Nikon or Sony). This clearly means that the advertisement benefits the recognition of the competitors' products.

The above example suggests that a big market place may well have a bigger spillover effect, since consumers will almost certainly encounter with products of the competitors there although the visit there (a big conventional or online shop) was triggered by an advertisement of a firm. Usually, a big market place is understood to generate more competitions among firms, and indeed that is still the case even in our model as far as the demand of consumers there is concerned. However, the competitiveness becomes valid only when the consumers are present in the market place. To avoid letting more consumers better informed and be present in such market places, firms have some incentives not to advertise. Hence, as our model predicts, products mainly supplied through big market places would not be advertised much unless the products are sufficiently differentiated and diversity in tastes is large.

The development of the internet obviously provides consumers with more opportunities to compare different products. It appears that the internet makes the environment more competitive by providing more information. However, it may well be the case that the spillover effect becomes larger with the development of the internet, and that, firms may well have less incentives to advertise. Nevertheless, this is not to predict that the internet will rule advertising out, since the model is not capturing other views of advertising such as the persuasive view.

Note that the model does not describe each individual consumer's problem, and thus, it is not capable of describing the determinants of the consumer behaviour. In other words, the model does not describe how advertising affects individual consumer's behaviour. Even if we confine our attention to the informative view of advertising, there are various ways in which advertising may influence the consumer's recognition, and consequently his behaviour. To see how advertising influences consumer's recognition, it is essential to describe how consumers acquire information. Our model does not address this question, and it only describes the consumer behaviour in a reduced form: the direct effect and the spillover effect.[7]

In fact, a description of information acquisition through advertising is not trivial. It does not make sense to assume expected utility, not to mention a common prior, in which case each consumers knows what piece of information (s)he is missing. Nakata (2006) deals with this issue, and adapts the results of Dekel et al. (2001) in the literature on unforeseen contingencies or unawareness in the context of information acquisition.[8] Nevertheless, it is not straightforward how to extend the results in an equilibrium model.

Furthermore, the issue of (lack of) recognition complicates the welfare analysis. Observe that the consumer surplus of a product defined above is really an ex post concept for consumers who recognise the product, which is not valid for consumers who do not recognise it. In fact, the preference of the same consumer changes in accord with his recognition of the product through advertisement, since the space on which the preference is defined when he does not recognise the product is a subset of the space on which the preference is defined when he recognises the product. In this sense, advertising affects the preferences of the consumers, but this is not the same as changes in tastes as the persuasive view claims.

Although it is common practice to measure welfare by ex post preferences, we argue that it may not be that simple. While such a measurement may be adequate in a partial equilibrium model, it may well not be the case in a dynamical general equilibrium model. In such a model, an apparent mistake due to the lack of recognition may result in a positive result. For example, because of the lack of recognition of some products, a consumer made an investment in a stock instead, although he would not have bought the stock, had he recognised the products. If the stock yields a very high return, the lack of recognition makes the consumer better off ex post. Hence, the welfare analysis is not very trivial when the analysis involves unawareness, or lack of recognition in general.

Another prediction the model provides is regarding relationship between advertising and the R&D expenditures. In our model, each firm makes R&D expenditures to win more consumers who recognise the products of both firms. Hence, an R&D race takes place between the firms, while the extent of the competitiveness of the race is determined by the proportion of consumers who recognise the products of both firms. Since the direct advertising effect increases the equilibrium recognition levels of the products, it makes the R&D race fiercer, while the spillover advertising effect makes the race less intense because it lowers the equilibrium recognition levels. The same logic applies to the effect of the degree of diversity in tastes and/or the product differentiation on the R&D expenditures; the model predicts larger

[7]Robert and Stahl (1993) describes a situation in which consumers proactively look for advertisement by a search model. However, it does not capture a situation of unawareness.

[8]See Dekel et al. (1998) for a literature review.

R&D expenditures when the consumers have more diverse tastes and/or the products are more differentiated.

5 Conclusion

We have examined the direct and spillover advertising effects on an economy. The results concerning the direct advertising effect is consistent with those of Grossman and Shapiro (1984). Namely, a larger direct effect makes the environment more competitive, i.e. lower prices and profits and higher demands. On the other hand, the spillover effect provides incentives for the firm to sustain a more monopolistic environment, i.e. higher prices and higher profits and lower demands. Moreover, a larger direct advertising effect also enhances R&D activities, while a larger spillover effect discourages them.

Although the analysis is based on a simple reduced form model, these results are quite striking and insightful. While the development of the internet appears to enhance competition as well as R&D activities, our results suggest that it may be the opposite if the internet magnifies the spillover advertising effects. Of course, the reality is far more complex than our reduced model describes, and thus, the results need to be understood with care. Yet, our results suggest that careful considerations are needed to design regulations on the internet, since the implications are not necessarily intuitively straightforward.

References

[1] Bagwell, K.: 'The economic analysis of advertising', working paper, Department of Economics, Columbia University 2005.

[2] Blackwell, D.: 'Equivalent comparisons of experiments', *Annals of Mathematical Studies*, **24**, 265-272 (1953).

[3] Boyer, M. C., and M. Moreaux: 'Strategic underinvestment in informative advertising: the cases of substitutes and complements', *Canadian Journal of Economics*, **32**, 654-672 (1999).

[4] Butters, G.: 'Equilibrium distributions of sales and advertising prices', *Review of Economic Studies*, **44**, 465-491 (1977).

[5] Cellini, R. and L. Lambertini: 'Advertising with spillover effects in a differential oligopoly game with differentiated goods', *Central European Journal of Operations Research*, **11**, 409-423 (2003).

[6] Dekel, E., B. L. Lipman and A. Rustichini: 'Recent developments in modeling unforeseen contingencies', *European Economic Review*, **42**, 523-542 (1998).

[7] Dekel, E., B. L. Lipman and A. Rustichini: 'Representing preferences with a unique subjective state space', *Econometrica*, **69**, 891-934 (2001).

[8] Dixit, A. and V. Norman: 'Advertising and welfare', *Bell Journal of Economics*, **9**, 1-17 (1978)

[9] Doraszelski, U., and S. Markovich: 'Advertising dynamics and competitive advantage', *RAND Journal of Economics*, forthcoming (2005).

[10] Dukes, A.: 'The advertising market in a product oligopoly', *Journal of Industrial Economics*, **52**, 327-348 (2004).

[11] Fershtman, C. and E. Muller: 'The benefits of being small: Duopolistic competition with market segmentation', *Review of Industrial Organization*, **8**, 101-111 (1993).

[12] Friedman, J. W.: 'Advertising and oligopolistic equilibrium', *Bell Journal of Economics*, **14**, 464-473 (1983).

[13] Grossman, G. M., and C. Shapiro: 'Informative advertising with differentiated products', *Review of Economic Studies*, **51**, 63-81 (1984).

[14] Hirshleifer, J.: 'The private and social value of information and the reward of inventive activity', *American Economic Review*, **61**, 561-574 (1971).

[15] Lynk, W. J.: 'Information, advertising, and the structure of the market', *Journal of Business*, **54**, 271-303 (1981).

[16] Nakata, H.: 'Modelling choice of information acquisition', Discussion Paper, Department of Accounting, Finance and Management, University of Essex (2006).

[17] Nelson, P.: 'Advertising as information' *Journal of Political Economy'*, **82**, 729-754 (1974).

[18] Nerlove, M. and K. J. Arrow: 'Optimal advertising policy under dynamical considerations', *Economica*, **29**, 129-142 (1962).

[19] Robert, J. and D. O. Stahl: 'Informative price advertising in a sequential search model', *Econometrica*, **61**, 657-686 (1993).

[20] Rogerson, W. P.: 'Price advertising and the deterioration of product quality', *Review of Economic Studies*, **55**, 215-229 (1988).

[21] Stahl, D. O.: 'Oligopolistic pricing and advertising', *Journal of Economic Theory*, **64**, 162-177 (1994).

[22] Stegeman, M.: 'Advertising in competitive markets', *American Economic Review*, **81**, 210-223 (1991).

[23] Sulganik, E. and I. Zilcha: 'The value of information: The case of signal-dependent opportunity sets', *Journal of Economic Dynamics and Control*, **21**, 1615-1625 (1997).

[24] Tirole, J.: *The Theory of Industrial Organization*, Cambridge, MA, MIT Press 1988.

A Proof of Proposition 4

$$
\begin{aligned}
\frac{\partial \pi^*}{\partial a} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial a} + \left(\frac{\partial \pi^*}{\partial \phi^k} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \right) \cdot \frac{\partial \phi^*}{\partial a} - \frac{\partial q^*}{\partial a} \\
&= (p^* - c) \cdot \frac{(\phi^*)^2}{\eta} \cdot \frac{\partial p^*}{\partial a} + (p^* - c) \cdot (1 - \phi^*) \cdot \frac{\partial \phi^*}{\partial a} - \frac{\partial q^*}{\partial a} \\
&= (p^* - c) \cdot \frac{(\phi^*)^2 (1 + \phi^*)}{2\eta} \cdot \frac{\partial p^*}{\partial a} - \frac{\partial q^*}{\partial a} \quad \text{(apply equation (12))} \\
&= \frac{\phi^*(2 - \phi^*)(1 + \phi^*)}{2} \cdot \frac{\partial p^*}{\partial a} - \frac{\partial q^*}{\partial a} \quad \text{(apply equation (9))} \\
&= \frac{1}{D^*} \left\{ \frac{\phi^*(2 - \phi^*)(1 + \phi^*)}{2} \cdot \frac{2(2 - \phi^*)\eta}{G^* q^*} - \frac{t}{2\phi^*(a+b)} + \frac{4t\phi^*[a(1 - \phi^*) + b(2 - \phi^*)]}{G^*(a+b)} \right\} \\
&= \frac{\eta}{D^*} \left\{ \frac{\phi^*(2 - \phi^*)^2 (1 + \phi^*)}{G^* q^*} - \frac{\phi^*}{2q^*(a+b)^2} + \frac{4\phi^*[a(1 - \phi^*) + b(2 - \phi^*)]}{G^*(a+b)} \right\} \\
&= \frac{\eta\phi^*}{D^*} \left\{ \frac{2(2 - \phi^*)^2(1 + \phi^*)(a+b)^2 - G^*}{2G^* q^*(a+b)^2} + F^* \right\} \\
&= \frac{\eta\phi^*}{D^*} \left\{ \frac{2(a+b)^2(\phi^*)^3 - 3(a+b)^2(\phi^*)^2 + 4(a+b)(2a+b)\phi^* + 8(a+b)^2 - 4a^2}{2G^* q^*(a+b)^2} + F^* \right\} \\
&< \frac{\eta\phi^*}{D^*} \left\{ \frac{2(a+b)^2(\phi^*)^3 - 3(a+b)^2(\phi^*)^2 + 3(a+b)^2\phi^* + 8(a+b)^2 - 4a^2}{2G^* q^*(a+b)^2} + F^* \right\} \\
&= \frac{\eta\phi^*}{D^*} \left\{ \frac{2(a+b)^2(\phi^*)^3 + 3(a+b)^2\phi^*(1 - \phi^*) + 8(a+b)^2 - 4a^2}{2G^* q^*(a+b)^2} + F^* \right\} \\
&< 0,
\end{aligned}
$$

where

$$
F^* = \frac{4[a(1 - \phi^*) + b(2 - \phi^*)]}{G^*(a+b)} > 0.
$$

Similarly,

$$
\begin{aligned}
\frac{\partial \pi^*}{\partial b} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial b} + \left(\frac{\partial \pi^*}{\partial \phi^k} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \right) \cdot \frac{\partial \phi^*}{\partial b} - \frac{\partial q^*}{\partial b} \\
&= (p^* - c) \cdot \frac{(\phi^*)^2}{\eta} \cdot \frac{\partial p^*}{\partial b} + (p^* - c) \cdot (1 - \phi^*) \cdot \frac{\partial \phi^*}{\partial b} - \frac{\partial q^*}{\partial b} \\
&= (p^* - c) \cdot \frac{(\phi^*)^2 (1 + \phi^*)}{2\eta} \cdot \frac{\partial p^*}{\partial b} - \frac{\partial q^*}{\partial b} \quad \text{(apply equation (13))} \\
&= \frac{\phi^*(2 - \phi^*)(1 + \phi^*)}{2} \cdot \frac{\partial p^*}{\partial b} - \frac{\partial q^*}{\partial b} \quad \text{(apply equation (9))} \\
&> 0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \pi^*}{\partial \eta} &= \frac{\partial \pi^*}{\partial p^{(k)}} \cdot \frac{\partial p^*}{\partial \eta} + \left(\frac{\partial \pi^*}{\partial \phi^k} \cdot \frac{\partial \phi^{k*}}{\partial q^{(k)}} + \frac{\partial \pi^*}{\partial \phi^{(k)}} \cdot \frac{\partial \phi^{(k)*}}{\partial q^{(k)}} \right) \cdot \frac{\partial q^*}{\partial \eta} \\
&= (2-\phi^*)\phi^* \frac{\partial p^*}{\partial \eta} + \left[(p^*-c) \cdot \frac{2-\phi^*}{2} \cdot \frac{b}{2\phi^*} - (p^*-c) \cdot \frac{\phi^*}{2} \cdot \frac{a}{2\phi^*} \right] \cdot \frac{\partial q^*}{\partial \eta} \\
&= (2-\phi^*)\phi^* \frac{\partial p^*}{\partial \eta} + \frac{2b-(a+b)\phi^*}{2a-(a+b)\phi^*} \cdot \frac{\partial q^*}{\partial \eta} \quad \text{(apply equation (10))} \\
&= (2-\phi^*)\phi^* \frac{\partial p^*}{\partial \eta} + \frac{2b-(a+b)\phi^*}{2a-(a+b)\phi^*} \cdot \frac{G^* q^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} \\
&= (2-\phi^*)\phi^* \frac{\partial p^*}{\partial \eta} + \frac{[2b-(a+b)\phi^*][2a-(a+b)\phi^*]q^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} \\
&= \frac{4a(2\phi^*-(\phi^*)^2) + 4abq^* - 2(a+b)^2\phi^* q^* + (a+b)^2(\phi^*)^2 q^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} \\
&= \frac{[(a+b)^2 q^* - 4a](\phi^*)^2 + 2[4a-(a+b)^2 q^*]\phi^* + 4abq^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} \\
&= \frac{[(a+b)(\phi^*)^2 - 4a](\phi^*)^2 + 2[4a-(a+b)(\phi^*)^2]\phi^* + 4abq^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} \\
&= \frac{[4a-(a+b)(\phi^*)^2]\phi^*(2-\phi^*) + 4abq^*}{4a} \cdot \frac{\partial p^*}{\partial \eta} > 0. \qquad \blacksquare
\end{aligned}
$$