Model uncertainty as partial-identification problems: Application to credible policy promises during crises *

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Abstract

I present a general equilibrium model of intermediary asset pricing where investors are struggling with uncertainty represented by a set of models about future asset return distribution. Each model is consistent with observable information and the understanding of the economic structure but has different implications for return distribution. Confronting such uncertainty, investors fear models predicting low future returns, reducing asset demand due to uncertainty aversion and amplifying the risk premium, especially during crises when capital within the intermediation sector is scarce. Following validation of subjective beliefs by various survey expectations, I evaluate credible policy promises that eliminate some adverse models from investors' set as inconsistent with announcements. I demonstrate the efficacy of announcements that eliminate pessimistic prospects for cash-flow growth and restore risk appetite. Methodologically, I develop agents' inference framework from endogenous variables, where subjective beliefs and other equilibrium dynamics are jointly determined.

Keywords: Asset prices, financial frictions, heterogeneous agent model, risk, robustness, model uncertainty

JEL Codes: C52, C54, D51, D52, D53, D81, D84, E44, E5, E6, G1, G2

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1 Introduction

A growing literature in macroeconomics and asset pricing has developed general equilibrium models with intermediaries who are marginal investors in financial markets. These models examine asset valuation and policy impacts during financial crises; however, they involve hard-to-measure parameters and states which agents inside these models can plausibly view as uncertain.

Existing theoretical frameworks overlook this uncertainty by assuming a rational expectations equilibrium, where agents fully trust the true equilibrium data generating process for asset returns. In these settings, government policies are limited to direct interventions such as recapitalizing distressed financial institutions, aimed at mitigation of heightened risk premia during crises. Such policy analysis ignores the possibility that parameter and state uncertainty are crucial contributors to risk premia, and that alternative policy prescriptions might exist to resolve the uncertainty. For instance, Haddad et al. (2023) emphasize that by altering the market perception, policymakers might do whatever it takes, going to much greater lengths to backstop markets if the situation worsens.

This paper proposes a new general methodology for agents to form alternative beliefs about plausible parameters and states, which must be consistent with observable endogenous objects; therefore, beliefs and other equilibrium dynamics are jointly determined. On the one hand, decisions under the subjective beliefs affect the equilibrium dynamics. On the other hand, observed data disciplines the subjective beliefs. Existing methodologies abstract from this feedback by studying simple environments where Pareto planners directly determine allocations and alternative beliefs are disciplined independently and statistically.

I apply this methodology to a general-equilibrium model featuring financial intermediaries, who are uncertain about parameters and states that are key return predictors. Such agents find many alternative combinations of predictors consistent with observable information and their understanding of the mapping from observable information to the uncertain predictors. I refer to this form of uncertainty as "structured" ambiguity. To guard against uncertainty, ambiguity-averse agents make cautious investment decisions under a worst-case scenario with the most adverse utility consequences, reducing risky investment.

My findings reveal that compensation for structured ambiguity accounts for a substantial portion of total risk premia, revealing the importance of policy interventions and regulations affecting this component of the risk premia. I use this framework as a laboratory, demonstrating the effectiveness of conditional policy promises aimed at resolving agents' uncertainty about return predictors. Such promises eliminate the worst-case scenario by shrinking the set of alternative return predictors consistent with observable information and understanding of the underlying economic structure.

The economic structure builds upon He and Krishnamurthy (2013) (hereafter HK), which includes risky and risk-free asset markets. The risky asset represents the claim on the aggregate dividend that stochastically evolves and has a fixed unit supply. The net supply of the risk-free asset is zero.

Two types of identical agents, intermediaries and households, exist. Households that trust the true data-generating process can invest in the risk-free asset market without friction; however, they cannot directly invest in the risky asset and must invest in the funds of financial intermediaries subject to margin constraints. The households' contribution is capped by a fraction of the intermediaries' own wealth, reflecting the tightness of the constraint. Intermediaries invest their own wealth and households' contribution in risky and risk-free assets without friction to maximize their lifetime utility.

In this economy, the total wealth share of intermediaries (or the financial sector's capitalization) and the tightness of the constraint, are key predictors for future risky asset returns. Low capitalization and tight financial constraints constrain households' contribution to the financial intermediary. In turn, intermediaries must borrow more risk-free funds from households to finance their risky asset holding, leading to a highly leveraged long position in this asset. The risk premium must increase in equilibrium to clear the market and compensate for this situation.

The expected cash-flow growth from the risky asset predicts the future risk-free rate; the higher expected cash flow growth predicts higher economic growth, increasing future risk-free rates.

Unlike HK, intermediaries under uncertainty aversion are uncertain about these three return predictors and make cautious decisions under the worst-case belief regarding the return predictors. This worst-case belief represents the scenario that minimizes their lifetime utility. In equilibrium, intermediaries fear lower expected returns on the risky asset, which would slow their wealth accumulation.

Intermediaries discipline the set of combinations of the three return predictors using observable information on the return volatility of the risky asset, realized risk-free rates, and their understanding of the mapping from those predictors to observable information. Intermediaries find that many combinations of predictors are consistent with the observable information, leading to a partial identification problem.

I illustrate the presence of multiple combinations of return predictors consistent with observable information with the following example. During a financial crisis, the financial sector's capitalization is low, and the margin constraint is binding. The economy experiences high return volatility and a lower risk-free rate; high return volatility arises because adverse shocks amplify reductions in intermediaries' wealth, reducing risky asset demand and asset prices. The risk-free rate drops due to the higher precautionary saving motive of the highly leveraged financial sector.

However, high return volatility and a low risk-free rate could also be consistent with high financial sector capitalization and lower cash flow growth. High capitalization would make the financial sector a more significant player, leading to considerable asset price fluctuations even with moderate leverage, indicating high return volatility. Lower cash flow growth prospects could reduce the risk-free rate, aligning it with observable information. This alternative scenario corresponds to each intermediary perceiving a lower expected return.

Since individual intermediaries cannot observe aggregate capitalization of the entire financial sector or the tightness of margin constraints of other intermediaries, they view both scenarios as possibly plausible and rationalizing available information. However, the latter scenario is more adverse from their perspective, since the lower perceived expected returns constitute less advantageous investment opportunities.

To guard against this alternative scenario, intermediaries reduce their demand for the risky asset compared to demand without structured ambiguity. In equilibrium, the difference in expected excess returns under the true data-generating process and under the worst-case belief accounts for approximately 40% of the total risk premia in this economy. This difference is termed as "the price of partial identification," representing the compensation for uncertainty induced by the partial identification problem.

This price of partial identification is especially amplified during crises, where the actual scenario with low capitalization is quite different from that with high capitalization perceived by individual intermediaries.

I further substantiate the model's predictions with survey evidence for subjective expectations, aligning with worst-case beliefs. In line with analysts' mean forecasts in the U.S. aggregate stock market (De la O and Myers (2021)), the price-dividend ratio is influenced more by subjective dividend forecasts than return forecasts. Moreover, investors consistently tend to underestimate future returns and dividend growth, particularly during crises. This concurs with various decision-makers' mean forecasts in alternative financial markets as reported by Nagel and Xu (2023), where the subjective risk premium demonstrates greater acyclicality than the objective risk premium. All these findings contradict the predictions of rational expectations equilibrium, establishing the current model as a natural laboratory for analyzing policies aimed at managing subjective expectations.

In policy experiments, conditional policy promises resolving uncertainty concerning cash flow growth are highly effective at mitigating heightened risk premia. Under this policy, like guaranteeing the cash flow from mortgage-backed securities, agents infer that the lower risk-free rate arises from the high precautionary saving motive of the sector associated with lower capitalization, not the lower cash flow growth, and that a lower capitalization causes the high return volatility. Agents then conclude that expected excess returns must be high, which increases their appetite for risky assets.

In contrast, policy promises to resolve uncertainty about the tightness of financial constraints such as capital requirements and deposit insurance, are not as effective. They do not eliminate the possibility of higher capitalization of the financial sector and less profitable opportunities in the market.

These examples provide this paper's main conceptual insight. Policymakers should be aware of how their policies alleviate uncertainty and shape the beliefs of market participants during financial crises to understand their efficacy. In particular, the understanding of how policy actions mitigate the worst-case scenario is crucial, which requires explicitly modelling the worst-case scenario in this paper.

Related Literature

This paper contributes to two strands of the literature. First, this paper provides a theoretical contribution to the ambiguity literature. Recent work by Hansen and Sargent (2022) refine the concept of ambiguity and distinguish between model misspecification concerns and structured ambiguity.

In the former, agents fear that all parametric economic theories are misspecified, leading to cautious decisions based on a worst-case model that is statistically close to alternative parametric baseline models but not necessarily a well-specified parametric model itself. The classic literature assumes the single parametric baseline model conceived by agents as in Hansen et al. (1999), Hansen and Sargent (2001), Anderson et al. (2003), Hansen et al. (2006), and Barillas et al. (2009), followed by many applications in more general environments[1]

In contrast, structured ambiguity involves concerns about identifying which parameter-

[1]See Pouzo and Presno (2016) and Bhandari et al. (2023) for applications of model misspecification concerns to general economic environments such as sovereign debt markets and labor markets. The optimal Ramsey policies under model misspecification concerns have been studied by Karantounias (2013), Ferriere and Karantounias (2019), and Karantounias (2023).

ized economic models represent the true data generating process, building on the static setting of Gilboa and Schmeidler (1989) and the dynamic extension by Chen and Epstein (2002). A challenge associated with incorporating structured ambiguity is how agents inside the models find alternative parametric models.

The existing applications of structured ambiguity are limited to simple environments involving representative agents' decision making, which is reduced to solving a planner's problem or where agents restrict the set of latent states from exogenous signals.[2] In this problem, agents or Pareto planners are uncertain about parameters characterizing exogenous processes in the economy and discipline the set of alternative parameters statistically, independent of equilibrium allocation and prices.

In contrast, this paper introduces structured ambiguity into a general environment where prices, allocations, and the worst-case belief are jointly determined. This approach necessitates that belief formation is consistent with observed endogenous prices. In the application in this paper, agents contemplate alternative parametric economic theories of financial intermediation with different values of parameters and states.

Second, it makes an applied contribution to the literature that develops general equilibrium models with financial frictions in macroeconomics and asset pricing initiated by Kiyotaki and Moore (1997) and Bernanke et al. (1999). This paper incorporates ambiguity over hard-to-measure parameters and states into a canonical model of this class and explores policy implications. Consistent with this paper's theoretical prediction, Bachmann et al. (2020) provide empirical findings documenting joint movements in perceived ambiguity and measured risk premia, such as those present in credit spreads.

The novel aspect of this paper's approach is incorporating this uncertainty following the ambiguity literature. This methodology offers a practical and manageable way to address multi-dimensional uncertainty without requiring tracking an extensive array of state variables

[2]For example, see Hansen and Sargent (2010), Boyarchenko (2012), and Han et al. (2022) for models with robust learning of unknown states from exogenous signals. Hansen and Sargent (2021) and Balter et al. (2023) study asset pricing in representative agent models with structured ambiguity and model misspecification concerns in the spirit of Hansen and Sargent (2022).

encompassing time-varying parameters and states[3]. Maintaining a small state space within the model is vital for the computational characterization of the equilibrium globally. This approach is particularly crucial when characterizing the essential state dependence observed in this class of models, as underscored by recent contributions by Adrian and Boyachenko (2012), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Di Tella (2017), and He and Krishnamurthy (2019).

This paper's focuses on the endogenous subjective beliefs during financial crises is close to the work examining multiple equilibria under rational expectations equilibrium in macro-finance models and implications of jumps across multiple equilibria for equilibrium dynamics such as Gertler and Kiyotaki (2015), Gertler et al. (2016), Gertler et al. (2020), and Khorrami and Mendo (2023). Like the classic study of bank runs by Diamond and Dybvig (1983), those models do not predict any subjective and objective probability over multiple equilibria.

In contrast, the set of the beliefs over alternative scenario and the worst-case scenarios are all endogenous in this paper, depending on the economic fundamentals, such as the capitalization of the financial sector. Given the empirical support from survey data for the subjective beliefs in the current paper, which is against rational expectations equilibrium models, I provide a natural framework to analyze how policy interventions alter the endogenous beliefs and implications for equilibrium dynamics.

More broadly, some recent work integrates the deviation from a rational expectations equilibrium in the form of behavioral expectation biases into macro-finance models. These works include Krishnamurthy and Li (2021) and Maxed (2023), which attempt to replicate the empirical boom-bust credit cycles surrounding financial crises. Furthermore, Fontanier (2022) studies the optimal policies in the presence of these expectations biases with financial frictions. This paper differs by explicitly modeling the endogenous formation of the subjective beliefs due to ambiguity concerns, predicting the joint movements in perceived ambiguity

[3]Caballero and Simsek (2013) also employ the Gilboa and Schmeidler (1989)'s min-max utilities to enhance the analytical tractability instead of expected utilities in a static model of banks' uncertainty consequence of financial network for counter-party risks during financial crises.

and measured risk premia, empirically documented in Bachmann et al. (2020).

The rest of this paper is structured as follows. Section 2 describes the model ingredients and the equilibrium in the sequential formulation. Section 3 and Section 4 characterize the set of alternative models and the worst-case beliefs in a Markovian setting. In Section 5, I calibrate the model and verify the worst-case model is statistically hard to distinguish from the true date generating process. Section 6 shows the impact of the partial identification challenge faced by investors on the risk premia, and Section 7 illustrates how government announcements can mitigate heightened risk premia in crisis episodes. Finally, Section 8 concludes with a discussion of future research.

2 Model

This section introduces ambiguity regarding alternative asset return processes into agents' preferences due to uncertainty regarding specific parameters and states in the intermediary asset pricing model in the spirit of He and Krishnamurthy (2013) (henceforce, HK). The HK model is one of the pioneering quantitative papers in the continuous-time intermediary asset pricing literature.

The central novel argument with the model of beliefs is introduced in subsection 2.5. I shortly describe the common elements with HK from subsection 2.1 to subsection 2.4. I adopt HK's notation when possible.

2.1 Model Set-Up

Time is continuous, denoted by $t \in [0, \infty)$ representing the current period. There are two distinct groups of agents: households and intermediaries. While households lack the expertise to invest in the risky asset market directly, intermediaries possess the required knowledge. Intermediaries can act on behalf of households in risky asset investments. This intermediary role is central to the model's structure: households demand intermediation services, and intermediaries supply these services.

Consequently, households face a portfolio decision, involving allocating their wealth between acquiring equity in the intermediaries and investing in risk-free bonds. Intermediaries, in turn, receive the equity contribution from households, combine them with their own wealth, and allocate the entire pool of managed funds between the risky asset and risk-free bonds. I will delve into a detailed examination of each component of this model in the forthcoming sections. First, the common elements shared with the HK model will be reviewed and then the novel element will described: intermediaries' preferences incorporating structured ambiguity about the true equilibrium data-generating process (DGP) due to the uncertainty about parameters and states.

2.2 Assets

The assets in this model adhere to the structure outlined in the Lucas (1978) tree economy, where a single perishable consumption good serves as the numeraire. I normalize the total supply of intermediated risky assets to one unit. Meanwhile, the riskless bond has zero net supply and is open for investment by both households and intermediaries.

The risky asset in this model yields a dividend flow D_t, which follows a geometric Brownian motion described by the following stochastic differential equation:

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t,$$

where D_0 is the initial condition. Here, g and σ represent the mean dividend growth and volatility.

In this paper, I work within the framework of a probability space denoted as $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ represents the true DGP. The stochastic process, Z_t, is established as a standard Brownian motion on this complete probability space.

Additionally, we define two key processes, P_t and r_t, which correspond to the risky asset price and interest rate processes, respectively. Furthermore, we define the total return on the risky asset, dR_t, which follows the equation:

$$dR_t = \frac{D_t dt + dP_t}{P_t} = (\pi_{R,t} + r_t)dt + \sigma_{R,t} dZ_t,$$

Here, $\pi_{R,t}$ represents the expected excess return and $\sigma_{R,t}$ is the return volatility under the true DGP determined in equilibrium.

2.3 Intermediary and Margin Constraint

At any given time t, each intermediary is randomly matched with a household. These interactions happen instantly, resulting in a continuum of identical bilateral relationships. Household j allocates a part of its wealth, $H_{j,t}$, to purchase equity issued by the intermediary. The wealth of intermediary i at time t, denoted as $w_{i,t}$. Intermediaries execute trades in a Walrasian risky asset and bond market, while households trade solely in the bond market. At $t + dt$, the match concludes, and the intermediation market repeats the process.

Considering a relationship between an intermediary i and household j, the intermediary's total funds comprise their wealth, $w_{i,t}$, and that allocated to the intermediary by the household, $H_{j,t}$. The intermediary makes all investment decisions for these total funds and faces no portfolio restrictions regarding buying or short-selling either the risky asset or the risk-free bond. Let $\alpha^I_{i,t}$ denote the ratio of the intermediary's risky asset holdings to its total funds, $w_{i,t} + H_{j,t}$. This ratio, capturing leverage, is typically larger than one. Therefore, the return on funds delivered by the intermediary is described by the following equation:

$$dR^I_t = r_t dt + \alpha^I_t (dR_t - r_t), \tag{1}$$

where dR_t represents the total return on the risky asset. When $\alpha_{i,t} > 1$, the intermediary invests more than 100 percent of the total funds in risky assets and borrows $(\alpha^I_{i,t} - 1)(w_{i,t} + H_{j,t})$ through the risk-free short-term bond market, thus making a leveraged investment in the risky asset.

The household is unwilling to invest more than $mw_{i,t}$ in the equity of a matched intermediary, where $m > 0$ is a constant parametrizing the financial constraint and $w_{i,t}$ is the wealth held by an intermediary i managing the intermediary. If an intermediary invests one dollar in his entire pool of managed funds, the household will invest at most m dollars of its own wealth. The margin constraint implies that the intermediary's demand, $H_{j,t}$, facing a household is at most:

$$H_{i,t} \leq mw_{i,t}. \tag{2}$$

A small m or $w_{i,t}$ restricts the household's ability to participate indirectly in the risky asset market. This constraint influences risk premia and asset prices in equilibrium[4].

2.4 Households: The Demand for Intermediation

In the HK model, the household sector is represented as an overlapping generation (OG) of agents, simplifying the household's decision problem. I explain the OG environment in a continuous-time model by indexing time as t, $t + \delta$, $t + 2\delta$, ... considering the continuous time limit when δ is of the order dt. A unit mass of generation t agents is born with wealth w_t^h and lives during periods t and $t + \delta$. Unlike intermediaries, households fully trust the true DGP and aim to maximize utility:

$$\rho\delta \log c_t^h + (1 - \rho\delta) E_t[\log w_{t+\delta}^h],$$

where c_t^h is the household's consumption rate in period t and w_t^h represents a bequest for generation $t + \delta$. Importantly, E_t is the expectations operator under the true DGP. Ad-

[4]This constraint, linking "net worth" and external financing, is standard in financial friction literature and can be justified by various agency or informational frictions. For instance, in He and Krishnamurthy (2012), hedge fund managers often have a significant portion of their wealth tied up in the fund. External investors require managers to have a substantial stake ("skin in the game") to align incentives. If the hedge fund sustains losses, depleting managers' stakes, investors are reluctant to contribute further capital due to concerns about mismanagement or more losses. This scenario, known as a "hedge fund capital shock," is captured in the model.

ditionally, generation t households are assumed to receive labor income at date t of $lD_t\delta$. Here, $l > 0$ is a constant, and recall that D_t is the dividend rate on the risky asset at time t: thus, labor income is proportional to the economy's aggregate output. This inclusion of labor income is crucial because it prevents scenarios where the household sector vanishes from the economy due to lack of income.

Households invest their wealth w_t^h from t to $t+\delta$ in financial assets. A fraction $(\lambda < 1)$ of households called debt households are required to keep in short-term debt issued by the intermediary sector, representing a baseline demand for holding a portion of household wealth in a risk-free asset in total λw_t^h. The household sector's demand for liquid balances is satisfied by issuing bank deposits. This feature is important as it generates leverage in the intermediary sector, even when the margin constraint does not bind, allowing for stochastic dynamics of the key state variable, the aggregate wealth share of intermediaries, in unconstrained states. The remaining fraction of households with total wealth $(1-\lambda)w_t^h$ called risky households can invest in matched intermediaries and risk-free bonds.

To summarize, a debt and risky asset household are born at generation t with wealth of w_t^h . The households receive labor income, choose consumption, and make savings decisions, respecting the restriction on their investment options. It is easy to verify that in the continuous time limit, i.e., when $\delta \to dt$, the households' consumption rule is

$$c_t^h = \rho w_t^h.$$

Debt households invest λw_t^h in the bond market at the interest rate r_t. A risky asset household with wealth $(1-\lambda)w_t^h$ decides the fraction $\alpha_t^h \in [0,1]$ to invest in intermediaries' equity. The remaining $1-\alpha_t^h$ of the risky asset household's wealth is allocated to the risk-free bond. Given the log utility, the risky asset households choose α_t^h to solve the following optimization problem:

$$\max_{\alpha_t^h \in [0,1]} \alpha_t^h E_t[dR_t^I - r_t dt] - \frac{1}{2}(\alpha_t^h)^2 Var_t[dR_t^I - r_t dt]$$

subject to the margin constraint (2)

$$H_t = \alpha_t^h (1-\lambda) w_t^h \leq m w_t,$$

where the dependence on identity i, j is omitted since households and intermediaries are identical within their classes ($w_{j,t}^h = w_t^h$) and ($w_{i,t} = w_t$). The evolution of w_t^h across generations is described by

$$dw_t^h = (lD_t - \rho w_t^h)dt + w_t^h r_t dt + \alpha_t^h (1-\lambda) w_t^h (dR_t^I - r_t dt).$$

2.5 Intermediaries

There exists a unit mass of identical and infinitely-lived intermediaries where households invest their resources. Intermediaries contemplate alternative stochastic processes of returns on the risky asset. Absolute continuity implies that the return volatility, $\sigma_{R,t}$, is observable in the continuous-time Brownian motion environment; thus, intermediaries are solely concerned about alternative specifications of expected excess return on the risky asset, $\pi_{R,t}$. They distrust the baseline true parameters and current state realization; therefore, they contemplate alternative expected excess returns which could prevail in equilibria with alternative true parameters and current state realizations.

I assume intermediaries doubt the baseline, or true values of margin constraint parameter m, the mean dividend growth g, and the aggregate wealth share of intermediaries $x_t \equiv w_t/P_t$, which are essential return predictors in this model, as discussed in section 4. From now on, I distinguish the true (baseline) parameter and state values with hats $(\hat{m}, \hat{g}, \hat{x}_t)$ from the general notation of these variables without hats (m, g, x_t).

2.5.1 Formulation of Alternative Models

Following Hansen and Sargent (2022), I describe a set of alternative models for the expected excess returns using convenient mathematical representations of positive martingales that modify a baseline probability model. For intermediaries, these martingales are likelihood ratios between the alternative and the baseline model. Starting from the intermediaries' baseline probability measure, I use martingales to represent probabilities that intermediaries consider as plausible alternative models.

For clarity, I use the following baseline model (or the true DGP) of the stochastic process governing the dynamics of excess returns:

$$dR_t - r_t dt = \hat{\pi}_{R,t} dt + \sigma_{R,t} dZ_t,$$

where $\hat{\pi}_{R,t}$ represents the expected excess return under the baseline model and is a measurable function with respect to the filtration $\mathcal{F}$.

Intermediaries contemplate alternative models for the excess return represented as likelihood ratios, which are strictly positive martingales with unit expectations under the baseline model. In the continuous-time Brownian information environment, owing to the Girsanov Theorem and related results, we can describe the evolution of a likelihood ratio denoted as M^S of an alternative process relative to the baseline specification as follows:

$$dM_t^S = M_t^S S_t dZ_t,$$

where S_t is progressively measurable with respect to the filtration $\mathcal{F}$. If

$$\int_0^t |S_\tau|^2 d\tau < \infty \tag{3}$$

with probability one, the stochastic integral $\int_0^t S_\tau dZ_t$ is well-defined. Imposing the initial condition $M_0^S = 1$, we express the solution of the stochastic differential equation (2) as a stochastic exponential:

$$M_t^S = \exp\left(\int_0^t S_\tau dZ_\tau - \frac{1}{2}\int_0^t |S_\tau|^2 d\tau\right). \tag{4}$$

Definition 1 *$\mathcal{M}$ denotes the set of all martingales M^S, constructed as stochastic exponentials via representation* (4) *with an S_t that satisfies* (3) *and is progressively measurable with respect to $\mathcal{F}$.*

In the subsequent discussion, I use the process S to represent alternative martingales of interest. Probabilities are implicitly described by delineating the family of conditional expectations associated with each such S process, namely:

$$E^S(B_t|\mathcal{F}_0) = E(M_t^S B_t|\mathcal{F}_0)$$

for any $t \geq 0$ and any bounded $\mathcal{F}_t$-measurable random variable B_t. This representation uses the positive random variable M_t^S as a Radon-Nikodym derivative for the date t conditional expectation operator $E^S(\cdot|\mathcal{F}_0)$. The martingale property for M^S ensures that the law of iterated expectations applies to the constructed probability measures. Subsequent sections will refer to this probability measure as being affiliated with the martingale M^S.

Under the alternative model, the evolution of expected excess returns follows:

$$dR_t - r_t = \pi_R^S dt + \sigma_{R,t} dZ_t^S,$$

where π_R^S represents the expected excess return under the alternative model, and

$$dZ_t = S_t dt + dZ_t^S,$$

where dZ_t^S is now a standard Brownian motion under the alternative model.

Importantly, intermediaries restrict the set of alternative specifications of the excess return processes in a structured way; thus, the alternative expected excess return must correspond to the equilibrium outcome in an alternative economy with alternative parameters (m, g) and aggregate state x_t, which is the aggregate wealth share of intermediaries, or the capitalization of the financial sector. This restriction disciplines the set of alternative models, S and M^S.

Definition 2 *$\mathcal{M}^S$ denotes the set of all martingales M^S in $\mathcal{M}$ that satisfies $S_t = \hat{\pi}_{R,t} - \pi_R(x_t, m, g)$. Here, $\pi_R(x_t, m, g)$ is the equilibrium expected excess return in an alternative economy with a wealth share of intermediaries x_t and parameters $m \in (0, \infty)$ and $g \in (-\infty, \infty)$.*

Under an alternative model in $\mathcal{M}^S$, the expected excess return evolves as follows:

$$dR_t - r_t = \pi_R^S(x_t, m, g)dt + \sigma_{R,t}dZ_t^S.$$

2.5.2 Restricting the Set of Structured Alternative Models

Without any further restrictions on alternative combinations of parameters and state, the set of alternative models $\mathcal{M}$ is too large. Many of those alternative combinations are inconsistent with the observable information from the realized excess returns, i.e., the return volatility and the risk-free rates,[5] along with the understanding of equilibrium relationships implied by the structure of the alternative economy, or the cross-equation restrictions. The following subsections elaborate on how intermediaries discipline the set of alternative models for expected excess returns by exploiting these restrictions.

In particular, at each period t, intermediaries form a set of (x_t, m, g) consistent with the observable return volatility, $\sigma_{R,t}$, and risk-free rate, r_t, by using the understanding of the equilibrium relationships between (x_t, m, g) and the observable σ_R and r, the so-called

[5] In continuous time with Brownian motion, observing returns on the risky asset allows for observing return volatility due to the continuity of the alternative models, or probability measures.

cross-equation restriction such that

$$\underbrace{\sigma_{R,t}}_{Observable\ information} \underbrace{=}_{Cross-equation\ restriction} \underbrace{\sigma_R(x_t, m, g)}_{Model}.$$

$$\underbrace{r_t}_{Observable\ information} \underbrace{=}_{Cross-equation\ restriction} \underbrace{r(x_t, m, g)}_{Model}.$$

$\sigma_R(x_t, m, g)$ and $r(x_t, m, g)$ are the equilibrium return volatility and the risk-free rate, respectively, in an alternative economy with the true value of margin constraint parameter m, current wealth share x_t, and the mean dividend growth rate g.

The collection of (x_t, m, g) forms the partially-identified set, with each element characterizing the alternative infinitesimal expected excess return $\pi^S_{R,t}$ and the associated local drift distortion S_t in Ξ_t. Because of the one-to-one correspondence between (x_t, m, g) and S_t. I denote the elements in Ξ_t as either (x_t, m, g) and S_t, Following Chen and Epstein (2002), I formulate the restricted set of $\mathcal{M}^O$ in terms of Ξ_t

$$\mathcal{M}^o \equiv \{M^S \in \mathcal{M}^{\mathcal{S}} : S_t \in \Xi_t \text{ for all t} \geq 0\}. \tag{5}$$

Discussion of dynamic consistency and admissibility of the worst-case belief: *Generally, the set Ξ_t is neither convex nor compact, which Epstein and Schneider (2003) suggested is sufficient to ensure the dynamic consistency of dynamic max-min preferences introduced later. To overcome this issue, I expand the original set Ξ_t to make it convex and compact such that the expanded set $\tilde{\Xi}_t$ includes all the alternative models S of expected excess returns that reside in the interval between the upper and lower bounds of the original set Ξ_t. Even in this expanded set, the minimization problem will choose the maximum or minimum S in the original set because the minimization problem has a linear objective function for S. Thus, the minimization problem under the expanded set still yields the admissible belief; therefore, assuming the formation* (5) *leads to the dynamic consistency of max-min preferences is not an issue in this model.*

To cope with uncertainty about alternative specifications, intermediaries choose the alternative model to minimize the expected lifetime utility to guard their decision rules against the structured ambiguity. More formally, the continuation value process $\{V_{i,t} : t \geq 0\}$ of an intermediary i is

$$\begin{aligned} V_{i,t} &= \min_{S_\tau \in \Xi_\tau : t \leq \tau < \infty} E\left(\int_0^\infty \exp(-\rho\tau) \left(\frac{M^S_{t+\tau}}{M^S_t} [u(c_{i,t+\tau}) d\tau | \mathcal{F}_\tau] \right) \right) \\ &= \min_{\mathcal{M}^o} E^S_t \left[\int_0^\infty \exp(-\rho\tau) u(c_{i,t+\tau}) d\tau \right], \end{aligned}$$

where $u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma}$.

Given the worst-case belief, the intermediary chooses his consumption rate and the portfolio decision of the intermediary to solve

$$\max_{c_{i,t+\tau}, \alpha^I_{i,t+\tau}} \min_{\mathcal{M}^o} E^S \left[\int_0^\infty \exp(-\rho\tau) u(c_{i,t+\tau}) d\tau \right]$$

s.t.

$$dw_{i,t} = -c_{i,t} dt + w_{i,t} r_t dt + w_{i,t} (dR^I_t(\alpha^I_{i,t}) - r_t dt).$$

where the intermediary return $dR_t(\alpha^I_{i,t})$ is given by (1). We can also rewrite the budget constraint in terms of the underlying return:

$$dw_{i,t} = -c_{i,t} dt + w_{i,t} r_t dt + \alpha^I_{i,t} w_{i,t} (\pi^S_{R,t} dt + \sigma_{R,t} dZ^S_t).$$

Note that the intermediary's portfolio choice of $\alpha^I_{i,t}$ effectively maximizes his lifetime utility. **Discussion of parameter and state uncertainty**: *As an illustration of uncertainty about the capitalization of the financial institutions in the intermediated asset markets, Adrian et al. (2014) and He et al. (2017) differ in their definitions of capital. He et al. (2017) include the capital of the bank holding companies to that of their US broker-dealers, whereas Adrian et al. (2014) do not. Controversy exists regarding whether the internal market within those* *banks allows for transferring capital from one subsidiary to another.*

Relatedly, the unobservability of the aggregate intermediaries' capitalization aligns with the assumption made in the Caballero and Simsek (2013). Individual banks cannot directly observe the balance sheet information of other banks and are uncertain about counter-party risks in the model of financial networks in crises.

Regarding the uncertainty about the tightness of constraint m, footnote 10 in He and Krishnamurthy (2013) documents the concern about the correct value of this parameter in their calibration.[6]*.*

Discussion of information and preferences: *I allow intermediaries to perceive that the unknown parameters and states could be time varying and that their evolution is too complex to conceive a well-specified parametric law of motion. This possibility hinders intermediaries from updating the set of alternative beliefs by Bayesian learning.*

The min-max preference specification is more suitable than the recursive smooth ambiguity preferences in the current information assumption since the model does not predict any unique prior over the parameters and states[7]*.*

2.6 Equilibrium

Definition 3 *The equilibrium parameterized by a baseline value of $(\hat{m}, \hat{g})$ must satisfy the following conditions. It comprises price processes $\{P_t\}$ and $\{r_t\}$, decisions $\{c_t, c_t^h, \alpha_t^I, \alpha_t^h\}$, and the set of alternative beliefs $\{\Xi_t\}$ such that:*

1. *Given the price processes and beliefs, decisions solve the consumption-savings problems of the debt household, the risky asset households and the intermediaries;*

2. *Decisions satisfy the intermediation constraint;*

[6] *"The m in our calibration applies to the entire intermediation sector, and as is evident in Table 1, there is functional heterogeneity across the modes of intermediation. In particular, it is not obvious what the m for the mutual fund or pension fund sector should be, which may lead one to worry about our choice of m based solely on considering the leveraged sector".*

[7] The min-max preferences can be viewed as a limit of the recursive smooth ambiguity preferences developed by Klibanoff et al. (2005) in a static and Klibanoff et al. (2009) in a dynamic environments as the degree of ambiguity aversion approaches infinity.

3. *The risky asset market clears*

$$\frac{\alpha_t^I(w_t + \alpha_t^h(1-\lambda)w_t^h}{P_t} = 1;$$

4. *The goods market clears;*

$$c_t + c_t^h = D_t(1+l);$$

5. *The alternative models Ξ_t must be consistent with the observed return volatility and the risk-free rate. $\sigma_R(x_t, m, g)$ and $r(x_t, m, g)$ must be implied by an equilibrium in the set of alternative economies parameterized by some (m, g).*

The following subsection focuses on a stationary Markov equilibrium, where the aggregate state variables consist of the aggregate dividend D_t and the aggregate wealth share of intermediaries $\hat{x}_t$. As standard in any economy with Constant relative risk aversion (CRRA) agents where endowments follow a geometric Brownian motion, I conjecture that the equilibrium risky asset price is

$$P_t = D_t p(\hat{x}_t),$$

where $p(\hat{x}_t)$ is the price-dividend ratio of the risky asset. Additionally, I conjecture that the equilibrium expected excess return $\pi_{R,t}$, return volatility $\sigma_{R,t}$, and risk-free rate r_t are all functions of only $\hat{x}_t$. Moreover, the aggregate intermediaries' wealth share evolves as

$$dx_t = \mu_x(\hat{x}_t)dt + \sigma_x(\hat{x}_t)dZ_t.$$

3 Recursive Representation of Preferences and Decisions

This section outlines the Markovian decision problem, corresponding to the sequential formulation in subsection 2.5, by characterizing a set of structured models and a continuation value process over consumption streams. The set of structured models, denoted as Ξ_t, is defined in terms of alternative parameters and the current realizations of the Markov state variable, x_t. In the context of Markovian decision problems, a Hamilton-Jacobi-Bellman (HJB) equation describes the evolution of continuation values.

3.1 Hamilton-Jacobi-Bellman Equation

The homothetic property of intermediaries' preferences implies their value function in the form:

$$J(w_{i,t}, Y(x_t)) = \frac{[w_{i,t}Y(x_t)]^{1-\gamma}}{1-\gamma},$$

where $Y(x_t)$ represents the future investment opportunity, evolving as

$$\frac{dY}{Y} = \mu_Y(x)dt + \sigma_Y(x)dZ.$$

Then the Hamilton-Jacobi-Bellman (HJB) equation of an intermediary i is formulated as:

$$\rho J_i = \max_{\alpha_i, c_i} \min_{S \in \Xi} \frac{c_i^{1-\gamma}}{1-\gamma} + (\partial_Y J_i)\mu_Y^S Y + (\partial_W J_i)(rw_i + \alpha_i \pi_R^S w_i - c_i) + \frac{1}{2}(Y\sigma_Y)^2(\partial_{YY}J_i)$$
$$+ \frac{1}{2}(\partial_{WW}J_i)w_i^2(\alpha_i\sigma_R)^2 + w_i(\alpha_i\sigma_R)Y\sigma_Y(\partial_{WY}J_i),$$

where

$$\pi_R^S = \pi_R - \sigma_R S;$$

$$\mu_Y^S = \mu_Y - \sigma_Y S.$$

S captures the differences between the subjective and true local drift. In particular,

$$\mu_Y - \mu_Y^S = (\partial_x Y)\hat{x}(\mu_x - \mu_x^S) = (\partial_x Y)\hat{x}(\hat{\alpha}^I - 1)(\pi_R - \pi_R^S),$$

as shown in the Appendix, where $\hat{\alpha}^I$ is the portfolio weight on the risky asset of the aggregate financial sector, which is greater than 1 in equilibrium. Therefore, alternative expected excess

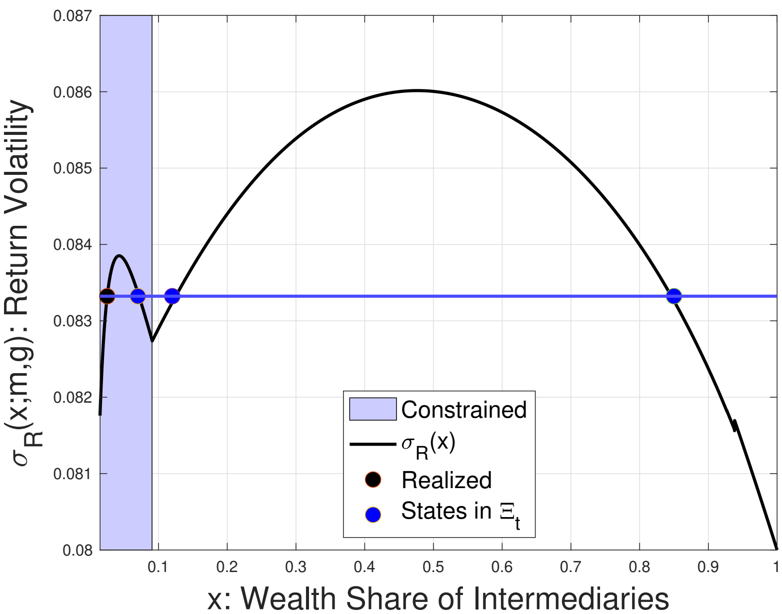

Figure 1: Return Volatility

The return volatility is graphed as a black curve against $x = w/P$, the intermediaries' aggregate wealth as a percentage of the assets in the economy. The black circle corresponds to the realized return volatility mentioned in the main text. Parameters are from Table 1 and Table 2. The shaded blue area corresponds to the constrained region.

returns on the risky asset affect the individual utility by accumulating individual intermediaries' wealth $w_{i,t}$ and the accumulation of aggregate intermediaries' wealth x_t. subsection 4.3 elaborates on the differential effects of alternative higher expected excess returns through these two channels, detailing how the worst-case parameter and state $(x^{worst}, m^{worst}, g^{worst})$ are determined.

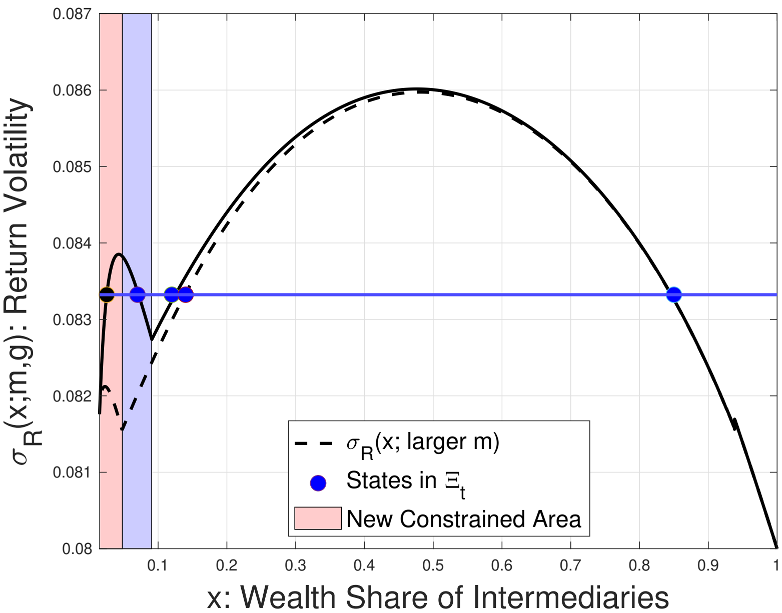

Figure 2: Return Volatility

The dashed curve represents the return volatility against $x = w/P$, the intermediaries' aggregate wealth as a percentage of the assets in the economy, when the underlying baseline value of $m = 2$. The blue circles are alternative combinations of return volatility and x. Other parameters are from Table 1 and Table 2. The shaded blue area corresponds to the constrained region. The black curve and circle are the same as in Figure 1.

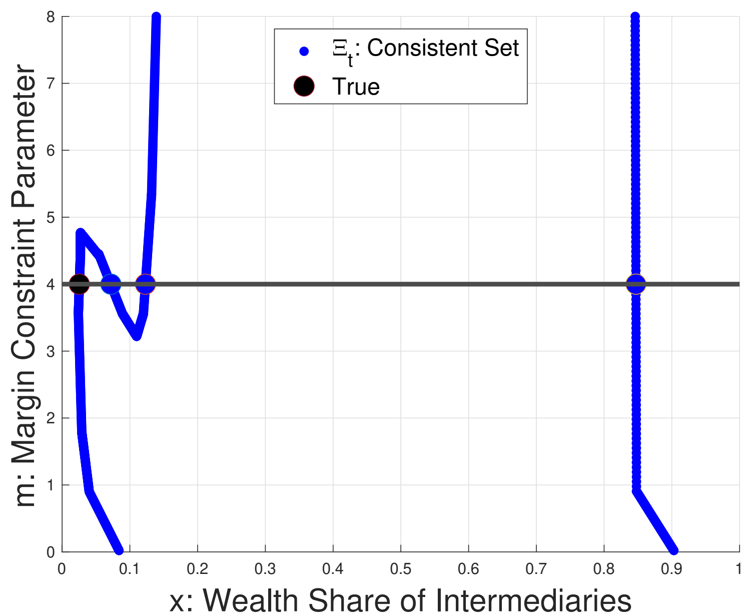

Figure 3: Set of Consistent (m, x)

The set of combinations (m, x) consistent with observable information is graphed in blue. The horizontal axis is the aggregate wealth share of intermediaries and the vertical axis is the margin constraint parameter. The black dot corresponds to the baseline model. Each point in blue has the corresponding value of g in Figure 4.

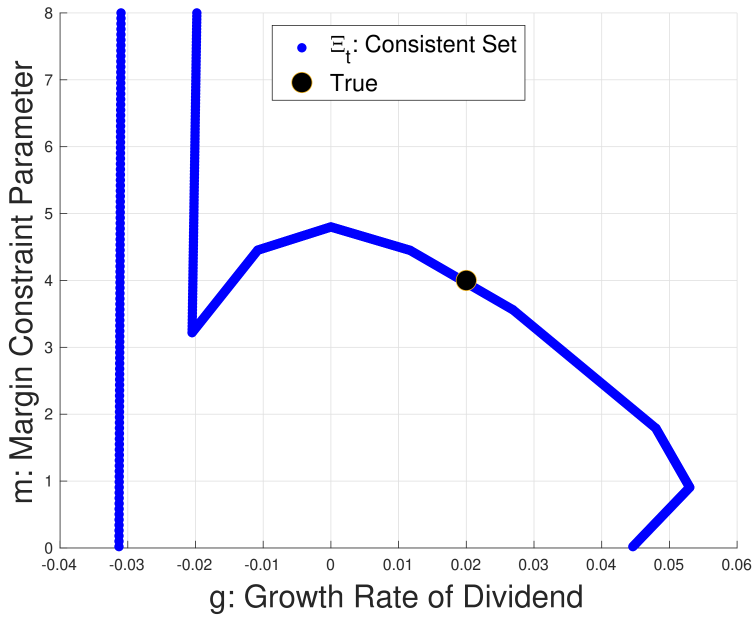

Figure 4: Set of Consistent (m, g)

The set of combinations (m, g) consistent with observable information is graphed in blue. The horizontal axis is the dividend growth and the vertical axis is the margin constraint parameter. The black dot corresponds to the baseline model. Each point in blue has the corresponding value of g in Figure 3

.

3.2 Markovian Characterization of Set of Alternative Models

The Markov property of the equilibrium objects allows for convenient characterizations of alternative models Ξ_t concerning (x, m, g). I illustrate how individual intermediaries identify (m, x) by focusing on a lower aggregate state where financial constraints are binding. Figure 1 plots return volatility as a function of the aggregate state x given a fixed m.

To interpret the shape of the return volatility, note that

$$\sigma_x = x \times vol_t\left(\frac{dw}{w} - \frac{dp}{p}\right) = \hat{x}(\hat{\alpha}^I - 1)\sigma_R,$$

where vol_t denotes the volatility component. On the one hand, as the wealth share of intermediaries x declines, the intermediation sector starts to increase the leverage $\hat{\alpha}^I >> 1$, which increases the relative volatility of the sector's wealth growth to the aggregate wealth growth of the entire economy; therefore, the volatility of the wealth share. On the other hand, as the wealth share of intermediaries decreases, the change in entire intermediaries' wealth level becomes relatively small compared to the entire economy, which decreases the volatility of the wealth share. More explicitly solving for σ_x, since $\sigma_R = vol_t(dp/p) = \frac{dp}{p}\sigma_x + \sigma$,

$$\sigma_x = \frac{\hat{x}(\hat{\alpha}^I - 1)\sigma}{1 - \hat{x}(\hat{\alpha}^I - 1)\frac{p'}{p}}.$$

The denominator of the right-hand side reflects the multiplier effects as in Brunnermeier and Sannikov (2014) and Di Tella (2017). The decline in the wealth share of intermediaries decrease the asset price due to their higher effective risk aversion, further decreasing the wealth share.

Starting from $\hat{x} = 1$, as the wealth share of intermediaries declines, the first effect dominates the second effect until $\hat{x} \approx 0.5$ and return volatility increases, while the second effect starts to dominate below $\hat{x} \approx 0.5$ and return volatility decreases. Without financial constraints, the volatility of wealth share $\hat{x}$ would converge to zero and return volatility to the fundamental volatility σ as $\hat{x} \downarrow 0$.

When the economy starts to be constrained, the first effect starts to dominate the second effect again since the leverage of the intermediation sector must increase rapidly. As further x goes down to 0, the second effect eventually dominates.

Intermediaries find three alternative aggregate state x's in blue circles consistent with the observed return volatility, in addition to the true $\hat{x}_t$ in the black circle. Without observing the current aggregate capitalization of the financial sector $\hat{x}_t$, individual intermediaries view those alternative capitalizations as plausible since they all rationalize available information.

Now consider the situation where intermediaries also contemplate an alternative margin constraint parameter m value. In Figure 2, the dashed line is the equilibrium return volatility in an alternative economy with the different m. Intermediaries find six combinations of (m, x) consistent with the observed return volatility. Figure 3 and Figure 4 plot all combinations of (m, x, g) consistent with the observed return volatility and risk-free rate. Since all these combinations are consistent with observable infromation, they constitute alternative models Ξ_t. Each intermediary regards all the elements in this set as equally plausible.

4 Characterization of Beliefs and Decisions

4.1 Equilibrium Risk Premium

This section characterizes the dynamics of the risk premium ($\pi_{R,t}$) first under the assumption of logarithmic utility for intermediaries and in the absence of ambiguity ($S = 0$). The risk premium represents the additional return required by investors for holding risky assets, which the Euler equation of intermediaries determines:

$$\pi_{R,t} = Cov_t\left[\frac{dw_t}{w_t}, dR_t\right].$$

This equation states that the risk premium is the covariance between the growth rate of the intermediation sector's capital (dw_t/w_t) and the risky asset returns (dR_t). It can be expressed as the product of the intermediary's exposure to the risky asset (α_t^I) and the variance of returns ($\sigma_{R,t}^2$):

$$\pi_{R,t} = \alpha_t^I \sigma_{R,t}^2.$$

The risk premium is influenced by the intermediary's exposure to the risky asset (α_t^I) and the variance of returns ($\sigma_{R,t}$). I primarily focus on the exposure term (α_t^I) since it is the main determinant of the risk premia in the calibration below.

Consider a scenario where the margin constraint binds, leading to entire intermediaries raising total funds of $(1+\hat{m})w_t$. This situation occurs when intermediaries' aggregate wealth share is low or the margin constraint is tight, represented by the condition $\hat{x}_t \leq x^c(\hat{m})$. Here, $x^c(\hat{m})$ denotes the critical wealth share below which the constraint binds, given by:

$$x^c(\hat{m}) = \frac{1-\lambda}{1-\lambda+\hat{m}}$$

In this constrained region, all risky assets are held through the intermediary. The equilibrium market clearing condition in this region implies:

$$\hat{\alpha}_t^{I,const}(w_t + mw_t) = P_t.$$

which, rearranged, gives the intermediary's exposure α_t^I in the constrained region as:

$$\hat{\alpha}_t^I = \frac{1}{\hat{x}_t}\frac{1}{1+\hat{m}},$$

where $\hat{\alpha}_t^I$ emphasizes the dependence on the baseline value of m. The risk premium rises as intermediaries' total capitalization $\hat{x}_t$ decreases within the constrained region. Moreover, when the parameter m is larger (indicating that the intermediaries can raise more equity capital from households for a given amount of their equity stake), the effect of decreasing intermediaries' capitalization on the risk premium is dampened.

This analysis highlights the intricate relationship between margin constraints, the intermediary's exposure to risky assets, and the resulting risk premium. It provides insights into how changes in the intermediaries' capitalization and margin constraint parameter m predict future market asset returns.

In the unconstrained region, where the margin constraint does not bind, the total funds of the intermediary sector is the sum of the intermediaries' wealth and the risky asset household's equity contribution to the intermediary sector. The market clearing condition for the risky asset in this region is given by:

$$\alpha_t^{I,unconst}(w_t + (1-\lambda)w_t^h\alpha_t^h) = P_t.$$

Here, α_t^h represents the risky asset household's share of wealth invested in the intermediaries, and $\alpha_t^{I,unconst}$ is the intermediary's exposure to the risky asset in the unconstrained region. In the calibration, I assume that the risky asset household chooses to invest 100% of their wealth in the intermediaries when there are no binding margin constraints ($\alpha_t^h = 1$). Consequently, the intermediary's exposure in the unconstrained region ($\alpha_t^{I,unconst}$) is calculated as:

$$\alpha_t^{I,unconst} = \frac{1}{1-\lambda(1-\hat{x}_t)}.$$

In the scenario where $\lambda = 0$ (indicating the absence of debt households in the economy), $\alpha_t^{I,unconst}$ is constant and equal to one. This outcome implies that the risk premium in the unconstrained region remains constant over time.

In contrast to the unconstrained region, as demonstrated earlier, the risk premium increases in the constrained region when the intermediaries' total funds fall due to binding margin constraints. This asymmetry in the response of risk premia to changes in intermediaries' capitalization is a key characteristic of the model. It is a central feature of the analysis, reflecting the intricate interplay between margin constraints, leverage effects, and risk premia in the intermediated asset markets.

For the case where $\lambda > 0$, which we consider in the calibration, the risk premium also rises in the unconstrained region because of a leverage effect; however, in the calibration, this effect in the unconstrained region is small compared to the constrained region.

Now, the concept of intermediaries' uncertainty regarding the baseline values of (x, m, g) is introduced, along with an exploration of how they evaluate alternative models of expected excess returns in light of this uncertainty. Intermediaries consider alternative values of (x, m, g), denoted as (x_t, m_t, g_t), within the set of possible models Ξ_t. Based on these alternative values, intermediaries calculate the expected excess return, denoted as $\pi^S_{R,t}(x_t, m_t, g_t)$, considering both their exposure to the risky asset and the compensation for parameter and state uncertainty. In equilibrium, the expected excess return under the true DGP is formulated as

$$\pi_{R,t}(\hat{x}_t, \hat{m}, \hat{g}) = \hat{\alpha}^I_t \sigma^2_{R,t} + UP_t,$$

where $\alpha^I_t = \alpha^{I,unconst}_t$ in the unconstrained region and $\alpha^I_t = \alpha^{I,const}_t$ in the constrained region. UP_t is the equilibrium compensation for parameter and state uncertainty known to intermediaries, whose definition is provided in (6). Each intermediary contemplates alternative models of expected excess return by exploring alternative values of $(x, m, g) \in \Xi_t$:

$$\pi^S_{R,t}(x_t, m_t, g_t) = \alpha^I(x_t, m_t)\sigma^2_{R,t} + UP_t,$$

where if $x \leq x^c(m)$

$$\alpha^I_t = \frac{1}{x}\frac{1}{1+m},$$

otherwise

$$\alpha^I_t = \frac{1}{1 - \hat{\lambda}(1-x)}.$$

Considering a general CRRA utility case, the expected excess return under the true DGP is formulated as

$$\hat{\pi}_{R,t} = \gamma \alpha_t^I(\hat{x}_t, \hat{m})\sigma_R^2 + (\gamma - 1)\sigma_R \sigma_Y(\hat{x}_t, \hat{m}, \hat{g}) + UP_t.$$

Here, the first term on the right-hand side represents the compensation for the exposure to the underlying aggregate shock. It quantifies the additional exposure to risk (quantity of risk) denoted by $\sigma_{R,t}$, multiplied by the price of risk, which is proportional to the volatility of the intermediaries' aggregate wealth. The second term signifies compensation for the intertemporal hedging motive, where σ_Y denotes the volatility of investment opportunities of the aggregate intermediary sector. The third term reflects compensation for the uncertainty about the state and the parameters.

Then each intermediary confronts alternative models for the expected excess return by contemplating alternative values of $(m, x, g) \in \Xi_t$:

$$\pi_{R,t}^S = \gamma \cdot \alpha_t^I(x, m)\sigma_R^2 + (\gamma - 1)\sigma_R \sigma_Y(x, m, g) + UP_t,$$

In this equation, the parameter and state uncertainty primarily affect the first term on the right side in the context of the calibration described below.

The compensation for the parameter and state uncertainty due to the partial identification problem, UP_t is defined as the difference between the expected excess returns under the true and the equilibrium worst-case DGP:

$$UP_t = \hat{\pi}_{R,t} - \pi_{R,t}^S(x_t^{worst}, m_t^{worst}, g_t^{worst}), \tag{6}$$

where $(x^{worst}, m^{worst}, g^{worst})$ is the equilibrium worst-case parameter, the solution to the minimization problem in equilibrium. In the subsequent sections, UP_t is called the "price of partial identification."

Algorithm 1: Fixed-Point Algorithm

Data: Guess for $\sigma_R(x,m,g)$ and $r(x,m,g)$, $x \in [0,1]$, $m \in (0,\overline{m})$, $g \in (\underline{g},\overline{g})$
Result: Equilibrium $\sigma_R(x,m,g)$ and $r(x,m,g)$
Initialization;
Set $n = 1$ and $\sigma_R^{(0)}(x,m,g) = \sigma_R^{REE}(x;m,g)$ and $r^{(0)}(x,m,g) = r^{REE}(x;m,g)$
while do
  for $(g_i, m_i) \in (\underline{g},\overline{g}) \times (0,\overline{m})$ **do**
    Compute a competitive equilibrium where
    - intermediaries form a set of beliefs $\{\Xi_t\}$ using $\sigma_R^{(n-1)}(x,m,g)$ and $r^{(n-1)}(x,m,g)$.
    - (g_i, m_i) is true (baseline) parameter value in this equilibrium.

  end
  $\Rightarrow \{\sigma_R^{(n+1)}(x,m,g)\},$, $x \in [0,1]$, $m \in (0,\overline{m})$, $g \in (\underline{g},\overline{g})$
  if $\max_{(x,m,g)\in[0,1]\times(0,\overline{m})\times(\underline{g},\overline{g})} |\sigma_R^{(n+1)}(x,m,g) - \sigma_R^{(n)}(x,m,g)| + |r^{(n+1)}(x,m,g) - r^{(n)}(x,m,g)| < \epsilon$ **then**
    break;
  end
  Set $n \Rightarrow n+1$
end

4.2 Equilibrium Computation

The iterative procedure for solving equilibria involves solving a sequence of ordinary differential equations for the equilibrium price-dividend ratios $p(x)$ in various economies with alternative baseline parameter values $(\hat{m},\hat{g})$. Intermediaries in these economies face uncertainty regarding parameter and state realizations, leading them to contemplate diverse stochastic processes for asset returns. Algorithm 1 displays the algorithm to find the equilibria. The Appendix outlines the computational details of this procedure.

The challenge lies in simultaneously solving for the price-dividend ratio and the local mean belief distortion S, ensuring the following conditions.

- In an economy with a baseline parameter value $(\hat{m},\hat{g})$, intermediaries construct a set of alternative beliefs Ξ_t using equilibrium return volatilities and risk-free rates from alternative economies with alternative baseline $(\hat{m},\hat{g})$ values.
- Given the worst-case beliefs, the economy's equilibrium must determine prices and

allocations.

The iterative solution process begins by gradually expanding the set of alternative models. Initially, the equilibrium price-dividend ratios are assumed to follow rational expectations, where the set of alternative models contains only a baseline model, restricting $S = 0$. Utilizing equilibrium solutions of price-dividend ratios from previous iterations, intermediaries form the set of alternative models Ξ_t for the excess return process. In the current iteration, the set Ξ_t is expanded regarding the magnitude of S, and the worst-case belief and price-dividend ratios are computed. This iterative process continues until the equilibrium price-dividends converge.

In equilibrium, the fraction of intermediaries' capitalization $\hat{x}_t$ ranges between 0 and 1. As $\hat{x}_t$ approaches zero, the economy predominantly comprises households. In this case, the boundary condition is as follows. Considering the intermediary's consumption from the goods market clearing condition:

$$c_t = D_t(1+l) - c_t^h = D_t(1+l) - \rho(1-\hat{x}_t)P_t = D_t[(1+l) - \rho(1-\hat{x}_t)p(\hat{x}_t)],$$

where the second equality arises from the myopic decision rule of households $c_t^h = \rho w_t^h$. Consequently, as $\hat{x}_t \to 0$:

$$p(0) = \frac{1+l}{\rho}.$$

Conversely, when $\hat{x}_t \to 1$, the economy behaves as if solely comprised of intermediaries. In this scenario, the ordinary differential equation in the limit $\hat{x}_t \to 1$ determines $p'(1)$ given the worst-case belief distortion at $x_t \to 1$, denoted as $S(1)$. The boundary condition for $S(1)$ is enforced by solving the minimization problem for intermediaries using equilibrium parameters at $x_t \to 1$ from the preceding iteration.

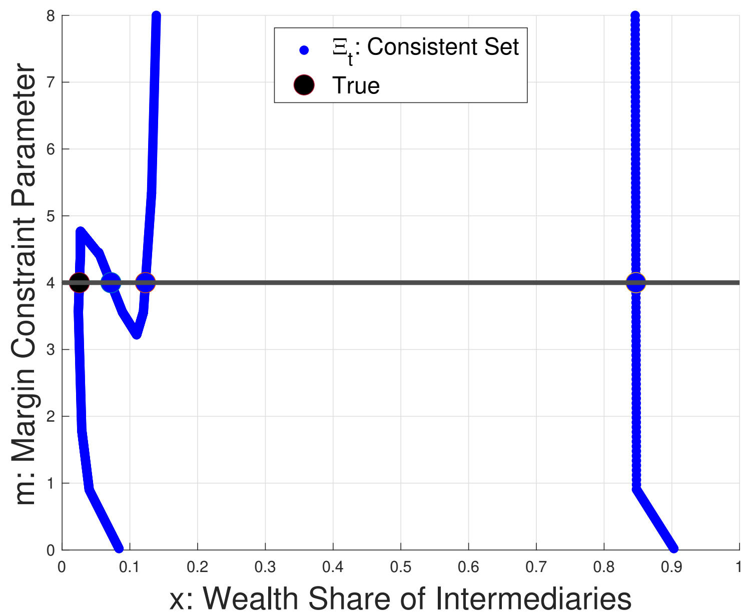

Figure 5: The worst-case combination and set of consistent (m, x)

The worst-case combination of (m, x) is in red. The set of combinations (m, x) consistent with observable information is graphed in blue. The horizontal axis represents the aggregate wealth share of aggregate intermediaries and the vertical axis is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 6

.

4.3 Characterizations of Worst-Case Model

In the minimization problem, the objective is to choose the expected excess return consistent with the parameterization in Ξ_t. Specifically,[8]

$$\min_{\pi_R^S} \underbrace{(\partial_Y J_i)(\partial_x Y)\hat{x}(\hat{\alpha}^I - 1)}_{<0} \pi_R^S + \underbrace{(\partial_W J_i)\alpha_i^I}_{>0} \pi_R^S,$$

This minimization is subject to

$$\pi_R^S \in \{\pi_R - \sigma_R S : S(m, x, g) \in \Xi_t\}.$$

[8]The dependence of α_i^I and c_i on S is eliminated due to the envelope condition.

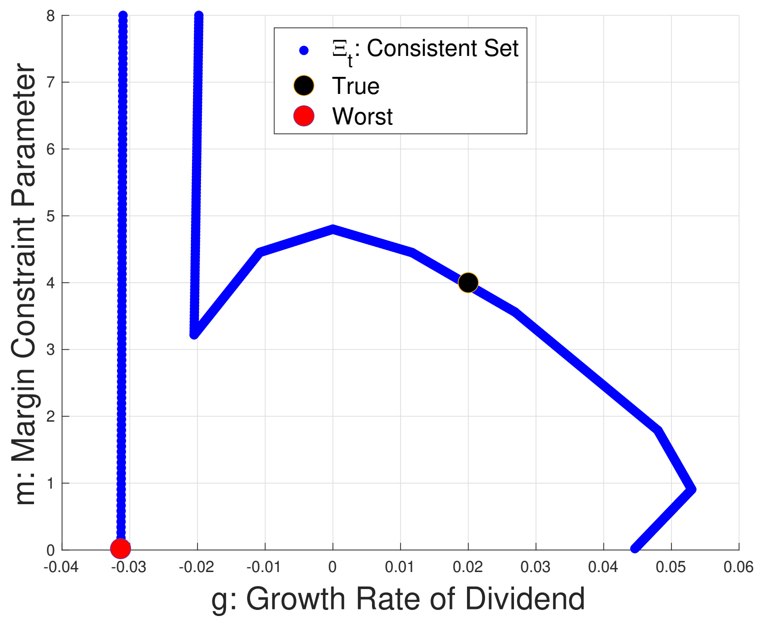

Figure 6: The worst-case combination and set of consistent (m, g)

The worst-case combination of (m, g) is in red. The set of combinations (m, g) consistent with observable information is graphed in blue. The horizontal axis is the dividend growth and the vertical axis is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 5.

A higher expected return has two opposing effects on individual lifetime utility. On the one hand, today's higher expected return improves individual utility by speeding up the accumulation of individual wealth. On the other hand, it also accelerates the accumulation of the entire financial sector, x_t. Then the excess returns tomorrow and after are expected to be low since the financial sector accumulates wealth and reduces the leverage, reducing future risk premia.

In equilibrium, intermediaries fear scenarios where the expected excess return is low, and their individual wealth growth is slow. In Figure 5, the worst-case combination of (m, x) in red is characterized by a higher aggregate wealth share than the actual. This situation would reduce the leverage of the whole intermediation sector and hence risk premium, predicting a lower future excess return.

The higher capitalization of the unconstrained sector could be still consistent with higher observed return volatility since a higher capitalization would make the financial sector a more significant player, significantly fluctuating asset prices even with moderate leverage, indicating high return volatility.

Figure 6 plots the implied g consistent with the observed risk-free rate. The worst-case expected growth rate of dividends is the lowest among alternatives. The lowest leverage implied by the worst-case (m, x) should imply the lowest precautionary saving motive by the aggregate financial sector, which would increase the implied risk-free rate. This is inconsistent with the observed lower risk-free rate driven by the actually higher saving motive due to the lower worst-case expected return on the risky asset. To be consistent, the expected growth rate of the aggregate dividend must be the lowest, which would induce the economy to grow slowly and reduce the implied risk-free rate.

4.4 Characterizations of Consumption-Saving and Portfolio Choice

Given the subjective expected return $\pi_R^S(x_t^{worst}, m_t^{worst}, g_t^{worst})$, which is the expected excess return under the worst-case alternative model, the maximization problem yields a standard consumption and portfolio choice rule:

$$c_{i,t} = \rho^{\frac{1}{\gamma}} Y^{\frac{\gamma-1}{\gamma}} \times w_{i,t};$$

$$\alpha_{i,t}^I = \underbrace{\frac{1}{\gamma} \frac{\pi_R^S(x_t^{worst}, m_t^{worst}, g_t^{worst})}{\sigma_{R,t}^2}}_{Myopic} + \underbrace{\frac{1-\gamma}{\gamma} \sigma_{R,t} \sigma_{Y,t}}_{Hedging\ motive}.$$

The myopic portfolio mainly drives the portfolio weight fluctuations, highlighting the importance of the future return predictability and, consequently, the uncertainty about return predictors (m_t, x_t, g_t) in the investment decision making.

Parameter	Description	Value	Target	Target value	Model
γ	Relative Risk Aversion	1.8	Average expected excess return of MBS	3.4	3.8
ρ	Discount Rate	0.08	Average risk-free rate	1	0.78
σ	Dividend Volatility	0.08	Return volatility of MBS	0.81	0.83
λ	Debt Household Share	0.6	Average Debt-to-Asset ratio in 2007	0.52	0.55
l	Labor Income Ratio	1.84	Share of Labor Income in Total Income	0.66	0.64

Table 1: Matched Moments and Internally Calibrated Parameters

Parameter	Description	Value	Source
m	Intermediation multiplier	4	HK (Share of managers compensation in intermediarys' profit)
g	Dividend Growth	2%	HK (Average real output growth in the U.S.)

Table 2: Fixed Parameters

5 Calibration and Verification of Amount of Uncertainty

In the calibration shown in Table 1, the model targets the same moments as in HK. The parameters of relative risk aversion and dividend growth volatility $(\gamma, \sigma) = (1.8, 0.08)$ are calibrated to match the model-implied average excess return (3.8%) and return volatility (0.78%) of mortgage-backed securities with the empirical moments (3.4% and 0.81% respectively). The discount rate $\rho = 0.08$ is set to match the average risk-free rate (1%), yielding the model-implied value of 0.78%. The fraction of households $\lambda = 0.6$ is chosen to align with the intermediary sector's model-implied average debt-assets ratio (0.55) with that of the financial sector in 2007 (0.52). The $l = 1.84$ value is based on the share of labor income to total income for the United States (66%). The model generates an average labor income share of 64%.

For fixed parameters in Table 2, the intermediation multiplier ($m = 4$) value and dividend growth rate ($g = 0.02$) are from HK. The choice of $m = 4$ aligns with the compensation of financial managers in the intermediary sector, and $g = 0.02$ corresponds to the average per-capita growth rate of the gross domestic product in the United States.

I employ detection error probabilities to quantify the plausibility of the worst-case parametric alternative as proposed by Anderson et al. (2003). Consider a random sample of independent draws indexed by i of time-series data of excess returns on the risky asset $\{dR_t^B - r_t^B dt\}_{t,i}$ drawn from the baseline distribution parameterized by the baseline values $f(\{dR_t - r_t dt\}_{t,i}; \hat{m}, \hat{g}, \{\hat{x_t}\}_t)$. Furthermore, consider a sample $\{dR_t^A - r_t^A dt\}_{t,j}$ indexed by j drawn from the alternative distribution $\tilde{f}(\{dR_t - r_t\}_{t,j}; \{m_t^{worst}\}, \{g_t^{worst}\}, \{x_t^{worst}\}_t)$ determined as the worst-case distribution under worst-case parameters. I evaluate the probability that the random sample is assigned a higher likelihood under the alternative distribution than under the correct benchmark distribution:

$$P\left(\sum_{i=1}^{I} \log \tilde{f}(\{dR_t^B - r_t^B dt\}_{t,i}) > \sum_{i=1}^{I} \log f(\{dR_t^B - r_t^B dt\}_{t,i})\right) = P\left(\sum_{i=1}^{I} \log m(\{dR_t^B - r_t^B dt\}_{t,i}) > 0\right). \tag{7}$$

Here, I is the number of samples. Conversely, the probability that the random sample $\{dR_t^A - r_t^A dt\}_{t,j}$ is assigned a higher likelihood under the benchmark distribution than under the correct alternative distribution is given by,

$$P\left(\sum_{i=j}^{I} \log f(\{dR_t^A - r_t^A dt\}_{t,j}) > \sum_{j=1}^{I} \log \tilde{f}(\{dR_t^A - r_t^A dt\}_{t,j})\right) = P\left(\sum_{i=1}^{I} \log m(\{dR_t^A - r_t^A dt\}_{t,j}) < 0\right). \tag{8}$$

The detection error probability is then defined as the average of the two probabilities above,

$$d(I) = \frac{1}{2}\left(P\left(\sum_{j=1}^{I} \log m(\{dR_t^A - r_t^A dt\}_{t,j}) < 0\right) + P\left(\sum_{i=1}^{I} \log m(\{dR_t^B - r_t^B dt\}_{t,i}) > 0\right)\right) \tag{9}$$

The detection error probability indicates a possibility that the likelihood ratio leads to the erroneous conclusion regarding which of the two distributions generated the random sample. The construction implies that $0 \leq d(I) \leq 1/2$, achieving the upper bound when $f(\{dR_t - r_t dt\}_t)$ and $\tilde{f}(\{dR_t - r_t dt\})$ are identical. The detection error probability is also decreasing in the sample size I, provided f and $\tilde{f}$ are statistically distinguishable.

When $I = 600$ months, which implies 50 years, the detection error probability is 32%, exceeding the commonly accepted threshold of 20% in the literature. Therefore, the worst-case model is statistically challenging for intermediaries to distinguish from the true DGP and the worst-case concern held by intermediaries is admissible.

6 Results

6.1 Model Predictions

Figure 7 illustrates the worst-case scenarios for mean dividend growth, margin constraint parameters, and intermediaries' capitalization (g, m, x) in relation to the economy's actual states. In the worst-case scenarios, the parameter m consistently approaches zero, suggesting minimal intermediation perceived by each individual intermediary. Furthermore, the worst-case capitalization x significantly exceeds the actual values especially when margin constraints are active in low capitalization situations. Despite the financial sector's high leverage during constrained periods, individual intermediaries perceive the aggregate leverage as low in the worst-case scenario. Consequently, the sector's precautionary saving motive should remain low in their perception, which would lead to a higher risk-free rate implied by the worst-case model compared to the actual values. The worst-case value for g must, therefore, be significantly lower than the actual value to align with these observed tendencies of a lower risk-free rate, especially in constrained regions.

Figure 8 plots the equilibrium returns as functions of actual state realizations. The objective risk premium, representing the expected excess return under the true DGP (baseline model), is amplified in the constrained region. This objective risk premium comprises the subjective risk premium and compensation for parameter and state uncertainty, termed as the price of partial identification. Both components contribute to amplifying the objective risk premium in the constrained region.

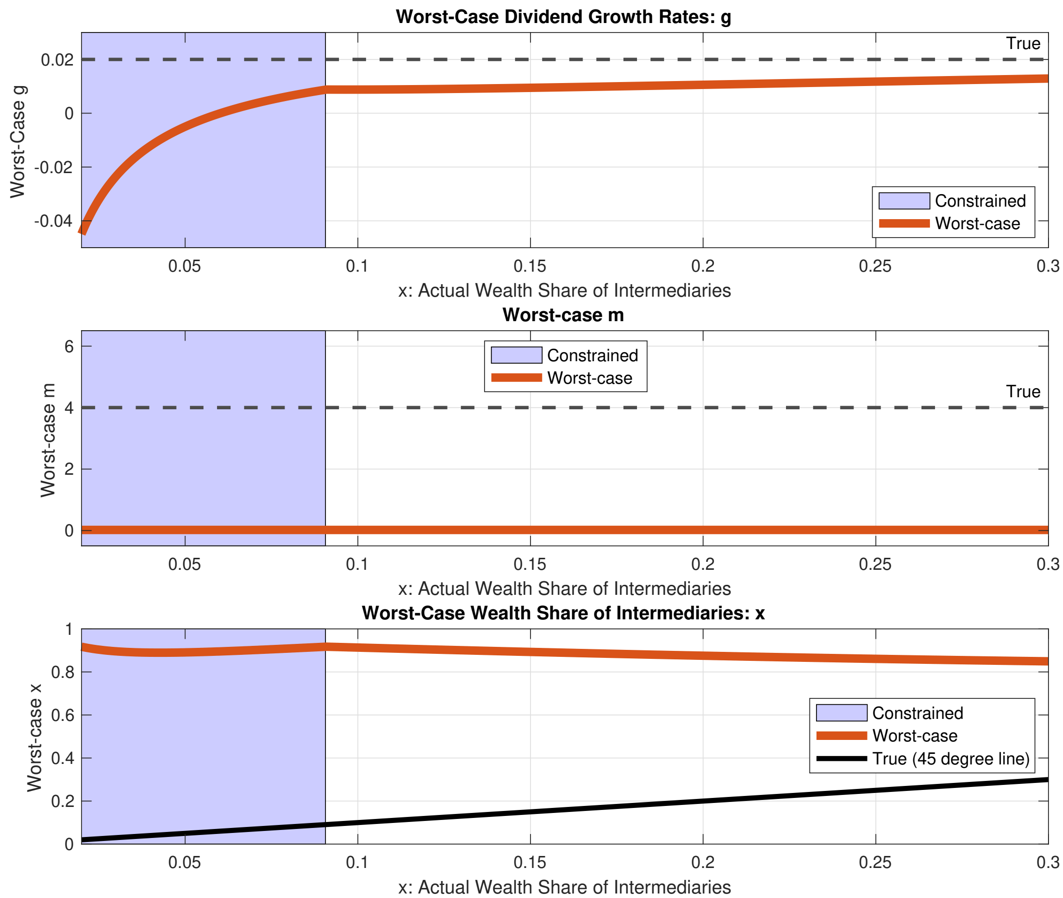

Figure 7: Worst-case parameters and state

The worst-case dividend growth rate, margin constraint parameter, and the aggregate wealth share of intermediaries are graphed in red against the actual realizations of the aggregate state. The flat dashed lines in the upper and middle panels correspond to the baseline values. The solid black line is a 45-degree line, corresponding to the baseline values of the aggregate state.

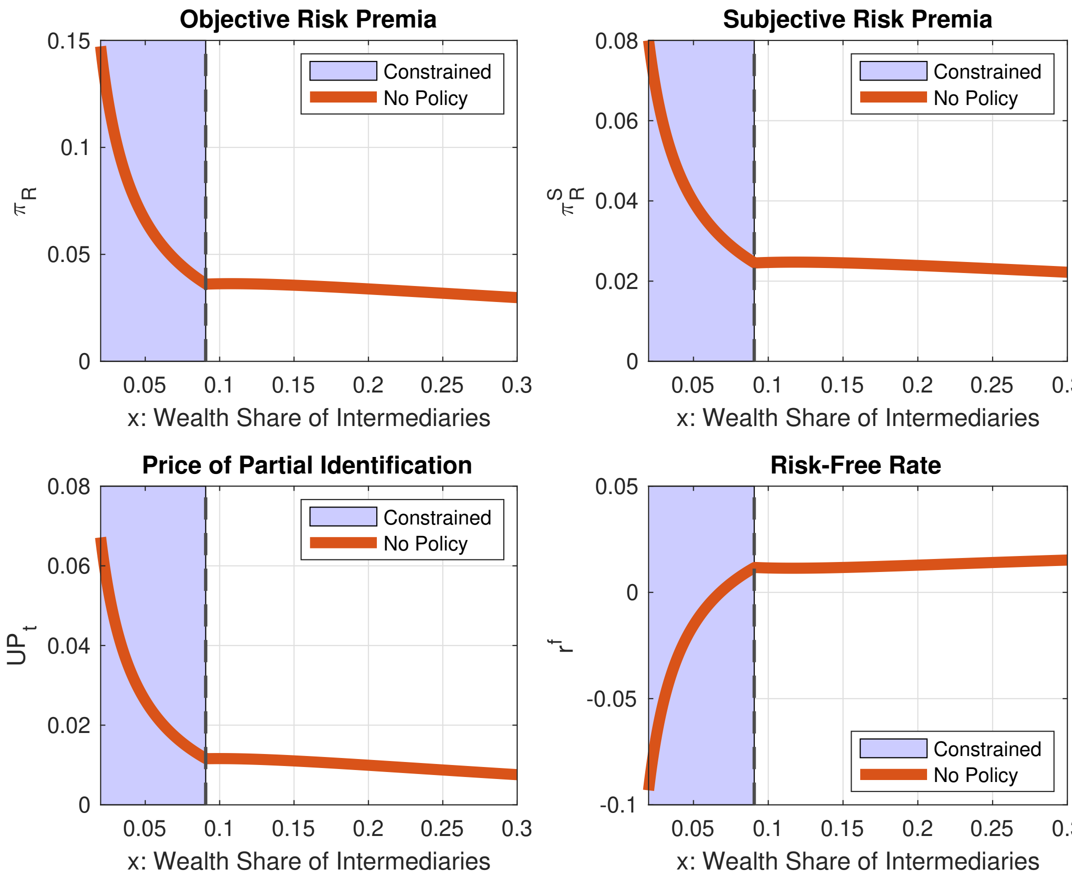

Figure 8: Equilibrium Returns as functions of actual state realization

Components of risk premium and risk-free rate are graphed against actual realizations of $x = w/P$. The objective risk premium is the expected excess return under the true DGP, while the subjective risk premium is that under the worst-case model. The price of partial identification is the difference of the compensation for the actual and worst-case leverage of the intermediation sector.

The subjective risk premium represents the expected excess return under the intermediaries' worst-case model. Demand for the risky asset declines when each individual intermediary faces higher leverage. Consequently, the current asset price decreases, causing the subjective risk premium to rise in equilibrium. This adjustment in the risk premium restores market clearing by stimulating demand for the risky asset. This concept of risk compensation parallels the rational expectations equilibrium presented by He and Krishnamurthy (2013).

The price of partial identification amplifies in the constrained region. Intermediaries perceive the aggregate leverage to be lower than it is. Consequently, the actual compensation for this leverage is significantly higher than perceived; this disparity is reflected in the elevated price of partial identification in the constrained region.

The risk-free rate becomes lower in the constrained region due to two reasons. First, individual intermediaries are exposed to higher leverage, which increases the precautionary saving motive. Second, the lower subjective expected excess return under the worst-case model induces intermediaries to save more to smooth consumption intertemporally. This situation causes the precautionary saving motive due to the uncertainty from the partial identification problem, or structured ambiguity.

Notably, the price of partial identification represents approximately 40% of the objective risk premia under the plausible value of the detection error probability. Moreover, the structured ambiguity amplifies the precautionary saving motive in the constrained region. This finding calls for government policies to mitigate the structured ambiguity during financial crises. He and Krishnamurthy (2013) explored several government interventions in the rational expectations equilibrium; however, those policies aimed to increase the financial sector's capitalization and do not directly speak to how policies can resolve the heightened uncertainty embedded in the price of partial identification and high objective risk premia. My framework provides a laboratory to examine the effects of government announcements on the intermediated asset markets.

6.2 Survey Evidence for Subjective Beliefs

I establish a connection between the model predictions of subjective expectations and the recent survey evidence for cash flow and return expectations observed in various asset markets, as provided by De la O and Myers (2021) and Nagel and Xu (2023). These studies present empirical evidence that challenges theoretical predictions made by rational expectations equilibrium. To assess the alignment between the model and empirical findings, I compare three different aspects of model-implied subjective beliefs to their empirical counterparts. The results verify that the worst-case beliefs in the current model are more consistent with the survey evidence than those assumed in the rational expectations equilibrium.

First, De la O and Myers (2021) presents empirical evidence derived from survey-based subjective expectations, constructed from aggregate stock market analyst forecasts in the U.S. According to this evidence, a significant portion of asset price fluctuations is attributed to cash flow growth forecasts rather than return forecasts. The subjective expectations are measured in terms of mean forecasts across analysts. A version of the Campbell-Shiller decomposition implies that changes in the log price-dividend ratio come from changes in k-year ahead expectations for future cash flows, discount rates (returns), and future price-dividend ratio.

$$\underbrace{\frac{cov(\mathbb{E}_t^{S}(\log D_{t+k}/D_t), \log P_t/D_t)}{var(\log P_t/D_t)}}_{CF_k} + \underbrace{\frac{-cov(\mathbb{E}_t^{S}(R_{t+k} - R_t), \log P_t/D_t)}{var(\log P_t/D_t)}}_{DR_k}$$
$$+ \frac{cov(\mathbb{E}_t^{S}(\log P_{t+k}/D_{t+k}), \log P_t/D_t)}{var(\log P_t/D_t)} = 1.$$

The second column in Table 3 indicates that the worst-case belief in the model correctly predicts improvements in cash flow forecasts and a tendency for discount rates (returns) forecasts to decline when the current log P-D ratio is lower. This aligns with the survey data in the third column across one- and two-year horizons[9].

[9]Note that one- and two-year ahead forecasts differ from the instantaneous drift, as they also consider

	Subjective (Model)	Survey data (De la O and Myers (2021))	Objective (Model)	Rational Model
CF_1	0.19	0.41	0	0
DR_1	-0.13	-0.05	0.06	0.12
CF_2	0.38	0.64	0	0
DR_2	-0.27	-0.08	0.12	0.19

Table 3: Variance decomposition

The table reports the variance decomposition of log price-dividend (P-D) ratio in terms of forecasts for future cash flow, return, and log price-dividend ratios. CF_k denotes the variance of log P-D ratio attributed to k-year ahead forecasts for future cash flow growth and DR_k corresponds to the variance attributed to k-year ahead forecasts for future returns.

	Model	Survey data (De la O and Myers (2021))	Rational
$Corr(FE^R_{t+1}, P_t/D_t)$	-0.83	-0.25	0
$Corr(FE^{\log(D_{t+1}/D_t)}_{t+1}, P_t/D_t)$	-0.67	-0.52	0

Table 4: Forecast error predictability

The table reports the correlations of forecast errors for one-year ahead future returns and dividend growth with the current price-dividend ratio. $FE^X_{t+1} \equiv X_{t+1} - \mathbb{E}^S_t(X_{t+1})$.

Conversely, the fourth column demonstrates that the variance decomposition with expectations under the true DGP implies a negative correlation between discount rates and the P-D ratio. Similarly, as observed in the last column, the rational expectations equilibrium (REE) model also exhibits a negative comovement between discount rate forecasts and the P-D ratio. This suggests limitations of REE models in capturing certain aspects of subjective expectations documented by survey data.

Furthermore, the worst-case belief in the current model underestimates expected returns and dividend growth during crises. This stems from the intermediaries' worst-case scenario, where other financial entities might hold a relatively large amount of wealth and be less risk-averse. The observed lower risk-free rates could arise due to excessively lower cash flow growth in their perceptions. Consequently, this worst-case concern leads to procyclical cash flow forecasts and countercyclical discount rate (return) forecasts concerning the P-D ratio.

The pessimistic forecasts during crises in the current model find additional support in

the evolving economic conditions during those periods. These longer-term expectations can be computed by leveraging the Feynman-Kac theorem and its connection to the solution for differential equations survey evidence for forecast error predictability from De la O and Myers (2021). In Table 4, the correlations of forecast errors for one-year ahead future returns and dividend growth with the current price-dividend ratio are reported. The second column demonstrates that the model-implied subjective belief underestimates future returns and dividend growth during crises when the current P-D ratio is low, aligning with survey evidence in the third column. The last column displays null correlations under the REE model.

Finally, I establish a connection between the model predictions concerning the cyclicalities of subjective and objective risk premia and survey evidence from various decision makers in alternative financial markets, as provided by Nagel and Xu (2023). Following their empirical specification, I run the following one-year ahead predictive regressions using the model's simulated data:

$$\mathbb{E}_t^S(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0^S - 0.17 \times \log(P_t/D_t) + u_t^S;$$

$$\mathbb{E}_t(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0 - 0.34 \times \log(P_t/D_t) + u_t.$$

The first specification regresses subjective expected excess returns onto a constant and log P-D ratio, while the second specification regresses objective expected returns. In the model, the subjective risk premium is more acyclical than the objective risk premium, consistent with Nagel and Xu (2023), who reports that the slope coefficient for the objective risk premium is approximately five times larger than for the subjective risk premium in magnitude. On the other hand, REE models imply the same slope across the two specifications.

This survey evidence verifies that the model-implied subjective beliefs are more consistent with the empirical properties of expectations. Thus, the current model serves as a natural laboratory for the analysis of policies aimed at managing subjective expectations through policy announcements.

7 Belief Management Policies

This section studies the equilibrium consequence of a government's credible promises to mitigate agents' uncertainty and heightened risk premia during crises. Private agents are assumed to trust the promises. These policy promises potentially affect the set of consistent beliefs and eliminate some beliefs inconsistent with policy announcements. Importantly, such promises can change the entire equilibrium dynamics but may not involve actual implementations since they eliminate the beliefs that will never realize in equilibrium.

This policy prescription is in contrast with those discussed in the existing rational expectations equilibrium of He and Krishnamurthy (2013) or even ambiguity literature involving the actual policy implementations[10]. Caballero and Simsek (2013), for example, examine the amplification effects of Knightian uncertainty about financial network structures in the form of Gilboa and Schmeidler (1989) but the set of alternative beliefs are exogenously given. Their policy implications are limited to the actual implementations of bailouts and asset price support to mitigate the crises.

7.1 Policy Experiments during Crises

I examine the efficacy of the two policies aimed at resolving uncertainty regarding the mean dividend growth rate g (referred to as the g policy) and the margin constraint parameter m (referred to as the m policy). These policies narrow down the set of parameters and states.

[10]See also Karantounias (2013), Ferriere and Karantounias (2019), and Karantounias (2023) for the optimal Ramsey policies under model misspecification concerns, which involves actual implementations of policies.

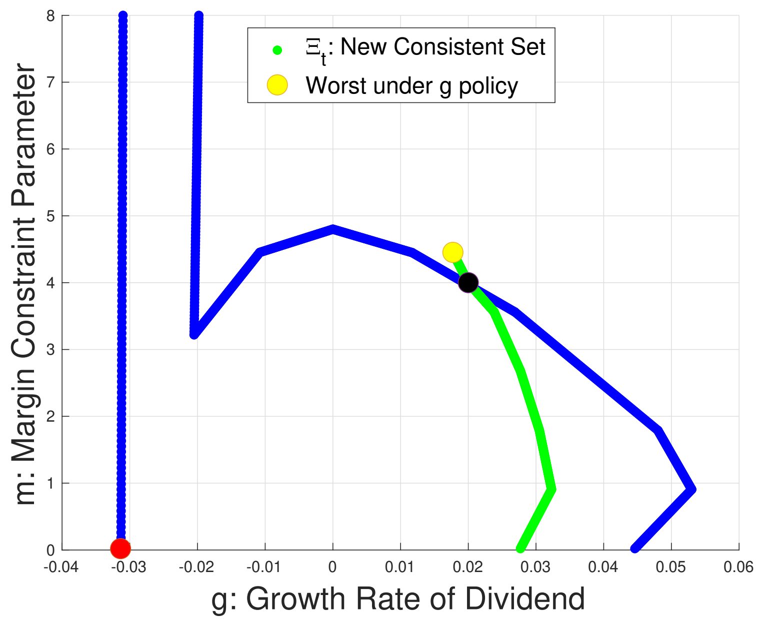

Figure 9: Set of (m, g) after g policy intervention

The combinations (m, g) under the g policy in green and without policies in blue are graphed. The worst-case combinations are plotted in yellow under the g policy and red represents those without policies. The baseline combination is graphed as a black dot.

The g policy restricts g in Ξ_t to be greater than 1%, similar to guaranteeing the cash flow from the intermediated risky asset. The m policy corresponds to the deposit insurance and capital requirement, which impose constraints that $3 < m$ and $m < 5$ in Ξ_t, respectively, reducing the potential perceived range of tightness of constraints m.

Following the spirit of HK's policy experiments, I measure the success of these policies regarding how they mitigate the heightened risk premia during financial crises when the financial sector is less capitalized and the constraint is binding.

At the beginning of period 0, the state is characterized by a risk premium of 12% before any announcements are made, and agents make their optimal decisions without prior knowledge of these announcements. Then the government announcements are made unexpectedly in period 0. The asset prices and the wealth share of intermediaries experience an immediate jump, indicating that these announcements alter the equilibrium dynamics.

The g policy is effective at mitigating the heightened risk premia. Figure 9 illustrates

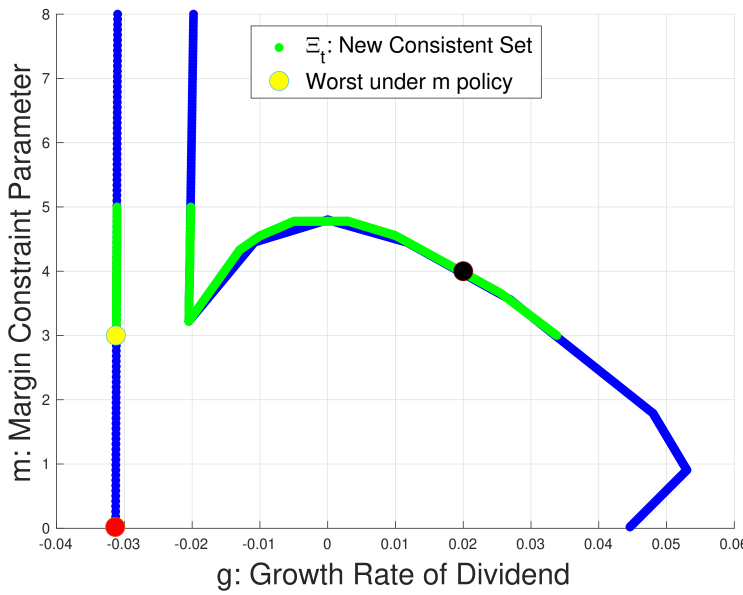

Figure 10: Set of (m, g) after m policy intervention

The combinations (m, g) under the m policy in green and without policies in blue are graphed. The worst-case combinations are plotted in yellow under the m policy and red represents those without policies. The baseline combination is graphed as a black dot.

the responses of the (m, g) set to the government's announcement of the g policy. The new worst-case belief, depicted in yellow, aligns much more closely with the actual scenario shown in black. This adjustment occurs because the g policy eliminates the possibility of the worst-case belief characterized by overly pessimistic cash flow growth consistent with observable information and the structure of an alternative economy. Each intermediary conjectures that the observed lower risk-free rate should arise due to the higher precautionary saving motive by the entire financial sector exposed to higher leverage, not because of the lower cash flow growth.

Moreover, individual intermediaries conclude that expected excess returns must be high, increasing their demand for the risky asset. This force reduces the price of partial identification and the risk premium under the true DGP in the new equilibrium.

In contrast, the m policy is less effective in this model. Figure 10 displays the response of the set of beliefs concerning (m, g) to the m policy announcement. In this scenario, the new

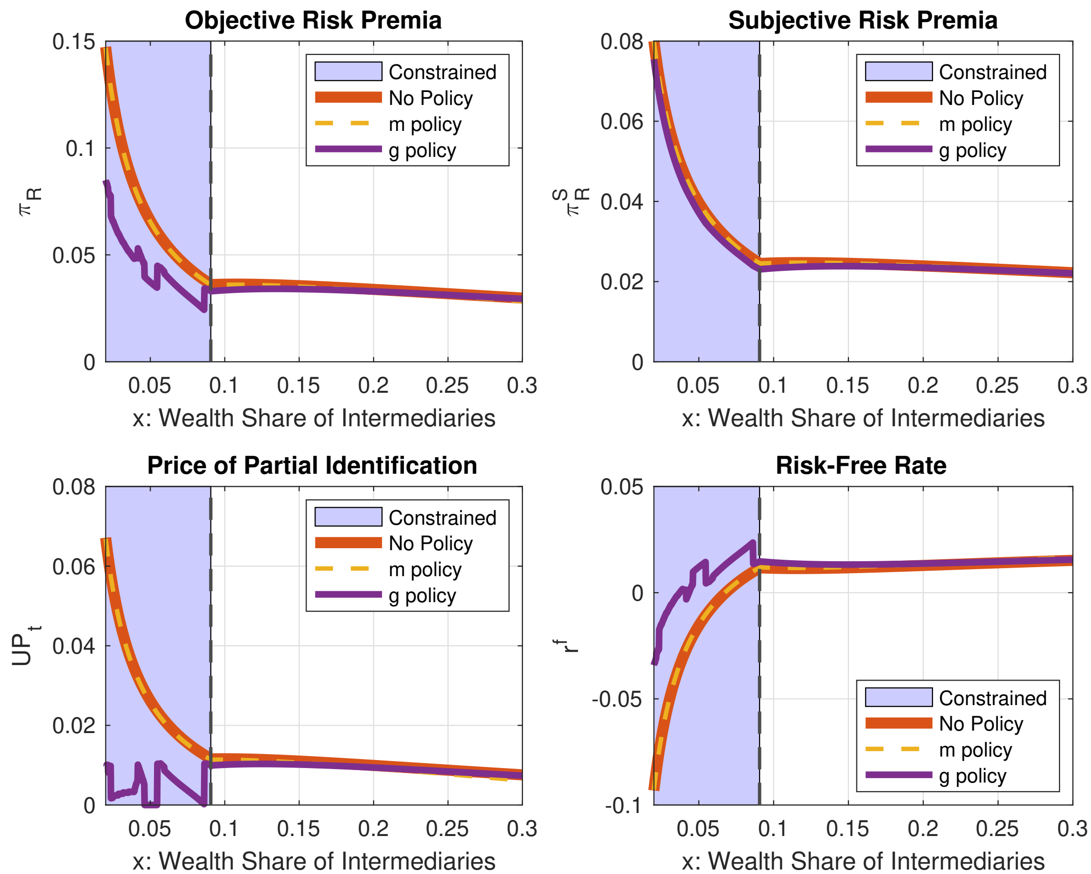

Figure 11: Equilibrium Returns under Policy Announcements

See the note in Figure 8 for a detailed description of the graph. The equilibrium objects under m policy (dashed yellow lines) overlap those without policy announcements (solid orange lines). The purple lines plot the equilibrium returns under the g policy announcement.

worst-case g remains near the previous value and significantly lower than the actual value. This result suggests that the financial sector's precautionary saving motive perceived by each intermediary would remain relatively low, and the aggregate sector leverage is perceived as low in the worst case. This policy prediction arises because, as illustrated in section 6, even without uncertainty regarding m, a higher capitalization of the unconstrained financial sector remains consistent with the observed higher return volatility. Individual intermediaries are concerned about the possibility of lower aggregate leverage and lower profitable opportunities in the market, which depresses their appetite for the risky asset.

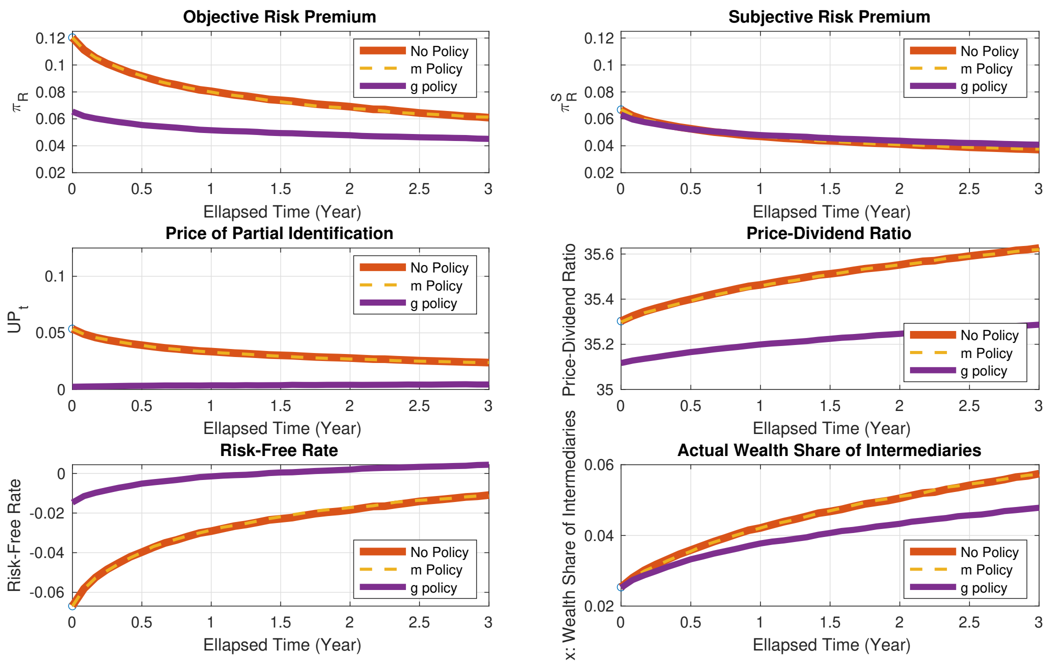

Figure 12: Mean Transition Dynamics

The mean transition dynamics with and without policy announcements are graphed against elapsed time (year). See the note in Figure 8 for the description of plotted objects. The mean transition paths are computed as the average of 5000 stochastic simulations starting at 0 from the state with an objective risk premium of 12% in equilibrium without policy announcements. Once announcements are made at time 0, the entire equilibrium dynamics immediately jump to the new equilibrium. The asset prices and wealth share jump but the portfolio holdings made in equilibrium without announcements are fixed at time 0 after announcements.

Transit to	10 (%)	7.5	6.0	5.0
No policy	0.45	1.38	2.68	4.7
m Policy	0.44	1.4	2.76	4.62
g Policy	0	0	0.52	1.11

Table 5: Mean Transition Time (Year)

Table 6: See Figure 12 for the construction of the mean transit time. This table shows how long the economy takes to hit the states with specific levels of objective risk premia in the first row.

Figure 11 displays the equilibrium returns as functions of the actual realizations of the aggregate state, similar to Figure 8. The m policy does not reduce the risk premium; however, the g policy effectively mitigates the heightened risk premia in the constrained region by decreasing the price of partial identification. The resolution of uncertainty concerning g addresses the uncertainty associated with the partial-identification problem of (m, x). The subjective risk premia remain unchanged in response to the policy announcement across different states. The announcements resolve uncertainty related to partial identification problems; however, they do not alleviate the scarcity of capitalization in the financial sector in the constrained region. This result contrasts the actual implementation of strategies such as capital injection, discount rate lending, or direct asset purchases analyzed in the rational expectations equilibrium by He and Krishnamurthy (2013).

Figure 12 illustrates the mean transition dynamics over time (in years) under different policy announcements. These paths are computed as the average of 5000 stochastic simulations, commencing at time 0 from the state with an objective risk premium of 12% in equilibrium without policy announcements. When announcements are made at time 0, the entire equilibrium dynamics promptly shift to the new equilibrium. While asset prices and wealth shares jump, the portfolio holdings established in the equilibrium without announcements remain fixed at time 0 after the announcements.

The g policy reduces the risk premium by reducing the price of partial identification, not the subjective risk premium. Although the announcement resolves uncertainty, it does not directly change the compensation for the high leverage individual intermediaries face.

In response to the g policy, the risk-free rate jumps as it resolves the precautionary saving motive arising from parameter and state uncertainty; this higher risk-free rate slightly decreases the price-dividend ratio. The wealth share of intermediaries does not experience a significant jump following the announcements; it grows slower due to the reduction in objective risk premia caused by the decreased price of partial identification. Table 6 displays the average transition time of the economy reaching specific levels of the objective risk premium, indicating that the g policy significantly accelerates the convergence of risk premia to lower levels.

A novel insight from this exercise is that policymakers should be cognizant of the endogenous linkages among different aspects of uncertainty. The exercise demonstrates that resolving one type of uncertainty about g can mitigate uncertainty concerning other dimensions (m, x). Conversely, resolving uncertainty regarding m alone does not have the same effect. Policymakers should be thoughtful of the worst-case scenario and the aspects of uncertainty that are crucial to sustain it for market participants.

8 Conclusion

This study introduced a model to investigate risk premia dynamics during crisis events, where intermediaries grapple with capital scarcity and fear low-profitable investment opportunities in asset markets. The worst-case parameterized model for asset returns is challenging to distinguish statistically from the baseline model. Compensation for parameter and state uncertainty contributes to approximately 40% of the total risk premium during crises. Evaluating the effectiveness of government announcements using the model reveals that guaranteeing the cash flow of risky assets is the most impactful policy to mitigate the heightened risk premium during crises.

In general, this paper's novel modeling methodology is broadly applicable in other contexts, providing a foundation to capture structured ambiguity in a broader and more general environment. One promising avenue for future research lies in introducing this methodology into the model of financial networks where banks are uncertain about the cross exposure and consequently, about counter-party risks. The existing theory such as Caballero and Simsek (2013) assumes the exogenous banks' set of alternative beliefs about financial network structure. Their policy discussion is limited to the actual implementation of distressed banks' bailouts and asset price supports. The endogenous formation of alternative beliefs in the current paper will speak to the equilibrium effects of credible policy promises and information disclosure that affect agents' beliefs during crises.

Another potential application involves investigating the impact of uncertainty arisen from the presence of multiple equilibria in New Keynesian models. There are ample empirical evidence for perceived uncertainty about the policy rule represented by the Taylor rule[11]; private agents plausibly view the policy rule coefficients as uncertain and are concerned about which equilibrium is the true DGP. My methodology allows for examining the implications of this structured policy uncertainty for macroeconomic and financial outcomes, highlighting the distinct role of central banks' communication policies.

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[11]See the recent contribution for identified shocks of market confidence shock in response to monetary policy announcements by Boyachenko et al. (2017), for empirical studies using survey data by Bauer et al. (2022), and for a comprehensive structural estimation by Bianchi et al. (2023).

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Appendix A. Technical Appendix

This Appendix details the derivations for the expressions in the main texts and the ODE derivations that characterize the equilibrium and boundary conditions.

A.1. Detailed Derivations

The homothetic property of intermediaries' preferences implies their value function in the form:

$$J(w_{i,t}, Y(x_t)) = \frac{[w_{i,t}Y(x_t)]^{1-\gamma}}{1-\gamma},$$

where $Y(x_t)$ represents the future investment opportunity, evolving as

$$\frac{dY}{Y} = \mu_Y(x)dt + \sigma_Y(x)dZ.$$

Then the Hamilton-Jacobi-Bellman (HJB) equation of an intermediary i is formulated as:

$$\begin{aligned}\rho J_i = \max_{\alpha_i, c_i} \min_{S \in \Xi} \frac{c_i^{1-\gamma}}{1-\gamma} + (\partial_Y J_i)\mu_Y^S Y + (\partial_W J_i)(rw_i + \alpha_i \pi_R^S w_i - c_i) + \frac{1}{2}(Y\sigma_Y)^2(\partial_{YY} J_i) \\ + \frac{1}{2}(\partial_{WW} J_i) w_i^2 (\alpha_i \sigma_R)^2 + w_i(\alpha_i \sigma_R) Y \sigma_Y (\partial_{WY} J_i),\end{aligned}$$

Given the belief distortion S from the minimization problem part due to the structure ambiguity, the first-order conditions for the maximization problem with respect to the share of wealth invested in the risky asset, α_i and consumption, c_i yields:

$$c_{i,t} = \rho^{\frac{1}{\gamma}} Y^{\frac{\gamma-1}{\gamma}} \times w_{i,t};$$

$$\alpha_{i,t}^I = \underbrace{\frac{1}{\gamma}\frac{\pi_R^S(x_t^{worst}, m_t^{worst}, g_t^{worst})}{\sigma_{R,t}^2}}_{Myopic} + \underbrace{\frac{1-\gamma}{\gamma}\sigma_{R,t}\sigma_{Y,t}}_{Hedging\ motive}.$$

Since the intermediaries are identical under the baseline model, $Y_{i,t} \equiv Y_t$ so that the (common) consumption-wealth ratio can be expressed as:

$$\frac{c_t}{w_t} \equiv \frac{c_{i,t}}{w_{i_t}} = \rho^{\frac{1}{\gamma}} Y^{\frac{\gamma-1}{\gamma}}.$$

Similarly, the shares are also identical under the baseline model: $\alpha_t \equiv \alpha_{i,t}$.

The (identical) intermediaries' budget constraint is:

$$\begin{aligned} \frac{dw_t}{w_t} &= -\frac{c_t}{w_t}dt + r_t dt + \alpha_t(dR_t - r_t) \\ &= (-\rho^{1/\gamma} Y_t^{1-\frac{1}{\gamma}} + r_t + \alpha_t \pi_{R,t})dt + \alpha_t \sigma - R, t dZ_t, \end{aligned}$$

where the second equality comes from eq. (11).

Recall the price-dividend ratio is denoted by $p(x_t) \equiv p_t \equiv \frac{P_t}{D_t}$. The Ito's Lemma implies:

$$\begin{aligned} \frac{dp_t}{p_t} &= \mu_p dt + \sigma_p dZ_t \\ &\equiv \left(\frac{p'}{p} x \mu_x + \frac{1}{2} \frac{p''}{p} (\sigma_x x)^2 \right) dt + \frac{p'}{p} x \sigma_x dZ_t. \end{aligned} \tag{10}$$

Invoking the identity $P_t = p_t D_t$, the Ito's Lemma again shows:

$$\begin{aligned} dP_t &= p(x_t) dD_t + D_t dp(x_t) + d[p(x_t), D_t] \\ &= (g p_t + P_t \mu_p + \sigma_p \sigma P_t)dt + (\sigma + \sigma_p) dZ_t; \end{aligned}$$

$$\frac{dP_t}{P_t} = (g + \mu_p + \sigma_p \sigma)dt + (\sigma + \sigma_p) dZ_t.$$

From the identity,

$$(\pi_R + r)dt = E_t\left[\frac{dP_t + D_t}{P_t}\right] = \left(g + \mu_p + \sigma_p \sigma + \frac{1}{p}\right)dt;$$

$$\sigma_R = vol_t\left[\frac{dP_t + D_t}{P_t}\right] = vol_t\left[\frac{dP_t}{P_t}\right] = \sigma + \sigma_R.$$

Trivially, the identify implies:

$$\pi_R + r - \frac{1}{p} = E_t\left[\frac{dP_t}{P_t}\right].$$

Now, I derive the law of motion for the aggregate state variable x_t in terms of endogenous variables. The aggregate wealth share of intermediaries is defined as $x_t \equiv w_t/P_t$ since the risk-free asset is zero-net supply. The Ito's Lemma implies:

$$\frac{dx_t}{x_t} = \frac{d\frac{w_t}{P_t}}{w_t/P_t} = \frac{dw_t}{w_t} - \frac{dP_t}{P_t} - \left(\frac{dw_t}{w_t}\right)\left(\frac{dP_t}{P_t}\right) + \left(\frac{dP_t}{P_t}\right)^2. \tag{11}$$

Substituting the expressions derived previously, the law of motion is given by

$$\begin{aligned} dx &= \mu_x dt + \sigma_x dZ_t \\ &\equiv \Big(- \rho^{1/\gamma} Y^{1-\frac{1}{\gamma}} + (\alpha-1)\pi_R + \frac{1}{p} - (\alpha-1)\sigma_R^2 \Big) dt + (\alpha-1)\sigma_R dZ_t. \end{aligned} \tag{12}$$

Under an alternative model S for the expected excess return π_R, the law of motion is given by:

$$\begin{aligned} dx &= \mu_x^S dt + \sigma_x dZ_t^S \\ &\equiv \Big(- \rho^{1/\gamma} Y^{1-\frac{1}{\gamma}} + (\alpha-1)\pi_R^S + \frac{1}{p} - (\alpha-1)\sigma_R^2 \Big) dt + (\alpha-1)\sigma_R dZ_t^S, \end{aligned} \tag{13}$$

where $\pi_R^S = \pi_R - \sigma_R S$.

Finally, I express the evolution of the intermediaries' investment opportunity in terms of equilibrium objects. The Ito's Lemma gives:

$$\begin{aligned} dY &= \mu_Y dt + \sigma_Y dZ_t \\ &= \Big(Y' x \mu_x + \frac{1}{2} Y'' (\sigma_x x)^2 \Big) dt + Y' x \sigma_x dZ_t. \end{aligned} \tag{14}$$

Under an alternative model S for expected excess return π_R^S, the law of motion is:

$$\begin{aligned} dY &= \mu_Y^S dt + \sigma_Y dZ_t \\ &= \left(Y' x \mu_x^S + \frac{1}{2} Y''(\sigma_x x)^2\right) dt + Y' x \sigma_x dZ_t^S. \end{aligned} \tag{15}$$

The difference of the investment opportunities' drifts between the baseline and the alternative model is:

$$\mu_Y - \mu_Y^S + Y' x(\mu_x - \mu_x^S) = Y' x(\alpha - 1)(\pi_R - \pi_R^S).$$

A.2. Derivation of the ODE for Price-Dividend Ratio

I have used x, the ratio of the aggregate intermediaries' wealth w to the price of risk asset price P, as the state variable so far. As in He and Krishnamurthy (2013), in writing the ODE, the expressions are simpler, leading to a computationally tractable boundary condition by changing variables to an alternative state variable $y \equiv w^h/D$, which is households' wealth w^h scaled by the current dividend level D. I denote the price-dividend ratio with the alternative state variable $F(y)$. Once we solve for the equilibrium price-dividend ratio $F(y)$, we can convert back to the original state variable x as follows:

$$x = 1 - y/F(y), \tag{16}$$

and the price-dividend ratio $p(x)$ as a function of the fraction of intermediaries' wealth x satisfies

$$F(y) = p(1 - y/F(y)).$$

When x ranges from 0 to 1, y takes value from $F(y_b)$ to 0, where the maximum household wealth is $y_b \equiv (1 + l)/\rho$.

Denote the dynamics of y_t as

$$dy_t = \mu_y dt + \sigma_y dZ_t, \tag{17}$$

for unknown functions μ_y and σ_y. We write dc_t/c_t and dR_t as functions of mu_y, σ_y and the $F(y)$ derivatives. Due to the market clearing, since $c_t = D_t(1 + l\rho y_t)$, I have

$$\begin{aligned}\frac{dc_t}{c_t} &= \frac{dD_t}{D_t} - \frac{\rho dy}{1+l-\rho y} - \frac{\rho}{1+l–\rho y} Cov_t\left[dy, \frac{dD}{D}\right] \\ &= \left(g - \frac{\rho}{1+l-\rho y}(\mu_y + \sigma_y \sigma)\right)dt + \left(\sigma - \frac{\rho\sigma_y}{1+l-\rho y}\right)dZ_t \\ &= \mu_c dt + \sigma_c dZ\end{aligned}$$

and

$$dR_t = \frac{dP_t + D_t dt}{P_t} = \left[g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma\right]dt + \left(\sigma + \frac{F'}{F}\sigma_y\right)dZ_t.$$

Given the solution S to the minimization problem in section 8, the Euler equation of intermediaries for the risky asset is given by

$$-\rho dt - \gamma E_t^S\left[\frac{dc_{i,t}}{c_{i,t}}\right] + \frac{1}{2}\gamma(\gamma+1)Var_t\left[\frac{dc_{i,t}}{c_{i,t}}\right] + E_t^S[dR_t] = \gamma Cov_t\left[\frac{dc_{i,t}}{c_{i,t}}, dR_t\right]. \tag{18}$$

Note that the worst-case alternative model differs from the true DGP only in terms of the first moment but not the second moment ($Var_t^S = Var_t$ and $Cov_t^S = Cov_t$).

Intermediary i's consumption evolves under the worst-case model as

$$\frac{dc_{i,t}}{c_{i,t}} = (\mu_{c,i} - \sigma_{c,i}S)dt + \sigma_{c,i}dZ^S.$$

Substituting this into the Euler equation above, I have:

$$-\rho dt - \gamma(\mu_{c,i} - \sigma_{c,i}S) + \frac{1}{2}\gamma(\gamma+1)\sigma_{c,i}^2 + E_t[dR_t] - \sigma_R S = \gamma\sigma_{c,i}\sigma_R \tag{19}$$

Since intermediaries are identical in equilibrium and $\mu_{c,i} = \mu_c$ and $\sigma_{c,i} = \sigma_c$,

$$-\rho dt - \gamma(\mu_c - \sigma_{c,i}S) + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + E_t[dR_t] - \sigma_R S = \gamma\sigma_c\sigma_R. \tag{20}$$

Substituting the expressions of dR_t and dc_t/c_t above, I obtain the ODE for F:

$$g - \sigma S + \frac{F'}{F}(\mu_y - \sigma_y S + \sigma_y\sigma) + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} = \rho + \gamma(g - \sigma S) -$$
$$- \frac{\gamma\rho}{1+l+\rho y}(\mu_y - \sigma_y S + \sigma_y\sigma) + \gamma\left(\sigma - \frac{\rho\sigma_y}{1+l+\rho y}\right)\left(\sigma + \frac{F'\sigma_y}{F}\right) - \frac{\gamma(\gamma+1)}{2}\left(\sigma - \frac{\rho\sigma_y}{1+l+\rho y}\right)^2. \tag{21}$$

Similarly, the Euler equation of intermediaries for the risk-free asset is given by

$$r_t dt = \rho dt + \gamma E_t\left[\frac{dc_{i,t}}{c_{i,t}}\right] - \frac{\gamma(\gamma+1)}{2}Var_t\left[\frac{dc_{i,t}}{c_{i,t}}\right].$$

I then have

$$r = \rho + \gamma(g - \sigma S) - \frac{\gamma\rho}{1+l-\rho\gamma}(\mu_y - \sigma_y S + \sigma\sigma_y) - \frac{\gamma(\gamma+1)\sigma^2}{2}\left(\sigma - \frac{\rho\sigma_y}{1+l-\rho y}\right).$$

Regarding the derivation of (μ_y, σ_y), refer to the Appendix of HK since the derivation is exactly the same. The results are

$$\sigma_y = -\frac{\theta_b}{1-\theta_s F'}\sigma$$

and

$$\mu_y + \sigma\sigma_y = \frac{1}{1-\theta_s F'}\left(\theta_s + l + (r-g)\theta_b - \rho y + \frac{1}{2}\theta_s F''\sigma_y^2\right)$$

where θ_s are the number of shares the risky asset household owns; $\theta_b D = w^h - \theta_s P$ is the amount of funds the households have invested in the risk-free bond. They depend on whether the economy is constrained or not. In the unconstrained region, $\theta_s = \frac{(1-\lambda)y}{F-\lambda y}$ and $\theta_b = \lambda y\frac{F-y}{F-\lambda y}$.

In the constrained region, $\theta_s = \frac{m}{1+m}$ and $\theta_b = y - \frac{m}{1+m}F$. The economy is unconstrained if $0 < y \leq y^c$, where $y^c = \frac{m}{1-\lambda+m}F(y^c)$.

The ODE for F can be derived by substituting these expressions into the Euler equations and then combining them to substitute the risk-free rate r.

A.3. Boundary Conditions

The upper boundary condition is given by $F(y_b) = y_b$ since the households hold the entire wealth in the economy. As in HK, I derive the lower boundary condition by taking the limit $y \to 0$ of the ODE:

$$F(0) = \frac{1 + F'(0)l}{\rho + g(\gamma - 1) + \frac{1}{2}\gamma(1-\gamma)\sigma^2 - \frac{l\gamma\rho}{1+l} - (\gamma+1)\sigma S(0)},$$

where I require $F(0) > 0$ so that

$$\rho + g(\gamma - 1) + \frac{1}{2}\gamma(1-\gamma)\sigma^2 - \frac{l\gamma\rho}{1+l} - (\gamma+1)\sigma S(0) > 0.$$

As documented in subsection 4.2, I approximate $S(0)$ by solving the minimization problem of intermediaries at $y = 0$ using the objective function from the previous iterations.

A.2. Numerical methods

I start the iterations with rational expectations equilibria with different values of (m, g), where $m \in (1e-1, 20)$ and $g \in (-0.2, 0.2)$. The space of (m, g) is discretized, where $m \in (1e-1, 20)$ and $g \in (-0.2, 0.2)$ are approximated by the equal-distant grids with 40 grid points.

Given the equilibrium objects from the previous iteration, I solve the new iteration as follows. I compute the belief distortion $S(0)$, using the previous equilibrium objects. I then implement the shooting algorithm that searches for $F'(0)$ that jointly satisfies the lower and upper boundary conditions and the ODE. Since both boundaries are singular, I truncate the state space as in HK and extrapolate the truncated solution of F following the same procedure in HK. Notice that each equilibrium can be computed in parallel during iterations.

To obtain the algorithm's stability, I gradually expand the set of alternative models by increasing the limit on the magnitude of the belief distortion S, starting $S = 0$ from the rational expectations equilibrium. The solution is regarded as converged if the solutions from the current iteration do not change from the previous iteration.

A.3. Long-Run Effects of Announcement Policies

This section examines the long-run implications of the aforementioned belief management policies. Figure 13 compares the stationary distributions with and without g policy announcements. Under the g policy, the economy frequently encounters constrained states. Table 7 presents measurements from simulations under various policy interventions, indicating that the g policy reduces the objective risk premia in the constrained region but not in the unconstrained region relative to the equilibrium without policy announcements.

The worst-case g becomes significantly lower than the baseline value endogenously only in the constrained region; therefore, the announcement policy does not alter asset demand in non-crisis states, preserving the risk premia comparable to those without government interventions and capital accumulation within the financial sector.

The higher probability of remaining in constrained states under the g policy arises from the lower objective risk premia in the constrained region, although to a lesser extent in the unconstrained region. Once the economy is constrained, these periods persist longer under the g policy, accompanied by moderately elevated risk premia. Additionally, resolving uncertainty under the g policy increases the risk-free rate due to the mitigated precautionary saving motive.

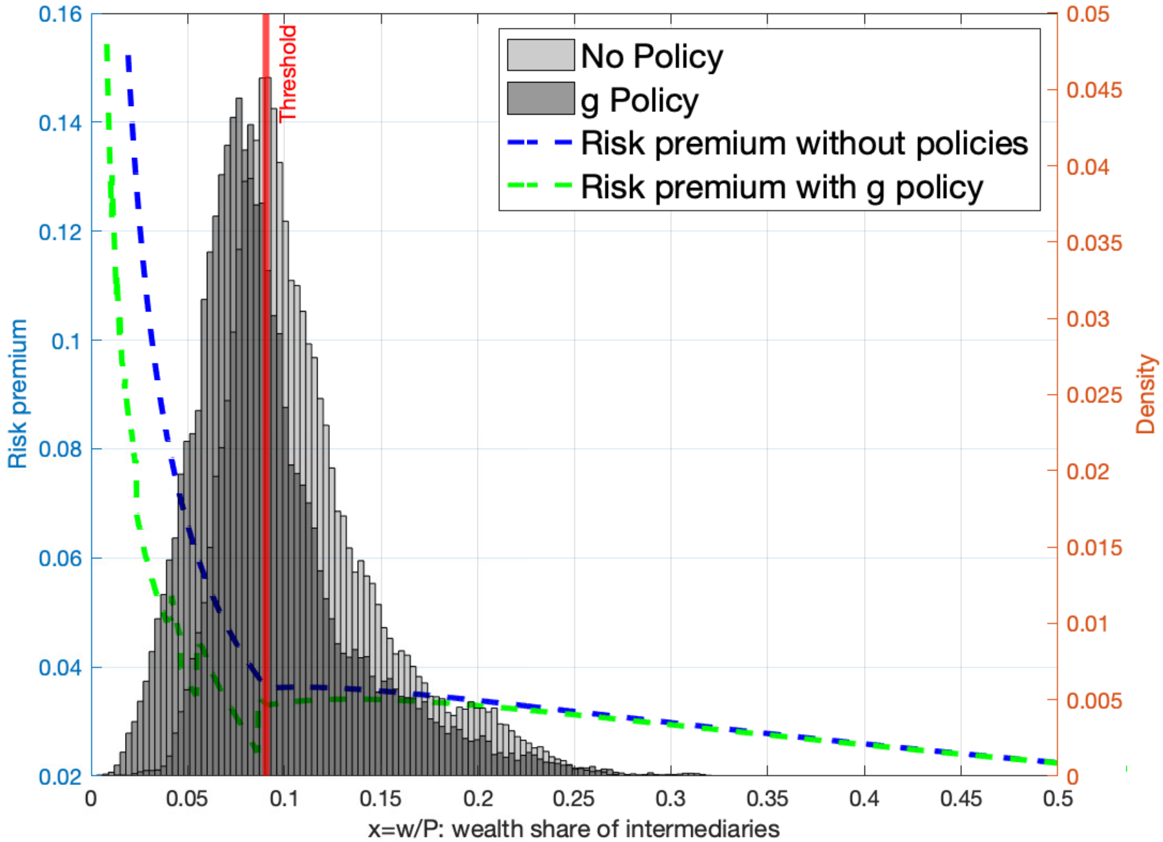

Figure 13: Stationary Distributions

The stationary distribution and objective risk premium against the actual realization of the aggregate state $x = w/P$ without and with policy announcements. The vertical red line is the constrained threshold. The threshold is not quantitatively different with and without policies. The darker histogram is the stationary distribution under the g policy, whereas the lighter one is without policies. The dashed blue line is the risk premia without policies, while the green represents those under the g policy.

	No Policy	m Policy	g Policy
Average risk premium (%)	3.84	3.88	3.6
Risk premium in the unconstrained region	3.58	3.51	3.36
Risk premium in the constrained region	4.61	4.53	3.73
Average Sharpe ratio	47.89	46.54	43.32
Average return volatility (%)	8.32	8.33	8.32
Average interest rate (%)	0.8	0.9	1.22
Average price-dividend ratio	36.16	36.24	35.68
Prob(unconstrained) (%)	60.36	64.33	36.09
Labor income ratio	0.64	0.64	0.64
Debt-to-asset ratio	0.56	0.55	0.62

Table 7: Measurements

The table presents several key moments from stochastic simulations in the models with and without policy announcements. I report the unconditional average risk premium, average risk premium in the unconstrained region, average risk premium in the constrained region, average Sharpe ratio, average return volatility, average interest rate, average price-dividend ratio, average labor income ratio, and debt-to-asset ratio. I also report the unconditional probability of the margin constraint not binding.