The Promises (and Perils) of Control-Contingent Forward Guidance*

He Nie[1] and Jordan Roulleau-Pasdeloup[2]

[1,2]Department of Economics, National University of Singapore

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Abstract

We develop a model with *control-contingent* forward guidance: the central bank explicitly anchors future policy announcements to short run inflation. Even though the model features past promises, we compute a closed form solution using a simple Markov chain representation. This allows us to show analytically that control-contingent forward guidance can rid the model of sunspot liquidity traps. The same holds for a policy of price level targeting, which emerges as a special case. Finally, we leverage this new framework to formally show that announced interest rates are only a means to an end: what truly matters is expected inflation.

Keywords: Zero Lower Bound, Forward Guidance, Sunspot Equilibria, Monetary Policy
JEL Codes: E52,E58,E61

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1 Introduction

What can central banks do when they have no choice but to set their nominal interest rate to its lower bound? Judging from recent experience, the answer seems to be: quite a lot. Indeed, central banks all over the world have been confronted to the interest rate lower bound and have responded by using quantitative easing, credit easing and forward guidance. The last one has been repeatedly used by the U.S. Federal Reserve and other central banks. Recently, Powell (2020) announced a new framework for the Federal Reserve monetary policy that amounts to a policy of forward guidance. Accordingly, forward guidance will be the focus of this paper. Loosely speaking, forward guidance involves a central bank announcing its future course of action. Therefore, such a policy has a dynamic feature to it and is announced to economic agents that are forward looking. In turn, this means that past promises arise as endogenous state variables and one has then to solve the model numerically. Because of this and despite its widespread use, little is known about the general properties associated with forward guidance.

Suppose that the central bank is forced to set its interest rate to zero, which is associated with a recession. What kind of forward guidance is advised to dampen the aforementioned recession? It turns out that the answer heavily depends on what exactly brought about the recession that forced the central bank to reach the lower bound. If this recession was brought about by an exogenous decrease in the efficient rate of interest, then we know since Eggertsson & Woodford (2003) that the central bank needs to keep its interest rate to zero even after the shock subsides. This creates anticipations of an overheated economy after the shock is over which, by anticipation, generates an increase in private demand early on. If the recession is brought about by a decrease in confidence that has nothing to do with fundamentals however, the central bank should implement an *increase* in interest rates instead —see Schmitt-Grohé & Uribe (2017) and Bilbiie (2018).

In this paper, we show that these—the presence of sunspot liquidity traps and forward guidance—are tightly linked. To do so, we develop a variant of the standard New Keynesian model along the lines of Eggertsson & Pugsley (2006) and Bilbiie (2019b), but where we allow the central bank to *explicitly*[1] anchor its announcements about future policy to current inflation: we call this a *control-contingent* policy stance. Interestingly, the framework is a close match to the recent monetary policy framework announced in Powell (2020).

We use a simple three-state Markov chain representation to solve the model in closed form.

[1] In our baseline formulation, announcements about interest rates in the medium run are anchored on realized inflation in the short run. This is in contrast to Bilbiie (2019b), where these are anchored to the fundamental shock. This seemingly innocuous difference turns out to be crucial for our results.

As a result, the model can be represented graphically using standard Aggregate Supply/Demand curves. This approach clarifies that control-contingent forward guidance changes the *slopes* of both these curves. In contrast, state-contingent forward guidance[2] that anchors announcements to the natural rate of interest only *shifts* those curves. The key is that control-contingent forward guidance *explicitly* ties expected inflation to realized inflation.

Using this new framework, we show analytically that a control-contingent policy stance that promises enough catch-up inflation can rid the model of sunspot liquidity traps. To do so, we consider a New Keynesian model that features sunspot liquidity traps in the absence of forward guidance. As has been shown in the literature (see Mertens & Ravn (2014),Wieland (2018, 2019) and Bilbiie (2018)), for a persistent enough sunspot the Euler and Phillips curves can cross a second time at the Zero Lower Bound (ZLB). By modifying the slopes of these two curves, a suitable policy of control-contingent forward guidance makes these curves rotate apart from each other: the sunspot liquidity trap cannot be an equilibrium. This arises because this policy makes households and firms expect a swifter recovery and thus counteracts contractionary income effects in the short run that become self-fulfilling in the absence of forward guidance. In contrast, a policy of state-contingent forward guidance does not modify the slopes of these curves and thus cannot rid the model of sunspot liquidity traps.

At first glance, this result seems to be at odds with Bilbiie (2018), who shows that forward guidance is contractionary in a sunspot liquidity trap. What our model makes clear is that a control-contingent policy of forward guidance has a inherent "*go big or go home*" feature: if the promised inflation in the medium run does not sufficiently make up for current below target inflation, then this policy actually worsens the sunspot induced recession. This arises because in such a recession households want to save instead of consume. In this context, a timid policy of control-contingent forward guidance makes the expected returns of saving less elastic to current savings through expected inflation. As a result, households rationally save more to reach their optimal expected returns on savings in the short run. These increased savings translate into lower aggregate demand and the sunspot liquidity trap is associated with a deeper recession as a consequence. Therein lies the spirit behind the title of the paper: while a timid policy of forward guidance can worsen a sunspot liquidity trap, a bold enough one can get rid of the sunspot liquidity trap altogether.

This framework also allows us to study in closed form a central bank policy that mimics Price Level Targeting (PLT). Indeed, a special case of a control-contingent policy stance is one where

[2]See Eggertsson & Pugsley (2006) and Bilbiie (2019b).

expected inflation/deflation in the medium run exactly offsets inflation/deflation in the short run. For example, assume that a sunspot liquidity trap generates a recession with deflation in the short run. If the central bank can credibly commit to generate enough inflation for a sufficiently long time in the medium run, then the expected cumulative sum of inflation during the simulation period can be made to be exactly zero. By construction,[3] the expected sum of inflation rates between time t and $t+T$ is equal to $\mathbb{E}_t p_{t+T} - p_{t-1}$. Therefore, a policy that equates this quantity to zero effectively stabilizes the price level to its starting point. We show that such a policy can prevent the occurrence of sunspots if it is aggressive enough. This sheds new light on the numerical findings in Holden (2019), where price level targeting is shown to almost always guarantee a unique equilibrium in models that feature an occasionally binding ZLB constraint. In addition, the fact that we can solve the model in closed form allows us to dig deeper. Accordingly, we can show that stabilizing the expected price level is *not enough*: the return of the price level has to be rapid enough to rule out sunspots. This result is also related to Armenter (2018), who shows that an economy with a discretionary central banker that targets the price level features multiple Markov-perfect equilibria. We show that allowing for some degree of commitment can rid the economy of sunspots, which are not considered in Armenter (2018).

It follows that these features turn out to be also warranted from a normative perspective: adopting a rapid return of the price level to its starting value when the economy is subject to sunspot liquidity traps can perfectly stabilize the economy: the announcements become a simple *off-equilibrium threat*.

Finally, we show that announcements of future interest rates do not matter, but expected inflation does. The results described in the previous paragraphs are derived in a model where the central bank directly announces expected inflation in the medium run as a function of short run inflation. In the last part of the paper, we show that this is the defining feature that makes monetary policy effective, *irrespective of how the interest rate is set in the medium run*. To show this, we study two extensions where the same path for inflation in the medium run can be either attained through zero, decreased or increased interest rates. Therefore, this framework makes clear that for an effective policy of forward guidance announced interest rates are mostly a mean to an end: *expected inflation*.

Related Literature.—The earliest formulation of what is now commonly known as forward guidance can be traced back to Krugman (1998). In this paper, the solution advanced to deal with a liquidity trap is for the central bank to commit to a *future* increase in the price level. This intu-

[3]Consider a central bank policy that ensures $\mathbb{E}_t \sum_{s=0}^{T} \pi_{t+s} = 0$. It then follows that, by the definition of price inflation $\mathbb{E}_t [p_t - p_{t-1} + p_{t+1} - p_t + \cdots + p_{t+T} - p_{t+T-1}] = \mathbb{E}_t p_{t+T} - p_{t-1} = 0$.

ition has further been formalized in a standard New Keynesian model in Eggertsson & Woodford (2003). There, the authors present numerical simulations of the optimal policy under commitment and show that the latter can be implemented through a suitable price level targeting rule. There is by now a burgeoning literature on optimal policy in fundamental liquidity traps: see Jung et al. (2005), Sugo & Teranishi (2005), Adam & Billi (2006), Nakov (2008), Sugo & Ueda (2008). Recent contributions also include Mendes (2011), Werning (2011), Schmidt (2013), Basu & Bundick (2015), Bilbiie (2019b), Armenter (2018), Hasui et al. (2016), Cochrane (2017), Nakata & Schmidt (2019a), Mertens & Williams (2019) and Duarte (2019). Except from Bilbiie (2019b), Mertens & Williams (2019) and Nakata & Schmidt (2019a), these papers present numerical solutions for optimal monetary policy after shock sends the economy at the interest rate lower bound. We will explain in more detail later how our paper differs from Bilbiie (2019b). Our paper differs from Nakata & Schmidt (2019a) in that these authors consider a central bank that cannot commit beyond the current period, which is crucial in our framework. Our paper is closely related to Mertens & Williams (2019). They focus on the ergodic distribution of interest rates/inflation in a New Keynesian model that potentially features commitment, but do not study how the latter influences the slopes of AS/AD schedules and gets rid of sunspots.

Since Benhabib et al. (2001), we know that the presence of the ZLB generates global multiplicity of equilibria in New Keynesian models featuring a Taylor rule. As a consequence, sunspots can send the economy at the ZLB without any fundamental shocks. The ensuing literature has shown that policy prescriptions differ markedly compared to a fundamental driven liquidity trap. For example, Mertens & Ravn (2014) show that the government spending multiplier is below 1 in this situation, which is in stark contrast with the results for a fundamental liquidity trap derived in Eggertsson (2010), Christiano et al. (2011) and Woodford (2011). This has been further clarified in Wieland (2018, 2019). Recently, both Bilbiie (2018) and Nakata & Schmidt (2019b) have studied the effect of different policies in a sunspot driven liquidity trap.[4] Finally, Aruoba et al. (2018) provide evidence for the fact that the U.S. has recently been stuck in a fundamental liquidity trap while Japan has been stuck in a sunspot driven liquidity trap. As a result, studying how monetary policy interacts with both types of liquidity traps should be high on the research agenda.

Compared to the existing literature, in this paper we show that taking the type of liquidity trap as given and studying the policy prescriptions is potentially misleading. Indeed, we formally show that the conduct of monetary policy in the medium to long run can either get rid of (the promise) or worsen (the perils) sunspot driven liquidity traps. As a result, this paper is also

[4] Relatedly, Garín et al. (2018), Uhlig (2018) and Uribe (2018) study the "Neo-Fisherian" properties of New Keynesian models which will arise in this framework in the context of sunspot liquidity traps.

related to Sugo & Ueda (2008) and Schmidt (2016). The former show that under optimal monetary policy with full commitment, the second steady state described in Schmitt-Grohe et al. (2001) disappears. The latter gets rid of short run sunspots with suitable Ricardian fiscal rules. Relatedly, Ono & Yamada (2018), Michaillat & Saez (2019), Nakata & Schmidt (2019b), Glover (2019), Cuba-Borda & Singh (2019), Diba & Loisel (2020) and Gabaix (2020) all find ways to get rid of the sunspot liquidity trap equilibrium. In a separate literature that departs from rational expectations, Benhabib et al. (2014) show that fiscal policy can get the economy out of an expectations-driven liquidity trap. To the best of our knowledge, none of these points to forward guidance as a potential solution.

This paper is also closely related to and builds heavily on Eggertsson & Pugsley (2006) and Bilbiie (2019b). The former considers a central bank that announces an *exogenous* medium run target inflation rate and applies this framework to make sense of the "mistake of 1937" in the context of the U.S. Great Depression. The latter is the first paper to analyse in closed form a policy of Optimal forward guidance (OFG). In this paper, the welfare maximizing policy after a large decrease in the efficient rate of interest is to make the *expected duration* of zero interest rates conditional on the shock. In the standard AS/AD representation (with inflation on the x axis and output gap on the y-axis) of a textbook New Keynesian model, the efficient rate shock shifts the AD curve down. A suitably specified OFG as in Bilbiie (2019b) shifts it back up and shifts the AS/Phillips curve down, generating inflation in the process. The same can be said of the policy considered in Eggertsson & Pugsley (2006).

In contrast, optimal monetary with commitment is control-contingent in the sense that it is anchored on current economic conditions. The same holds for the famous catch-up rule proposed in Reifschneider & Williams (2000). To capture this, we assume that announcements about medium run policy by the central bank are *explicitly* dependent on current economic conditions. As a result, in the same AS/AD representation the announced stance[5] in the medium run will modify the slopes of both AS and AD curves.[6] Therein lies the intuition as to why such a policy can worsen/prevent sunspots.

Finally, this paper is closely related to a recent paper by Eggertsson & Giannoni (2020). In this paper, the authors show that monetary neutrality in the medium/long run is prone to generate non-existence of equilibria and contractionary black holes. The same result arises in the standard New Keynesian model if efficient rate shocks are close to a threshold level of persistence. In a

[5]In this sense, the paper is also related to Hills et al. (2018), who study optimal monetary policy using promised values for output and inflation.

[6]Nakata & Schmidt (2019b) show that optimal discretionary government spending also amounts to a change in AD's slope when the ZLB is binding. If this change is large enough, sunspot liquidity traps can be ruled out.

flexible price version of the model, the authors show that a price level targeting policy can restore uniqueness of equilibrium in this context. The results in our paper therefore complements these as we show formally how a policy of forward guidance (or its price level targeting special case) can get rid of sunspot ZLB equilibria.

The rest of the paper is structured as follows. We lay out the baseline model without forward guidance in Section 2 to highlight the different types of liquidity traps that can arise. In Section 3, we describe the general model where the central bank makes announcement about medium run policy. We use this to study forward guidance in sunspot liquidity traps. In Section 4, we show two examples with different paths for medium term interest rates, but which yield the same inflation rate and are thus isomorphic to the mode of Section 3. We conclude in Section 5.

2 Liquidity traps in a model without forward guidance

We consider a standard New Keynesian model linearized around its non stochastic steady state. A representative household consumes, supplies labor elastically and saves in one period government bonds. The representative household's utility function depends on the logarithm of consumption and a convex function of labor with curvature $\eta > 0$. Household members discount future consumption at a rate $\beta < 1$. Absent government spending, consumption c_t equals output. We express the model in deviations from its flexible price counterpart and let y_t denote the output gap. Households supply labor services to a continuum of monpolistically competitive firms that sell a differentiated good with an elasticity of substitution across goods given by $\theta > 1$. Firms face quadratic Rotemberg (1982)-type adjustment costs with constant parameter ψ. In the case of $\psi = 0$, the model features perfectly flexible prices. We let π_t denote net inflation. The equilibrium conditions of the private sector are as follows[7]

$$y_t = \mathbb{E}_t y_{t+1} - [R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t] \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t, \tag{2}$$

where ϵ_t is the efficient rate of interest, *i.e* the one that prevails under flexible prices.[8] Turning to the Aggregate Supply equation, $\kappa \equiv \theta(1+\eta)/\psi$ is the elasticity of current inflation to the output

[7]Due to the limited space, we only show the linearized equilibrium aggregate demand and supply equations here and other derivatives are presented in Appendix A.

[8]For empirical estimates about this efficient rate of interest, see Laubach & Williams (2003), Holston et al. (2017) and Christensen & Rudebusch (2019).

gap. For this Section only, monetary policy follows the following Taylor (1993)-type rule:

$$R_t = \max\left[0; -\log(\beta) + \phi_\pi \pi_t\right],$$

where we assume that $\phi_\pi > 1$. We assume that the natural rate shock has a simple two-state Markov structure with each respective steady state being absorbing. The persistence of the natural rate shock is given by p. We also assume that there is potentially a sunspot shock with the same persistence. Both the sunspot and the natural rate shock revert to their steady state value of zero with probability $1-p$. Using the fact that the model is purely forward looking, the allocation will be constant for a stochastic number of periods. We let variables with a subscript 'S' denote the allocation during the short run period. For example, the output gap in the short run is given by y_S. Letting $\mathbb{E}_S$ denote expectations conditional on the economy being in regine 'S', we can write $\mathbb{E}_S y_{t+1} = p \cdot y_S$ and likewise for inflation. As a result, we can rewrite the Phillips Curve as

$$y_S = \frac{1-\beta p}{\kappa}\pi_S, \tag{3}$$

where we label the coefficient on the right hand side the slope of the Phillips curve. For the Euler equation, we have to differentiate two cases depending on whether the decrease in the efficient rate of interest is large enough or a persistent sunspot arises. Then we have the following Euler equations:

$$y_S = \begin{cases} -\frac{\phi_\pi - p}{1-p}\pi_S + \frac{\epsilon_S}{1-p} & \text{if } R_S > 0 \\ \frac{p}{1-p}\pi_S + \frac{\epsilon_S - \log(\beta)}{1-p} & \text{if } R_S = 0. \end{cases} \tag{4}$$

The coefficient multiplying π_S on the right hand side of the second equation will be referred as the slope of the Euler equation. We will refer back to these when we will derive the corresponding slopes for the model with forward guidance. Using the two equations (3)-(4) and assuming that the ZLB is binding, we can get the following solution for y_S and π_S:

$$y_S = \frac{1-\beta p}{(1-p)(1-\beta p) - p\kappa}(\epsilon_S - \log(\beta))$$
$$\pi_S = \frac{\kappa}{(1-p)(1-\beta p) - p\kappa}(\epsilon_S - \log(\beta)).$$

As is made clear in Bilbiie (2018) and Wieland (2018, 2019), the sign of the denominator will be crucial here. Let us define this denominator as a function $\mathcal{D} : \mathbb{R} \to \mathbb{R}$ with argument p. It is straightforward to check that $\mathcal{D}(\cdot)$ is a quadratic function of p with a discriminant that is strictly positive —see Appendix B for details. As a result, this function has exactly two zeros in $\mathbb{R}$ and we show in the Appendix B that one of these is strictly higher than 1 while the other is in $(0,1)$. Since p is a probability, we focus on the second of these roots which we call $\overline{p}$.

Since $\mathcal{D}(0) = 1$ by construction, we know that $\mathcal{D}(p) > 0$ for $p < \overline{p}$. In this case, the economy features deflation and a fall in the output gap if ϵ_S is lower than some threshold. As is shown in Bilbiie (2018) and Wieland (2018, 2019), another possibility arises when $p > \overline{p}$. In this case, in the absence of an efficient rate shock there will be two short run equilibria, which opens the door to sunspot fluctuations taking the economy to an equilibrium that again features deflation and a fall in the output gap.[9] To better illustrate these two configurations visually, I will use a standard calibration of the model to plot the two curves. The calibration is described in Table 1 and the two curves are represented in Figure 1.

Table 1: Parameter Values

Discount factor	$\beta = 0.99$
Preference parameter	$\eta = 1$
Elasticity of inflation w.r.t real marginal cost	$\kappa = 0.2$
Inflation feedback parameter	$\phi_\pi = 1.5$
Persistence (fundamental)	$p = \overline{p} - 0.1$
Persistence (sunspot)	$p = \overline{p} + 0.1$

From Figure 1, one can see that there is a threshold level of deflation such that, if deflation passes this threshold then the central bank has no choice but to set its interest rate to zero. One can see that the slope of the Euler/AD curve is crucial. What really matters here is whether or not the slope of the AS curve is higher/lower than its AD counterpart when the economy is at the ZLB. For a fundamental ZLB episode, the slope of the AS curve is bigger than its AD counterpart. The reverse holds for the sunspot-driven liquidity trap. As a result of this, the AS and AD curves cross twice which opens the door to a sunspot Zero Lower Bound equilibrium. Notice that increasing p generates the second crossing by *(i)* increasing the AD slope and *(ii)* decreasing the AS slope simultaneously. Observe also that if a fundamental demand shock persists with probability $p > \overline{p}$ and is large enough, the AD curve can shift down so much that there exists no stable equilibrium in this economy. This echoes the findings reported in Mendes (2011).

Figure 1 is also instructive to understand how forward guidance can dampen (resp. worsen) a recession in Bilbiie (2019b) (resp. Eggertsson & Pugsley (2006)). In these papers, a policy of forward guidance is best viewed as a shock that potentially *shifts* both curves. In Eggertsson &

[9] Strictly speaking, it has been argued that this second equilibrium could just be a theoretical curiosity since it is not E-stable —see Christiano et al. (2018). However, Mertens & Ravn (2014) have shown that under different learning schemes this equilibrium is relevant since the economy can spend a long period of time around this equilibrium before converging back to a vicinity of the intended steady state. In addition, Arifovic et al. (2018) show that this sunspot liquidity trap equilibrium is stable under social learning.

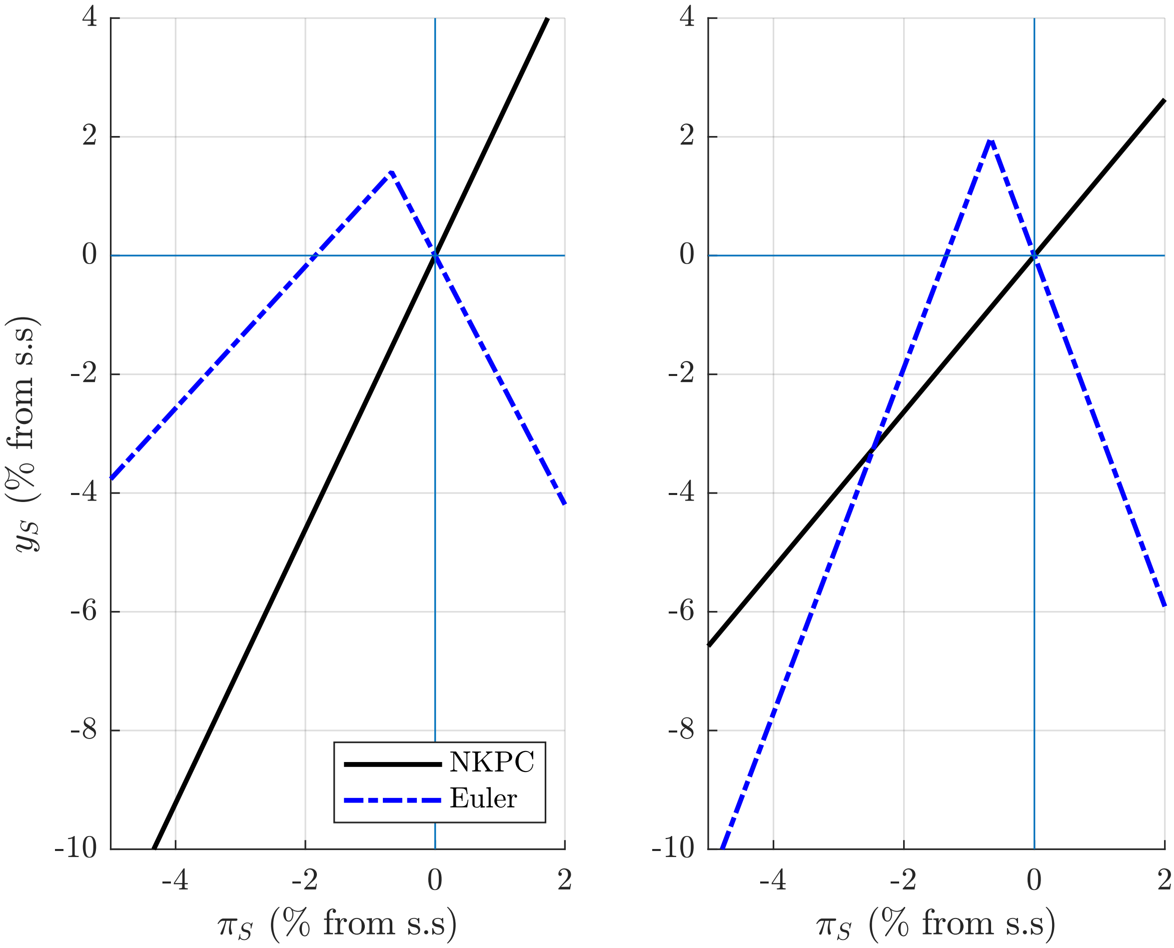


Figure 1: Fundamental (left) and Sunspot (right) Liquidity Traps

Pugsley (2006), the central bank is assumed to credibly announce a positive inflation target in the medium run. This generates both higher expected inflation and output. As a result, this policy will shift the AD/Euler curve up and the AS/Phillips curve down through sticky prices and rational expectations. In Bilbiie (2019b), forward guidance amounts to keep the interest rate at zero for an (stochastic) extended period of time after the demand shock subsides in order to increase expected consumption in the future. Therefore, while the initial demand shock shifts the AD curve down, forward guidance shifts it back up. Through sticky prices and rational expectations, it also shifts the Phillips curve down and generate more inflation today. As shown in Bilbiie (2018), this type of forward guidance makes things worse in a sunspot liquidity trap: in this case, the AD curve only moves up as a result of the forward guidance policy and generates further deflation and a more negative output gap. Bilbiie (2018) shows that the optimal policy in this context is to *increase* interest rates during the trap and target zero inflation after.

Finally, before we move on to the setup with forward guidance, we can get more intuition about the two different cases if we re-cast the Euler equation. The following discussion will build heavily on Bilbiie (2018), who discusses how competing income and inter-temporal substitution effects interact in this framework. To avoid confusion, let us define expected output gap next period, conditional on being in the short run state with low efficient rate or pessimistic expectations by $\mathbb{E}_S y_{t+1}$. Following Bilbiie (2018), it can be shown that the output gap in the short run can be expressed as

$$y_s = \frac{1 - \beta p + \kappa}{1 - \beta p} \times \mathbb{E}_S y_{t+1} + \text{t.i.p}, \tag{5}$$

where t.i.p denotes terms independent of policy. The coefficient multiplying expected output is called the *elasticity to future news shocks on aggregate demand* in Bilbiie (2018) and is strictly larger than 1. In this simple framework without forward guidance, one can write $\mathbb{E}_S y_{t+1} = p \cdot y_S$ so that whenever there is a short run variation in the output gap, the latter is expected to stay away from its steady state deviation of 0 with probability p. In other words, if there is a liquidity trap today so that $y_S < 0$, one expects the output gap to be $p \cdot y_S < 0$ tomorrow as well.

Assume that there is a recession with zero interest rates in the short run. Due to shock persistence, agents expect the economy to still be in a recession next period so that $\mathbb{E}_S y_{t+1} < 0$. This negative news about future aggregate demand is amplified by the larger than 1 elasticity and delivers a further decrease in aggregate demand today if $p < \overline{p}$. Using $\mathbb{E}_S y_{t+1} = p \cdot y_S$, this condition on p is such that the coefficient multiplying y_S on the right hand side is strictly lower than 1. When that is the case, one can solve this expression backward and recover the effect of the efficient rate shock on y_S. In this case, news of low aggregate demand next period triggers low aggregate demand today and thus low inflation —which also persists tomorrow. Lower expected inflation tomorrow in turn triggers inter-temporal substitution effects that further brings aggregate demand down through higher expected real interest rates. This is the famous vicious feedback loop described in Eggertsson (2010). When $p \to \overline{p}$, these effects go unboundedly large and the economy enters what Eggertsson & Giannoni (2020) call a contractionary black hole.

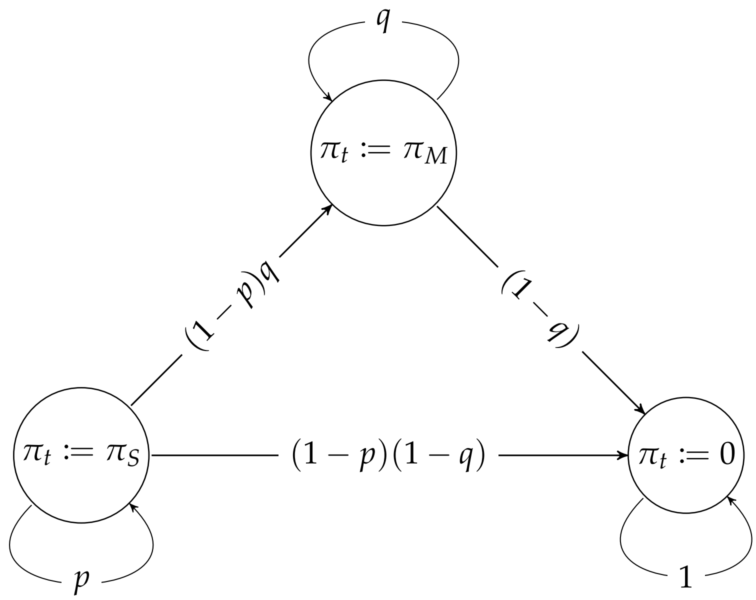
When $p > \overline{p}$ however, the coefficient multiplying y_S on the right hand side is strictly larger than 1. This opens the door to sunspot fluctuations. Indeed, assume that there are no fundamental shocks but only a sunspot here so that the t.i.p term is only proportional to $\log(\beta)$. In this case, pessimistic news about future aggregate demand trigger a *more than proportional* decrease in aggregate demand today. These again feed into expected aggregate demand tomorrow because of the high persistence of the sunspot. As a result, the output gap as well as inflation end up negative, validating the initial pessimistic expectations. All in all, what brings a liquidity trap here is an expected wealth effect that feeds into itself.

We will study how forward guidance modifies this representation of the AD/Euler equation later on to get a better intuition into how the former actually works here. To that end, we now describe the model with forward guidance.

3 A model with central bank announcements

In this Section, we generalize the model from Section 2 to allow for the central bank to make announcements about its policy stance in the medium run. The economy starts in state 'S' and can transition to two different states from there. It can go back to steady state as in last Section. Another possibility is to go through state 'M' before going back to steady state. As a result, the economy follows a 3-state Markov chain. The latter is illustrated in Figure 2 with the associated transition probabilities. In particular, we assume that the economy moves from regime 'S' to

Figure 2: forward guidance: a graphical representation



regime 'M' with probability $(1-p)q \in (0,1)$. Once in regime 'M', the economy stays there with probability q or goes back to steady state with complementary probability $1-q$. Another possibility starting from regime 'S' is to go back directly to steady state as in Last section. This now happens with probability $(1-p)(1-q)$. Once in steady state, the economy stays there with probability 1. Notice that for $q = 0$, the top-part of the graph in Figure 2 does not matter and we go back to the analysis of last section. With this in mind, we now need to describe the relationship between medium-run inflation π_M and its short run counterpart π_S which will be crucial. Central bank announcements about medium run inflation are given by:

$$\begin{cases} \pi_M = -\zeta_\pi \pi_S - \zeta_\epsilon \epsilon_S & \text{with probability } q \\ \pi_M = 0 & \text{with probability } 1-q, \end{cases}$$

where both ζ_π and ζ_ϵ govern the stance of monetary policy in the medium run.[10] We will use the following definition throughout the paper:

Definition 1. *A vector $(q, \zeta_\pi, \zeta_\epsilon) \in [0,1] \times \mathbb{R}^2$ is called a **policy stance**. The policy stance is:*

1. *neutral if $q \times \zeta_\pi = q \times \zeta_\epsilon = 0$*
2. *control-contingent if $q \times \zeta_\pi > 0$ and $\zeta_\epsilon = 0$*
3. *state-contingent if $q \times \zeta_\epsilon > 0$ and $\zeta_\pi = 0$.*

For example, if $\zeta_\pi = \zeta_\epsilon = 0$ the central bank acts as a discretionary policy maker by returning inflation to its target value of 0 as soon as the shock subsides. As has been pointed out by Eggertsson & Woodford (2003), this type of policy is far from optimal and usually results in a deep recession after a large decrease in the natural rate of interest. If $q > 0$, then the Central bank makes an announcement about future inflation. We distinguish the cases where announcements about future inflation are conditioned on the short run natural rate shock ($\zeta_\epsilon > 0$) or the short run inflation rate ($\zeta_\pi > 0$). Given that the first one depends on the exogenous state of this economy, we call this policy stance *state-contingent*. Likewise, since the second one depends on inflation, which is a control variable we call this policy stance *control-contingent*. Of course, one can also think about a hybrid policy stance where announced inflation depends on both the exogenous state and the control variable.[11] Since the goal of this paper is to highlight the benefits of a control-contingent stance compared to a state-contingent one, we focus on the two polar cases.

Both the *state-* and *control-contingent* policy stances are similar in spirit to the catch-up rule advocated in Reifschneider & Williams (2000). More specifically, Reifschneider & Williams (2000) advocate for the use of a Taylor rule that makes up for past undershooting in inflation by keeping the interest rate at zero for an extended period of time. This stimulates the economy both in the medium run and in the short run through rational expectations. Both the *state-* and *control-contingent* policy stances exhibit a similar feature of history dependence as the inflation target *in*

[10] To avoid the possibility of sunspots in the medium run, we assume throughout the paper that $q < \overline{q}$, where $\overline{q}$ is such that $\overline{q} < 1$ and $\mathcal{D}(\overline{q}) = 0$.

[11] One could also think about a policy stance where either ζ_π or ζ_ϵ are strictly negative. This would amount to the central bank promising to decrease inflation in the medium run. Since this is generally the opposite of what central banks try to do in the context of a recession, we do not consider this possibility here.

the medium run is adjusted proportionally to short run outcomes. Among the two, the control-contingent policy stance is closer in spirit to Reifschneider & Williams (2000) since it makes this dependence explicit.

We remain agnostic—for now—about exactly how the central bank achieves this medium run inflation rate to focus on the implications of the latter. In principle, any desired value of inflation could be engineered by the central bank by setting its interest rate to zero for some time. We show in Section 4 that the actual value of the central bank interest rate in the medium run is essentially immaterial and what truly matters is expected inflation. This is the reason why we focus on expected inflation throughout this section.

With this in mind, this announcement by the central bank is anticipated from the start so that we can write expected inflation (conditional on the economy being in regime S) as follows:

$$\begin{aligned} \mathbb{E}_S \pi_{t+1} &= p\pi_S + (1-p)q\pi_M \\ &= p\pi_S - (1-p)q(\zeta_\pi \pi_S + \zeta_\epsilon \epsilon_S) \\ &= (p - (1-p)q\zeta_\pi)\pi_S - (1-p)q\zeta_\epsilon \epsilon_S \end{aligned}$$

The last equation clarifies the main difference between control- and state-contingent forward guidance. On the one hand, control-contingent forward guidance modifies how short run inflation feeds into expected inflation next period—see first term on the right hand side. On the other hand, state-contingent forward guidance modifies expected inflation *irrespective* of the short run inflation rate. Crucially, π_S will be a function of ϵ_S in equilibrium. But that is a feature of the solution and not of individual optimization behavior. This distinction will be important for how these two policy stances influence dynamics with an occasionally binding Zero Lower Bound. Naturally, a similar expression can be derived for the expected output gap.

Observe that if either $q\zeta_\pi$ or $q\zeta_\epsilon$ is sufficiently large, then expected inflation can actually be positive even though $\pi_S < 0$ after a recession in the short run. Using these expressions for expectations to solve for the short run equilibrium, it will be clear that a control-contingent policy stance will change the slope of both the AS and AD curves. In contrast, state-contingent will shift both AS and AD curves. If the goal is to generate a desired level of inflation, then both policy stances can do the trick. However, the main difference is that a suitably defined control-contingent stance can potentially rid the model of sunspot liquidity traps in the short run. To clarify this, notice that the AS/Phillips curve is now written as follows:

$$y_S = \frac{1-\beta p + \beta(1-p)q\zeta_\pi}{\kappa}\pi_S + \frac{\beta(1-p)q\zeta_\epsilon}{\kappa}\epsilon_S. \tag{6}$$

Now consider plotting this AS/Phillips curve in a (π_S, y_S) graph. From equation (6), it is clear that a control-contingent stance will modify the slope of this curve. In contrast, a state-contingent stance will generate a shift in the same curve. Now remembering Figure 1, what opens the door to sunspot liquidity traps is that the slope of this curve is too low. From equation (6), this can be readily remedied by setting $q\zeta_\pi$ high enough: making sure that catch-up inflation in the medium run sufficiently corrects for below target inflation in the short run.

Beyond the mathematical requirement, the economic intuition is as follows. A control-contingent policy stance counteracts short run variations in inflation by generating the opposite in the future *in a proportional manner*. As a result and because of sticky prices, expected inflation matters less for current inflation. In equilibrium, this implies that actual inflation in the short run moves less for a given realization of y_S. The flip side of this is that the output gap reacts relatively more to each variation of inflation, which explains the higher slope. In contrast, for a state-contingent policy stance the central bank can generate expected inflation, but the latter will *not be perceived* by rational households/firms as being proportional to short run inflation. Eventually, inflation will be proportional to the natural rate shock, but that is an ex-post, general equilibrium result.

All the features that have been highlighted here depend on expectations. Therefore, these will also be featured in the AD/Euler equation. As before, for the Euler equation two cases must be considered. With this in mind, the Euler equation can now be written as follows:

$$y_S = \begin{cases} \frac{p-\phi_\pi-(1-p)q\zeta_\pi\left(1+\frac{1-\beta q}{\kappa}\right)}{1-p}\pi_S + \frac{1-(1-p)q\zeta_\epsilon\left(1+\frac{1-\beta q}{\kappa}\right)}{1-p}\epsilon_S & \text{if } R_S > 0 \\ \frac{p-(1-p)q\zeta_\pi\left(1+\frac{1-\beta q}{\kappa}\right)}{1-p}\pi_S + \frac{1-(1-p)q\zeta_\epsilon\left(1+\frac{1-\beta q}{\kappa}\right)}{1-p}\epsilon_S - \frac{\log(\beta)}{1-p} & \text{if } R_S = 0. \end{cases}$$

From these two equations, it is clear that a control-contingent policy stance will generate a *lower* AD/Euler slope. In contrast and in line with the results presented for the AS/Phillips curve, a state-contingent stance will not modify the slope but will mitigate the effects of a natural rate shock instead. In a nutshell, with a neutral stance the natural rate shock shifts the AD/Euler curve down. With a state-contingent stance, anticipated monetary policy announcements shift it back up. In addition, these announcements also shift the AS/Phillips curve down—this curve does not shift in the absence of announcements.

To illustrate this, we report two experiments in Figure 3. First, we report the AS/AD curves for the control-contingent stance after a fall in the natural interest rate in the left panel. As a benchmark, we also report the AS/AD curves for the model with a neutral stance. In this context, the fall in the natural rate of interest is calibrated so that it generates a sizable recession and a binding Zero Lower Bound in the short run. In the right panel, we report the AS/AD curves for the state-contingent stance after the same fall in the natural interest rate. In both cases, we assume an expected duration of $1/(1-0.5) = 2$ quarters for the catch-up inflation period. For simplicity, we also assume that $\zeta_\epsilon = \zeta_\pi = 1$.

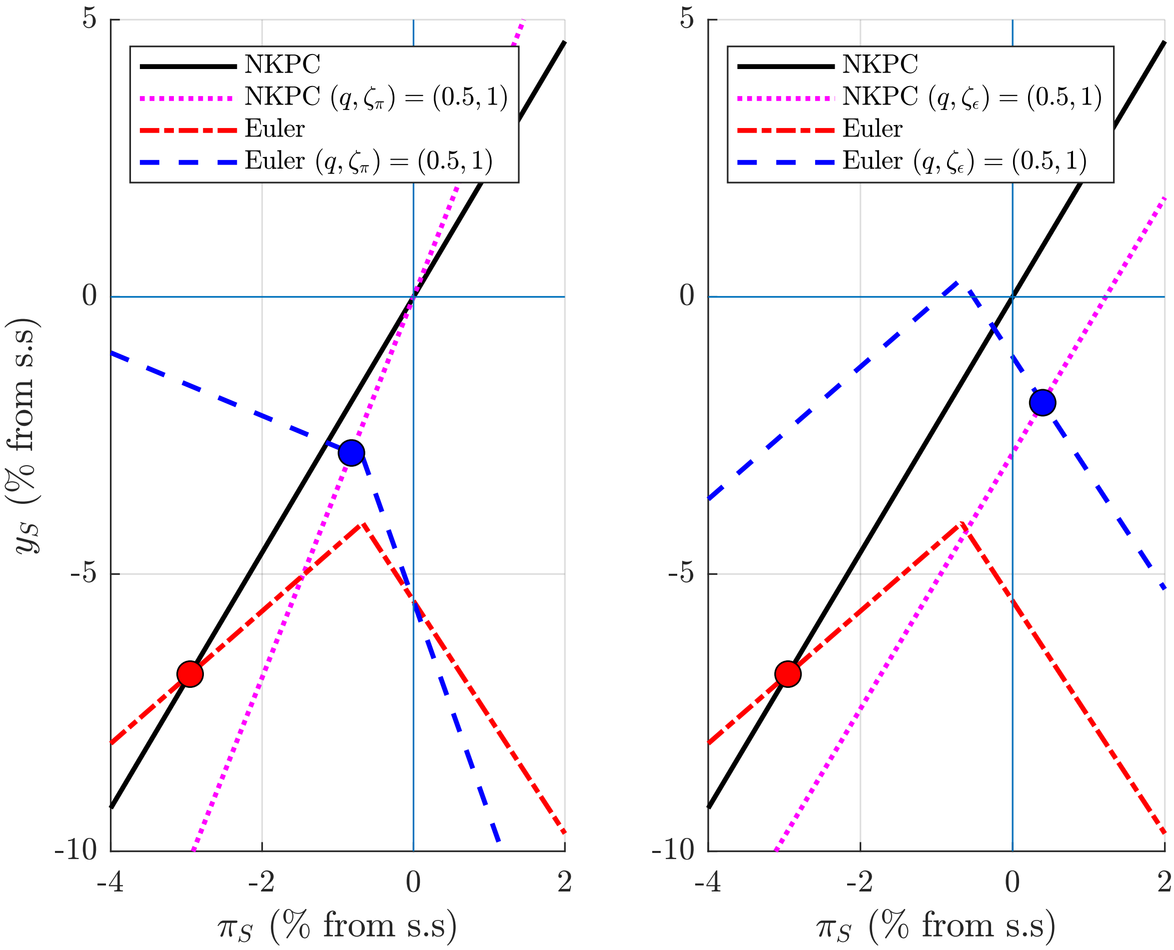


Figure 3: AS/AD with control- and state-contingent stances

The key take-away from Figure from Figure 3 is that both control- and state-contingent policy stances mitigate the impact of the decrease in the natural rate of interest. However, they do it in a very different manner. Focus first on the control-contingent policy stance on the left panel. One can see that a control-contingent policy stance increases the slope of the Phillips Curve. For the Euler equation, it decreases the slope. In normal times, this results in a steeper AD/Euler curve. At the ZLB however, in this case the AD/Euler actually switches signs and becomes negative. As a result, the infamous switch in the AD/Euler curve at the ZLB (see Eggertsson (2010)) does not happen in this case and the relationship between inflation and the output gap is negative throughout the plane.

Turning to the state-contingent policy stance, one notices that it does not change any of the slopes but instead shifts both the AS and AD curves. Since the central bank counteracts the fall in the natural rate of interest by announcing more inflation in the future, both of these shifts deliver higher inflation today through sticky prices and rational expectations. Notice that the upward shift in AD is enough by itself to get the economy out of the Zero Lower Bound. On top of that, increased inflation from the Phillips curve acts like a cost push shock: it increases short term inflation while decreasing short term output.

From these two panels, it is clear that state-contingent forward guidance as in Bilbiie (2019b) can mitigate the effects of a large decrease in the natural rate of interest. However, it cannot rid the new Keynesian model of sunspot liquidity traps since it leaves the slopes of AS/AD curves unchanged. For this reason, we will mainly focus on the control-contingent policy stance in the remainder of the paper. Accordingly, unless specified otherwise we now assume that $\zeta_\pi > 0$, $\zeta_\epsilon = 0$ and $\epsilon_S = 0$.

To set the stage for the following discussion, we re-derive the equivalent of equation (5) relating current aggregate demand with respect to news about the future, but allowing for central bank announcements this time. We obtain the following expression:

$$\begin{aligned} y_S &= \mathcal{E}(q, \zeta_\pi, \cdots)\mathbb{E}_S y_{t+1} - \log(\beta) \\ &= \mathcal{E}(q, \zeta_\pi, \cdots)\left[p - (1-p)q\zeta_\pi \frac{1-\beta q}{\kappa}\right] y_S - \log(\beta), \end{aligned} \tag{7}$$

where the elasticity $\mathcal{E}(q, \zeta_\pi, \cdots)$ is a complicated function of model parameters and is reported in Appendix C. Naturally, $\mathcal{E}(0, 0, \cdots)$ reduces to the slope derived in equation (5) for the model with a neutral policy stance. From equation (7), one can see that the central bank's stance can affect the equilibrium allocation through two channels: *(i)* by modifying the elasticity with respect to news about future aggregate demand and *(ii)* by modifying the *expected* persistence of y_S. While the impact of the policy stance on the elasticity is *a priori* ambiguous, one can see that increasing ζ_π has a clear negative impact on $\mathbb{E}_S y_{t+1}$. Regarding q, the persistence of y_S is first decreasing —if $q \leq 1/(2\beta)$, then increasing. Notice that if ζ_π is large enough, then expected output gap tomorrow can even switch signs: in this case, the stance of monetary policy is such that households/firms do not anticipate a recession anymore but a recovery/boom next period.

The result that forward guidance can mitigate the negative impact of a short term decrease in the natural rate of interest has been known since Eggertsson & Woodford (2003) and the literature that has followed. We don't know yet whether forward guidance can rid the model of sunspot liquidity traps and much less how that can happen. Accordingly, we will maintain the assumption that $p > \overline{p}$ so that the economy can fall into a sunspot Zero Lower Bound episode under a neutral policy stance. The goal will now be to study under which conditions a control-contingent policy stance can rid the model of sunspot ZLB episodes.

3.1 Forward guidance with persistent sunspots

In this subsection, we will focus on the second (right) panel of Figure 1 where p is high enough so that the AS/AD curves cross twice. The main result of this subsection can be understood readily from this Figure. Given the results derived previously, remember that a control-contingent monetary policy stance both *(i)* increases the slope of the AS/Phillips curve and *(ii)* decreases the slope of the AD/Euler curve. To facilitate the discussion, we will use the following definition:

Definition 2. *Assume that we have two control-contingent policy stances* $(q^{(1)}, \zeta_\pi^{(1)})$ *and* $(q^{(2)}, \zeta_\pi^{(2)})$*. We say that policy stance* (1) *is strictly more pro-active compared to policy stance* (2) *if*

$$q^{(1)} \times \zeta_\pi^{(1)} > q^{(2)} \times \zeta_\pi^{(2)}.$$

Given the reported effects on the respective slopes, it follows that starting from a neutral stance and adopting a slightly more pro-active control-contingent stance will make the sunspot equilibrium worse: more deflation and a larger output gap. As a result, to get rid of the sunspot equilibrium the monetary policy stance will have to be pro-active enough. For simplicity, we take $q > 0$ as given here and study equilibrium multiplicity as a function of ζ_π. The effect of varying ζ_π on the two equilibria is illustrated in Figure 4 where we plot the AS/AD curves for $q = 0.5$ and $\zeta_\pi = \{0, 0.2, 1\}$.

The first panel essentially reproduces the left panel of Figure 1. The middle panel of Figure 4 illustrates what happens after a small increase in ζ_π: both slopes rotate apart from each other. This causes a larger deflation associated with a larger decrease in output in the sunspot equilibrium. To understand the intuition behind this result, we will use once again equation (7), which we

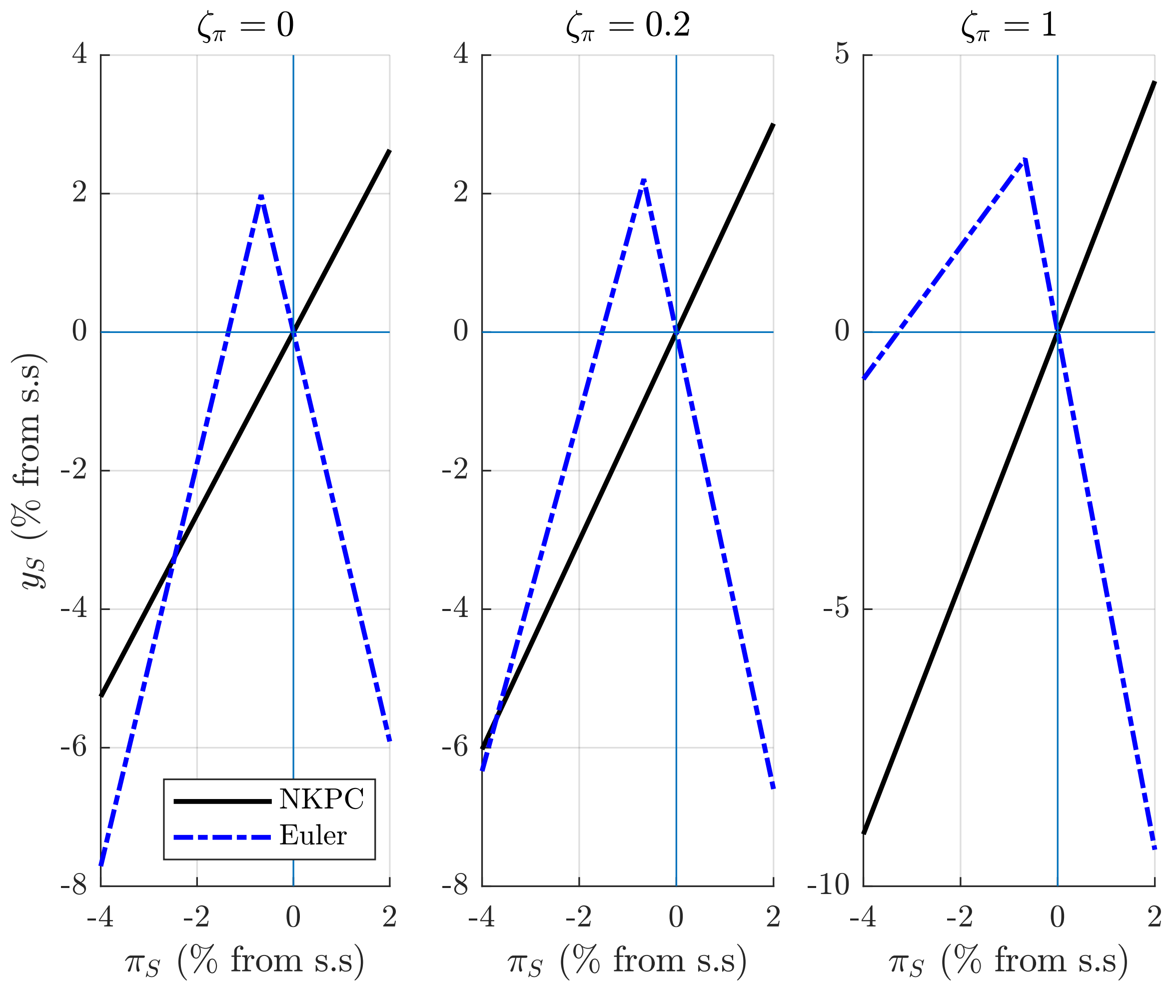


Figure 4: AS/AD with more pro-active state-contingent stances

reproduce here for convenience:

$$y_S = \underbrace{\mathcal{E}(q, \zeta_\pi, \cdots) \left[p - (1-p)q\zeta_\pi \frac{1-\beta q}{\kappa} \right]}_{>1} y_S - \log(\beta),$$

Assume initially that $y_S = 0$ so that output is equal to its steady state value in the short run. From this equation, this cannot be an equilibrium: the marginal benefit from consuming today (left hand side) is 0, while the marginal benefit of saving today (right hand side) is positive since $-\log(\beta) > 0$. To restore equilibrium, household members will therefore save more and consume less. This will reduce aggregate demand so that $y_S < 0$. Now, the lower is the coefficient on the right hand side —the closer it is to 1 since it has to be strictly larger than 1, the lower will be the effects of increased savings on the expected returns to saving. If the coefficient on the right hand side is close to 1, a large increase in savings is required to restore equilibrium. This explains why adopting a marginally more pro-active policy stance in a sunspot liquidity trap will make things worse. This result echoes the findings in Bilbiie (2018) where it is found that a policy of forward guidance in such a situation entails implementing a current increase in interest rates and announcing zero inflation in the medium/long run.

That being said, the third panel of Figure 4 illustrates that a control-contingent policy stance that is sufficiently pro-active can rid the model of the sunspot ZLB equilibrium. Indeed, if this stance is pro-active enough then the coefficient multiplying y_S on the right hand side is strictly lower than 1 and the income effects from the Euler equation are effectively dampened by monetary policy announcements. In this case, one can use the Euler equation and solve it forward to get an expression for y_S as a function of the natural rate of interest, which is assumed to be zero: the steady state is the only stable equilibrium. This is the reason why this equilibrium is usually labeled the fundamental equilibrium. When the sunspot ZLB equilibrium arises, the income effects from the Euler equation are strong enough so that iterating the Euler equation forward generates a cumulative sum that diverges: the only possible equilibrium at the ZLB is a sunspot, non-fundamental one. These intuitions gleaned from Figure 4 are formalized in Proposition 1.

Proposition 1. *Assume $0 < q < \overline{q}$. There exists a threshold level $\underline{\zeta_\pi} > 0$ such that*

1. *Output gap/inflation in a sunspot equilibrium is strictly decreasing in ζ_π for $\zeta_\pi < \underline{\zeta_\pi}$.*

2. *The sunspot equilibrium does not arise for $\zeta_\pi > \underline{\zeta_\pi}$.*

Proof. See Appendix D. □

From Figure 4, one can see that the presence of pro-active monetary policy is enough to rid the model of the sunspot equilibrium. Indeed, observe that the slope of the AS/Euler equation at the ZLB is *lower* than the AS/NKPC. Note that this particular monetary policy stance is more than enough to get rid of the sunspot. Using the baseline calibration and the formula from Proposition 1, for $q = 0.5$ we find that $\zeta_\pi \geq 0.67$ is enough. Therefore, the policy stance needed to get rid of the sunspot equilibrium does not involve the central bank to promise runaway inflation in the medium run. In addition, the expectation of accommodative monetary policy in the medium run will decrease the magnitude of deflation in the short run, which will further decrease the amount of inflation actually needed in the medium run.[12]

[12] One drawback of this kind of policy is that, for given q it is proportional to the expected duration of the sunspot $1/(1-p)$. For permanent sunspot so that $p \to 1$, ruling out the latter implies $\zeta_\pi \to \infty$ and is thus infeasible. The same will hold for a policy of Price Level Targeting.

3.2 Price Level Targeting as a special case

A longstanding debate in monetary economics revolves around the following question: is price level targeting (PLT) a policy that better stabilizes macroeconomic aggregates[13] compared to inflation targeting? This question has mostly been answered with an affirmative answer. See for example Bernanke (2017) for a recent overview in light of the Zero Lower Bound. The reason for this largely has to do with the fact that a PLT-style policy mimics the optimal policy with commitment.[14] The technical issue that arises with such a rule is that it introduces past prices into the model, which now features an endogenous state variable. This complicates the solution of the model such that a closed form one is not attainable in general. To remedy this here, we will consider a rule that features one of the central properties of PLT rules: the price level is expected to go back to its initial level after a shock.

Let us consider the following experiment: the economy starts with a given price level p_{t-1}. Then a sunspot happens at time t. Using the simple Markov chain structure of the current framework, we show in the Appendix E that one can write expected cumulative inflation as follows:

$$\begin{aligned}
\mathbb{E}_S \sum_{k=0}^{T} \pi_{t+k} &= \frac{1}{1-p}\pi_S + \frac{q}{1-q}\pi_M \\
&= \frac{1}{1-p}\pi_S - \zeta_\pi \frac{q}{1-q}\pi_S \\
&= \left(\frac{1}{1-p} - \zeta_\pi \frac{q}{1-q}\right)\pi_S,
\end{aligned}$$

where we have assumed that the economy is back at steady state at time T so that $\pi_t = 0$ for $t \geq T$. It is then straightforward to see that expected cumulative inflation is exactly zero if ζ_π is set up so that the coefficient in front of π_S is zero. When that is the case, then it follows that the price level returns to its initial, steady state level after a shock to the efficient rate of interest since we have

$$\mathbb{E}_t \left[p_t - p_{t-1} + p_{t+1} - p_t + \cdots + p_{t+T} - p_{t+T-1}\right] = \mathbb{E}_t p_{t+T} - p_{t-1} = 0$$

For this reason, we will refer to the value of ζ_π that makes the expected cumulative sum equal to

[13]See Svensson (1996) and the references therein for an early contribution.

[14]See Vestin (2006), Giannoni (2014) and Eggertsson & Woodford (2003) for a model with an occasionally binding ZLB.

zero as ζ_π^{PLT}. The latter is given by the following expression:

$$\zeta_\pi^{PLT} = \frac{1}{q}\frac{1-q}{1-p}.$$

With this result in mind, the fact that the price level eventually stabilizes is just one property of a PLT-type rule. Another important feature relates to *how fast* is the price level expected to return to its initial level. Based on previous discussions, this expected speed will be governed by parameter q. Still assuming a constant natural rate of interest and using the expression for ζ_π^{PLT}, we can write the expression for equilibrium output gap y_S as follows:

$$y_S = -\frac{1+\beta(1-p-q)}{(1-\beta p)(1-p) - p\kappa + (1-q)\Big[1+\kappa+\beta\,(1-p-q)\Big]}\log(\beta), \tag{8}$$

so that the policy stance only enters through parameter q. Importantly, this is an equilibrium if and only if the denominator is strictly negative. We can then use this expression to see how stabilization through Price Level Targeting depends on the actual adjustment speed of the price level. The lower q, the faster the price will adjust back to its starting point. From equation (8), one can observe that increasing q decreases both the numerator and the denominator. Therefore, the effect of varying q is *a priori* ambiguous on the resulting output gap in a sunspot ZLB.

Is there a threshold value of q such that the sunspot liquidity trap doesn't arise even if $p > \overline{p}$? In other words, we are looking for the value of q that guarantees an AD/Euler slope which is strictly *lower* compared to the AS/Phillips. This is detailed in the following proposition.

Proposition 2. *The short run sunspot ZLB equilibrium does not arise if*

$$(1-q)\,[\beta\kappa(1-p) + \kappa + 1 - \beta q] > p\kappa - (1-\beta p)(1-p) > 0. \tag{9}$$

Let us define $\overline{q}^Z$ as the value of q for which the left hand side of (9) equals the middle term. We have

$$\overline{q} < \overline{q}^Z,$$

so that $q < \overline{q}$ is a necessary and sufficient condition for the sunspot to be ruled out in the short and long run.

Proof. See Appendix F. □

The restriction that $p > \overline{p}$ guarantees that the term in the middle is strictly positive. Further, it is clear that the left hand side is strictly decreasing in $q \in [0, 1]$ and reaches 0 as $q \to 1$. Therefore, there exists a threshold level $\bar{q}^Z$ such that if $q < \bar{q}^Z$, then the sunspot ZLB does not arise. The key take-away from this exercise is the following: stabilizing the price level is not enough in its own right. This stabilization must not take too long, otherwise the sunspot equilibrium will remain a possibility.

3.3 Discussion and comparison with the literature

At first glance, this result appears to be in contradiction with the results reported in Armenter (2018). In this paper, the author considers a *discretionary* policymaker and shows that if such a policymaker targets the price level then there exists multiple Markov-perfect equilibria. However, a key assumption is that the policymaker acts with discretion. In the current paper, the policymaker explicitly makes announcements about future inflation and this is what allows her to ensure that there are no sunspots ZLB equilibria.

On a more technical note, both Armenter (2018) and Nakata & Schmidt (2019a) assume a two-state Markov process that has no absorbing state and features a stationary distribution. In the current paper, we have followed Eggertsson (2010) in assuming that once the economy goes back to steady state, it stays there forever. For a second Markov-perfect equilibrium to exist, Nakata & Schmidt (2019a) show that the persistence of the state with a positive natural interest rate needs to be strictly lower than some threshold. If this persistence is equal to 1 (as in our framework), then the second Markov-perfect equilibrium does not exist. While uniqueness of a Markov-Perfect equilibrium is challenging to establish with recurring Markov states, it is much easier with a transient and absorbing state as in the current framework.

In addition, the types of equilibria considered here and in Armenter (2018) are different. Indeed, Armenter (2018) does not consider sunspots and focuses on Markov-perfect equilibria. In the context of this paper, there is only one Markov-Perfect equilibrium. However, there can be another sunspot equilibrium that will feature a binding Zero Lower Bound. The main result of the current paper is that this sunspot equilibrium can be ruled out with a suitable policy stance that requires commitment from the central bank.

To sum up, the results presented in this paper are not in contradiction with Armenter (2018) or Nakata & Schmidt (2019a). The emphasis of this paper is on the presence (or not) of a *sunspot* ZLB equilibrium which is not treated in these two papers. Qualitatively however, the current paper somewhat concurs with Armenter (2018) in that a discretionary policymaker ($q = 0$) cannot ensure a unique equilibrium. This result should be viewed as complementary with Armenter (2018). The latter shows that a discretionary policy maker facing recurring cycles cannot guarantee a unique equilibrium with an occasionally binding ZLB. Our paper shows that a discretionary policymaker faced with a one time shock cannot rule out the existence of a sunspot ZLB equilibrium.

Finally, the results presented in this paper are consistent with and shed light on the results presented in Holden (2019). In this paper, the author develops tools to investigate the presence of multiple equilibria in monetary models that feature an occasionally binding Zero Lower Bound constraint. The main result in this paper is that multiple equilibria are pervasive in this kind of models. On the flip side, the author shows that a policy of price level targeting guarantees the existence of a unique equilibrium across the board. In a more recent contribution, Roulleau-Pasdeloup (2020) studies a new Keynesian model with a central bank with loose commitment and monetary policy that switches between active and passive stances. The main result is that there exists a sufficient degree of commitment that rules out the existence of sunspots. This is because the central bank can commit for a longer time period after the passive spell.Therefore, this result is qualitatively similar with Proposition 1, which shows that a central bank that is accomodative enough rules out sunspot ZLB episodes.

3.4 Welfare analysis

We now want to know whether engaging in a pro-active monetary policy stance is warranted from a welfare point of view. This turns out to be rather straightforward. Indeed, we know from Proposition 1 that there exists values for q such that the sunspot equilibrium ceases to exists under PLT. When that happens and in the absence of fundamental shocks to the natural rate of interest, the unique equilibrium is the intended steady state where $\pi_S = y_S = 0$. Therefore, welfare is maximized for this policy. As a result, it is always welfare-maximizing to adopt a suitably pro-active monetary policy stance in this context. In addition, even though the policymaker promises *ex-ante* to deviate from the zero inflation target in the medium run, the actual deviation turns out to be nil in the medium run. If well executed then, such a policy becomes an *off-equilibrium* threat. These features lend further credence to the policy change recently announced in Powell (2020), which moves the U.S monetary policy framework closer to one of price level targeting.

4 Does the path of interest rates matter?

The reader will have noticed that we have framed all of the discussion in terms of future expected inflation rather than future interest rates as is common in the literature. Indeed, the early contribution of Eggertsson & Woodford (2003) and the literature that has followed has mostly framed forward guidance at the ZLB in terms of the central bank engaging in a lower for longer policy: commit to keep interest rates at zero even after the decrease in the efficient rate of interest has subsided. The question that we want to ask here is whether the actual path of the central bank policy rate matters for the results presented in Section 3. We will see that the answer is actually negative: the same results can be obtained if the central bank either commits to increasing or decreasing the policy rate in the medium run.[15]

To illustrate this, we will study two variants of the model that can deliver dynamics that are essentially isomorphic to the setup described in Section 3. The first one will use variations in the medium run inflation target to generate catch-up inflation/deflation along the lines of Garín et al. (2018). The second one will use a zero interest rate in the medium run along the lines of Bilbiie (2019b), but with a duration that endogenously depends on short run inflation.

4.1 A model with endogenous medium run inflation target

In this subsection, we study an extension of the baseline model where the instrument through which the central bank engineers inflation is made explicit. To do so, we keep the same formulation for the AS/AD curves, but we now assume that monetary policy is such that *(i)* the central bank targets zero inflation in the short run and *(ii)* it follows

$$R_M = \begin{cases} \max\left[0; -\log(\beta) + \phi_\pi(\pi_M - \pi^*)\right] & \text{with probability } q \\ \max\left[0; -\log(\beta) + \phi_\pi \pi_M\right] & \text{with probability } 1-q \end{cases}$$

in the medium run.[16] We can then solve for inflation in the medium run. This results in the following expression

$$\pi_M = \frac{\kappa\phi_\pi}{(1-q)(1-\beta q) + \kappa(\phi_\pi - q)}\pi^*$$

[15] In a recent contribution, Eggertsson et al. (2020) use the framework introduced in Cochrane (2017) and show that in their simulations the unifying feature that rids the New Keynesian model of its "paradoxes" is expected inflation at the end of the ZLB spell and not the path of interest rates.

[16] In the absence of natural rate shocks and with $q < \overline{q}$, the ZLB will not be binding in the medium run.

Therefore, the central bank can announce a raise in the inflation target π^* to generate higher inflation in the medium run. In addition, we assume that the central bank ties this announcement to short run inflation π_S as in Section 3. Formally, we assume that

$$\pi^* \equiv -\vartheta \pi_S.$$

Therefore, if we define

$$\zeta_\pi = \vartheta \frac{\kappa \phi_\pi}{(1-q)(1-\beta q) + \kappa(\phi_\pi - q)},$$

then we can write expected inflation conditional on being in the short run regime as follows

$$\mathbb{E}_S \pi_{t+1} = p\pi_S - (1-p)q\zeta_\pi \pi_S$$

just as before, and likewise for the output gap. As a consequence, all the results involving the existence of a monetary policy stance such that sunspots do or do not apply carry through here. What is left to answer is the behavior of the interest rate that implements this allocation. Using the Taylor rule, it is straightforward to show using the solution for inflation that the interest rate in the contingency where the central bank deviates from zero inflation targeting in the medium run is given by

$$\max \left[0; -\log(\beta) + \phi_\pi \frac{q - (1-q)(1-\beta q)}{(1-q)(1-\beta q) + \kappa(\phi_\pi - q)} \pi^* \right].$$

Therefore, raising inflation in the medium run requires an *increase* in the policy rate in the medium run if the numerator of the coefficient in front of target inflation is positive. This coefficient is an increasing function of q that goes from -1 to $+1$. In fact, given that $\beta < 1$ it is straightforward to observe that for $q = 0.5 = 1 - q$, the numerator is equal to $\beta q(1-q) = \beta/4$. As a result, if the central bank commits to setting the interest rate higher for an expected duration that is greater or equal to just *two* periods, then raising actual inflation entails an increase in the policy rate in the medium run.[17]

This is consistent with the intuition developed in Garín et al. (2018) whereby a higher persistence of their target inflation process makes it more likely for the model to display "Neo-Fisherian" properties.[18] This proves that the results derived in Section 3 can be implemented by a central bank announcing an *increase* in interest rates in the medium run. We now move on to describe a second extension of the model where the central bank implements the inflation target

[17]For most calibrations, $\kappa < 1$ so that the value of q that makes the numerator positive is strictly lower than the value of q that opens the door to sunspots in the medium run.

[18]See also Cochrane (2016).

by setting the interest rate to zero in the medium run.

4.2 A model with an endogenous duration of ZLB in the medium run

Assume now that the central bank commits to keep the interest rate at zero in the medium run in the event that the economy is in a sunspot liquidity trap in the short run. Importantly, the economy can be in a liquidity trap because of a sunspot or a fundamental shock as in Bilbiie (2019b). Even more important is the fact that we allow the duration of the medium run ZLB episode to depend on short run inflation. Formally, the probability that the medium run interest rate goes back to its steady state value is now given by $1 - q(\pi_S)$. Conditional on the economy being in the medium run regime then, expected medium run output gap/inflation under forward guidance will be:

$$
\begin{aligned}
q(\pi_s))y_M &= -q(\pi_s)\frac{1-\beta q(\pi_s)}{(1-q(\pi_s))(1-\beta q(\pi_s)) - \kappa q(\pi_s)}\log(\beta) \\
q(\pi_s))\pi_M &= -q(\pi_s)\frac{\kappa}{(1-q(\pi_s))(1-\beta q(\pi_s)) - \kappa q(\pi_s)}\log(\beta),
\end{aligned}
$$

both of which are positive if $q(\pi_S) < \bar{q}$ which is the maintained assumption throughout this paper. From these two expressions, it is clear that if we use these to solve for the short run equilibrium we will end up with AS/AD curves that will *not* be piece-wise linear as before. To remedy this, we will use one simplification to show that this model can be considered isomorphic (up to a constant) to the one in Section 3. The simplification relies on a first order Taylor approximation. More specifically, let us define $q(\pi_S) \cdot \pi_M(\pi_S) \equiv \mathcal{P}(\pi_S)$. Then, we will use the fact that

$$
q(\pi_S) \cdot \pi_M(\pi_S) \sim \mathcal{P}(0) + \mathcal{P}'(0) \cdot \pi_S,
$$

around zero steady state inflation and where the second coefficient is given by

$$
\mathcal{P}'(\pi_S) = -\frac{q'(\pi_S)\mathcal{D}(q(\pi_S)) + \left[2\beta q'(\pi_S)q(\pi_S) - (1+\beta+\kappa)q'(\pi_S)\right] q(\pi_S)}{\mathcal{D}(q(\pi_S))^2}\kappa \log(\beta)
$$

evaluated at $\pi_S = 0$ and where $\mathcal{D}(q(\pi_S))$ denotes the denominator in the expression for expected output gap/inflation in the medium run. Importantly, notice that $\mathcal{P}(0) = 0$ if $q = 0$ but $\mathcal{P}'(0) \neq 0$ in this case. For simplicity, we then assume that $q = 0$ so that the probability of staying at the ZLB in the medium run only depends on π_S. Let us assume further that the elasticity of the probability q with respect to π_S is constant and given by $-\iota$. Setting ι such that $\mathcal{P}'(0) = -\zeta_\pi$, we can now write expected inflation conditional on being in the short run regime as

$$\mathbb{E}_S \pi_{t+1} \sim p\pi_S - (1-p)\zeta_\pi \pi_S,$$

which is the same as before, save for the fact that $q\zeta_\pi$ is replaced by only ζ_π here. This is largely inconsequential as we can redefine $\zeta_\pi = \tilde{q}\tilde{\zeta}_\pi$ to have a model that is more in line with the one from Section 3. We can proceed in the same manner for output which yields a similar expression. Therefore, this example shows that the results that we have derived before can be obtained by setting the nominal interest rate to zero in the medium run.

4.3 Taking stock

What is the main characteristic of an effective forward guidance Policy? By effective, we mean that such a policy effectively shields the economy from self-fulfilling prophecies that yield a binding ZLB. The main message of this Section is that what actually matters for forward guidance to be an effective policy is expected inflation in the medium run. We have shown that the expected path of interest rates is largely irrelevant as a given level of medium run inflation can be achieved either by increasing, decreasing, or even keeping interest rates at zero in the medium run.

One of the important features of our control-contingent policy stance is that it is straightforward to communicate: the policy maker just has to announce (i) how much of short run deflation will be made up through higher medium run inflation and (ii) for how long. This does not require the central banker to know the extent of the fall in the efficient interest rate, which is itself unobserved. All that is needed is to observe the short run rate of net price inflation, which is readily available, save for a short lag.

In addition, for the special case of Price Level Targeting we have found that the required *expected* duration of catch-up inflation to get rid of the sunspot ZLB equilibrium is pretty short. Given that most central bankers have a fixed tenure, it might be hard for an incumbent central banker to constrain the actions of his/her successor. If the duration of catch-up inflation is short then, it is more probable that this will fall into the term of the central banker who initially made the promise/commitment.

5 Conclusion

In this paper, we have proposed a new framework that allows one to think about forward guidance in closed form. The framework yields new insights about the relationship between sunspot driven liquidity traps and forward guidance. We have shown that, by modifying the slopes of both AS/Phillips and AD/Euler curves, a control-contingent and sufficiently pro-active monetary policy can rid the New Keynesian model of sunspot liquidity traps in the short and medium run.

Finally, the analysis in this paper has been carried out in the simple New Keynesian model for transparency. However, the framework can be easily modified to include Heteorogeneity along the lines of Werning (2015), Acharya & Dogra (2018), Bilbiie (2019a, 2020), Broer et al. (2020), Holm (2020), Ravn & Sterk (2020) and Acharya et al. (2020); all while retaining analytical tractability. In addition, one can introduce bounded rationality along the lines of Gabaix (2020), again while retaining analytical tractability. These issues are interesting avenues for future research.

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Online Appendix

A Details for the linear model of Section 2

As in a standard New Keynesian Model, the household has the following utility function:

$$\mathcal{U}(C_t, N_t) = \log(C_t) - \chi \frac{N_t^{1+\eta}}{1+\eta}, \quad \chi, \eta > 0$$

where households work N_t hours, consume amount C_t, trade government bonds B_t, pay lump-sum tax $\mathcal{T}_t$ and wage income tax τ_t^n so that their budget constraint reads

$$C_t + \frac{B_t}{P_t} = W_t N_t (1 - \tau_t^n) + \mathcal{D}_t - \mathcal{T}_t + \exp(\varsigma_{t-1}) \frac{1 + R_{t-1}}{P_t} B_{t-1}.$$

where wage income is subject to a tax (we assume $\tau^n = 0$ at steady state) and ς_t is a "risk premium" shock. The maximization program can now be set as a Lagrangian as follows:

$$\begin{aligned}\mathcal{L}_0^H \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \left[\log(C_t) - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] \\ & - \lambda_t \left[C_t + \frac{B_t}{P_t} + \mathcal{T}_t - W_t N_t (1 - \tau_t^n) - \mathcal{D}_t - \exp(\varsigma_{t-1}) \frac{1 + R_{t-1}}{P_t} B_{t-1} \right] \Bigg\}.\end{aligned}$$

Taking the first order condition (FOC) of $\mathcal{L}_0^H$ with respect to N_t we get:

$$\begin{aligned}\frac{\partial \mathcal{L}_0^H}{\partial N_t} &= 0 \\ \Leftrightarrow \beta^t \left\{ -\chi N_t^{\eta} + \lambda_t W_t (1 - \tau_t^n) \right\} &= 0 \\ \Leftrightarrow N_t^{\eta} &= \frac{\lambda_t W_t (1 - \tau_t^n)}{\chi} \\ \Leftrightarrow W_t &= \chi \frac{C_t N_t^{\eta}}{1 - \tau_t^n}.\end{aligned}$$

FOC of $\mathcal{L}_0^H$ with respect to C_t gives:

$$\begin{aligned}\frac{\partial \mathcal{L}_0^H}{\partial C_t} &= 0 \\ \Leftrightarrow \beta^t \left\{ C_t^{-1} - \lambda_t \right\} &= 0 \\ \Leftrightarrow C_t^{-1} &= \lambda_t.\end{aligned}$$

FOC with respect to B_t gives:

$$\frac{\partial \mathcal{L}_0^H}{\partial B_t} = 0$$
$$\Leftrightarrow \frac{\lambda_t}{P_t} - \beta \exp(\varsigma_t)(1+R_t)\mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} = 0.$$

The Euler equation is obtained:

$$C_t^{-1} = \beta \exp(\varsigma_t)(1+R_t)\mathbb{E}_t \left\{ \frac{C_{t+1}^{-1}}{1+\Pi_{t+1}} \right\}.$$

After approximating the Euler equation around the steady state, we can get the linearized equilibrium Euler equation:

$$c_t = \mathbb{E}_t c_{t+1} - \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t \right],$$

where $\epsilon_t = -\varsigma_t$ is the efficient rate of interest, *i.e* the one that prevails under flexible prices with constant labor taxes.

We assume that here is a large number of firms indexed by z which produce a differentiated intermediate good sold to a final good producer. Each firm z has the following production function

$$Y_t(z) = N_t(z).$$

The demand coming from the final good producer for each firm z is given by

$$Y_t(z) = Y_t \left(\frac{P_t(z)}{P_t} \right)^{-\theta},$$

where $\theta > 1$ is the elasticity of substitution across different goods. By minimizing the total cost we can obtain the real marginal cost equation dictating their optimal labor demand:

$$MC_t(z) = W_t = \chi \frac{C_t N_t^{\eta}}{1-\tau_t^n}.$$

We show the following intermediate good firm's profits equation:

$$\mathcal{D}_t(z) = (1+\tau) \left(\frac{P_t(z)}{P_t} \right) Y_t^D(z) - MC_t(z) \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t - \Xi(z),$$

where we adopt the Rotemberg (1982) adjustment cost $\Xi(z) \equiv \frac{\psi}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 Y_t$ and τ is a constant subsidy.

The maximization program for firms to have the expected discounted sum of real profits reads:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \frac{C_t}{C_{t+s}} \mathcal{D}_{t+s}(z).$$

We will focus on a symmetric equilibrium in this model so that we can safely ignore dependence on z and FOC with respect to $P_t(z)$ gives the New Keynesian Phillips Curve below:

$$(1-\theta)(1+\tau) + \theta MC_t - \psi d(\Pi_t) + \psi \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \frac{Y_{t+1}}{Y_t} d(\Pi_{t+1}) = 0,$$

where $d(\Pi_t) \equiv \Pi_t(1+\Pi_t)$.

After approximating the New Keynesian Phillips Curve around the steady state, we can get the linearized equilibrium condition:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - \Gamma \tau_t^n),$$

where $\kappa \equiv \theta(1+\eta)/\psi$ and $\Gamma = -1/(1+\eta)$.

In addition, by using $y_t = c_t$ the Euler equation can be approximated as

$$y_t = \mathbb{E}_t y_{t+1} - \left[R_t + \log(\beta) - \mathbb{E}_t \pi_{t+1} - \epsilon_t\right].$$

B Derivations for function $\mathcal{D}$

Let's reproduce $\mathcal{D}$ as a quadratic function of p here,

$$\begin{aligned}\mathcal{D}(p) &= (1-p)(1-\beta p) - p\kappa \\ &= \beta p^2 - p(1+\beta+\kappa) + 1.\end{aligned}$$

We can show the discriminant for function $\mathcal{D}$:

$$\begin{aligned}\triangle(\mathcal{D}) &= (1+\beta+\kappa)^2 - 4\beta \\ &= (1+\beta)^2 + \kappa^2 + 2\kappa(1+\beta) - 4\beta \\ &= (1-\beta)^2 + \kappa^2 + 2\kappa(1+\beta) \\ &> (1-\beta)^2 + \kappa^2 + 2\kappa(1-\beta) \\ &= (1-\beta+\kappa)^2 > 0.\end{aligned}$$

In this case, the discriminant for function $\mathcal{D}$ is strictly positive. Hence this function has two zeros in $\mathbb{R}$ and further the solutions are

$$p = \frac{(1+\beta+\kappa) \pm \sqrt{(1+\beta+\kappa)^2 - 4\beta}}{2\beta}.$$

Since $(1+\beta+\kappa)^2 - 4\beta > (1-\beta+\kappa)^2$, we have one solution which is strictly higher than 1:

$$\begin{aligned}p_1 &= \frac{(1+\beta+\kappa) + \sqrt{(1+\beta+\kappa)^2 - 4\beta}}{2\beta} \\ &> \frac{(1+\beta+\kappa) + (1-\beta+\kappa)}{2\beta} \\ &= \frac{1+\kappa}{\beta} > 1.\end{aligned}$$

For another solution, we can show that,

$$\begin{aligned}p_2 &= \frac{(1+\beta+\kappa) - \sqrt{(1+\beta+\kappa)^2 - 4\beta}}{2\beta} \\ &< \frac{(1+\beta+\kappa) - (1-\beta+\kappa)}{2\beta} \\ &= 1.\end{aligned}$$

According to Vieta's Theorem, we see $p_1 \times p_2 = \frac{1}{\beta}$ and $p_1 > 1$ thus p_2 is a positive, furthermore p_2 is in $(0, 1)$.

C Derivation for equation (7)

Using the fact that the economy is at the ZLB so that the interest rate is zero, we can write

$$y_S = \mathbb{E}_S y_{t+1} + \mathbb{E}_S \pi_{t+1} - \log(\beta).$$

Notice that we can rewrite $\mathbb{E}_S \pi_{t+1}$ as

$$\begin{aligned}\mathbb{E}_S \pi_{t+1} &= [p - (1-p)q\zeta_\pi]\,\pi_S \\ &= \kappa \frac{p-(1-p)q\zeta_\pi}{1-\beta p + \beta(1-p)q\zeta_\pi} y_S.\end{aligned}$$

Further we can write

$$\begin{aligned}[p-(1-p)q\zeta_\pi]\,y_S &= p y_S - (1-p)q\zeta_\pi y_S + (1-p)q\zeta_\pi \frac{1-\beta q}{\kappa} y_S - (1-p)q\zeta_\pi \frac{1-\beta q}{\kappa} y_S \\ &= \left[p - p(1-p)q\zeta_\pi \frac{1-\beta q}{\kappa}\right] y_S + (1-p)q\zeta_\pi \left[\frac{1-\beta q}{\kappa} - 1\right] y_S \\ &= \mathbb{E}_S y_{t+1} + (1-p)q\zeta_\pi \left[\frac{1-\beta q - \kappa}{\kappa}\right] y_S.\end{aligned}$$

Using the last two relations, we can rewrite the AD/Euler equation as

$$y_S = \mathbb{E}_S y_{t+1} + \frac{\kappa}{1-\beta p + \beta(1-p)q\zeta_\pi}\left[\mathbb{E}_S y_{t+1} + (1-p)q\zeta_\pi \left[\frac{1-\beta q-\kappa}{\kappa}\right] y_S\right] - \log(\beta).$$

Collecting terms and re-arranging, we obtain

$$\begin{aligned}y_S &= \frac{1-\beta p + \kappa + \beta(1-p)q\zeta_\pi}{1-\beta p + (1-p)q\zeta_\pi\,[\beta(1+q)+\kappa-1]}\mathbb{E}_S y_{t+1} - \log(\beta) \\ &\equiv \mathcal{E}(q, \zeta_\pi, \dots)\mathbb{E}_S y_{t+1} - \log(\beta),\end{aligned}$$

where

$$\mathbb{E}_S y_{t+1} = \left[p - (1-p)q\zeta_\pi \frac{1-\beta q}{\kappa}\right] y_S.$$

D Proofs for Proposition 1

D.1 Part 1

Absent any risk premium shock and with forward looking monetary policy it is straightforward to combine the Euler equation and Phillips Curve to obtain

$$y_S = -\frac{1-\beta p+\beta(1-p)q\zeta_\pi}{(1-\beta p)(1-p)-p\kappa+(1-p)q\zeta_\pi\Big[1+\kappa+\beta(1-p-q)\Big]}\log(\beta)$$
$$\pi_S = -\frac{\kappa}{(1-\beta p)(1-p)-p\kappa+(1-p)q\zeta_\pi\Big[1+\kappa+\beta(1-p-q)\Big]}\log(\beta).$$

The sunspot equilibrium does arise when there is a fall in GDP/inflation. Since $-\log(\beta) > 0$, $\kappa > 0$ and $1-\beta p+\beta(1-p)q\zeta_\pi$ is positive, the following condition needs to be met:

$$(1-\beta p)(1-p)-p\kappa+(1-p)q\zeta_\pi\Big[1+\kappa+\beta(1-p-q)\Big] < 0$$
$$\Rightarrow\ \zeta_\pi < \frac{p\kappa-(1-\beta p)(1-p)}{(1-p)q\Big[1+\kappa+\beta(1-p-q)\Big]} = \underline{\zeta_\pi}.$$

For $\zeta_\pi < \underline{\zeta_\pi}$, we can show the sign of derivative of y_S with respect to ζ_π:

$$\text{sign}\left(\frac{\partial y_S}{\partial \zeta_\pi}\right) = \text{sign}\Big\{\beta(1-p)q\mathcal{D}^M(p)-(1-p)q\Big[1+\kappa+\beta(1-p-q)\Big]\mathcal{N}^M(p)\Big\} < 0,$$

where $\mathcal{D}^M(p) = (1-\beta p)(1-p)-p\kappa+(1-p)q\zeta_\pi\left[1+\kappa+\beta(1-p-q)\right] < 0$ and $\mathcal{N}^M(p) = 1-\beta p+\beta(1-p)q\zeta_\pi > 0$.

It follows that one can obtain the sign of derivative of π_S with regard to ζ_π:

$$\text{sign}\left(\frac{\partial \pi_S}{\partial \zeta_\pi}\right) = \text{sign}\Big\{-(1-p)q\zeta_\pi\Big[1+\kappa+\beta(1-p-q)\Big]\Big\} < 0.$$

Therefore output gap/inflation in a sunspot equilibrium is strictly decreasing for $\zeta_\pi < \underline{\zeta_\pi}$.

D.2 Part 2

This part can be proved directly by the result of part 1, we show that there is **no** fall in GDP/inflation if

$$
\begin{aligned}
&(1-\beta p)(1-p) - p\kappa + (1-p)q\zeta_\pi\Big[1+\kappa+\beta(1-p-q)\Big] > 0 \\
&\Rightarrow\ \zeta_\pi > \frac{p\kappa - (1-\beta p)(1-p)}{(1-p)q\Big[1+\kappa+\beta(1-p-q)\Big]} = \underline{\zeta_\pi}.
\end{aligned}
$$

Thus, the sunspot equilibrium does not arise for $\zeta_\pi > \underline{\zeta_\pi}$.

E Derivations for the expected cumulative inflation

Using the simple Markov structure, we can assume that the probability for the inflation to stay in the regime S at time $t+s$ ($s \geq 0$) is p^s, while if the state changes to the medium run inflation, there is one nature shock subsidy with probability $1-p$, and then it follows by the contingency that the the regime M binds with probability q. Therefore with regard to the rest $s-1$ periods, the n different histories of inflation regime with the probabilities $p^n q^{s-1-n}$ ($0 \leq n \leq s-1$). We show one simple example, if the shock ends at the period $t+2$, the only possibility of the history with the medium run inflation (regime M) in period 3 is $p \times (1-p) \times q \times q$.
The expression for the expected cumulative inflation is

$$
\begin{aligned}
\mathbb{E}_t \pi_{t+s} &= p^s \pi_S + (1-p) q \sum_{n=0}^{s-1} p^n q^{s-1-n} \pi_M \\
&= p^s \pi_S + (1-p) q^s \frac{1-(p/q)^s}{1-(p/q)} \pi_M \\
&= p^s \pi_S + (1-p) \frac{q^s - p^s}{1-(p/q)} \pi_M \\
&= p^s \pi_S + (1-p) q \frac{q^s - p^s}{q-p} \pi_M.
\end{aligned}
$$

Using this derivation, one can compute the expected cumulative sum of inflation:

$$
\begin{aligned}
\mathbb{E}_t \sum_{s=0}^{T} \pi_{t+s} &= \mathbb{E}_t \sum_{s=0}^{T} \left[p^s \pi_S + (1-p) q \frac{q^s - p^s}{q-p} \pi_M \right] \\
&= \frac{1}{1-p} \pi_S + \frac{(1-p)q}{q-p} \sum_{s=0}^{T} \left(q^s - p^s \right) \pi_M \\
&= \frac{1}{1-p} \pi_S + \frac{(1-p)q}{q-p} \left(\frac{1}{1-q} - \frac{1}{1-p} \right) \pi_M \\
&= \frac{1}{1-p} \pi_S + \frac{q}{1-q} \pi_M.
\end{aligned}
$$

We can employ the the expected cumulative sum of inflation to obtain the result in the main text.

F Derivation of PLT threshold

For the sunspot equilibrium to be ruled out, the slope of the AS/Phillips curve has to be strictly higher than the one for the AD/Euler curve. This requires that the monetary policy stance (q, ζ_π) satisfies

$$\frac{1-\beta p+\beta(1-p)q\zeta_\pi}{\kappa} > \frac{p-(1-p)q\zeta_\pi\left[1+\frac{1-\beta q}{\kappa}\right]}{1-p}.$$

Multiplying both sides by $\kappa(1-p)$ and re-arranging we obtain

$$(1-p)(1-\beta p)+\beta\kappa(1-p)^2 q\zeta_\pi > p\kappa-(1-p)q\zeta_\pi\left[1+\frac{1-\beta q}{\kappa}\right]$$
$$\Leftrightarrow\ q\zeta_\pi(1-p)[\beta\kappa(1-p)+\kappa+1-\beta q] > p\kappa-(1-p)(1-\beta p).$$

Using the definition of $\zeta_\pi^{PLT} \equiv (1-q)/(q(1-p))$ to substitute for ζ_π, we obtain the following condition for the absence of a sunspot liquidity trap:

$$(1-q)[\beta\kappa(1-p)+\kappa+1-\beta q] > p\kappa-(1-p)(1-\beta p).$$

Let us define the difference between the left and right hand sides of this equation as $\mathcal{L}^{PLT}(q)$:

$$\mathcal{L}^{PLT}(q)=(1-q)[\beta\kappa(1-p)+\kappa+1-\beta q]-p\kappa+(1-p)(1-\beta p).$$

We can obtain the derivative of $\mathcal{L}^{PLT}(q)$ with regard to q:

$$\frac{\partial\mathcal{L}^{PLT}(q)}{\partial q}=-[\beta\kappa(1-p)+\kappa+1-\beta]+2\beta q<0.$$

It is then clear that

1. $\mathcal{L}^{PLT}(q)$ is strictly decreasing in q.
2. $\mathcal{L}^{PLT}(0)>0$.
3. $\mathcal{L}^{PLT}(1)<0$.

Part 2 follows from the fact that

$$\mathcal{L}^{PLT}(0)=\beta\kappa(1-p)+(1-p)(1-\beta p)+1+\kappa(1-p)>0.$$

Part 3 follows from the fact that

$$\mathcal{L}^{PLT}(1) = (1-p)(1-\beta p) - p\kappa,$$

which is negative since we maintain the assumption that $p > \overline{p}$. This proves the result in the main text. On the other hand, to show $\overline{q}^Z > \overline{q}$, first we have $\overline{q}$ in the medium run such that

$$(1-q)(1-\beta q) - q\kappa = 0.$$

The only feasible root which is less than 1 is

$$\overline{q} = \frac{(1+\beta+\kappa) - \sqrt{(1+\beta+\kappa)^2 - 4\beta}}{2\beta}.$$

Second we have $\overline{q}^Z$ such that

$$(1-q)[\beta\kappa(1-p) + \kappa + 1 - \beta q] - p\kappa + (1-p)(1-\beta p) = 0.$$

The only feasible root which is less than 1 is

$$\begin{aligned}\overline{q}^Z =& \frac{(1+\beta+\kappa+\beta\kappa(1-p))}{2\beta} \\ &- \frac{\sqrt{(1+\beta+\kappa+\beta\kappa(1-p))^2 - 4\beta(1+\beta\kappa(1-p)+\kappa-p\kappa+(1-p)(1-\beta p))}}{2\beta}\end{aligned}$$

Since we assume that $(1-p)(1-\beta p) < p\kappa$, one can prove

$$\begin{aligned}\mathcal{N} =& 2(1+\beta+\kappa)(\beta\kappa(1-p)) - 4\beta(\beta\kappa(1-p)+\kappa-p\kappa+(1-p)(1-\beta p)) \\ &< 2(1+\beta+\kappa)(\beta\kappa(1-p)) - 4\beta(\beta\kappa(1-p)+\kappa) \\ &= 2\beta\kappa(1-p)(1-\beta-\frac{2}{1-p}+\kappa)\end{aligned}$$

If $\overline{q}^Z > \overline{q}$, we need to show

$$\begin{aligned}&\sqrt{(\beta\kappa(1-p))^2} - \sqrt{(1+\beta+\kappa)^2 + (\beta\kappa(1-p))^2 - 4\beta + \mathcal{N}} > -\sqrt{(1+\beta+\kappa)^2 - 4\beta} \\ &\Rightarrow 2\sqrt{(\beta\kappa(1-p))^2}\sqrt{(1+\beta+\kappa)^2 - 4\beta} > \mathcal{N}\end{aligned}$$

One can trivially show this holds.