Education, Wage Dynamics and Wealth Inequality

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Abstract

To what extent does heterogeneity in education contribute to wealth inequality and life-cycle savings, and through which pathways? Using the PSID data, I estimate skill-specific wage processes, allowing for both *deterministic* between-group wage dispersion and *stochastic* within-group wage dispersion. I evaluate the quantitative implications of these wage processes using an incomplete-markets overlapping-generations general equilibrium model in which households choose their education and labor supply. I find that allowing wage processes to vary by skills is crucial for understanding the wealth inequality and life-cycle savings of skilled and unskilled households. Importantly, the *deterministic* between-group wage difference is vital for college attainment choice, while a relatively more volatile persistent component of wage shocks for the skilled plays a key role in explaining the top percentile distribution of wealth and the large difference in the life-cycle savings between the two skill groups.

Keywords: Education, wage differentials, wealth inequality, life-cycle savings

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1 Introduction

Education is an important determinant of earnings capacity and of the earnings risk that households face. Then, to what extent do differences in education contribute to wealth inequality and life-cycle savings, and through which pathways? That is, are highly educated workers different because, on average, they make more (the deterministic component of wages) or because they face different wage risk (the stochastic component of wages)? This is an important question as, in the incomplete-market models that we use, the nature of wage differences across households largely determines households' savings behavior and, thus, the distribution of wealth.

To answer this question, I first estimate skill-specific wage processes for college- and non-college educated workers using the PSID data. Here, I allow both *deterministic* between-group and skill-varying *stochastic* within-group wage dispersion. Between-group wage dispersion includes college wage premia and skill-specific labor market experience premia. Within-group wage dispersion consists of both persistent and transitory wage shocks. The estimated results show that, in the benchmark year 2004, skilled households received a college wage premium and faced steeper age-wage profiles than unskilled households. Moreover, skilled workers faced more volatile persistent wage shocks, compared to unskilled workers, while the persistence of shocks and the variance of transitory shocks are similar across education groups.

I study the implications of these estimated wage processes in an incomplete-markets overlapping-generations general equilibrium model with college education choices and elastic labor supply. Households make their college education decisions before the start of their working life. Education is costly and households have access to college loans. After their college choice, they face wage processes, over their working lives, that are specific to their education levels. Specifically, skilled workers benefit from a permanently higher hourly wage – the college wage premium and labor market experience premium – and face more volatile wage risk compared to unskilled households. After retirement, households receive social security benefits proportional to their earnings.

The calibrated economy successfully reproduces the joint distribution of wealth and college attainment seen in the 2004 SCF. In particular, while untargeted, the benchmark economy explains the relatively high fraction of skilled households in the top distribution of wealth and the high probability of being wealthy for college graduates. Moreover, a borrowing limit and the resource costs of college give rise to total annual education cost around $7,380, and 79 percent of college graduates holding student loan debt, similar to the data.[1]

I find that explicitly modeling skill-specific between- and within-group wage differentials is important for understanding the wealth inequality seen in the data. For instance, the benchmark economy with skill-specific wage processes generates a wealth Gini of 0.73 and 53 percent of total wealth held by the top 10 percent of households. By comparison, in the 2004 SCF data, the corresponding numbers are 0.77 and 63 percent, respectively. In contrast, in an alternative model with a common wage process, essentially an Aiyagari model with elastic labor supply, the wealth Gini drops to 0.64, and the share of wealth held by the wealthiest 10 percent of households falls by 9 percentage points from the benchmark economy.

The more skewed distribution of wealth in the benchmark economy is mainly driven by the savings of skilled households. Skill-specific wage processes affect the saving behavior of skilled households through two channels. First, the deterministic between-group wage difference – the higher hourly wage and steeper age-wage profile for the skilled –leads to a high level of earnings, increasing their savings. Second, a higher within-group wage dispersion provides a better opportunity for the skilled to become wealthy. This is because of the following. A more volatile persistent component of wage shock for the skilled implies a higher probability of both favorable and unfavorable wage shocks. As elastic labor supply allows skilled households to insure themselves against downside wage risk by increasing their hours worked, their more volatile wage risk leads to a higher level of earnings and thus savings, driving further inequality.

Allowing wage risk to differ by skills is also crucial to understand the large difference in

[1] By comparison, in data, the average annual college education cost for the full-time undergraduate students is around $13,993 and 67 percent of seniors graduate from four-year institutions with student loan debt.

the life-cycle wealth of skilled and unskilled households seen in the data. When, instead, a common wage risk is assumed, unskilled households have counterfactually high volatility in the persistent component of their wage shocks. This allows them to realize a significant amount of earnings without a college education, when they are lucky. Thus, common wage risk for skilled and unskilled households places too much emphasis on luck in determining individual wealth. This narrows the gap in realized earnings and wealth across education groups. In contrast, by deviating from this, the benchmark economy partly endogenizes uninsurable wage risk through the discrete choice of skill and better explains the difference in the life-cycle savings of skilled and unskilled households.

To explore the relative importance of between and within-group wage differentials for the distribution of wealth and life-cycle savings, I also conduct two main experiments. In the first, I abstract from between-group wage dispersion, but households still face skill-specific wage shock processes. In the second experiment, I assume that all households face the same wage risk but allow for college wage premia and different age-wage profiles. Using the first experiment, I show that between-group wage dispersion is critical for the college education decision. Without the observed benefits of college, households do not pursue a college degree, and wealth inequality sharply falls. In the second experiment, I find that a more volatile post-education persistent wage shock for the skilled is an important driver for the distribution of wealth at the top as it provides a better opportunity to become wealthy for the skilled.

Given that wage inequality has been rising in the U.S., the natural question that arises is how the rising wage inequality has affected wealth inequality. To shed light on this, I also solve the model economy with the 1989 skill-specific wage processes. In 1989, the college wage premium was half of that in 2004, and the persistent component of the wage shock was much less volatile for the skilled than in 2004.[2] With these 1989 estimates, wealth inequality sharply falls. For instance, the 1989 economy only explains a wealth Gini of 0.67 compared to 0.73 for the 2004 economy. The share of wealth held by the top 10 percent of households also falls by 9 percentage points compared to the 2004 economy. I further explore the relative importance of between- and within-group wage differentials

[2]The variances of other shocks are similar between two years.

for such differences. I find that the higher college wage premium in 2004 increases the college attainment rate by 3 percentage points relative to 1989. However, the more skewed distribution of wealth in 2004 is mainly the result of a more dispersed persistent wage shock for the skilled.

This paper provides several contributions. First, I estimate skill-specific wage processes using the PSID data to show that households with different education levels face different wage processes. Second, I introduce these rich wage dynamics that vary by education in a framework whose crucial ingredients are college education choice and elastic labor supply. For example, elastic labor supply is important as it provides a channel for households to insure themselves against negative wage shocks, making wage volatility less risky for consumption. Third, this is the first paper to study the implications of skill-specific wage processes with both between- and within-group dispersion on wealth inequality and life-cycle savings. Fourth, in contrast to Huggett et al. (2011), by allowing idiosyncratic shock processes to differ by education, I find that differences in both deterministic and stochastic components of wages are important determinants for wealth inequality and life-cycle savings.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents empirical analysis, including the estimation of wage processes. Section 4 summarizes the model economy. Section 5 discusses the calibration. Section 6 presents the quantitative results. Section 7 shows the sensitivity of results to initial distribution and borrowing limits. Section 8 shows the implications of the rise in wage inequality, and Section 9 concludes.

2 Related literature

This work is related to the literature that allows rich specifications in individual or household earnings dynamics. Guvenen (2007, 2009) underlines the importance of heterogeneous income growth profiles when estimating earnings processes and their consistency with consumption dynamics. A growing body of empirical evidence also suggests nonlinearities in earnings dynamics: persistence and skewness of earnings shocks that depend on the history of shocks (Arellano et al. (2017)), negative skewness and high kurtosis in earning changes and asymmetric persistence of shocks over the distribution of earnings (Guvenen et al. (2019)), and non-normality and nonlinearity in earnings shocks over the distribution of earnings and age (De Nardi et al. (2018)).

Though important advances have been made in the literature on earnings dynamics, with the exceptions of Guvenen (2007) and De Nardi et al. (2018), not many papers quantitatively evaluate the implications of rich earnings processes on consumption and saving behavior using structural models. My work complements these two papers by studying the implications of education-varying wage dynamics on wealth inequality and life-cycle savings behavior in an incomplete-markets OLG GE model.

My work contributes to the literature of incomplete-markets macroeconomic models that study the distribution of wealth. A large literature including Aiyagari (1994), Cagetti and De Nardi (2006), Huggett (1996), Krusell and Smith (1998), Castaneda et al. (2003), Benhabib et al. (2015) and Lusardi et al. (2017) explores different margins that determine wealth inequality. Relative to this work, I study the role of education differences.

My model is closely related to Heathcote et al. (2010b), which explores the implications of rising wage inequality for the cross-sectional distribution of labor, earnings, and consumption. They build an incomplete-markets overlapping-generations small open economy with elastic labor supply and an education decision. In their economy, workers are differentiated by education levels and genders. In contrast to Heathcote et al. (2010b), in my model, workers receive not only skill-specific hourly wages, but also face skill-specific labor market experience premia and idiosyncratic shock processes.

This paper is also in line with Huggett et al. (2011), who quantify the relative significance of initial conditions established in early life and subsequent shocks over working life for lifetime inequality. Studying a risky human capital overlapping-generations model with heterogeneity in ability, initial human capital, and initial wealth, they find that differences in initial conditions are more important drivers for lifetime earnings and wealth compared to shocks over the agents' life-cycle. I also show that an initial human capital decision – college attainment – is an important determinant of households' lifetime wealth. However, it is because initial human capital decision determines the post-education wage processes households face over their working lives.

Lastly, my model economy introduces skill-specific wage processes in a heterogeneous-agent incomplete-markets model. Skill-specific earnings processes are also introduced in Fuster et al. (2008) and Angelopoulos et al. (2019). The former studies the welfare effects of tax reforms in a dynastic model while the latter explores how savings externalities increase wealth inequality.

3 Empirical analysis

3.1 Data description

PSID To estimate wage processes, I used the 1968-2011 Panel Study Income Dynamics (PSID) data. The PSID is a longitudinal survey of a sample of US individuals and families conducted annually from 1968 to 1997, and biennially since 1997. The original 1968 PSID sample combines the Survey Research Center (SRC), which is representative of the U.S. population, and the Survey of Economic Opportunities (SEO), which oversamples the poor. In this work, I use the core samples (SRC) that include 3,000 households.[3] I use the hourly wage of the male head of households who are between 25 and 59 years old. Here, the hourly wage is defined as the gross annual earnings of the male head divided by his total annual hours worked.[4] Gross annual earnings include all pre-tax income from wages, salaries, bonuses, overtime, commissions, professional practice, and the labor part of the farm and business income. For the top-coded observations of earnings, I multiply them by 1.5 following Katz et al. (1999).[5] I select male heads of households with no missing values for education and self-employment status that satisfy the following criteria: 1) their age is between 25 and 59 years old, 2) the individual does not have positive labor income with zero annual hours worked, 3) the individual works at least 260 hours per year, 4) their hourly

[3]A sample design generates 3,000 completed households from 2,930 actual interviews taken in 1968 from the SRC sample. The SEO sample consists of approximately 2,000 households.

[4]Note that, in the PSID, total annual earnings and annual hours worked variables are retrospective. Thus, those surveyed in year t refer to year t-1.

[5]There are only 97 top-coded observations of earnings before sample selection, and most of these observations happened in the years before 1983.

wage is not less than half of the minimum wage, 5) income is not from self-employment.[6] After sample selection, I have 99,037 observations, with an average of 2,677 individuals per year. All the variables are deflated and expressed in 2013 dollars using the CPI. Finally, I define the college-educated as those with a college degree or more and non-college graduates as those without a college degree.

SCF The wealth information is from the 2004 Survey of Consumer Finances (SCF). SCF is a triennial household data conducted by the Board of Governors of the Federal Reserve System in cooperation with the Statistics of Income Division of the IRS since 1983. Based on its sample design, the SCF oversamples the rich and may better capture the concentration of wealth in the top and among college graduates.[7]

I select households where the head of households' age is between 25 and 84 years and is not self-employed. All the variables are expressed in 2013 dollars, and full sample weights are used. I define the college-educated as those with a college degree or more and non-college graduates as those without a college degree. After sample selection, I have a total of 15,628 observations with five imputations.[8] Net worth is defined as total assets minus total debt. Total assets include financial assets and nonfinancial assets. Financial assets include current values and characteristics of deposits, cash accounts, securities traded on exchanges, mutual funds and hedge funds, annuities, cash value of life insurance, tax-deferred retirement accounts, and loans made to other people. Nonfinancial assets include current values of principal residences, other real estates not owned by a business, corporate and non-corporate private businesses, and vehicles. Total debt includes outstanding balances on credit cards, lines of credit and other revolving accounts, mortgages, installment loans for vehicles and education, loans against pensions and insurance policies, and money owed to a business owned at least in part by the family.

[6]These sample selection criteria are broadly consistent with those used in the previous studies. (Guvenen (2009) and Heathcote et al. (2010b))

[7]The SCF employs a dual-frame sample design. One frame is a multi-stage national area probability design, which provides information on the characteristics of the population, and the other is a list sample to provide a disproportionate representation of wealthy households.

[8]Thus, I have approximately 3,000 households interviewed.

3.2 Estimation of skill-specific wage processes

I estimate skill-specific wage processes using the PSID data between 1968 and 2011.[9] I first run an OLS regression to estimate time-varying college wage premia and skill-varying labor market experience premia over the sample period. These *between-group* wage differences capture the deterministic component of wage inequality.[10] Next, I use minimum distance estimation to find wage shock processes for each skill group, allowing for both persistent and transitory shocks. This skill-specific *within-group* wage dispersion describes the stochastic component of wage inequality.[11]

In the following, $w_{i,j,t,e}$ represents the hourly wage of an individual i with age j and education level e in year t. I run an OLS regression of log hourly wage on time dummies (D_t); an interaction term with time dummies and the college education dummy ($D_{h,t}$); an interaction term with education and labor market experience, θ; and an interaction term with education and experience-squared, θ^2. $D_{e,t}$ are the education dummies which take the value 1 if the education level is $e \in \{l, h\}$, with l representing the non-college educated and h representing the college-educated.

$$\log w_{i,j,t,e} = \sum_{t=1}^{T} \beta_{t,0} D_t + \sum_{t=1}^{T} \beta_{t,1} D_t D_{h,t} + \sum_{e=l,h} \left(\beta_{e,2} D_{e,t} \theta_{i,j,t,e} + \beta_{e,3} D_{e,t} \theta^2_{i,j,t,e} \right) + \widehat{r}_{i,j,t,e}$$

Here, labor market experience, θ, is measured as age minus years of schooling minus 6.[12] Note that $\beta_{e,2}$ and $\beta_{e,3}$ capture education-specific labor market experience premium.

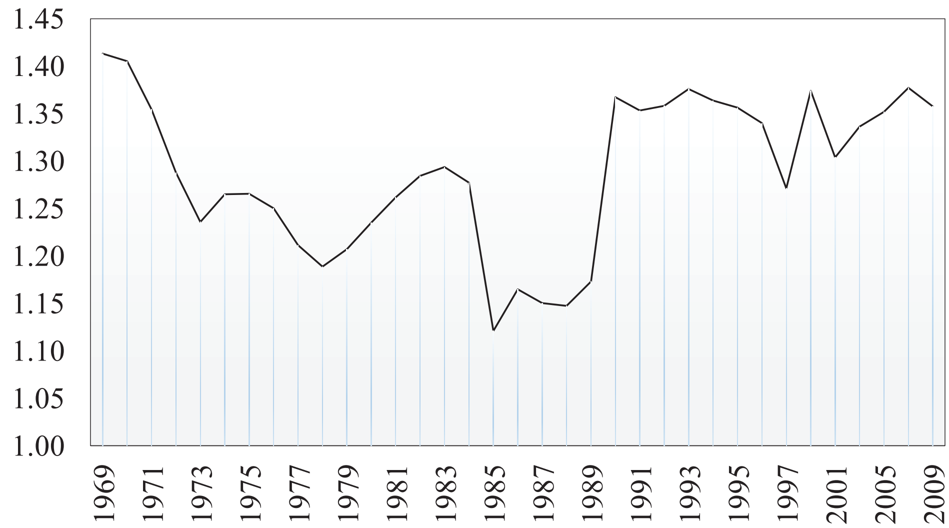
[9] Generally, I follow the estimation strategy in Heathcote et al. (2010b) but allow labor market experience premia and wage shock processes to be conditional on skills.

[10] The education-specific labor market experience premia allow each education group to have different age-wage growth profiles, partly capturing the permanent heterogeneity in income profiles in Guvenen (2009). Guvenen (2009) estimates the distribution of coefficients that characterize the labor market experience premia and allows heterogeneity in labor income growth profiles across individuals. Here, for simplicity, I only allow different deterministic age-wage profiles by education levels.

[11] Krueger and Perri (2006) documented that 36 percent of the rise in income inequality between 1980 and 2003 is driven by the change in the between-group income dispersion while 40 percent and 24 percent are driven by the rise in the variances of persistent shocks and transitory shocks, respectively. This underlines the importance of both between-group and within-group wage dispersion in the wage estimation.

[12] In years missing the variable for years of schooling, I proxy years of schooling using the median of education brackets for individuals with less than a college degree. For example, if the individual responded that they finished 6-8 grades, I approximate years of schooling for this individual as 7. For individuals with a college degree or more, I proxy their years of schooling as 16.

Figure 1: Male college wage premium



Source: PSID data (1968-2011)

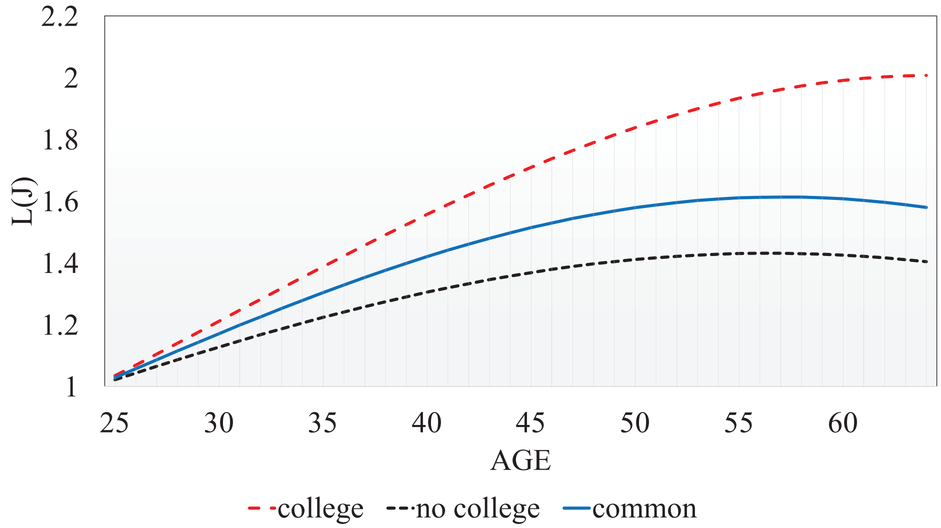
Figure 1 shows the estimated returns to college over the sample period. The college wage premium decreases until the 1980s; thereafter, it rapidly increases in 1990.[13] Figure 2 shows the estimated potential labor market experience function for each education group. This shows that wage growth is much steeper for the college-educated compared to that for non-college educated workers. For example, the labor market experience more than doubles hourly wages for college-educated workers through the first 35 years of work (red line), while it only increases those for the lower education group by 30 percent relative to their initial levels (black line). Note that efficiency units of labor depend on both labor market experience, $l^e(j) = exp(\beta_{e,3}\theta + \beta_{e,4}\theta^2)$, and idiosyncratic wage (productivity) shocks.

The regression residuals $\widehat{r}_{i,j,t,e}$ are assumed to be the sum of idiosyncratic wage shocks, $\epsilon_{i,j,t,e}$, and measurement error, $\widetilde{v}_{i,j,t,e}$.[14] Idiosyncratic shocks consist of both a persistent

[13] The estimated time-varying college wage premia are overall lower than the estimates in Heathcote et al. (2010b). This is because allowing different labor market experience premium for the skilled captures some of the college wage premia between education groups.

[14] Following Heathcote et al. (2010b), I use French (2002)'s estimate for the variance of a measurement error in log hourly wages of 0.02.

Figure 2: Labor market experience



Source: PSID data (1968-2011)

component, η, and transitory component, ϵ^v. To be specific,

$$\epsilon_{i,j,t,e} = \eta_{i,j,t,e} + \epsilon^v_{i,j,t,e}$$

$$\eta_{i,j,t,e} = \rho^e \eta_{i,j-1,t-1,e} + \epsilon^p_{i,j,t,e}$$

where $\epsilon^p_{i,j,t,e} \sim N(0, \sigma^2_{pt,e})$ and $\epsilon^v_{i,j,t,e} \sim N(0, \sigma^2_{vt,e})$.

I estimate skill-specific within-group year-varying shock variances $\{\sigma^2_{pt,e}, \sigma^2_{vt,e}\}$, the persistence of the shock $\{\rho^e\}$, and the variance of the initial value for the persistent shock $\sigma^2_{\pi_e}$, using minimum distance methods. These estimates are used as distinct idiosyncratic productivity shock processes, across skill groups, in the structural model. I use survey data from 1968 to 2011, but only estimate the variances through 2008 because of the finite sample bias at the end of the sample period.[15] I separate samples by education groups,

[15]Given that the PSID has conducted a biennial survey starting from 1997, the estimation of annual shock processes there must confront the problem of observations missing for every other year. As Heathcote et al. (2010b) point out, although the variance for the persistent shock for the missing years can be theoretically found using the available information from adjacent years, the resulting estimates are downward-biased because of insufficient information. Therefore, I follow their approach and estimate variances for missing years by taking the weighted average of the two closest surrounding years.

college graduates and non-college graduates, to estimate two separate productivity shock processes. I assume that wage shocks between these two groups are orthogonal to each other. I estimate $L = 86$ parameters for each education group. The parameter vector is denoted by $\mathcal{P}_{L\times 1}$. From now on, I abstract from education variables for ease of notation.

The theoretical moment is defined as

$$m^j_{t,t+n}(\mathcal{P}) = E(r_{i,j,t}r_{i,j+n,t+n})$$

which is the covariance between the wages of individuals at age j in year t and $t+n$. To calculate empirical moments, I group individuals into 44 years and 26 overlapping age groups. For example, the first age group contains all observations between 25 and 34 years old, and the second group contains those between 26 and 35 years old. The empirical moment conditions are

$$\hat{m}^j_{t,t+n} - m^j_{t,t+n}(\mathcal{P}) = 0$$

where $\hat{m}^j_{t,t+n} = \frac{1}{I_{j,t,n}}\sum_{i=1}^{I_{j,t,n}} \widehat{r}_{i,j,t}\widehat{r}_{i,j+n,t+n}$ and $I_{j,t,n}$ is the number of observations of age j at year t existing n periods later.

The minimum distance estimator solves

$$\min_{\mathcal{P}}[\hat{\mathbf{m}} - \mathbf{m}(\mathcal{P})]'[\hat{\mathbf{m}} - \mathbf{m}(\mathcal{P})]$$

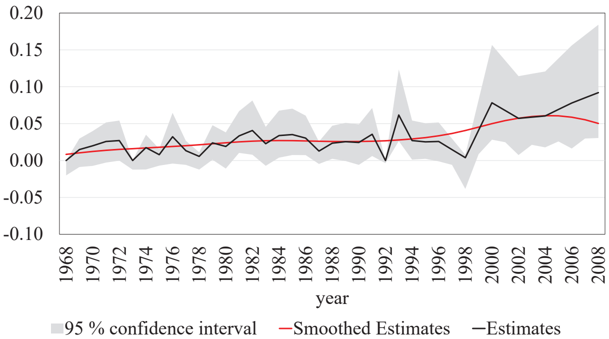
where the vectors $\hat{\mathbf{m}}$ and $\mathbf{m}$ represent empirical and theoretical moments of dimension $9,474 \times 1$. The identity matrix is used as the weighting matrix.

Figure 3 displays the estimates of the variances of persistent and transitory wage shocks for college and non-college graduates between 1968 and 2008..[16] The overall wage residual becomes more dispersed over time, especially for college-educated workers, compared to non-college educated workers. Importantly, more volatile wage shocks for college graduates

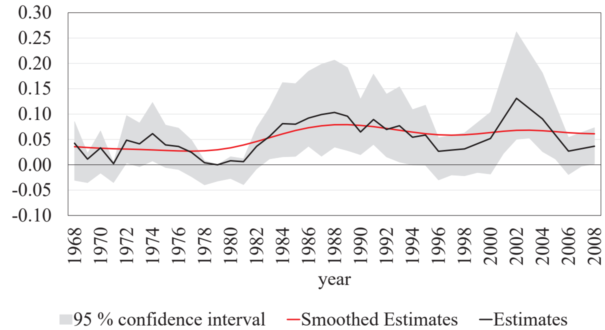
[16] These estimates are broadly consistent with those in Guvenen (2009). When I average the variance of shocks for each education group over 1968 -1993 to make them comparable to estimates in Guvenen (2009), the average variance of the persistent shock is 0.0202, and that of transitory shock is 0.0491, for college graduates while they are 0.0133 and 0.0586, respectively, for non-college graduates.

Figure 3: Variance of persistent and transitory shocks for college and non-college graduates

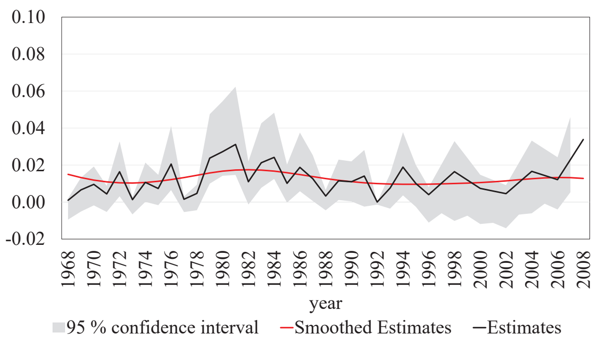
(a) Persistent shock for college graduates



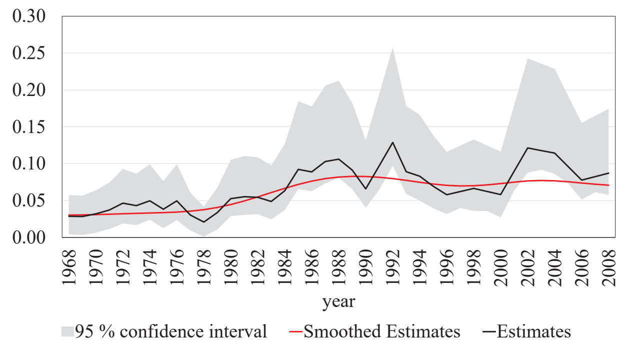
(b) Transitory shock for college graduates



(c) Persistent shock for non-college graduates



(d) Transitory shock for non-college graduates



Note: Minimum distance estimates of the shocks for college and non-college graduates. Smoothed series are generated using a Hodrick-Prescott trend with a smoothing parameter of 100. Confidence intervals are estimated using block bootstrapping with 300 replications.

are mainly driven by the rise in the variance of persistent shock.[17] For instance, the persistent shock variance increases to 0.06 in 2004 from 0.02 in 1983 for college graduates, while, over the same period, it stays around 0.01 for non-college graduates.

Table 1 shows the smoothed estimates of the skill-specific shock processes in 2004, which are used in the subsequent quantitative analysis. Notice that the persistent shock variance for skilled workers is 0.061, higher than that of 0.012 for unskilled workers. The transitory shock variances are relatively similar across education groups. As explained in section 6, the more volatile wage shock process provides a better opportunity for college-educated workers

[17] The rise in within-group wage or earnings dispersion in the U.S. among college graduates is also documented in Lemieux (2010) and Lee et al. (2014).

Table 1: Minimum distance estimates in 2004

education	ρ^e	$\sigma^2_{p_t,e}$	$\sigma^2_{v_t,e}$	$\sigma^2_{\pi,e}$
skilled	0.975	0.061	0.068	0.165
	(0.0094)	(0.0217)	(0.0330)	(0.0209)
unskilled	0.981	0.012	0.077	0.151
	(0.0043)	(0.0116)	(0.0143)	(0.0083)

Note: Estimates for the persistence of wage shock ρ^e, the variance of persistent and transitory shock, $\sigma^2_{p_t,e}$, $\sigma^2_{v_t,e}$, and the initial age persistent shock variance, $\sigma^2_{\pi,e}$ for each skill group. Standard errors are calculated using block bootstrapping with 300 replications and are reported in parentheses.

if they can insure themselves against downside wage risk but have a higher probability of favorable shocks. Lastly, we see that productivity shocks are persistent both for college and non-college graduates over the sample period.

To show how well the estimated wage processes replicate observed wages in the PSID, I simulate 50,000 agents with a college degree and 135,000 agents without a college degree using the estimated wage processes.[18] Figure 4 compares the life-cycle profiles of hourly wages for skilled and unskilled workers from simulated data to those in the 2005 PSID.[19] Note that the estimated wage processes reproduce the observed life-cycle average wages for skilled and unskilled workers reasonably well. There is a relatively large wage gap, however, between simulated and actual data in late working life. This is because, in the simulations, I assume that workers face the same estimated wage processes over the entire working life, while, in data, the realized wage is a result of time-varying college wage premia and wage shock processes. Thus, old workers in data experienced a smaller college wage premia and less volatile wage shock when they were young compared to simulated individuals. This results in a large gap between simulated and actual data late in working life.

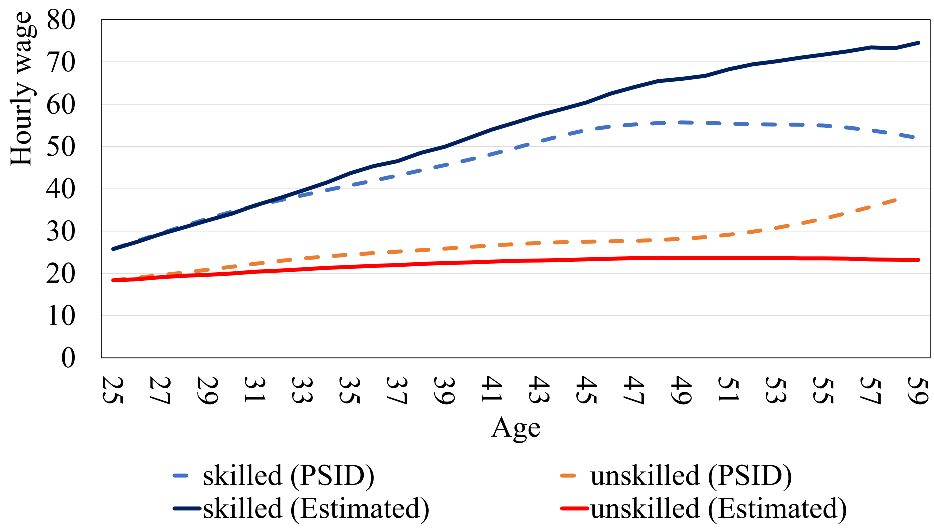
In Table 2, I also compare the distributions of earnings between simulated and the PSID data. To construct earnings in simulation, I assume that college workers spend 30 percent of their time working and non-college workers spend 34 percent of their time.[20] As seen

[18]This matches the 27 percent of college-educated workers to non-college educated workers in the 2004 SCF. This is used as a calibration target in the benchmark economy in section 4.

[19]As earnings and hours worked data are retrospective in the PSID, wage information in the survey year 2005 represents 2004 hourly wages.

[20]These are average hours worked for skilled and non-skilled workers in the benchmark economy.

Figure 4: Hourly wages over the life-cycle



Note: For the PSID data, I smoothed the average level of hourly wage over age using a Hodrick-Prescott trend with a smoothing parameter of 100. (Source: 2005 PSID data)

in Table 2, the estimated wage processes successfully reproduce the observed percentile distributions of earnings as well as earnings Gini coefficients.

Table 2: Distribution of earnings

Year	1%	5%	10%	50%	90%	Gini
2005 PSID	12.1	23.8	33.5	76.7	98.1	0.418
estimated	9.8	23.7	34.0	76.6	97.5	0.416

Note: Table 2 shows the share of earnings held by the top 1, 5, 10, 50, and 90 percent of richest households and the earnings Gini coefficient in the 2005 PSID and from the simulated data using the estimated wage processes.

3.3 Estimation of common wage process

Using a similar approach in section 3.2, I also estimate a common wage process.[21] This common wage process is also used in section 6 to explore the implications skill-specific wage

[21] I run an OLS regression of log hourly wage on time dummies; an interaction term with education and time dummies; labor market experience; and labor market experience-squared. Note that, in this common wage process case, labor market experience terms no longer interact with education dummies. Thus, there are no skill-varying age-wage profiles as in section 3.2. Moreover, I do not introduce a college wage premium in the common wage process economy. Finally, I use regression residuals to estimate a common wage shock process applied to both skill groups.

processes on the distribution of wealth.

The estimated common labor market experience premium is shown in Figure 2. This shows that highly educated workers experience a much flatter age-wage profile compared to the skill-specific wage processes. Moreover, the common labor market experience premium overestimates the wage growth rate for non-college graduates over their working lives.

In Table 3, I summarize the 2004 estimates for a common wage shock process. Crucially, these estimates imply far greater volatility of the persistent shock for unskilled households than the skill-specific wage shock process in Table 1, while the persistence and the volatility of transitory shocks are similar. For instance, the estimated variance for the persistent shock is 0.037 for unskilled households with a common wage shock, compared to 0.012 in the skill-varying wage shock process.[22]

Table 3: A common wage shock process in 2004

ρ	$\sigma^2_{p_t}$	$\sigma^2_{v_t}$	σ^2_{π}
0.976	0.037	0.076	0.155
(0.0049)	(0.0125)	(0.0138)	(0.0086)

Note: Table 3 shows the persistence of wage shock ρ, the variance of persistent and transitory shocks, $\sigma^2_{p_t}$, $\sigma^2_{v_t}$, and the initial age persistent shock variance, σ^2_{π}. Standard errors are calculated using block bootstrapping with 300 replications and are reported in parentheses.

3.4 Wealth inequality and life-cycle savings

In the quantitative analysis, I study the effects of the above estimated wage processes on wealth. Thus, in this section, I summarize the empirical distribution of wealth measured using the SCF. For greater consistency with the PSID data used to estimate wage processes, I report wealth, excluding self-employed households. It is worth noting that, even after excluding entrepreneurs, Table 4 shows significant wealth inequality among households.[23]

As seen in Table 4, the U.S. wealth distribution is highly concentrated and skewed to the right. In 2004, more than 28 percent of total wealth was held by the top 1 percent of

[22]These variance estimates are similar to those in Heathcote et al. (2010b). Their estimates for 2000, which is the last year of their sample period, are 0.212 for persistent shock and 0.0872 for transitory shock, respectively.

[23]For the importance of entrepreneurship for wealth inequality, see Cagetti and De Nardi (2006).

Table 4: Wealth inequality in the U.S. economy

Year	1%	5%	10%	50%	90%	≤ 0	Gini
2004 SCF	28.7	50.5	63.5	96.6	100	8.5	0.771

Note: Table 2 shows the share of wealth held by the top 1, 5, 10, 50, and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings in the U.S. economy. (Source: 2004 SCF data)

the wealthiest households and approximately 64 percent of the wealth in the economy was held by the top 10 percent of households. In contrast, the bottom 50 percent of households only held 3-4 percent of total wealth, and more than 8 percent of households had zero or negative assets.[24] The wealth Gini coefficient is 0.77.[25]

Figure 5: Life-cycle wealth accumulation

Note: I smoothed the average level of wealth over age using a Hodrick-Prescott trend with a smoothing parameter of 100. (Source: 2004 SCF data)

Figure 5 describes the average level of net worth of households with a college degree, and those without a college degree, over age. Though both skill groups have a relatively

[24]If we include self-employed households, the wealth distribution is more concentrated. The top 1 percent of households held around 33 percent of total wealth and the top 10 percent of households hold 69 percent of total wealth. The wealth Gini coefficient is 0.80, and the share of zero or negative asset holdings becomes 8.5 percent.

[25]To accurately measure the wealth Gini with households holding negative assets, I use the wealth Gini measure in Chen et al. (1982).

small amount of wealth at age 25, households with a college degree rapidly accumulate wealth, reaching an average of 1.4 million dollars by age 61. In contrast, the mean level of savings of households without a college degree, at age 61, is less than $\$300,000$ dollars.[26] This great disparity in wealth between skilled and unskilled households would at first seem plausible. However, as seen in section 6.3, a model economy without skill-varying wage risk fails to explain these results.

4 The model

4.1 Overview

In the model economy, there are three sets of agents: households, firms, and government. Household demographic structure involves J overlapping generations. Each generation has a fraction of μ_j of the population, and the total population is normalized to one. Households at age j survive until next age with a probability, ζ_j. At each date, a new cohort of measure $\mu_1 = \sum_{j=2}^{J} \mu_j(1-\zeta_j)$ enters the economy. Each period, households, who are endowed with a unit of time, value consumption and leisure and discount future utility by β.

There are three life-cycle stages: education, work, and retirement. In an education stage, with labor productivity at the initial working age $j = 1$ known, a household decides whether to go to college or not. College education involves a fixed cost denominated in units of output. Households also have access to college loans. With the known initial productivity, college loans may encourage productive households to invest in higher education, leading to more income and wealth dispersion among households with different education levels.

Once households have a college education, they become skilled workers. Otherwise, they enter the labor market as unskilled workers. Skilled workers earn wage w^h higher than w^l, the wage earned by unskilled households. They also face a higher labor market

[26]The importance of skills in the accumulation of wealth has become more pronounced in recent years. To show this, I ran two ordinary least squares regressions using the 1983 and 2004 SCF data. I regressed the log of real net worth on a college education dummy, the log of income, and age, race, and work status dummies. Table A1 shows that the coefficient on the education dummy increases from 0.24 to 0.53 between 1983 and 2004, while coefficients and standard errors for other variables are similar across the two regressions. This increase in the coefficient of college education demonstrates that the tendency for skilled labor to hold a higher level of assets relative to unskilled labor has increased across the sample period.

experience premium, $l^h(j)$, over their working lives, compared to the non-college educated, $l^l(j)$.[27] Workers also face skill-specific wage shock processes over their working lives. After retirement at age J_r, households receive social security benefits proportional to their last earnings shock, $s^e(\varepsilon^{Jr-1})$. Finally, I introduce a warm-glow bequest motive, $v^b(a')$, such that households do not de-accumulate their wealth counterfactually fast after retirement. Accidental bequests from those that do not survive are re-distributed to the newly born households as their initial assets. Any remaining amount, τ_r, is paid lump-sum to households.

A representative firm produces output with aggregate capital, K, and labor, N, through a strictly concave, constant returns to scale production function $F(K,N) = K^\alpha N^{1-\alpha}$, where $0 < \alpha < 1$. Aggregate labor N is itself constant elasticity of substitution function $H(H^h, H^l)$ of two types of effective units of skilled and unskilled labor.

$$H(H^h, H^l) = [\lambda(H^h)^{\frac{\theta-1}{\theta}} + (1-\lambda)(H^l)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{1}$$

Above, H^e is the aggregate labor input associated with education level $e \in \{l, h\}$, θ is the elasticity of substitution between the two types of labor, and λ reflects skill-biased demand shift. The rate of depreciation for capital is $\delta \in (0,1)$.

Finally, following Heathcote et al. (2017), the government uses non-linear tax and transfer system $T(y) = y - \tau_1 y^{1-\tau_2}$ to finance social security payments to retirees. Here, τ_2 determines the degree of progressivity of the tax system, while τ_1 determines the level of taxation. Note that y is the sum of both labor (or social security income) and capital income. The government budget is balanced.

4.2 College education decision

Before the initial age of working life, households complete an education decision. They can choose either a college degree or a high school diploma. Agents who pursue a college degree have to pay an idiosyncratic output cost, $\kappa \in [\kappa_l, \kappa_h]$, which is drawn from the distribution, $Q(\kappa)$. However, agents with a college degree benefit from a college wage

[27]Following Heathcote et al. (2010b), I assume a return to age as a proxy for labor market experience.

premium and higher experience premium, compared to those without a college degree. These increase their lifetime earnings faster than unskilled workers. Furthermore, the wage shock process differs by education groups.

Individuals decide to go to college if the value of pursuing a college degree is higher than their value without a college degree. A college education is possible only when individuals have sufficient resources, including borrowing, to finance their education costs. An output cost, unlike a utility cost, captures the fact that individuals may be unable to go to college because of insufficient income. This introduces a direct role for borrowing constraints, and implicitly, college loans.

Let $\mathrm{v}^h(1, a, \varepsilon, \kappa)$ be the value function of a household who pays an education cost κ at age $j = 1$, with assets a and an idiosyncratic productivity shock ε. Given a and κ, the household chooses consumption c, saving a', and labor supply n. The maximization problem of such a household is

$$\mathrm{v}^h(1, a, \varepsilon_i, \kappa) = \max_{c, a', n} \left\{ u(c, 1-n) + \beta\zeta_1 \sum_{m=1}^{N_\epsilon} \pi_{im} v^h(2, a', \varepsilon_m) + \beta(1-\zeta_1) v^b(a') \right\} \tag{2}$$

$$\text{subject to } c + a' = (1+r)a + w^h l^h(1)\varepsilon_i n + \tau_r - \kappa - T(y)$$

$$y = ra + w^h l^h(1)\varepsilon_i n$$

$$a' \geq \underline{a},\ c \geq 0,\ n \in [0, 1]$$

where $\underline{a}$ is the borrowing loan limit.

The problem of a household who does not pay a fixed education cost is

$$v^l(1, a, \varepsilon_i) = \max_{c, a', n} \left\{ u(c, 1-n) + \beta\zeta_1 \sum_{m=1}^{N_\epsilon} \pi_{im} v^l(2, a', \varepsilon_m) + \beta(1-\zeta_1) v^b(a') \right\} \tag{3}$$

$$\text{subject to } c + a' = (1+r)a + w^l l^l(1)\varepsilon_i n + \tau_r - T(y)$$

$$y = ra + w^l l^l(1)\varepsilon_i n$$

$$a' \geq \underline{a},\ c \geq 0,\ n \in [0, 1]$$

Note that this problem is the same as that of a working household of any other age.

The optimal education decision, $e(a, \varepsilon, \kappa)$, can be summarized as

$$e(a, \varepsilon, \kappa) = \begin{cases} h & \text{if } \mathrm{v}^h(1, a, \varepsilon, \kappa) \geq v^l(1, a, \varepsilon) \\ l & \text{otherwise} \end{cases} \tag{4}$$

subject to $m = (1 + r)a + w^h l^h(1)\varepsilon_i n + \tau_r - \kappa - T(y) \geq \underline{a}$.

4.3 A household in the working life

Now, I describe the behavior of households during their working life, after the college education decision has been made. Let $v^e(j, a, \epsilon)$ represent the expected discounted value of a household, with education level $e \in \{l, h\}$, at age j with assets a and an idiosyncratic productivity shock ε. During their working ages, households earn both labor and capital income. The problem of a working household is

$$v^e(j, a, \varepsilon_i) = \max_{c, a', n} \left\{ u(c, 1 - n) + \beta\zeta_j \sum_{m=1}^{N_\epsilon} \pi_{im} v^e(j + 1, a', \varepsilon_m) + \beta(1 - \zeta_j)v^b(a') \right\} \tag{5}$$

$$\text{subject to } c + a' = (1 + r)a + w^e l^e(j)\varepsilon_i n + \tau_r - T(y)$$

$$y = ra + w^e l^e(j)\varepsilon_i n$$

$$a' \geq \underline{a},\ c \geq 0,\ n \in [0, 1]$$

4.4 A household after retirement

After retirement, households receive social security benefits from the government as a function of their last working age's earnings shock and education level.

$$v^e(j,a,\varepsilon^{Jr-1}) = \max_{c,a'} \left\{u(c,1) + \beta\zeta_j v^e(j+1,a',\varepsilon^{Jr-1}) + \beta(1-\zeta_j)v^b(a')\right\} \tag{6}$$

$$\text{subject to } c + a' \leq (1+r)a + s^e(\varepsilon^{Jr-1}) + \tau_r - T(y)$$

$$y = ra + w^e s(\varepsilon^{Jr-1})$$

$$a' \geq \underline{a},\ c \geq 0$$

4.5 Recursive equilibrium

The distribution of households varies over age, wealth, labor productivity, and education level. Let $\mathbf{J} = \{1,\ldots,J\}$ represent the set of indices for household age. Households' wealth is $a \in \mathbf{A} = [\underline{a},\infty)$, and their education level is $e \in \{l,h\}$, where $e = l$ if a household is a non-college graduate and $e = h$ otherwise. $\mathbf{E} = \{\varepsilon_1,\ldots,\varepsilon_{N_\varepsilon}\}$ defines the support for productivity shocks.[28] Lastly, $\mathbf{\Gamma} = \{\kappa_l,\ldots,\kappa_h\}$ is the space of fixed education costs. The product space, $\mathbf{S} = \mathbf{J}\times\mathbf{A}\times\mathbf{E}\times\{l,h\}$, describes the space for the distribution of households. Define $\mathcal{S}$ as the Borel algebra generated by the open subsets of $\mathbf{S}$. We define $\psi : \mathcal{S} \to [0,1]$ as a probability measure over households.

Initial wealth is drawn from $a_0 \sim \chi(a_0)$. Initial productivity is drawn from $\pi^0 \sim logN(0,\sigma_\pi^2)$, the invariant distribution for $\{\pi_{im}\}_{i,m=1}^{N_\varepsilon}$.[29] Let μ_1 be the number of households at age $j = 1$. The distribution of non-college and college educated households at the initial age is described by

$$\psi(1,a_0,\varepsilon_i,l) = \int_{\{\kappa|e(a_0,\epsilon_i,\kappa)=l\}} \pi_i^0\mu_1 Q(d\kappa)\chi(a_0)$$

$$\psi(1,a_0,\varepsilon_i,h) = \int_{\{\kappa|e(a_0,\epsilon_i,\kappa)=h\}} \pi_i^0\mu_1 Q(d\kappa)\chi(a_0)$$

[28] I fix the support for shocks to that of the skilled and use the Tauchen algorithm to discretize the distribution implied by the variance of shocks for the unskilled onto this support. The initial value for the shock is also drawn from the same support.

[29] I assume a distribution for initial labor productivity using the value estimated for the skilled, $\sigma_{\pi,h}^2$, for both education groups.

In subsequent periods, the distribution of households with each education level is given by the following.

$$\psi(j+1, A, \varepsilon_m, l) = \sum_{i=1}^{N_\varepsilon} \pi_{im}^l \int_{\{a|g(j,a,\varepsilon_i,l)\in A\}} \zeta_j \psi(j, da, \varepsilon_i, l)$$

$$\psi(j+1, A, \varepsilon_m, h) = \sum_{i=1}^{N_\varepsilon} \pi_{im}^h \int_{\{a|g(j,a,\varepsilon_i,h)\in A\}} \zeta_j \psi(j, da, \varepsilon_i, h)$$

A *recursive competitive equilibrium* is a set of functions (v^l, v^h, g, n, c, e) and prices (w^l, w^h, r) such that:

(i) (v^l, v^h) solve (2), (3), (5) and (6). $g : \mathbf{J}\times\mathbf{A}\times\mathbf{E}\times\{l,h\} \to \mathbf{A}$ is the associated optimal policy for saving. Note that $g(1, A, E, h) = \int_{\{\kappa|e(a,\varepsilon,\kappa)=h\}} g_1^h(A, E, \kappa)Q(d\kappa)$ where $g_1^h : \mathbf{A}\times\mathbf{E}\times\mathbf{\Gamma} \to \mathbf{A}$ is the associated optimal policy for (2). $n : \mathbf{J}\times\mathbf{A}\times\mathbf{E}\times\{l,h\} \to [0,1)$ is the associated optimal policy for labor supply, $c : \mathbf{J}\times\mathbf{A}\times\mathbf{E}\times\{l,h\} \to \mathbf{R}_+$ is the associated optimal policy for consumption, and $e : \mathbf{A}\times\mathbf{E}\times\mathbf{\Gamma} \to \{l,h\}$ is the education decision rule for paying the fixed cost to get a college degree.

(ii) Markets clear

$$H^l = \sum_{j=1}^{J}\sum_{i=1}^{N_\varepsilon} \int_{\mathbf{A}} l^l(j)\varepsilon_i n(j, a, \varepsilon_i, l)\psi(j, da, \varepsilon_i, l)$$

$$H^h = \sum_{j=1}^{J}\sum_{i=1}^{N_\varepsilon} \int_{\mathbf{A}} l^h(j)\varepsilon_i n(j, a, \varepsilon_i, h)\psi(j, da, \varepsilon_i, h)$$

$$K = \sum_{j=1}^{J}\sum_{i=1}^{N_\varepsilon} \left\{ \int_{\mathbf{A}} g(j, a, \varepsilon_i, l)\psi(j, da, \varepsilon_i, l) + \int_{\mathbf{A}} g(j, a, \varepsilon_i, h)\psi(j, da, \varepsilon_i, h) \right\}$$

$$C + \delta K = F(K, H) - \sum_{i=1}^{N_\varepsilon} \int_{\{(a,\kappa)|e(a,\varepsilon_i,\kappa)=h\}} \kappa\psi(1, da, \varepsilon_i, h)Q(d\kappa),$$

where $C = \sum_{j=1}^{J}\sum_{i=1}^{N_\varepsilon} \left\{ \int_{\mathbf{A}} c(j, a, \varepsilon_i, l)\psi(j, da, \varepsilon_i, l) + \int_{\mathbf{A}} c(j, a, \varepsilon_i, h)\psi(j, da, \varepsilon_i, h) \right\}$

(iii) The government budget is balanced

$$\sum_{j=J_r}^{J}\sum_{i=1}^{N_\varepsilon}\sum_{e=l}^{h}\int_{\mathbf{A}} s^e(\varepsilon_i^{Jr-1})\psi(j,da,\varepsilon_i,e) =$$
$$\sum_{j=1}^{J}\sum_{i=1}^{N_\varepsilon}\sum_{e=l}^{h}\int_{\mathbf{A}} T\left(ra + \mathbb{1}_{j<J_r} w^e l^e(j)\varepsilon_i n^e + \mathbb{1}_{j\geq J_r} s^e(\varepsilon_i^{Jr-1})\right)\psi(j,da,\varepsilon_i,e)$$

where $\mathbb{1}_{j<J_r}$ is an indicator function for workers and $\mathbb{1}_{j\geq J_r}$ is an indicator function for retirees.

(iv) Prices are competitively determined

$$w^l = D_2F(K,H)D_2H(H^h,H^l)$$

$$w^h = D_2F(K,H)D_1H(H^h,H^l)$$

$$r = D_1F(K,H) - \delta$$

5 Calibration

The benchmark model is calibrated to the 2004 U.S economy. The model period is one year. Households are assumed to enter the labor market at age 25 and retire at age 60, and their last possible age is 84.

5.1 College education cost and borrowing limit

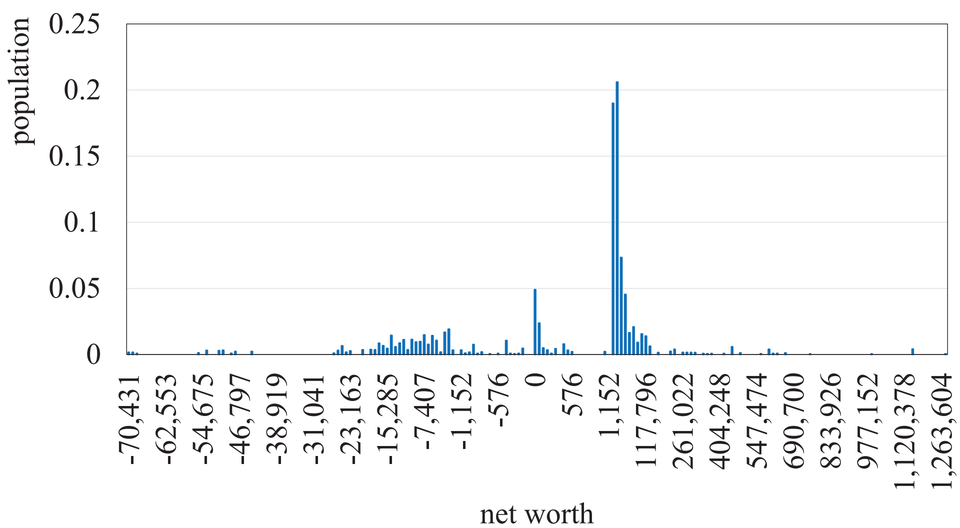
I assume that the college education cost is drawn from the uniform distribution, $\kappa \sim U(0,\kappa_u)$. I choose the parameter κ_u to match the college completion rate of 27 percent for males between 25- and 59- years old in 2004 SCF. Assuming four years of college education, the calibrated education cost implies the annual college education cost of around $7,380. This is close to an average annual net education cost of $6,700 measured by Abbott et al. (2019) using full-time full-year students who are enrolled in non-profit private and public four-year colleges in 2000.

Households can borrow up to the borrowing limit, $\underline{a}$. I choose this borrowing limit to match the share of households with zero or negative net worth in the 2004 SCF. Importantly, while I have not targeted, the calibrated parameters for education cost and borrowing limits give rise to 79 percent of households entering the labor market with student loan debt. This is similar to three quarters of graduating seniors from four-year institutions with student loan debt reported by the National Postsecondary Student Aid Study (NPSAS).

5.2 Initial wealth distribution

The distribution of initial asset holdings, $\chi(a_0)$, is directly estimated from the 2004 SCF data. Specifically, I use the net worth of households whose heads are between 20 and 25 years old. Figure 6 describes the estimated distribution of households over 200 wealth bins with more points around zero.[30] I introduce the normalized distribution of this into the model economy.[31] I assume that initial asset holdings are financed by accidental bequests.

Figure 6: Distribution of wealth across households between 20 and 25 years old



Source: 2004 SCF data

Note that the distribution of initial assets, alongside idiosyncratic education costs, may determine the effect of liquidity constraints on households' college education decisions.

[30] Given the wealth concentrated around zero, I put more points around zero wealth value.
[31] I normalize it as the units of output in the model are different from those in data.

Specifically, it can interact with borrowing constraints, discouraging some households who pursue a college degree from gaining skills because of the insufficient resources.

5.3 Preference, bequest, and social security benefits

The household period utility function is CRRA

$$u(c_t, n_t) = \frac{{c_t}^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n_t)^{1-\sigma}}{1-\sigma} \tag{7}$$

where the relative risk aversion parameter, $\gamma = 2$.[32] The parameter for leisure ψ is 2.2 to match average male hours worked of 30% of time endowment. Given this, $\sigma = 4.3$ matches a 0.48 Frisch elasticity of labor supply for men.[33] I set $\beta = 0.952$ to replicate the ratio of average wealth to average pre-tax income in 2004 SCF.

Following De Nardi et al. (2010), the utility of leaving bequest a is

$$v^b(a) = \chi \frac{(a+\xi)^{1-\gamma}}{1-\gamma}$$

where χ is the strength of the bequest motive, and ξ determines the curvature of the bequest function. The calibrated parameters of bequest imply the marginal propensity to bequeath of 0.99 and the consumption values of $\$17,328$ at which households find it worthwhile to leave bequests.[34] These are in the range of the estimates in Lockwood (2018) where the marginal propensity to bequeath ranges between 0.93 and 0.99 and the threshold value of consumption between $\$12,500$ and $\$30,000$.

The social security benefit is proportional to a worker's last labor income, $s^e(\epsilon^{Jr-1}) = \theta_s w \epsilon^{Jr-1} \overline{n}$, where $\overline{n} = 0.333$ is the average level of labor supply in the economy. The replacement rate of social security benefits, θ_s, is chosen to match 45 percent of average pre-tax earnings in Hosseini (2015).

[32] Attanasio (1999) estimates this parameter between one and two.

[33] The Frisch elasticity of labor supply is $(\frac{1}{\sigma})\frac{(1-n)}{n}$, where n is the average aggregate hours worked.

[34] Following De Nardi et al. (2010), I calculate the optimal bequest level in consumption units and marginal propensity to bequest for a household at the last age who starts the period with cash x. The optimal level of bequest is $a' = \frac{fx-\xi}{1+f}$ and the marginal propensity to bequest is $\frac{\partial a'}{\partial x} = \frac{f}{1+f}$, where $f = \chi^{\frac{1}{\gamma}}$.

5.4 Remaining model parameters

The production parameter θ that governs the elasticity of substitution between skilled and unskilled labor is set to 1.67 following Krusell et al. (2000). The capital share of output is $\alpha = 0.36$, and the depreciation rate δ is 0.06. Following Heathcote et al. (2010b), λ is calibrated to match the observed college wage premium of 1.34 in 2004, as measured in section 3.2. Lastly, the progressivity of taxes is set to $\tau_2 = 0.181$ using the estimate in Heathcote et al. (2017), and τ_1 is calibrated to balance the government budget.[35]

6 Results

In this section, I evaluate the quantitative implications of skill-specific wage processes estimated in section 3.2 on wealth inequality and life-cycle savings. To compare, I also present results from an alternative economy with a common wage process estimated in section 3.3.[36] Finally, I evaluate the relative importance of the deterministic between-group and stochastic within-group wage inequality by removing each of them one at a time in the model economy.

6.1 Wealth inequality

In Table 5, I summarize moments of the distribution of wealth and the share of households with a college degree in the 2004 SCF and the two economies – the benchmark economy with skill-specific wage processes and the economy with the common wage process.

As seen in Table 5, the empirically consistent skill-specific wage processes reproduce a significant amount of the wealth inequality seen in the data. For example, the wealth Gini coefficient in the benchmark economy is 0.73 compared to 0.77 in the data, and the top 10 percent of households hold 53 percent of the total wealth in the economy compared

[35] Heathcote et al. (2017) estimate U.S. tax progressivity τ_2 using pre-tax gross household income that includes labor earnings, self-employment income, private transfers, and income from interest, dividends, and rents, using 2000-2006 PSID data.

[36] In the common wage process, there is no between-group wage dispersion, and all households face the same wage shock process during their working lives.

Table 5: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
2004 SCF	28.7	50.5	63.5	96.6	100	8.5	0.77	0.27
benchmark	16.0	38.7	53.0	99.3	100	10.9	0.73	0.27
common	8.3	27.8	43.6	94.3	100	5.5	0.64	0.0

Note: Table 5 shows the share of wealth held by the top 1, 5, 10, 50, and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

to 64 percent in the data. Although the model economy cannot generate enough asset accumulation among the wealthiest 1 percent of households, it is important to note that introducing skill-specific wage processes helps the model explain a significant fraction of wealth inequality in the absence of strong saving motives arising from the counter-factually high variance of income shocks (Castaneda et al. (2003)), entrepreneurship with collateral constraints (Cagetti and De Nardi (2006)), preference shocks (Krusell and Smith (1998)), stochastic life cycles (Castaneda et al. (2003)) or heterogeneity in return to savings (Benhabib et al. (2011)). Rather, I introduce empirically consistent distinct wage processes across households with different skill levels into a dynamic structural model and show that these wage processes themselves go a long way to explain additional inequality in wealth.

The dispersion in wealth in the benchmark economy is mainly driven by the top distribution of households who are skilled.[37′38] Skill-specific wage processes increase skilled households' savings through the following two channels. First, the higher hourly wage and steeper age-wage growth profile result in a high level of earnings for the skilled. Second, a relatively more dispersed persistent component of wage risk for the skilled leads to a higher probability of both favorable and unfavorable wage shocks compared to unskilled households. Importantly, as skilled households can smooth their earnings against downside wage risk by increasing their hours worked, a large variance of the persistent wage shock

[37]The rise in wealth inequality driven by the high-earnings group is in line with the empirical evidence that shows a relatively important role of the upper distribution of households driving wealth and income inequality (see Saez and Zucman (2016) and Heathcote et al. (2010a)).

[38]See figure C1 for the distribution of wealth over the log of labor productivity and asset holdings for each education group in the benchmark economy.

provides a better opportunity for the skilled to be wealthy, driving further inequality.[39]

By comparison, the economy with a common wage process explains much less wealth inequality than the benchmark economy, as seen in the last row of Table 5. Without wage differentials across skill groups, households do not pursue a college degree, making the model equivalent to an Aiyagari economy with elastic labor supply. Note that, in this economy, households who would be skilled in the benchmark economy face lower hourly wages, lower labor market experience premia and less volatile persistent component in the wage shock while those who would be unskilled receive higher hourly wages, higher labor market experience premia, and more volatile persistent wage shock.

Given that wealth inequality is mainly driven by the highly educated group, the change in the wage structure for the would-be skilled households sharply decreases inequality in wealth. These households realize a smaller level of earnings compared to the benchmark economy as they are paid much lower hourly wages and labor market experience premia. Besides, the probability of a favorable wage shock for them decreases as they face a less dispersed persistent component with a common wage shock process.[40] As a result, the wealth Gini drops by 9 points, and the share of wealth held by the top 10 percent of households decreases by around 10 percentage points compared to the benchmark economy.

Tables 6 and 7 summarize the over-identified predictions of the benchmark economy regarding the joint distribution of wealth and college attainment. Table 6 summarizes the fractions of skilled and unskilled households across wealth percentiles both in the 2004 SCF and the benchmark economy. The model can explain a significant fraction of skilled households at the top of the distribution of wealth. For example, the model predicts 70 percent of the top 5 percent of households being skilled compared to 78 percent in the 2004 SCF. Table 7 also shows the probability of being in each wealth percentile for college and non-college graduates. In the model economy, 22 percent of college graduates reach the

[39]Such an interaction between elastic labor supply and wage risk is also explored in Heathcote et al. (2008). They show that as long as the coefficient of relative risk aversion is greater than 1 with separable preferences, income effects of a rise in wage dispersion dominate substitution effects, and individuals increase their labor supply in times with adverse shocks. They also show that, with sufficiently elastic labor supply, a rise in wage dispersion can be indeed welfare-improving.

[40]The variance of the persistent wage shock process for the skilled, in the benchmark, is 0.069 while that in the common wage process is 0.036.

top 5 percent of the wealth distribution, while only 3 percent of unskilled households can become the wealthiest 5 percent of households. These are similar to 11 percent of college graduates and 2 percent of non-college graduates comparing the wealthiest 5 percent of households in the data.

Table 6: The fraction of the skilled and unskilled in the top percentiles of the wealth distribution

2004 SCF	1%	5%	10%	50%	90%
skilled households	0.85	0.78	0.74	0.50	0.38
unskilled households	0.15	0.22	0.26	0.50	0.62
benchmark economy	1%	5%	10%	50%	90%
skilled households	0.99	0.70	0.49	0.29	0.27
unskilled households	0.01	0.30	0.51	0.71	0.73

Table 7: The probability of being in the top percentiles of the wealth distribution

2004 SCF	1%	5%	10%	50%	90%
skilled households	0.02	0.11	0.20	0.67	0.93
unskilled households	0.00	0.02	0.04	0.40	0.89
benchmark economy	1%	5%	10%	50%	90%
skilled households	0.04	0.22	0.33	0.81	0.91
unskilled households	0.00	0.03	0.12	0.73	0.90

6.2 The relative importance of between- and within-group wage dispersion

To show the relative importance of between-group and skill-varying within-group wage inequality for understanding the distribution of wealth, I further explore the distribution of households in the three alternative economies. In the first economy, I assume that there is no college wage premium, but households still face skill-varying labor market experience premium and shock processes (*no college wage premium*). In the second economy, I further remove the skill-varying labor market experience premium from the first alternative economy. This leaves both skilled and unskilled households receiving the same hourly wage and labor market experience premium but facing skill-specific wage shock processes (*no between-group wage dispersion*). In the third economy, college wage premium and skill-specific labor market experience premium exist, but workers face the same wage shock process as in the common wage process economy (*common wage shock*). Table 8 compares the resulting distribution of wealth from these three economies to that in the benchmark economy.

The deterministic between-group wage heterogeneity – college wage premia and labor market experience premia – is crucial for understanding college attainment decisions and the distribution of wealth seen in the data. The second row of Table 8 shows that, without a college wage premium, only 13 percent of households pursue a college degree, and the wealth Gini decreases by 5 points relative to the benchmark economy. The decrease in college-educated households is intuitive, given that there is a much lower incentive for a college education. When I further remove the skill-varying labor market experience premium, as seen in the third row of Table 8, the wealth Gini falls by an additional 5 points from the *no college wage premium* economy and the share of wealth held by the top 10 percent of households decreases to 40 percent. Note that, without observed benefits of a college education, all households remain unskilled as in the common wage economy in Table 5. Consequently, households realize a much lower level of earnings and face less dispersed wage shocks as unskilled workers. This sharply decreases wealth inequality.

Table 8: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
benchmark	16.0	38.7	53.0	99.3	100	10.9	0.73	0.27
no college wage premium	11.9	31.2	45.9	99.4	100	8.3	0.68	0.13
no between-group wage difference	8.1	25.7	40.3	93.8	100	6.1	0.63	0.0
common wage shock	11.5	32.6	49.2	99.5	100	9.7	0.70	0.27

Note: Table 8 shows the share of wealth held by the top 1, 5, 10, 50, and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

The last row of Table 8 shows the distribution of wealth when there is no skill-varying wage *shock* process. With only between-group wage differences, the wealth Gini falls to 0.70 from 0.73 in the benchmark economy despite the same fraction of skilled households. More importantly, the common wage shock process reduces the concentration of wealth at the top of the distribution of households. For example, the share of wealth held by the wealthiest 1 percent of households falls from 16 percent to 11 percent, which is equivalent to more than a 30 percent drop.[41] As mentioned before, with the same wage shock process, skilled households face the wage shock with a less volatile persistent component. In contrast, unskilled households face wage shock with a more volatile persistent component, compared to skill-specific wage shock processes. A less volatile persistent wage shock implies a lower probability of favorable shock for the skilled and dampens wealth accumulation of the high-education group. As savings of the skilled are main drivers for wealth inequality, the model economy generates the less skewed distribution of wealth compared to the benchmark economy.

In sum, these results show that understanding between-group and within-group heterogeneity in wages, across households with different education levels, is important for explaining a significant fraction of the observed wealth inequality. Specifically, the deterministic component of wage differential is essential for college education decisions, while the stochastic component of wage differential drives a higher concentration of wealth at the top of the distribution.

6.3 Life-cycle wealth accumulation

Understanding the sources of wage inequality is not only important for aggregate wealth inequality but also for understanding the life-cycle savings of households across skill groups. To show this, I simulate 40,000 households for each education group in the two model economies in section 6.2.[42] The first model is the benchmark economy with skill-specific wage shock processes, and the second is the model with the same wage shock process across skill groups (*no within-group wage dispersion*).[43]

Figure 7 compares life-cycle wealth profiles across education groups from the two model economies to those from the 2004 SCF data.[44] As seen in the top panel of Figure 7, the

[41]Though not present in the paper, when the model is instead solved with a linear tax system, the top 1 percent share of wealth decreases by 49 percent without within-group wage dispersion.

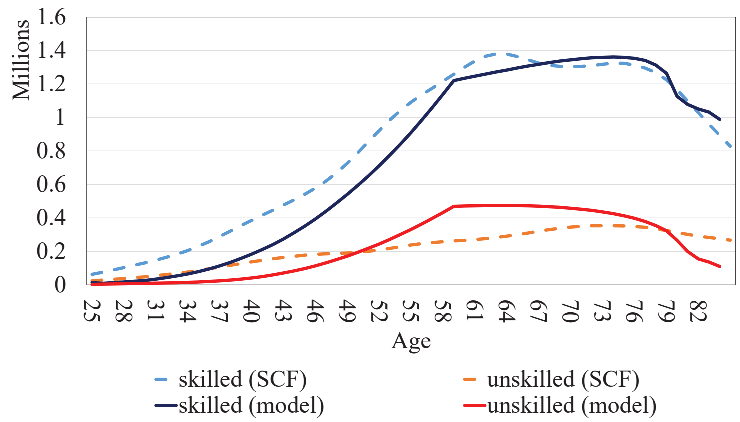
[42]I use simulation method instead of explicitly aggregating households' decisions at each age to avoid potential small sample bias.

[43]I abstract from the model without between-group wage dispersion as all households become unskilled, as shown in Table 8.

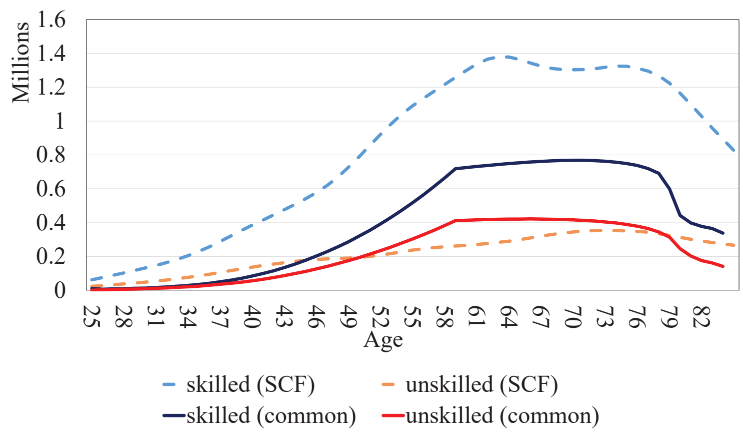
[44]For comparability between model and data, I calculate a factor which makes the average level of wealth of 50-year-old unskilled households in the model equal to that in the data, then adjust other series accordingly. This adjustment is needed as the units in the model are different from those in the data.

Figure 7: Life-cycle wealth accumulation for skilled and unskilled households

(a) Skill-specific wage shock



(b) Common wage shock



Note: For the SCF data, smoothed series are generated using a Hodrick-Prescott trend with a smoothing parameter of 100.

benchmark economy predicts a difference in wealth across education groups similar to that in the data. This was not targeted in the calibration. In contrast, with the common wage shock process, the disparity in wealth across education groups sharply reduces, reproducing half of the gap explained by the benchmark economy. This is a striking result given that the common wage shock economy still produces a wealth Gini of 0.7 (see Table 8).

In the common wage shock economy, the entire working population faces the same wage risk. In particular, unskilled workers face a wage risk with higher variance in their persistent shock component compared to that of the benchmark economy. This implies that if an unskilled household is lucky, he can have a significant amount of earnings without a college education, compared to the benchmark economy. This narrows the gap in earnings and thus wealth, over education levels. This also shows that assuming the same wage risk across households puts too much emphasis on luck to determine households' wealth accumulation.[45]

6.4 The implications of elastic labor supply and education choice

In this section, I examine the implications of elastic labor supply and education choice on the resulting wealth inequality in my model economy. In the first experiment, I fix labor supply for each skill group to the average level of hours worked by skills in the benchmark economy.[46]

Table 9: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
benchmark	16.0	38.7	53.0	99.3	100	10.9	0.73	0.27
no elastic labor supply	15.2	36.4	51.0	99.4	100	8.7	0.72	0.18
no education choice	15.5	37.7	52.0	99.5	100	9.8	0.72	0.27

Note: Table 9 shows the share of wealth held by the top 1, 5, 10, 50, and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

The second row of Table 9 shows that, without elastic labor supply, a smaller fraction of households pursue a college degree. Only 18 percent of households become skilled without elastic labor supply compared to 27 percent in the benchmark economy. Given access to college loans, elastic labor supply allows productive but financially constrained households to become college-educated. This is because these households can increase hours worked during their working lives to repay their college loans. Thus, without labor supply choice,

[45]The top panel of Figure 7 also suggests that college attainment, which is made early in life, plays a crucial role in determining households' lifetime wealth. Such a positive correlation between college attainment and lifetime wealth is consistent with findings in Huggett et al. (2011). For example, Huggett et al. (2011) show that variation in initial human capital, as of age 23, determines 62 percent of the variation in lifetime wealth. However, in contrast to Huggett et al. (2011), by allowing unobserved wage heterogeneity to vary by education levels, I show that shocks over the life-cycle also play a substantial role in determining differences in lifetime wealth across education groups.

[46]Results are robust when I fix the labor supply for all households to the economy-wide average level of hours worked.

these financially constrained households find it more difficult to pursue a college degree, decreasing the fraction of skilled households in the economy. Moreover, without elastic labor supply, skilled households are no longer able to insure themselves against unfavorable wage shocks, making volatile persistent wage shocks less favorable. However, it is worth noting that, though inelastic labor supply discourages college education, the resulting aggregate wealth inequality is similar to that in the benchmark economy. This is because the primary determinants of the aggregate wealth inequality are the post-education wage differentials households face over their working lives, not the composition of the skilled and the unskilled.

To make this point clearer, in the second experiment, I randomly assign 27 percent of households to become skilled instead of allowing them to make their college education choices.[47] Table 9 shows that the model without educational choice explains the similar distribution of wealth to that in the benchmark economy. This emphasizes that the additional wealth inequality explained in the economy with skill-specific wage processes, compared to a common wage process, is not driven by high-ability households selectively going to the college. Rather, it is between-group and within-group wage dispersion high-ability households face after their education decisions that shape the distribution of wealth in the economy.

6.5 Bequests

In the benchmark economy, the bequest motive plays a key role in reproducing enough wealth concentration at the top as well as a slow de-accumulation of wealth after retirement seen in the data.

As shown in Table 10, in the absence of a bequest motive, the aggregate wealth inequality sharply falls. For instance, wealth Gini drops by 4 points from that in the benchmark economy, and the share of wealth held by the top 10 percent of households decreases to 45 percent, compared to 53 percent in the benchmark economy. In the benchmark economy, wealthy college-educated households are more likely to leave bequests for their kids, leading to a higher concentration of wealth at the top. Without this motive, wealthy skilled

[47] To see the net effects of each channel, I fix the price and wages to the steady-state levels in the benchmark economy.

Table 10: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
no bequest	9.4	28.2	45.0	99.3	100	12.1	0.69	0.25

Note: Table 10 shows the share of wealth held by the top 1, 5, 10, 50, and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

households decrease their wealth accumulation, especially with progressive income taxes, reducing aggregate wealth inequality. Importantly, the model economy with a common wage process and the same bequest motive fails to reproduce as much concentration of wealth and inequality as the benchmark economy (see Table 5.) This suggests that it is not just the bequest motive that generates a significant amount of wealth inequality but the wage differentials across education groups that interact with the bequest motive.

7 Sensitivity analysis

7.1 Initial distribution

In section 6.4, I show that the post-education wage processes are crucial channels to shape the distribution of wealth for the top, not the composition of skill and unskilled households. However, this might be driven by the lack of enough heterogeneity at the initial age as the model economy abstracts from direct bequests or inheritance of ability.

To explore this, I build the model economy where households draw their initial assets and labor productivity from the distribution of households who do not survive.[48] Although this still has a lack of forward-planning compared to a full dynastic model, when combined with a warm-glow bequest motive, it does capture some desire of parents to leave bequests for their descendants. Moreover, this allows a strong positive correlation between initial labor productivity and asset holdings.[49]

[48]This model no longer has a lump-sum transfer, τ_r. All of it is re-distributed to households at the initial age.

[49]Figure C2 shows this initial distribution over the log of labor productivity and assets. Given that skilled households are more likely to leave bequests, the shape of this initial distribution is similar to that for the distribution of skilled households in the benchmark economy in Figure C1.

I solve this alternative model and re-calibrate it to the same moments as in the benchmark economy. Next, I repeat the counterfactual experiments in sections 6.2 and 6.4. Table 11 summarizes the results. As seen in the first row, this economy generates the distribution of wealth close to the benchmark economy. Moreover, the counterfactual experiments generate very similar quantitative results as those in the benchmark economy. This suggests that the post-education wage differentials still play a significant role in shaping the distribution of wealth despite a stronger selection effect on education choice in this alternative economy.[50]

Table 11: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
full model w/ new initial distribution	16.2	39.7	53.5	99.8	100	11.8	0.70	0.27
common wage process	8.4	27.7	43.0	91.5	100	7.4	0.62	0.0
no between-group wage difference	8.9	26.1	39.7	90.0	100	7.3	0.59	0.0
common wage shock	10.8	31.5	46.6	94.8	100	9.5	0.66	0.35
no education choice	14.5	35.6	49.4	95.1	100	10.7	0.67	0.27

Note: Table 11 shows the share of wealth held by the top 1, 5, 10, 50 and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

7.2 Borrowing limits

There is a growing literature that studies the effects of credit constraints on college education attainment. For example, using the National Longitudinal Survey of Youth 1997, Belley and Lochner (2007) document the empirical evidence of the important role of family income for credit-constrained individuals on their educational attainments.[51] Hai and Heckman (2017) also emphasize the role of credit constraints on human capital accumulation and education attainment.[52]

[50] The only case that has a significant difference from the benchmark economy is when I randomly assign the education choice. As shown in the last row of Table 11, the wealth Gini drops to 0.67 without education choices, compared to 0.70 in the full model. This is in contrast to the benchmark economy, where random education choice rarely changes the distribution of wealth. This is driven by a stronger selection effect on education decisions with the new initial distribution.

[51] The early literature finds little evidence on the effects of family income in college attendance for the early cohort. (See Carneiro and Heckman (2002) and Keane and Wolpin (2001).)

[52] In their economy, credit constraints are the natural debt limits that are determined by elastic labor supply, human capital, and asset holdings.

In this section, I study how the borrowing limit affects college education decisions and wealth inequality.[53] To do so, I solve the counterfactual economy where agents can borrow up to their idiosyncratic tuition costs at the initial age.[54] Table 12 shows the resulting distribution of wealth in this economy. We can see that the relaxed borrowing limit encourages more agents to pursue a college degree, leading to a total of 56 percent of skilled households. A larger fraction of wealthy skilled households is likely to increase wealth inequality. However, Table 12 shows that, despite a higher fraction of skilled households, the distribution of wealth is similar to that in the benchmark economy. This is because marginal college-graduates who pursue a college degree with the relaxed borrowing limit are less productive and in a larger debt than skilled households in the benchmark economy. Thus, they are less likely to accumulate a substantial amount of wealth, offsetting the effects of a high fraction of skilled households.

Table 12: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
relaxed borrowing constraint	14.7	39.5	55.1	99.9	100	12.3	0.73	0.56

Note: Table 12 shows the share of wealth held by the top 1, 5, 10, 50 and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

8 Implications of rising wage inequality on wealth inequality: 1989 vs. 2004

Given that wage inequality has been increasing in the U.S., the natural question that arises is how the rising wage inequality in the U.S. has affected the aggregate wealth inequality. To shed light on this, I compare the distribution of wealth in the two economies – the one with the 1989 wage process and the other with the 2004 wage process (benchmark

[53] Though explicitly modeling a student loan is important for college education decision, given that the focus of my work is to explore the effects of post-education wage differentials that vary by skills on the wealth inequality, I model the credit constraint as an *ad-hoc* fixed loan limit.

[54] These borrowing can not be used to finance consumption.

economy).[55] Given that this is a static comparison where agents experience the same wage process over their entire working lives, it provides the upper bound for the effect of the increased wage inequality for wealth inequality.

Table 13 compares the estimates of the college wage premia and skill-specific shock processes for 1989 to those for 2004.[56] First, it shows that the college wage premium in 1989 is half of that in 2004. Second, the variance of the persistent shock for the skilled has significantly increased from 1989 to 2004, while the volatilities of other shocks stay similar between two periods.

Table 13: Minimum distance estimates in 1989 and 2004

year	$\frac{w^h}{w^l}$	skilled		unskilled	
		persistent	transitory	persistent	transitory
1989	1.17	0.026	0.079	0.012	0.083
2004	1.34	0.061	0.068	0.012	0.077

Note: Estimates for the college wage premium and variances of persistent and transitory shocks for each skill group for 1989 and 2004.

Table 14 compare the resulting distribution of wealth in the two economies. The first two rows in Table 14 show that the increased wage inequality, including both between-group and within-group wage dispersion, raises the wealth Gini from 0.67 in 1989 to 0.73 in 2004. Moreover, the share of wealth held by the top 10 percent of households increases by more than 10 percentage points. Lastly, college-educated households increase by 4 percentage points. This shows that the increased wage inequality between 1989 and 2004 leads to the more dispersed distribution of wealth.

To further study the relative importance of the change in between- and within-group wage inequality for the wealth inequality in 2004, I also show the distributions of wealth when the college wage premium decreases to the 1989 level from the 2004 benchmark economy and when the shock processes change to the 1989 estimates.

The third row of Table 14 shows that, with half of the college wage premium of 2004, the

[55] I choose the year 1989 as this is the most recent year with a significantly lower college wage premium. As seen in Figure 1, after 1989, college wage premia stay around 1.35, similar to the value for 2004.

[56] Note that skill-varying labor market experience premia and initial age shock process are time-invariant.

Table 14: Distribution of wealth

	1%	5%	10%	50%	90%	≤ 0	Gini	share
2004 benchmark	16.0	38.7	53.0	99.3	100	10.9	0.73	0.27
1989 wage processes	8.8	27.7	42.9	99.4	100	9.3	0.67	0.24
1989 college wage premium	14.7	36.0	50.3	99.5	100	10.0	0.71	0.23
1989 shock process	9.5	28.9	44.5	99.4	100	10.0	0.68	0.27

Note: Table 14 shows the share of wealth held by the top 1, 5, 10, 50 and 90 percent of the wealthiest households, the wealth Gini coefficient, the share of households with zero or negative asset holdings, and the share of households with a college degree.

fraction of skilled households decreases to 23 percent. Given that only 4 percentage points additional households receive a lower college wage premium, the resulting distribution of wealth does not change much from that in the 2004 benchmark economy.

The last row of Table 14 shows the distribution of wealth when the economy faces 1989 wage shock processes, which are overall less volatile than those in the 2004 benchmark economy. This reduces the wealth Gini from 0.73 to 0.68 and the wealth held by the top 10 percent of households from 53 to 44 percent. Ultimately, these results suggest that the increased college wage premium from 1989 to 2004 encourages more households to become skilled, but the concentration of wealth for the top is largely driven by the more volatile persistent wage shock for the skilled.

9 Conclusions

In this paper, I study the effects of heterogeneity in education on wealth inequality and life-cycle savings. To do so, I first estimate skill-specific wage processes that allow both between-group and within-group wage dispersion, using the PSID data. Then, I quantitatively evaluate the implications of these estimated wage processes in an incomplete-markets overlapping-generations model with a discrete college education choice, labor supply decisions, and borrowing limits.

I find that understanding the sources of wage differentials across households with different education levels is critical to explain both aggregate wealth inequality and their life-cycle savings. For example, when allowing the wage processes to vary by education, the benchmark economy explains the wealth Gini of 0.73 as well as the observed life-cycle wealth profiles of skilled and unskilled households. In contrast, when I assume that all households face the same wage process, the model economy only explains the wealth Gini of 0.64 and fails to account for a large difference in the life-cycle wealth across the two education groups. In these results, I find that the deterministic between-group wage differential plays an important role in college education decisions. Moreover, the more volatile persistent wage shock for the skilled is crucial to understand the distribution of wealth at the top as well as the difference in wealth accumulation between skilled and unskilled households.

Broadly speaking, this paper emphasizes the importance of understanding heterogeneity in wages across households and challenges the standard assumption of a common wage process in incomplete-market macro models. The presence of heterogeneous wage processes across households implies distinct saving behavior attributable to different earnings processes that may vary with demographic characteristics, occupation, and education. Though my focus has been on the role of education-specific wage processes in explaining the distribution of wealth, the results more generally suggest the importance of different earnings processes for different groups of households to understand households saving behavior and wealth inequality.

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A Regression analysis

Table A1: Determinants of log of net worth (SCF data)

Year	1983	2004
Log of income	1.20^{***}	1.15^{***}
	(0.02)	(0.02)
Age	0.11^{***}	0.09^{***}
	(0.01)	(0.01)
Age squared	-0.62^{***}	-0.49^{***}
	(0.10)	(0.10)
College	0.24^{***}	0.53^{***}
	(0.06)	(0.05)
Race: white	0.36^{*}	0.21^{*}
	(0.21)	(0.12)
Race: black	-0.30	-0.63^{***}
	(0.23)	(0.14)
Race: hispanic	-0.47^{*}	-0.29^{*}
	(0.26)	(0.15)
Work status : working for other	-0.10	-0.30^{**}
	(0.13)	(0.14)
Work status : self-employed	0.91^{***}	0.49^{***}
	(0.15)	(0.14)
Work status : retired	*na*	0.07
	(0.36)	(0.35)
constant	-5.58^{***}	-4.82^{***}
	(0.36)	(0.35)
observations	3,540	3,992
R-square	0.65	0.78

College education dummy has a value of one if a household head has a college degree. Race dummies control for white, black, Hispanic and the other race. Work status dummies include working for some else, retired, and not working. Note: Standard errors in parentheses. * Significant at 10%; ** significant at 5%; *** significant at 1%. Work status-retired is not available in 1983. (Source: 1983 and 2004 SCF data)

B Numerical method

Stationary equilibria are standard finite-horizon dynamic programming problems. The state space is five-dimensional: age, education, wealth, persistent and transitory shocks. I determine decision rules for each skill group backward by age, given a static problem for the last age J. Each iteration of the golden section search algorithm involves bisection to solve for labor-leisure choice for workers. I allow age- and skill-varying grids for the beginning period of assets starting with the borrowing limit for the previous age. I log-spaced these asset grids to have finer grids at a low level of wealth where the value functions have more curvature.

Solving for stationary equilibria involves three prices (w^l, w^h, r). Following Heathcote et al. (2010b), I use a ratio of the marginal products of skilled and unskilled labor to pin down λ, which implies the observed college wage premium of 1.34:

$$1.34 = \frac{w^h}{w^l} = \frac{\lambda}{(1-\lambda)}\left(\frac{H^h}{H^l}\right)^{-\frac{1}{\theta}} \tag{8}$$

This implies that I can solve the stationary equilibria only with two prices (w^l, r) subject to $w^h = 1.34w^l$. Given the initial guess of prices, I compute decision rules and distribution. The distribution of households is determined using a large grid; weights are used to place decision rules onto this grid. I updated prices using Brodyen's method which begins with the identity matrix as the initial guess of the Jacobian. The Jacobian is updated using successive evaluations of the objective function and its gradient. I iterate the above steps until prices converge.

C Additional figures

Figure C1: Distribution of wealth for skilled and unskilled households

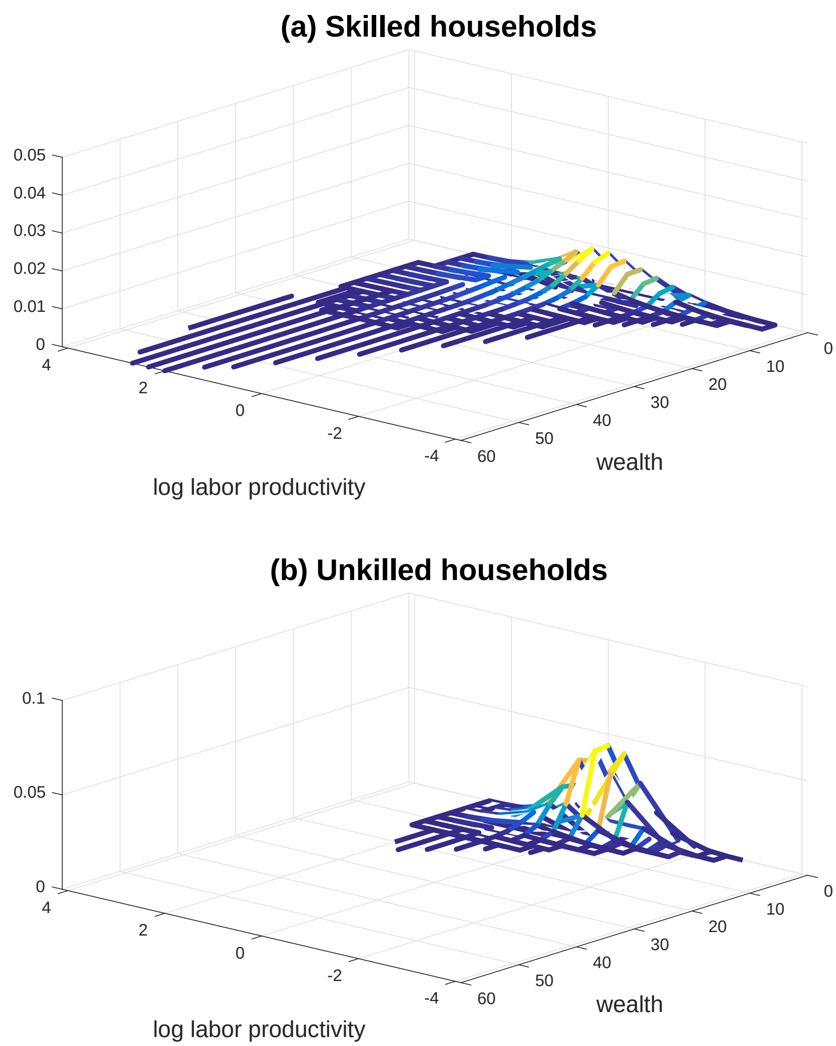

Figure C2: Initial distribution of households