Liquidity Crises and The Lender of Last Resort in a Monetary Economy

Tarishi Matsuoka & Makoto Watanabe

Tokyo Metropolitan Univ. & VU Amsterdam, Tinbergen Institute

June 25, 2020 @ U of Tokyo

Motivation

- *"While there is considerable agreement on the need for a domestic lender of last resort, some disagreements persist about what the lender of last resort should do" (Fisher, 1999)*
- Classical doctrine (Bagehot, 1873)
    - lend liquidity freely to illiquid but solvent financial institutions at a penalty rate
- Moral hazard problem
    - LLR may produce an incentive for banks to behave risky.
    - *"since the Franklin National in 1974, the Fed has bailed out insolvent institutions which were deemed 'too big to fail'. This has led to moral hazard."(Bordo, 2014)*

- Conventional views
  - *"the penalty rate is a way of reducing moral hazard."(Solow, 1982)*
  - *"the lender of last resort should seek to limit moral hazard by imposing costs on those who have made mistakes. Lending at a penalty rate is one way to impose such costs." (Fischer, 1999)*

Motivation

- Questions
  - How does the existence of the LLR affect bank's portfolio choice?
  - Does the LLR increase a probability of a crisis?
  - Does the LLR induce financial institutions to take more risk?
  - Does the penalty rate prevent moral hazard?

Objectives

- What We Do
  - To construct a monetary model where money and banking are essential.
  - To examine effects of the LLR on banks' portfolio and baking crises.
- How We Do
  - We extend Williamson (2012, AER) by introducing
    - aggregate uncertainty of money demand
    - risky asset
    - LLR

Key Ingredients

- Banking and liquidity (Williamson, 2012, 2016)
  - Non-monitored exchanges; money only
  - Monitored exchanges; money plus credit
- Aggregate uncertainty about the total demand for money, may impede the smooth functioning of banks' liquidity provision.

Key Ingredients

- Banking Crisis
  - bank reserve shortage and suspensions of convertibility
  - Champ, Smith, & Williamson (1996)

Related Literature

- Non-Monetary Banking Model with LLR
  - Flannery (1996), Freixas & Jorge (2007), Freixas, Parigi & Rochet (2000), Allen & Gale(1998, 2004), Allen, Carletti & Gale (2007), Heider, Hoerova, & Holthausen (2015), Acharya, Gromb & Yorulmazer, (2010)
- OLG model with Banking and LLR
  - Champ, Smith & Williamson (1996), Smith (2002), Antinolfi, Huybens & Keister (2001), Antinolfi & Keister (2006), Matsuoka (2012)
- Monetary Search with Banking
  - Berentsen, Camera, & Waller (2007), Ferraris & Watanabe (2008, 2011), Bencivenga & Camera (2011), Williamson (2012, 2016), Gu, Mattesini, & Wright (2013), Matsuoka & Watanabe (2017), Andolfatto, Berentsen & Martin (2017)

Outline

1. The Environment
2. Equilibrium without LLR (baseline)
3. Introducing LLR
4. Conclusions

The Environment

The Model

- Lagos & Wright (2005), Williamson (2012)
- Time: discrete, infinite; two sub-periods (*day* and *night*)
- Agents: buyers, sellers; homogeneous, unit mass, infinitely lived
- Goods: *special* goods, *general* goods (numéraire)

Preferences

- Discount factor $\beta \in (0,1)$
- Period utility:
  - buyer
$$u(q^b) + U(x) - h \tag{1}$$
  - seller
$$-q^s + U(x) - h \tag{2}$$

where:

- q^b: quantity of special good consumed
- q^s: quantity of special good produced
- x: quantity of general good consumed
- h: quantity of general good produced (if > 0)

Preferences

- Assumptions
  - $u'(q) > 0 > u''(q)$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$
  - $\xi \equiv -\frac{qu''(q)}{u'(q)} > 0$ and $u'(q^*) = 1$
  - $U'(x) > 0 > U''(x)$, and $U'(x^*) = 1$

Assets

- Fiat money:
  - ϕ: price of money in terms of general goods.
  - it grows (or shrinks) at a constant rate, $M_+ = \pi M, \ \pi > \beta$
- Safe asset:
  - one unit of general good into $R > 1$ units for sure next period.
  - $\beta R < 1$
- Risky asset:
  - one unit of general good into λR units with probability η and zero with probability $1 - \eta$ next period,
  - $\lambda > 1$ and $\lambda\eta \leq 1$.
  - $(\lambda - 1)R$: unobservable ("private benefit"); R: observable

Night market (Walrasian market)

- Agents can consume, produce and trade general goods.
- Any credit contracts are settled.
- Fiat money is traded at market price ϕ.
- Buyers (or banks) choose a portfolio.

Day Market (Decentralized Market)

- Search market (pairwise trade and bargaining)
- Buyers wish to consume special goods produced by sellers.
- Take-it-or-leave-it offer
- Non-monitored exchange (α)
  - No "memory"
  - money is essential as a medium of exchange
- Monitored exchange ($1 - \alpha$)
  - Record-keeping & commitment
  - Money and Credit

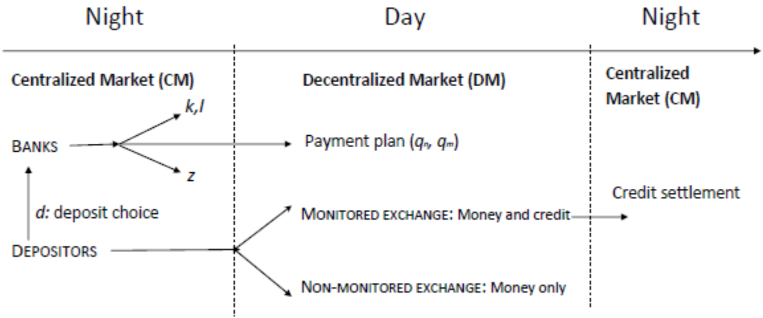
Day Market (Decentralized Market)

- $\alpha \in (0, 1)$ is a random variable
  - $F(\alpha)$: distribution function; $f(\alpha)$: density function
  - aggregate uncertainty about money demand ("liquidity shock")

Diamond-Dybvig Bank

- Buyers form a competitive bank
  - banks live for one period
  - zero profit
- Buyers deposit $d > 0$
- Given d, the bank chooses:
  - a portfolio (z, k, l)
    - $z = \phi m$: amount of real cash balances
    - k: amount of the safe asset
    - l: amount of the risky asset
  - a consumption plan (q^n, q^m)
    - q^n: consumption of a non-monitored buyer
    - q^m: consumption of a monitored buyer

Timing


Night
Day
Night
Centralized Market (CM)
Decentralized Market (DM)
Centralized
Market (CM)
k,l
BANKS
Payment plan (q_n, q_m)
z
Credit settlement
d: deposit choice
MONITORED EXCHANGE: Money and credit
DEPOSITORS
NON-MONITORED EXCHANGE: Money only

Equilibrium without the Lender of Last Resort

Banks Payment Plan (given d; z; k; l; α)

- In a monitored trade, $q^m = q^*(= u^{-1\prime}(1))$: efficient quantity
- The bank's problem

$$\max_{\theta \in [0,1]} \alpha u(q^n) + (1-\theta)\frac{z}{\pi} \tag{3}$$

subject to

$$\alpha q^n = \frac{\theta z}{\pi} \tag{4}$$

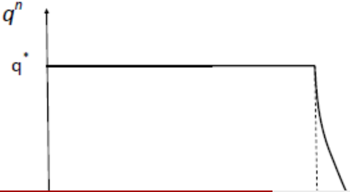
  - θ: proportion of real cash reserves to non-monitored buyers

- First order condition

$$\frac{z}{\pi}\left\{u'(q^n) - 1\right\} \geq 0 \quad (= \text{if } \theta < 1). \tag{5}$$

- Two situations are possible
  - $\theta < 1$: $q^n = q^*$ consumption smoothing.
  - $\theta = 1$: $q^n < q^*$ a banking crisis.

Consumption in a Non-Monitored trade



Lemma (Banks' Optimal Payment Plan)

Given cash reserves $z > 0$, the optimal payment plan of banks is described by $q^m = q^*$ and

$$q^n(\alpha) = \begin{cases} q^* & \text{if } 0 < \alpha < \alpha^*, \\ \frac{z}{\alpha\pi} & \text{if } \alpha^* \leq \alpha < 1, \end{cases}$$

where $\alpha^* \equiv \frac{z}{\pi q^*} > 0$, and

$$\theta(\alpha) = \begin{cases} \frac{\alpha}{\alpha^*} & \text{if } 0 < \alpha < \alpha^*, \\ 1 & \text{if } \alpha^* \leq \alpha < 1. \end{cases}$$

Bank's Portfolio Choice

- Given the total deposit d and the repayment plan $q^n = q^n(z, \alpha)$ and $\theta = \theta(z, \alpha)$, the banks' portfolio choice problem in the CM is

$$V(d) = \max_{z,k,l \geq 0} \int_0^1 \Bigg[\alpha \left\{ u(q^n) + W(0, k, l, 0) \right\} + (1-\alpha) \left\{ u(q^*) + W\left(\frac{(1-\theta)z}{(1-\alpha)\pi}, k, l, q^* \right) \right\} \Bigg] f(\alpha) d\alpha,$$

subject to

$$d = z + k + l.$$

Bank's Portfolio Choice

- The risky asset is not selected, i.e., $l = 0$, because $\eta\lambda R < R$.
- FOC

$$\frac{1}{\pi}\Upsilon(z) - R \geq 0 \quad (= \text{ if } z < d) \tag{6}$$

where

$$\Upsilon(z) \equiv F(\alpha^*) + \int_{\alpha^*}^{1} u'(q^n) f(\alpha) d\alpha$$
$$\alpha^* = \frac{z}{\pi q^*}$$

Lemma (Banks' Optimal Portfolio Choice)

Given deposit $d > 0$, the optimal portfolio of banks is described by $k = d - z \geq 0$, $l = 0$, and

$$z = \begin{cases} z(d) & \text{if } \Upsilon(d) < \pi R, \\ d & \text{if } \Upsilon(d) \geq \pi R, \end{cases}$$

where $z(d) \in (0, d)$ is a solution to $\frac{1}{\pi}\Upsilon(z) = R$.

Deposit Choice

- Deposit choice

$$\max_{d \geq 0} \{-d + \beta V(d)\},$$

- The Euler equation

$$\frac{\pi}{\beta} = \Upsilon(d), \tag{7}$$

or

$$\frac{\pi}{\beta} = 1 + \underbrace{\int_{\alpha^*}^{1} \left\{ u'\left(\frac{d}{\pi\alpha}\right) - 1 \right\} f(\alpha) d\alpha}_{\text{liquidity premium}}$$

Theorem (Monetary Equilibrium without LLR)

A monetary equilibrium exists without LLR, and is unique, in which

- $d_N = z_N \in (0, \pi q^*)$
- *a banking crisis occurs with probability* $1 - F(\alpha^*) \in (0, 1)$
- *the probability of a crisis is strictly increasing in inflation*
- *the level of deposits,* d_N*, is decreasing in inflation*

The Friedman rule can eliminate a crisis and achieve the first best,

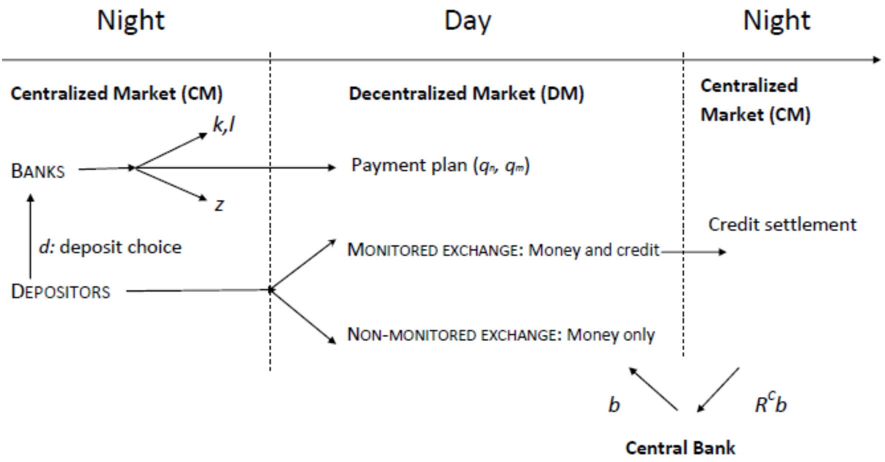
- $1 - F(\alpha^*) \to 0$ *and* $q^n \to q^*$ *as* $\pi \to \beta$

The Lender of Last Resort

The Lender of Last Resort

- During a day, the central bank opens a discount window
- The discount window loan is
  - an intraday cash loan with a gross interest rate R^C $(> \max\{\pi R, 1\})$
  - used for non-monitored buyers and repaid in the following CM
  - fully collateralized

Timing with LLR



The Lender of Last Resort

- **Note**: the safe and risky assets are substitute.
- Consider the two extreme cases;
  - one case with $k \geq 0 = l$; the other with $l \geq 0 = k$.
- Depositors compare the expected utilities of these two cases and choose the higher one in equilibrium.
- Assumption
$$\frac{1}{\pi} < \min\{\eta \underline{R}^C, R\},$$
where $\underline{R}^C \equiv R/\{1 - \beta\eta(\lambda - 1)R\} > R$.

Safe Asset

Banks payment plan with Safe asset (given α, $z > 0$, $k > 0 = l$)

- Again, $q^m = q^* (= u^{-1\prime}(1))$: efficient quantity
- The bank's problem

$$\max_{\theta \in [0,1],\ b \geq 0} \alpha u(q^n) + (1-\theta)\frac{z}{\pi} - R^C b,$$

subject to

$$\begin{aligned} \alpha q^n &= \frac{\theta z}{\pi} + b \\ R^C b &\leq Rk \end{aligned}$$

- θ: proportion of bank monetary reserve to non-monitored buyers
- b: real cash balances borrowing from the central bank

Lemma (Banks' Optimal Payment Plan with LLR and Safe Asset)

Given $z > 0,\ k \geq 0 = l$, the optimal payment plan of banks with safe asset in the presence of LLR is described by $q^m = q^*$ and $\theta = \theta(\alpha)$ just the same as in Lemma 1, and

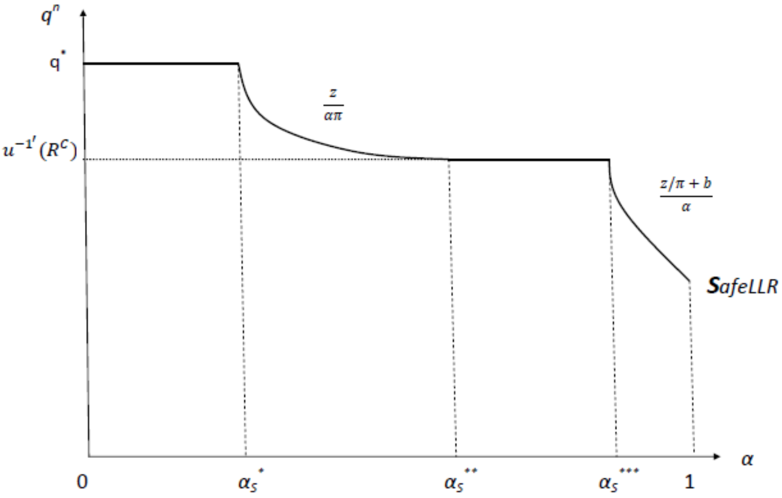
$$b(\alpha) = \begin{cases} 0 & \text{if } 0 < \alpha \leq \alpha^{**}, \\ \alpha u^{-1\prime}(R^C) - \frac{z}{\pi} & \text{if } \alpha^{**} < \alpha < \alpha^{***}, \\ \frac{Rk}{R^C} & \text{if } \alpha^{***} \leq \alpha < 1, \end{cases}$$

$$q^n(\alpha) = \begin{cases} q^* & \text{if } 0 < \alpha < \alpha^*, \\ \frac{z}{\alpha\pi} & \text{if } \alpha^* \leq \alpha \leq \alpha^{**}, \\ u^{-1\prime}(R^C) & \text{if } \alpha^{**} < \alpha < \alpha^{***}, \\ \frac{\frac{R^C}{\pi}z + Rk}{R^C\alpha} & \text{if } \alpha^{***} \leq \alpha < 1, \end{cases}$$

where

$$\alpha^* \equiv \frac{z}{\pi q^*}, \quad \alpha^{**} \equiv \min\left\{\frac{z}{\pi u^{-1\prime}(R^C)}, 1\right\}, \quad \text{and} \quad \alpha^{***} \equiv \min\left\{\frac{\frac{R^C}{\pi}z + Rk}{R^C u^{-1\prime}(R^C)}, 1\right\}.$$

Consumption in a Non-Monitored trade



- There are two cases.

1. For $z = z(d) \in (0, d)$ (an interior solution), the Euler equation is

$$\frac{1-\beta R}{\beta R} = \underbrace{\int_{\alpha^{***}}^{1} \left\{ \frac{u'(q^n)}{R^C} - 1 \right\} f(\alpha) d\alpha}_{\text{liquidity premium}} \tag{8}$$

2. For $z = d$ (the corner solution), the bank does not hold any long-term assets, it cannot borrow from the LLR, $b = 0$.

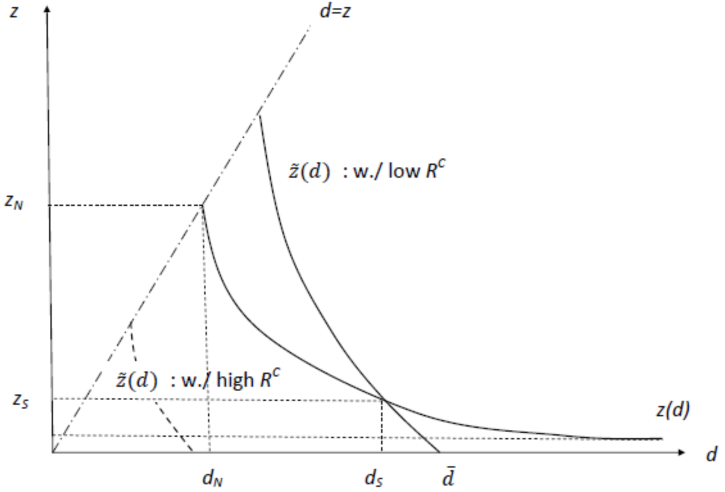
Proposition (Monetary Equilibrium with LLR and Safe Asset)

With the LLR and safe asset, a monetary equilibrium with bank deposit exists and is unique in which the cash reserves, denoted by z_S, and the bank deposit, denoted by d_S, satisfy

$$z_S = \begin{cases} z(d_S) < d_S & \text{for } R^C \in (\pi R, R^{C*}], \\ d_N & \text{for } R^C \in (R^{C*}, \infty), \end{cases}$$

with some critical value $R^{C*} \in (\pi R, \infty)$, and $z_S \leq z_N$ and $d_S \geq d_N$. Further, whenever $R^C < R^{C*}$, it holds that $\alpha^{***} < 1$.

Monetary Equilibrium with LLR and Safe Asset



Proposition (Effects of Inflation and Loan Rate)

Suppose that $\xi \equiv -\frac{qu''(q)}{u'(q)} > 0$ *is not too big. Then, the interior solutions with the safe asset* (z_S, d_S) *satisfies*

$$\frac{\partial z_S}{\partial \pi} < 0, \quad \frac{\partial d_S}{\partial \pi} > 0, \quad \frac{\partial z_S}{\partial R^C} > 0, \quad and \quad \frac{\partial d_S}{\partial R^C} < 0.$$

Furthermore,

$$\begin{aligned} \frac{\partial \alpha^*}{\partial \pi} < 0, \quad \frac{\partial \alpha^{**}}{\partial \pi} < 0, \quad \frac{\partial \alpha^{***}}{\partial \pi} = 0, \\ \frac{\partial \alpha^*}{\partial R^C} > 0, \quad \frac{\partial \alpha^{**}}{\partial R^C} > 0, \quad and \quad \frac{\partial \alpha^{***}}{\partial R^C} = 0. \end{aligned}$$

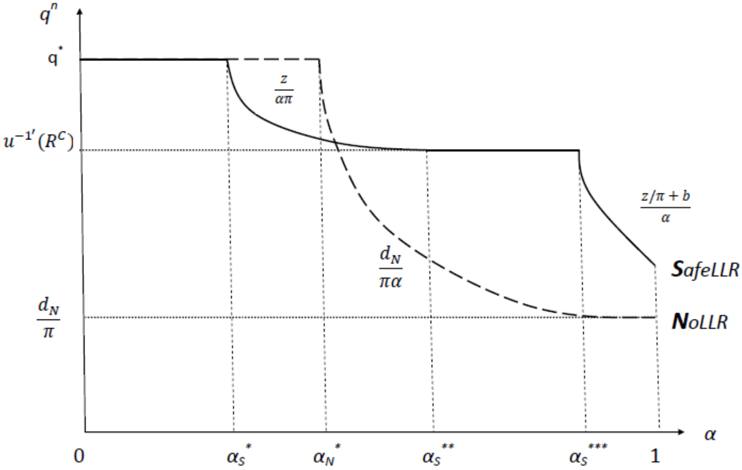
Implications

- $\alpha^{***} < 1$
- d is higher
- z is lower

Corollary

The LLR is welfare improving, but increases the probability of a banking crisis.

Consumption in a Non-Monitored trade



Risky Asset

Banks payment plan with risky asset (given z, $l \geq 0 = k$)

- Essentially the same as before except that now with risky assets,
- private banks can honor their promise **only when the project becomes successful (limited liability)**, which happens with probability η.
- expected payment rate is ηR^C rather than R^C

$$\max_{\theta \in [0,1],\ b \geq 0} \alpha u(q^n) + (1-\theta)\frac{z}{\pi} - \eta R^C b,$$

subject to

$$\begin{aligned} \alpha q^n &= \frac{\theta z}{\pi} + b \\ R^C b &\leq Rl \end{aligned}$$

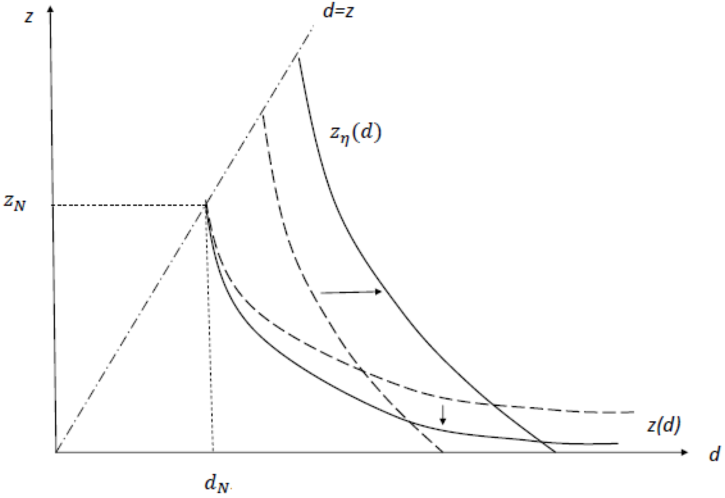
Proposition (Monetary Equilibrium with LLR and Risky Asset)

With the LLR and risky asset, a monetary equilibrium with bank deposit exists and is unique in which the cash reserve balances and the bank's deposit satisfy

$$z = \begin{cases} z_R \ (< d_R) & \textit{for} \ \ R^C \in (\underline{R}^C, \hat{R}^{C*}], \\ z_N \ (= d_N) & \textit{for} \ \ R^C \in (\hat{R}^{C*}, \infty), \end{cases}$$

with some critical value $\hat{R}^{C*} \in (\underline{R}^C, \infty)$*, and* $z_S < z_R$ *and* $d_R > d_N$ *for any* $\eta \in (0, 1)$*. Further, whenever* $R^C < \hat{R}^{C*}$*, it holds that* $\alpha_\eta^{***} < 1$*.*

Monetary Equilibrium with LLR and Risky Asset



Proposition (Effects of Inflation and Loan Rate)

Suppose that $\xi \equiv -\frac{qu''(q)}{u'(q)} > 0$ *is not too big. Then, the interior solutions with the risky asset satisfies*

$$\frac{\partial z_R}{\partial \pi} < 0, \quad \frac{\partial d_R}{\partial \pi} > 0, \quad \frac{\partial z_R}{\partial R^C} > 0, \quad and \quad \frac{\partial d_R}{\partial R^C} < 0.$$

Furthermore,

$$\frac{\partial \alpha^*}{\partial \pi} < 0, \quad \frac{\partial \alpha_\eta^{**}}{\partial \pi} < 0, \quad \frac{\partial \alpha_\eta^{***}}{\partial \pi} = 0,$$
$$\frac{\partial \alpha^*}{\partial R^C} > 0, \quad \frac{\partial \alpha_\eta^{**}}{\partial R^C} > 0, \quad and \quad \frac{\partial \alpha_\eta^{***}}{\partial R^C} = 0.$$

Asset Choice

Asset Choice

- Let us compare the two cases, with the safe and risky asset
- Deposit choice:

$$\max\{-d_S + \beta V^s(d_S),\ -d_R + \beta V^r(d_R)\}$$

- Define

$$\Delta(R^C, \eta) \equiv (1-\beta)[\{-d_S + \beta V^s(d_S)\} - \{-d_R + \beta V^r(d_R)\}].$$

$$\Delta > 0 \implies \text{the safe asset is selected}$$
$$\Delta < 0 \implies \text{the risky asset is selected}$$

Lemma

1. *If* $R^C > \max\{R^{C*}, \hat{R}^{C*}\}$*, then* $\Delta(R^C, \eta) = 0$.
2. *If* $R^C \in [R^{C*}, \hat{R}^{C*})$*, then* $\Delta(R^C, \eta) < 0$.
3. *If* $R^C \in [\hat{R}^{C*}, R^{C*})$*, then* $\Delta(R^C, \eta) > 0$.
4. $\lim_{\eta \to 1} \Delta(R^C, \eta) = 0$.
5. $\lim_{\eta \to 1} \frac{\partial \Delta}{\partial R^C}(R^C, \eta) = 0$.
6. *If* $\eta\lambda$ *and* βR *are very close to 1, then* $\lim_{R^C \to \underline{R}^C} \Delta(R^C, \eta) < 0$.

Proposition (Moral Hazard)

In a monetary equilibrium with LLR, the discount window is activated if and only if the lending rate is low, $R^C < \max\{R^{C*}, \hat{R}^{C*}\}$. *Whenever the LLR lending is used, private banks will invest in a risky asset, rather than a safe asset, if the expected return of the risky asset is sufficiently high and the cost of holding the collateral is sufficiently small.*

- Compared to safe asset, risky asset leads to
  - higher d
  - lower z
- $\Rightarrow$ banking panics and banking defaults are closely intertwined!

Conclusion

- The LLR reduces bank's cash reserves and increases the likelihood of a banking crisis. However, the magnitude of a crisis is mitigated.
- The LLR may create moral hazard:
  - private banks may take more financial risks ex ante.
- The occurrence of moral hazard is determined mainly by
  - the expected return on the risky asset
  - asymmetric information about the quality of bank's assets

$\Rightarrow$

  - a penalty rate may not have enough powers against moral hazard.
  - a lower real interest rate on discount window loans can be preferred.

Extensions

- Liquidity Requirement

$$z \geq \kappa d, \quad \kappa \in [0, 1].$$

- Constructive Ambiguity
  - *"the task of curbing moral hazard appears to have been performed largely by constructive ambiguity,"(Giannini, 1999, p.14)*
  - *"Constructive ambiguity supposedly constrains excessive risk taking by banks." (Schwartz, 2002, p.452)*
  - Discount window lending is available with probability $\rho \in [0, 1]$.