Disclosure and Efficiency in Takeover Markets*

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Abstract

We study welfare implications of disclosure schemes in takeover markets. Firms heterogeneous in their non-tradeable "skills" trade their "projects" of different qualities amongst themselves. Given the complementarity between the two factors, the first best reallocation features positive assortative matching between skills and projects among a subset of firms with unbalanced factor endowments. We analyze two types of disclosure schemes: (i) a full disclosure provided by a non-competitive intermediary and (ii) a free but partial disclosure. We first show that the welfare loss in the first scheme is 28.5% of the first best welfare gains, while that in the second scheme is 70.6%, which can be lowered to 17.4% by setting a minimum quality standard for projects. We also study a hybrid market structure where the two schemes coexist, and show that making partial disclosure available limits the distortion of the non-competitive intermediary, lowering the overall welfare loss to 6.7%. The analysis suggests that a partial disclosure scheme, although inefficient on its own, might be a useful regulatory tool to indirectly control non-competitive behaviors of intermediaries in takeover markets.

Keywords: Asymmetric information, Disclosure, Market segmentation, Takeovers.

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1 Introduction

Recent studies show that a significant amount of productive resources are reallocated in takeover markets (Eckbo 2014, David 2017, Levine 2017, Xu 2017). How should we design takeover markets to achieve the maximum efficiency gains from reallocation? What is the magnitude of efficiency losses in suboptimally designed takeover markets? We present a simple model of takeover markets to answer these questions.

We formulate our results in a trading model subject to information asymmetries. Each firm can manage one indivisible project. Firms are heterogenous in the quality of projects they initially hold as well as in their skills to manage projects. Both project qualities and skills are firms' private information. A firm which purchases a new project must abandon the old one, while a firm which sells a project must leave the market with the proceed it receives. Firms cannot be a buyer and a seller simultaneously. Given the complementarity between the project quality and the skill level, the first best reallocation features the transfer of projects among firms with unbalanced factors: high quality projects held by low skill firms should be transferred to high skill firms which initially had low quality projects.

We first show that the efficient reallocation is achieved by opening many markets indexed by the quality of projects for sale, if and only if sellers can credibly disclose the quality of their projects. We call these markets *"full" disclosure markets* to be compared with a less efficient *"partial" disclosure market* studied later.

We extend our model of takeover markets by incorporating two sources of inefficiency: a full disclosure facilitated by a non-competitive intermediary and a free but partial disclosure. We first investigate each source of inefficiency independently. As an opposite extreme to a costless full disclosure, we study a single monopoly intermediary who charges a fee to target firms for its disclosure service. While the sum of fees paid by target firms to the intermediary is a pure transfer, monopoly pricing distorts reallocation by affecting firms' participation decisions. We show that the associated welfare loss is 28.5% of the first best welfare gains.

Next, we study a situation where only a free but a partial disclosure is available. In this market, a choice to be targets reveals some information but projects of different qualities must be traded at the same price. We show that, in equilibrium where projects of *all* qualities are traded, the welfare loss is 70.6%. We then study a more effective partial disclosure scheme where a regulatory body sets a minimum standard for projects to be traded.[1] We show that with the optimally chosen minimum standard the welfare loss is reduced to 17.4%. Intuitively, the partial disclosure can be superior to fine disclosure markets if the benefit of

[1]For example, this can be implemented by introducing a large ex post penalties for firms selling projects of qualities below the minimum standard.

reducing the entry distortion imposed by the fee outweighs the cost of pooling.

Finally, we construct an equilibrium with a hybrid market structure. When both types of disclosure schemes coexist, the regulatory body must choose the minimum standard to prevent firms in the coarse market from going to fine markets. Because the presence of the partial disclosure makes demands for full disclosure more elastic, in equilibrium the intermediary sets a lower fee. This leads to not only larger volume but also the higher average quality of projects in the full disclosure markets, relative to those in the absence of the partial disclosure. As a result, the overall welfare loss drops to 6.7%. Our analysis indicates that the partial disclosure scheme, although suboptimal on its own, might be a useful regulatory tool to indirectly control non-competitive behaviors of intermediaries who provide disclosure services in takeover markets.

Contribution to the literature. The closest to this paper is Jovanovic and Braguinsky (2004). They show in a model similar to ours that target premia and bidder discounts are consistent with the efficient reallocation. However, in their model with only two types of projects, a single price can perfectly reveal information about projects. We incorporate inefficiency due to two sources and analyze how to improve the efficiency of takeover markets. Our richer environment is more suitable for studying the issue of market design. Levine (2017) and Wang (2018) embed a similar reallocation mechanism in a dynamic model to assess its quantitative magnitude, but they do not study the market design issue. Our simple static framework allows us to identify a key trade-off facing a regulatory authority to improve the efficiency of takeover markets.

This paper is organized as follows. Section 2 describes a model and characterizes the first best reallocation. Section 3 studies efficient takeover markets. Section 4 studies inefficient takeover markets. Section 5 concludes. All proofs are gathered in the Appendix.

2 Model

Firms utilize two factors of production to produce output Y. The first factor is tradeable but indivisible, while the second factor is non-tradeable. We call the first factor *a project*, and call the second factor *skill*. Projects are indivisible, and each firm can manage at most one project. Both factors vary in their quality.[2] We assume a simple form of complementarity

[2] A project represents any productive asset which will not entirely lose its use when its ownership changes. Examples include a large plant and a customer base. Skill represents any productive asset whose usefulness cannot be transferred beyond the firm boundary. Examples include organizational knowledge and well-aligned motivation of workers.

between the two factors. A firm with a project of quality A and the skill level X can produce

$$Y = AX.$$

We assume that A and X are independently uniformly distributed in $[0, 1]$, and that there is a continuum measure one of firms. Then, the aggregate production without reallocation is

$$\int_0^1 \int_0^1 (AX)\, dAdX = \frac{1}{4}.$$

Planner's solution. Given the complementarity between the two factors, the first best allocation features positive assortative matching between skills and projects: a firm with skill level $X = z$ should have a project of quality $A = z$ to produce $Y = AX = z^2$. The maximum aggregate production then is $\int_0^1 z^2 dz = \frac{1}{3}$. Using the aggregate output as our welfare measure, the maximum welfare gains are $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$, which is 33.3% of the aggregate production without reallocation.

However, to achieve the maximum aggregate production, all firms (except measure zero firms endowed with $A = X$) must "buy and sell" their projects. In the later analysis of takeover markets we will assume that each firm can either buy or sell. Accordingly, we modify our efficiency benchmark as follows. Transferring a project from a firm with (A_1, X_1) to another firm with (A_2, X_2) yields net gains $A_1X_2 - (A_1X_1 + A_2X_2)$. Taking this as a constraint, the planner will implement the reallocation. Consider transferring projects of quality $A = z$ to firms with skill level $X = z$. Because "target" firms' original production would be lost, it is better to collect projects of quality z from firms with lower skills. Also, because "bidder" firms' original production would be lost as well, it is better to transfer the collected projects to firms with lower project quality. Suppose that the planner collects projects of quality z from firms with skill level in $[0, y]$ ("targets"), and transfer them to firms with skill level $X = z$ and initial project quality in $[0, y]$ ("bidders"). The new production achieved by this reallocation is yz^2. The lost production by the targets is $\int_0^y AzdA$, while the lost production by the bidders is $\int_0^y zXdX$. The planner chooses $y \in [0, 1]$ to maximize the net gain from the reallocation:

$$\max_{y\in[0,1]} \left\{ yz^2 - \left(\int_0^y AzdA + \int_0^y zXdX \right) \right\}.$$

The solution to this problem is $y = \frac{z}{2}$.[3] The planner conducts this reallocation for each $z \in [0, 1]$. According to this reallocation, firms whose (A, X) satisfy $A \geq 2X$ will give away

[3] This result generalizes to the case where A and X have an identical (but independent) distribution.

their projects, while firms whose (A,X) satisfy $A \leq \frac{X}{2}$ will obtain a new project of quality X and produce X^2. Other firms will keep their initial (A,X). The welfare gains achieved by this reallocation is

$$\underbrace{\int_0^1 \frac{z}{2} z^2 dz}_{\text{New production}} - \underbrace{\int_0^1 \left(A \int_0^{\frac{A}{2}} X dX \right) dA}_{\text{Lost production by "targets"}} - \underbrace{\int_0^1 \left(X \int_0^{\frac{X}{2}} A dA \right) dX}_{\text{Lost production by "bidders"}} = \frac{1}{16},$$

which is 25% of the aggregate production without reallocation. This is summarized in **Lemma 1**.

Lemma 1 (efficient reallocation) *The social planner will reallocate projects from firms whose initial (A,X) satisfies $\frac{A}{X} \geq 2$ to firms whose initial (A,X) satisfies $\frac{A}{X} \leq \frac{1}{2}$ such that the latter firms with skill level X produce X^2. This achieves welfare gains of $\frac{1}{16}$.*

This reallocation and associated welfare gains are our first best benchmark. The efficient reallocation is illustrated in **Figure 1**.

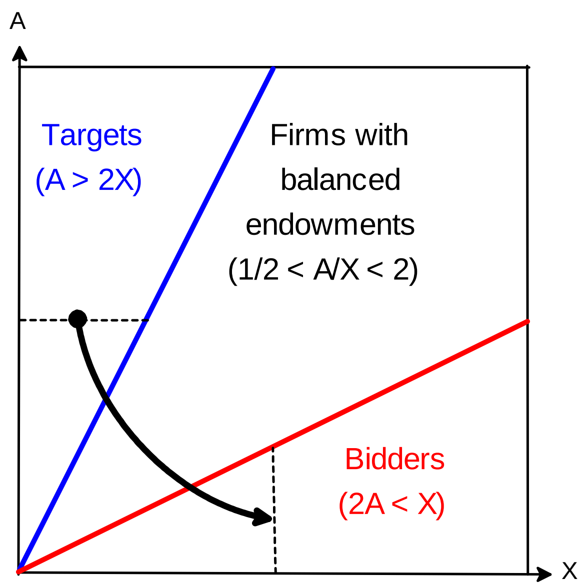

Figure 1. Efficient reallocation

The efficient reallocation has two features. First, there is positive assortative matching. Firms with $A \leq \frac{X}{2}$ receive new projects $A' = X$ and produce X^2. Second, some firms do not trade. Firms with relatively balanced endowments of (A,X) such that $\frac{1}{2} < \frac{A}{X} < 2$ should keep their initial projects, while firms with unbalanced initial allocations, those with $\frac{A}{X} \geq 2$ and the others with $\frac{A}{X} \leq \frac{1}{2}$, should trade projects among themselves.

In the next section, we study takeover markets that implement the efficient reallocation.

3 Efficient takeover markets

In this section we study competitive takeover markets, where each firm chooses either (i) to buy one project, in which case the firm abandons the initial project, or (ii) to sell one project, in which case the firm produces nothing and its payoff is the proceed from the sales of its project, or (iii) not to trade, in which case the firm produces with its initial project. We focus on a competitive pricing mechanism where firms take prices as given.

To achieve the efficient reallocation, projects must be transferred from firms with $A = z$ and $X \in \left[0, \frac{z}{2}\right]$ to firms with $X = z$ and $A \in \left[0, \frac{z}{2}\right]$, for each $z \in [0, 1]$. A natural way to achieve this is to open a continuum of markets indexed by $z \in [0, 1]$ such that firms with $A = z$ and firms with $X = z$ participate in market z. Taking this sorting pattern of firms across markets as given, we first show that market-clearing equilibrium exists in each market. Then we investigate firms' incentive with respect to their sorting across markets. In particular, we investigate the nature of disclosure of (A, X) that is necessary for the right sorting pattern.

3.1 Market-clearing

Consider a market indexed by $z \in [0, 1]$, where firms with $A = z$ and firms with $X = z$ are potential participants. Let P_z be a price of projects sold in market z. Then a firm with $(A, X) = (z, X)$ is willing to be a seller in market z only if

$$P_z \geq zX, \tag{1}$$

where P_z is the payoff as a seller and zX is its stand-alone value without trading. Similarly a firm with $(A, X) = (A, z)$ is willing to be a buyer in market z only if

$$z^2 - P_z \geq Az, \tag{2}$$

where $z^2 - P_z$ is the payoff as a buyer and Az is its stand-alone value without trading. Finally, firms that satisfy both (1) and (2) consider the following selection condition:

$$P_z > z^2 - P_z \Leftrightarrow P_z > \frac{z^2}{2} \Leftrightarrow \text{Target} \succ \text{Bidder}. \tag{3}$$

We compute the market-clearing price P_z using (1) and (2), and then verify (3) at the market-clearing price. Because targets have $A = z$ and satisfy $\frac{P_z}{z} \geq X$, a measure of targets in market z is

$$S_z(P_z) = \frac{P_z}{z}.$$

Buyers have $X = z$ and satisfy $z - \frac{P_z}{z} \geq A$, so a measure of bidders in market z is

$$B_z(P_z) = z - \frac{P_z}{z}.$$

A market-clearing condition $S_z(P_z) = B_z(P_z)$ defines a unique market-clearing price

$$P_z^* = \frac{z^2}{2}. \tag{4}$$

Substituting (4) into (1) and (2) shows that firms with $A = z$ and $X \leq \frac{z}{2}$ are willing to be targets while firms with $X = z$ and $A \leq \frac{z}{2}$ are willing to be bidders. Finally, at the market-clearing price (4), targets and bidders have the same payoff

$$P_z^* = z^2 - P_z^* = \frac{z^2}{2} \equiv \Pi_z, \tag{5}$$

i.e., targets and bidders split gains from trade equally. More importantly, taking the type of firms in market z as given, a competitive equilibrium supports the efficient reallocation. In the next subsection, we investigate incentive constraints with respect to sorting of firms across markets.

3.2 Sorting across markets

We start by checking if sellers in market z have an incentive to be sellers in other markets. Recall that targets in market in z have $A = z$ and $X \in \left[0, \frac{z}{2}\right]$, where (A, X) is their private information. Because targets' payoff $P_z^* = \frac{z^2}{2}$ increases in z, they can increase their payoff by pretending $A = z' > z$. However, if project quality can be credibly disclosed, a standard unravelling result ensues: all but the lowest type targets have an incentive to disclose their project quality. It turns out that this disclosure scheme on the targets' side is sufficient for all other incentive constraints to be satisfied.

Proposition 1 (efficient takeover markets) *Competitive prices $P_z^* = \frac{z^2}{2}$ for $z \in [0,1]$ implement the efficient reallocation if and only if sellers can credibly disclose their project quality.*

Proposition 1 shows that disclosure of heterogenous project qualities is necessary and sufficient for implementing the efficient reallocation. The importance of revealing the project quality is consistent with a typical due diligence process in takeover deals which mainly scrutinizes a target firm rather than a bidder firm.

4 Inefficient takeover markets

In the benchmark established in the previous section, disclosure was free and perfect. In practice, credible and detailed disclosure may need to rely on a third party with an expertise to evaluate project quality. Accordingly, we introduce two sources of inefficiency: (i) a full disclosure facilitated by a non-competitive intermediary and (ii) a free but coarse disclosure scheme. In the next two subsections, we first investigate each source of inefficiency independently. In the last subsection, we study how the two sources interact in equilibrium.

4.1 Non-competitive intermediary

In the previous section, we assumed that target firms can credibly disclose their project quality at no cost. As an opposite extreme to this free disclosure, we study a single monopoly intermediary who charges a fee to target firms for its disclosure service.[4] Suppose that the intermediary charges each target a fee $\phi \geq 0$ for their service of facilitating disclosure. Taking ϕ as given, the previous analysis of fine disclosure markets go through with a minor modification. In particular, targets' participation constraint in market z becomes $P_z - \phi \geq zX$. A measure of sellers in market z is

$$S_z\left(P_z;\phi\right) = \frac{P_z - \phi}{z}.$$

A measure of bidders in market z is same as before. A market-clearing condition $S_z\left(P_z;\phi\right) = B_z\left(P_z\right)$ defines a unique market-clearing price. In market z the market-clearing price and the volume of projects traded are

$$P_z\left(\phi\right) = \frac{z^2 + \phi}{2} \qquad \text{and} \qquad Q_z\left(\phi\right) = \frac{1}{2}\left(z - \frac{\phi}{z}\right). \tag{6}$$

Importantly, with a positive ϕ, markets for small z cannot open.

$$Q_z\left(\phi\right) > 0 \Leftrightarrow z > \sqrt{\phi}.$$

[4] For the analysis of efficiency, only the sum of fees charged on both sides matters. Therefore, assuming only one side pays a fee is without loss of generality.

In each active market $z > \sqrt{\phi}$, bidders and targets have the same profit

$$\Pi_z(\phi) = \frac{z^2 - \phi}{2}.$$

Next we turn to the monopoly intermediary's choice of ϕ. We assume that the intermediary cannot price-discriminate across markets. This implies that the aggregate demand the intermediary faces is

$$Q(\phi) \equiv \int_{\sqrt{\phi}}^{1} Q_z(\phi)\, dz = \frac{1}{4}(1 - \phi + \phi \ln \phi).$$

The intermediary chooses $\phi \in (0,1)$ to maximize $\phi Q(\phi)$. $Q'(\phi) < 0$ and $Q(1) = 0$ imply the existence of a unique solution $\phi_M \equiv \arg\max_{\phi} \phi Q(\phi)$. We calculate the welfare gains using ϕ_M in (6) for each active market $z > \sqrt{\phi_M}$. The reallocation pattern is illustrated in **Figure 2** below.

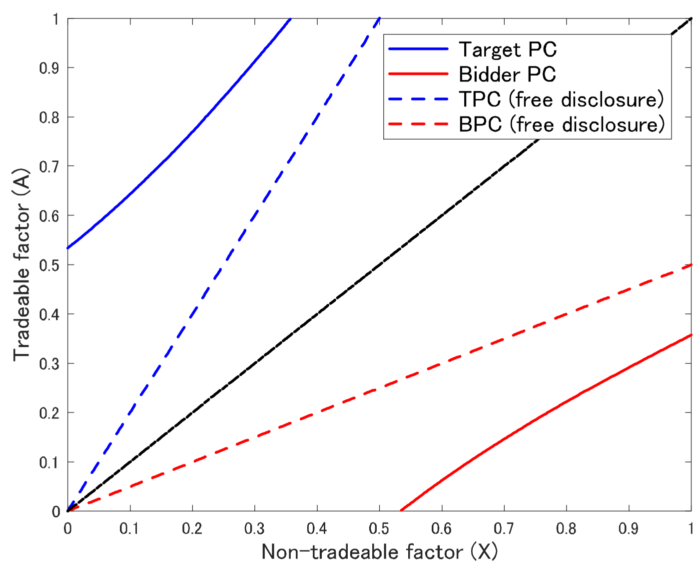

Figure 2. Reallocation with a monopoly intermediary

Solid lines represent firms' participation constraint with ϕ_M, while dashed lines represent participation constraint in the efficient benchmark (i.e., $\phi = 0$). While the sum of fees $\phi_M Q(\phi_M)$ paid by targets firms to the intermediary is a pure transfer, monopoly pricing distorts reallocation by affecting firms' participation decisions. First, small markets with $z \leq \sqrt{\phi_M}$ are inactive. Second, for each active market $z > \sqrt{\phi_M}$, smaller number of firms participate. Given this reallocation we compute the welfare gains and the associated welfare loss relative to the first best welfare gains of $\frac{1}{16}$.

Proposition 2 *The welfare gains with a monopoly intermediary is* $\frac{1}{16}(1-\phi_M)$, *where* ϕ_M *is a solution to* $\phi(1-2\ln\phi)=1$.

Proposition 2 shows that ϕ_M is a measure of welfare loss as a fraction of the first best welfare gains. Because $\phi_M = 0.285$, the welfare loss due to a monopoly intermediary is 28.5% of the first best welfare gains.

4.2 Free partial disclosure

In this subsection we study a partial disclosure market where projects of different qualities are traded at the same price. We assume that a regulatory body can ban trading of projects with quality below a specified minimum standard $A_{\min} \in [0,1]$. We start with a case without the minimum standard, i.e., $A_{\min} = 0$, and then explain how the equilibrium changes with a general $A_{\min}$. The full analysis of the general case is relegated to the Appendix.

Suppose that projects for sale are traded at a single price P. Because the expected quality of projects for sale is endogenous, we denote it by

$$a \equiv E\left[\widetilde{A}|\widetilde{A} \text{ is for sale}\right].$$

For a firm with (A,X), participation constraints as sellers and buyers are given by

$$AX \leq P, \tag{7}$$

$$AX \leq aX - P. \tag{8}$$

In **Figure 3** below, the targets' participation constraint (7) is illustrated by a blue line decreasing in X, while the bidders' participation constraint (8) is illustrated by a red dotted line increasing in X. The intersection of the two lines exhibits a threshold skill level $X^* \equiv \frac{2P}{a}$ at which firms with $A \leq A^* \equiv \frac{a}{2}$ are indifferent between being sellers and buyers. The indifferent firm with the highest project quality has $(X^*, A^*) \equiv \left(\frac{2P}{a}, \frac{a}{2}\right)$.

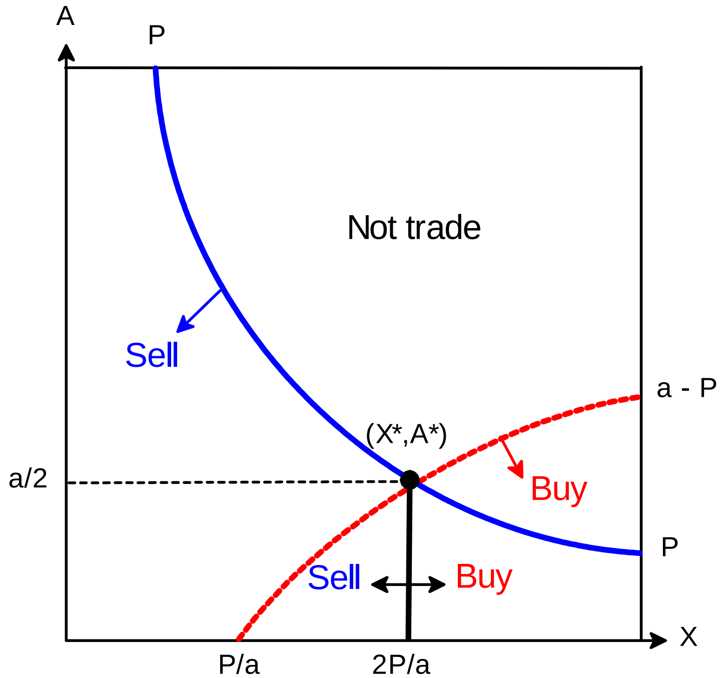

Figure 3. Sorting in a coarse market

Given this selection, a measure of sellers and a measure of buyers are given by

$$
\begin{aligned}
S(P) &= A^*X^* + \int_{A^*}^{1} \frac{P}{A} dA = P(1 - \ln A^*), \\
B(P) &= \int_{X^*}^{1} \left(a - \frac{P}{X}\right) dX = a - P\left(2 - \ln \frac{2P}{a}\right),
\end{aligned}
$$

and a market-clearing condition $S(P) = B(P)$ is equivalent to

$$
a = P(3 - \ln P). \tag{9}
$$

The market-clearing condition (9) defines a unique market-clearing price for a given $a \in (0, 1)$, which we denote by $P(a)$. Finally, given the conjectured a, the expected quality of projects for sale is

$$
\Gamma(a) \equiv \frac{\int_0^{A^*} AX^* dA + \int_{A^*}^{1} A \frac{P(a)}{A} dA}{S(P(a))} = \frac{1 - \frac{a}{4}}{1 - \ln \frac{a}{2}}.
$$

In the Appendix we show that $\Gamma(a) = a$ has a unique solution $a^* \in (0,1)$. This characterizes the expected quality of projects for sale in equilibrium. Using a^* and $P^* \equiv P(a^*)$, we can compute equilibrium objects such as volume and the expected welfare gains in the single market without any certification.

The above analysis can be generalized with a minimum standard $A_{\min} \in [0,1]$ for projects for sale. **Figure 4** shows the two possible cases. The left panel (a) illustrates the first case, $A_{\min} < A^*$. The right panel (b) illustrates the second case, $A_{\min} \geq A^*$.

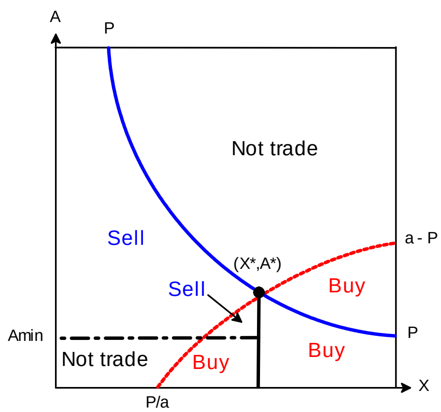

Figure 4 (a) $A_{\min} < A^*$

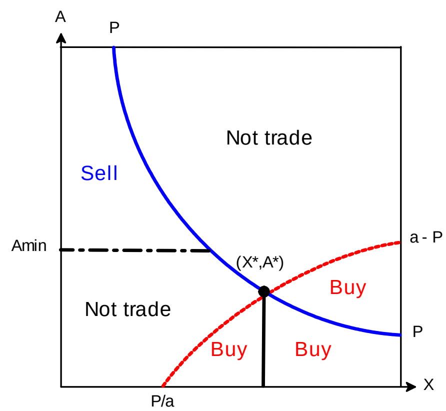

Figure 4 (b) $A_{\min} \geq A^*$

A qualitative difference between the two cases is that in the left panel targets and bidders are separated by the indifference condition $X = X^*$, while in the right panel the two groups are separated by non-trading firms. For each case, we follow the steps described above to characterize equilibrium values of (a, P). Because $A^* = \frac{a}{2}$ and P are both endogenous and depend on $A_{\min}$, which case in Figure 4 is relevant depends on the value of $A_{\min}$. The next result summarizes this analysis.

Proposition 3

(a) *A unique market-clearing equilibrium exists for all* $A_{\min} \in [0,1]$.
The expected welfare gains are $\left(\frac{a-P}{2}\right)^2$.

(b) *There is an optimal* $A^*_{\min} \in (A_0, 1)$ *that maximizes the expected welfare gains, where* $A_0 \in (0,1)$ *is a smaller solution to* $1 = A(1 - 2\ln A)$ *such that* $A_{\min} \lesseqgtr A^*$ *if and only if* $A_{\min} \lesseqgtr A_0$.

Three panels below show how the sorting pattern changes as $A_{\min}$ changes. In each panel, the minimum standard $A_{\min}$ is represented by a horizontal dashed line. From the left to the right panel, $A_{\min}$ increases.

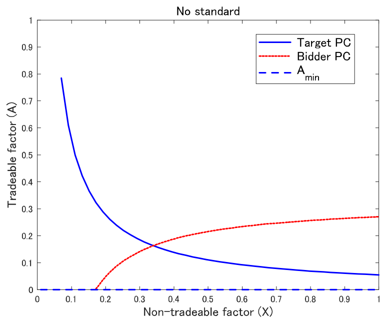

Figure 5 (a)

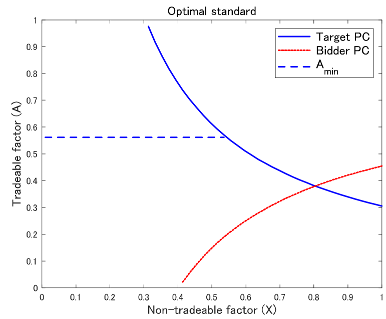

Figure 5 (b)

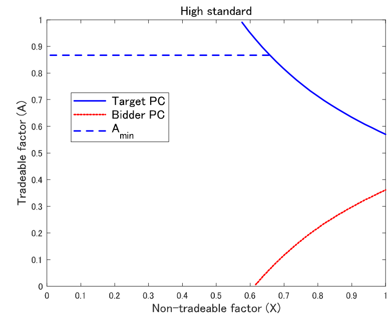

Figure 5 (c)

The left panel (a) is the case with $A_{\min} = 0$ such that projects of all qualities are traded in equilibrium. The middle panel (b) shows the case with the optimal standard $A_{\min} = A^*_{\min}$. The right panel (c) shows the case with a higher standard. Targets are located below a decreasing solid blue line, which represents their participation constraint $AX \leq P$, but above the horizontal dashed line that corresponds to the minimum standard $A_{\min}$. Bidders are located below an increasing red dotted line, which represents their participation constraint $A \leq a - \frac{P}{X}$. As shown in **Figure 5**, as $A_{\min}$ increases, a mass of sellers move up and also spread to the right, while a mass of buyers becomes compressed to the right. We show in the Appendix that both the average quality a and price P increase in $A_{\min}$, while $\frac{a}{A_{\min}}$ and $\frac{a}{P}$ decrease in $A_{\min}$ (see **Lemma A**). Intuitively, a higher standard reduces the quality uncertainty, hence it leads to smaller price discount.

The left panel (a) of **Figure 6** shows how the welfare loss (as a percentage of the first best welfare gains of $\frac{1}{16}$) changes as we we change $A_{\min}$. It shows that with $A_{\min} = 0$, the welfare loss is 70.6%, but it can be lowered to 17.4% by setting the minimum standard optimally ($A^*_{\min} \approx 0.56 > A_0 \approx 0.28$).

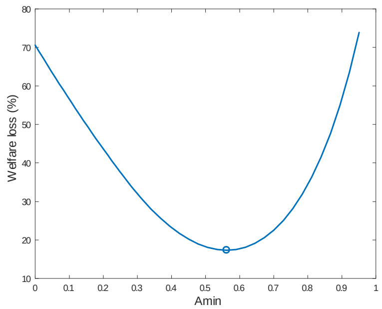

Figure 6 (a)

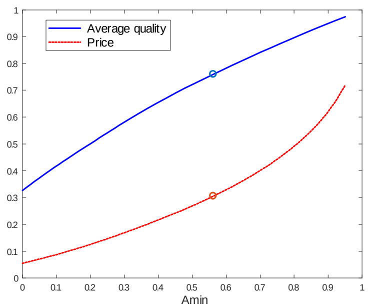

Figure 6 (b)

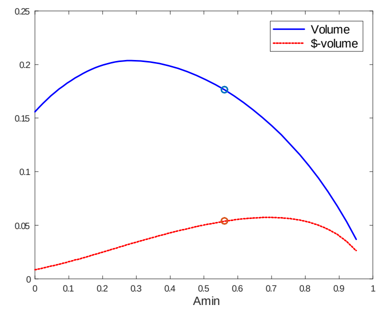

Figure 6 (c)

To better understand what drives the optimal standard $A^*_{\min}$, the panel (b) plots average quality a as well as the price of projects P. The panel (c) plots volume (i.e., a measure of targets or bidders, $S(P^*) = B(P^*) = Q^*$) and dollar volume (i.e., P^*Q^*). Markers indicate the optimal standard $A_{\min} = A^*_{\min}$. **Proposition 3(a)** shows that the welfare gains are monotonic in $a - P$, which is the expected net value of a project for sale. The panel (b) shows that both the average quality of projects for sale and the price increase in $A_{\min}$, but as $A_{\min}$ becomes sufficiently large, the price increases at the faster rate, reflecting the reduced quality uncertainty and the demand from more skilled bidders. Accordingly, $a - P$ has a unique maximum. The panel (c) shows that both volume and dollar-volume exhibit a hump-shaped pattern. Interestingly, it also shows that maximizing volume or dollar-volume does *not* maximize gains from trade. In fact, the optimal standard lies in between: the minimum standard should be higher than the value that maximizes volume, but it should be lower than the value that maximizes dollar volume. In other words, if $A_{\min}$ were chosen to maximize the *number* of takeover deals, the average quality and price of deals would be too low, while if it were chosen to maximize the *monetary value* of deals, the average quality and price would be too high.

4.3 A hybrid market structure

In this subsection we study how the presence of partial disclosure affects the behavior of monopoly intermediary who provides fine disclosure services. Let a price in a coarse market be P_0, and those in fine markets $\{P_z\}_{z \geq \overline{A}}$, where z indexes different fine markets as before. We conjecture that projects of quality equal to or above $\overline{A} \in (0, 1)$ are traded in fine markets, while projects of quality below $\overline{A}$ (but no below the minimum quality $A_{\min}$) are traded in the coarse market. The quality threshold $\overline{A}$ will be endogenously determined in equilibrium. Also, we will show that $A_{\min}$ cannot be arbitrary. It must be chosen to be consistent with firms' choice over different markets.

The characterization of full disclosure markets are the same as before. Recall from Section 4.1 that the payoff in market z is $\Pi_z(\phi) = \frac{z^2-\phi}{2}$. The analysis of a partial disclosure market follows the same step described in the Appendix, but we require two modifications. First, the best type on both sides of the market is $\overline{A}$ in stead of 1. Second, the best bidder with $X = \overline{A}$ in the coarse market must be indifferent about going to a fine market with $P_{\overline{A}}$. The same must be true for the best target with $A = \overline{A}$. Importantly, since the payoff is identical for all targets in the coarse market, there cannot be firms that are indifferent between becoming targets and becoming bidders. So we can focus on the case where targets and bidders are separated by non-trading firms, i.e., $A_{\min} > A^*$. Accordingly, a supply curve and a demand curve are given by

$$\begin{aligned} S(P_0) &= \int_{A_{\min}}^{\overline{A}} \frac{P_0}{A} dA = P_0 \ln \frac{\overline{A}}{A_{\min}}, \\ B(P_0) &= \int_{\frac{P_0}{a}}^{\overline{A}} \left(a - \frac{P_0}{X} \right) dX = a\overline{A} - P_0 - P_0 \ln \frac{\overline{A}a}{P_0}. \end{aligned}$$

A market-clearing condition $S(P_0) = B(P_0)$ yields

$$a\overline{A} = P_0 \left(1 - \ln P_0 + \ln \frac{a}{A_{\min}} + 2 \ln \overline{A} \right). \tag{10}$$

The right hand side is increasing in P_0 and is greater than $a\overline{A}$ at $P_0 = a\overline{A}$. This implies the existence of a unique market-clearing price $P_0 \in (0, a\overline{A})$. In the Appendix we show that the expected quality of projects for sale is

$$a^* = \frac{\overline{A} - A_{\min}}{\ln \overline{A} - \ln A_{\min}}. \tag{11}$$

A hybrid market structure

We use the indifference conditions to pin down $\overline{A}$ and $A_{\min}$. Because the marginal target with project $A = \overline{A}$ and the marginal bidder with skill $X = \overline{A}$ obtain the same payoff $\Pi_{\overline{A}}(\phi) = \frac{\overline{A}^2-\phi}{2}$ by going to the fine market $z = \overline{A}$, the following two conditions must hold:

$$\frac{\overline{A}^2 - \phi}{2} = P_0 \quad \text{and} \quad \frac{\overline{A}^2 - \phi}{2} = a\overline{A} - P_0.$$

These conditions imply

$$P_0 = \frac{a\overline{A}}{2}, \tag{12}$$

$$\overline{A}^2 - a\overline{A} - \phi = 0. \tag{13}$$

Substituting (12) into the market-clearing condition (10) yields

$$A_{\min} = \frac{2}{e}\overline{A}, \qquad \text{where } \frac{2}{e} \approx 0.736. \tag{14}$$

Intuitively, to keep the relative balance between the fine markets and the coarse market, $A_{\min}$ cannot be chosen independent of $\overline{A}$. By substituting (14) into (11) to eliminate $A_{\min}$,

$$a^* = \kappa\overline{A}, \qquad \text{where } \kappa \equiv \frac{1-\frac{2}{e}}{\ln\frac{e}{2}} \approx 0.861. \tag{15}$$

Note that $A_{\min} < a^* < \overline{A}$. Finally, we use (13) and (15) to derive $\overline{A}$ as a function of ϕ.

$$\overline{A}^2 - \left(\kappa\overline{A}\right)\overline{A} - \phi = 0 \Leftrightarrow \overline{A} = \sqrt{\frac{\phi}{1-\kappa}}. \tag{16}$$

Because $\frac{2}{e}\sqrt{\frac{1}{1-\kappa}} > 1$, this implies that

$$A_{\min} = \frac{2}{e}\sqrt{\frac{\phi}{1-\kappa}} > \sqrt{\phi}, \tag{17}$$

i.e., the minimum standard must be strictly greater than the quality of the marginal fine market that would have opened in the absence of a coarse disclosure scheme. In the following analysis, we use the notation $A_{\min}(\phi)$ and $\overline{A}(\phi)$ to make their dependence on ϕ explicit.

Monopoly intermediary in a hybrid market structure.

Next we turn to the monopoly intermediary's choice of ϕ. We assume that the regulatory body uses a minimum disclosure scheme $A_{\min}(\phi)$ given in (17). In other words, the regulatory body commits to support a hybrid market structure such that a marginal fine market for a given ϕ is $\overline{A}(\phi) > \sqrt{\phi}$. Taking these as a constraint, a monopoly intermediary chooses ϕ. The aggregate demand for the intermediary in the hybrid market structure is

$$Q^H(\phi) \equiv \int_{\overline{A}(\phi)}^{1} Q_z(\phi)\,dz = \frac{1}{4}\left(1 - \frac{\phi}{1-\kappa} + \phi\ln\frac{\phi}{1-\kappa}\right).$$

The super script H stands for hybrid. Note that $Q^H(1-\kappa) = 0$. Therefore, the intermediary chooses $\phi \in (0, 1-\kappa)$ to maximize $\phi Q^H(\phi)$. A unique solution $\phi_H \equiv \arg\max_{\phi} \phi Q(\phi)$ is characterized by

$$\phi\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi}{1-\kappa}\right) = 1. \tag{18}$$

We calculate the welfare gains using$A_{\min}(\phi_H)$ and $\overline{A}(\phi_H)$, where ϕ_H is characterized by (18). The reallocation pattern is illustrated in **Figure 7** below.

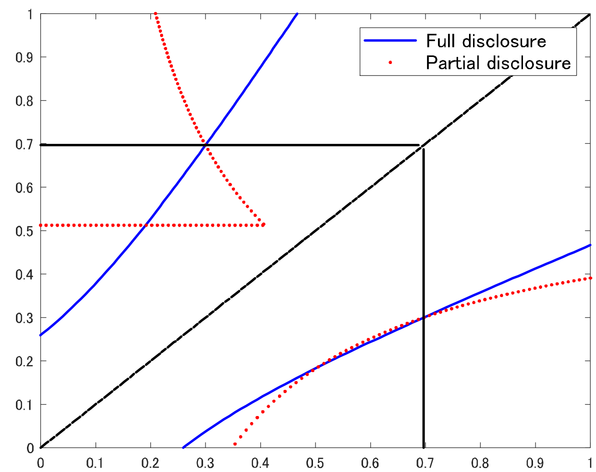

Figure 7. Reallocation in hybrid markets

A solid black square represents $\overline{A}(\phi_H)$, i.e., the boundary between the partial disclosure market and the full disclosure markets. Firms on this line are indifferent between the two disclosure schemes. A dotted red line shows the selection in the partial disclosure market. In the black square, bidders reside in the bottom-right region, while targets reside in the top-left region. The horizontal part of the dotted red line corresponds to $A_{\min}(\phi_H)$, which is greater than $\sqrt{\phi_H}$ (the point at which a solid blue line starts on the vertical axis). Finally, a solid blue line shows sorting and selection in the fine markets. Outside of the black square, the positive assortative matching between bidders and targets is achieved. **Proposition 4** summarizes this analysis.

Proposition 4 *Let $\kappa \equiv \frac{1-\frac{2}{e}}{\ln\frac{e}{2}}$. For any $\phi \in (0, 1-\kappa)$ a hybrid equilibrium with $A_{\min} = \frac{2}{e}\sqrt{\frac{\phi}{1-\kappa}}$ and $\overline{A} = \sqrt{\frac{\phi}{1-\kappa}}$ exists. A monopoly intermediary chooses $\phi_H \in (0, 1-\kappa)$, where ϕ_H is a solution to $\phi\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi}{1-\kappa}\right) = 1$. The welfare gains are $\frac{1}{16}(1-\phi_H)$.*

Because $\phi_H = 0.067$, the welfare loss is 6.7% of the first best welfare gains. This is a significant improvement over $\phi_M = 0.285$, i.e., 28.5% when the coarse market does not exist. The improvement is significant even relative to 17.4% loss when the minimum standard is set optimally (i.e., $A^*_{\min}$ in **Proposition 3**) in the absence of the monopoly intermediary. This indicates that the coarse disclosure scheme, although suboptimal on its own, might be a useful regulatory tool to indirectly control non-competitive behaviors of intermediaries who can provide useful disclosure services in takeover markets.

5 Conclusion

We studied welfare implications of different disclosure schemes in takeover markets. We studied two schemes, each with different source of inefficiency: disclosure facilitated by a for-profit intermediary and a free but partial disclosure. We showed that in equilibrium the two sources of inefficiency offset each other: the availability of free partial disclosure scheme limits the distortion by the monopoly intermediary, lowering the overall welfare loss to 6.7%. The analysis suggests that the coarse disclosure scheme, although inefficient on its own, might be a useful regulatory tool to indirectly control non-competitive behaviors of intermediaries in takeover markets.

In this paper we remained in a static model. Also, the coarse disclosure market in our model was restricted to use a single price. How much do we gain by opening a second market simultaneously, or allowing a second round of trading? These are important questions left for future research.

Appendix

Proof of Proposition 1

The "only if" part is obvious from the paragraph above **Proposition 1**. For the "if" part, first we check if buyers in market z have an incentive to be buyers in other markets. Buyers in market in z have $X = z$ and $A \in \left[0, \frac{z}{2}\right]$. By pretending to have $X = z'$ to be a buyer in market z', they can buy a project of quality z' at price $P_{z'}$. The payoff from this deviation would be

$$\Pi^B_{z,z'} \equiv z'z - P_{z'}.$$

Therefore, buyers in market z have no incentive to be buyers in other markets if and only if $\Pi_z > \Pi^B_{z,z'}$ for all $z' \neq z$. Because $\Pi_z = z^2 - P_z$,

$$\Pi_z > \Pi^B_{z,z'} \Leftrightarrow z\left(z - z'\right) > P_z - P_{z'} \Leftrightarrow \begin{cases} z > \frac{z+z'}{2} & \text{for } z' < z \\ z < \frac{z+z'}{2} & \text{for } z' > z \end{cases}.$$

Because this is always satisfied, buyers in market z have no incentive to to be buyers in other markets. Recall that these buyers have projects of quality at most $\frac{z}{2}$. The certification scheme prevents these buyers from becoming sellers in market $z' > \frac{z}{2}$. These buyers clearly have no incentive to be sellers in market $z' \leq \frac{z}{2}$, because doing so would lower their payoff to $\Pi_{z'} \leq \Pi_{\frac{z}{2}} < \Pi_z$.

Next, we check if targets in market z have an incentive to be buyers in other markets. Recall that these sellers have skill level $X \in \left[0, \frac{z}{2}\right]$. From the above argument, the best they could do as bidders is to go to market X and have Π_X. Because $\Pi_X < \Pi_z$ for all $X \in \left[0, \frac{z}{2}\right]$, sellers in market z have no incentive to be buyers in other markets.

It remains to check that non-trading firms have no incentive to participate in any market. Consider a firm with $(A, X) = (a, x)$ that participates neither in market a as a seller nor in market x as a buyer. Then it must be that

$$ax > \max\left\{\Pi_a, \Pi_x\right\}.$$

The certification scheme prevents this firm from becoming a seller in market $a' > a$. There is no other profitable deviation because $\Pi_{a'} < \Pi_a$ for $a' < a$ and $\Pi^B_{x,x'} < \Pi_x$ for $x' \neq x$. ■

Proof of Proposition 2

For a given ϕ, a measure of traded projects in market z is $Q_z(\phi) = \frac{1}{2}\left(z - \frac{\phi}{z}\right)$. Total gains from trade are

$$\int_{\sqrt{\phi}}^{1} z^2 Q_z(\phi)\, dz - \int_{\sqrt{\phi}}^{1} \left(A \int_0^{Q_A(\phi)} X dX \right) dA - \int_{\sqrt{\phi}}^{1} \left(X \int_0^{Q_X(\phi)} A dA \right) dX.$$

Due to symmetry, the second term and the third term have the same value. The first term, the new production, is

$$\begin{aligned}
\frac{1}{2}\int_{\sqrt{\phi}}^{1} z^2 \left(z - \frac{\phi}{z} \right) dz &= \frac{1}{2}\left[\frac{1}{4}z^4 - \frac{\phi}{2}z^2 \right]_{\sqrt{\phi}}^{1} \\
&= \frac{1}{8}\left\{ 1 - \phi^2 - 2\phi(1-\phi) \right\} \\
&= \frac{1}{8}(1-\phi)^2.
\end{aligned}$$

The second term, the lost production for targets, is

$$\begin{aligned}
\frac{1}{2}\int_{\sqrt{\phi}}^{1} \left(A \{Q_A(\phi)\}^2 \right) dA &= \frac{1}{8}\int_{\sqrt{\phi}}^{1} A \left(A - \frac{\phi}{A} \right)^2 dA \\
&= \frac{1}{8}\left[\frac{1}{4}\left(1-\phi^2\right) - \phi(1-\phi) - \phi^2 \ln\sqrt{\phi} \right] \\
&= \frac{1}{8}\left[\frac{1}{4} - \phi + \frac{3}{4}\phi^2 - \phi^2 \ln\sqrt{\phi} \right].
\end{aligned}$$

Combining these together, gains from trade are

$$\begin{aligned}
G(\phi) &\equiv \frac{1}{8}\left[(1-\phi)^2 - 2\left\{ \frac{1}{4} - \phi + \frac{3}{4}\phi^2 - \phi^2 \ln\sqrt{\phi} \right\} \right] \\
&= \frac{1}{8}\left(\frac{1}{2} - \frac{1}{2}\phi^2 + \phi^2 \ln\phi \right) \\
&= \frac{1}{16}\left\{ 1 - \phi^2 (1 - 2\ln\phi) \right\}.
\end{aligned}$$

Because ϕ_M satisfies $\phi_M(1 - 2\ln\phi_M) = 1$, $G(\phi_M) = \frac{1}{16}(1-\phi_M)$. ■

The following results are used to prove **Proposition 3**.

Lemma A *Define $A_0 \in (0,1)$ by a smaller solution to $A(1 - 2\ln A) = 1$.*

(a) *If $A_{\min} \in [0, A_0)$, then $a^* \in (0, 2A_0)$ is given by a unique solution to $a = \Gamma(a; A_{\min})$, where*

$$\Gamma(a; A_{\min}) \equiv \frac{1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2}{1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}}, \tag{19}$$

and $A_{\min} < A^* = \frac{a^*}{2}$ *holds. A market-clearing price is a unique solution to*

$$a^* = P\left\{3 - \ln P - \left(4\frac{A_{\min}}{a^*} + \ln\left(1 - \frac{A_{\min}}{a^*}\right)\right)\right\}. \tag{20}$$

(b) *If* $A_{\min} \in [A_0, 1)$*, then* $a^* = -\frac{1-A_{\min}}{\ln A_{\min}} \in [2A_0, 1)$ *and* $A_{\min} \geq A^* = \frac{a^*}{2}$ *holds. A market-clearing price is a unique solution to*

$$a^* = P\left(1 - \ln P + \ln\frac{a^*}{A_{\min}}\right). \tag{21}$$

(c) a^* *is increasing in* $A_{\min}$*. It satisfies* $a^* = 2A_0$ *when* $A_{\min} = A_0$ *and* $\lim_{A_{\min}\nearrow 1} a^* = 1$*.*

(d) P^*, $\frac{A_{\min}}{a^*}$ *and* $\frac{P^*}{a^*}$ *are increasing in* $A_{\min}$*. Also,* $\lim_{A_{\min}\nearrow 1} P^* = 1$*.*

Proof of Lemma A

(a) Conjecture $A_{\min} < A^* = \frac{a}{2}$. We verify later that this occurs if and only if $A_{\min} < A_0$. Given $A_{\min} < A^*$ and for a given (a, P) such that $0 < P < a < 1$, the sorting pattern implies that targets satisfy $X \leq \frac{P}{A}$, $X \leq X^*$, and $A \geq A_{\min}$. Hence, a supply curve is

$$S(P) = \int_{A_{\min}}^{A^*} X^* dA + \int_{A^*}^{1} \frac{P}{A} dA = X^*(A^* - A_{\min}) - P\ln A^*.$$

Substituting $A^* = \frac{a}{2}$ and $X^* = \frac{2P}{a}$,

$$S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right).$$

Bidders satisfy $A \leq a - \frac{P}{X}$, and additionally, if $A \in [A_{\min}, A^*]$, $X > X^*$. Note that $a - \frac{P}{X} = 0$ defines a skill threshold $X = \frac{P}{a}$ (below which no bidder exists), while $a - \frac{P}{X} = A_{\min}$ defines another threshold $X = \frac{P}{a-A_{\min}} \in \left(\frac{P}{a}, X^*\right)$ (above which $A \geq A_{\min}$ becomes a binding constraint for some firms). Using these, a demand curve is

$$\begin{aligned} B(P) &= \int_{\frac{P}{a}}^{1}\left(a - \frac{P}{X}\right)dX - \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(a - \frac{P}{X} - A_{\min}\right)dX \\ &= P\left\{\frac{a}{P} - \ln\frac{a}{P} + \ln\left(2\frac{a - A_{\min}}{a}\right) - 2\frac{a - A_{\min}}{a}\right\}. \end{aligned}$$

For $P > 0$, a market-clearing condition $S(P) = B(P)$ is equivalent to

$$a = P\left\{3 - \ln P - \left(4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right)\right\},$$

which is (20). Denote the right hand side of (20) by $\Phi_1(P; a, A_{\min})$. A brief inspection of Φ_1 yields

$$\frac{\partial \Phi_1}{\partial P} = 2\left(1 - 2\frac{A_{\min}}{a}\right) + \ln\frac{a}{a - A_{\min}} - \ln P.$$

This is positive for any $0 < P < a < 1$ such that $A_{\min} < A^* = \frac{a}{2}$, because

$$2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln P - \ln\frac{a}{a - A_{\min}}.$$

Also, $\Phi_1(0; a, A_{\min}) = 0$ and $\Phi_1(a; a, A_{\min}) = a\left\{3 - 4\frac{A_{\min}}{a} - \ln(a - A_{\min})\right\}$. Note that $\Phi_1(a; a, A_{\min}) > a \Leftrightarrow 2 - 4\frac{A_{\min}}{a} > \ln(a - A_{\min})$ holds because $A_{\min} < A^* = \frac{a}{2}$ implies $2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln(a - A_{\min})$. This establishes a unique solution $P(a) \in (0, a)$ to (20).

Given the sorting pattern, the expected quality of projects for sale is

$$\Gamma(a; A_{\min}) \equiv \frac{\int_{A_{\min}}^{A^*}(AX^*)\,dA + \int_{A^*}^{1}\left(A\frac{P}{A}\right)dA}{S(P)}.$$

The numerator can be evaluated as

$$P\left\{\frac{1}{2}\frac{2}{a}\left(A^{*2} - A_{\min}^2\right) + 1 - A^*\right\} = P\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right).$$

Combining this with $S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right)$ yields (19). We prove the following three properties for $A_{\min} < A_0$:

$$\Gamma(2A_{\min}; A_{\min}) > 2A_{\min}, \qquad \Gamma(1; A_{\min}) < 1, \qquad \frac{\partial \Gamma(a; A_{\min})}{\partial a}|_{a=a*} < 1.$$

Taken together, these imply that $\Gamma(a; A_{\min}) = a$ has a unique solution $a^* \in (2A_{\min}, 1)$. For the first property,

$$\Gamma(2A_{\min}; A_{\min}) = \frac{1 - \frac{A_{\min}}{2} - \frac{A_{\min}}{2}}{-\ln A_{\min}} > 2A_{\min}$$
$$\Leftrightarrow 1 > A_{\min}(1 - 2\ln A_{\min}) \Leftrightarrow A_{\min} < A_0.$$

For the second property,

$$\Gamma(1; A_{\min}) = \frac{\frac{3}{4} - A_{\min}^2}{1 + \ln 2 - 2A_{\min}} < 1 \Leftrightarrow \frac{3}{4} - \ln 2 < (1 - A_{\min})^2.$$

This holds because $\frac{3}{4} - \ln 2 = 0.057 < (1 - A_0)^2 = (1 - 0.285)^2 = 0.511 < (1 - A_{\min})^2$. For the third property, let $N \equiv 1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2$ and $D \equiv 1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}$ so that $\Gamma(a; A_{\min}) = \frac{N}{D}$.

Then

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial a}<1\Leftrightarrow\frac{\partial N}{\partial a}D-N\frac{\partial D}{\partial a}<D^{2}\Leftrightarrow\frac{\partial N}{\partial a}-D<\frac{N}{D}\frac{\partial D}{\partial a}$$

$$\Leftrightarrow\left(\frac{A_{\min}}{a}\right)^{2}-\frac{1}{4}-D<\frac{N}{D}\left(\frac{2A_{\min}}{a}-1\right)\frac{1}{a}.$$

Because both sides are negative for $A_{\min}<\frac{a}{2}$, this is equivalent to

$$\frac{N}{D}=\Gamma\left(a;A_{\min}\right)<a\frac{1-\ln\frac{a}{2}-\frac{2}{a}A_{\min}+\frac{1}{4}\left\{1-\left(\frac{2A_{\min}}{a}\right)^{2}\right\}}{1-\frac{2A_{\min}}{a}}.$$

The right hand side can be written as $a+a\frac{\ln\frac{2}{a}+\frac{1}{4}\left(1-\frac{2A_{\min}}{a}\right)\left(1+\frac{2A_{\min}}{a}\right)}{1-\frac{2A_{\min}}{a}}$, so

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial a}<1\Leftrightarrow\frac{\Gamma\left(a;A_{\min}\right)-a}{a}<\frac{\ln\frac{2}{a}}{1-\frac{2A_{\min}}{a}}+\frac{1}{4}\left(1+\frac{2A_{\min}}{a}\right).$$

Because the right hand side is positive for $A_{\min}<\frac{a}{2}$ while the left hand side is zero at $a=a^{*}$, this implies that $\frac{\partial\Gamma(a;A_{\min})}{\partial a}<1$ holds at $a=a^{*}$.

To show that a^{*} is increasing in $A_{\min}$, it suffices to show that $\frac{\partial\Gamma(a;A_{\min})}{\partial A_{\min}}|_{a=a^{*}}>0$.

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial A_{\min}}>0\Leftrightarrow\frac{\partial N}{\partial A_{\min}}D>N\frac{\partial D}{\partial A_{\min}}.$$

Because $\frac{\partial N}{\partial A_{\min}}=-\frac{2A_{\min}}{a}$ and $\frac{\partial D}{\partial A_{\min}}=-\frac{2}{a}$ are both negative,

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial A_{\min}}>0\Leftrightarrow\frac{N}{D}=\Gamma\left(a;A_{\min}\right)>A_{\min}.$$

This holds at $a=a^{*}$, because $A_{\min}<\frac{a^{*}}{2}<a^{*}=\Gamma\left(a^{*};A_{\min}\right)$.

Finally, we already know that, for $A_{\min}<1$, $\Gamma\left(2A_{\min};A_{\min}\right)=2A_{\min}\Leftrightarrow1=A_{\min}\left(1-2\ln A_{\min}\right)\Leftrightarrow A_{\min}=A_{0}$. Therefore, $\lim_{A_{\min}\nearrow A_{0}}a^{*}=2A_{0}$.

(b) Conjecture $A_{\min}\geq A^{*}=\frac{a}{2}$. We verify later that this occurs if and only if $A_{\min}\geq A_{0}$. A supply curve is

$$S\left(P\right)=\int_{A_{\min}}^{1}\frac{P}{A}dA=-P\ln A_{\min}.$$

A demand curve is

$$B\left(P\right)=\int_{\frac{P}{a}}^{1}\left(a-\frac{P}{X}\right)dX=a-P+P\ln\frac{P}{a}.$$

A market-clearing condition $S(P) = B(P)$ is equivalent to $a = P\left(1 - \ln P + \ln \frac{a}{A_{\min}}\right)$ as shown in (21). Denote the right hand side of (21) by $\Phi_2(P; a, A_{\min})$. A brief inspection of Φ_2 yields

$$\frac{\partial \Phi_2}{\partial P} = \ln \frac{a}{P A_{\min}}.$$

This is positive if and only if $\frac{a}{P} > A_{\min}$, which is true for any $A_{\min} < 1$ and $P < a$. Also, $\Phi_2(0; a, A_{\min}) = 0$ and $\Phi_2(a; a, A_{\min}) = a(1 - \ln A_{\min}) > a$ for any $A_{\min} < 1$. This establishes a unique solution $P(a) \in (0, a)$ to (21).

Given the sorting pattern, the expected quality of projects for sale is

$$\frac{\int_{A_{\min}}^{1} \left(A \frac{P}{A}\right) dA}{S(P)} = \frac{P(1 - A_{\min})}{-P \ln A_{\min}} = \frac{A_{\min} - 1}{\ln A_{\min}}.$$

Therefore, given $A_{\min} \geq A^* = \frac{a}{2}$, $a^* = \frac{A_{\min}-1}{\ln A_{\min}}$. To verify the conjecture $A_{\min} \geq A^* = \frac{a}{2}$,

$$A_{\min} \geq \frac{1}{2} \frac{A_{\min} - 1}{\ln A_{\min}} \Leftrightarrow 1 \leq A_{\min}(1 - 2\ln A_{\min}) \Leftrightarrow A_{\min} \geq A_0.$$

That $a^* = \frac{A_{\min}-1}{\ln A_{\min}}$ is increasing in $A_{\min}$ is immediate from

$$\ln A_{\min} - \frac{A_{\min} - 1}{A_{\min}} > 0 \Leftrightarrow 1 > A_{\min}(1 - \ln A_{\min}),$$

where the right hand side is increasing in $A_{\min}$ and approaches one as $A_{\min} \nearrow 1$. Note also that $\lim_{A_{\min} \nearrow 1} \frac{A_{\min}-1}{\ln A_{\min}} = 1$. At $A_{\min} = A_0$, $a^* = \frac{A_0 - 1}{\ln A_0} = 2A_0$ holds because this is equivalent to $1 = A_0(1 - 2\ln A_0)$. This means that a^* is continuous in $A_{\min} \in [0, 1)$.

(c) This was proved in the proof of (a)(b).

(d) [For $A_{\min} < A_0$] For $\frac{A_{\min}}{a^*}$, consider

$$\frac{\Gamma(a; A_{\min})}{A_{\min}} = \frac{a}{A_{\min}} \Leftrightarrow \frac{1}{A_{\min}}\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right) = \frac{a}{A_{\min}}\left(1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}\right)$$

$$\Leftrightarrow \frac{1}{a}\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right) = 1 - \ln\frac{a}{2} - 2\frac{A_{\min}}{a}$$

$$\Leftrightarrow \frac{1}{a} - \frac{1}{4} + \ln\frac{a}{2} = \left(1 - \frac{A_{\min}}{a}\right)^2.$$

Because the left hand side is decreasing in a, $\frac{A_{\min}}{a}$ must be increasing in a. Since a^* is increasing in $A_{\min}$, $\frac{A_{\min}}{a^*}$ must be increasing in $A_{\min}$ as well.

For P^*, consider a market-clearing condition (20) with a^*, i.e., $a^* = \Phi_1(P; a^*, A_{\min})$. The left hand side a^* increases in $A_{\min}$. We want to show that for a fixed P the right hand side $\Phi_1(P; a^*, A_{\min})$ decreases in $A_{\min}$. It suffices to show that $4\frac{A_{\min}}{a^*} + \ln\left(1 - \frac{A_{\min}}{a^*}\right)$ increases in $\frac{A_{\min}}{a^*}$. This is true because

$$\frac{\partial\{4X + \ln(1-X)\}}{\partial X} = 4 - \frac{1}{1-X} = \frac{3-4X}{1-X}$$

and $\frac{3-4\frac{A_{\min}}{a^*}}{1-\frac{A_{\min}}{a^*}} > 0$ holds for $A_{\min} < \frac{a^*}{2} < \frac{3}{4}a^*$.

For $\frac{P^*}{a^*}$, rewrite (20) as

$$\frac{a^*}{P^*} = 3 - \ln P^* - \left(4\frac{A_{\min}}{a^*} + \ln\left(1 - \frac{A_{\min}}{a^*}\right)\right).$$

Clearly the right hand side decreases in $A_{\min}$.

[For $A_{\min} \geq A_0$] For $\frac{P^*}{a^*}$, rewrite a market-clearing condition (21) as

$$\frac{a}{P} = \ln\frac{a}{P} + \ln\frac{e}{A_{\min}}.$$

For the domain $\frac{a}{P} > 1$, the above equation in $\frac{a}{P}$ has a unique solution and it decreases in $A_{\min}$. Hence $\frac{P^*}{a^*}$ increases in $A_{\min}$ with $\lim_{A_{\min}\nearrow 1}\frac{P^*}{a^*} = 1$.

Because a^* increases in $A_{\min}$, $P^* = a^*\frac{P^*}{a^*}$ increases in $A_{\min}$. To show that $\frac{A_{\min}}{a^*} = \frac{A_{\min}}{-\frac{1-A_{\min}}{\ln A_{\min}}} = \frac{A_{\min}\ln A_{\min}}{A_{\min}-1}$ increases in $A_{\min}$, take its derivative:

$$\frac{(\ln A_{\min} + 1)(A_{\min} - 1) - A_{\min}\ln A_{\min}}{(A_{\min} - 1)^2} = \frac{A_{\min} - \ln A_{\min} - 1}{(A_{\min} - 1)^2}.$$

The numerator is decreasing in $A_{\min} < 1$ and takes zero when $A_{\min} = 1$. So $A_{\min} - \ln A_{\min} - 1 > 0$ for any $A_{\min} < 1$. Finally, $\lim_{A_{\min}\nearrow 1} a^* = 1$ and $\lim_{A_{\min}\nearrow 1}\frac{P^*}{a^*} = 1$ imply $\lim_{A_{\min}\nearrow 1} P^* = 1$. ■

Proof of Proposition 3(a)

For notational simplicity, we use (a, P) instead of (a^*, P^*). The expected welfare gains are given by $NP - (TL + BL)$, where NP is the expected value of new production by bidders, TL is targets' lost production, and BL is bidders' lost production. These are given by

$$NP = a\int_{X^*}^{1} X\left(a - \frac{P}{X}\right)dX = a\left(\frac{a}{2} - P\right),$$

$$
\begin{aligned}
TL &= \int_0^{A^*}\int_0^{X^*}(AX)\,dAdX+\int_{A^*}^1\left(A\int_0^{\frac{P}{A}}XdX\right)dA \\
&= \frac{A^{*2}}{2}\frac{X^{*2}}{2}+\frac{P^2}{2}\int_{A^*}^1\frac{1}{A}dA \\
&= \frac{P^2}{2}\left(\frac{1}{2}-\ln\frac{a}{2}\right).
\end{aligned}
$$

$$
\begin{aligned}
BL &= \int_{X^*}^1\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX \\
&= \frac{1}{2}\int_{X^*}^1\left(a^2X-2aP+\frac{P^2}{X}\right)dX \\
&= \left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right).
\end{aligned}
$$

Hence, the lost production is

$$
TL+BL=\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\frac{1}{2}-\ln P\right).
$$

Finally, the expected welfare gains are

$$
NP-(TL+BL)=\left(\frac{a}{2}-P\right)\left(\frac{a}{2}+P\right)+\frac{P^2}{2}\left(\ln P-\frac{1}{2}\right)\equiv G\left(a,P\right). \tag{22}
$$

From the market-clearing condition (9),

$$
a=P\left(3-\ln P\right)\Leftrightarrow\ln P=3-\frac{a}{P}.
$$

Substituting this into $G\left(a,P\right)$,

$$
\begin{aligned}
NP-(TL+BL) &= \left(\frac{a}{2}-P\right)\left(\frac{a}{2}+P\right)+\frac{P^2}{2}\left(\frac{5}{2}-\frac{a}{P}\right) \\
&= \left(\frac{a}{2}\right)^2+\left(\frac{P}{2}\right)^2-\frac{Pa}{2} \\
&= \left(\frac{a-P}{2}\right)^2.
\end{aligned}
$$

General case.

For a general $A_{\min}\in[0,1]$, we proceed in two steps. First, we compute the expected welfare gains as a function of $(a, P, A_{\min})$. Second, we show that using a market-clearing condition the expected welfare gains can always be expressed as $\left(\frac{a-P}{2}\right)^2$.

[Step 1. The expected welfare gains as a function of $(a, P, A_{\min})$]

For $A_{\min} < A^*$, the expected value of new production is given by

$$NP_1 \equiv a \int_{X^*}^{1} X \left(a - \frac{P}{X}\right) dX + a \int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} X \left(a - \frac{P}{X}\right) dX + a \int_{\frac{P}{a-A_{\min}}}^{X^*} (X A_{\min})\, dX.$$

The first term is evaluated to be $a\left(\frac{a}{2} - P\right)$ as in the case with $A_{\min} = 0$. The second term is

$$\begin{aligned} a \int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} X \left(a - \frac{P}{X}\right) dX &= a \left[\frac{a}{2} X^2 - PX\right]_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} \\ &= aP^2 \left(\frac{1}{a - A_{\min}} - \frac{1}{a}\right) \left(\frac{1}{2} \frac{a}{a - A_{\min}} - \frac{1}{2}\right) \\ &= \frac{P^2}{2} \left(\frac{A_{\min}}{a - A_{\min}}\right)^2. \end{aligned}$$

The third term is

$$\begin{aligned} a \int_{\frac{P}{a-A_{\min}}}^{X^*} (X A_{\min})\, dX &= a \frac{A_{\min}}{2} \left\{ X^{*2} - \left(\frac{P}{a - A_{\min}}\right)^2 \right\} \\ &= a \frac{A_{\min}}{2} P^2 \left\{ \left(\frac{2}{a}\right)^2 - \left(\frac{1}{a - A_{\min}}\right)^2 \right\} \\ &= \frac{P^2}{2} \frac{A_{\min}}{a} \left(4 - \left(\frac{a}{a - A_{\min}}\right)^2\right). \end{aligned}$$

Therefore,

$$\begin{aligned} NP_1 &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2} \frac{A_{\min}}{a} \left\{ \frac{a}{A_{\min}} \left(\frac{A_{\min}}{a - A_{\min}}\right)^2 + 4 - \left(\frac{a}{a - A_{\min}}\right)^2 \right\} \\ &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2} \frac{A_{\min}}{a} \left(4 - \frac{a}{a - A_{\min}}\right) \\ &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2} \frac{A_{\min}}{a} \left(\frac{3 - 4\frac{A_{\min}}{a}}{1 - \frac{A_{\min}}{a}}\right). \end{aligned}$$

Targets' lost production is

$$\begin{aligned} TL_1 &= \int_{A_{\min}}^{A^*}\int_0^{X^*}(AX)\,dAdX+\int_{A^*}^{1}\left(A\int_0^{\frac{P}{A}}XdX\right)dA \\ &= \frac{1}{2}\left(A^{*2}-A_{\min}^2\right)\frac{X^{*2}}{2}-\frac{P^2}{2}\ln\frac{a}{2} \\ &= \frac{P^2}{2}\left(\frac{1}{2}-\ln\frac{a}{2}\right)-P^2\left(\frac{A_{\min}}{a}\right)^2. \end{aligned}$$

Bidders' lost production is

$$BL_1=\int_{X^*}^{1}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX+\int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX-\int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}}AdA\right)dX.$$

The first term is $\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right)$. To compute the remaining two terms,

$$\begin{aligned} \int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX &= \frac{1}{2}\int_{\frac{P}{a}}^{X^*}X\left(a-\frac{P}{X}\right)^2dX \\ &= \frac{1}{2}\int_{\frac{P}{a}}^{X^*}\left(a^2X-2aP+\frac{P^2}{X}\right)dX, \end{aligned}$$

$$\begin{aligned} \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}}AdA\right)dX &= \frac{1}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*}X\left\{\left(a-\frac{P}{X}\right)^2-A_{\min}^2\right\}dX \\ &= \frac{1}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*}\left(a^2X-2aP+\frac{P^2}{X}\right)dX-\frac{A_{\min}^2}{2}\int_{\frac{P}{a-A_{\min}}}^{X^*}XdX. \end{aligned}$$

Therefore,

$$\begin{aligned}
& \int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}} AdA\right)dX - \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}} AdA\right)dX \\
= \; & \frac{1}{2}\int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}}\left(a^2X - 2aP + \frac{P^2}{X}\right)dX + \frac{A_{\min}^2}{4}\left\{X^{*2} - \left(\frac{P}{a-A_{\min}}\right)^2\right\} \\
= \; & \frac{P^2}{2}\left[\frac{A_{\min}}{(a-A_{\min})\,a}\left\{\frac{a}{2}\frac{a}{a-A_{\min}} - \frac{3}{2}a\right\} + \ln\frac{a}{a-A_{\min}} + \frac{A_{\min}^2}{2}\left(\frac{2}{a} + \frac{1}{a-A_{\min}}\right)\left(\frac{2}{a} - \frac{1}{a-A_{\min}}\right)\right] \\
= \; & \frac{P^2}{4}\left[\left(\frac{A_{\min}}{a-A_{\min}}\right)^2 - 2\frac{A_{\min}}{a-A_{\min}} + 2\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right) + \left(\frac{A_{\min}}{a}\right)^2\left(4 - \left(\frac{a}{a-A_{\min}}\right)^2\right)\right] \\
= \; & \frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2 - \frac{A_{\min}}{a-A_{\min}} + \ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right].
\end{aligned}$$

Bidders' lost production then is

$$\left(\frac{a}{2} - P\right)^2 + \frac{P^2}{2}\left(\ln\frac{a}{2} - \ln P\right) + \frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2 - \frac{A_{\min}}{a-A_{\min}} + \ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right].$$

Total lost production is

$$\begin{aligned}
TL_1 + BL_1 \;=\; & \frac{P^2}{2}\left(\frac{1}{2} - \ln\frac{a}{2}\right) - P^2\left(\frac{A_{\min}}{a}\right)^2 + \left(\frac{a}{2} - P\right)^2 + \frac{P^2}{2}\left(\ln\frac{a}{2} - \ln P\right) \\
& + \frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2 - \frac{A_{\min}}{a-A_{\min}} + \ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right] \\
=\; & \left(\frac{a}{2} - P\right)^2 + \frac{P^2}{2}\left(\frac{1}{2} - \ln P\right) + \frac{P^2}{2}\left[\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right) - \frac{A_{\min}}{a-A_{\min}}\right].
\end{aligned}$$

The expected welfare gains are

$$\begin{aligned}
NP_1 - (TL_1 + BL_1) \;=\; & a\left(\frac{a}{2} - P\right) - \left\{\left(\frac{a}{2} - P\right)^2 + \frac{P^2}{2}\left(\frac{1}{2} - \ln P\right)\right\} \\
& + \frac{P^2}{2}\frac{A_{\min}}{a}\left(\frac{3 - 4\frac{A_{\min}}{a}}{1 - \frac{A_{\min}}{a}}\right) - \frac{P^2}{2}\left[\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right) - \frac{A_{\min}}{a-A_{\min}}\right] \\
=\; & G(a,P) + \frac{P^2}{2}\left\{\frac{A_{\min}}{a}\left(\frac{3 - 4\frac{A_{\min}}{a}}{1 - \frac{A_{\min}}{a}}\right) + \frac{A_{\min}}{a-A_{\min}} - \ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right\} \\
=\; & G(a,P) + \frac{P^2}{2}\left\{4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right\}.
\end{aligned}$$

For $A_{\min} \geq A^*$, the expected value of new production is given by

$$\begin{aligned} NP_2 &\equiv a\int_{\frac{P}{a}}^{1} X\left(a-\frac{P}{X}\right)dX = a\left[\frac{a}{2}\left\{1-\left(\frac{P}{a}\right)^2\right\}-P\left(1-\frac{P}{a}\right)\right] \\ &= \frac{(a-P)^2}{2}. \end{aligned}$$

Targets' lost production is

$$\begin{aligned} TL_2 &= \int_{A_{\min}}^{1}\left(A\int_{0}^{\frac{P}{A}} XdX\right)dA = \frac{1}{2}\int_{A_{\min}}^{1} A\left(\frac{P}{A}\right)^2 dA \\ &= -\frac{P^2}{2}\ln A_{\min}. \end{aligned}$$

Bidders' lost production is

$$\begin{aligned} BL_2 &= \int_{\frac{P}{a}}^{1}\left(X\int_{0}^{a-\frac{P}{X}} AdA\right)dX = \frac{1}{2}\int_{\frac{P}{a}}^{1} X\left(a-\frac{P}{X}\right)^2 dX \\ &= \frac{1}{2}\int_{\frac{P}{a}}^{1}\left(a^2X-2aP+\frac{P^2}{X}\right)dX \\ &= \frac{1}{4}(a-P)(a-3P)-\frac{P^2}{2}\ln\frac{P}{a}. \end{aligned}$$

Total lost production is

$$\begin{aligned} TL_2+BL_2 &= \frac{1}{4}(a-P)(a-3P)-\frac{P^2}{2}\ln\frac{P}{a}-\frac{P^2}{2}\ln A_{\min} \\ &= \frac{1}{4}(a-P)(a-3P)-\frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right). \end{aligned}$$

The expected welfare gains are

$$\begin{aligned} NP_2-(TL_2+BL_2) &= \frac{(a-P)^2}{2}-\frac{1}{4}(a-P)(a-3P)+\frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right) \\ &= \frac{1}{4}(a-P)\{2(a-P)-(a-3P)\}+\frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right) \\ &= \frac{1}{4}(a-P)(a+P)+\frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right), \end{aligned}$$

where $A_{\min} \geq A^* = \frac{a}{2}$.

[Step 2. Use a market-clearing condition to get rid of $A_{\min}$]

We use **Lemma A** to show that the expected welfare gains are always $\left(\frac{a-P}{2}\right)^2$.

For $A_{\min} < A_0$, from the market-clearing condition (20),

$$4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right) = 3 - \ln P - \frac{a}{P}. \tag{23}$$

Recall that $NP_1 - (TL_1 + BL_1) = G(a, P) + \frac{P^2}{2}\left\{4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right\}$, where $G(a, P) = \left(\frac{a}{2} - P\right)\left(\frac{a}{2} + P\right) + \frac{P^2}{2}\left(\ln P - \frac{1}{2}\right)$. Eliminating $A_{\min}$ using (23) yields

$$\begin{aligned} NP_1 - (TL_1 + BL_1) &= \left(\frac{a}{2}\right)^2 - P^2 + \frac{P^2}{2}\left(\ln P - \frac{1}{2} + 3 - \ln P - \frac{a}{P}\right) \\ &= \left(\frac{a-P}{2}\right)^2. \end{aligned}$$

For $A_{\min} \geq A_0$, from the market-clearing condition (21),

$$\ln\left(A_{\min}\frac{P}{a}\right) = -\frac{a-P}{P}. \tag{24}$$

Recall that $NP_2 - (TL_2 + BL_2) = \frac{1}{4}(a-P)(a+P) + \frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right)$. Eliminating $A_{\min}$ using (24) yields

$$\begin{aligned} NP_2 - (TL_2 + BL_2) &= \frac{1}{4}\left(a^2 - P^2\right) - \frac{P}{2}(a-P) \\ &= \left(\frac{a-P}{2}\right)^2. \quad \blacksquare \end{aligned}$$

Proof of Proposition 3(b)

It suffices to show that $a - P$ monotonically increases in $A_{\min} \leq A_0$.

(To be written). $\blacksquare$

Proof of Proposition 4

Denote the right hand side of the market-clearing condition (10) by $\Phi_0(P_0) \equiv P_0\left(1 - \ln P_0 + \ln\frac{a}{A_{\min}} + 2\right.$
This is increasing in $P \leq a\overline{A}$ because

$$\Phi_0'(P_0) = \ln\left(\frac{a}{A_{\min}P_0}\overline{A}^2\right) \geq \ln\frac{\overline{A}}{A_{\min}} > 0.$$

$\Phi_0(0) = 0$ and $\Phi_0\left(a\overline{A}\right) = a\overline{A}\left(1 + \ln\frac{\overline{A}}{A_{\min}}\right) > a\overline{A}$ imply the existence of a unique solution

$P_0(a) \in \left(0, a\overline{A}\right)$ to (10). Next, the expected quality of projects for sale is

$$\frac{\int_{A_{\min}}^{\overline{A}} A\frac{P_0}{A}dA}{S(P_0)} = \frac{P_0\left(\overline{A} - A_{\min}\right)}{P_0 \ln\frac{\overline{A}}{A_{\min}}} = \frac{\overline{A} - A_{\min}}{\ln\overline{A} - \ln A_{\min}}.$$

The derivation of $\overline{A}$, $A_{\min}$, and a^* as functions of ϕ were explained in the main text.

Gains from trade in the fine markets are

$$G^F(\phi) \equiv \int_{\overline{A}}^{1} z^2 Q_z(\phi)\,dz - \int_{\overline{A}}^{1}\left(A\int_0^{Q_A(\phi)} X dX\right)dA - \int_{\overline{A}}^{1}\left(X\int_0^{Q_X(\phi)} A dA\right)dX.$$

Due to symmetry, the second term and the third term have the same value. The first term, the new production, is

$$\begin{aligned}
\frac{1}{2}\int_{\overline{A}}^{1} z^2\left(z - \frac{\phi}{z}\right)dz &= \frac{1}{8}\left\{1 - \overline{A}^4 - 2\phi\left(1 - \overline{A}^2\right)\right\} \\
&= \frac{1}{8}\left(1 - \frac{\phi}{1-\kappa}\right)\left(1 + \phi\frac{2\kappa - 1}{1-\kappa}\right).
\end{aligned}$$

The second term, the lost production for targets, is

$$\begin{aligned}
\frac{1}{2}\int_{\overline{A}}^{1}\left(A\left\{Q_A(\phi)\right\}^2\right)dA &= \frac{1}{8}\int_{\overline{A}}^{1}\left(A^3 - 2\phi A + \frac{\phi^2}{A}\right)dA \\
&= \frac{1}{8}\left[\frac{1}{4}\left(1 - \overline{A}^4\right) - \phi\left(1 - \overline{A}^2\right) - \phi^2\ln\overline{A}\right] \\
&= \frac{1}{8}\left[\frac{1}{4} - \phi + \left(\frac{\phi}{1-\kappa}\right)^2\frac{3-\kappa}{4} - \phi^2\ln\sqrt{\frac{\phi}{1-\kappa}}\right].
\end{aligned}$$

Combining these together,

$$\begin{aligned}
G^F(\phi) &= \frac{1}{8}\left[\left(1 - \frac{\phi}{1-\kappa}\right)\left(1 + \phi\frac{2\kappa-1}{1-\kappa}\right) - 2\left\{\frac{1}{4} - \phi + \left(\frac{\phi}{1-\kappa}\right)^2\frac{3-\kappa}{4} - \phi^2\ln\sqrt{\frac{\phi}{1-\kappa}}\right\}\right] \\
&= \frac{1}{16}\left\{1 - \left(\frac{\phi}{1-\kappa}\right)^2 + 2\phi^2\ln\frac{\phi}{1-\kappa}\right\}.
\end{aligned}$$

Next, we compute gains from trade in the coarse market. The expected value of new production is

$$NP_0 \equiv a\int_{\frac{P_0}{a}}^{\overline{A}} X\left(a - \frac{P_0}{X}\right)dX = \frac{\left(\overline{A}a - P_0\right)^2}{2}.$$

Targets' lost production is

$$TL_0 = \int_{A_{\min}}^{\overline{A}} \left(A \int_0^{\frac{P_0}{A}} X dX \right) dA = \frac{P_0^2}{2} \ln \frac{\overline{A}}{A_{\min}}.$$

Bidders' lost production is

$$BL_0 = \int_{\frac{P_0}{a}}^{\overline{A}} \left(X \int_0^{a-\frac{P_0}{X}} A dA \right) dX = \frac{1}{4} \left(\overline{A}a - P_0\right) \left(\overline{A}a - 3P_0\right) + \frac{P_0^2}{2} \ln \frac{\overline{A}a}{P_0}.$$

Total lost production is

$$TL_0 + BL_0 = \frac{1}{4} \left(\overline{A}a - P_0\right) \left(\overline{A}a - 3P_0\right) + \frac{P_0^2}{2} \ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right).$$

The expected welfare gains are

$$\begin{aligned} G^C(\phi) &\equiv NP_0 - (TL_0 + BL_0) \\ &= \frac{\left(\overline{A}a - P_0\right)^2}{2} - \frac{1}{4} \left(\overline{A}a - P_0\right) \left(\overline{A}a - 3P_0\right) - \frac{P_0^2}{2} \ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right) \\ &= \frac{1}{4} \left(\overline{A}a - P_0\right) \left(\overline{A}a + P_0\right) - \frac{P_0^2}{2} \ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right). \end{aligned}$$

Using the market-clearing condition (10), $\ln \left(\frac{a}{P_0 A_{\min}} \overline{A}^2 \right) = \frac{\overline{A}a}{P_0} - 1$. Substituting this into the above expression of $G^C(\phi)$,

$$G^C(\phi) = \frac{1}{4} \left(\overline{A}a - P_0\right) \left(\overline{A}a + P_0\right) - \frac{P_0^2}{2} \left(\frac{\overline{A}a}{P_0} - 1 \right) = \left(\frac{\overline{A}a - P_0}{2} \right)^2.$$

Using $P_0 = \frac{\overline{A}a}{2}$ and $a = \kappa \overline{A}$,

$$G^C(\phi) = \frac{\kappa^2 \overline{A}^4}{16} = \frac{1}{16} \left(\frac{\kappa \phi}{1 - \kappa} \right)^2.$$

Finally, the total gains from trade in a hybrid market structure is

$$\begin{aligned} G^{H}(\phi) &\equiv G^{F}(\phi) + G^{C}(\phi) \\ &= \frac{1}{16}\left\{1 - \left(\frac{\phi}{1-\kappa}\right)^{2} + 2\phi^{2}\ln\frac{\phi}{1-\kappa} + \left(\frac{\kappa\phi}{1-\kappa}\right)^{2}\right\} \\ &= \frac{1}{16}\left\{1 - \phi^{2}\left(\frac{1-\kappa^{2}}{(1-\kappa)^{2}} - 2\ln\frac{\phi}{1-\kappa}\right)\right\} \\ &= \frac{1}{16}\left\{1 - \phi^{2}\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi}{1-\kappa}\right)\right\}. \end{aligned}$$

Because ϕ_H satisfies $\phi_H\left(\frac{1+\kappa}{1-\kappa} - 2\ln\frac{\phi_H}{1-\kappa}\right) = 1$, $G^{H}(\phi_H) = \frac{1}{16}(1-\phi_H)$. ■

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