Asset Prices in a Small Production Network^{*}

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Abstract

This paper constructs an asset-pricing model where firms in heterogeneous sectors interact with each other in a network as producers and consumers of materials and investment goods. Idiosyncratic sectoral shocks are transmitted through the network with the dynamics being affected by the heterogeneity in production functions and capital adjustment costs. The model is estimated using sectoral and aggregate U.S. data. Results show that 1) shocks to the primary sector account for a substantial part of the equity premium in all sectors because their volatility is much higher than that of shocks to the other sectors, and 2) the model endogenously generates volatility clusters despite the fact that shocks are conditional homoskedatic. These results depend crucially on the presence of network effects.

JEL Classification: E44, G12

Key Words: Asset-pricing, production economy, stock returns, networks, input-output, sectoral shocks.

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1. Introduction

This paper studies asset pricing in a production economy where firms in heterogenous sectors interact with each other in a network. Interactions take place directly from the production and consumption of materials and investment goods, and indirectly from the fact that the consumption bundle purchased by households is composed of goods produced by all sectors. The focus is on the implications of sectoral heterogeneity and network interactions for sectoral stock returns and on the transmission of sectoral disturbances to real and financial variables of all sectors through the network.

I consider here the simplest possible network with three sectors, namely a primary sector that produces raw materials, a manufacturing sector, and a service sector. Three is the minimal number of sectors with nontrivial interactions between sectors. The deliberate choice of working with a small network allows me to construct and estimate a rich model with multiple sources of heterogeneity and to fully examine its time-series implications.¹ Estimates show substantial heterogeneity in production functions, capital-adjustment costs, and the volatility of productivity innovations across sectors. In particular, the standard deviation of shocks to the primary sector is two orders of magnitude larger than the standard deviation of shocks to manufacturing and services.

Using impulse-response analysis, I trace out the propagation of sectoral shocks through the network and their effect on sectoral and aggregate variables. I find that heterogeneity in capital-adjustment costs and in network interactions induce different dynamics across sectors in response to the same shock. The large volatility of shocks to the primary sector and their propagation through the network play a key role in the financial variables of all sectors despite the fact that this sector is small and it does not produce capital goods. I show that shocks to the primary sector account for around about 20% of the equity risk premia in both manufacturing and services. In contrast, in a model without network interactions shocks to the primary sector account for about 1% of the equity risk premia in these sectors. Finally, I find that the nonlinear asset-pricing model can endogenously generate volatility clusters in sectoral stock returns despite the fact that shocks are conditionally homoskedastic.

This paper contributes to two branches of the literature on asset pricing. First, this paper contributes to the macro-finance literature that studies asset pricing in production economies. This literature is generally concerned with aggregate stock returns and models production as taking place in a representative firm subject to an aggregate productivity shock. Important contributions

¹As part of this research agenda, a companion paper (Ruge-Murcia, 2018) studies the cross-section implications of the model using data at a higher level of disaggregation with 31 sectors at the two-digit level of the North American Industry Classification System (NAICS).

include Cochrane (1991), Rouwenhorst (1995), Jermann (1998), Tallarini (2000), Campanale et al. (2010), and Croce (2014). Ready (2018) studies the effect of oil shocks on stock returns in a partial equilibrium setup. Compared to this literature, the focus here is on sectoral, rather than on aggregate, stock returns. To that end, I relax the assumption that all firms are identical and subject to the same shocks and assume instead that firms belong to one of a finite number of sectors. Firms in the same sector are identical, but firms in different sectors have different production functions, use different combinations of materials and investment goods to produce their output, face difference costs to adjust their capital stock, and are subject to idiosyncratic productivity shocks with different persistence and volatility. Firms in different sectors interact with each other in the market for intermediate inputs and investment goods in a manner consistent with the U.S. input-output accounts. This model builds on research that studies business cycles in multi-sector economies (e.g., Horvath, 1998, and, specially, Bouakez et al., 2009), but focuses instead on the asset-pricing implications of sectoral heterogeneity and differs methodologically by going beyond standard linear solutions. I show that nonlinearity is important, for instance, to generate conditional heteroskedasticity endogenously.

Second, this paper contributes to the finance literature on network effects on asset prices. Recent contributions include Buraschi and Porchia (2012), Ahern (2013), Herskovic (2015), and Ramirez (2017). These papers work at a very high level of disaggregation and focus the role of centrality (that is, the number of connections with other sectors) in asset prices. Ahern finds that more central firms have higher stock returns because they are more exposed to sectoral shocks through inter-sectoral trade. Ramirez finds that more central firms command a lower risk premia because of a greater diversification in customers and suppliers. My research complements their work by exploring the role of sources of heterogeneity other than network centrality on sectoral stock returns. For instance, I show that heterogeneity in the volatility of sectoral shocks is important to understand equity risk premia in different sectors.

The paper is organized as follows. Section 2 presents the production and consumption network, and describes the nonlinear solution method used to solve model. Section 3 presents the data and econometric strategy, and reports the parameter estimates. Section 4 reports the results from the analysis. Finally, Section 5, concludes.

2. The Network

This section describes an economy where firms in different sectors interact directly with each other in the market for intermediate goods and indirectly in the market for final goods consumed by households.

2.1 Production and Intermediate Consumption

Production is carried out by perfectly competitive firms in each of S = 3 heterogenous sectors. For concreteness, think of these sectors are producers of raw materials, manufactured goods, and services, respectively. The representative firm in sector $s \in S$ uses the technology

$$y_t^s = z_t^s (z_t n_t^s)^{\eta^s} (K_t^s)^{\alpha^s} (M_t^s)^{\theta^s},$$
(1)

where y_t^s is output, z_t^s and z_t are productivity shocks, n_t^s is labor, K_t^s is capital, M_t^s is materials, and $\eta^s, \alpha^s, \theta^s \in (0, 1)$ are parameters. The shock z_t^s is sector-specific and affects only the firms in sector s, while the shock z_t is aggregate and affects all firms in all sectors simultaneously. Note that the sector-specific shock is a total-factor productivity (TFP) shock and the aggregate shock is a labor-augmenting productivity shock. The technology is constant returns to scale and, thus, $\eta^s + \alpha^s + \theta^s = 1$. The capital stock is owned by firms while labor is rented from households at the rate w_t^s .

Materials are purchased from all sectors and combined according to

$$M_t^s = \prod_{i=1}^S \zeta_{is}^{-\zeta_{is}} (m_{i,t}^s)^{\zeta_{is}},$$
(2)

where $m_{i,t}^s$ is the quantity of good purchased from sector i and $\zeta_{is} \ge 0$ are weights that satisfy the restriction $\sum_{i=1}^{S} \zeta_{is} = 1$. The weights vary across purchasing sectors and, thus, each sector uses a different combination of materials to produce its output. The production structure is round-about meaning that in principle all sectors use materials from all sectors. In the empirical part of this paper, the weights ζ_{is} are computed from the Use table of the U.S. Input-Output (I-O) accounts and, hence, the flows of materials across sectors will be in line with those observed in the data. The price of M_t^s is $Q_t^{M^s} = \prod_{i=1}^{S} (p_t^i)^{\zeta_{is}}$, where p_t^i is the price of good i.

Investment goods are also purchased from all sectors and combined according to

$$X_t^s = \prod_{i=1}^S \kappa_{is}^{-\kappa_{is}} (x_{i,t}^s)^{\kappa_{is}},\tag{3}$$

where $x_{i,t}^s$ is the quantity of good purchased from sector *i* and $\kappa_{is} \ge 0$ are weights that satisfy the restriction $\sum_{i=1}^{S} \kappa_{is} = 1$. Since the weights vary across purchasing sectors, the aggregate X_t^s is a sector-specific combination of investment goods. The special case where $\kappa_{is} = 0$ covers the situations where 1) the sector *i* does not produce any investment good or 2) it produces investment goods but they are not useful in the production of good *s*. In the empirical part of this paper, the weights κ_{is} are computed from the Capital Flow table of the I-O accounts and, thus, the flows of investment goods across sectors and the composition of sectoral capital stocks will be in agreement with those in the data. The price of X_t^s is $Q_t^{X^s} = \prod_{i=1}^{S} (p_t^i)^{\kappa_{is}}$.

The investment aggregate X_t^s is added to the current capital stock (net of depreciation) to form the capital that will be used in production in the next period. That is,

$$K_{t+1}^{s} = (1 - \delta)K_{t}^{s} + X_{t}^{s} - \Gamma_{t}^{s},$$
(4)

where $\delta \in (0, 1)$ is the rate of depreciation. The function Γ_t^s represents the cost of installing or uninstalling additional units of capital and it is assumed to have the convex form

$$\Gamma_t^s = (\chi^s/2) \left(X_t^s / K_t^s - \delta^* \right)^2 K_t^s, \tag{5}$$

where $\chi^s \ge 0$, $\delta^* = \delta + \varsigma - 1$, and $\varsigma > 1$ is the gross rate of growth of the economy. Capital-adjustment costs permit variations in Tobin's q over time and across sectors and limit the households' ability to smooth the volatility of marginal rates of substitution (see Jermann, 1998). The functional form (5) implies that capital-adjustment costs in the steady state are zero.

Sectoral interactions are not summarized here using a single network (e.g., as in Ahern, 2013 who constructs a social accounting matrix that incorporates trade flows across all industries, consumers, and the government.) Instead, sectors interact differently in the markets for materials, investment goods, and consumption goods. I follow this modeling strategy for two reasons. First, technology and the nature of the goods i and s require the weights ζ_{is} and κ_{is} to be different. In particular, we will see in the empirical section of the paper that the Use table has relatively large diagonal entries—meaning, for example, that the service sector is a large provider of materials for the production of services—, while the Capital Flow table is sparse and has large entries in the row corresponding to manufacturing because most investment goods are produced by this sector. Second, the difference in sectoral interactions in the markets for material and investment goods has dynamic implications because materials are used within the period but investment goods can be transferred intertemporally in the form of capital.

The representative firm maximizes

$$E_{\tau} \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Lambda_{\tau,t} d_t^s, \tag{6}$$

where E_t is the expectation conditional on information available at time τ , $\beta \in (0, 1)$ is the discount factor, $\Lambda_{\tau,t}$ is the ratio of the shareholders' marginal utilities between periods t and τ , and d_t^s is profits. Profits are total revenue minus total costs,

$$d_t^s = p_t^s \left(c_t^s + \sum_{j=1}^S x_{s,t}^j + \sum_{j=1}^S m_{s,t}^j \right) - \left(w_t^s n_t^s + \sum_{i=1}^S p_t^i x_{i,t}^s + \sum_{i=1}^S p_t^i m_{i,t}^s + \Gamma_t^s Q_t^{X^s} \right),$$
(7)

where c_t^s is final consumption by households, and $x_{s,t}^j$ and $m_{s,t}^j$ is intermediate consumption by sector j in the form of investment good and materials input, respectively. Notice that final and intermediate goods are physically the same good and differ only by whether they are consumed by households or by firms to produce other goods.² The firm takes as given the demand functions by households and other firms. The costs are the wage bill, total expenditures on capital goods, total expenditures on materials inputs, and the cost incurred by adjusting the capital stock. Firms do not issue new shares and all investment is financed through retained earnings. Since the production function is constant returns to scale and firms are perfectly competitive, profits are simply the return on capital net of investment and adjustment costs. In every period, profits are transferred to shareholders in the form of dividends.

The consumption of materials produced by sector i is the solution to

$$\max_{\{m_{i,t}^{s}\}} \prod_{i=1}^{S} (\zeta_{is})^{-\zeta_{is}} (m_{i,t}^{s})^{\zeta_{is}},$$
(8)

subject to the constraint that $\sum_{i=1}^{S} p_t^i m_{i,t}^s$ equals a given expenditure level. The solution is

$$m_{i,t}^s = \zeta_{is} Q_t^{M^s} M_t^s / p_t^i.$$

$$\tag{9}$$

Note that $\sum_{i=1}^{S} p_t^i m_{i,t}^s = Q_t^{M^s} M_t^s$. Similarly, the consumption of investment goods produced by sector i is

$$x_{i,t}^s = \kappa_{is} Q_t^{X^s} X_t^s / p_t^i, \tag{10}$$

with $\sum_{i=1}^{S} p_t^i x_{i,t}^s = Q_t^{X^s} X_t^s$. Due to the assumption of Cobb-Douglas aggregators in (2) and (3), the expenditure shares of capital goods and materials inputs produced in sector *i* are constant and equal to the weights κ_{is} and ζ_{is} , respectively.

2.2 Final Consumption

Households are identical, infinitely-lived, and their total number is normalized to be one. The representative household has recursive preferences over consumption (Epstein and Zin, 1989),

$$U_t = \left((1-\beta) (C_t)^{1-1/\psi} + \beta \left(E_t \left(U_{t+1}^{1-\gamma} \right) \right)^{(1-1/\psi)/(1-\gamma)} \right)^{1/(1-1/\psi))},$$
(11)

 $^{^{2}}$ For instance, in the same way that a car is a final good if purchased by a household and an intermediate good if purchased by a leasing firm for the purpose of producing car rentals.

where C_t is final consumption, γ is the coefficient of relative risk aversion, and ψ is the intertemporal elasticity of substitution (IES). Consumption is a composite of goods produced in all sectors,

$$C_t = \prod_{s=1}^{S} (\xi^s)^{-\xi^s} (c_t^s)^{\xi^s},$$
(12)

where c_t^s is consumption of good s and $\xi^s \ge 0$ are weights that satisfy $\sum_{s=1}^{S} \xi^s = 1$. The household supplies its time endowment in a competitive labor market in every period. For convenience, the time endowment is normalized to be one. Labor is completely mobile between sectors.

The financial assets in this economy are shares and one-period bonds, both of which can be traded costlessly. Shares are claims on the profits made by the firms in each sector and are bundled into a mutual fund for that sector. Bonds are riskless in the sense that they pay one unit of consumption at maturity regardless of the state of nature. The household's budget constraint is

$$\sum_{s=1}^{S} p_t^s c_t^s + q_t^b b_t + \sum_{s=1}^{S} q_t^s a_t^s = \sum_{s=1}^{S} w_t^s n_t^s + b_{t-1} + \sum_{s=1}^{S} (d_t^s + q_t^s) a_{t-1}^s,$$
(13)

where q_t^b is the price of a bond, b_t is the number of bonds, and q_t^s and a_t^s are respectively the price of a share and the number of shares of the mutual fund of sector s. The aggregate price index is

$$P_t = \prod_{s=1}^{S} (p_t^s)^{\xi^s}.$$
 (14)

The aggregate price index serves as the numeraire in this model and, hence, $P_t = 1$ for all t.

The final consumption of the good produced in sector s is the solution to

$$\max_{\{c_t^s\}} \prod_{s=1}^{S} (\xi^s)^{-\xi^s} (c_t^s)^{\xi^s}, \tag{15}$$

subject to the constraint that $\sum_{s=1}^{S} p_t^s c_t^s$ equals a given expenditure level. The solution is

$$c_t^s = \xi^s C_t / p_t^s, \tag{16}$$

which implies $\sum_{s=1}^{S} p_t^s c_t^s = C_t$. Since ξ^s varies across sectors, expenditure shares will vary across sectors as well. The assumption of a Cobb-Douglas aggregator in (12) implies that the expenditure share of goods produced in sector s in total consumption is equal to the weight ξ^s .

2.3 Asset Pricing

The Euler equations that describe the household's utility maximization are

$$q_t^b = \beta E_t \left(\Lambda_{t,t+1} \right), \tag{17}$$

$$q_t^s = \beta E_t \left(\Lambda_{t,t+1} (d_{t+1}^s + q_{t+1}^s) \right), \tag{18}$$

for s = 1, 2, ..., S, where

$$\Lambda_{t,t+1} = (V_{t+1}/W_t)^{1/\psi - \gamma} (C_{t+1}/C_t)^{-1/\psi}, \qquad (19)$$

 $V_t \equiv \max U_t$ is the value function, and $W_t \equiv E_t V_{t+1}$ is the certainty-equivalent future utility. As usual, Euler equations compare the marginal cost of acquiring an additional unit of the financial asset (thus, sacrificing some current consumption) with the discounted expected marginal benefit of keeping the asset until next period.

Define the gross return on shares of sector s as $r_{t+1}^s = (d_{t+1}^s + q_{t+1}^s)/q_t^s$ and rewrite equation (18) as

$$1 = E_t \left(\beta \Lambda_{t,t+1} \right) E_t (r_{t+1}^s) + R_t^s \tag{20}$$

where

$$R_t^s = cov_t \left(\beta \Lambda_{t,t+1}, r_{t+1}^s\right) \tag{21}$$

is the risk premium. Then, using equation (17) and defining the gross yield of the riskless bond as $r_{t+1}^b = 1/q_t^b$, equation (20) can be written as

$$E_t(r_{t+1}^s) - r_{t+1}^b = -r_{t+1}^b R_t^s, (22)$$

where the left-hand side is the excess return of equity in sector s. Equation (22) has the usual implication that the excess return is positive when the return, r_{t+1}^s , is negatively correlated with the pricing kernel, $\beta \Lambda_{t,t+1}$, which means that the return is high when the marginal utility of consumption is high. Note that if sectoral stock returns covary in a quantitatively different way with the pricing kernel, the risk premium and excess return will vary systematically across sectors.

2.4 Shocks

The sector-specific shock follows the process

$$\ln z_t^s = \rho_s \ln z_{t-1}^s + \epsilon_{s,t},\tag{23}$$

where $\rho_s \in (-1, 1)$ is the autocorrelation coefficient and $\epsilon_{s,t}$ is an independent and identically distributed (i.i.d.) innovation with mean zero and standard deviation σ_{ϵ_s} . The persistence and

variance of this shock may vary across sectors, which is why the autocorrelation coefficient and the standard deviation of the innovation are indexed by the subscript s.

The aggregate shock follows the process

$$\ln z_t = (1 - \rho)\varsigma + \ln z_{t-1} + \rho(\ln z_{t-1} - \ln z_{t-2}) + \epsilon_t, \qquad (24)$$

where ς is the mean gross rate of growth of the economy, $\rho \in (-1, 1)$, and ϵ_t is an i.i.d. innovation with mean zero and standard deviation σ . This shock specification is attractive because 1) it helps capture the high persistence in the data, 2) it permits the identification of the aggregate shock, and 3) it delivers a balanced-growth path where all variables grow at a constant rate while allowing heterogeneity in sectoral shocks. Because labor productivity is non-stationary and there is long-run growth in this economy, the model will be rendered stationary by rescaling all variables by z_{t-1} .

2.5 Aggregate Resource Constraint

Using the facts that share holdings in each sector add up to 1, that net bond holdings are zero (because agents are identical), and that wages are the same in all sectors (because labor is completely mobile across sectors), the aggregate counterpart of the household's budget constraint is

$$\sum_{s=1}^{S} p_t^s c_t^s = w_t + \sum_{s=1}^{S} d_t^s,$$
(25)

Write profits in sector s as

$$d_t^s = Y_t^s - w_t n_t^s - Q_t^{X^s} X_t^s - \Gamma_t^s Q_t^{X^s},$$

where

$$Y_t^s = p_t^s \left(c_t^s + \sum_{s=1}^S x_{s,t}^i + \sum_{i=1}^S m_{s,t}^i \right) - \sum_{i=1}^S p_t^i m_{i,t}^s$$
(26)

is the value added in sector s (i.e., gross output minus the cost of materials inputs). Then, aggregate profits are

$$\sum_{s=1}^{S} d_t^s = \sum_{s=1}^{S} Y_t^s - w_t - \sum_{s=1}^{S} Q_t^{X^s} X_t^s - \sum_{s=1}^{S} \Gamma_t^s Q_t^{X^s},$$
(27)

where I have used the assumption that the total time endowment is equal to one. Substituting (27) into (25) and rearranging yields

$$C_t + \sum_{s=1}^{S} Q_t^{X^s} X_t^s + \sum_{s=1}^{S} \Gamma_t^s Q_t^{X^s} = \sum_{s=1}^{S} Y_t^s.$$
 (28)

Equation (28) is the aggregate resource constraint whereby the sum of aggregate consumption and aggregate investment (including adjustment costs) equals aggregate output measured in terms of value added.

2.6 Solution Method

Since this model does not have an exact analytical solution, I use a perturbation method to compute an approximate nonlinear solution. Perturbation methods consist of 1) the exact solution to a simplified form of the original problem and 2) a power series that characterizes deviations from the exact solution.³ In this paper, the exact solution are the allocations and prices in the deterministic steady state of the model and the power series is the second-order expansion of the policy functions around the steady state (see Jin and Judd, 2002, and Schmitt-Grohé and Uribe, 2004). This perturbation method is computationally faster than projection methods like value function iteration and it performs as well in terms of Euler equation accuracy (see Caldara et al., 2012). The former is an important advantage for this project because, as discussed below in section 3.5, the econometric estimation of the model requires solving the model in each iteration of an optimization routine.

The approximate solution consists of linear and quadratic terms in the state variables and a constant risk-adjustment factor. The state variables are the capital stocks in all sectors, the sector-specific productivity shocks, and the aggregate productivity shock. The risk-adjustment factor is a linear combination of the variances of the shock innovations (see Andreasen, 2012). I exploit this observation below to quantify the relative contribution of each shock to the equity premium in each sector.

3. Estimation

This section describes the data and the econometric strategy used to estimate the model, and report the parameter estimates.

3.1 Data

The data used to estimate the model are quarterly observations of the growth rate of consumption, the growth rate of investment, the real return on 3-month Treasury bills (T-bills), and real returns on stocks of three broad sectors on the U.S. economy, namely the primary sector (which produces raw materials), manufacturing, and services. The sample period is from 1966Q1 to 2015Q4. The sample starts in 1966 because before this date there are missing observations in the data for stock returns in raw materials. The sample ends with the latest available observations at the time the data was collected.

Consumption is measured by personal consumption expenditures and investment is measured by private nonresidential fixed investment. The raw data are seasonally-adjusted and reported at

³For an introduction to perturbation methods in economics see Judd (1998).

the annual rate. The series are converted into real per-capita terms at the quarterly rate dividing by four, by the seasonally-adjusted consumer price index (CPI), and by civilian non-institutional population. The CPI and population are the average of the three monthly observations in each quarter. The 3-month Treasury bill serves as empirical counterpart of the one-period bond in the model and its real return is computed as the ratio of the nominal return on the bill and the realized CPI inflation rate, both measured in gross quarterly terms. By construction, this is an *ex-post* return and I assume that it differs from the *ex-ante* return in the model by a serially uncorrelated (measurement) error with mean zero and constant standard deviation. The former two assumptions—serial uncorrelation and zero mean—are satisfied under the assumption of rational expectations. The standard deviation is one of the parameters estimated below. The data were taken from the website of the Federal Reserve Bank of St. Louis (www.stlouisfed.org).

The data on stock returns by industry are constructed by Kenneth French using raw data from the Center for Research in Security Prices (www.crsp.com).⁴ Each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio at the end of June of each year depending on its four-digit Standard Industrial Classification (SIC) code at that time. Equally-weighted returns on industry portfolios are available at several level of disaggregation, but it easy to aggregate up to the three industries used in this project as follows: the primary sectors is SIC codes 0100 to 1499, which include agriculture, mining, and oil and gas extraction; manufacturing is SIC codes 1500 to 3999, which includes construction and durable and nondurable manufacturing; and services is SIC codes 4000 to 8999, which include transportation, communications, utilities, trade, and finance.⁵ The raw returns in the database are monthly and I compute the quarterly returns as the product of the three gross monthly returns of each quarter.

3.2 Production Functions

The parameters of the production functions in the three sectors are estimated using the sectoral input-output database (KLEM) constructed by Dale Jorgenson and described in Jorgenson and Stiroh (2000).⁶ The database contain quantities and producer prices of total output, capital services, labor inputs, and material inputs for U.S. sectors disaggregated at the two-digit level of the SIC for the period 1960 to 2005. Aggregation up to the three sectors used here is consistent with the one for stock returns. Thus, the primary sector is SIC codes 1 to 14, manufacturing is SIC codes 15 to 39, and services is SIC codes 40 to 89. The first-order conditions that describe the optimal

⁴The data are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html.

⁵In preliminary work, I also estimated the model using value-weighted returns. Results are very similar to those reported here and support the same conclusions.

⁶The data are available at http://scholar.harvard.edu/jorgenson/data.

choice of labor and materials imply

$$\eta^s = w_t^s n_t^s / P_t y_t^s, \tag{29}$$

$$\theta^s = Q_t^{M^s} M_t^s / P_t y_t^s, aga{30}$$

where $Q_t^{M^s} M_t^s = \sum_{i=1}^{S} p_t^i m_{i,t}^s$. Aggregating the KLEM data as described above allows me to compute the wage bill, total expenditures on materials, and the value of total output for each of the three sectors for each year in the sample. In turn, the ratios (29) and (30) deliver estimates of η^s and θ^s for each sector and year of the sample. Under the assumption that the production function is constant returns to scale, an estimate of α^s for each sector and year is computed as $1 - \eta^s - \theta^s$. The final estimates of η^s , θ^s , and α^s are the sample averages of these yearly estimates and their standard deviations are $\sqrt{\sigma^2/T}$ where T = 46 is the sample size and σ^2 is the variance of the yearly observations. The estimates are reported in table 1. Three observations follow from table 1. First, the primary sector and manufacturing are intensive in materials inputs, while services is intensive in labor. Second, the production function parameter are quantitatively and statistically different across sectors. Finally, materials inputs are a large share of productive inputs in all sectors and, thus, network interactions are likely to be quantitatively important for the understanding stock returns at the sectoral level.

3.3 Consumption Weights

The estimation of the consumption weights uses the implication of the Cobb-Douglas aggregator (12) that the optimal expenditure share on goods from each sector is constant and equal to ξ_s . I estimate these shares using the data from the column "Personal Consumption Expenditures" in the final-user part of the 1992 Use table of the Input-Output (I-O) accounts.⁷ Producing sectors are aggregated up to the three sectors examined here and the shares are computed as the ratio of the purchases by consumers of commodities from sector *s* over total consumption expenditures. By construction, $\xi_s \in [0,1]$ and $\sum_{s=1}^{S} \xi_s = 1$ for all *s*. These shares are reported in the last column of table 1 and show that final consumption consists mostly of services (79.2 percent) and manufactured goods (20.1 percent), and that (not surprisingly) households consume limited quantities of raw materials directly. This means that the key interaction in the final goods market is that between services and manufacturing.

 $^{^{7}}$ I use the 1992 tables because they are roughly in the middle of the sample and, thus, capture the average interaction between sectors during the period. To evaluate whether result may depend on the tables used, I performed the same calculations described in sections 3.3 and 3.4 using the tables for 1982. However, results are very similar because the expenditure shares are relatively stable at the level of disaggregating that I consider here.

3.4 Materials and Investment-Good Weights

The optimal choice of materials inputs and investment goods in equations (9) and (10) imply that the expenditure share on goods purchased from sector i is constant and equal to the weights ζ_{is} and κ_{is} , respectively. Thus, I estimate these weights using expenditures shares computed using data from the I-O accounts as follows.

For the materials inputs weights ζ_{is} the raw data come from the 1992 Use table of the I-O accounts. This table reports the total use of commodities by intermediate users in producer prices, with rows containing commodities and columns containing users. The table is produced at different levels of disaggregation but it is easy to aggregate up to the three sectors studied here. I compute the expenditure shares as the ratio of the purchases by sector s of commodities from sector i over the total purchases by sector s. By construction, $\zeta_{is} \in [0, 1]$ and $\sum_{i=1}^{S} \zeta_{is} = 1$ for all s. The fact that purchases are in producer prices is consistent with the model, where there are no taxes and, hence, consumer and producer prices coincide.

I equate commodities with sectors as in the model, where good s is produced only by sector s. This amounts to assuming that the Make table of the I-O accounts is diagonal and permits the estimation of the weights ζ_{is} employing the Use table alone. The Make table reports the value of each commodity produced by each industry and it is not perfectly diagonal because there are commodities classified under one sector in the I-O accounts despite that fact that they are physically produced in another sector. An example of such a commodity is printed advertisement, which is classified in the I-O accounts as a service even though it is produced by printing and publishing. I examine the quantitative importance of the off-diagonal elements are all above 0.99, I conclude that treating the Make table as diagonal is a reasonable approximation at this level of disaggregation. The weights (shares) ζ_{is} computed using the Use table are reported in table 2 and show that, at this level of disaggregation, all sectors use materials from all sectors, as in the round-about structure assumed in the model.

For the investment goods weights κ_{is} , the raw data come from 1992 capital flow table (CFT). The CFT is a matrix with 163 commodities (rows) and 64 purchasing industries (columns). The 163 commodities (equipment and structures) are classified by commodity number, but it is trivial to match the commodity number with the SIC code of the producing industry. The 64 purchasing industries are classified by SIC code. The entries in the table are total flows in producer prices. I compute the expenditure shares as the ratio of the purchases by sector s of equipment and structures from sector i over the total purchases by sector s. By construction, $\kappa_{is} \in [0, 1]$ and $\sum_{i=1}^{S} \kappa_{is} = 1 \text{ for all } s. \text{ The weights (shares) are reported in table 3 and show that most of the U.S. capital stock is produced in the manufacturing sector, which here includes construction and durable manufacturing. The fact that the primary sector produces some of its own capital reflects the fact that oil and gas extraction (SIC code 13) produces a substantial part of its own capital stock. The small, but non-negligible, proportion of investment goods produced in the services sector is due to the fact that this sector includes services that are ancillary to investment (e.g., engineering services).$

3.5 SMM

The remaining parameters of the model are estimated by the simulated method of moments (SMM). Define by $\theta \in \Theta$ the $q \times 1$ vector of structural parameters. The SMM estimator is

$$\widehat{\theta} = \arg\min_{\{\theta\}} \left((1/T) \sum_{t=1}^{T} m_t - (1/\lambda T) \sum_{\iota=1}^{\lambda T} m_\iota(\theta) \right)' \mathbf{W} \left((1/T) \sum_{t=1}^{T} m_t - (1/\lambda T) \sum_{\iota=1}^{\lambda T} m_\iota(\theta) \right), \quad (31)$$

where **W** is a $q \times q$ weighting matrix, T is the sample size, λ is a positive integer, m_t is a $p \times 1$ vector of empirical observations on variables whose moments are of interest to us, and $m_t(\theta)$ is a counterpart of m_t with elements obtained from the simulation of the model. In words, the SMM estimator minimizes the weighted distance between the unconditional moments predicted by the model and those computed from the data, where the moments predicted by the model are obtained using artificial data simulated from the model. Lee and Ingram (1991) and Duffie and Singleton (1993) show that the SMM estimator is consistent and asymptotically normal with distribution

$$\sqrt{T}(\widehat{\theta} - \theta) \to N(\mathbf{0}, (1 + 1/\lambda)(\mathbf{J}'\mathbf{W}^{-1}\mathbf{J})^{-1}\mathbf{J}'\mathbf{W}^{-1}\mathbf{\Sigma}\mathbf{W}^{-1}\mathbf{J}(\mathbf{J}'\mathbf{W}^{-1}\mathbf{J})^{-1}),$$
(32)

where

$$\boldsymbol{\Sigma} = \lim_{T \to \infty} Var\left((1/\sqrt{T}) \sum_{t=1}^{T} \mathbf{m}_t \right)$$
(33)

and $\mathbf{J} = E(\partial m_{\iota}(\theta)/\partial \theta)$ is a finite Jacobian matrix of dimension $p \times q$ and full column rank. Estimation is computationally demanding because the model needs to be solved in each iteration of the minimization routine that solves (31). An additional computational cost arises from the fact the deterministic steady state depends nontrivially on the some of the model parameters contained in θ (e.g., the intertemporal elasticity of substitution) and, thus, the system of nonlinear equations that determines prices and allocations in steady state needs to be solved in each iteration of the minimization routine as well.

In this project, the weighting matrix is the identity matrix, the matrix Σ is computed using the Newey-West estimator with a Bartlett kernel and bandwidth given by the integer of $4(T/100)^{2/9}$,

where T = 200 is the sample size, and the matrix **J** is computed by taking numerical derivatives with respect the elements of θ at the optimum. The number of simulated observations is 100 times larger than the sample size (that is, $\lambda = 100$). Since the asymptotic variance of the SMM estimator differs from the one of the GMM (generalized method of moments) estimator by the factor $(1 + 1/\lambda)$, the use of the large value $\lambda = 100$ implies that the statistical efficiency of SMM here is basically the same as that of GMM. To limit the effect of starting values on the results, the simulated sample contains 5000 additional "training" observations that are discarded for the purpose of computing the moments. The dynamic simulations of the nonlinear model are based on the pruned version of the solution. I use here the pruning scheme proposed by Andreasen et al. (2017). The moments used to estimate the model are the means, variances, and the first-order autocovariances of all six series—consumption growth, investment growth and the real return on 3-month T-bills and stock portfolios in the primary sector, manufacturing, and services—. Thus, the total number of moments is 18.

The estimated parameters are the intertemporal elasticity of substitution (ψ) , the coefficient of risk aversion (γ) , the discount factor (β) , the capital-adjustment cost parameter in all sectors (χ^s) , the processes of the sectoral and aggregate productivity shocks, and the standard deviation of the measurement error of the safe-asset return. Thus, the total number of estimated parameters is 15. During the estimation procedure the depreciation rate is fixed to 0.025 (that is, 10% per year); the gross rate of aggregate productivity growth is fixed to 1.003 (that is, 1.2% per year); and the consumption weights, materials weights, investment-goods weights, and production function parameters are fixed to the estimates reported in tables 1 through 3.

The local identification of the model parameters requires that $rank(E(\partial m_{\iota}(\theta)/\partial \theta)) = q$, where θ is the point in the parameter space Θ where the rank condition is evaluated. I verified that this rank condition is indeed satisfied at the optimum $\hat{\theta}$ for both versions of the model.

3.6 Parameter Estimates

Estimates of the parameters of the model are reported in table 4, along with standard deviations computed using a parametric bootstrap with 199 replications.⁸ The estimate of the intertemporal elasticity of substitution (IES) is 0.75, which is statistically different from zero but not statistically different from 1. This estimate is quantitatively similar to values reported in the literature. For instance, Epstein and Zin (1991) reports values between 0.18 and 0.87 depending on the measure of consumption and on the set of instruments used to estimate the model, and Vissing-Jørgensen

⁸I use bootstrap rather than asymptotic standard errors because Monte-Carlo results in Ruge-Murcia (2012) suggest that the latter are not always a good approximation to the actual variability of SMM estimates of nonlinear models in small samples.

(2002) reports values between 0.30 and 1 depending on the households' asset holdings. Havranek (2015) performs a meta-analysis of 169 studies that estimate this parameter and concludes that the IES for asset holders is around 0.35. The risk aversion parameter is 6.64, which is moderately large and statistically different from zero. Since risk aversion is larger than the inverse of the IES, households prefer an early resolution of uncertainty. The subjective discount rate is 0.9996, which together with the IES and the mean of aggregate productivity growth, imply a gross quarterly rate of return of $1/(\beta \zeta^{1-1/\psi}) = 1.0009$ (that is, about 0.4% per year) in the deterministic steady state. As should be expected, this certainty-equivalent value is smaller from the average T-bill return of 0.8% per year observed in the data.

There is some variation in the estimate of the capital adjustment-cost parameter across sectors with the estimates for manufacturing and the primary sector (which includes mining) being much larger than the estimate for services. All estimates are significantly different from zero and the hypothesis that the cost parameter is the same in all sectors is rejected by the data. The persistence of sectoral productivity shocks is relatively high and similar across sectors, but there is a large difference between the standard deviation of productivity innovations in the primary sector and in the other two sectors. In particular, the standard deviation in the primary sector (0.17) is two orders order of magnitude larger than in manufacturing and services (0.0029 and 0.0020, respectively), and also two orders of magnitude larger than the standard deviation of aggregate productivity innovations (0.0085). As we will see below, the large standard deviation of shocks to the primary sector, combined with network interactions, will give this sector a key role in asset pricing despite the fact that it is relatively small (e.g., in terms of gross output).

The estimate of the standard deviation of the measurement error of the safe-asset return is 0.0076. Recall that this measurement error arises because the 3-month Treasury bill is a nominal bond whose *ex-post* real return was computed by adjusting for the realized CPI inflation rate. Instead, the safe asset in the model is a real bond whose return is known by the household at the time of purchase. Thus, the discrepancy between the safe return in the model and the measured return in the data arises from the error in forecasting future inflation. To examine whether the estimate of the standard deviation of the measurement error is plausible, I estimate a simple forecasting model of inflation—i.e., autoregressive process of order one—and compute the standard deviation of the forecasting error. The standard deviation is 0.0052, which is quantitatively close to the SMM estimate of 0.0076 from the structural model. Thus, I conclude that the SMM estimate of the measurement error is plausible.

4. Results

This section examines the time-series implications of the model. In particular, this section examines the effects of productivity shocks on real and financial variables at the sectoral and aggregate levels, and it quantifies the relative contribution of all shocks to the equity risk premia. This section also shows that the nonlinear model with network effects endogenously generates conditional heteroskedasticity.

4.1 Model Fit

Panel A of figure 1 reports the fit of the model by comparing the moments predicted by the model (vertical axis) with those computed using the U.S. data (horizontal axis). The moments are the mean, variance, and first-order autocovariance of consumption growth, investment growth, the real return of the 3-month T-bill, and the real return of stock portfolios in the primary, manufacturing, and service sectors. These were the moments used to estimate the model. The continuous line is the 45 degree line. If the model were to perfectly match the moments of the data, all dots would lie on this line. We can see in panel A that the predicted moments are quantitatively close to those of the data and that the correlation between the two set of moments is quite high: 0.997. Panel B of figure 1 performs a similar comparison for moments that were not used in the estimation of the variance-covariance matrix of the variables above. As before, the predicted and data moments are very close and their correlation is high (in this case, 0.977). Thus, in broad terms, the model successfully replicates key moments of the U.S. data, including some that were not used in its estimation.

A key moment of interest in the literature is the mean return of stocks compared with the mean return of bonds. The equity premium in the primary sector is 3.57%, in manufacturing is 6.75%, and in services is 6.47%. The model respectively predicts equity premia of 2.74%, 1.94%, and 1.91% for these sectors. Hence, although the model does generate sizable equity premia with moderate levels of risk aversion, they are generally smaller and less heterogenous than in the data.

4.2 The Transmission of Shocks between Sectors

Figures 2 through 4 report the responses of selected sectoral variables to a positive productivity shock in the primary, manufacturing, and service sectors, respectively. The size of the shock is one standard deviation of the respective innovation. Thus, by construction, the shock to the primary sector is larger than the shock to the other two sectors. The shock is assumed to take place when all variables are equal to the mean of their ergodic distribution. In all figures, the vertical axis is the percentage deviation from the ergodic mean and the horizontal axis is periods. The key observation from these figures is that a shock in one sector affects real and financial variables of all sectors through the production network. There are, however, differences in the dynamics as a result of differences in the persistence of the shocks, capital-adjustment costs, and the combinations of materials and investment goods used by each sector.

Consider first figure 2, which plots sectoral responses to a productivity shock in the primary sector. The shock leads to a large increase in the production of raw materials and a large reduction in its real price. The fact that manufacturing and services are inputs to the production of raw materials means that output in both of these sectors increases as well. The increase in output requires additional labor and capital and, hence, investment rises in all sectors. Since most investment goods are produced in the manufacturing sector (see table 3), the rise in investment in all sectors contributes further to the increase of output in manufacturing. In the end, figure 2 shows that the increase of output in manufacturing is larger than that in services. Part of the reason is that the proportion of expenditures on investment goods that goes into manufacturing is much larger than the proportion that goes into services: 57.5% versus 13.4% (see the first column in table 3). This effect is amplified as the shock propagates through the network. The increase of output in manufacturing is large enough that its real price decreases, while the real price of services increases.⁹ The response of investment is different across sectors, in part, because capital-adjustment costs are different. In particular, investment in manufacturing responds more sluggishly than in the other two sectors because it features large adjustment costs. Dividends are the return on capital net of investment and of adjustment costs and they increase in all sectors following the positive productivity shock to the primary sector. The increase in dividends is more pronounced in the primary and service sectors than in manufacturing. In contrast, share prices in all sectors increase by about the same proportion.

Figure 3 plots sectoral responses to a productivity shock in manufacturing. The shock leads to an increase in manufacturing output and a decrease in the real price of manufactured goods. Since the service and primary sectors supply inputs to manufacturing, their output increases as well. The output increase in services is one order of magnitude larger than in the primary sector because the former is a larger supplier of materials and investment goods to manufacturing than the latter. To see this note in the second column of table 2 that the proportion of manufacturing expenditures on materials that goes into services is 32% while the proportion that goes into raw

⁹Recall that the aggregate price index is the numeraire in this model. Thus, by construction, if one or more real prices decrease, then at least one real price must increase.

materials is only 10.5%. Also note in second column of table 3 that the primary sector is not a producer of investment goods for manufacturing while services receives 15.7% of manufacturing expenditures on investment goods. The real price of services and raw materials rise, but in the case of raw materials the response is relatively muted. In order to increase in output, firms in all sectors must hire additional labor and build up their capital stock and, thus, investment rises in all sectors. Interestingly, the increase in the capital stock is larger in services than in manufacturing reflecting both lower adjustment costs and higher capital intensity (see table 1). Dividends increase in all sectors (notably in services) because the value of the marginal return to capital increases in all sectors after the shock, and, consequently, share prices increase as well.

Finally, figure 4 plots sectoral responses to a productivity shock in services. The general pattern uncovered in figures 2 and 3 is present here as well. Thus, output in services increases but its real price decreases. The transmission of the shock via the production network induces an increase in output and real prices in the other sectors and a generalized increase in investment and the capital stock. The increase in the value of the marginal return to capital leads to increases in dividends and share prices in all sectors.

4.3 Dynamics of Consumption and Bond Prices

Figure 5 reports the effects on aggregate consumption, the real wage, bond prices, and bond yields of productivity shocks to the primary sector (column 1), manufacturing (column 2), and services (column 3). As before, the size of the shock is one standard deviation of the respective innovation and the shock takes place when all variables are equal to the mean of their ergodic distribution.

All productivity shocks lead to an increase in consumption and the real wage. The former is driven by a generalized increase in output (with prices adjusting to clear the goods markets), while the latter reflects the increase in the marginal product of labor associated with the increase in total factor productivity. The shock to the primary sector has the largest initial effect on both variables, but the response decreases quickly and undershoots, reaching the ergodic mean from below. The initial effects on bond prices are different for the different sectoral shocks. The shock to the primary sector leads to a large initial increase in the bond price and, hence, a large decrease in its yield, but its effects are relatively short-lived. Instead, shocks to manufacturing and services lead to smaller, but more persistent, increases in the bond price and decreases in its yield. In all case, the variables return monotonically to their ergodic mean.

4.4 Endogenous Volatility Clustering

A well-known feature of stock returns is that their conditional variance changes over time, with several periods of high volatility followed by several periods of lower volatility. This volatility clustering is modeled in the literature using stochastic volatility models or versions of the autoregressive conditional heteroskedasticity (ARCH) model due to Engle (1982). In both cases the process for the conditional variance of the shocks is modeled as time-varying and it is typically assumed to be exogenous. This section shows that the nonlinear asset-pricing model studied here can endogenously generate volatility clustering even though the shocks are conditionally homoskedastic.

Table 5 reports results of Lagrange Multiplier (LM) tests of the hypothesis of no conditional heteroskedasticity (Engle, 1982) applied to the quarterly U.S. data. The test is carried out on the residuals of a first-order autoregression and the statistic is calculated as the product of the number of observations and the uncentered R^2 of the ordinary least squares (OLS) regression of squared residuals on a constant and three of its lags. Under the null hypothesis, the statistic is distributed chi-square with three degrees of freedom. The *p*-values in table 5 show that the hypothesis of no conditional heteroskedasticity can be rejected at the 5% significance level for stock returns in raw materials and consumption growth and at the 10% for stock returns in manufacturing and services, but that it cannot be rejected for bond returns and investment growth.

The table also reports results of tests applied to artificial observations of series generated from the model. The hypothesis of no conditional heteroskedasticity can be rejected at the 5% level for all sectoral stock returns and for consumption growth, and the 10% for investment growth, but it cannot be rejected for bond returns. With the exception of investment growth, the implications from the model are in line with those from the U.S. data.

The time-varying volatility in stock returns predicted by the model is illustrated in figure 6, which plots 20000 observations of returns in the primary, manufacturing, and service sectors simulated from the model using the parameters reported in table 4. The facts that the model generates volatility clusters in the stock returns and that they tend to be synchronous across sectors are apparent in this figure.

In contrast with stochastic volatility or ARCH models where the conditional heteroskedasticity of stock returns follows from the conditional heteroskedasticity of the shocks, the conditional heteroskedasticity in this model is endogenous. In particular, conditional heteroskedasticity arises from the nonlinear propagation mechanism of the model and despite the fact that shocks are conditionally homoskedastic.¹⁰

¹⁰To my knowledge, Granger and Machina (2006) where the first to suggest that a nonlinear model can generate ARCH effects even when shocks are i.i.d. and parameters are time-invariant.

4.5 Composition of the Risk Premia

The equity risk premia in this model may be expressed as a linear combination of the variance of the innovations to the structural shocks. The shocks are an aggregate shock and three sectoral shocks. The aggregate shock hits simultaneously all firms in all sectors. A sectoral shock hits only the firms in one sector but affects firms in the other sectors via the network. In this section, I decompose the contribution of each shock to the risk premia in all sectors and report the results in the first column of table 6. (Results in the other columns will be discussed below in section 4.6.)

Results show that the contribution of the aggregate shock to the equity risk premia is large and accounts about three-quarters of the premia in all sectors, with the rest of the premia accounted for by sectoral shocks. Standard asset-pricing models predict that idiosyncratic risk generates no compensation or risk adjustment. However, in this model sectoral shocks are idiosyncratic only in the narrow the sense that the sectoral shock initially hits only firms in one sector. Thereafter, a sectoral shock affects all sectors because firms interact with each other in the production network. Hence, a sectoral shock contributes to aggregate risk and the risk associated with it is priced by the market. Results show that shocks to the primary sector are by far the most important sectoral shock and account for somewhat less than one-quarter of the risk premia in all sectors. This finding is driven by the large volatility of productivity shocks to this sector and by the network structure that transmits this shock to the other sectors. The contributions of the other sectoral shocks to the equity premia are negligible.

4.6 Counterfactual Experiments

In order to explore further the role of the network in the composition of the equity premia, I perform the decomposition under counterfactual assumptions about the network structure. I consider two cases. In the first case the input-output and capital-flow tables are diagonal and, thus, each sector produces its own investment goods and materials inputs. Notice that sectors do not interact in the market for intermediate goods, but they interact indirectly in the market for final goods consumed by households. This case understates sectoral interaction compared with the actual input-output and capital-flow tables. In the second case the input-output and capital flow tables are completely symmetric meaning that all entries in the tables are 1/3. This case overstates sectoral interaction compared with the actual input-output and capital-flow tables.

Consider now the decomposition of the equity premia under these two alternative scenarios. Results are reported in the second and third columns of table 6. In the version with no network interactions, the aggregate shock accounts for basically all the equity premia in the manufacturing and service sectors and for about 88% of the premia in the primary sector. Sectoral shocks account for little of the equity premia in all sectors, except in the primary sector where its own productivity shock accounts for 11% of the premia. Overall, this specification overestimates the importance of the aggregate shocks, and underestimates the importance of shocks to the primary sectors, in the determination of equity premia.

In the version with a symmetric network, the primary sector is a larger supplier of investment and materials goods to the other sectors than in the actual U.S. economy. As a result its volatile idiosyncratic shocks are strongly transmitted to the real and financial variables of other sectors. In particular, column 3 of table 6 shocks that shocks to the primary sector now account for around 85% of the equity premia in all sectors, while the aggregate shock accounts for somewhat less than 15%. The other sectoral shocks account for a negligible proportion of the equity premia. Overall, this specification overstates the exposure of manufacturing and services to the shocks in the primary sector and, consequently, overestimates the importance of primary shocks in the determination of their equity premia.

Figures 7 through 9 plot respectively the responses of sectoral variables to a positive productivity shock in the primary, manufacturing, and service sectors in the counterfactual networks. The thick lines are the responses in the case where the input-output and capital-flow tables are diagonal. The thin lines are the responses in the case where the tables are symmetric and all entries are equal to 1/3. Figure 7 shows that when sectors do not interact in the market for intermediate goods (thick lines) the effects of a shock to the primary sector on manufacturing and services is small or zero. Hence, the finding that shocks to the primary sector affect the financial and real variables in the other sectors depends crucially on network interactions in production. Figure 8 and 9 show that this conclusion also applies to sectoral shocks to manufacturing and services, but the effects are somewhat larger in these cases because both sectors also interact in the market for final goods. Comparing figures 7 through 9 with figures 2 through 4 shows that the effect of a sectoral shock on other sectors is always larger in the latter case where sectors buy and sell goods to each other in the production network.

Figures 7 through 9 also show that when the input-output and capital flow tables are symmetric (thin lines), the effects of a sectoral shock on other sectors is large. In particular, the effects are larger than those reported in figures 2 through 4 for realistic input-output and capital flow tables. This result indicates that the production network is a powerful mechanism that amplifies the effects of shocks in one sector on the financial and real variables of other sectors. However, this result also means that a realistic specification of the input-output and capital flow tables is important to obtain quantitatively accurate estimates of network effects.

5. Conclusions

This paper constructs a nonlinear asset-pricing model where heterogenous sectors interact with each other in a production network. The model is estimated using aggregate and sectoral U.S. data and impulse-response analysis shows how sectoral shocks are transmitted through the economy and affect dividends and stock prices in all sectors. By deliberately focusing on a small network, I am able to construct and estimate a rich model with multiple sources of heterogeneity and to fully characterize its time-series implications. Results show that sectoral shocks to the primary sector play an important role in the stock returns of all sectors because it is subject to very volatile shocks that are transmitted to the other sectors via the production network. Results also show that the nonlinear model endogenously generates conditional heteroskedasticity in stock returns despite the fact the shocks are conditionally homoskedastic. The comparison with counterfactual models shows that the production network amplifies the effects of shocks in one sector on the financial and real variables of other sectors above and beyond the effects that arise from sectoral interactions in the final goods market.

	Production Function Parameters						Consumption
	η^s		α^s		θ^s		Shares
Sector	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	ξ^s
Primary Manufacturing Services	0.203* 0.289* 0.395*	$0.030 \\ 0.011 \\ 0.011$	0.273* 0.104* 0.218*	$0.035 \\ 0.015 \\ 0.007$	0.524* 0.607* 0.387*	$\begin{array}{c} 0.024 \\ 0.012 \\ 0.014 \end{array}$	0.007 0.201 0.792

Table 1. Production Function Parameters and Consumption Shares

Note: The table reports estimates of final consumption shares and production function parameters for each sector. s.e. denotes standard error and * denotes significance at the 5 percent level.

	Consumer			
Producer	Primary	Manufacturing	Services	
D :	0.979	0.105	0.024	
Primary	0.372	0.105	0.034	
Manufacturing	0.258	0.575	0.229	
Services	0.370	0.320	0.738	

 Table 2. Input-Output Matrix

Notes: This table reports the share of total expenditures on materials inputs by the consuming sector that goes into goods from the producing sector. The shares were computed by the author using the table "The Use of Commodities by Industries" for 1992 produced by the BLS. Columns may not add up to one due to rounding.

	Consumer			
Producer	Primary	Manufacturing	Services	
Primary	0.291	0.000	0.000	
Manufacturing	0.575	0.843	0.896	
Services	0.134	0.157	0.104	

Table 3. Capital-Flow Table

Notes: This table reports the share of total expenditures on investment goods by the consuming sector that goes into goods from the producing sector. The shares were computed by the author using the table "Distribution of New Equipment and Structures to Using Industries in Producers' Prices" for 1992 produced by the BLS. Columns may not add up to one due to rounding.

Parameter	Estimate	b.s.e.
Preferences		
Elasticity of substitution	0.7486^{*}	0.1516
Risk aversion	6.6434^{*}	1.2870
Discount factor	0.9996^{*}	0.0001
Primary		
Capital-adjustment cost	8495.0^{*}	1211.4
Autoregressive coefficient	0.8922^{*}	0.0962
Standard deviation	0.1737^{*}	0.0414
Manufacturing		
Capital-adjustment cost	8135.4^{*}	1155.0
Autoregressive coefficient	0.8547^{*}	0.1038
Standard deviation	0.0029^{*}	0.0006
Services		
Capital-adjustment cost	3934.6^{*}	1241.8
Autoregressive coefficient	0.9500^{*}	0.0182
Standard deviation	0.0020^{*}	0.0003
Aggregate productivity		
Autoregressive coefficient	0.2185^{*}	0.0286
Standard deviation	0.0085^{*}	0.0024
Measurement error		
Standard deviation	0.0076^{*}	0.0014
	-	

Table 4. SMM Estimates

Note: The table reports SMM estimates of the model parameters under different network structures, b.s.e. denotes standard errors computed using a parametric bootstrap with 199 replications, and * denotes significance at the five percent level.

	Data	Model
	0.010	0.000
Consumption growth	0.013	0.023
Investment growth	0.406	0.084
3-month T-bill rate	0.231	0.539
Stock returns		
Primary	< 0.001	< 0.001
Manufacturing	0.096	< 0.001
Services	0.077	< 0.001

Table 5: Test of No Conditional Heteroskedasticity

Notes: This table reports *p*-values of Lagrange Multiplier tests of the hypothesis of no conditional heteroskedasticity (Engle, 1982).

		Counterfactuals		
		No Network	Symmetric	
$\operatorname{Sector}/\operatorname{Shock}$	Model	Interactions	Network	
Primary				
Aggregate	74.71	88.02	13.59	
Primary	24.64	11.08	86.32	
Manufacturing	0.35	0.18	0.04	
Services	0.30	0.73	0.05	
Manufacturing				
Aggregate	79.93	98.52	14.52	
Primary	19.36	0.83	85.39	
Manufacturing	0.39	0.17	0.04	
Services	0.33	0.48	0.05	
Services				
Aggregate	79.20	97.83	12.57	
Primary	20.07	1.17	87.34	
Manufacturing	0.40	0.17	0.04	
Services	0.32	0.82	0.05	

Table 6. Composition of the Equity Risk Premia (in %)

Note: The table reports the contribution of each shock to the equity risk premia for the model and counterfactuals.

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Figure 1: Model Fit



Figure 2: Responses to a Shock in the Primary Sector



Figure 3: Responses to a Shock in Manufacturing



Figure 4: Responses to a Shock in Services



Figure 5: Responses of Aggregate Variables to Sectoral Shocks



Figure 6: Volatily Clustering in Sectoral Stock Returns



Figure 7: Counterfactual Responses to a Shock in the Primary Sector



Figure 8: Counterfactual Responses to a Shock in Manufacturing



Figure 9: Counterfactual Responses to a Shock in Services

