Income Distribution Shock, Liquidity Trap and Aggregate Demand

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Abstract

This paper analyzes how future income distribution shock may lead to aggregate demand drop via trigger liquidity trap. Specifically, I analyzes how future income distribution shock affects the possibility of liquidity trap, and how the shock affect the aggregate demand and output when liquidity trap occurs. In my model, there is one borrower and one lender, and the borrower is less patient than the lender. The borrower is faced with a borrowing constraint each period, which is determined by his income or value of house he owns at that period. We also assume nominal rigidity and zero lower bound in this economy. In the baseline model I discussed four types of shocks: unanticpated temporary shock, anticipated temporary shock, unanticipated permanent shock, anticipated permanent shock. I study how these shocks trigger liquidity trap and during liquidity trap how these shocks reduce the aggregated demand and output, and compare the different results under different shocks. In the extension, I discussed how liquidity trap will happen due to an unanticipated income distribution shock in the future when there is a house market, and how the shock affects the output and asset price in the equilibrium when liquidity trap occurs. The main story behinind my model is that assuming the heterogeneity among households,income distribution shock reduces/increases the consumption demand of the households. However, due to the different MPCs among households, the total change of consumption demand is not zero. This model is still at its preliminary stage, trying to illustrate one channel by which the income inequality increases the possiblity of crises, and throw light upon the implication on redistribution policy.

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1 Introduction

Since the Global financial crises in 2008, there has long been a debate on the linkage between Economic inequality and financial instability. Data reveals that over the 30 years up to the 2008's financial crisis, inequality has been rising in a number of countries, and some literature suggests that widening inequality may has played a destabilizing role in the economy, especially in the recent crisis. Although there is a small but growing body of academic research that has attempted to formally analyze the relationship empirically and theoretically, there is no systematical model on the how household income inequality will trigger financial instability. Moreover, those theoretical papers failed to take the 'overborrowing' and 'externality' into consideration, which are considered as significant macroeconomic problems in terms of causing financial crises (Bianchi and Mendoza(2011)).

In response to this, this paper constructs a infinite model with nominal rigity and zero lower bound to illustrate how income distribution shock may trigger household to 'overborrow' and lead to financial crises,and how, during liquidity trap, the income distribution shock affects the aggregate demand and output. The crises this paper focuses on here is liquidity trap, an awkward condition in which monetary policy loses its effect because the nominal interest rate is essentially zero. In the baseline model , I assume there is one borrower and one lender, with the former being less patient than the latter. The borrower is faced with a borrowing constraint, which is determined by his income at the current period. I studies four types of shocks, unanticpated temporary shock, anticipated temporary shock, unanticipated permanent shock, anticipated permanent shock. Under the first three shocks I reached the similar conclusions, that is ,the increasing inequality may trigger liquidity trap and lead to output drop when liquidity trap occurs. However, under the final shock, the decreasing inequality but not increasing inequality may trigger liqudity trap.

In the extension model, I consider an economy with housing market, and the borrowing constraint is determined not by the borrower's income but by the value of housing that the borrower holds. The analysis shows that an unanticipated shock of income distribution (borrower becomes poorer and lender becomes richer) at $t = 1$lead to a increase in the housing price and drop in the interest rate in the equilibrium under proper conditions, and when the shock is sufficiently large, the borrower will 'overborrow' even though they are not intentionally doing so, and thus triggers liquidity trap. Moreover, the larger the shock, the larger the drop of aggregate demand in the equilibrium, and also the drop of housing price.

The mechanism that is described in this paper corresponds to the first channel of the four channels via which inequality trigger crises, channels that are put up by Stcockhammer (2013). In his paper, Stockhammer points out that the inequality may reduce consumption expenditures and thus on aggregate demand since the poorer has a higher MPC compared to the richer, Although he didn't construct a channel to illustrate it. One key ingredient in my paper is that the borrowing constraint depends on the income or asset value of the household, so the income distribution shock is the only force that may drive the economy into a deleveraging episode. The second key ingredient is the ZLB and nominal ridity. Given this assumption , when natural interest rate falls to negative, ZLB makes the the consumption expenditure fall, and the noniminal ridity makes the output drop.

The contribution of this paper to existing knowledge is twofold. First, I try to set up a hypothesis to reveal how income and wealth inequality may bring financial instability through overborrowing and externality. Although the overborrowing is one key factor that leads to financial instability, there is few literature that tried to explore this channel. Second, based on my result, I also explore the implication for macroprudential policies and redistribution policies. in Kronek Anton(2016)'s settings, the macroprudential policy can be used to correct overborrowing and achieve a constrained efficiency. In this paper, I explored whether the designing of macroprudential policy is dependent on the income distribution. Moreover, when discussing the case of permanent income distribution shock, I illustrated the how a future permament transfer from rich to poor may trigger overborrowing at the current period and pointed out that macroprudential policy is necessary to eliminate the possibility of overborrowing in this case.

The remaining parts of the thesis go as follows: Section 2 is a selected review of literature. Section 3 presents the baseline model. Section 4 presents the extensions of the basic model. Section 5 concludes.

2 Literature Review

This paper belongs to the strand of the literature that try to explore how income inequality affects macroeconomy via financial frictions. It's closely related to the following two strands of literature.

First strand is the literature on the link between the inequality and crises, which provides the background and motivation for this paper. So far, some empirical studies, by panel regressions, have explored the effect of income inequality on financial crisis. their methods are similar in that they do regressions in two steps. The first step is to provide evidence on the link between credit expansion and periods of financial crisis. Credit booms usually tend to develop into financial crises, which is agreed by several empirical studies. The second step is to assess the impact of inequality and deregulation on credit expansion.Christiano and Jens(2015) found a direct and significant coefficient of the inequality (top 1%) in the regression model. The result is robust when trying different measures of inequality. However, Bordo and Meissner(2012) found that inequality does not appear to be a significant determinant of credit growth once they introduce other macroeconomic aggregates into regression. Aside from empirical studies, there are also some literature that tried to reveal the mechanism by which inequality affects the probability of crises.One representative example is by Kumhof and Ranciere(2010), whose conclusion is that the decrease of income of workers relative to investors increases the possibility of worker's debt default. Another representative paper by Iacoviello (2008) constructed and simulated a heterogeneous agents model that mimics the distribution of income and household debt in the United States in the period 1963–2003. My paper is similar to his in that I also let two types of the agents exist in the economy. They are different in their time discount factors and MPCs. based on this setting I studied how the income distribution affects agents with different MPCs.

The second strand is the literature on liquidity trap. This paper is related to the literature that studies the cause and effect of liquidity traps (for example, Krugman (1998) and Eggertsson and Woodford (2003, 2006)). In these literature, highly stylized model are studied to reveal the mechanism by which the liquidity trap emerges and damage the economy. In addition to analytical analysis, Guerrieri and Lorenzoni (2011) analyzed during a DSGE model how the tightening of borrowing constraint lead to a decline in interest rates, which in turn can trigger a liquidity trap. The paper that is mostly related to this paper is by Korinek(2016), which analyzed the conditions under which the borrower chooses to overborrow to thus trigger a liquidity trap, and the ex ante macroprudential policy that can be designed to prevent overborrowing from happening.

3 Model

In this section, I set up a simple model to see how the income share can act as a driving of deleveraging. The model framework is the combination of Korinek (2016) and Bianchi(2010). There are one borrower and one lender. The borrower maximize his discounted sum of utility:

$$\max \sum_{t=0}^{\infty} \beta_b^t u\left(c_{b,t}\right)$$

We assume that $u(.)$ is strictly increasing and strictly concave. The budget constraint is:

$$c_{b,t} + d_t = \left(\frac{1}{2} - \lambda_t\right) e_t + \frac{d_{t+1}}{1+r_1}$$

We assume that the borrowing can not exceed a fraction of the borrower's income . thus we have

$$d_{t+1} \leq \phi \left(\frac{1}{2} - \lambda_t\right) e_t$$

In this model, we refer to the borrower as 'poor' and the lender as 'rich'. This is motivated by Auclert (2016), who finds out that low-income agents have high MPCs, and high-income agents have low MPCs. Thus, the transfer of income from the borrower (lender) toward the lender (borrower) can be regarded as an increase (decrease) in inequality. The assumption on borrowing constraint captures the case that the borrowing constraint is determined by the current income of the household, which is also applied by Bianch (2010). Thus, the deleveraging is driven by the income distribution shock. Once the borrower is given less share of income, not only does his income decreases, but also the amount he can borrow up to decreases.[1].

Similarly, the problem of the lender is :

[1]This assumption also makes sense in that the we can regard the shock of income distribution as an exogenous government income redistribtuion policy.

$$\max \sum_{t=0}^{\infty} \beta_l^t u\left(c_{l,t}\right)$$

subject to the following budget constraint:

$$c_{l,t} = \lambda_t e_t - \frac{d_{t+1}}{1+r_1} + d_t$$

The income of the household, e_t, is defined by the following:

$$e_t = \left(w_t n_t\right) + \frac{1}{2}\int_0^1 \Gamma\left(v\right) dv$$

In which the w_t is wage rate, the n_t is the labor the household supply to intermediate firms.We also denote l as the largest labor the household can supply in the economy.We also assume that the household can get profit from the intermediate firms , which is denoted by the last term. Suppose there is a government; he collects the e_t and redistribute it to the household according to a certain rule. One key assumption is the nominal rigidity and monopolistic firm setting. This assumption insures that output is demand-determined. Specifically, we assume that a competitive final good sector uses intermediate varieties v to produce the consumption good according to the Digit-Stiglitz technology. It faces the following problem:

$$\min_{y_t(v)} \int_0^1 p_t\left(v\right) y_t\left(v\right) dv$$

$$s.t\ y_t = \left(\int_0^1 y_t\left(v\right)^{\frac{\varepsilon-1}{\varepsilon}} dv\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$y_t\left(v\right)$, input, is produced by monopolistic firms. $p_t\left(v\right)$ is the nominal price level for the monopolist for variety v at time t. The y_t and $p_t\left(v\right)$ are given for the the final good sector. The optimal condition gives the following demand function:

$$y_t\left(v\right) = y_t\left(\frac{p_t\left(v\right)}{P_t}\right)^{-\varepsilon}$$

In which P_t is given by

$$P_t = \left(\int_0^1 y_t\left(v\right)^{\frac{\varepsilon-1}{\varepsilon}} dv\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

In our baseline model , we assume that monopolists have preset nominal prices that are equal to each other and that never change. This means that $P_t\left(v\right) = P$ for each t. This implies that the final good price is also constant, $P_t = P$ for each t. The problem of the monopolistic firm is :

$$\max_{n_t(v)} P y_t\left(v\right) - w_t n_t\left(v\right)$$

$$s.t\ y_t\left(v\right) = n_t\left(v\right) \text{and}\ y_t\left(v\right) \leq y_t\left(\frac{p_t\left(v\right)}{P_t}\right)^{-\varepsilon} = y_t$$

We assume that $P > w_t$. Since the supply of labor is inelastic, there is no equation to determine the wage. thus the optimal condition for the monopolistic firm is $n_t\left(v\right) = y_t$. This induces an employment of $n_t = y_t$.Thus the profit of the monopolistic firm is $Py_t - w_t y_t$, and e_t is equal to Py_t.

Given the problem of the monopolistic firms and the definition of the e_t, we define the efficient level of net income as

$$e_t^* \equiv \max_{n_t} n_t$$

The other key assumption is a lower bound on the nominal interest rate.

$$i_{t+1} \geq 0$$

The nominal interest rate cannot fall significantly below zero, since households would otherwise hold cash instead of keeping their wealth in interest-bearing accounts. Given the assumption that price is sticky, we can write this condition as

$$r_{t+1} \geq 0$$

Following these two assumptions, we can see that the output is ultimately determined by the aggregate demand for the final consumption. The aggregate demand , in turn, depends on monetary policy, which controls the nominal and the real interest rate. Given the fixed price assumption, we assume that monetary policy focuses on output stabilization and try to replicate the 'natural' real interest, which is determined in the market. Before the equilibrium analysis, we introduce the following lemma, which is also introduced by Korinek(2016).

Lemma 1. *(i) if* $r_{t+1} > 0$*, then* $e_t = e^*$*. (ii) if* $r_{t+1} = 0$*, then* $e_t = c_t^b + c_t^l \leq e^*$

The equilibrium is $\left\{c_{b,t}, c_{l,t}, i_{t+1}, d_{t+1}, y_t\right\}$ that satisfies the following conditions:

$$c_{b,t} + d_t = (1 - \lambda_t)\, y_t + \frac{d_{t+1}}{1 + i_{t+1}} \qquad \text{(budget constraint of borrower)}$$

$$c_{l,t} + c_{b,t} = y_t \qquad \text{(market clearing of goods)}$$

$$u'\left(c_{l,t}\right) = (1 + i_{t+1})\, \beta_l u^{'}\left(c_{l,t+1}\right) \qquad \text{(Euler equation)}$$

$$d_{t+1} \leq \phi \left(\frac{1}{2} - \lambda_t\right) y_t \qquad \text{(borrowing constraint)}$$

$$i_{t+1} \geq 0 \qquad \text{(zero lower bound)}$$

$$y_t = l \;\; if \;\; i_{t+1} > 0$$

4 Income Distribution Shock

Based on the model constructed in Section 3,it's reasonable to study how the economy will behave given the exogenous process of $\{\lambda_t\}$. From this section and on I study the how idfferent types of income distribution shocks affect the possibility of liquidty, as well as how these shocks affect the output when liqudity trap occurs.

4.1 Unanticipated Temporary Shock

In this subsection I study how a temporary income distribution shock affect the economy. Before the numerical analysis, I first make an extreme simplification. Specifically, I assume following income distribution profile:

- Endowment share profile of the lender : $\frac{1}{2}, \left(\frac{1}{2} + \lambda\right), \frac{1}{2}, \frac{1}{2}, \ldots.$
- Endowment share profile of the borrower: $\frac{1}{2}, \left(\frac{1}{2} - \lambda\right), \frac{1}{2}, \frac{1}{2}, \ldots.$

i.e., there is a temporary income distribution shock at $t = 1$. We first study the case that the income share shock is unanticipated. This exercise is useful to understand the dffect of an unexpeted shock that has no persistence[2]. Using guess and verify, We focus on such an equilibrium that from $t = 2$ and on , the economy converges into steady state, in which the borrower keeps borrowing $\beta_l d_2$ and paying back d_2. We also assume that the borrowing constraint is binding from $t = 1$ and on .

[2]For example, we can consider think of a situation about that government suddenly put into a policy that may change the income distribution between different agents, but the policy only lasts for a short period of time due to some exogenous reasons.

Assume that the borrowing constraint at $t=1$ is binding[3]. We first see the Euler equation of the lender at $t=1$:

$$u'\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\phi\frac{1}{2}e^{*}-\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_{2}}\right)=(1+r_{2})\beta_{l}u'\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e^{*}-\beta_{l}\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

In which we can find out that when λ increases, r_2 decreases. When λ is sufficiently large, r_2 falls to zero, and liquidity trap occurs. The intuition is as follow. when $\lambda>0$, the lender is faced with an unanticipated inccrease in current income and decrease in future income. The increase in current comes from two part. One is the exogenous increase in the share of income, the other is the decrease in the lending to the borrower since the borrower are allowed to borrow up to a less amount. The decrease in the future income also comes from the tightening of the borrowing limit. Therefore, the lender values its future income more, has more motivation to lend more, and thus makes the real interest rate fall. We can see that the shock of income distribution reduces the real interest rate via two channel; The increase in the lender's income and the decrease in the borrower's income (and thus the tightening of borrowing constraint) together drag the real interest down.

When $r_2<0$, due to zero lower bound, the interest rate can not fall below 0. In order for the above Euler equation to hold, mathematically the e_1 may not neccessarily be equal to e^*, and it is determined by the Euler equation:

$$u'\left(\left(\frac{1}{2}+\lambda\right)e_{1}+\phi\frac{1}{2}e^{*}-\phi\left(\frac{1}{2}-\lambda\right)e_{1}\right)=\beta_{l}u'\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e_{1}-\beta_{l}\phi\left(\frac{1}{2}-\lambda\right)e_{1}\right)$$

For simplicity we assume a log uiltity function. thus we have:

$$\frac{1}{2}(1-\beta_{l}\phi)e^{*}=\left[\beta_{l}\left(\frac{1}{2}+\lambda\right)-\phi\left(\frac{1}{2}-\lambda\right)\right]e_{1}$$

Here we assume that $\frac{1}{2}-\beta_l\phi>0$ and $\beta_l\left(\frac{1}{2}+\lambda\right)-\phi\left(\frac{1}{2}-\lambda\right)>0$. It's obvious that the increase in λ will lead to an decrease in e_1. Mathematcally, the increase in λ raises the coefficient of e_1 via two parts. One is the rise of $\beta_l\left(\frac{1}{2}+\lambda\right)$, which comes from the increase of the lender's income. The other is the decrease of $\phi\left(\frac{1}{2}-\lambda\right)$, which comes from the decrease of the borrower's income and thus his borrowing constraint. To furthure explain the intuition of the result, recall that:

$$e_{1}=c_{l,1}+c_{b,1}=\frac{1}{\beta_{l}}\left[\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e_{1}-\beta_{l}\phi\left(\frac{1}{2}-\lambda\right)e_{1}\right]+\left(\frac{1}{2}-\lambda\right)e_{1}-\phi\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e_{1}$$

Consider a marginal increase in the λ. the consumption of the lender at $t=1$ decreases by $\phi(1-\beta_l)\frac{1}{\beta_l}e_1$, while the consumption of the borrower at $t=1$ decreases by $(1+\phi)e_1$. thus the aggregate demand decreases by $\left[\phi(1-\beta_l)\frac{1}{\beta_l}+(1+\phi)\right]e_1$. Thus the output decrease by the same amount, and the income given to the lender decreases by $\left(\frac{1}{2}+\lambda\right)\left[\phi(1-\beta_l)\frac{1}{\beta_l}+(1+\phi)\right]e_1$, while the income given to the borrower decreases by $\left(\frac{1}{2}-\lambda\right)\left[\phi(1-\beta_l)\frac{1}{\beta_l}+(1+\phi)\right]e_1$, and this in turn lead to the consumption drop.... The result is summarized in the following proposition. The increase of λ decreases the consumption demand of lender via a tightened borrowing constraint and decreases the consumption demand of borrower via both a tightened borrowing constraint and a lessened share of income.

Proposition 2. *consider an unanticipated temporary income distribution shock at $t=1$. that is , at $t=1$, the lender witnessed an unticipated shock of share of income from $\frac{1}{2}e^*$ to $\left(\frac{1}{2}+\lambda\right)e^*$, while the borrower witnessed an unticipated shock of income share from $\frac{1}{2}e^*$ to $\left(\frac{1}{2}-\lambda\right)e^*$, in which $\lambda>0$. The larger the λ, the lower the real interest rate r_2; r_2 in the equilibrium falls to zero, liquidity trap occurs, and e_1 falls below its efficient level, e^*, and the larger the λ the small the λ in the equilibrium.*

[3] From now on , we assume that at $t=1$ the borrowing constraint is binding. This assumption captures the situation under which the deleveraging episode is sufficiently serious so that the borrower want to borrower more than his borrowing limit. The binding of borrowing constraint may come from the high impatience of borrower relative to that of the lender.

4.2 Anticipated Temporary Shock

In this part, we assume that the income distribution shock at $t = 1$ can be perfectly foreseen by the household. Therefore, at $t = 0$, the borrower takes the income distribution shock into consideration when making borrowing decisions . We see under what condition will the borrower choose to 'overborrow', and how to design the macroprudential policy to prevent the borrower from overborrowing. The description of the equilibrium is in Appendix A.

4.2.1 Equilibrium Analysis

We still focus on such an equillibrium that from $t = 2$ and on , the economy converges into steady state, in which the borrower keeps borrowing $\beta_l d_2$ and paying back d_2.Assume that the borrowing constraints at $t = 1$ are binding. We first see the Euler equation of the lender:

$$u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+d_1-\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}\right)=(1+r_2)\beta_l u^{'}\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e^{*}-\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right) \quad (*)$$

In which we can find out that when d_1 increases, r_2 decreases. The threshold value of debt, $\bar{d}_1$, is thus determined by:

$$u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d}_1-\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)=\beta_l u^{'}\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e^{*}-\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

It's obvious that the threshold value of debt will decrease when λ increases, which implies that the more share of income at $t = 1$ the lender, the easier the liquidity trap occurs. The intuition is obvious; the more wealth held by the lender, the less it values the consumption at $t = 1$ period. That is, the lender tends to save more when he is richer, thus leads to the decrease of real interest rate.

Proposition 3. *Threshold value, the debt level under which the real interest rate, r_2, becomes zero, is dependent on the value λ. Specifically, the larger the value of λ, the lower the value of $\bar{d}_1$.*

What we are interested in is under which condition will the household choose to overborrow. In fact, we have the following result ,whose proof can be seen in the Appendix A.

Proposition 4. *Assume that $\bar{d}_1 \geq \phi\frac{1}{2}e^{*}$,i.e, that is , the threshold value of the debt d_1 does not exceed the borrowing limit at $t = 0$. Then, the exists a threshold value $\bar{\lambda}\left(e^{*},\phi,\beta_l,\beta_b\right)$ such that when $\lambda > \bar{\lambda}\left(e^{*},\phi,\beta_l,\beta_b\right)$, liquidity trap will definite occurs.*

The intuition is that, if the borrower perfectly foresee at $t = 0$ that there is a big tmporal income share shock at $t = 1$, he will know that at time $t = 1$ he will experience a large income drop, due to both the drop of shares and the tightening of borrowing constraint.This makes the borrower want have less desire to borrow at $t = 1$. On the other hand, the lender knows that his income at $t = 1$ will increase due to the increase of the share and the drop of the amount he can lend to the borrower at $t = 1$. And at $t = 2$, his income will decrease due to the tightening of borrowing constraint. This will make the lender want to lend more at $t = 1$. Thus the r_2 in the equilibrium decreases. if λ is larger enough , then the real interest rate becomes zero.

Up to now the analysis is nothing new, since it's a simple analysis of the household's intertemoral consumtption and saving choice. What is really interesting the situation when liquidity trap occurs. We next see given a debt level that leads to liquidity trap, how the λ affects the output in the economy. We can see when λincrease in the equilibrium, the e_1 decreases. The proof is Appendix A.

Proposition 5. *During liquidity trap , when λ increases, $e_1$in the equilibrium decreases.*

The intuition can be explained as follows: when the λ increases, borrower has to cut his consumption at $t = 1$. On the other hand , the lender faces higher income at $t = 1$ and less consumption at $t = 2$(since his income at $t = 2$ becomes less due to tightening borrowing constraint). Since the interest the household faces , in order for his consumption to obey the Euler rule, the lender will cut their consumtpion at $t = 1$. Thus the aggregate demand in the society will drop, which futher leads to the decrease in the income held by borrower. Moreover, the borrowing constraint gets even tighter, not only because λgets larger, but

Table 1: Calibration: Parameter setting

parametres	definition	value
β_l	the discount rate of the lender	0.98
β_b	the discount rate of the borrower	0.7
ϕ	LTV ratio limit	0.3
e^*	the efficient output	1

also because that $e_1$decreases. Thus the borrower has to cut his consumption expenditure further more .....Here we can see that although at first glance the increase in λ hurts the borrower and benefits the lender, under zero lower bound both agents will reduce their consumtption expenditure, and through the nominal rigidity this will affect the total ouput in the economy . The income distribution shock leads to a deleveraging, making borrower cut consumption the total output drop, and the recession in turn makes the deleverging more severe i.e, income distribution shock make deleveraging and output drop reinforce each other.[4]

To give a full understanding of the whole model , I use numerical method to see how the consumption , interest rate , output and externality behave in the equilibrium when changing the value of λ. The decription of the whole model is given in the Appendix A. The result of calibration is shown in Appendix C. the horizontal axis is the value of λ.

We can see from the figure 6.1 that the before the the r_2 in the equilibrium reaches 0, when the λ increases, r_2, $r_1$and d_1 increases in the equilibrium. The consumptions of the borrower at $t = 0$ and $t = 1$ decrease, while the consumptions of the lender at $t = 0$ and $t = 1$ increase. The result shows that the the borrowing constraint is not binding under the parameter settings. Although the d_1 is decreasing in the equilibrium, the difference between the d_1 , and its threshold value, $\bar{d}_1$, decreases rapidly in the equilibrium, as can be seen in the 'debt difference' chart. When λ is sufficiently large, $r_2$reaches 0, and the zero lower bound makes liquidity trap occurs. During the liquidity trap, as λ in the equilibrium increase, both r_1, d_1 , output $e_1$and consumptions of the household decrease. the binding0 and binding1 chart show whether the borrowing constraints at $t = 0$ and $t = 1$ binding or not. 1 represents that the borrowing constraint is binding, and 0 represents that the borrowing constraint is not binding. We can see from the chart that at $t = 0$ the borrowing constraint is not binding, while at $t = 1$ the borrowing constraint is binding all the time.

4.2.2 Externality and Macroprudential Policy

We next discuss the externality arises from over borrowing and the proper macroprudential policy that can prevent overborrowing from happening. The conclusion is that although constrained efficient allocation is independent of λ, the pigovian tax, which is used to correct the overborrowing, depends on λ. Suppose a Ramsey planner that are faced with the following problem.

$$\max_{c_{b,0},c_{l,0},d_1} \gamma_b \left[u\left(c_{b,0}\right) + v_b\left(d_1\right)\right] + \gamma_l \left[u\left(c_{l,0}\right) + v_l\left(d_1\right)\right]$$

$$s.t.\ c_{b,0} + c_{l,0} = e_0 = e^*$$

In which

$$v_b\left(d_1\right) = u\left(\left(\frac{1}{2} - \lambda\right)e_1\left(d_1\right) - d_1 + \frac{\phi(\frac{1}{2} - \lambda)e_1\left(d_1\right)}{1 + r_2\left(d_1\right)}\right) + \beta_b u\left(e^* - \phi(\frac{1}{2} - \lambda)e_1\left(d_1\right) + \beta_l \phi(\frac{1}{2} - \lambda)e_1\left(d_1\right)\right)$$

$$v_l\left(d_1\right) = u\left(\left(\frac{1}{2} + \lambda\right)e_1\left(d_1\right) + d_1 - \frac{\phi(\frac{1}{2} - \lambda)e_1\left(d_1\right)}{1 + r_2\left(d_1\right)}\right) + \beta_l u\left(e^* + \phi(\frac{1}{2} - \lambda)e_1\left(d_1\right) - \beta_l \phi(\frac{1}{2} - \lambda)e_1\left(d_1\right)\right)$$

[4]Up to now , all these conclusion are sensitive to whether the income difference between the $t = 1$ and $t = 2$ is large enough. Since the analysis is based on the Euler equation of the lender, the income profile will largely affect the saving and consumption behavior of the lender. Since this paper studies the temporary income shock, we can assume that income held by the lender at $t = 1$ is sufficiently higher than

That is,the Ramsey planner determines the consumption and borrowing at $t = 0$, From $t = 1$ and on ,the market still exists, i.e the household react given the debt level d_1. Thus the allocation under the Ramsey planner is at least as good as the competitive equilibrium. We next discuss how the macroprudential policy should be designed. First, we derive the allocation under social planner. The calculation can be seen in the appendix A. The result is summarized in the following proposition.

Proposition 6. *Under the framework, the allocation of the Ramsey planner is*

(i)$d_1 < \bar{d}_1$ *and*

$$\frac{\beta_b u'\left(c_{b,1}\right)}{u'\left(c_{b,0}\right)} = \frac{\beta_l u'\left(c_{l,1}\right)}{u'\left(c_{l,0}\right)}$$

(ii)$d_1 = \bar{d}_1$ *and*

$$\frac{\beta_b u'\left(c_{b,1}\right)}{u'\left(c_{b,0}\right)} \leq \frac{\beta_l u'\left(c_{l,1}\right)}{u'\left(c_{l,0}\right)}$$

Therefore the optimal allocation has nothing to do with the parameter λ. In the figure 6.3, I present the welfare loss of the economy when changing the value of λ. We can see in the chart that when before the liqudity trap occurs, the allocation under competitive equilibrium is as good as that under ramsey planner, and there is no welfare loss. However, when the λ is large enough so that the liquidity trap occurs in the equilibirum, due to aggregate demand externality there is welfare loss under conpetitive equilibrium, and the larger the λ is , the larger the welfare loss is. This result implies that in an economy with nominal rigidity and ZLB, a anticipated income distribution shock may lead to welfare loss due to aggregate demand externality.

We next discuss whether macroprudential policy should take the income distribution into consideration. Here we focus on how the pigovian tax , which is imposed on borrower's borrowing, should be designed. We suppose that given the other parameters, the λ is larger than its threshold value, $\bar{\lambda}$, and thus we have $d_1 > \bar{d}_1$. now we calculate the tax rate that is needed to make the borrower choose $d_1 = \bar{d}_1$, which is one of the optimal allocation under social planner. The tax that is collected from the borrower is given to the lender as a lump-sum transfer. the tax rate is designed to induce the borrower to borrow at $\bar{d}_1$.

$$u'(\frac{1}{2}e^* + \frac{(1-\tau)\bar{d}_1}{1+r_1}) = \beta_b \frac{1+r_1}{(1-\tau)} u'\left(\left(\frac{1}{2} - \lambda\right)e^* + \phi(\frac{1}{2} - \lambda)e^* - \bar{d}_1\right)$$

$$u'(\frac{1}{2}e^* - \frac{\bar{d}_1}{1+r_1} + T) = \beta_l (1+r_1) u'\left(\left(\frac{1}{2} + \lambda\right)e^* - \phi(\frac{1}{2} - \lambda)e^* + \bar{d}_1\right)$$

The Euler equation for the borrower holds here because we assume that $\bar{d}_1 < \frac{1}{2}\phi e^*$,i.e, the borrowing constraint at $t = 0$ is not binding. Plugging in the expression of $\bar{d}_1$ and noticing that $T = \frac{\tau \bar{d}_1}{1+r_1}$we can get the following results if we assume a log utility function:

$$(1-\tau)\left[\left(e^* - \tilde{c_{l,1}}(\lambda)\right)\frac{1}{\beta_b} - 2\bar{d}_1(\lambda)\right] = \frac{\tilde{c_{l,1}}(\lambda)}{\beta_l}$$

In which $\tilde{c_{l,1}}(\lambda) = \left(\frac{1}{2} + \lambda\right)e^* - \phi(\frac{1}{2} - \lambda)e^* + \bar{d}_1$. From the (3.1) we can see that $\tilde{c_{l,1}}(\lambda)$ is decreasing with λ.Now Suppose λ increases. Thus the left hand side increases, while the right hand decreases. Therefore, the τ increases. The intuition of the result is obvious: since we have already know that given the other parameters, the larger the λ, the more likely the overborrowing will happen. Therefore, more tax should be imposed to prevent borrowers from overborrowing.

Proposition 7. *Under the current framework , the macroprudential policy (pigovian tax rate), τ, is given by the following equation if we assume a logarithm utility function.*

$$(1-\tau)\left[\left(e^* - \tilde{c_{l,1}}(\lambda)\right)\frac{1}{\beta_b} - 2\bar{d}_1(\lambda)\right] = \frac{\tilde{c_{l,1}}(\lambda)}{\beta_l}$$

Under this tax rate, the borrower will choose to borrow $d_1 = \bar{d}_1$.

4.3 Unanticipated Permanent Shock

In this section we discuss how the persistence of the income distribution shock will affect our conclusion. That is , we assume the following income share profile.

- Endowment share profile of the lender : $\frac{1}{2}, \left(\frac{1}{2}+\lambda\right), \left(\frac{1}{2}+\lambda\right), \left(\frac{1}{2}+\lambda\right), ...$
- Endowment share profile of the borrower: $\frac{1}{2}, \left(\frac{1}{2}-\lambda\right), \left(\frac{1}{2}-\lambda\right), \left(\frac{1}{2}-\lambda\right), ...$

We still first consider the case that income distribution shock is unanticipated. the Euler equation for the lender now becomes:

$$u'\left(\left(\frac{1}{2}+\lambda\right)e^* + \phi\frac{1}{2}e^* - \frac{\phi\left(\frac{1}{2}-\lambda\right)e^*}{1+r_2}\right) = (1+r_2)\beta_l u'\left(\left(\frac{1}{2}+\lambda\right)e^* + \phi\left(\frac{1}{2}-\lambda\right)e^* - \beta_l\phi\left(\frac{1}{2}-\lambda\right)e^*\right)$$

if we assume a logrigthm utility function form ,we can then calulate the r_2 :

$$1+r_2 = \frac{(1-\phi)e^*\lambda + \left(\frac{1}{2}e^* + \frac{1}{2}\phi e^*\right)}{e^*\lambda + \left(\frac{1}{2}e^* + \frac{1}{2}\phi e^*\right)}$$

We can see from that under an unticipated permanent distribtuion shock, the larger the shock is , the smaller the $r_2$is. When λ increases, both future income and current income increases; they have opposite effects on the real interest rate. Under the logrigthm utility function form assumption , the net change of the real interest rate goes down.

When λ is sufficiently large, the liquidity trap occurs, and the e_1 is determined by :

$$u'\left(\left(\frac{1}{2}+\lambda\right)e_1 + \phi\frac{1}{2}e^* - \phi\left(\frac{1}{2}-\lambda\right)e_1\right) = \beta_l u'\left(\left(\frac{1}{2}+\lambda\right)e^* + \phi\left(\frac{1}{2}-\lambda\right)e_1 - \beta_l\phi\left(\frac{1}{2}-\lambda\right)e_1\right)$$

the expression for the e_1 is

$$e_1 = \frac{\left(\frac{1}{2}+\lambda\right)e^* - \frac{1}{2}\beta_l\phi e^*}{\beta_l\left(\frac{1}{2}+\lambda\right) - \phi\left(\frac{1}{2}-\lambda\right)}$$

It's easy to see that the e_1 in the equilibrium decreases with λ. Therefore, In summary, suppose the shock is unanticipated, conclusions under the temporary income shock and permanent shock are similar if we assume a logrithm utility function form.

4.4 Anticipated Permanent Shock

We next consider the case when the income distribution shock is anticipated. We still assume that that borrowing constraint is binding from $t = 1$ and on . However, at $t = 0$ whether the borrowing constraint is binding or not is not certain. Thus we discuss the following two cases.

The first case is that there is no borrowing constraint at $t = 0$. The numerical result in figure 3 shows that, if the borrower anticipated that in the future their income increases, they will try to borrow more at the current period. if the borrower can borrow as much as they can , they may trigger a liquidity trap and lead to drop in aggregate demand. We can think of the decrease of λ as a transfer payment from the richer (lender) toward to the poorer (borrower). If the borrower predicts that in the future such a transfer policy will be put into effect, they will be more likely to overborrow if there is no proper borrowing constraint at the current period.

The second case is that there is a borrowing constraint at $t = 0$. The borrower is allowed to borrow up to a fraction of the income they are given. As the numerical result in figure 4 implies, the existence of the borrowing constraint at $t = 0$ can prevent the liquidity trap from happening. If the poorer gets richer and the richer gets poorer (which can regarded as a kind of reducing inequality) permanently and the agents know this, in order to prevent borrowing from happening, it's necessary to set a borrowing limit at current period.

But this results, on the other hand, states an implication that contradicts the conclusions under the previous shocks. The result implies if the income inequality gets severe forever in the future and the agents know this, the liquidity trap will be less likely to happen. Intuitively speaking, standing at the $t = 0$, if the borrower knows that he will be poorer and the lender knows that he will be richer, then the borrowing at $t = 0$, namely d_1, will decrease. The Euler of the lender between $t = 1$ and $t = 2$ implies that he is less willing to lend at $t = 1$, since the d_1 he receives at $t = 1$ decreases. Thus the equilibrium real interest rate increases. The reason why the conclusion is so different with that under temporary income distribution shock can be explained by the following: consider a shock during which the lender get richer and the borrower gets poorer. if the shock is temporary, then it means that the income of the lender at $t = 1$ increase largely compared to the income at $t = 2$. Thus , the lender has a large desire to increase his lending, and since the borrower is constraint, the real interest rate , r_2 will thus decrease. On the contrary , if the shock is permanent, both the income at $t = 1$ and the income at $t = 2$ increase, thus the lender's desire to lend at $t = 1$ does not increase much. Moreover, the decrease in d_1 also prevent the lender to lend more at $t = 1$ Thus, the net effect may be that the lender cut lending at $t = 1$, thus the real interest rate, $r_2$increases.

5 Extension: Housing Markets

In this section, I modify the model settings to allow the borrowers to trade house. The borrowing constraint is based on the value of house the borrower owns. I analyze how the asset price and output behavior if there is an unanticipated income share shock, asumming that the economy will converge to a steady state immediately after the shock.

5.1 Model

The model is a combination of Korinek (2016) and Iacoviello (2008)and We can think of the borrowers are those people who want to borrow to buy house. This model is more realistic in that It includes the durable good consumption (for example ,house). Consider the case with the consider the following economy.The borrower maximizes its life-time utility:

$$\sum_{t=0}^{\infty} \beta_b^t \left[u\left(c_{b,t}\right) + \varphi v\left(H_t\right) \right]$$

In which H is the purchase of housing. As before, we assume that both the $u(.)$ and $v(.)$ are both strictly increasing and strictly concave. At each period, the borrower are faced with the following budget constraint:

$$d_t + c_{b,t} + p_t H_t = (1-\lambda_t) e_t + p_t H_{t-1} + \frac{d_{t+1}}{1+r_{t+1}}$$

That is , at each period, the borrower chooses the debt he wants to borrow, d_{t+1} and the amount of house he wants to hold, H_t. There is no depreciation of housing here. The borrower is also faced with a borrowing constraint, which is given by:

$$d_{t+1} \leq \phi p_t H_t$$

Symmetrically, the problem of the lender is:

$$\sum_{t=0}^{\infty} \beta_l^t \left[u\left(c_{l,t}\right) \right]$$

$$s.t.\ c_{l,t} = \lambda_t e_t - \frac{d_{t+1}}{1+r_{t+1}} + d_t$$

In this model I made several simplifications. One is that only the borrower is allowed to trade house. Second is that the supply of housing is fixed and we do not consider the depreciation of the house.

5.2 Unanticipated Temporary Shock

We first consider the steady state. By plugging the binding constraint and market clearing condition into the first order condition and assuming a steady state, we can get:

$$\varphi v^{'}(\bar{H}) = pu^{'}((1-\lambda)e^{*} - (1-\beta_l)\phi p\bar{H})(1-\beta_l\phi + \beta_b(\phi-1))$$

This equation determines the asset price at the steady state. Consider an increase in the λ. Then p increases. If we assume the logarithm form, the expression for the p is

$$p = \frac{\varphi(1-\lambda)e^{*}}{[1-\beta_l\phi + \beta_b(\phi-1) + \varphi(1-\beta_l)\phi]\bar{H}} \tag{5.1}$$

And the borrowing constraint becomes:

$$\phi p\bar{H} = \frac{\varphi\phi(1-\lambda)e^{*}}{1-\beta_l\phi + \beta_b(\phi-1) + \varphi(1-\beta_l)\phi}$$

The calculation can be seen in the appendix B.This result implies that p decreases with an increasing λ. The concentration of wealth toward the lender may lead to deleveraging. The intuition is that the decrease of wealth held by the borrower makes him cut the purchase on house, and this leads to a drop in the housing price and thus the credit constraint. The tightening of credit constraint in turn leads to the drop in borrower's house purchase... In summary, the tightening borrowing constraint and the falling asset price reinforce each other through a loop. The consumption of the household under steady state is respectively:

$$c_b = (1-\lambda)e^{*}\frac{1-\beta_l\phi + \beta_b(\phi-1)}{1-\beta_l\phi + \beta_b(\phi-1) + (1-\beta_l)\phi} \text{ and } c_l = e^{*} - c_b$$

It's obvious to see that the larger the λ, the small the c_b, and thus the larger the c_l.

Proposition 8. *Under the current frame work , a steady state with higher income share of lender has lower housing price, a tighter borrowing constraint, a higher consumption of lender and lower consumption of borrower.*

Now consider what will happen during such a income distribution shock. Suppose that at $t = 1$ there is an unexpected[5] temporary shock in the income distribution. Following Korinek(2016), We consider an equilibrium such that from $t = 2$ and on the system converges to a steady state, in which the borrower keeps borrowing at d_2 level. For the lender , the Euler equation can be written as

$$u^{'}\left(\lambda^{'}e^{*} + \phi p\bar{H} - \frac{d_2}{1+r_2}\right) = (1+r_2)\beta_l u^{'}(\lambda e^{*} + d_2 - \beta_l d_2)$$

Suppose that the borrowing constraint is binding. Thus

$$u^{'}\left(\lambda^{'}e^{*} + \phi p\bar{H} - \frac{\phi p_1\bar{H}}{1+r_2}\right) = (1+r_2)\beta_l u^{'}(\lambda e^{*} + \phi p_1\bar{H} - \beta_l\phi p_1\bar{H}) \tag{5.2}$$

In which p is given by (5.1). The another equation that describes the equilibrium is

$$\varphi v^{'}(\bar{H}) = p_1 u^{'}\left((1-\lambda^{'})e^{*} + \frac{\phi p_1\bar{H}}{1+r_2} - \phi p\bar{H}\right)\left(1 - \frac{\phi}{1+r_2}\right) +$$

$$\beta_b u^{'}((1-\lambda)e^{*} + \beta_l\phi p_1\bar{H} - \phi p_1\bar{H})(\phi p_1 - p_2(p_1)) \tag{5.3}$$

The p_2 is given by the following equation:

$$\varphi v^{'}(\bar{H}) = p_2 u^{'}((1-\lambda)e^{*} - (1-\beta_l)\phi p_1\bar{H})(1-\beta_l\phi + \beta_b(\phi-1))$$

So It's easy to see that the p_2 is a decreasing function of p_1.The (5.2) and (5.3) together determine the r_2 and p_1. In (5.2), there is a positive relationship between the p_1 and r_2. The equation (5.3) can be written as

[5] I make this assumption to focus on how the income share may affect interest and make the analysis simpler. if the the income distribution shock can be anticipated,the reaction of the household may pollute the affect of income on interest and the calculation is messy. In the future work, I plan to consider the case in which the shock of income distribution can be anticipated

$$p_1 = \frac{\varphi v^{'}(\bar{H}) + \beta_b u^{'}(c_{b,2})\, p_2(p_1)}{u^{'}(c_{b,1})\left(1 - \frac{\phi}{1+r_2}\right) + \phi \beta_b u^{'}(c_{b,2})} \equiv f(p_1)$$

In order for this equation to have a well-defined solution, we suppose $\frac{\partial f(p_1)}{\partial p_1} < 1$. Under this condition, there is at least one solution of this equation. Also, noticing that $\frac{\partial f(p_1)}{\partial r_2} < 0$, we have

$$1 = \frac{\partial f(p_1)}{\partial p_1} + \frac{\partial f(p_1)}{\partial r_2}\frac{\partial r_2}{\partial p_1}$$

Then it's obvious that $\frac{\partial r_2}{\partial p_1} < 0$. Thus in (5.3), there is a negative relationship between r_2 and p_1. Consider an increase in the $\lambda^{'}$. Both the (5.2) and (5.3) shift down , making the r_2 in the new equilibrium smaller than that in the previous equilibrium. However, the sign of shock of housing price is ambiguous. We can set proper parameter condition to make housing price goes up when λ goes up in the equilibrium. [6].In summary, similar to the case in the baseline model , the temporary shock in the income distribution decreases the real interest rate via two channels. One is via the increase in the lender's income. with more income at $t = 1$, the lender wants to lender more, leading to an decrease in the real lending rate. The other one is the via the decrease in the borrower's income. Given the price of the house, the decrease in borrower's income will make the the household purchase less housing. Since we always assume a binding borrowing constraint , the decreasing in the value of house the borrower purchases tightens the borrowing constraint and reduces the borrowing in the equilibrium. Therefore, the real interest rate in the economy drops further. Thus we have the following results:

Proposition 9. *In the framework ,consider an unexpecte d temporary income distribution shock. Specifically, at $t = 1$ there is an unexpected endowment transfer from borrower to lender. Therefore, at $t = 1$, the real interest rate, r_2 , will fall below the steady state value $\frac{1}{\beta_l} - 1$. and the more share of wealth held by the lender, the smaller r_2 will be in the equilibrium.*

When $\lambda^{'}$ is large enough, r_2 will drop to zero and the economy will experience liquidity trap. Although the borrower in the previous period did not mean to 'overborrow', the unanticipated shock in the income share makes the debt that they should pay back more than the threshold value, a situation that can be regarded that the borrower 'unintentionally' over-borrowed. When liquidity trap occurs, the equilibrium is given by

$$u^{'}\left(\lambda^{'} e_1 + \phi p \bar{H} - \phi p_1 \bar{H}\right) = \beta_l u^{'}(\lambda e^* + \phi p_1 \bar{H} - \beta_l \phi p_1 \bar{H}) \tag{5.4}$$

$$\varphi v^{'}(\bar{H}) = p_1 u^{'}\left((1-\lambda^{'})e_1 + \phi p_1 \bar{H} - \phi p \bar{H}\right)(1-\phi) +$$

$$\beta_b u^{'}((1-\lambda)e^* + \beta_l \phi p \bar{H} - \phi p_1 \bar{H})(\phi p_1 - p_2(p_1)) \tag{5.5}$$

In the (5.4), there is a positive relationship between e_1 and p_1. This equation illustrates the affect of asset price on aggregate demand. Intuitively speaking, the larger the asset price, the more the borrower can borrow, and thus the larger the aggregate demand of the household. In the equation (5.5), there is also a positive relationship between e_1 and p_1. The intuition illustrates the income that is required to support a given asset price. Higher net income raises borrower consumption and therefore supports a higher asset price. Any intersection of (5.4) and (5.5) is an equilibrium. However, since aggregate demand determines aggregate output, only those intersections that satisfy the slope of (5.4) is smaller than that of curve (5.5) are stable equilibrium[7].

Now consider an increase in the $\lambda^{'}$. This makes the curve (5.4) shift down and the curve (5.5) shift up. Therefore, both $e_1$and p_1 are smaller than their counterparts in the original equilibrium. The shock of income distribution,on the one hand, reduces the income that is distributed to the borrower and makes

[6]This can be regarded as an approximation to the Japan before the liquidity trap, when the housing price in Japan kept going drastically

[7]Consider an equilibrium point at which the slope of (5.4) is larger than that of (5.5). Suppose an marginal increase in the asset price p. This makes the aggregate demand larger than the output that is required to support the asset price. to make demand and supply clear, the asset price must increase again to make the output catch up with the aggregate demand. However, this makes the aggregate demand increases again. Thus, this equilibrium is not stable.

Table 2: Calibration: Parameter setting

parameter	definition	value	source
e^*	the efficient output level	1	standardized to 1
β_l	the time discount rate of the lender	0.99	Iacoviello(2005)
β_b	the time discount rate of the borrower	0.96	Iacoviello(2005)
ϕ	the LTV ratio limitation	0.95	Mian and Sufi(2011)
$\bar{H}$	house supply	1	standarized to 1
φ	weight on house consumption	0.049	Ngo(2013)
λ	The fraction of total income held by lender	0.5	

him cut the purchase on the house, and thus the house price fall. The drop in the house price in turn tightens the borrowing constraint and thus decreases the aggregate demand (i.e. output) of the economy. Therefore, the income of the borrower decreases furthermore and the price decreases further more. On the otherhand, the tightening of borrowing implies that the amount that the borrower pays back to the lender at $t = 2$ is decreases. Therefore, the lender, anticipating the decrease in income in future, has to cut its consumption demand at $t = 1$, since now the interest rate is already zero. In a word, via price drop, the income distribution reduces the consumption demands of both the lender and the borrower, leading to a drop in aggregate demand and output.

Another interesting conclusion is that the asset price from $t = 2$, i.e., p_2, will increase in the equilibirum if $\lambda^{'}$ increases. The intuition is obvious: at $t = 1$, and income distribution shock makes the borrower cut his borrowing, and due to the some reason he keeps this low borrowing level (maybe due to laws and regulations.) However, the borrower's income from $t = 2$ is larger than it is at $t = 1$, meaning that the borrower has more resources. Therefore, they will spend more on house purchasing, leading to a price boom from $t = 2$.

Proposition 10. *In the equilibrium when $r_2 < 0$. the decrease of the share of income held by borrower will lead to decrease in both output and asset price in the equilibrium.*

Finally, to verify the above analysis, I use calibration to study the behavior of endogenous variables in the equilibrium when changing the value of λ. The parameters for calibration are in the following:

The Figure 6.2 shows the behabavior of endogenous variables. The 'binding' is to indicate whether the the borrowers borrows up to limit at $t = 1$, with 1 being 'yes' and 0 being 'no'. The result shows that borrower always borrows up to the limit given the value of $\lambda^{'}$ in the chart. once the $\lambda^{'}$is larger than at around 0.57, the binding condition will no longer hold, so I only report the result before the 'binding' becomes 0. We can see that the the calibration matches the analysis above; before the r_2 reaches zero, if $\lambda^{'}$ is larger than λ and $\lambda^{'}$ increases, the real interest rate and the consumption of the borrower at $t = 1$ decreases, while the housing price and the consumption of the lender increases. After the r_2 reaches zero, due to zero lower bound and the nominal rigidity, housing price, household's consumption and aggregate output all decreases. This is to say, if the unanticipated income distribution (more income is concentrated to the lender) is sufficient large, liquidity trap will happen, and asset price and output will drop drastically in the equilibrium

6 Conclusions

Via a simple heterogeneous agent model, This paper explores how income distribution shock will trigger liquidity trap and output drop under nominal rigidity and zero lower bound. In the baseline model, there is no asset market , and the borrowing limit of the borrower is determined by the income he is given. I discussed how the agents will behave when there is a income distribution shock in the future. Specifically, I studies four types of shocks, Unanticpated temporary shock, anticipated temporary shock, unanticipated permanent shock, anticipated permanent shock. Under the first three shocks I reached the similar conclusions, that is ,the increasing inequality may trigger liquidity trap and lead to output drop when liquidity trap occurs. However, under the final shock, the decreasing inequality but not increasing inequality may trigger liqudity trap. This implies that a well-expected persistent redistribution policy (a transfer of income from rich to poor) in the futrue may trigger the poorer to overborrow at current period if no macroprudential policy is put into effect. In the extension, in which borrower can trade housing and the borrowing constraint is determined by the value of the house the borrower owns, I analyzed how an unanticipated income distribution shock will lead to liquidity trap. The conclusion is that during the equilibrium , before the liquidity trap, the larger shock is, the lower the interest rate, and under proper parameter conditions, the higher the housing price and borrowing. When the shock is large enough so that real interest rate drops below zero and liquidity trap occurs, the larger the shock, the lower the housing price, output and borrowing constraint.

This is the very first stage of my research. Several extensions can be done in the future :First, through this paper we treat the distribution shock as exogenous. In this paper I only assume that the shock of income distribution shock is totally exogenous[8]However, the income distribution may not be exogenous since it is affected by aggregate economy. Household with different endowment and patience may react differently to the aggregate economy, and thus the income and wealth distribution will evolve endogenous. Therefore, it is interesting to consider the interaction between wealth inequality/income inequality and aggregate economy. Second, it is interesting to see how the introduction of financial intermediatary will change will affect the conclusions the paper has so far reached. Intuitively speaking, when income inequality is larger, the poor want to borrow more and the rich want to lend more. Therefore, the demand for financial service increases, and lead to the increasing scale and leveraging of banks and other financial intermediaries. It's interesting to study how income inequality affects the possiblity of crises via the financial system.

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Appendix A: Baseline Model

We give a full description of the equilibrium of the baseline model (anticipated income distibution shock) in this appendix. We assume that at $t = 0$, the borrowing constraint is not binding, while at $t = 1$ the borrowing constraint is binding.

[8]This is consistent with the Iacoviello(2008) and Kumhof and Ranciere(2010), which both assume exogenous income distribution shock.

When $d_1 < \bar{d_1}$ The equilibrium condition is

$$u^{'}(\frac{1}{2}e^{*}+\frac{d_1}{1+r_1})=\beta_b(1+r_1)u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}+\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}-d_1\right) \tag{A.1}$$

$$u^{'}(\frac{1}{2}e^{*}-\frac{d_1}{1+r_1})=\beta_l(1+r_1)u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}-\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}+d_1\right) \tag{A.2}$$

$$u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+d_1-\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}\right)=(1+r_2)\beta_l u^{'}\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e^{*}-\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right) \tag{A.3}$$

The three equations together determine the d_1,$r_1$and r_2.The condition for the borrowing constraint to hold at $t=1$ and not hold at $t=0$ are respectively:

$$u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}+\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}-d_1\right)>(1+r_2)\beta_b u^{'}\left(\frac{1}{2}e^{*}-\phi\left(\frac{1}{2}-\lambda\right)e^{*}+\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

$$u^{'}(\frac{1}{2}e^{*}+\frac{\phi\frac{1}{2}e^{*}}{1+r_1})<\beta_b(1+r_1)u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}+\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}-\phi\frac{1}{2}e^{*}\right)$$

Now we characterize the equilibrium at $t=0$. We assume that, during this period, liquidity trap does not happen. We characterize under what condition will the over borrowing will happen. First we define $\bar{r_1}$as:

$$\frac{1}{1+\bar{r_1}}=\frac{\beta_l u^{'}(c_{l,1})}{u^{'}(c_{l,0})}=\frac{\beta_l u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d_1}-\phi\left(1-\lambda^{'}\right)e^{*}\right)}{u^{'}\left(\frac{1}{2}e^{*}-\frac{\bar{d_1}}{1+\bar{r_1}}\right)} \tag{3.2}$$

Recall the Euler equation of the lender at $t=1$ and $t=2$:

$$u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d_1}-\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)=\beta_l u^{'}\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e^{*}-\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

Which implies that the $u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d_1}-\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$ is increasing with λ. constant. Thus, (3.2) implies that $\bar{r_1}$ is a decreasing function of the λ, since $\bar{d_1}$is decreasing with λ. we also assume that $\frac{\bar{d_1}(\lambda)}{1+\bar{r_1}(\lambda)}$ is also an decreasing function of λ.

$$\frac{1}{1+\bar{r_1}}=\frac{\beta_l u^{'}\left(\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d_1}-\phi\left(1-\lambda^{'}\right)e^{*}\right)}{u^{'}\left(\frac{1}{2}e^{*}-\frac{\bar{d_1}}{1+\bar{r_1}}\right)}>\frac{\beta_b u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}-\bar{d_1}+\phi\left(1-\lambda^{'}\right)e^{*}\right)}{u^{'}\left(\frac{1}{2}e^{*}+\frac{\bar{d_1}}{1+\bar{r_1}}\right)} \tag{3.3}$$

It's obvious that the left hand side is increasing with λ since $\bar{r_1}$ is a decreasing function of the λ. On the other hand, the right hand side decreases with λ[9]. Therefore, there exists a threshold value $\bar{\lambda}(e^{*},\phi,\beta_l,\beta_b)$ such that when $\lambda>\bar{\lambda}(e^{*},\phi,\beta_l,\beta_b)$, the inequality (3.3) holds. However, considering the borrowing constraint in the $t=0$ period, if $\bar{d_1}\geq\phi\frac{1}{2}e^{*}$, the overborrowing will not happen even if the $\lambda>\bar{\lambda}(e^{*},\phi,\beta_l,\beta_b)$ holds. Thus, in order for the overborrowing to be possible, we should assume that $\bar{d_1}>\phi\frac{1}{2}e^{*}$

When $d_1>\bar{d_1}$, the equilibrium is described by:

$$u^{'}(\frac{1}{2}e^{*}+\frac{d_1}{1+r_1})=\beta_b(1+r_1)u^{'}\left(\left(\frac{1}{2}-\lambda\right)e_1+\phi\left(\frac{1}{2}-\lambda\right)e_1-d_1\right) \tag{A.4}$$

[9]On the right hand side, the nominator decreases since $\left(\frac{1}{2}+\lambda\right)e^{*}+\bar{d_1}-\phi\left(\frac{1}{2}-\lambda\right)e^{*}$decreases with λ. Thus $\left(\frac{1}{2}-\lambda\right)e^{*}-\bar{d_1}+\phi\left(1-\lambda^{'}\right)e^{*}$ increase. On the other hand, the denominator increases since $\frac{\bar{d_1}(\lambda)}{1+\bar{r_1}(\lambda)}$ decreases with λ

$$u^{'}(\frac{1}{2}e^{*}-\frac{d_1}{1+r_1})=\beta_l(1+r_1)u^{'}\left(\left(\frac{1}{2}+\lambda\right)e_1-\phi\left(\frac{1}{2}-\lambda\right)e_1+d_1\right) \tag{A.5}$$

$$u^{'}\left(\left(\frac{1}{2}+\lambda\right)e_1+d_1-\phi\left(\frac{1}{2}-\lambda\right)e_1\right)=\beta_l u^{'}\left(\frac{1}{2}e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e_1-\beta_l\phi\left(\frac{1}{2}-\lambda\right)e\right) \tag{A.6}$$

This three equations together determines the d_1,$r_1$and e_1.The condition for the borrowing constraint to hold at $t=1$ and not hold at $t=0$ are respectively:

$$u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}+\phi\left(\frac{1}{2}-\lambda\right)e_1-d_1\right)>\beta_b u^{'}\left(\frac{1}{2}e^{*}-\phi\left(\frac{1}{2}-\lambda\right)e_1+\beta_l\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

$$u^{'}(\frac{1}{2}e^{*}+\frac{\phi\frac{1}{2}e^{*}}{1+r_1})<\beta_b(1+r_1)u^{'}\left(\left(\frac{1}{2}-\lambda\right)e^{*}+\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{1+r_2}-\phi\left(\frac{1}{2}-\lambda\right)e^{*}\right)$$

suppose an marginal increase in λ. if we assume that e_1 increase, then $d_1$decreases, and thereofre either (A.4) or (A.5) will not hold. Thus e_1 must decrease in the equilibrium

We next derive the allocation under planner.take derivatives w.r.t $c_{b,0},c_{l,0},d_1$we have

$$\gamma_b u'(c_{b,0})=\gamma_l u'(c_{l,0})\text{ and }\beta_b\gamma_b\frac{\partial v_b(d_1)}{\partial d_1}+\beta_l\gamma_l\frac{\partial v_l(d_1)}{\partial d_1}=0$$

recall that

$$v_b(d_1)=u\left((\frac{1}{2}-\lambda)e(d_1)-d_1+\frac{\phi(\frac{1}{2}-\lambda)e(d_1)}{1+r_2(d_1)}\right)+\beta_b u\left(e^{*}-\phi(\frac{1}{2}-\lambda)e(d_1)+\beta_l\phi\frac{1}{2}e^{*}\right)$$

$$v_l(d_1)=u\left(\left(\frac{1}{2}+\lambda\right)e(d_1)+d_1-\frac{\phi(\frac{1}{2}-\lambda)e(d_1)}{1+r_2(d_1)}\right)+\beta_l u\left(e^{*}+\phi(\frac{1}{2}-\lambda)e(d_1)-\beta_l\phi\frac{1}{2}e^{*}\right)$$

When $d_1>\bar{d}_1$:

$$\frac{\partial v_b(d_1)}{\partial d_1}=u^{'}(c_{b,1})\left((\frac{1}{2}-\lambda)\frac{\partial e(d_1)}{\partial d_1}-1\right)+\phi(\frac{1}{2}-\lambda)\frac{\partial e(d_1)}{\partial d_1}\left(u^{'}(c_{b,1})-\beta_b u^{'}(c_{b,2})\right)$$

$$\frac{\partial v_l(d_1)}{\partial d_1}=u^{'}(c_{l,1})\left((\frac{1}{2}+\lambda)\frac{\partial e(d_1)}{\partial d_1}+1\right)-\phi(\frac{1}{2}-\lambda)\frac{\partial e(d_1)}{\partial d_1}\left(u^{'}(c_{l,1})-\beta_b u^{'}(c_{l,2})\right)=u^{'}(c_{l,1})\left((\frac{1}{2}+\lambda)\frac{\partial e(d_1)}{\partial d_1}+1\right)$$

Recall that $\frac{\partial e_1}{\partial d_1}=\frac{-\beta_l}{\beta_l\left(\frac{1}{2}+\lambda\right)-(\beta_l+1)\phi\left(\frac{1}{2}-\lambda\right)}$. thus both $(\frac{1}{2}+\lambda)\frac{\partial e(d_1)}{\partial d_1}+1<0$ and $(\frac{1}{2}-\lambda)\frac{\partial e(d_1)}{\partial d_1}-1<0$ hold. Plug these two equations into the second conditions and we can find out that the equation can not hold.

When $d_1<\bar{d}_1$.

$$\frac{\partial v_b(d_1)}{\partial d_1}=(-1+\eta)u^{'}(c_{b,1})\text{ and }\frac{\partial v_l(d_1)}{\partial d_1}=(1-\eta)u^{'}(c_{l,1})$$

in which [10]$\eta=\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{(1+r_2)^2}\frac{\partial r_2}{\partial d_1}\in[-1,0]$

[10]Reconsider the Euler equation of the lender, (*). Taking derivative of r_2 w.r.t d_1 we can get :

$$u^{''}(c_{l,1})\left[1+\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{(1+r_2)^2}\frac{\partial r_2}{\partial d_1}\right]=\frac{\partial r_2}{\partial d_1}\beta_l u^{'}(c_{l,2})$$

It's obvious to see that $\frac{\phi\left(\frac{1}{2}-\lambda\right)e^{*}}{(1+r_2)^2}\frac{\partial r_2}{\partial d_1}\in[-1,0]$

Thus it's obvious to see that the allocation is

$$\frac{\beta_b u^{'}\left(c_{b,1}\right)}{u'\left(c_{b,0}\right)} = \frac{\beta_l u^{'}\left(c_{l,1}\right)}{u'\left(c_{l,0}\right)}$$

When $d_1 = \bar{d}_1$, we notice that

$$\frac{\nabla_{sub} v_b\left(d_1\right)}{\partial d_1} \in \left[u^{'}\left(c_{b,1}\right)\left(\left(\frac{1}{2}-\lambda\right)\frac{\partial e\left(d_1\right)}{\partial d_1}-1\right)+\phi\left(\frac{1}{2}-\lambda\right)\frac{\partial e\left(d_1\right)}{\partial d_1}\left(u^{'}\left(c_{b,1}\right)-\beta_b u^{'}\left(c_{b,2}\right)\right), \left(-1+\eta\right)u^{'}\left(c_{b,1}\right)\right] \text{ and}$$

$$\frac{\nabla_{sub} v_l\left(d_1\right)}{\partial d_1} \in \left[u^{'}\left(c_{l,1}\right)\left(\left(\frac{1}{2}+\lambda\right)\frac{\partial e\left(d_1\right)}{\partial d_1}+1\right), \left(1-\eta\right)u^{'}\left(c_{l,1}\right)\right]$$

Thus, for the second condition to hold, we must have:

$$\frac{\beta_b u^{'}\left(c_{b,1}\right)}{u'\left(c_{b,0}\right)} \leq \frac{\beta_l u^{'}\left(c_{l,1}\right)}{u'\left(c_{l,0}\right)}$$

Appendix B The Model with Housing Market

In this Appendix I derive the the steady state in the model with housing market. The first order condition for the lender is:

$$u^{'}\left(\lambda_t e_t + d_t - \frac{d_{t+1}}{1+r_{t+1}}\right) = \left(1+r_{t+1}\right)\beta_l u^{'}\left(\lambda_{t+1}e_{t+1} + d_{t+1} - \frac{d_{t+2}}{1+r_{t+2}}\right)$$

The condition for the borrower is:

$$\beta^t u^{'}\left((1-\lambda_t)e_t + q_t H_{t-1} + \frac{d_{t+1}}{1+r_{t+1}} - q_t H_t - d_t\right)(-p_t) +$$

$$\beta^{t+1} u^{'}\left((1-\lambda_{t+1})e_{t+1} + q_{t+1}H_t + \frac{d_{t+2}}{1+r_{t+2}} - p_{t+1}H_{t+1} - d_{t+1}\right)p_{t+1} + \beta^t v^{'}\left(H_t\right) + \eta\phi p_t = 0$$

$$\beta^t u^{'}\left((1-\lambda_t)e_t + q_t H_{t-1} + \frac{d_{t+1}}{1+r_{t+1}} - q_t H_t - d_t\right)\frac{1}{1+r_{t+1}}$$

$$-\beta^{t+1} u^{'}\left((1-\lambda_{t+1})e_{t+1} + q_{t+1}H_t + \frac{d_{t+2}}{1+r_{t+2}} - q_{t+1}H_{t+1} - d_{t+1}\right) - \eta = 0$$

Combining the two equations we can get:

$$v^{'}\left(H_t\right) = A q_t\left(1 - \frac{\phi}{1+i_{t+1}}\right) + B\beta_b\left(\phi q_t - q_{t+1}\right)$$

In which

$$A = u^{'}\left((1-\lambda_t)e_t + q_t H_{t-1}^b + \frac{d_{t+1}}{1+i_{t+1}} - q_t H_t^b - d_t\right)$$

$$B = u^{'}\left((1-\lambda_{t+1})e_{t+1} + q_{t+1}H_t^b + \frac{d_{t+2}}{1+r_{t+2}} - q_{t+1}H_{t+1}^b - d_{t+1}\right)$$

Appendix C Figures

Figure 6.1: Calibration: Baseline Model

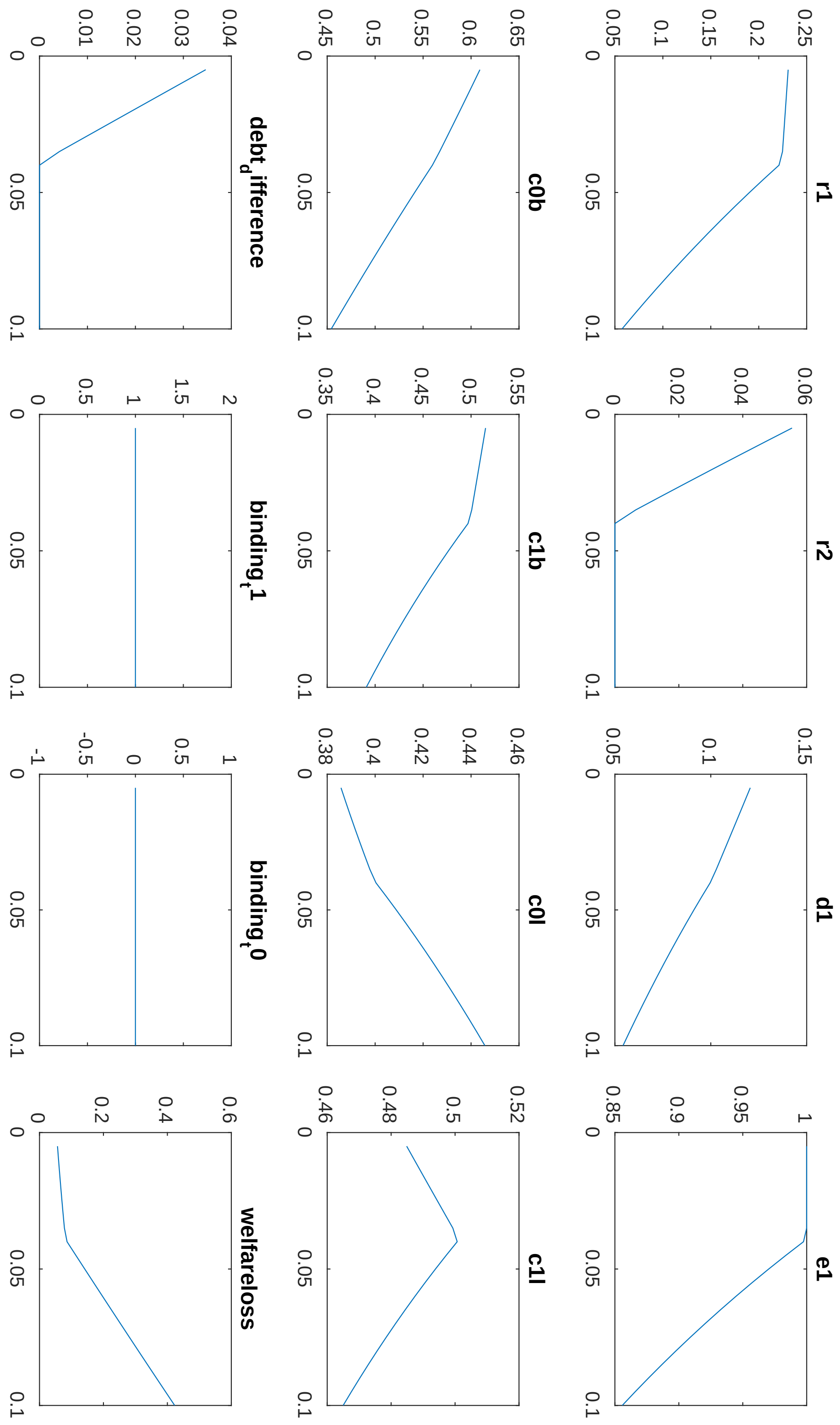

Figure 6.2: Calibration: Extension

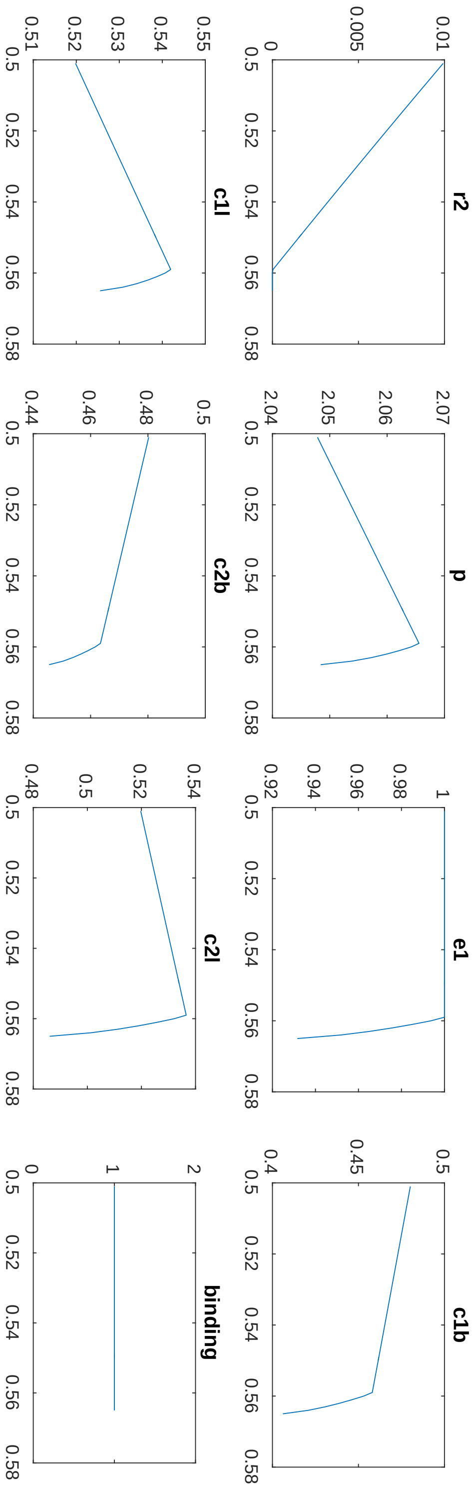

Figure 6.3: Anticipated Permanent Shock: No borrowing constraint at $t = 0$

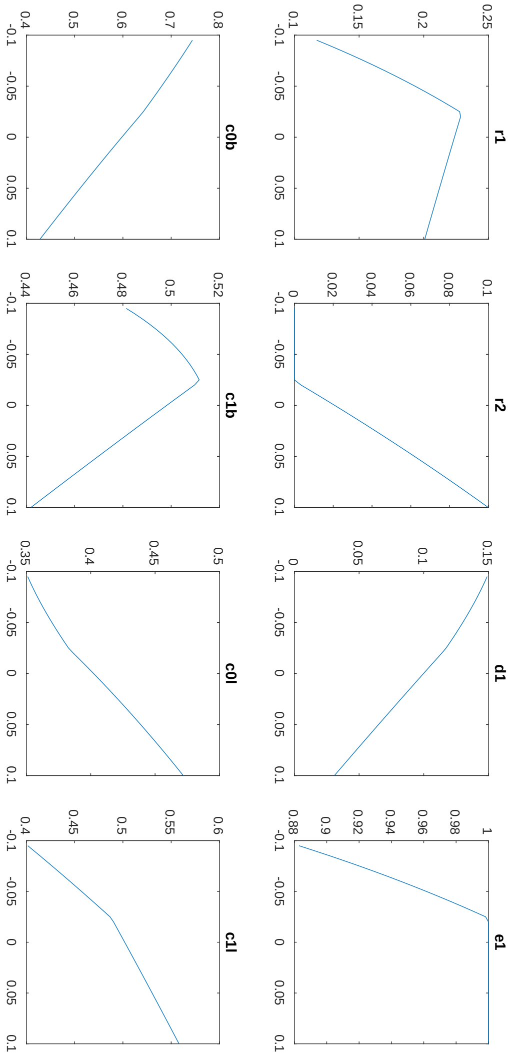

Figure 6.4: Anticipated Permanent Shock: borrowing constraint at $t = 0$