Investors' Subjective Beliefs and the Cross-Section of Stock Returns*

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Abstract

This paper empirically investigates how investors' subjective beliefs drive the cross-section of stock returns. Using a data set of real-time professional survey forecasts, I first estimate belief wedges, defined as the difference between the professional survey forecasts and the *Factor-Augmented Vector Autoregression* (*FAVAR*) model implied conditional rational forecasts. I then construct empirical measures of investors' subjective beliefs as latent factors from the estimated belief wedges. Next, I show that the subjective belief factors exhibit significant explanatory power with large and significant coefficients for expected returns across eight stock portfolio groups separately and jointly. Finally, a potential theoretical explanation for the origins of belief disparities is rendered based on the robust preference model.

JEL: G12, G41, E71

Keywords: Asset Pricing, Behavioral Finance, Survey Forecast

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1 Introduction

Economists have long recognized the importance of beliefs in determining macroeconomic outcomes. The rational expectation hypothesis, which has been the mainstream since the 1970s, assumes that economic agents understand every aspect of the economy so that their subjective beliefs coincide with the objective probability that governs macroeconomic dynamics. Even after decades of work, the rational expectation hypothesis faces many difficulties in explaining macroeconomic and financial data[1]. To overcome these difficulties, recent theories focus on exploring deviations from rational expectations by emphasizing investors' subjective beliefs. Inspired by the recent development in theory, this paper empirically investigates whether investors' belief wedges, defined as the difference between survey forecasts and rational forecasts under the data-generating measure, are able to explain observed cross-sectional variations in expected stock returns.

Why do investors' *subjective* beliefs matter for asset-pricing? The price of a claim to a random future payoff equals the conditional expectation of the inner product of a stochastic discount factor and the random future payoff, evaluated using the investors' *subjective* probability measure. In general, the investors' *subjective* probability measure differs from the *physical* probability measure used by the econometrician. Hence, if the value of the claim is evaluated under the econometrician's physical probability measure, we need to adjust the investor's stochastic discount factor by the disparity between the investors' subjective beliefs and the data-generating probability measure inferred by the econometrician. Fluctuations in this disparity induce fluctuations in aggregate market valuation, and this disparity therefore acts as a non-diversifiable risk factor that is priced in the cross-section of stock returns.

The first contribution of this paper is to construct a novel empirical measure of the

[1]For example, the equity premium puzzle, found by Hansen and Singleton (1983) and Mehra and Prescott (1985), shows that the high equity premium observed in data is hard to justify by a standard rational, representative-agent model with a reasonable degree of risk aversion. Weil (1989) documented a low risk-free rate resulting in the risk-free rate puzzle, and Shiller (1981) found that equities are too volatile compared to fundamental macro variables. Fama (1984) documented the forward premium puzzle, in which high interest rate currencies tend to appreciate.

theoretical belief disparity between the investors' *subjective* probability measure and the econometrician's *physical* probability measure. To construct the empirical measure, I first define the investors' belief disparity as the belief wedge between real-time professional survey forecasts and conditional rational forecasts. A crucial step here is to identify the conditional rational expectation under the data-generating process to obtain an empirical measure of belief disparities prevailing in the market. I fulfill this purpose by setting up a *Factor-Augmented Vector Autoregression (FAVAR)* model proposed by Bernanke et al. (2005) which can make an effective use of an information-rich environment in a relatively simple framework. To be specific, I estimate the information factors used to construct the conditional rational forecasts from a large data set consisting of 135 monthly macro variables and compute the *FAVAR* model implied conditional rational expectation for each of the eleven survey forecast series. After obtaining the belief wedge for each of the eleven series, I estimate two latent common factors from the eleven series of constructed belief wedges by the static principle component analysis. The two latent common factors are treated as a representation of investors' belief disparities between their *subjective* probability measure and the data-generating probability measure inferred by the econometrician. The first subjective belief factor loads heavily on inflation with a marginal contribution of 21% to the common variation of the eleven belief wedges, while the second subjective belief factor concentrates on real activity and labor explaining 19% of variation in the eleven series. Overall, these two factors account for 40% of variation contained in the eleven series.

Using the two constructed subjective belief factors, the second contribution is to study their impact on eight groups of stock portfolios double-sorted on measures such as Size, Book to market, Investment, Operating profit, Short-term reversals and Long-term reversals separately and in a pooled estimation of all portfolios jointly. I find that exposures to the subjective belief factors have strong explanatory power in explaining the cross-sectional variations in expected excess returns in all these portfolio groups separately and jointly, and the estimated R^2 ranges from 0.36 to 0.66. In particular, the portfolios that comove more with the first subjective belief factor, which concentrates on inflation, have higher average excess returns. The risk price of the second subjective belief factor loading heavily on real activity and labor, whenever significant, is positive as well. This result matches the interpretation that the subjective belief factors correlate positively with aggregate market valuation, and the risk prices of these factors are positive when pricing risky assets whose payoffs covary systematically with aggregate good and bad times.

Notice that a crucial assumption made in constructing the subjective belief factors is that the belief wedge between professional survey forecasts and model-implied conditional rational forecasts is a sole representation of investor's subjective belief disparities. However, in principle, the belief wedge may also come from the information set discrepancy between professional survey forecasters and the econometrician in the sense that the professional forecasters have a larger information set compared to the econometrician when making the forecast. Section 6.1 provides a thorough analysis to justify the assumption.

Theoretically, what preferences result in investors' belief disparities? One potential explanation is rendered by the robust preference model (Hansen and Sargent (2001a), Hansen and Sargent (2001b), and Hansen and Sargent (2008)). Consider a representative agent with robust preferences who is concerned about model misspecification. Compared with a preference without a concern for model misspecification, this concern for model misspecification adds an extra term (a Radon-Nikodým derivative) in the stochastic discount factor, which overweights bad states. In equilibrium, fluctuations in this term should be a source of priced risk.

This paper contributes to a small but rapidly growing literature that shows measuring beliefs directly from survey data helps explain asset returns. Using survey data to measure interest rate forecasts, Froot (1989) studied the expectations hypothesis for long-horizon bonds, and Piazzesi et al. (2015) documented less volatile and not very cyclical subjective bond risk premia. Focusing on foreign currency markets, Gourinchas and Tornell (2004) showed that the belief distortion, directly measured from survey data, is useful in explaining well-known puzzles in international finance. Different from the existing literature, this paper extracts subjective belief information as latent common factors from a set of real-time professional survey forecast data, and show that the information obtained from investors' beliefs does help explain the cross-sectional variation in stock returns, providing novel evidence from the stock market.

This paper is also related to the literature that studies the impact of information contained in survey data on macroeconomic dynamics. Ang et al. (2007) and Barsky and Sims (2012) showed that survey data contain useful information about future economic activity. In this paper, I show that the information on subjective beliefs obtained from survey forecast data is useful in explaining the cross-section of stock returns.

The third contribution of this paper is to provide a theory-based risk factor that is priced in the cross-section of stock returns. There is a large literature constantly seeking for risk factors that can explain the cross-section of asset returns. Unlike the previous literature that builds on models under rational expectations, this paper departs from the rational expectation hypothesis and derives a risk factor from a representative agent who evaluates her utility under her *subjective* probability measure.

The rest of the paper is structured as follows. Section 2 provides a detailed literature review. Section 3 sets up a theoretical model guiding the following empirical analysis. The econometric framework and estimation for subjective belief factors are provided in Section 4. Section 5 investigates the impact of investors' subjective beliefs on the cross-section of stock returns. Section 6 provides two related discussions. Section 7 concludes. The detailed data explanation and additional results are provided in Appendix.

2 Previous Literature

The rational expectation hypothesis, the workhorse in macroeconomics and finance, assumes that there exists an objective probability law governing the state process, and economic agents know this law which coincides with their subjective beliefs. Ever since Hansen and Singleton (1983) and Mehra and Prescott (1985) documented the equity risk premium puzzle, a large number of empirical papers found various difficulties of the rational expectation hypothesis when analyzing asset returns and links between asset markets and the macroeconomy[2]. Since then, researchers began to depart from the rational expectation hypothesis and to explore various deviations from it. Cecchetti et al. (2000) considered a two-state Markov switching process for the consumption growth rate and modeled a randomized subjective transition matrix which is on average less persistent than the true transition matrix in an otherwise standard Lucas asset pricing model. They showed that the model is able to match the first and second moments of the equity premium and the risk-free rate, as well as the persistence and predictability of excess returns found in the data. Ju and Miao (2012) developed a more generalized model with recursive smooth ambiguity and hidden Markov states, and showed that the model could replicate many puzzles observed in data. The robust control theory (Hansen and Sargent (2001a), Hansen and Sargent (2001b), and Hansen and Sargent (2008)) models a representative agent with an *approximating* model constantly concerned with model misspecification (a *worst-case* model). As illustrated in Hansen and Sargent (2008), agents' cautious responses to possible model misspecification raise the theoretical values of risk aversion, which helps to explain many asset-pricing puzzles.

Inspired by the recent development in theory, the goal of this paper is to construct estimates of belief disparities and estimate their risk prices for the cross-section of stock returns. There is a small but rapidly growing literature that shows measuring belief directly from survey data helps explain asset returns. Using survey data to measure interest rate forecasts, Froot (1989) argued that the failure of the expectations hypothesis for long-horizon bonds may be attributed to the failure of rational expectation assumption imposed in the tests. Gourinchas and Tornell (2004) showed that the belief distortion directly measured

[2]See, for example, Shiller (1981), Fama and French (1988a), Fama and French (1988b), Fama and French (1989)), Campbell and Shiller (1988a), Campbell and Shiller (1988b)), Poterba and Summers (1988), Fama (1984) and Backus and Smith (1993).

from survey data could explain both the foreign exchange forward-premium and delayed-overshooting puzzles. Bacchetta et al. (2009) documented that the predictability of excess returns may be due to the predictability of expectational errors in various financial markets. Piazzesi et al. (2015) and Szőke (2017) found that measuring interest rate forecasts by survey data results in less volatile and not very cyclical subjective bond risk premia in contrast to a common statistical measure. One concern with survey data is that they may contain too much noise. Greenwood and Shleifer (2014) compared survey data from six different sources and found highly positive correlation between the six sources. They further showed that the investors' expectations are strongly negatively correlated with the model-based expectations, casting doubt on a representative agent model under rational expectations. The result was confirmed by Koijen et al. (2015) using international data.

This paper is also related to the literature that studies the impact of information contained in survey data on macroeconomic dynamics. Ang et al. (2007) showed that survey data have superior forecasting power on inflation over alternative methods using macro variables or asset market data. Barsky and Sims (2012) documented that innovations to consumer confidence convey incremental information about economic activities far into the future. Carroll (2003) analyzed survey expectation of households and professional forecasters and found that the "stickyness" in survey expectation has essential impacts on macro dynamics. Coibion and Gorodnichenko (2012) confirmed information rigidities from survey data. Leduc and Liu (2016) extracted information on uncertainty from the Michigan survey of consumers and showed that the uncertainty shock acts like a negative demand shock.

3 Theoretical Motivation

In this section, I provide a theoretical background for the following empirical analysis. I set up a general economic framework of a representative agent who evaluates her utility under a *subjective* probability measure which may, in general, differ from the *physical* probability measure used by the econometrician. I derive the stochastic discount factor and consider the implications for asset pricing.

3.1 Belief Distortion

I model the belief distortion as a Radon-Nikodým derivative denoted by ξ_{t+1}/ξ_t, where ξ_{t+1}/ξ_t is a strictly positive martingale with $E_t\left[\xi_{t+1}/\xi_t\right] = 1$ and $\xi_0 = 1$. This stochastic process measures the disparity between the investors' *subjective* probability measure $\mathcal{P}^*$ and the *physical* probability measure $\mathcal{P}$ which is defined from the econometrician' s perspective. For any random variable x_{t+1},

$$E_t^*\left[x_{t+1}\right] \equiv E_t\left[\frac{\xi_{t+1}}{\xi_t} x_{t+1}\right]. \tag{1}$$

The belief wedge Δ_t between a forecast of x_{t+1} under the *subjective* probability measure $\mathcal{P}^*$ and the one under the *physical* probability measure $\mathcal{P}$ is thus defined as

$$\begin{aligned}\Delta_t \equiv E_t^*\left[x_{t+1}\right] - E_t\left[x_{t+1}\right] &= E_t\left[\frac{\xi_{t+1}}{\xi_t} x_{t+1}\right] - E_t\left[x_{t+1}\right] \\ &= cov_t\left(\frac{\xi_{t+1}}{\xi_t}, x_{t+1}\right). \end{aligned} \tag{2}$$

The last equality follows from $E_t\left[\xi_{t+1}/\xi_t\right] = 1$.

3.2 Implications for Asset Pricing

To explain the role of fluctuations in investors' subjective beliefs, consider a representative agent model over periods $t = 0, 1, 2$. The representative agent evaluates her utility under her *subjective* probability measure $\mathcal{P}^*$. There is a set of risky assets indexed by i that are traded in periods 0 and 1, and pay out dividends $D_{i,2}$ in the terminal period 2[3]. The price

[3] For the ease of illustration, I present the simplest model here. But the conclusion can be easily extended to a model with infinitely lived assets which pay out a dividend in every period.

of the asset i at period $t = 1$ is given by

$$P_{i,1} = E_1^* \left[M_2 D_{i,2} \right] = E_1 \left[\frac{\xi_2}{\xi_1} M_2 D_{i,2} \right] = E_1 \left[M_2 D_{i,2} \right] + Cov_1 \left(\frac{\xi_2}{\xi_1}, M_2 D_{i,2} \right), \tag{3}$$

where $P_{i,1}$ is the price of asset i at $t = 1$ and M_1 is the stochastic discount factor at $t = 1$. Fluctuations in subjective beliefs at time $t = 1$ about outcomes in period 2, imply movements in the covariance term and induce fluctuations in the price $P_{i,1}$. Equation (3) also holds for the aggregate market dividend D_2,

$$P_1 = E_1 \left[\frac{\xi_2}{\xi_1} M_2 D_2 \right] = E_1 \left[M_2 D_2 \right] + Cov_1 \left(\frac{\xi_2}{\xi_1}, M_2 D_2 \right), \tag{4}$$

and hence fluctuations in subjective beliefs affect aggregate market valuation P_1. When the random variable x_2 in equation (2) is correlated with the dividend process D_2, we can proxy the covariance with the belief wedge Δ_1. Valuation of the asset i in period 0 is then given by

$$P_{i,0} = E_0 \left[\frac{\xi_1}{\xi_0} M_1 P_{i,1} \right].$$

Since the subjective belief wedge Δ_1 affects aggregate conditions in period 1, the stochastic discount factor M_1 is a function of Δ_1. The cross-section of expected excess returns on assets between period 0 and 1,

$$E_0 \left[R_{i,1}^e \right] = -E_0 \left[\frac{\xi_1}{\xi_0} M_1 \right]^{-1} Cov_0 \left(\frac{\xi_1}{\xi_0} M_1, R_{i,1}^e \right)$$

therefore depends on Δ_1 through the exposure of M_1 to Δ_1. If Δ_1 correlates positively with aggregate market conditions, then the associated price of risk should be positive.

4 Construction of Subjective Belief Factors

In this section, I describe the econometric framework used to construct the investor's subjective belief factors and present the results.

I construct the investor's subjective belief factors as hidden common factors from a set of eleven series of belief wedges, which are defined as the difference between the survey forecast of a macro variable and the model-implied conditional rational expectation of the same macro variable. Let us denote $\tilde{y}^t_{j,t-l}$ as the survey forecast of a macro variable y_j made in the month t for its realization in the month $t-l$, e.g., $y_{j,t-l}$, right before its early release. The macro news is usually released with delays. Among the eleven series of macro news releases that are analyzed in this paper, ten series are published with a one-month lag ($l=1$). This implies that the announcement made in April 1999 is, in fact, the realization in March 1999. The remaining one series is published without delays ($l=0$) . Hence, in this sense, ten series of professional survey forecasts are actually backcasts, and the remaining one series is a nowcast. Following (2), I formally define the investor's belief wedge in the month t of the macro variable $y_{j,t-l}$ as

$$\Delta_{j,t} = \tilde{y}^t_{j,t-l} - E\left[y_{j,t-l}|\mathcal{I}_{t-1}\right], \tag{5}$$

where $\tilde{y}^t_{j,t-l}$ is the survey estimate for y_j formulated in the month t for the realization of the month $t-l$ before its early release, and $E\left[y^t_{j,t-l}|\mathcal{I}_{t-1}\right]$ is the conditional rational expectation of $y_{j,t-l}$ based on the information set $\mathcal{I}_{t-1}$. $\mathcal{I}_t$ denotes the information set that covers all the information up to the end of month t. A crucial assumption is embedded in (5). The assumption is that the information set used by professional survey forecasters is the same as the one used by the econometrician. In other words, following (2), it is necessary to assume

$$\tilde{y}^t_{j,t-l} = E^*\left[y_{j,t-l}|\mathcal{I}_{t-1}\right]. \tag{6}$$

This is a relatively strong assumption given the complexity of the information set used by professional survey forecasters. First of all, the professional survey forecasts analyzed in this paper are real-time forecasts in the sense that professional survey forecasters are allowed to revise their forecasts of a macro news announcement up to the night before its first release. Thus, even within a month, the forecast of a macro news announcement published later in the month should be based on a slightly larger information set than the one released early in the month. Hence, in principle, this implies that the forecasts of macro news announcements published later in the month are more accurate than those released early in the month[4]. This hypothesis is tested in Section 4.2.1. The result shows that no such systematic bias is observed in the data, which validates the assumption. More importantly, professional survey forecasters may have a larger information set than the econometrician, and this superior information set helps them to make better forecasts than the model-implied rational forecast. In other words, the belief wedges defined in (5) may also contain the information set discrepancy between professional survey forecasters and the econometrician. Section 6.1 provides empirical analysis to investigate this concern and finds no evidence of the informatoin set difference. Therefore, I proceed with the assumption stated by (6).

4.1 Econometric Framework

A crucial first step in my construction is to have a conditional rational expectation estimate of the macro announcement $y_{j,t-l}$, e.g., $E\left[y_{j,t-l}|\mathcal{I}_{t-1}\right]$ in (5), from which I construct belief wedges that serve as the basis of the investor's subjective belief factors. To identify the conditional rational expectation, it is important to build the forecast based on an information set that is as rich as possible so that the difference between the survey forecast and the model-implied conditional rational expectation can represent belief disparities prevailing in the market. I fulfill this purpose by setting up a *Factor-Augmented Vector Autoregression (FAVAR)* model,

[4] However, in practice, some news annnouncements, e.g., unemployement rate, are easier to predict than others.

which can capture substantial observed variations in "big data" using a reduced number of unobserved common factors extracted from the data and thus allows us to make the forecast based on the information set as close as possible to market participants (Bernanke et al. (2005), Stock and Watson (2016), and Jurado et al. (2015)). The data set used for the estimation of factors consists of 135 series of macro variables spanning the period of 1959/01 to 2017/04. The details of the data set are described in Section 4.2 and Appendix. Compared with conventional *VAR* analysis, the *FAVAR* model avoids the dimensionality curse and thus facilitates data-rich analysis in empirical macroeconomics.

Before moving on to the description of the econometric framework, two comments are in order regarding the data choice. The first question is whether it is desirable to use real-time data or historical data. The difference between real-time data and historical data is that historical data record all the final revised values while real-time data, usually indexed by time vintages, limit the information set only to the time indicated by the time vintages. Since earlier estimates of many macro variable series are often inaccurate and will be revised in the following months, the value of many macro variables is different between these two data sets for recent months. At first glance, the real-time data set may appear more appropriate as it clearly traces the changes in the information set. Nevertheless, the goal of setting up and estimating a *FAVAR* model is to determine the true dynamics of the underlying macro variables. For this reason, I use actual historical data for the following analysis.

Second, I use real-time professional survey forecast data downloaded from Bloomberg Financial Services. The real-time survey data can timely reflect the belief information prevailing in the market participants. More fundamentally, as emphasized by the literature of the limited participation in the stock market (Mankiw and Zeldes (1991), Vissing-Jørgensen (2002), Guvenen (2009) and Malloy et al. (2009)), participation in the stock market is heavily bounded toward more sophisticated and professional investors, and most households do not hold stocks that are actively managed. In fact, as shown by Bhandari et al. (2016) among others, households' expectations are systematically pessimistically biased relative to professional forecasters'. Given that the goal here is to identify the subjective beliefs of investors who are marginal investors and actively participate in the stock market, it is more appropriate to use professional survey forecast data in the following analysis.

Let $X_t = (X_{1,t}, \cdots, X_{N,t})'$ be an $N \times 1$ vector of observed time-series data at time t that are pre-processed to ensure stationarity detailed in Section 4.2. X_t is a set of *information series* from which I estimate the latent common factors that drive the dynamics of the whole macroeconomic system. To be specific, I assume that X_t has an approximate factor structure following

$$X_{i,t} = \Lambda_i' F_t + e_{i,t}, \tag{7}$$

where the information factor F_t is an $r \times 1$ vector of unobserved or latent factors and Λ_i is an $r \times 1$ vector of factor loadings. $e_{i,t}$ is the innovation, which is allowed to be serially correlated in the approximate factor structure. One crucial feature of the dynamic factor model is that the number of observed time-series data X_t, N, is significantly larger than the implied number of factors, r.

Let $y_t = (y_{1,t}, \cdots, y_{n,t})'$ be an $n \times 1$ vector of variables that need to be forecasted. In general, y_t may be a subset of *information series,* or a mixture of a subset of *information series* and some other series not included in the data set of *information series*, or a set of series other than series in the data set of *information series.* Based on the discussion above, the belief wedge of a macro variable y_j in the month t is formally rewritten as

$$\Delta_{j,t} = \tilde{y}^t_{j,t-l} - E\left[y_{j,t-l} | \mathcal{I}_{t-1}\right], \tag{8}$$

where $\tilde{y}^t_{j,t-l}$ is the survey estimate for y_j formulated in the month t for its realization in the month $t-l$ before its early release and $E\left[y_{j,t-l}|\mathcal{I}_{t-1}\right]$ is the conditional rational expectation of $y_{j,t-l}$ based on the information set $\mathcal{I}_{t-1}$.

The forecasting model takes a standard form of *Factor-Augmented Vector Autoregression*

(Stock and Watson (2002a), Stock and Watson (2002b), Stock and Watson (2012), Stock and Watson (2016), Bai and Ng (2008), and Jurado et al. (2015) among others)

$$y_{j,t+1} = \alpha + \beta(L) F_t + \gamma(L) y_{j,t} + \epsilon_{t+1}, \tag{9}$$

where $\beta(L)$ and $\gamma(L)$ are lag polynomials with nonnegative and finite powers of L and ϵ_{t+1} is assumed to satisfy $E_t[\epsilon_{t+1}|y_t, X_t, F_t, y_{t-1}, X_{t-1}, F_{t-1}, \cdots] = 0$. Only using the lags to predict future realizations may suffer from a limited information problem as pointed out by Bernanke et al. (2005), (9) solves this problem by including additional factors estimated from a large data set of macro variables. Thus, if $\beta(L), \gamma(L)$ and $\{F_t\}$ and its lags were known, the minimum mean square forecast of $y_{j,t+1}$ would be $\alpha + \beta(L) F_t + \gamma(L) y_{j,t}$. The fitted value $\hat{y}_{j,t-l}$ obtained by estimating (9) is used to replace the conditional rational expectation $E[y_{j,t-l}|\mathcal{I}_{t-1}]$ in (8) and the belief wedge is computed for each of the professional survey series accordingly. Given a set of belief wedges obtained by (8), I construct subjective belief factors as latent common factors from the data set of belief wedges. In particular, the subjective belief factors are modeled as

$$\hat{\Delta}_{j,t} = \Lambda_j^{*\prime} F_t^* + u_{j,t}, \tag{10}$$

where $\hat{\Delta}_{j,t}$ is the constructed belief wedge in time t for the macro variable $y_{j,t-l}$ from (8), and F_t^* is a $q \times 1$ vector of unobserved or latent factors and Λ_j^* is a $q \times 1$ vector of factor loadings. $u_{j,t}$ is the innovation, which is allowed to be serially correlated in the approximate factor structure.

4.1.1 Estimation and Forecasting

Following Stock and Watson (2002b), I adopt a two-step procedure to estimate the rational forecast $E[y_{j,t-l}|\mathcal{I}_{t-1}]$ in (9), or the fitted value $\hat{y}_{j,t-l}$ in (9). The first step is to use static principal component estimation allowing for missing values to obtain $\hat{\Lambda}$ and $\hat{F}_t$. This is the EM algorithm proposed by Stock and Watson (2002b). The algorithm is implemented as follows. Initially, missing observations are replaced by the unconditional mean based on the non-missing values, and I demean and standardize all the series. These balanced panel data are used to estimate factors F_t and factor loadings Λ by the static principal component method. The missing value of X_{it} is updated as $\hat{\Lambda}_i'\hat{F}_t$. To obtain the original unstandardized value, I multiply this estimated value by the standard deviation of the series and add the mean to it. The resulting value is treated as a new observation for X_{it}. I demean and standardize the new data set and use this dataset to re-estimate F_t and Λ. I iterate these steps until the factor estimates converge. Stock and Watson (2002a) show that the principal component estimator is pointwise (for any date t) consistent and has a limiting mean squared error (MSE) over all t that converges in probability to zero. The second step is to estimate (9) by OLS. Bai and Ng (2006) show that if $\sqrt{T}/N \to 0$, the estimates $\hat{F}_t$ can be treated as if they were observed in the subsequent regression. After obtaining the estimates of rational forecast $\hat{y}_{j,t-l}$, I compute belief wedges by (8) and estimate Λ_j^* and F_t^* by the same EM algorithm described above.

4.2 Data Description

This section briefly describes the data. A more detailed description including data sources and necessary transformation is provided in Appendix. I use two data sets to construct the subjective belief factors.

The first data set is a monthly data set of *information series*, consisting of 135 macro variables spanning the period 1959/01 to 2017/04. This data set is used to estimate hidden factors for the *FAVAR* model. The data set includes all the 128 data series in the FRED-MD data set of the vintage 2017-05 downloaded from the website of the Federal Reserve Bank of St. Louis. In addition to these data series, I manually update seven series of macro variables which have been removed from the FRED-MD data set since the vintage 2016-06. The resulting data set is classified into eight categories: (1) output and income, (2) labor market, (3) housing, (4) consumption and orders, (5) prices, (6) interest rates and exchange rates, (7) money and credit, and (8) stock market. I transform each data series by taking the first difference, the second difference, logarithm, the first difference in logarithm, the second difference in logarithm, and the difference in growth rate when necessary to render the series stationary. Before estimating factors, I also demean and standardize every data series[5].

The second data set is a monthly data set consisting of eleven series of macro variables for both professional survey forecast value and historical value. I choose monthly regular macro releases (once per month) for which I have reliable survey forecast data and historical data. This data set is used in the estimation of (9) and in the construction of belief wedges defined in (8). The resulting eleven series are: Change in Nonfarm Payrolls (NFP TCH), Conference Board Consumer Confidence (CONCCONF), ISM Manufacturing (NAPMPMI), Durable Goods Orders (DGNOCHNG), Consumer Price Inflation (CPI CHNG), Retail Sales (RSTAMOM), Unemployment Rate (USURTOT), Industrial Production MoM (IP CHNG), Capacity Utilization (CPTICHNG), Housing Starts (NHSPSTOT), and Producer Price Inflation (PPI CHNG). All of the macro news is released with a one-month lag, with only one exception of Conference Board Consumer Confidence which is released without any delays. For example, except Conference Board Consumer Confidence, all others series published in April 2011 are the value in March 2011, while the value of Consumer Confidence released at the end of April 2011 is the value in April 2011. The medians of survey forecast data for the above eleven series spanning the period 1999/01 to 2017/04 are downloaded from the Bloomberg Financial Services. The historical data of the above eleven series are downloaded from the website of the Federal Reserve Bank of St. Louis whenever available, otherwise they are downloaded from original publishers[6]. The historical data of the series above span the period 1959/01 to 2017/04 except the Conference Board Consumer Confidence, for which only short time span, from 1977/06 to 2017/04 is available. Before estimating (9), I conduct

[5]Detailed data description as well as necessary transformation can be found at https://research.stlouisfed.org/econ/mccracken/fred-databases/Appendix_Tables_Update.pdf

[6]Original publishers include Bureau of Labor Statistics (BLS), Bureau of the Census (BC), Bureau of Economic Anal- ysis (BEA), Federal Reserve Board (FRB), Conference Board (CB), Employment and Training Administration (ETA), and Institute for Supply Management (ISM).

necessary transformation to render the series stationary detailed in Appendix.

4.2.1 First Glance at Survey Data

As discussed above, (6) assumes that all the survey forecasts made in month t are based on the same information set $\mathcal{I}_{t-1}$. However, since professional survey forecasters are allowed to update their predictions up to the night before the official release of the macro news announcement, the survey forecasts of macro news published later in the month, in principle, are made on a larger information set than those of earlier releases. Although some macro announcements, e.g., unemployment rate, are easier to predict than others, no obvious evidence is observed on the larger information set used to predict later macro releases. For example, Consumer Confidence is one of the best-performance forecasts, thought it is a nowcast in the sense that it is published without any delays. On the other hand, Durable Goods Orders, which is published in the third or fourth week of next month, delivers the worst performance. More details regarding the quality of professional survey forecasts are provided in Appendix.

4.3 Estimates of Investors' Subjective Belief Factors

In this section, I present the result from estimation of investors' subjective belief factors. I first show the estimates of the information factors F_t, and then I move on to the estimation results for the forecasting equation (9). Finally, I discuss the estimates of subjective belief factors F_t^*.

4.3.1 Estimation of Factors

I estimate the information factors F_t by the static principal component method as described in the Section 4.1. To select the number of significant factors, I follow the existing literature to use the PC_{p2} criterion developed in Bai and Ng (2002), which is a generalization of Mallow's C_p criteria for large dimensional panel data. This criterion suggests eight factors in this sample[7].

	$\mathbf{mR^2\,(1)}$	0.157	$\mathbf{mR^2(2)}$	0.068	$\mathbf{mR^2(3)}$	0.067
Top 10	USGOOD	0.731	T10YFFM	0.606	CUSR0000SAC	0.746
	PAYEMS	0.724	AAAFFM	0.605	DNDGRG3M086*	0.734
	MANEMP	0.684	BAAFFM	0.579	CUSR0000SA0L2	0.711
	IPMANSICS	0.644	T5YFFM	0.554	CPIAUCSL	0.680
	NAPM	0.640	TB3SMFFM	0.464	CUSR0000SA0L5	0.649
	DMANEMP	0.624	TB6SMFFM	0.452	CPITRNSL	0.627
	INDPRO	0.610	T1YFFM	0.397	PCEPI	0.614
	NAPMNOI	0.602	COMPAPFFx	0.227	CPIULFSL	0.560
	NAPMPI	0.600	BUSINVx	0.225	WPSFD49502	0.489
	IPFPNSS	0.565	HOUST	0.198	WPSID61	0.477
	$\mathbf{mR^2\,(4)}$	0.050	$\mathbf{mR^2\,(5)}$	0.041	$\mathbf{mR^2\,(6)}$	0.036
Top 10	AAA	0.395	T1YFFM	0.243	IPCONGD	0.227
	GS1	0.386	GS5	0.236	NAPMEI	0.200
	GS5	0.386	GS1	0.218	AWHMAN	0.182
	BAA	0.369	GS10	0.217	NAPMII	0.171
	GS10	0.360	PERMITW	0.215	IPDCONGD	0.168
	TB6MS	0.353	PERMIT	0.214	ISRATIOx	0.167
	HOUST	0.240	HOUSTW	0.201	IPFINAL	0.158
	TB3MS	0.239	TB6MS	0.188	NAPM	0.148
	HOUSTW	0.237	TB6SMFFM	0.181	IPFPNSS	0.142
	CP3Mx	0.234	HOUST	0.181	CES0600000007	0.141

Table 1 – Summary of Factors. This table provides a list of top eleven series that load heavily on each of the first six estimated factors. $mR^2\,(k)$ reports the average marginal R^2 for Factor k. For instance, $mR^2\,(1) = 0.157$ indicates the first factor explains 15.7% variations among 135 macro data series. $USGOOD = 0.731$ means that the first factor explains 73.1% variaitons of $USGOOD$. The full series Fred code is DNDGRG3M086SBEA. The sample spans the period 1959/01 to 2017/04.

After the factors are estimated, I compute $mR^2\,(k)$, where $k = 1, \ldots, 8$ to evaluate the marginal contribution of each factor in explaining the variation of each data series. In particular, I regress the i-th series in the data set on the first k factors and record $R_i^2\,(k)$. This results in eight R^2s and I define the marginal contribution of the factor k in explaining variation of data series i as $mR_i^2\,(k) = R_i^2\,(k) - R_i^2\,(k-1)$, with $mR_i^2\,(1) = R_i^2\,(1)$. The average importance of the factor k is computed as the average of marginal contribution of

[7] The criterion finds nine factors in this sample if no outlier adjustment is performed. I define a data point as an outlier if it deviates from the sample median by more than eleven interquartile ranges. The outliers are removed and treated as missing values.

all data series,

$$mR^2(k) = \frac{1}{N}\sum_{i=1}^{N} mR_i^2(k).$$

Table 1 and Table 2 reports average marginal contributions for each of the eight estimated factors and top eleven series with the highest marginal contribution of each factor and associated contribution. The first information factor contributes to 15.7% of the variation in the data set and can be interpreted as a labor and real activity factor since the $mR_i^2(1)$ associated with the employment series and industrial production is close to 0.7. The second information factor explains 6.8% of the variation in the data set and loads heavily on term spreads in the bond market. The third information factor has a marginal contribution of 6.7% and exhibits highest explanatory power among price variables and hence can be interpreted as an inflation factor. The fourth and the fifth information factor are a mix of housing and interest rate variables and have marginal contributions of 5% and 4.1%, respectively. Similar to the first factor, the sixth information factor with a marginal contribution of 3.6% can also be interpreted as a real activity and labor factor. The seventh information factor summarizes the variation in stock market variables while the eighth information factor concentrates on exchange rates.

	$\mathbf{mR^2(7)}$	0.029	$\mathbf{mR^2(8)}$	0.024
Top 10	S&P 500	0.546	TIXMMTH	0.465
	S&P: indust	0.544	EXSZUSx	0.313
	S&P div yield	0.420	EXUSUKx	0.276
	S&P PE ratio	0.315	EXJPUSx	0.197
	UMCSENTx	0.149	EXCAUSx	0.113
	VXOCLSx	0.147	SRVPRD	0.107
	EXCAUSx	0.105	CES0600000008	0.093
	TB3MS	0.064	CES3000000008	0.088
	M2SL	0.063	USTRADE	0.080
	IPCONGD	0.061	USGOVT	0.074

Table 2 – Summary of Factors, continued. This table provides a list of top eleven series that load heavily on each of the seventh and eighth estimated eight factors. $mR^2(k)$ reports the average marginal R^2 for Factor k. For instance, $mR^2(7) = 0.029$ indicates the first factor explains 2.9% variations among 135 macro data series. $S\&P = 0.546$ means that the first factor explains 54.6% variaitons of $S\&P$. The sample spans the period 1959/01 to 2017/04.

Turning to the explanatory power of the eight information factors on each of data series in the data set, Figure 1 reports $R_i^2(8)$ for the regression of the i-th data series on the eight information factors ordered by categories. The x-axis is the ID number of data series and the y-axis plot the value of $R_i^2(8)$. Overall, these eight information factors explain 47% of variation in the data set. The explanatory power of these eight information factors varies across different data series. There are 67 data series that have an R^2 over 0.5 and 32 data series whose R^2 ranges between 0.2 and 0.5. However, there are 23 series that have more than 90% idiosyncratic variation that cannot be explained by the eight information factors. Looking at the category-wise evidence, Housing and Interest Rates, and Exchange Rates are the top two groups that are best explained by the eight information factors while these factors have lowest explanatory power for the group of Money and Credit mainly because of the extreme values that it took during the recent recession.

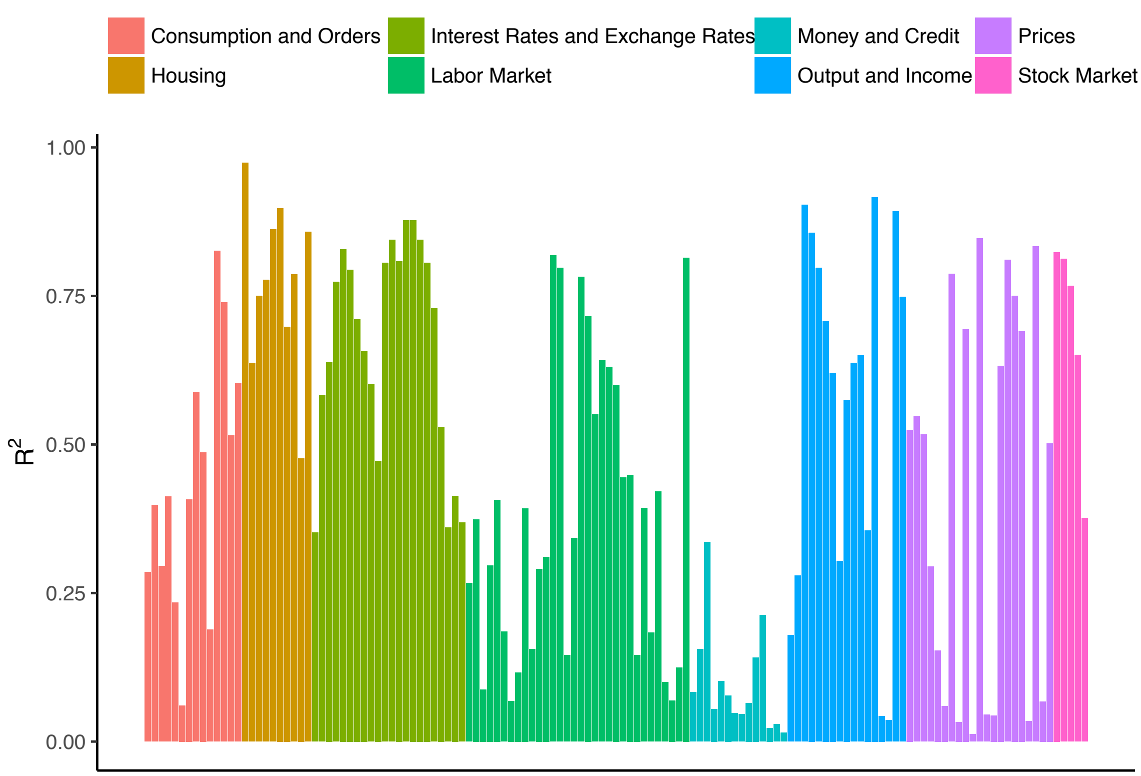

Figure 1 – Importance of Factors: R^2. This figure provides $R^2(8)$ for the regression of the i-th data series on eight factors. The sample spans the period 1959/01 to 2017/04.

4.3.2 Forecasting

Moving on to the forecast equation (9), I estimate this equation by using the historical value of eleven macro variables from the second data set described in Section 4.2 and the eight estimated information factors. I conduct the in-sample estimation by OLS to best align the econometrician's information set with the survey forecaster's information set. I choose the number of autoregressive lags ($1 \leq$ lags ≤ 4) and the number of lags for the information factors ($1 \leq$ lags ≤ 4) by using Bayesian information criterion (BIC). Table 3 summarizes optimal lags suggested by BIC and R^2 for each regression. Except ISM Manufacturing, all other series choose one lag for the information factors. Change in Nonfarm Payrolls has the highest R^2 thanks to the first and the sixth information factor which concentrate on labor/real activity variables. Consumer Confidence scores the lowest R^2 suggesting that confidence index is difficult to predict by macro factors. The key point here, however, is not to find the best forecasting model but to gather information as much as possible to mimic what professional survey forecasters did when they reported their forecast value.

	Nonfarm Payrolls	**ISM Manufacturing**	**Durable Goods Orders**	**CPI Inflation**	**Unemployment Rate**	**Retail Sales**
Lags of $\mathbf{y_t}$	2	4	2	4	1	2
Lags of $\mathbf{\hat{F}_t}$	1	2	1	1	1	1
$\mathbf{\bar{R}^2}$	0.49	0.26	0.23	0.28	0.30	0.12
	Industrial Production	**Capacity Utilization**	**Housing Starts**	**PPI Inflation**	**Consumer Confidence**	
Lags of $\mathbf{y_t}$	1	1	2	3	1	
Lags of $\mathbf{\hat{F}_t}$	1	1	1	1	1	
$\mathbf{\bar{R}^2}$	0.25	0.24	0.21	0.31	0.09	

Table 3 – Optimal Lags Implied by BIC. R^2 is reported for the forecasting regression. The sample spans the period 1959/01 to 2017/04, except consumer confidence data, which has a shorter time span of the period 1977/06 to 2017/04.

4.3.3 Estimates of Subjective Belief Factors

After obtaining the model-implied rational forecast for each of the eleven macro variables, I compute the belief wedges as the difference between the survey forecasts and model-implied rational forecasts according to (5). I demean and standardize the eleven estimated belief wedges and estimate (10) by the static principle component approach. The number of subjective belief factors is chosen as two. The loadings of the subjective belief factors are reported in Table 4. Table 5 shows the summary of these subjective belief factors.

Series	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$
Nonfarm Payrolls	0.121	0.488
ISM Manufacturing	0.010	0.305
Durable Goods Orders	0.055	0.203
CPI Inflation	0.226	−0.381
Unemployment Rate	−0.086	−0.344
Retail Sales	0.119	0.111
Industrial Production	0.075	0.601
Capacity Utilization	0.135	0.353
New House Starts	−0.001	0.333
PPI Inflation	0.219	−0.410
Consumer Confidence	0.127	−0.259

Table 4 – Loadings for Subjective Belief Factors. Factors are estimated by $\hat{F}_t^* = \hat{\Lambda}^{*\prime}\Delta_t$, where $\hat{\Lambda}^*$ is the matrix of eigenvectors of the sample variance matrix of Δ_t. The table reports the loadings $\hat{\Lambda}^*$ which has been normalized so that the sum of the loading for the factor equals one. Optimal lags are chosen by BIC. The sample spans the period 1999/01 to 2017/04.

The first subjective belief factor contributes to 21% of the common variation in the data set and can be interpreted as an inflation factor since the $mR_i^2(1)$ associated with CPI Inflation and PPI Inflation are more than 0.6. The second subjective belief factor explains 18% of the variation in the data set and loads heavily on real activity and labor market. Overall, these two factors explain 40% of the sample variation in the constructed belief wedges.

Figure 2 plots the estimates of the subjective belief factors against time. The blue line is the original factor value, and the red line is a trend estimator by a simple average of previous six months. NBER recessions are shaded by grey. The first subjective belief factor

	$\mathbf{mR^2\,(1)}$	0.206	$\mathbf{mR^2(2)}$	0.178
Top 3	CPI Inflation	0.648	Industrial Production	0.479
	PPI Inflation	0.609	Nonfarm Payrolls	0.316
	Capacity Utilization	0.231	PPI Inflation	0.223

Table 5 – Summary of Subjective Belief Factors. This table provides a list of top three series that load heavily on each of the estimated belief factors. $mR^2\,(k)$ reports the average marginal R^2 for Factor k. The sample spans the period 1999/01 to 2017/04.

increases gradually before each recession and starts to decrease at the beginning of each recession. During the second half of the recent Great Recession, the first subjective belief factor dramatically increases, suggesting a substantial change in investor's subjective beliefs. Another spike in the first subjective belief factor is observed around 2016[8]. The second subjective belief factor also displays slow increase before each recession. However, unlike the first factor, the second one does not exhibit a visible decrease in the recent recession.

4.4 Robustness Check

How are the estimated subjective belief factors sensitive to different model specifications? In this subsection, I conduct robustness exercises by using the Akaike information criterion (AIC) to choose the autoregressive legs ($1 \leq$ lags ≤ 4) and the number of lags for information factors ($1 \leq$ lags ≤ 4) in the forecasting equation (9). Different from BIC, AIC imposes a smaller size of penalty on the number of independent variables, so AIC tends to choose a more complex model than BIC does. Here I re-estimate the forecasting equation (9) by using the optimal lags suggested by AIC. The results of optimal lags and associated R^2 are reported in Table 15 in Appendix. Except for unemployment rate, AIC chooses more lags than BIC does. Nevertheless, except for consumer confidence, whose adjusted R^2 increases from 0.08 to 0.18, increased lags do not improve model's fitting significantly. Table 16 in Appendix reports coefficients for the estimated subjective belief factors. Although the magnitude of coefficients is different across the two models, the relative importance of each data series in

[8]This is perhaps due to oil price plunge from the beginning of 2016.

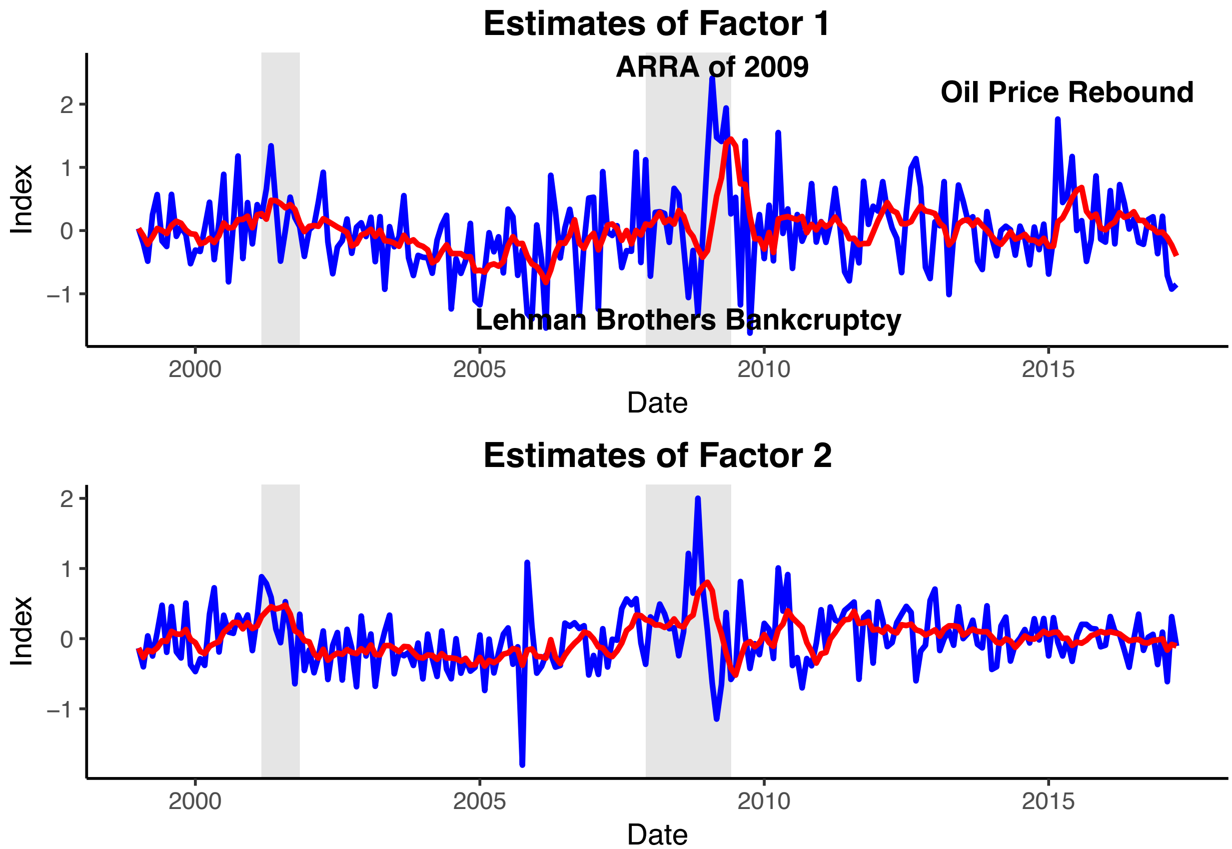

Figure 2 – Estimates of Subjective Belief Factors. Optimal lags are chosen by BIC. The blue line is the original factor value and the red line is a trend estimator by simple moving average of previous six months. NBER recessions are shaded by grey. The sample spans the period 1999/01 to 2017/04.

forming the subjective belief factors remains unchanged. The correlation between the two models for the first and the second subjective belief factor is 0.985 and 0.986, respectively. These almost perfect correlations show that the estimates of the subjective belief factors are robust to different model specifications. Figure 3 in Appendix plots estimates of subjective belief factors along with a six months simple moving average, which confirms the robustness of the estimates.

5 The Impact of Investors' Subjective Beliefs on Stock Returns

In this section, I investigate the impact of the subjective belief factors on explaining the cross-section of stock returns by using stock portfolio returns double-sorted on various measures. I first present the econometric framework, and then describe the stock portfolio data and present the estimation results in the end.

5.1 Econometric Framework

The empirical investigation is guided by a linear approximation of M^*_{t+1}, with the subjective belief factors as systematic risk factors:

$$M^*_{t+1} \approx a + b_1 F^*_{1,t+1} + b_2 F^*_{2,t+1}, \tag{11}$$

where $M^*_{t+1} = (\xi_{t+1}/\xi_t)\, M_{t+1}$. In principle, M^*_{t+1} depends on the disparities between the subjective probability measure and physical probability measure as well as the consumption growth or more generally speaking, a set of state variables that drive dynamics of the whole macroeconomic system. In practice, however, since the aggregate consumption growth rate or, generally speaking, macro state variables are far less volatile than the subjective belief factors, exposures to the subjective belief factors explain a much larger fraction of the test portfolios studied in this paper. Hence, in this section, to highlight the impact of the subjective belief factors, I only focus on the approximation with the subjective belief factors as systematic risk factors and present the full-fledged model with additional macro risk factors in Section 6.1.2 in the context of justifying the assumption (6) made in constructing the belief wedges.

It is known that survey data, which are the survey responses of individuals, are much noisier and sometimes unreliable, so researchers have been prevented from applying them to economic analysis for a long time. To deal with the noise and make a more effective use of the subjective belief factors estimated from the professional survey data, I estimate trends of the constructed subjective belief factors by a simple moving average of previous six months. In addition, both investors' *subjective* beliefs and macro variables are generally persistent. Using trend estimates instead of original estimates should better capture the comovement between investors' *subjective* beliefs and macro annoucements. In fact, compared to the original estimates, the resulting trend estimates of the subjective belief factors exhibit much stronger explanatory power in explaining the cross-section of stock returns.

Exposures to the subjective belief factors are estimated by time-series regressions for each of the assets $j = 1, \ldots, N$ in investigation:

$$R^e_{j,t} = \alpha_j + \beta_{1j} F^*_{1t} + \beta_{2j} F^*_{2t} + \epsilon_{j,t}, \qquad t = 1, \ldots, T, \tag{12}$$

where β_{1j} and β_{2j} measure exposures to the first and the second subjective belief factor, respectively. The prices of subjective belief factors that measure to which extent the subjective belief exposures explain cross-sections of stock returns are then estimated by the cross-sectional regressions:

$$E_T\left[R^e_{j,t}\right] = \lambda_0 + \lambda_1 \hat{\beta}_{1j} + \lambda_2 \hat{\beta}_{2j} + v_j, \qquad j = 1, \ldots, N, \tag{13}$$

where $E_T\left[R^e_{j,t}\right]$ is the average excess return of the asset j, and λ_0 is the risk price of "zero beta" accounted for the inaccuracy of the risk-free rate proxy. λ_1 and λ_2 are the risk prices of the subjective belief factors. Hat in (13) indicates that these are the values estimated by (12). (12) and (13) are jointly estimated by the General Moment Method (GMM) proposed by Hansen (1982). Standard errors are estimated by the block bootstrap method intended to correct for the first-stage estimate of the risk exposures $\hat{\beta}$ as well as the serial correlation in the time series regression. Horowitz (2001) provided a useful survey for bootstrap methods.

Along with the estimation above, two measures of fit are reported. The first measure is

R^2 which is defined as

$$R^2 \equiv 1 - \frac{\sum_{j=1}^{N}\left(E_T\left[R_{j,t}^e\right] - \hat{R}_{j,t}^e\right)^2}{\sum_{j=1}^{N}\left(E_T\left[R_{j,t}^e\right] - \bar{R}_{j,t}^e\right)^2},$$

and the second measure is the root-mean-squared pricing error (RMSE) as a fraction of the root-mean-squared return (RMSR) on the portfolios being priced. This measure is also reported in Lettau et al. (2017), i.e.,

$$RMSE \equiv \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(E_T\left[R_{j,t}^e\right] - \hat{R}_{j,t}^e\right)^2}$$

$$RMSR \equiv \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(E_T\left[R_{j,t}^e\right]\right)^2},$$

where $\hat{R}_{j,t}^e = \hat{\lambda}_0 + \hat{\lambda}_1\hat{\beta}_{1j} + \hat{\lambda}_2\hat{\beta}_{2j}$ and $\bar{R}_{j,t}^e = \sum_{j=1}^{N}\left(E_T\left[R_{j,t}^e\right]\right)/N$.

5.2 Data Description

The data for portfolio returns are downloaded from Professor Kenneth R. French's website at Dartmouth College[9]. The portfolio groups are double-sorted on Size and Book to market (Size-BM), Size and Investment (Size-Inv), Size and Operating profit (Size-OP), Book to market and Investment (BM-Inv), Book to market and Operating profit (BM-OP), Operating profit and Investment (OP-Inv), Size and Short-term reversal (Size-SH) and Size and Long-term reversal (Size-LO), respectively[10]. The sample spans the period of 1999/01 to 2017/04.

[9] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[10] At the end of each June, stocks are allocated to five Size groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five B/M groups (Low to High), again using NYSE break-points. The intersections of the two sorts produce twenty-five Size-B/M portfolios. In the sort for June of year t, B is book equity at the end of the fiscal year ending in year $t-1$ and M is the market cap at the end of December of year $t-1$, adjusted for changes in shares outstanding between the measurement of B and the end of December. Other portfolios are formed in the same way, except that sort variables are different. Operating profitability, OP, in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, Inv, is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets. Short-term reversal, SH, is the prior one month return. Long-term reversal, LO, is the prior 13 to 60 months return.

Table 6 shows average monthly excess returns for twenty-five portfolios independently double-sorted on various measures. Panel (A) reports results for portfolios sorted on Size and Book to market. In each BM column of Panel (A), average return typically falls from small stocks to big stocks. This is the so called "size effect". The first column (low BM extreme growth stocks) is the only exception, and there is no apparent relation between Size and average return. The relation between average return and BM, called the "value effect", can also be observed in Panel (A). Especially in the microcap row, average returns increase with BM value. The size effect appears more consistently in portfolios sorted on Size and Investment in Panel (B) with one exception of the highest investment column in which the microcap portfolio has the second lowest average excess returns. For every Size row of Panel (B) average returns in the lowest investment column are much higher than those in the highest investment column, which is mainly because of substantially low returns in the highest investment column. In Panel (C) the size effect is observed in every OP column. Similar patterns of BM, Inv and OP can be observed in Panel (D) and Panel (E). In Panel (F) portfolios with the lowest investment (first column) have consistently higher average excess returns than those with the top investment (last column) as observed in other Panels. No obvious relation, however, is present regarding OP. Panel (G) and Panel (H) report the results for portfolios sorted on Size and prior returns. Although the size effect can be observed in most of the columns, no apparent patterns are present in prior returns. The only observation regarding the long-term reversal is that average excess returns in the lowest prior return column are consistently higher than those in the highest prior return column with some mixings in the middle three columns.

5.3 Results

Table 7 reports results from estimating the cross-sectional regressions (13) on portfolio groups double-sorted on Size and Book to market (Size-BM), Size and Investment (Size-Inv), Size and Operating profit (Size-OP), Book to market and Investment (BM-Inv), Book to market

	1	2	3	4	5		1	2	3	4	5
Size\BM			**Panel (A)**			**Size\Inv**			**Panel (B)**		
1	0.50	0.86	0.98	1.09	1.30	**1**	1.37	1.19	1.14	0.89	0.41
2	0.68	0.97	1.05	0.88	0.92	**2**	0.94	0.98	1.16	1.10	0.48
3	0.62	0.97	0.88	0.94	1.15	**3**	0.99	1.02	0.96	1.00	0.49
4	0.79	0.82	0.75	0.91	0.69	**4**	0.90	0.85	0.91	0.90	0.55
5	0.46	0.66	0.77	0.57	0.69	**5**	0.79	0.73	0.73	0.64	0.22
Size\OP			**Panel (C)**			**BM\Inv**			**Panel (D)**		
1	1.01	1.13	0.97	1.10	0.94	**1**	0.89	0.90	0.92	0.76	0.23
2	0.61	1.04	1.01	1.05	1.06	**2**	1.16	0.96	1.00	0.89	0.69
3	0.58	0.87	0.94	0.95	1.09	**3**	1.25	0.95	1.09	0.88	0.63
4	0.45	0.94	0.76	0.93	0.91	**4**	1.30	0.96	1.14	1.06	0.62
5	0.15	0.60	0.66	0.64	0.65	**5**	1.59	1.34	1.11	1.10	0.52
BM\OP			**Panel (E)**			**OP\Inv**			**Panel (F)**		
1	0.42	0.67	0.79	0.70	0.81	**1**	1.30	1.10	1.04	0.73	0.19
2	0.80	0.96	0.85	0.93	1.07	**2**	1.12	1.17	1.23	1.11	0.65
3	1.00	1.02	0.84	1.00	1.10	**3**	1.27	0.95	1.01	0.97	0.66
4	1.10	0.90	1.04	0.98	0.85	**4**	1.53	1.08	1.04	0.94	0.65
5	1.17	1.33	1.13	1.53	1.21	**5**	1.31	0.96	0.89	0.91	0.77
Size\SH			**Panel (G)**			**Size\LO**			**Panel (H)**		
1	1.31	0.80	0.94	1.01	0.56	**1**	1.13	0.94	1.04	0.99	0.84
2	0.99	1.08	0.90	0.78	0.75	**2**	1.22	0.98	1.02	0.94	0.91
3	0.70	0.95	0.83	0.79	0.69	**3**	0.99	0.99	0.95	0.74	0.77
4	0.44	1.00	0.87	0.76	0.50	**4**	0.79	0.79	0.83	0.83	0.70
5	0.55	0.66	0.61	0.55	0.39	**5**	0.62	0.52	0.62	0.57	0.42

Table 6 – Average Monthly Excess Returns. Portfolios are formed on Size and BM, Size and Inv, Size and OP, BM and Inv, BM and OP, OP and Inv, Size and SH, and Size and LO. In the Panel (A) of Size\BM, different rows corresponds to different Size value and different column corresponds to different Book to market value. "1" represents lowest value, while "5" represents highest value. The sample spans the period of 1999/01 to 2017/04.

and Operating profit (BM-OP), Operating profit and Investment (OP-Inv), Size and Short-term reversal (Size-SH) and Size and Long-term reversal (Size-LO), respectively, and a pooled estimation of all the portfolios jointly. For each portfolio group, I report the estimated risk prices of the subjective belief factors in the cross-sectional regressions (13) and two measures of model fit: R^2 and $RMSE/RMSR$. Block bootstrap standard errors are reported in parentheses. Table 8 provides $RMSE/RMSR$ for each of the portfolios in the pooled estimation to show which groups are best priced by the subjective belief factors and which groups are most mispriced.

Table 7 shows that the risk prices of the first subjective belief factor are positive and strongly significant (P-value < 0.01) in the cross-sectional regressions for all the portfolios under our consideration. The risk prices of the second subjective belief factor are all positive as well with only one exception of the portfolio group sorted on Size and Long-term reversal whose risk price is slightly negative but insignificant. The risk prices of the second subjective belief factor are strongly significant (P-value < 0.01) for the portfolio groups sorted on Size and Book to market, and Size and Short-term reversal, and remain moderately significant ($0.01 <$ P-value < 0.1) for the portfolio group sorted on Size and Operating profit, and in the pooled estimation of all the portfolios. The positive risk price of the subjective belief factors matches the interpretation in Section 3.2. The subjective belief factors correlate positively with aggregate market valuation, so the risk prices of these factors are positive when pricing risky assets. Exposures to these two subjective belief factors explain sizable variation in these cross-sectional risk premia. R^2 ranges from 0.36 to 0.66, with top two value obtained on the portfolio groups sorted on Size and Operating profit, and Size and Investment. Intercepts are significant for most of the portfolio groups with two exceptions of portfolio groups sorted on Size and Investment, and Operating profit and Investment, implying that one-month T-bill rate may not be an accurate proxy for risk-free rates in the sample. Turning to the pooled estimation in Panel (J), both risk prices are significant with strong significance for the first subjective belief factor, matching the results of cross-sectional regressions run for each portfolio group. Table 8 provides $RMSE/RMSR$ in the pooled estimation. The best-priced portfolio group is the one sorted on Size and Operating profit, while the portfolio group sorted on Book to market and Investment is the most mispriced. This result is also consistent with the findings of each portfolio group.

Figure 4 in Appendix plots fitted excess returns obtained from the the time-series regression (12) against realized excess returns for each group of portfolios. Figure 5 in Appendix plots fitted excess returns against realized excess returns for all the porfolios jointly. The blue line shows the 45° line. Ideally, all the portfolios should lie on the 45° line.

Panel (A): Size/Book to Market				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.84***	0.28***	0.43***	0.40	0.18
(0.22)	(0.07)	(0.16)		
Panel (B): Size/Investment				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.61	0.40***	0.45	0.63	0.18
(0.38)	(0.10)	(0.29)		
Panel (C): Size/Operating Profit				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.85**	0.30***	0.47*	0.66	0.16
(0.36)	(0.09)	(0.24)		
Panel (D): Book to Market/Investment				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.55**	0.36***	0.31	0.36	0.23
(0.27)	(0.10)	(0.20)		
Panel (E): Book to Market/Operating Profit				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.68***	0.13***	0.05	0.44	0.17
(0.16)	(0.05)	(0.12)		
Panel (F): Operating Profit/Investment				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.41	0.35***	0.23	0.49	0.19
(0.37)	(0.10)	(0.27)		
Panel (G): Size/Short-term Reversal				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.83***	0.30***	0.44***	0.52	0.18
(0.24)	(0.07)	(0.17)		
Panel (H): Size/Long-term Reversal				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.50***	0.13***	−0.03	0.46	0.16
(0.17)	(0.04)	(0.14)		
Panel (J): All Portfolios				
Constant	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$	$\mathbf{R^2}$	$\frac{\textbf{RMSE}}{\textbf{RMSR}}$
0.69***	0.26***	0.28**	0.45	0.20
(0.16)	(0.04)	(0.11)		

Table 7 – Expected Return-beta Regression with Two Subjective Belief Factors. Twenty-five Portfolios are sorted on various variables. Bootstrap standard errors are reported in the parentheses. $*$ denotes $p < 0.1$. $**$ denotes $p < 0.05$. $***$ denotes $p < 0.01$. The sample spans the period 1999/01 to 2017/04.

Size-BM	Size-Inv	Size-OP	BM-Inv	BM-OP	OP-Inv	Size-SH	Size-LO
0.19	0.20	0.17	0.25	0.20	0.20	0.19	0.19

Table 8 – RMSE/RMSR Ratio with Two Subjective Belief Factors. The sample spans the period 1999/01 to 2017/04.

$MiBj, MiIj, MiOj, BiIj, BiOj, OiIj, MiSj, MiLj$ with $i = 1, \ldots, 5$ and $j = 1, \ldots, 5$ denotes the portfolios sorted on Size and Book to market, Size and Investment, Size and Operating profit, Book to market and Investment, Book to market and Operating profit, Operating profit and Investment, Size and Short-term reversal, and Size and Long-term reversal, respectively, where "1" denotes the lowest value and "5" denotes the highest value.

The visual presentation confirms the results summarized in Table 7. Exposures to belief factors have best explanatory power in explaining the cross-sectional variations of the portfolio groups sorted on Size and Operating profit, and Size and Investment. In Panel (C) the only outlier is $M5O1$ whose excess return is much lower than the model-predicted excess returns. Table 6 Panel (C) also confirms this result with the average monthly yield equal to 0.15 which is much lower than other portfolios in this group. Moving on to the portfolio group with a sorting measure of investment in Panel (b), Panel (d) and Panel (f), the model performs least well for portfolios with the highest investment, e.g., $XXI5$. As argued by Fama and French (2015), probably these are the firms invest a lot regardless of profitability. In Panel (e), the most massive spread is observed in the portfolio with the lowest Book to market value and the lowest Operating profit. In Panel (h), portfolios in the largest category of size, e.g., $S5LX$, are overpriced by the model with all the portfolios in this category lying above the 45° line. The results of pooled estimation is plotted in Figure 5. In general, the portfolio group sorted on Book to market and Investment is the least well priced in the pooled estimation with large spreads in portfolios such as $B1I5$, $B5I1$, $B5I5$, $B3I1$ and $B2I2$.

6 Discussion

In this section, I provide two relevant discussions. First, I provide empirical evidence for the assumption made in Section 4 when constructing subjective belief factors that the subjective belief factors, constructed as latent factors from belief wedges, are a good representation of investor's subjective beliefs. Second, I describe a framework of a representative agent with robust preferences to render a potential explanation for the belief disparity between the subjective probability measure and the physical probability measure.

6.1 Justification of Subjective Belief Factors

The empirical counterpart of the theoretical belief wedge defined in (2) is obtained by (5), which assumes that the professional survey forecast and the model-implied conditional rational forecast are based on the same information set as stated in (6). However, in principle, the professional forecasters may have a larger information set compared to the econometrician when making the forecast, which implies

$$\tilde{y}^t_{j,t-l} = E^* \left[y_{j,t-l} | \mathcal{I}^*_{t-1} \right], \ \mathcal{I}_{t-1} \subseteq \mathcal{I}^*_{t-1}. \tag{14}$$

If it is the case that the professional forecasters have a larger information set, the empirical measure of belief wedges should also contain macro risk that cannot be captured by the *FAVAR* model with the smaller information set of the econometrician. This implies that the subjective belief factors, constructed as latent common factors of belief wedges, should reflect the macro risk as well.

To confirm that the constructed subjective belief factors are a good representation of investors' subjective beliefs rather than macro risk, I include estimated contemporary macro risk factors in the stochastic discount factor to explicitly account for macro risk. Recall that except for the data series of Consumer Confidence, all the macro news is released with a one-month delay. Thus, even if these belief wedges contain macro risk, they should cover the risk with a one-month lag. More importantly, compared to the professional survey forecasts, realizations of the macro variables should contain more accurate information about the contemporary macro risk. Since asset prices are a timely reflection of contemporary macro risk, when a measure of contemporary macro risk, constructed from the realizations of the macro variables, is introduced into the stochastic discount factor, the constructed subjective belief factors should become insignificant if they are a representation of macro risk.

6.1.1 Macro Risk Factors

The contemporary macro risk factors are constructed using realizations of the same macro variables as in the construction of subjective belief factors. At first glance, it may seem more desirable to use the data set of *information series*, e.g., X_t, as this large data set should, in general, summarize the fundamental macro risk better. However, to identify whether the subjective belief factors contain macro risk, we need to introduce macro risk factors that capture the same risk along as the subjective belief factors, and macro risk factors extracted from the large data set of *information series* may capture macro risk other than the subjective belief factors, which invalidates the following empirical analysis.

I assume that y_t has an approximate factor structure following

$$y_{j,t} = \Lambda_j^{M\prime} F_t^M + \nu_{j,t}, \tag{15}$$

where F_t^M is a $m \times 1$ vector of unobserved or latent factors and Λ_j^M is a $m \times 1$ vector of factor loadings. $\nu_{j,t}$ is the innovation, which is allowed to be serially correlated. Before estimation, y_t is demeaned and standardized. The estimation method follows Section 4.1.1.

Two macro risk factors are estimated from the eleven series of macro news realizations. Table 9 reports loadings for each of the estimated macro risk factors, and Table 10 summarizes the importance of each macro risk factors. The first macro risk factor, F_1^M, contributes

Series	$\mathbf{F_1^M}$	$\mathbf{F_2^M}$
Non-farm Payrolls	0.205	−0.011
ISM Manufacturing	0.095	0.148
Durable Goods Orders	0.132	−0.047
CPI Inflation	0.015	0.404
Unemployment Rate	−0.166	0.033
Retail Sales	0.105	0.088
Industrial Production	0.241	−0.050
Capacity Utilization	0.241	−0.031
New House Starts	0.069	0.022
PPI Inflation	0.003	0.417
Consumer Confidence	0.059	0.027

Table 9 – Loadings for Macro Risk Factors. Factors are estimated by $\hat{F}_t^M = \hat{\Lambda}^{M\prime} y_t$, where $\hat{\Lambda}^M$ is the matrix of eigenvectors of the sample variance matrix of y_t. The table reports the loadings $\hat{\Lambda}^M$ which has been normalized so that the sum of the loading for the factor equals one. The sample spans the period 1959/01 to 2017/04, except consumer confidence data, which has a shorter time span of the period 1977/06 to 2017/04.

to 29% of the variation in the data set and can be interpreted as a real activity and labor factor as the $mR_i^2(1)$ associated with real activity and labor are 0.79 and 0.57, respectively. The second macro risk factor, F_2^M, explains 14% of the variation in the data set and loads heavily on inflation variables. Overall, these two factors explain 43% of the variation in the data set. In comparison with the constructed subjective belief factors, both sets of factors exhibit similar patterns in factor loadings. The real activity and labor factor uses information mainly from Industrial Production and Nonfarm Payrolls, and the inflation factor relies heavily on CPI Inflation and PPI inflation with similar associated marginal contribution mR^2. In general, these two sets of factors represent similar macro risk[11].

[11] Another way to construct macro risk factors is to use the same loadings as the subjective belief factors so that these two sets of factors represent the risk along the exactly same dimension. The test result with these two constructed macro risk factor, available upon request, shows the same conclusion. Nevertheless, since the loadings are inherited from the subjective belief factors, these two macro risk factors are not a good representation of variation in the data set.

	$\mathbf{mR^2\,(1)}$	0.287	$\mathbf{mR^2(2)}$	0.144
Top 3	**Industrial Production**	0.791	**PPI Inflation**	0.734
	Capacity Utilization	0.790	**CPI Inflation**	0.690
	Non-farm Payrolls	0.574	**ISM Manufacturing**	0.093

Table 10 – Summary of Macro Factors. This table provides a list of top three series that load heavily on each of the estimated belief factors. $mR^2\,(k)$ reports the average marginal R^2 for Factor k. The sample spans the period 1959/01 to 2017/04, except consumer confidence data, which has a shorter time span of the period 1977/06 to 2017/04.

6.1.2 Tests with Stock Portfolio Groups

The empirical investigation is guided by a linear approximation of M^*_{t+1} with the subjective belief factors and macro risk factors as systematic risk factors:

$$M^*_{t+1} \approx a + b_1 F^*_{1,t+1} + b_2 F^*_{2,t+1} + b_3 F^M_{1,t+1} + b_4 F^M_{2,t+1}. \tag{16}$$

Exposures to the subjective belief factors and macro risk factors are estimated by time-series regressions for each of the assets $j = 1, \ldots, N$ in investigation for:

$$R^e_{j,t} = \alpha_j + \beta_{1j} F^*_{1t} + \beta_{2j} F^*_{2t} + \beta_{3j} F^M_{1t} + \beta_{4j} F^M_{2t} + \epsilon_{j,t}, \qquad t = 1, \ldots, T, \tag{17}$$

where β_{1j} , β_{2j}, β_{3j} and β_{4j} measure exposures to the subjective belief factors and the macro risk factors, respectively. The price of the factors that measures to which extent these factor exposures explain the cross-section of stock returns are then estimated by the cross-sectional regressions:

$$E_T\left[R^e_{j,t}\right] = \lambda_0 + \lambda_1 \hat{\beta}_{1j} + \lambda_2 \hat{\beta}_{2j} + \lambda_3 \hat{\beta}_{3j} + \lambda_4 \hat{\beta}_{4j} + v_j, \qquad j = 1, \ldots, N, \tag{18}$$

where $E_T\left[R^e_{j,t}\right]$ is the average return of the asset j, and λ_0 is the risk price of "zero beta" accounting for the inaccuracy of the risk-free rate proxy. λ_1 and λ_2 are the risk prices of the subjective belief factors, and λ_3 and λ_4 are the risk prices of the macro risk factors. Hat in (18) indicates the value estimated by (17).

Table 11 reports results from estimating the cross-sectional regressions (18) on portfolio groups double-sorted on Size and Book to market (Size-BM), Size and Investment (Size-Inv), Size and Operating profit (Size-OP), Book to market and Investment (BM-Inv), Book to market and Operating profit (BM-OP), Operating profit and Investment (OP-Inv), Size and Short-term reversal (Size-SH), and Size and Long-term reversal (Size-LO), respectively, and a pooled estimation of all the portfolios jointly. For each portfolio group, I report the estimated risk prices of the subjective belief factors and macro risk factors in the cross-sectional regressions (18) and two measures of model fit: R^2 and $RMSE/RMSR$. Block bootstrap standard errors are reported in parentheses.

Table 11 shows that even after the macro risk factors are introduced in linear approximation of M^*_{t+1}, the exposures to the subjective belief factors remain highly significant in explaining the cross-section of stock returns, and in most of the portfolio groups the magnitude of the risk prices is similar to the one without macro risk factors. Exceptions are observed in the portfolio group sorted on Size and Operating Profit where the risk price of the second subjective belief factor F_2^* becomes highly significant after macro risk is accounted for by macro risk factors. The same result can also be observed in the pooled estimation of all portfolios jointly. In the portfolio group sorting on Size and Long Term Reversal, the risk price of the first subjective belief factor F_1^* becomes slightly less signifcant after the introduction of macro risk factors. The risk price of the second subjective belief factor turns insignificant in the portfolio group sorted on Size and Short Term Reversal, and slightly negative but insignificant in the portfolio group sorted on Operating Profit and Investment.

Regarding the risk prices of macro risk factors, the result is mixed. In more than half of the portfolio groups, the macro risk factors are not significant indicating that macro risk factors have limited explanatory power in explaining fluctuations in asset returns in short run. In the porfolio groups where macro risk factors are significant, only the risk price of the first macro risk factor, mainly identified from real activity and labor, is significant. Nevertheless, the sign of the risk prices of the first macro risk factor is inconsistent. A

Constant	F_1^*	F_2^*	F_1^M	F_2^M	R^2	$\frac{RMSE}{RMSR}$
Panel (A): Size/Book to Market						
0.86***	0.35***	0.63***	−0.01	0.01	0.43	0.17
(0.25)	(0.08)	(0.19)	(0.01)	(0.02)		
Panel (B): Size/Investment						
0.53	0.42***	0.50	−0.00	0.00	0.66	0.17
(0.41)	(0.10)	(0.33)	(0.02)	(0.03)		
Panel (C): Size/Operating Profit						
1.10***	0.33***	0.78***	−0.03	0.02	0.71	0.15
(0.37)	(0.09)	(0.27)	(0.02)	(0.02)		
Panel (D): Book to Market/Investment						
0.51	0.31***	0.34	0.02	0.01	0.45	0.21
(0.32)	(0.10)	(0.23)	(0.02)	(0.02)		
Panel (E): Book to Market/Operating Profit						
0.58***	0.14***	0.06	0.02**	0.00	0.52	0.15
(0.20)	(0.05)	(0.14)	(0.01)	(0.02)		
Panel (F): Operating Profit/Investment						
0.07	0.22***	−0.01	0.07***	0.01	0.75	0.13
(0.38)	(0.09)	(0.27)	(0.02)	(0.03)		
Panel (G): Size/Short Term Reversal						
0.93***	0.20***	0.12	−0.04***	−0.03	0.76	0.13
(0.28)	(0.08)	(0.21)	(0.01)	(0.02)		
Panel (H): Size/Long Term Reversal						
0.69***	0.06**	−0.10	−0.04***	−0.00	0.75	0.11
(0.18)	(0.03)	(0.15)	(0.01)	(0.01)		
Panel (J): All Portfolios						
0.69***	0.28***	0.38***	−0.01	0.01	0.45	0.20
(0.16)	(0.04)	(0.11)	(0.07)	(0.01)		

Table 11 – Expected Return-beta Regression with Two Subjective Belief Factors and Two Macro Factors. Twenty-five Portfolios are sorted on various variables. Bootstrap standard errors are reported in the parentheses. $*$ denotes $p < 0.1$. $**$ denotes $p < 0.05$. $***$ denotes $p < 0.01$. The sample spans the period 1999/01 to 2017/04.

more thorough framework is needed to understand the impact of macro risk factors on the cross-section of asset returns.

Figure 6 and Figure 7 in Appendix provide visual representation of the model fit for estimations of the portfolio groups sorted on various measures separately, and a pooled estimation of all portfolio jointly. Both of the figures plot fitted excess returns in (18) against the realized excess returns. A 45° line is plotted to help identify the model fit. Ideally, all the points should lie on the 45° line if the model is perfect. The figures show that the model with macro risk factors improves its performance over the one without macro risk factors whenever macro risk factors are significant.

With a few minor exceptions, it is safe to conclude that the explanatory power of the subjective belief factors remains unaffected after the introduction of macro risk factors, which suggests that the subjective belief factors indeed capture the belief disparities between investor's subjective probabilities and the econometrician's physical probabilities rather than the macro risk due to the econometrician's limited information set.

6.2 Robust Preference

Theoretically, what are the origins of investor's subjective belief disparities? In this section, I provide one potential theoretical explanation based on the robust preference. I set up a general economic framework of a representative agent with an approximating model being constantly concerned for model misspecification. The model builds on Hansen and Sargent (2001a), Hansen and Sargent (2001b), Hansen and Sargent (2008) and a more recent paper Bhandari et al. (2016).

Consider a representative agent with robust preferences whose continuation value satisfies

$$V_t = \min_{m_{t+1}>0, E_t[m_{t+1}=1]} u\left(x_t\right) + \beta E_t\left[m_{t+1} V_{t+1}\right] + \frac{\beta}{\theta} E_t\left[m_{t+1} \log m_{t+1}\right], \tag{19}$$

with one-period unility $u\left(x_t\right)$. Here, x_t is a $n \times 1$ vector containing exogenous and endogenous state variables as well as control variables. x_t follows a Markovian law of motion

$$x_{t+1} = \psi\left(x_t, w_{t+1}\right), \tag{20}$$

where $w_{t+1} \sim N(0_{k\times 1}, I_{k\times k})$ is an i.i.d vector of normally distributed shocks under the data-generating probability measure $\mathcal{P}$. The agent treats the measure $\mathcal{P}$ as an *approximating* model[12] and considers potential stochastic deviations from this model, denoted by the strictly positive, mean-one random variable m_{t+1}. The agent, concerned with model misspecification of his *approximating* model, makes robust decisions by searching for a *worst-case* model captured by the minimizaiton problem in (19). The models considered by the agent are difficult to distinguish statistically from the *approximating* model, and the degree of statistical similarity is controlled by the entropy penalty $E_t\left[m_{t+1}\log m_{t+1}\right]$, scaled by the penalty parameter θ. Substantial deviations from the *approximating* model represented by m_{t+1} deliver a larger value of the penalty entropy. As the parameter $\theta \to 0$, the resulting preference approches a utility-maximizing agent under rational expectation. The solution of (19) is

$$m_{t+1} = \frac{\exp\left(-\theta V_{t+1}\right)}{E_t\left[\exp\left(-\theta V_{t+1}\right)\right]} \tag{21}$$

and the stochastic discount factor is

$$M^*_{t+1} \equiv m_{t+1}\frac{u'(x_{t+1})}{u'(x_t)} = \frac{\exp\left(-\theta V_{t+1}\right)}{E_t\left[\exp\left(-\theta V_{t+1}\right)\right]}\frac{u'(x_{t+1})}{u'(x_t)}. \tag{22}$$

m_{t+1} here chacterizes the disparity between the *approximating* model and the *worst-case* model, and compared to the *approximating* model it overweights bad states as V_{t+1} is lower in bad states. Using m_{t+1} one can construct a strictly positive martingale $\xi_{t+1} = m_{t+1}\xi_t$

[12]Here, for ease of description of the preferences, I take the process for x_{t+1} as given. In many cases where one needs to solve for control variables, the preferences become min-max preferences.

with $\xi_0 = 1$ which defines a new probability measure $\mathcal{P}^*$. For any random variable x_{t+1},

$$E_t^* [x_{t+1}] \equiv E_t \left[\frac{\xi_{t+1}}{\xi_t} x_{t+1} \right] = E_t [m_{t+1} x_{t+1}] \,. \tag{23}$$

Hence, the concern of model misspecification provides a source of belief disparities between the probability measure $\mathcal{P}^*$ and $\mathcal{P}$. This belief disparity depends on the continuation value of the representative agent as well as the penalty parameter. Consequently, fluctuations in the belief disparity are a risk factor that should be reflected in the asset returns.

7 Conclusion

This paper empirically investigated how the investor's subjective beliefs drive the cross-section of stock returns. I first constructed two subjective belief factors as latent common factors from a set of eleven series of belief wedges. The belief wedge is defined as the difference between the real-time professional survey forecast and the *Factor-Augmented Vector Autoregression* model (*FAVAR*) implied conditional rational expectation whose factors are estimated from a large data set consisting of 135 series of historical macro data. Next, I estimated the risk prices of these two subjective belief factors from various portfolio groups double-sorted on Size and Book to market (Size-BM), Size and Investment (Size-Inv), Size and Operating profit (Size-OP), Book to market and Investment (BM-Inv), Book to market and Operating profit (BM-OP), Operating profit and Investment (OP-Inv), Size and Short-term reversal (Size-SH) and Size, and Long-term reversal (Size-LO), respectively, and from a pooled estimation of all the portfolios jointly. I found that both of the risk prices are strongly significantly positive, and the explanatory power of these two subjective belief factors R^2 ranges from 0.36 to 0.66. A crucial assumption made in constructing the subjective belief factors is that the belief wedge between professional survey forecasts and model-implied conditional rational forecasts is a sole representation of the investor's subjective belief disparity. A concern here is that the belief wedge may contain macro risk arising from the difference between professional forecaster's information set and the econometrician's. I addressed this concern by including contemporary macro risk factors in the stochastic discount factor, and found evidence in support of the assumption. A potential theoretical explanation for the origin of belief disparities between the subjective probability measure and physical probability measure is rendered based on the robust preference model. A more rigorous analysis using restrictions imposed by the robust preference model is left for the future research.

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Appendix

This appendix provides detailed data sources, results of robustness check, and figures of model-fit in Section 5 and Section 6.1.

Data

Data for Estimating Macro Factors

The data set used to estimate hidden factors for the *FAVAR* model is a monthly data set consisting of 135 macro variables spanning the period 1959/01 to 2017/04. The data set includes all the 128 data series in the FRED-MD data set of the vintage 2017-05 downloaded from the website of the Federal Reserve Bank of St. Louis. In addition to these data series, I manually update seven series of macro variables which have been removed from the FRED-MD data set since the vintage 2016-06. The seven series are NAPMPI, NAPMEI, NAPM, NAPMNOI, NAPMSDI, NAPMII, and NAPMPRI downloaded from the website of the Institute for Supply Management. The resulting data set is classified into eight categories: (1) output and income, (2) labor market, (3) housing, (4) consumption and orders, (5) prices, (6) interest rates and exchange rates, (7) money and credit, and (8) stock market. The number of series contained in each category is 17 in output and income, 32 in labor market, 10 in housing, 14 in consumption and orders , 14 in prices, 22 in interest rates and exchange rates, 21 in money and credit and 6 in the stock market. I transform each data series by taking the first difference, the second difference, logarithm, the first difference in logarithm, the second difference in logarithm and the difference in growth rate when necessary to render the series stationary. Before estimating factors, I also demean and standardize every data series[13].

[13] Detailed data description as well as necessary transformation can be found at https://research.stlouisfed.org/econ/mccracken/fred-databases/Appendix_Tables_Update.pdf

Data for Constructing Belief Wedges

The data set is a monthly data set consisting of eleven series of macro variables for professional survey forecast value and historical value. The data set includes eleven series: Change in Nonfarm Payrolls (NFP TCH), Conference Board Consumer Confidence (CONCCONF), ISM Manufacturing (NAPMPMI), Durable Goods Orders (DGNOCHNG), Consumer Price Index Inflation (CPI CHNG), Retail Sales (RSTAMOM), Unemployment Rate (USURTOT), Industrial Production MoM (IP CHNG), Capacity Utilization (CPTICHNG), Housing Starts (NHSPSTOT), Producer Price Index Inflation (PPI CHNG). The original publishers for the eleven series are: BLS for Change in Nonfarm Payrolls (NFP TCH), BC for Conference Board Consumer Confidence (CONCCONF), ISM for ISM Manufacturing (NAPMPMI), BC for Durable Goods Orders (DGNOCHNG), BLS for Consumer Price Index Inflation (CPI CHNG), BC for Retail Sales (RSTAMOM), BLS for Unemployment Rate (USURTOT), FRB for Industrial Production MoM (IP CHNG), FRB for Capacity Utilization (CPTICHNG), BC for Housing Starts (NHSPSTOT), and BLS for Producer Price Index Inflation (PPI CHNG)[14]. The necessary transformation to achieve stationarity is shown in Table 12.

	Nonfarm Payrolls	**ISM Manufacturing**	**Durable Goods Orders**	**CPI Inflation**	**Unemployment Rate**	**Retail Sales Advance**
TCode	5	5	5	2	2	5
	Industrial Production	**Capacity Utilization**	**Housing Starts**	**PPI Inflation**	**Consumer Confidence**	
TCode	5	2	5	2	5	

Table 12 – Necessary Transformation. The row TCode denotes the following data transformation for a series x: (1) no transformation; (2) Δx_t; (3) $\Delta^2 x_t$; (4) $\log(x_t)$; (5) $\Delta \log(x_t)$; (6) $\Delta^2 \log(x_t)$; (7) $\Delta(x_t/x_{t-1} - 1)$. Survey value of Nonfarm Payrolls in the month t is constructed as the sum of the survey value of Change in Nonfarm Payrolls in the month t and the historical value of Nonfarm Payrolls in the month $t-1$.

[14] Bureau of Labor Statistics (BLS), Bureau of the Census (BC), Federal Reserve Board (FRB), Conference Board (CB), Institute for Supply Management (ISM),

First Glance at Survey Data

Bloomberg Financial Services surveys professional economists on their expectations of macroeconomic announcements every month. Professional survey forecasters are allowed to submit or update their predictions up to the night before the official release of the macro news announcement. Consequently, in principle, the survey forecast should include all the information up to the time point before the release of macro news, which may imply that the survey forecasts of macro news published later in the month are more accurate than those of earlier releases since later forecasts can gather more information as new macro releases become available.

To investigate the quality of the professional survey forecasts, I perform the following time-series regresssion:

$$y_{j,t-l} = \beta_0 + \beta_1 \widetilde{y}^t_{j,t-l} + \epsilon_t, \tag{24}$$

where $y_{j,t-l}$ is the realization of macro news y_j in the month $t-l$ and $\widetilde{y}^t_{j,t-l}$ is the corresponding survey forecast in the month t. In principle, if $\widetilde{y}^t_{j,t-l}$ is a perfect forecast for $y_{j,t-l}$, one should expect $\beta_0 = 0$ and $\beta_1 = 1$.

Table 13 compares the mean of survey forecasts with the corresponding mean of their realizations, as well as release dates for each marco news. Table 14 reports the results for the time-series regression (24). Together with the estimation results, the T-test results for $\beta_0 = 0$ and $\beta_1 = 1$, are also summarized in Table 14.

Among eleven series of professional survey forecasts, CPI Inflation performs exceptionally well with a statistically insignificant intercept and a slope statistically indifferent from one, consistent with the model prediction. Moreover, survey forecasts of CPI Inflation are on average correct for its realizations. Although the average survey forecast of PPI Inflation and Retail Sales hits their average realization, the time-series analysis reveals that both of the series, in fact, underestimate their realizations. A similar pattern is also observed in the series of Durable Goods Order, whose survey forecasts underestimate its realizations even though the average survey forecast is higher than the average realization. The survey forecasts of Unemployment Rate make a good estimate of its realizations along the time-series dimension, though the average survey forecast is slightly higher than its average realization. The evidence from both tables shows that the survey forecasts of Industrial Production and ISM Manufacturing seem to overestimate their realizations. The performance of survey forecasts of Change in Nonfarm Payrolls is inconclusive. High R^2 is observed in the series whose intercept and slope are not statistically different from 0 and 1, respectively.

As discussed above, since professional survey forecasters are allowed to update their predictions up to the night before the official release of the macro news announcement, the survey forecasts of macro news published later in the month may be more accurate than those of earlier releases. However, this systematic bias is not observed in the data. Among the survey forecasts of best performance, Consumer Confidence is a nowcast in the sense that it is published without any delays. So in principle, professional survey forecasters should have the smallest information set when making the forecast for it. Nevertheless, the forecasts are almost as accurate as its realizations. Not surprisingly, Unemployment Rate, released on the first Friday of each month, performs best among all the series along the time-series dimension. On the other hand, Industrial Production and Durable Goods Orders deliver worst performance, though they are published in the mid-month, and in the third or fourth week, respectively. Hence, although some macro announcements are easier to predict than others, the evidence that later releases are based on a larger information set is not observed in the sample.

	Change in Nonfarm Payrolls	ISM Manufacturing	Growth Rate of Durable Goods Orders	CPI Inflation
Mean of Forecast	94.89	52.49	0.18%	0.18%
Mean of Realization	84.54	52.18	0.11%	0.18%
Release Date	First Friday	First Day	Third or Fourth Week	Mid-month
	Unemployment Rate	**Growth Rate of Retail Sales**	**Growth Rate of Industrial Production**	**Capacity Utilization**
Mean of Forecast	6.11%	0.27%	0.17%	77.78%
Mean of Realization	6.08%	0.27%	0.07%	77.29%
Release Date	First Friday	Mid-month	Mid-month	Mid-month
	Housing Starts	**PPI Inflation**	**Consumer Confidence**	
Mean of Forecast	1283.04	0.19%	88.95	
Mean of Realization	1291.06	0.19%	89.62	
Release Date	Third or Fourth Week	Mid-month	Last Tuesday (No Lags)	

Table 13 – Summary of Professional Survey Forecasts. The sample spans the period of 1999/01 to 2017/04.

Robustness Check by Akaike Information Criterion (AIC)

This section reports the results of robustness check by Akaike information criterion. Tables and Figures are in parallel to those in Section 4.3.3.

	Change in Nonfarm Payrolls	ISM Manufacturing	Growth Rate of Durable Goods Orders	CPI Inflation
$\hat{\beta}_0$	-26.41^{**}	3.92^{***}	-0.00	-0.00
$\hat{\beta}_1$	1.17^{**}	0.92^{***}	1.36^{***}	1.08
$\mathbf{R^2}$	0.80	0.88	0.49	0.82
	Unemployment Rate	**Growth Rate of Retail Sales**	**Growth Rate of Industrial Production**	**Capacity Utilization**
$\hat{\beta}_0$	0.00	-0.00	-0.13^{**}	-0.00
$\hat{\beta}_1$	0.99	1.20^{*}	1.21	1.00
$\mathbf{R^2}$	0.99	0.66	0.49	0.90
	Housing Starts	**PPI Inflation**	**Consumer Confidence**	
$\hat{\beta}_0$	0.28	-0.00	0.55	
$\hat{\beta}_1$	1.01	1.34^{***}	1.00	
$\mathbf{R^2}$	0.97	0.72	0.97	

Table 14 – Time-series Regression of Macro Realizations on Macro Forecasts . The t-test result for the test $H_0 : \beta_0 = 0$ v.s. $H_1 : \beta_0 \neq 0$ is reported with $\hat{\beta}_0$ and the t-test result for the test $H_0 : \beta_1 = 1$ v.s. $H_1 : \beta_1 \neq 1$ is reported with $\hat{\beta}_1$. $*$ denotes $p < 0.1$. $**$ denotes $p < 0.05$. $***$ denotes $p < 0.01$. Standard errors are computed with the Newey-West correction of four lags. The sample spans the period of 1999/01 to 2017/04.

	Change in Nonfarm Payrolls	ISM Manufacturing	Durable Goods Orders	CPI Inflation	Unemployment Rate	Retail Sales Advance
Lags of $\mathbf{y_t}$	1	4	2	4	1	2
Lags of $\mathbf{\hat{F}_t}$	2	4	3	3	1	2
$\mathbf{\bar{R}^2}$	0.50	0.28	0.27	0.31	0.29	0.12
	Industrial Production	**Capacity Utilization**	**Housing Starts**	**PPI Inflation**	**Consumer Confidence**	
Lags of $\mathbf{y_t}$	2	4	3	4	4	
Lags of $\mathbf{\hat{F}_t}$	2	4	3	2	4	
$\mathbf{\bar{R}^2}$	0.28	0.28	0.24	0.32	0.18	

Table 15 – Optimal Lags Implied by AIC. R^2 is reported for the forecasting regression. The sample spans the period 1959/01 to 2017/04, except consumer confidence data, which has a shorter time span of the period 1977/06 to 2017/04.

Series	$\mathbf{F_1^*}$	$\mathbf{F_2^*}$
Nonfarm Payrolls	0.115	0.500
ISM Manufacturing	−0.002	0.215
Durable Goods Orders	0.008	0.106
CPI Inflation	0.240	−0.336
Unemployment Rate	−0.077	−0.301
Retail Sales	0.132	0.024
Industrial Production	0.065	0.556
Capacity Utilization	0.105	0.427
New House Starts	−0.002	0.251
PPI Inflation	0.237	−0.348
Consumer Confidence	0.103	−0.092

Table 16 – Loadings for Subjective Belief Factors. Factors are estimated by $\hat{F}_t^* = \hat{\Lambda}^{*\prime} X_t$, where $\hat{\Lambda}^*$ is the matrix of eigenvectors of the sample variance matrix of X_t. The table reports the loadings $\hat{\Lambda}^*$ which has been normalized so that the sum of the loading for the factor equals one. Optimal lags are chosen by AIC. The sample spans the period 1999/01 to 2017/04.

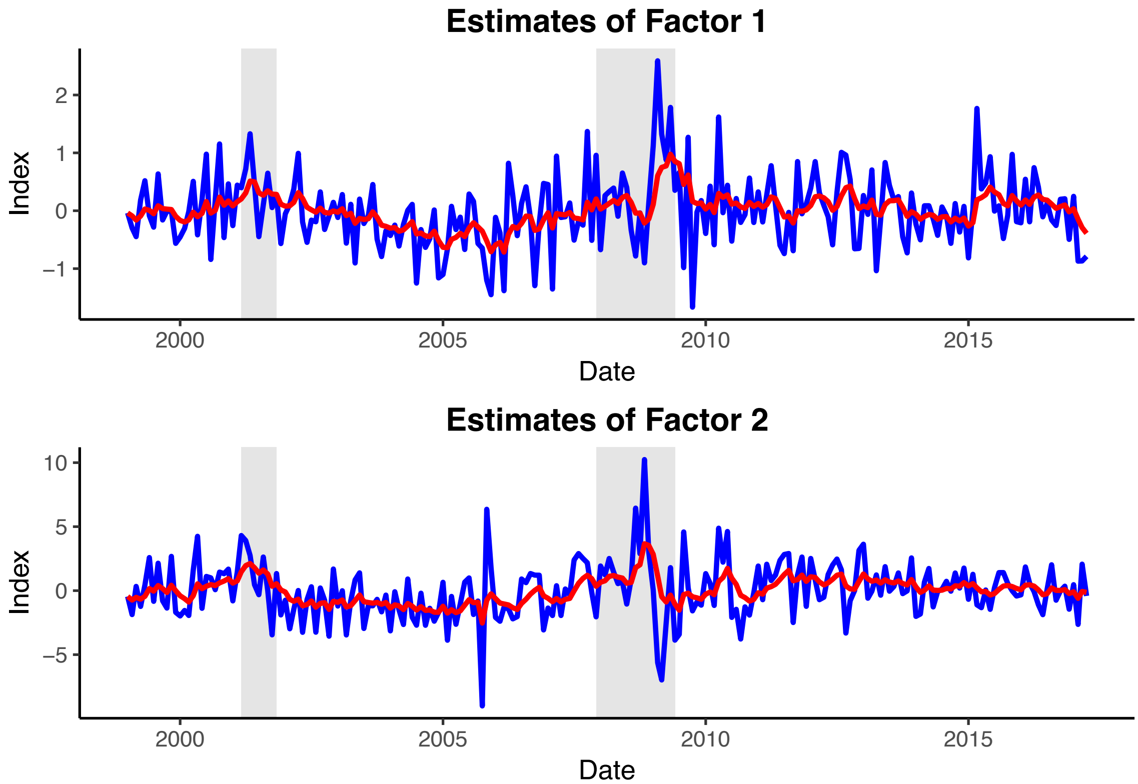

Figure 3 – Estimates of Subjective Belief Factors. Optimal lags are chosen by AIC. The blue line is the original factor value and the red line is a trend estimator by simple moving average of previous six months. NBER recessions are shaded by grey. The sample spans the period 1909/01 to 2017/04.

Figures of Model fit

Thses two figures provide visual impression of the cross-sectional regressions (13) and (18) for the portfolio groups double-sorted on Size and Book to market (Size-BM), Size and Investment (Size-Inv), Size and Operating profit (Size-OP), Book to market and Investment (BM-Inv), Book to market and Operating profit (BM-OP), Operating profit and Investment (OP-Inv), Size and Short-term reversal (Size-SH) and Size and Long-term reversal (Size-LO), respectively, and a pooled estimation of all the portfolios jointly.

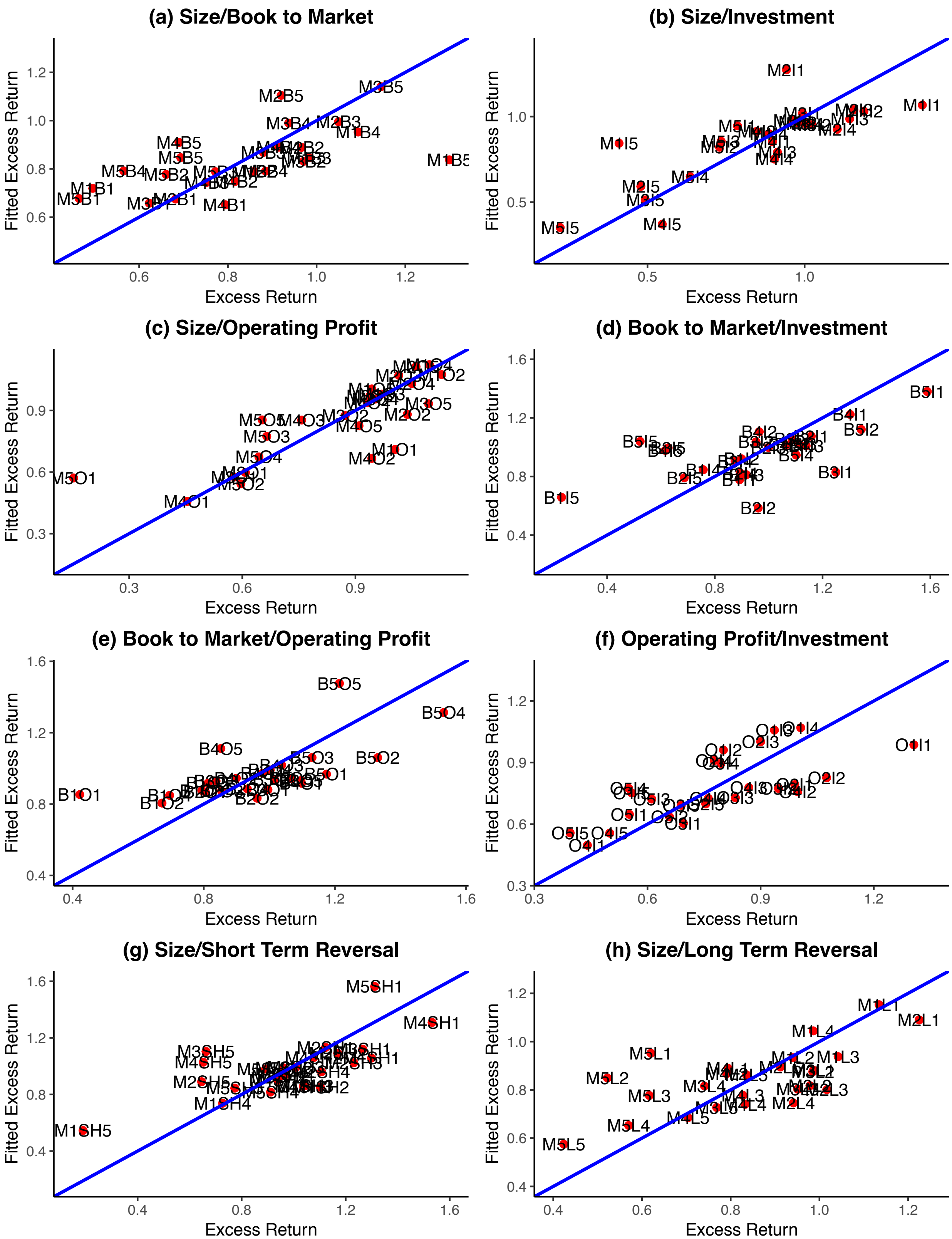

Figure 4 – Excess Return v.s. Fitted Excess Return with Subjective Belief Factors. "1" represents lowest value, while "5" represents highest value. The sample spans the period of 1999/01 to 2017/04.

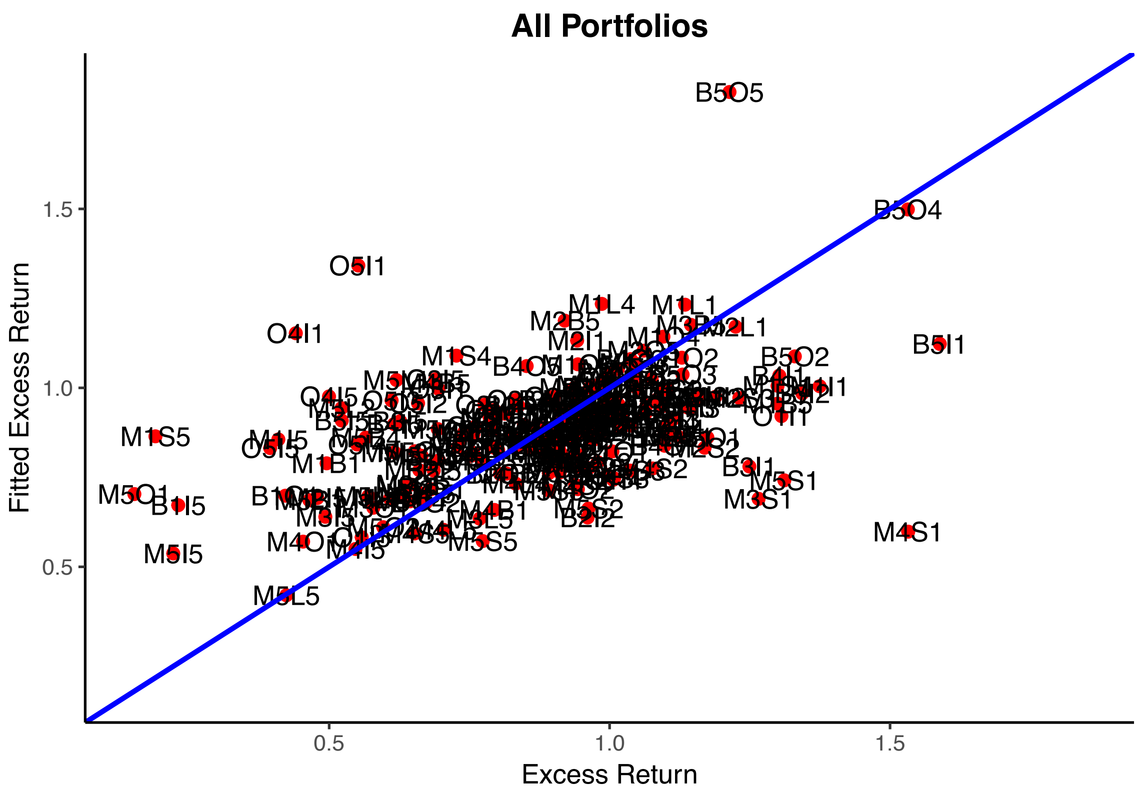

Figure 5 – Excess Return v.s. Fitted Excess Return with Subjective Belief Factors, continued. "1" represents lowest value, while "5" represents highest value. The sample spans the period of 1999/01 to 2017/04.

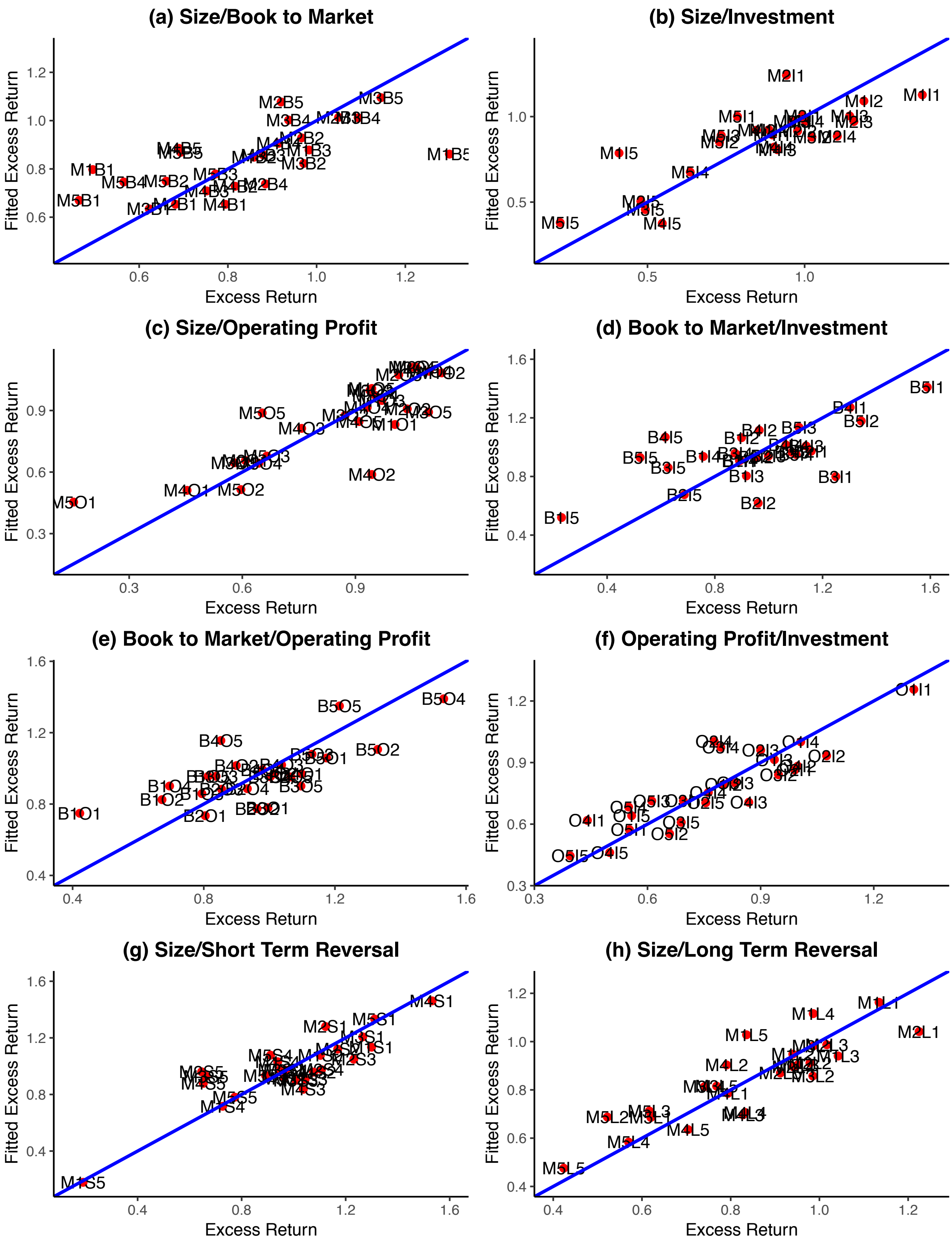

Figure 6 – Excess Return v.s. Fitted Excess Return with Subjective Belief Factors and Macro Factors. "1" represents lowest value, while "5" represents highest value. The sample spans the period of 1999/01 to 2017/04.

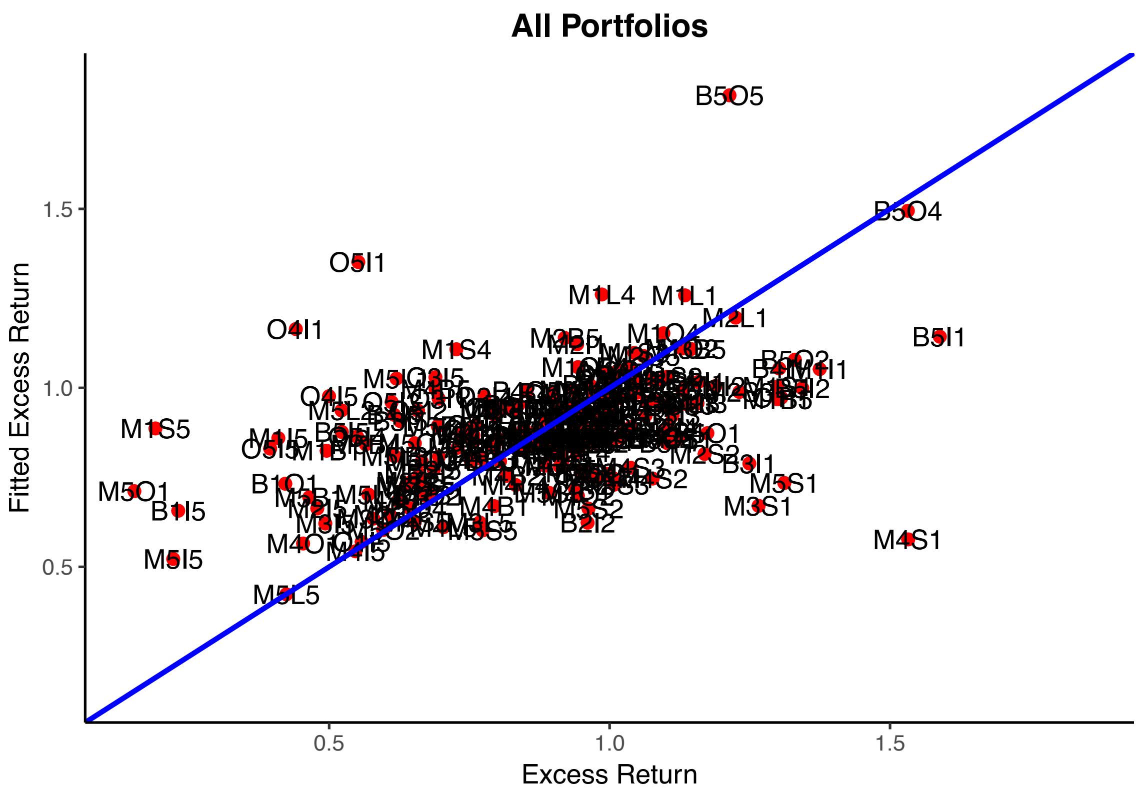

Figure 7 – Excess Return v.s. Fitted Excess Return with Subjective Belief Factors and Macro Factors, continued. "1" represents lowest value, while "5" represents highest value. The sample spans the period of 1999/01 to 2017/04.