Employment and Hours over the Business Cycle in a Model with Search Frictions[1]

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Abstract

This paper studies a labor market search-matching model with multi-worker firms to investigate how firms utilize the extensive and intensive margins over the business cycle. The earnings function derived from the Stole-Zwiebel bargaining acts as an adjustment cost function for employment and hours. We calibrate the model to match the Japanese labor market, in which the intensive margin accounts for 79% of the variations in total working hours. The model replicates the observed cyclical behavior of hours of work, but fails to generate employment volatility of realistic magnitude. Additional penalties for longer hours of work do not resolve this issue. Wage rigidity and persistent shocks are promising lines of further investigations.

JEL classification: E32, J20, J64.

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1 Introduction

Firms adjust their labor inputs over the business cycle through the intensive margin (hours of work per employee) and the extensive margin (the number of employees). While the two margins play distinct roles in labor adjustment, the dominant approach to business cycle research adopts models which places no significance in the *composition* of labor demand over the business cycle: in the frictionless framework, firms are indifferent between the two margins; and in the textbook search-matching framework, each production unit consists of a single worker who supplies a fixed unit of labor. In this paper, we study employment and hours over the business cycle by focusing on how firms utilize the two margins in labor adjustment.

In their survey, Hall *et al.* (2000) found that 62% of firms consider overtime as the primary reaction to a demand boom. In a perfectly competitive labor market, however, firms do not need to utilize overtime, which usually requires extra wage payment, because they can employ extra workers instantly at the going wage rate. This suggests the importance of *frictions* in understanding how firms utilize the intensive and extensive margins over the business cycle. The point of departure of this paper is the observation that firms need to engage in time-consuming search-matching process when hiring new employees while changes in hours of work per employee are instantaneous.

Our basic model builds on a recent development in labor market search models with *multi-worker firms*.[1] A novel property of our model is that the earnings function derived from the Stole-Zwiebel (1996) bargaining between each firm and the workers is a convex function of hours of work, and this function specifies the marginal cost for the firm of choosing longer hours of work, which acts as an *endogenous* labor adjustment cost function (Kudoh and Sasaki, 2011).

We calibrate our model to match the Japanese labor market facts. An important characteristic about the Japanese labor market fluctuations is that the intensive margin accounts for a partic-

[1] A partial list of the related contributions is Bertola and Caballero (1994), Bertola and Garibaldi (2001), Smith (1999), Cahuc and Wasmer (2001), Cooper *et al.* (2007), Koeniger and Prat (2007), Cahuc *et al.* (2008), Fang and Rogerson (2009), Felbermayr and Prat (2011), Kudoh and Sasaki (2011), Acemoglu and Hawkins (2013), Elsby and Michaels (2013), Fujita and Nakajima (2013).

ularly large proportion of cyclical fluctuations in the aggregate labor input. In our empirical analysis, we find that variations in hours of work per employee account for *79%* of the variations in the aggregate labor input, while variations in the number of employees account for 21% of the variations. This is in sharp contrast with the labor market fluctuations in the U.S., in which 79% of the variations are accounted for by the *extensive* margin.

In our numerical analysis, we show that our model replicates the Japanese labor market fact that much of cyclical fluctuations in total hours of work is accounted for by the intensive margin. In addition, it predicts that firms adjust the both margins in the same direction over the business cycle, which is consistent with the evidence. We also obtain realistic impulse response functions for unemployment, vacancies, employment and hours of work. Furthermore, the magnitude of volatility in hours of work per worker for the calibrated model matches that of the data.

However, the model fails to replicate the *magnitude of volatility in employment*. As a result, fluctuations in total hours of work are mostly explained by fluctuations in hours per worker: 94% of the variations in the labor input are accounted for by the intensive margin, which is too high compared with the data. We also note that the Shimer (2005) puzzle, the inability of the standard search-matching model to generate sufficient cyclical fluctuations in unemployment and vacancies, applies to our model. Consistent with the literature, our model possesses the properties that lower unemployment benefit is associated with lower volatility for unemployment and vacancies, and countercyclical separation shocks generate a positively-sloped Beveridge curve.

A natural question is whether firms in our model utilize the extensive margin more heavily over the business cycle *if it is more costly to extend hours of work per employee*. This issue is important for understanding why firms in some countries utilize the intensive margin more heavily than those in other countries. To address this important issue, we consider extensions of the basic model in which hours of work is bargained, and overtime wage premium is imposed, and there is an efficiency loss from longer hours of work. These extensions lead us to conclude that introduction of additional penalties for longer hours of work does not help resolve the low employment volatility result at all.

We find that a model with (ad-hoc) wage rigidity resolves the issue. Our modeling strategy is to introduce an earnings function that does not fully reflect the change in TFP. The model with a rigid earnings function generates fluctuations in employment as well as unemployment and vacancies with somewhat realistic magnitudes. We also find that a model with (unrealistically) high persistence of TFP shocks is successful in many dimensions.

Since this paper deals with a general issue, there is a long list of related studies. Therefore, the description of the related literature must necessarily be partial.

We intend our empirical analysis to be a contribution to the growing literature on re-examination of hours of work using new models and new datasets. This includes Rogerson (2006) and Ohanian and Raffo (2012), to name a few. Particularly relevant is Ohanian and Raffo (2012), who find that the intensive margin is increasingly more important for labor adjustment in 14 OECD countries they studied. Using the dataset we built for the Japanese labor market, our empirical analysis confirms the importance of the intensive margin.

This paper is certainly related to the early real business cycle literature. Using a frictionless, competitive business cycle model, Cho and Cooley (1994) study hours and employment over the business cycle, and calibrate it to match the US labor market facts. Braun *et al.* (2006) adopt the Cho-Cooley framework to study the labor market in Japan. We show that, in steady state, our model nests a version of the Cho-Cooley model if the worker's bargaining power is zero and the matching function takes a certain form.

In the labor hoarding literature and more generally the factor utilization literature, it is often argued that real business cycle models with costly labor adjustment display realistic cyclical fluctuations (Burnside et al., 1993; Bils and Cho, 1994; Cogley and Nason, 1995; Burnside and Eichenbaum, 1996). Burnside et al. (1993), for instance, assume that it takes one period to adjust labor input. Note that our model naturally possesses the same feature because the level of employment increases only after firms post vacancies. Contrary to the factor utilization literature, in which the cost function for factor utilization is exogenously given, in our model, the cost that a firm faces when choosing longer hours of work is derived from the Stole-Zwiebel bargaining.

In terms of the structure of the model, particularly related to our model are Cooper *et al.* (2007) and Kudoh and Sasaki (2011), in which determination of hours of work is considered in the context of labor market search-matching with multi-worker firms. Cooper *et al.* (2007) study both employment and hours of work over the business cycle using a model similar to ours. They emphasize the importance of nonlinear cost of posting vacancies, and as a result, the wage bargaining is simplified by assuming the take-it-or-leave-it offer protocol. While we restrict our analysis to the linear vacancy cost, our model is more general in that we adopt the Stole-Zwiebel bargaining to derive the earnings function.

Although our basic model is similar to Kudoh and Sasaki (2011) for many dimensions, we take a step further to study the model's business cycle properties. While Kudoh and Sasaki (2011) focus on the model's steady-state properties and efficiency, we focus on cyclical fluctuations around the steady state. A subtle, yet important, difference between Kudoh and Sasaki (2011) and our model is that we employ a constant-returns-to-scale (CRS) production technology with labor and capital. Introduction of capital with a CRS production function serves two purposes. One is that it makes calibration more comparable to the existing business cycle research. The other is that with CRS technology, we can focus on the two margins, employment and hours of work, by safely ignoring the third margin, entry and exit of firms over the business cycle.

From a theoretical perspective, this paper is a contribution to the growing literature on labor market search-matching models with multi-worker firms mentioned above. An alternative approach that can be used to address the same issue is the framework developed by Merz (1995) and Andolfatto (1996), in which there is a representative household with a continuum of family members. The advantage of this framework is that a rich set of assumptions about the household's decisions can easily be introduced. The advantage of our framework is that firms' dynamic decisions can easily be introduced.

The remainder of the paper is organized as follows. In Section 2, we empirically examine how labor inputs are adjusted over the business cycle in Japan, and present some cyclical characteristics of the Japanese labor market. Section 3 describes our basic model. Section 4 characterizes the equilibrium of the model. In Section 5, we calibrate the model parameters and present our main results. Section 6 concludes. Proofs and some additional results are found in the Appendix.

2 Labor Market Facts

2.1 Labor Market Fluctuations in Japan

In this section, we present some key empirical results. We obtain the series of the number of employed workers and the number of labor force from the Labour Force Survey (LFS), conducted by the Statistics Bureau and the Director-General for Policy Planning. The series of the average monthly hours worked per worker are obtained from the Monthly Labour Survey (MLS) conducted by the Ministry of Health, Labour and Welfare (MHLW). We construct our measure of the aggregate labor input (i.e., total hours of work) as the product of the average monthly hours worked per worker and the number of employed workers, normalized by the labor force. These measures are consistent with Ohanian and Raffo (2012).

Our data are quarterly, which, when necessary, are obtained by averaging or aggregating the corresponding monthly series. The sample covers 1980Q2–2010Q4. All series are seasonally adjusted using the Census Bureau's X12 ARIMA procedure and transformed by taking natural logarithms. Since our focus is on cyclical fluctuations in hours and employment, the low-frequency movements in the data are filtered out by using the Hodrick-Prescott (HP) filter with smoothing parameter 1600.

Figure 1 plots the cyclical components of the aggregate labor input (i.e., total hours of work), hours worked per worker, and the number of employed workers. The figure shows that the aggregate labor input and its components fluctuate significantly over the business cycle, and both hours and employment display comovement with total hours worked. It also indicates that the aggregate labor input comoves more closely with hours of work per worker than with employment, and that employment is less volatile than the others.

Table 1 quantifies what we see in Figure 1 by summarizing cyclical characteristics of the key labor market variables, which are as follows: u is the unemployment rate, v is the vacancy rate,

Table 1: Summary statistics, quarterly Japanese data, 1980-2010

Variables	$\hat{u}$	$\hat{v}$	$\hat{f}$	$\hat{s}$	$\hat{w}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	$\hat{A}$
Standard deviation	0.059	0.096	0.091	0.092	0.010	0.009	0.008	0.004	0.011
Correlation matrix									
$\hat{u}$	1	-0.777	-0.420	0.449	-0.453	-0.526	-0.325	-0.558	-0.281
$\hat{v}$	-	1	0.320	-0.517	0.656	0.728	0.644	0.378	0.470
$\hat{f}$	-	-	1	-0.344	0.192	0.201	0.139	0.185	0.005
$\hat{s}$	-	-	-	1	-0.365	-0.416	-0.422	-0.108	-0.371
$\hat{w}$	-	-	-	-	1	0.610	0.587	0.222	0.406
$\hat{t}$	-	-	-	-	-	1	0.902	0.485	0.359
$\hat{h}$	-	-	-	-	-	-	1	0.060	0.392
$\hat{l}$	-	-	-	-	-	-	-	1	0.038
$\hat{A}$	-	-	-	-	-	-	-	-	1

f is the job-finding rate, s is the separation rate, w is the real wage rate, t is total hours of work, h is hours worked per employee, l is the number of employed workers, and A is the total factor productivity (TFP). All variables are with hat, meaning that they are in percentage deviations from their trend levels.

The TFP series are from Braun *et al.* (2006). We obtain the unemployment rate series from the LFS. The vacancy rate series are obtained from the Monthly Report on Employment Service (Shokugyo Antei Gyomu Tokei) conducted by the MHLW. Following Miyamoto (2011) and Lin and Miyamoto (2012), we construct the job finding and separation rates from the LFS. Real wages are taken from the Monthly Labour Survey (MLS) conducted by the MHLW.[2] All data are seasonally adjusted by the Census Bureau's X12 filter. To focus on cyclical fluctuations, we detrend the logged data using the HP filter with smoothing parameter 1600.

[2] The MHLW calculates real wage indices by using the consumer price index (CPI).

Hours and Employment Since the series are in natural logarithms, the standard deviations can be interpreted as mean percentage deviations from their trend levels. For instance, the standard deviation of total hours worked is 0.9 percent, that of hours worked per worker is 0.8 percent, and that of employment is 0.4 percent. The results quantify our observation from Figure 1. Consistent with Figure 1, Table 1 shows a strong positive relationship between total hours worked and hours worked per worker. The correlation between them is 0.90. We also find a positive relationship between total hours and employment, with a correlation of 0.49, which is significantly smaller than the correlation between total hours and hours per worker. However, as Braun et al. (2006) pointed out, there is no strong correlation between hours worked per worker and employment. The correlation between them is 0.06. Table 1 also shows that the aggregate labor input and its components comove positively with TFP and negatively with the unemployment rate, indicating that they are all pro-cyclical. Interestingly, the correlation between employment and TFP is very weak.

Unemployment and Vacancy Table 1 shows that the unemployment rate is counter-cyclical and the vacancy rate is pro-cyclical. The correlation between the unemployment rate and TFP is -0.28. The correlation between the vacancy rate and TFP is 0.47. Since the unemployment rate is counter-cyclical and the vacancy rate is procyclical, these two series comove negatively. The correlation between them is -0.78. The negative correlation between unemployment and vacancy reflects the Beveridge curve. Both the unemployment rate and the vacancy rate are more volatile than TFP. While the standard deviation of TFP is 1.1 percent, the standard deviations of the unemployment and vacancy rates are 5.9 percent and 9.6 percent, respectively. Thus, the unemployment rate is about five times as volatile as TFP and the vacancy rate is about nine times as volatile as TFP.

Job Finding Rate and Separation Rate The job finding rate is acyclical and the separation rate is counter-cyclical. The correlation between the job finding rate and TFP is 0.05. The correlation between the separation rate and TFP is -0.37. The standard deviations of job finding and separation rates are 9.1 percent and 9.2 percent, respectively. Similar to the unemployment and vacancy rates, both the job finding and separation rates fluctuate much more than TFP does. Volatilities of these two series are roughly eight times as large as that of TFP.

Real Wage Rate The real wage rate is procyclical. The correlation between the real wage rate and TFP is 0.41. The standard deviation of the real wage rate is 1 percent, and thus the real wage rate is as volatile as TFP.

2.2 Decomposition of Fluctuations

We now study the relative contributions of the intensive and extensive margins to fluctuations in the total hours worked. With $\hat{t} = \hat{h} + \hat{l}$, variance of total hours worked can be decomposed as

$$Var(\hat{t}) = Var(\hat{h}) + Var(\hat{l}) + 2Cov(\hat{h}, \hat{l}) = Cov(\hat{t}, \hat{h}) + Cov(\hat{t}, \hat{l}). \tag{1}$$

The term $Cov(\hat{t}, \hat{h})$ gives the amount of variations in $\hat{t}$ that derived from variations in $\hat{h}$ and through its comovement with $\hat{l}$. Similarly, the term $Cov(\hat{t}, \hat{l})$ is the amount of variations in $\hat{t}$ that derived from variations in $\hat{l}$ and through its comovement with $\hat{h}$. By dividing the both sides of (1) by $Var(\hat{t})$, we obtain

$$1 = \frac{Cov(\hat{t}, \hat{h})}{Var(\hat{t})} + \frac{Cov(\hat{t}, \hat{l})}{Var(\hat{t})} = \beta^h + \beta^l, \tag{2}$$

where β^h and β^l are the relative contributions of variations in $\hat{h}$ and $\hat{l}$ to variations in $\hat{t}$. These measures are an application of the "beta value" in finance.[3]

From the data, we find that $\beta^h = 0.79$ and $\beta^l = 0.21$. In other words, the intensive margin explains 79% of variations in total hours worked and the extensive margin accounts for 21% of the variations. This implies that over the business cycle, Japanese firms adjust labor inputs both by the intensive and extensive margins, but they use the intensive margin more heavily than the extensive margin.

This result is in sharp contrast with what we find from the U.S. data. We utilize the dataset constructed by Ohanian and Raffo (2012) and find that $\beta^h = 0.21$ and $\beta^l = 0.79$ for the U.S. labor

[3]Petrongolo and Pissarides (2008) and Fujita and Ramey (2009) apply this measure to decompose unemployment fluctuations into inflow fluctuations and outflow fluctuations.

market. Thus, U.S. firms adjust labor inputs mainly through the extensive margin. Figure 2 is reassuring. It indicates that the aggregate labor input comoves more closely with employment than with hours of work per worker, and that hours per worker is less volatile than the others.

3 The Model

In this section, we present our basic model for explaining the observed high volatility of hours worked per worker in Japan. The key feature of the model is that it captures the fact that while hours of work per worker can change instantly, firms need to open (costly) vacancies in a frictional labor market to hire new workers. To focus on the composition of the labor demand at each firm, we build a labor market search-matching model with multi-worker firms.

3.1 Environment

Consider an economy consisting of a large number of workers and firms. The measure of workers is normalized to unity. Both workers and firms are homogeneous. Time is discrete and all agents discount the future at the common discount rate r.

The production technology for each firm is given by $A_t L_t^{\alpha} k_t^{1-\alpha}$, where $0 < \alpha < 1$ is the labor share, A_t denotes the level of total factor productivity (TFP), which is exogenous, k_t denotes the stock of capital at each firm, and L_t denotes the total labor input (i.e., total hours worked). Assuming homogeneous labor, we postulate that $L_t \equiv h_t l_t$, where h_t is hours of work per employee and l_t is the number of employees at each firm. Thus, output y_t is given by $y_t = A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha}$.

It will be shown that, with this constant-returns-to-scale technology, the (steady-state) value of operating a firm exactly cancels out the (appropriately-chosen) cost of firm entry, and as a result, firms are indifferent between entry and exit, as in the zero-profit result in the standard neoclassical model (Proposition 2). Thus, we adopt the conventional remedy for indeterminacy of the number of firms by normalizing the measure of firms to be unity. In effect, we can ignore entry and exit of firms over the business cycle and focus on our main theme, hours versus employment over the business cycle.[4]

Each firm possesses a technology that converts one unit of the final consumption good into a unit of investment good. Let x_t be the level of investment made in period t. Then the stock of capital evolves according to $k_{t+1} = (1-\delta_k)k_t + x_t$, where δ_k is the rate of capital depreciation.

All workers are risk neutral, and maximize the expected lifetime utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[I_t - e\left(h_t\right) \right],$$

where $\delta \equiv 1/(1+r)$ is the discount factor, I_t denotes income, and $e(h_t)$ represents disutility of work. We assume that $e'(\cdot) > 0$, $e''(\cdot) > 0$ and $\lim_{h\to\infty} e(h) = \infty$. Our specification of the disutility function is

$$e\left(h\right) = e_0 \frac{h^{1+\mu}}{1+\mu}, \tag{3}$$

where $e_0 > 0$ and $\mu > 0$. It is evident that $e''(h)h/e'(h) = \mu$, where $1/\mu$ is the Frisch elasticity.

The labor market is frictional. The number of matches in period t is determined by the constant returns to scale matching technology $m(U_t, V_t)$, where U_t is the total number of job seekers and V_t is the number of aggregate job vacancies. A vacancy is matched to a worker during a period with probability q_t, where $q_t \equiv m(U_t, V_t)/V_t = m(U_t/V_t, 1)$. This is the vacancy filling rate. It is easy to verify that an increase in labor market tightness $V_t/U_t \equiv \theta_t$ decreases this probability. Similarly, the probability that a worker is matched with a vacancy, or the job finding rate, is given by $m(U_t, V_t)/U_t = m(1, V_t/U_t) = \theta_t q(\theta_t)$, where $q(\theta) \equiv m(U/V, 1)$.

Our specification of the matching function is $m(U, V) = m_0 U^{\xi} V^{1-\xi}$, where $0 < \xi < 1$. With this specification, we obtain

$$q_t = q\left(\theta_t\right) = m_0 \theta_t^{-\xi}. \tag{4}$$

Evidently, $q'(\theta)\theta/q(\theta) = -\xi$.

[4] At the empirical level, extry and exit play relatively minor roles in job creation and destruction. For instance, for the 1973–1988 period in the U.S., job creation associated with startups accouts for 8.4% of total quarterly job creation, and job destruction associated with shutdowns accounts for 11.6% of total quarterly job destruction (Davis et al., 1996, p.29).

We assume exogenous separations. At the end of each period, a fraction λ of the current employees are assumed to leave the firm. Since the firm creates v_t units of vacancies, the number of new employees for the next period is $q(\theta_t)v_t$. These new employees are not hit by the separation shock. Thus, the number of employees at each firm evolves according to $l_{t+1} = (1-\lambda)l_t + q(\theta_t)v_t$.

3.2 Timing

Let $S_t = (A_t, l_t, k_t, \theta_t)$ be the set of state variables. Among the state variables, the level of A_t is revealed at the beginning of each period. Given the state variables, each firm and its employees bargain over earnings. Under our assumption of homogeneous workers, all employees earn W_t in each period. The bargaining outcome is summarized by $W_t = W(S_t, h_t)$ in each period, where $W(S_t, h_t)$ is the earnings function to be determined as the outcome of bargaining. The earnings function specifies the transfers from the firm to its employees and it depends on hours of work. With the earnings function, the firm chooses the demand for hours of work (h_t), vacancies to create (v_t), and the level of capital investment (x_t). Then, production takes place and output y_t is realized. Finally, λl_t of the current employees leave the firm, and $q(\theta_t)v_t$ workers are newly employed.

3.3 Firms

We solve the firm's optimization problem by stationary dynamic programming. The instantaneous payoff to a firm is given by $y - Wl - \gamma v - x$, where $\gamma > 0$ is the cost of posting each vacancy. Let l_{+1} and k_{+1} denote the levels of employment and capital in the next period, respectively. The value of a firm $J(S)$ satisfies the following Bellman equation:

$$J(S) = \max_{h,v,x} \left\{ Ah^{\alpha} l^{\alpha} k^{1-\alpha} - W(S,h)l - \gamma v - x + \delta \mathbb{E} J(S_{+1}) \right\}, \tag{5}$$

where

$$l_{+1} = (1-\lambda)l + q(\theta)v, \tag{6}$$

$$k_{+1} = (1-\delta_k)k + x. \tag{7}$$

The first-order conditions with respect to h, v, and x imply

$$\alpha A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha} = W_h(S,h), \tag{8}$$

$$\delta \mathbb{E} J_l\left(S_{+1}\right) = \frac{\gamma}{q(\theta)}, \tag{9}$$

$$\delta \mathbb{E} J_k\left(S_{+1}\right) = 1. \tag{10}$$

The envelope conditions yield

$$J_l\left(S\right) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(S,h) - W_l(S,h)l + (1-\lambda)\,\delta \mathbb{E} J_l\left(S_{+1}\right), \tag{11}$$

$$J_k\left(S\right) = (1-\alpha)\,A h^{\alpha} l^{\alpha} k^{-\alpha} - W_k(S,h)l + (1-\delta_k)\,\delta \mathbb{E} J_k\left(S_{+1}\right). \tag{12}$$

Substitute (9) and (10) into (11) and (12) to obtain

$$J_l\left(S\right) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(S,h) - W_l(S,h)l + \frac{(1-\lambda)\,\gamma}{q(\theta)}, \tag{13}$$

$$J_k\left(S\right) = (1-\alpha)\,A h^{\alpha} l^{\alpha} k^{-\alpha} - W_k(S,h)l + 1 - \delta_k, \tag{14}$$

respectively.

3.4 Workers

The value of being employed, $J^E(S)$, satisfies

$$J^E\left(S\right) = W(S,h) - e\left(h\right) + \lambda\delta \mathbb{E} J^U\left(S_{+1}\right) + (1-\lambda)\delta \mathbb{E} J^E\left(S_{+1}\right), \tag{15}$$

where $J^U(S)$ is the value of being unemployed. Note that hours of work h is determined by the firm, and therefore the worker takes h and therefore the level of disutility $e(h)$ as given.

Consider the unemployed. The probability that a job seeker finds a job is $m(U,V)/U = \theta q(\theta)$. Thus, the value of being unemployed can be written as

$$J^U\left(S\right) = b + \theta q(\theta)\delta \mathbb{E} J^E\left(S_{+1}\right) + (1-\theta q(\theta))\delta \mathbb{E} J^U\left(S_{+1}\right), \tag{16}$$

where b is the (exogenous) unemployment benefit.

3.5 Wage Bargaining

At the beginning of each period, workers and a firm engage wage bargaining. We assume that workers are not unionized, and each worker is treated as a marginal worker (Stole and Zwiebel, 1996). Consider a bargaining process between a firm and a group of workers of measure Δ. The threat point for the firm is $J\left(A, l-\Delta, k, \theta\right)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(A,l,k,\theta) - J(A,l-\Delta,k,\theta) + \Delta(J^E(S) - J^U(S))$. If the firm's share of the surplus is given by $1-\beta \in [0,1]$, then we have $\beta[J(A,l,k,\theta) - J(A,l-\Delta,k,\theta)] = (1-\beta)\Delta[J^E(S) - J^U(S)]$. In the limit as $\Delta \to 0$,

$$\beta J_l\left(S\right) = \left(1-\beta\right)\left[J^E(S) - J^U(S)\right]. \tag{17}$$

This is the key equation for rent sharing. Note that this amounts to maximizing the asymmetric Nash product $[J_l(S)]^{1-\beta}[J^E(S) - J^U(S)]^{\beta}$ with respect to $W(S,h)$.

Proposition 1 *The earnings function is given by*

$$W(S,h) = \frac{\alpha\beta A h^{\alpha} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + \left(1-\beta\right)\left[e\left(h\right) + b\right] + \beta\gamma\theta. \tag{18}$$

Proof. See Appendix A. ■

The earnings function (18) is an extension of the one derived in Kudoh and Sasaki (2011). From (18), we obtain

$$W_h\left(S,h\right) = \frac{\alpha^2\beta A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + \left(1-\beta\right)e'\left(h\right) > 0, \tag{19}$$

$$W_{hh}\left(S,h\right) = -\left(1-\alpha\right)\frac{\alpha^2\beta A h^{\alpha-2} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + \left(1-\beta\right)e''\left(h\right), \tag{20}$$

$$W_l\left(S,h\right) = -\left(1-\alpha\right)\frac{\alpha\beta A h^{\alpha} l^{\alpha-2} k^{1-\alpha}}{\alpha\beta + 1 - \beta} < 0, \tag{21}$$

$$W_k\left(S,h\right) = \left(1-\alpha\right)\frac{\alpha\beta A h^{\alpha} l^{\alpha-1} k^{-\alpha}}{\alpha\beta + 1 - \beta} > 0. \tag{22}$$

The key result here is that the marginal hourly wage rate is nonlinear, and is influenced by the marginal product of hours per worker ($\alpha A h^{\alpha-1} l^{\alpha} k^{1-\alpha}/l$) and the marginal disutility from longer hours of work. (20) suggests that, when the disutility function is sufficiently convex, the earnings function itself becomes convex in hours of work ($W_{hh} > 0$). Another key result is that the earnings function is decreasing in the number of employees ($W_l < 0$). This property induces the firm to employ too many workers in order to cut the wage rate, known as the overemployment effect (Smith, 1999, Kudoh and Sasaki, 2011). It is interesting to observe from (18) and (19) that the labor market tightness θ has no effect on the marginal hourly wage rate, while it influences the level of earnings.

Using (19)–(22), we rewrite (13) and (14) as

$$J_l(S) = \frac{(1-\beta)\alpha}{\alpha\beta+1-\beta} Ah^{\alpha} l^{\alpha-1} k^{1-\alpha} - (1-\beta)\left[e(h)+b\right] - \beta\gamma\theta + \frac{(1-\lambda)\gamma}{q(\theta)}, \tag{23}$$

$$J_k(S) = \frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} Ah^{\alpha} l^{\alpha} k^{-\alpha} + 1 - \delta_k. \tag{24}$$

4 Equilibrium

4.1 Definition

We look for a rational expectations equilibrium in which TFP follows an exogenous stochastic process. Below, we define equilibrium of the model as a system of stochastic difference equations. From (8) and (19), we obtain

$$\frac{\alpha}{\alpha\beta+1-\beta} A_t h_t^{\alpha-1} l_t^{\alpha-1} k_t^{1-\alpha} = e'(h_t), \tag{25}$$

which governs h_t. Substitute (23) into (9) to obtain the Euler-type equation:

$$\mathbb{E}_t \left\{ \frac{(1-\beta)\alpha}{\alpha\beta+1-\beta} A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha-1} k_{t+1}^{1-\alpha} - (1-\beta)\left[e(h_{t+1})+b\right] - \beta\gamma\theta_{t+1} + \frac{(1-\lambda)\gamma}{q(\theta_{t+1})} \right\} = \frac{(1+r)\gamma}{q(\theta_t)}, \tag{26}$$

which determines the demand for l_t. Equations (25) and (26) summarize the firm's optimal choice regarding hours of work and employment. The evolution of employment follows $l_{t+1} = (1-\lambda)l_t + q(\theta_t)v_t$, which determines the the firm's vacancy v_t.

Similarly, substitute (24) into (10) to obtain

$$\mathbb{E}_t \left\{ \frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha} k_{t+1}^{-\alpha} + 1 - \delta_k \right\} = 1 + r. \tag{27}$$

This determines the demand for capital. The evolution of capital stock is given by $k_{t+1} = (1 - \delta_k) k_t + x_t$, which determines investment x_t.

The aggregate variables are determined as follows. In this economy, the number of the unemployed U_t equals the the rate of unemployment because the labor force is normalized to unity. In each period, $\theta_t q(\theta_t) U_t$ job seekers find jobs. Similarly, the aggregate number of employees is $1 - U_t$, from which the aggregate number of separations is $\lambda(1 - U_t)$. Thus, the number of the unemployed evolves according to

$$U_{t+1} - U_t = \lambda (1 - U_t) - \theta_t q(\theta_t) U_t. \tag{28}$$

In any steady state, the flow into employment $\theta q(\theta) U$ must equal the flow into unemployment $\lambda(1 - U)$, or $m(U, V) = \lambda(1 - U)$, which defines the Beveridge curve. Labor market tightness is given by $\theta_t = V_t / U_t$.

Since the number of firms is normalized to unity, we obtain

$$\frac{1 - U_t}{l_t} = 1, \tag{29}$$

where the numerator is the aggregate number of employees and the denominator is the number of employees at each firm, so the ratio defines the number of firms in the economy. For the same reason, the aggregate number of vacancies equals the number of vacancies created by each firm, or $V_t = v_t$. Similarly, the aggregate output, or GDP of the economy Y_t, is given by $Y_t = y_t$.

4.2 Steady State Equilibrium

For brevity of exposition, let $K \equiv k/hl$ denote the steady state capital-labor ratio. A non-stochastic steady state is given by the solution to the following system of equations:

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta}AK^{-\alpha} = r+\delta_k, \tag{30}$$

$$\frac{\alpha}{\alpha\beta+1-\beta}AK^{1-\alpha} = e'(h), \tag{31}$$

$$\frac{(r+\lambda)\gamma}{q(\theta)}+\beta\gamma\theta = (1-\beta)\left[e'(h)h-e(h)-b\right], \tag{32}$$

$$l = 1-\frac{\lambda}{\lambda+\theta q(\theta)}, \tag{33}$$

$$k = Khl. \tag{34}$$

For existence of a steady-state equilibrium, parameters must be chosen to satisfy $e'(h)h - e(h) - b > 0$, or $\mu(1+\mu)^{-1}e_0h^{1+\mu} > b$. Uniqueness of the steady state is verified as follows. First, the steady-state capital-labor ratio K is determined by (30), which comes from (27). Given K, the steady-state hours of work h is determined by (31), which is from (25). Given h, (32), which is from (26), determines θ. Given θ, the steady state level of l is determined by (33), which comes from (28) and (29). Finally, given the values of K, h, and l, we can derive the value of k by (34).

Proposition 2 *For an appropriately chosen cost of entry, in any steady-state equilibrium, firms are indifferent between entry and exit.*

This result verifies that our assumption of a unit measure of firms causes no loss of generality. The cost of entry is chosen to make the size distribution of firms degenerate, as in Smith (1999) and Kudoh and Sasaki (2011). The detail is found in Appendix B.

Proposition 3 *(a) An increase in A increases K, h, θ, l, and k. (b) An increase in β decreases K, h, θ, l, and k. (c) An increase in γ has no effect on K and h, and decreases θ, l, and k. (d) An increase in λ has no effect on K and h, and decreases θ, l, and k. (e) An increase in m_0 has no effect on K and h, and increases θ, l, and k.*

Proof of Proposition 3 is straightforward, and is omitted. Among the results, the most notable is that changes in γ, λ, and m_0, which are the key parameters determining the labor market frictions, have no effect on the steady state level of hours of work, while these parameters influence the steady-state level of employment and labor market tightness. An important implication of the result is that search frictions are *irrelevant* for understanding the long-run trend of hours of work.

To understand the role of search frictions and wage bargaining, it is helpful to make a comparison between our model and a model with a perfectly competitive labor market. The benchmark for our comparison is a version of Cho and Cooley (1994), in which there is a representative household who maximizes

$$\sum_{t=0}^{\infty} \delta^t \left[u\left(c_t\right) - e\left(h_t\right) l_t + b\left(1 - l_t\right) + g\left(1 - l_t\right) \right]$$

subject to $k_{t+1} = Ah_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha} + (1 - \delta_k) k_t - c_t$, where $g(.)$ is an increasing function of non-employment $1 - l_t$. In this model, l_t measures the days of work or the number of family members who participate in the labor market. As l_t increases, more workers incur the utility cost $e\left(h_t\right)$ and lose the opportunity to receive b. All other costs of labor market participation is summarized by $g(.)$. To make this frictionless economy comparable with our model economy, we assume that g is log-linear: $g(x) = e_1(\ln x - x)$, where e_1 is a positive parameter.

Proposition 4 *Suppose that utility is linear in consumption. (a) If $\beta > 0$, then the capital-labor ratio of the economy with search and wage bargaining is less than that for the frictionless economy, and hours of work per employee in the economy with search and wage bargaining is longer than that for the frictionless economy. (b) The steady-state levels of the capital-labor ratio, hours of work, and employment for the two economies coincide if $\beta = 0$, $\xi = 1/2$, and*

$$e_1 = \frac{r + \lambda}{m_0} \gamma \frac{\lambda}{m_0}. \tag{35}$$

Proof of Proposition 4 is found in Appendix C. Condition (35) equates the marginal benefit of non-employment and an index of search costs. A higher discount rate, a higher separations rate, a higher vacancy cost, or a smaller matching coefficient, each of which implies greater search frictions, increases the term of the right-hand side of (35). If the right-hand side of (35) is greater than e_1, then the level of employment for the economy with search frictions is below the one for the frictionless economy even when $\beta = 0$ and $\xi = 1/2$. Proposition 4 indicates that our model contains a competitive, frictionless economy as a special case.

5 Quantitative Analysis

In this section, we study a quantitative version of our baseline model. Specifically, we calibrate the model to match some long-run Japanese labor market facts. We then solve the quantitative model by approximating the equilibrium conditions around the non-stochastic steady state, and simulate it to obtain the model's cyclical properties. Our objective is to investigate whether the model replicates the cyclical properties of the Japanese labor market presented in Section 2.

5.1 Calibration

The parameters of the model are chosen to match some long-run Japanese labor market facts. The resulting parameter values are summarized in Table 2.

We choose the model period to be a quarter and set the discount rate to be $r = 0.01$, which implies the discount factor to be $\delta = 1/(1+r) = 0.996$. This choice of the parameter is somewhat a priori, but is consistent with other studies such as Braun et al. (2006).

The matching function is Cobb-Douglas, given by $m(U, V) = m_0 U^{\xi} V^{1-\xi}$, where m_0 is the matching constant and ξ is the matching elasticity with respect to the number of job-seekers. Lin and Miyamoto's (2014) estimate of the elasticity ξ for the Japanese labor market is 0.6.[5] We adopt the Lin-Miyamoto estimate to set $\xi = 0.6$. This value lies in the plausible range of 0.5–0.7, which is reported by Petrongolo and Pissarides (2001).

Using the panel property of the monthly LFS, Miyamoto (2011) and Lin and Miyamoto (2012) construct the job-finding rate and the separation rate in Japan. Miyamoto (2011) also reports the mean value of the vacancy-unemployment ratio is 0.78. We use the monthly job-finding rate 0.142 and the vacancy-unemployment ratio 0.78 to pin down the scale parameter m_0. In particular, m_0 is

[5] See also Kano and Ohta (2002).

Table 2: Parameter values

Parameter	Description	Value	Source/Target
r	Interest rate	0.01	Data
δ	Discount rate $1/(1+r)$	0.996	Data
α	Parameter in production function	2/3	Labor share
m_0	Matching efficiency	0.471	Monthly job-finding rate
ξ	Matching elasticity	0.6	Lin and Miyamoto (2014)
λ	Exogenous separation rate	0.014	Monthly separation rate
e_0	Parameter in disutility function	15.666	$h = 1/3$
δ_k	Depreciation rate	0.028	Esteban-Pretel et al. (2010)
μ	Inverse of Frisch elasticity	2.0	Kuroda and Yamamoto (2008)
b	Unemployment benefits	0.347	Replacement rate = 0.6
β	Worker's bargaining power	0.6	$\beta = \xi$ (Hosios condition)
γ	Vacancy cost	0.031	$v - u$ ratio = 0.78
A	Productivity	1.0	Normalization
ρ	AR-coefficient of shock	0.612	Data
ε	Standard deviation of the shock	0.0085	Data

the solution to $m_0(0.78)^{1-0.6} = 3 \times 0.142$. We also set the exogenous separation rate $\lambda = 0.014 = 3 \times 0.0048$ from Miyamoto (2011) and Lin and Miyamoto (2012).

Following the convention, we use the Hosios (1990) condition to pin down the worker's bargaining power, so $\beta = \xi$. In the production function, we set the labor share $\alpha = 2/3$. Following Esteban-Pretel et al. (2010), we set the deprecation rate to be $\delta_k = 0.028$. The Frisch elasticity is given by $1/\mu$. Kuroda and Yamamoto (2008) estimate the labor supply elasticity in Japan and find that the elasticity for males is in the range of 0.2 to 0.7. In our benchmark, we set μ equal to 2, which implies the Frisch elasticity of 0.5.

We assume that productivity follows a first order autoregressive process. Specifically, $\log A_t$ satisfies $\log A_t - \log A = \rho_A (\log A_{t-1} - \log A) + \varepsilon_{A,t}$,where $0 < \rho_A < 0$ and $\varepsilon_{A,t} \sim N(0, \sigma^2)$. We set $\rho_A = 0.612$ and $\varepsilon = 0.0085$ to match the first-order autocorrelation and standard deviation of total factor productivity (TFP) in the data.

Given this, we target the vacancy-unemployment ratio to be $\theta = 0.78$, from Miyamoto (2011). According to Martin (2000), the replacement rate, the ratio of the unemployment benefit to the average wage, in Japan is about 0.6. We adopt this estimate to target the unemployment benefit b to be 60% of the average wage of employed workers in the economy. Finally, we target the steady-state value of hours worked to be 1/3. We thus have three target moments and three model parameters: b, γ, and e_0. We choose the parameter values that match the three target moments.

5.2 Results

Figure 3 plots impulse response functions of selected variables to positive one-standard deviation shock to productivity. Following the productivity shock, the unemployment rate falls and vacancies increase, and thus the labor market tightness rises. A higher productivity encourages the firm to post more vacancies as it increases the expected returns to hiring a worker. The increase in vacancies leads to less unemployment. The pattern of responses to the unemployment rate, vacancies, and labor market tightness are in line with the empirical result found in Lin and Miyamoto (2012).

The positive productivity shock increases total hours worked. The increase in total hours worked is due to increases in both hours per worker and the number of employees. Thus, hours and employment are both pro-cyclical. Firms not only ask their existing employees to work longer, but also hire more in response to a positive productivity shock. This result is consistent with what we found in Section 2, namely, firms adjust labor inputs in the same direction both along the intensive and extensive margins.

We now turn to comparing business cycle statistics from the simulated time series with those from the corresponding time series data. Table 3 reports standard deviations of variables of interest obtained from both our model and the data. From the table, we obtain the following two

Table 3: Volatility

	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	$\hat{A}$
Data	0.0585	0.0960	0.0103	0.0086	0.0075	0.0037	0.0107
Baseline model	0.0188	0.0557	0.0195	0.0087	0.0081	0.0006	0.0108
1. Fixed hours $h = 1/3$	0.0226	0.0669	0.0117	0.0008	-	0.0008	0.0108
2. Model with $b/\bar{W} = 0.4$	0.0047	0.0140	0.0196	0.0083	0.0081	0.0002	0.0108
3. Model with $\mu = 2.2$	0.0146	0.0432	0.0188	0.0078	0.0074	0.0005	0.0108
4. Model with $\mu = 1.8$	0.0286	0.0847	0.0203	0.0099	0.0090	0.0010	0.0108

important results.

First, the model succeeds in replicating the magnitudes of fluctuations in total hours of work and hours per worker. Specifically, both in the model and in the data, the standard deviations of total hours worked and hours per worker are 0.009 and 0.008, respectively. The model also captures the observation that employment is significantly less volatile than hours per worker. In the data, the standard deviation of employment is 0.004, while it is 0.001 in the model.

To quantify the relative contributions of the intensive and extensive margins in the model, we apply (2) to decompose the variations in total hours of work. We find that $\beta^h = 0.94$ and $\beta^l = 0.06$. While the model captures the observation that much of fluctuations in total hours worked is explained by fluctuations in hours per worker, it fails to replicate the magnitudes of the relative contributions of hours per worker and employment. The root of the problem is that *the model's employment volatility is too low*.

The second important result is that the model fails to account for the high volatility of the unemployment rate and the vacancy rate that are observed in the data. The standard deviations of unemployment and vacancies in the data are 0.059 and 0.096, respectively. The standard deviation of unemployment generated from the model is 0.019, which is only 1/3 of the standard deviation of unemployment in the data. Similarly, the standard deviation of vacancies generated from the model is 0.056, which is 1/2 of the standard deviation of vacancies in the data. The model's inability to generate high volatilities of unemployment and vacancies is common to many equilibrium search-matching models and is considered as an important challenge in the literature.[6]

Table 4: Correlation

	$(\hat{U},\hat{V})$	$(\hat{h},\hat{W})$	$(\hat{h},\hat{t})$	$(\hat{h},\hat{A})$	$(\hat{h},\hat{l})$
Data	-0.777	0.587	0.902	0.582	0.060
Baseline model	-0.715	1.000	0.999	1.000	0.884
Stochastic λ	0.049	1.000	0.985	1.000	0.576

Table 4 reports correlations between some key variables. As is clear from the correlation between $\hat{U}$ and $\hat{V}$, our model performs quite well (-0.777 in the data vs. -0.715 in the model). The model also captures the signs of correlations between hours per worker and several labor market variables. However, their magnitudes are much larger than what we observe in the data. In particular, the correlation between hours per worker and the number of employees obtained from the model is far from that of the data. This weak correlation between hours and employment found in the Japanese data is not observed in the U.S. data. Braun et al. (2006) considered this fact as a major puzzle in the Japanese business cycle.

Role of the Intensive Margin To clarify the importance of the intensive margin, we present the cyclical properties of a model that is otherwise identical to the benchmark model, except that hours of work per employee is fixed at 1/3. The results are found in Figure 3 and in Table 3. Qualitatively, the models with and without the intensive margin display a similar pattern in response to a productivity shock. However, quantitative effects of the productivity shock on the economy are different. Figure 3 shows that the response of the number of employees in the model with fixed hours of work is larger than that in our benchmark model. A positive productivity shock encourages a firm to hire more workers as it increases the match surplus. Without the intensive

[6]Shimer (2005) demonstrates that the standard search-matching model cannot generate the observed unemployment and vacancy fluctuations in response to productivity shock of reasonable size. Recently, a number of papers study whether the Shimer puzzle holds for the Japanese labor market (Esteban-Pretel et al., 2011; Miyamoto, 2011). These papers reach the conclusion that the Shimer puzzle hold for the Japanese economy.

margin, the firm achieves a new optimal labor input by adjusting the number of vacancies. As a result, the responses of the number of employees and vacancies are larger than those in the model with the intensive margin. This also leads to a larger response of unemployment to the productivity shock in the model with fixed hours of work. Surprisingly, the response of total hours of work in the model with the intensive margin is much larger than that in the model with fixed hours of work, although the response of the number of employees in the former model is smaller than that in the latter. This finding implies that the contribution of the intensive margin to variations in total labor input is larger than that of the extensive margin. Table 3 (row 1) reports standard deviations of variables of interest obtained from the model with fixed hours of work. As is expected, the volatility of the unemployment rate and the vacancy rate in the model with fixed hours of work is closer to the data than that in the benchmark model. However, it cannot account for the volatility of total hours of work. While the standard deviation of total hours work in the data is 0.009, the corresponding value generated from the model is 0.001.

Replacement Rate The choice of the parameter for the unemployment benefit, b, has been the subject of discussion in the literature on cyclical properties of search-matching models. For the U.S. labor market, Shimer (2005) sets b so that the replacement rate is 0.4, while Hagedorn and Manovskii (2008) argue that Shimer's b is too low and that with a higher b, search-matching models can replicate unemployment and vacancy fluctuations of realistic magnitude. With this debate in mind, we briefly discuss the sensitivity of our results to the choice of b. In our benchmark calibration, we adopt Martin (2000) to set b so that the replacement rate is 0.6, which is higher than the conventional value for the US labor market. As an alternative, we consider b that is consistent with the replacement rate of 40%. As Tables 3 (row 2) shows, with a smaller b, unemployment and vacancies are less volatile. This is in line with Hagedorn and Manovskii (2008). Interestingly, for other variables, the model's cyclical properties are quite similar under the two different replacement rates.

Frisch Elasticity The labor supply elasticity, in our case μ, has a wide range of estimation results. It is therefore important to present results under other values for μ to see whether greater disutility of longer hours of work increases employment volatility. Interestingly, Table 3 (row 3 and row 4) shows that the impact of productivity shock on labor market variables is magnified as μ *decreases*. This is because a decrease in convexity in the disutility of labor decreases the marginal cost of production, encouraging a firm to post more vacancies and to lengthen hours of work. What we learn from this exercise is that while a greater disutility of longer hours of work does imply less volatility in hours per worker, it doe not imply greater volatility in employment. In particular, for greater volatility in employment, μ must be smaller, in which case the volatilities of unemployment and vacancies also improve.

5.3 Employment Volatility

The basic model replicates the key property of the Japanese labor market fact that labor adjustment takes place mainly through the intensive margin. However, the model's employment volatility is too low compared to the data, and our analysis so far is silent about the key elements generating the excessive reliance of the intensive margin. An important challenge is to identify the source of low volatility of employment.

In what follows, we present several directions for extending the model to account for employment volatility of realistic magnitude. Our suspects are, in the basic model, hours of work per employee is chosen by the firm; there is no overtime wage premium; there is no productivity loss from longer hours of work; the wage rate is too flexible; the separations rate is constant over the business cycle.

Bargained Hours of Work A possible criticism of the basic model is that it is the firm who chooses hours of work per employee. As a result, the firm may excessively depend on the intensive margin. We show that this intuition is correct only in terms of the steady-state level of hours per worker. Consider the basic model in which h is determined in the bargaining stage. Specifically, we assume that both the earnings function W and hours per employee h are determined so

Table 5: Model Extensions

	$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	$\hat{A}$
Data	0.0585	0.0960	0.0103	0.0086	0.0075	0.0037	0.0107
Baseline model	0.0188	0.0557	0.0195	0.0087	0.0081	0.0006	0.0108
1. Bargained hours	0.0089	0.0265	0.0189	0.0084	0.0081	0.0003	0.0108
2. Overtime premium	0.0101	0.0299	0.0199	0.0084	0.0081	0.0003	0.0108
3. Efficiency ($\eta = 0.8$)	0.0099	0.0294	0.0174	0.0077	0.0074	0.0003	0.0108
4. Efficiency ($\eta = 0.6$)	0.0069	0.0204	0.0157	0.0070	0.0068	0.0002	0.0108
5. Wage rigidity	0.0381	0.1128	0.0169	0.0093	0.0081	0.0013	0.0108
6. Stochastic λ	0.0593	0.0704	0.0193	0.0093	0.0080	0.0020	0.0107

as to maximize the following asymmetric Nash product:

$$[J_l(S)]^{1-\beta}\left[J^E(S) - J^U\right]^{\beta}.$$

The first-order conditions with respect to W and h are $\beta J_l(S) = (1-\beta)[J^E(S) - J^U(S)]$ and $(1-\beta)[J^E(S) - J^U][\partial J_l(S)/\partial h] + \beta J_l(S)[\partial J^E(S)/\partial h] = 0$, which reduces to

$$\frac{\partial\left[J_l(S)\right]}{\partial h} + \frac{\partial\left[J^E(S)\right]}{\partial h} = 0,$$

where

$$\begin{aligned}
\frac{\partial\left[J_l(S)\right]}{\partial h} &= \frac{(1-\beta)\,\alpha^2}{\alpha\beta + 1 - \beta} A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha} - (1-\beta)\, e'(h)\,, \\
\frac{\partial\left[J^E(S)\right]}{\partial h} &= \frac{\alpha^2 A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha}}{\alpha + \frac{1-\beta}{\beta}} - \beta e'(h)\,.
\end{aligned}$$

Thus, (25) is replaced with

$$\frac{\alpha^2}{\alpha\beta + 1 - \beta} A_t h_t^{\alpha-1} l_t^{\alpha-1} k_t^{1-\alpha} = e'(h_t)\,. \tag{36}$$

As is now clear, (25) and (36) are nearly identical. Thus, we expect very little change in the quantitative results. Table 5 (row number 1) verifies that there is no improvement. Even worse, unemployment and vacancies are less volatile than those in the basic model.

Overtime Wage Premium Since absence of overtime wage premium is an important omission for understanding the tradeoff between longer hours and more employees, it is important to study how cyclical behavior of our model is influenced by the presence of legal overtime premium. To see the impact of the wage premium within a simple framework, we introduce an exogenously specified transfer scheme from the firm to its employees that depends on hours of work. In particular, we assume that there is a transfer $p(h)$ from the firm to each worker, where $p(h)$ is the legal wage premium, which satisfies $p'(h) > 0$ and $p''(h) > 0$. Our modeling strategy is to avoid any kink or discontinuity in the premium schedule.[7] With this transfer scheme, the value of a firm satisfies

$$J(S) = \max_{h,v,x} \left\{ Ah^{\alpha} l^{\alpha} k^{1-\alpha} - W(S,h)l - p(h)\, l - \gamma v - x + \delta \mathbb{E} J(S_{+1}) \right\},$$

and the value of being employed satisfies

$$J^E(S) = W(S,h) + p(h) - e(h) + \lambda \delta \mathbb{E} J^U(S_{+1}) + (1-\lambda)\delta \mathbb{E} J^E(S_{+1}),$$

It is easy to show that the earnings function for this economy is given by

$$W(S,h) = \frac{\alpha\beta A h^{\alpha} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] - p(h) + \beta\gamma\theta.$$

Interestingly, the wage is reduced exactly by $p(h)$. In other words, the wage bargaining neutralizes the transfer scheme *by reducing the normal pay by the amount exactly equals to the overtime payment*. For our numerical study, we assume $p(h) = h^2/2$ to see how the introduction of transfer that depends on hours per worker change the business cycle properties of the economy. Table 5 (row number 2) shows that the overtime wage premium does not introduce any additional volatility in hours per worker. Moreover, unemployment, vacancies, and employment are significantly less volatile than those in the basic model.

Decreasing Efficiency Another possible scenario that might explain why longer hours of work is costly for firms is that longer hours of work per worker cause a loss of efficiency in labor input.

[7] An alternative formulation is found in Hansen and Sargent (1988), in which straight-time and overtime are treated as different inputs in production.

To capture this possibility, we modify our basic model such that the effective labor input is concave in hours per employee while it is linear in the number of employees. In this sense, firms are no longer indifferent between hours and employment. To be more specific, we assume that total labor input satisfies

$$L = h^{\eta} l,$$

where $\eta \leq 1$. Clearly, $\eta = 1$ corresponds to the benchmark model. The value of a firm is given by

$$J(S) = \max_{h,v,x} \left\{ A h^{\alpha\eta} l^{\alpha} k^{1-\alpha} - W(S,h) l - \gamma v - x + \delta \mathbb{E} J(S_{+1}) \right\},$$

and the optimization conditions need to be modified accordingly. The earnings function for this economy is given by

$$W(S,h) = \frac{\alpha\beta A h^{\alpha\eta} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] + \beta\gamma\theta.$$

Table 5 (row 3 and row 4) summarizes the result from this extension. It shows that unemployment and vacancies are even less volatile than those in the basic model. Once again, a greater penalty for longer hours of work does not generate a higher employment volatility.

Wage Rigidity Shimer (2005) argues that the textbook search-matching model fails to replicate the volatility of unemployment and vacancies. An important current debate is whether wage rigidity resolves the volatility puzzle. Here, we study whether wage rigidity helps explain the low employment volatility result in our model. Our modeling strategy is to modify the model as little as possible, rather than to write down a full-fledged micro-founded model of wage rigidity. To be more specific, we introduce an ad-hoc wage function with rigidity that possesses the following two properties. One is that the rigid wage function is identical to benchmark wage function (18) in the steady state. In other words, while ad-hoc, the rigid wage function does not alter the steady state of the basic model. The other is that the current wage level does not fully reflect the current TFP (Pissarides, 2009). Specifically, let $S_t^P = (A_t^P, l_t, k_t, \theta_t)$ be the *perceived* state of the economy in period t, where A_t^P is the perceived level of TFP which is assumed to satisfy

$$A_t^P = \phi A_t + (1-\phi) A. \tag{37}$$

Thus, the perceived TFP is given by the weighted average of the true current TFP and its steady-state level.[8] We assume that in the bargaining stage, the true state is not verifiable, and as a result, the wage bargaining is conditional only on the perceived state. The resulting earnings function in period t is given by

$$W(S_t^P, h_t) = \frac{\alpha\beta A_t^P h_t^\alpha l_t^{\alpha-1} k_t^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e\left(h_t\right) + b\right] + \beta\gamma\theta_t. \tag{38}$$

The wage is rigid in the sense that the earnings function does not fully reflects the current TFP. For our numerical analysis, we choose ϕ to be 0.359 from a structural estimation by Lin and Miyamoto (2014). Table 5 (row 5) shows that the model with wage rigidity generates fluctuations of unemployment and vacancies of realistic magnitudes. Further, the volatility of employment is doubled (0.0006 in the benchmark model vs. 0.0013). However, while there is a significant improvement, the volatility of employment is about one third of the employment volatility in the data.

Stochastic Separations Recent empirical studies demonstrate that both unemployment inflow and outflow significantly contribute the unemployment dynamics in Japan (Miyamoto, 2011; Lin and Miyamoto, 2012). In particular, in the data, TFP and the separation rate are negatively correlated. In order to assess the importance of the unemployment inflow channel in generating unemployment fluctuations, here we study a model that incorporates a persistent shock to the separation rate of workers into unemployment. Specifically, we assume that $\log\lambda_t$ follows a first-order autoregressive process of the form $\log\lambda_t - \log\lambda = \rho_\lambda\left(\log\lambda_{t-1} - \log\lambda\right) + \varepsilon_{\lambda,t}$, where $0 < \rho_\lambda < 0$ and $\varepsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2)$. We choose $\rho_\lambda = 0.1575$ and $\rho_{\lambda A} = -0.3706$, both of which are from the data. Table 5 (row 6) shows that the model with stochastic separations accounts for the volatilities of unemployment and vacancies. In addition, the model accounts for a half of the observed fluctuations in employment, which outperforms the result obtained from the benchmark model. These

[8] Our formulation of wage rigidity is inspired by Hall (2005) and in particular Krause and Lubik (2007). These authors assume an ad-hoc wage equation in which the actual current wage rate is given by the weighted average of the Nash bargained wage rate and a reference wage rate, such as the past wage rate and the steady-state level.

results are obtained without losing the model's performance in terms of hours of work per worker and total hours worked. However, as is clear from Table 4, the model fails to generate a negative relationship between unemployment and vacancies, i.e. the Beveridge curve. This is because a positive productivity shock substantially reduces the number of job seekers (unemployed workers) by lowering job separation, which in turn makes vacancy posting less attractive. Fujita and Ramey (2012) show that search-matching models cannot generate the observed strong negative correlation between unemployment and vacancies when the separation rate is counter-cyclical. It it now clear that their result extends to a model with the intensive margin.

5.4 Persistent Productivity Shocks

In this section, we study the importance of persistence of productivity shocks. The idea is that firms can safely rely on the intensive margin as long as the productivity shock is transient, but, with significantly persistent shocks, firms need to rely more on the extensive margin. The purpose of the exercise here is to study the importance of persistent shocks in understanding the role of extensive margin. Our choice of ρ is 0.987 to target the standard deviation of vacancies in the data. Since a change in ρ changes the size of the shock, which makes model evaluation less clear, in what follows we look at the standard deviation of vacancies normalized by the standard deviation of TFP.

Table 6: Persistent Productivity Shocks

		$\hat{U}$	$\hat{V}$	$\hat{W}$	$\hat{t}$	$\hat{h}$	$\hat{l}$	$\hat{A}$
Data	$\sigma(x)/\sigma(A)$	5.364	8.727	0.909	0.818	0.727	0.364	1.000
Baseline model ($\rho = 0.612$)	$\sigma(x)/\sigma(A)$	1.739	5.146	1.801	0.802	0.750	0.059	1.000
$\rho = 0.897$	$\sigma(x)/\sigma(A)$	5.301	8.727	1.999	0.928	0.750	0.179	1.000

Table 6 shows the result. We apply (2) to decompose the variations in total hours of work. We find that $\beta^h = 0.81$ and $\beta^l = 0.19$. Introduction of (unrealistically) persistent productivity shocks resolves Shimer's (2005) puzzle, and significantly improves the low employment volatility result.

6 Conclusion

In this paper, we showed that while both employment and hours per employee are important for explaining cyclical fluctuations in the Japanese labor market, the intensive margin explains 79% of variations in total labor input. To understand the fact, we developed a labor market search-matching model with multi-worker firms to investigate how firms utilize both the extensive and intensive margins over the business cycle. The calibrated version of our model replicated the observed cyclical behavior of hours of work in Japan, but it failed to replicate the employment volatility.

We attempted to resolve the low employment volatility result in our theory by extending the model in several directions. Greater penalties for longer hours of work are not the answer. A model with wage rigidity is partially successful, but the magnitude of employment fluctuations is still far below the employment volatility in the data. High persistence of productivity shock generates high volatilities of unemployment, vacancies and employment.

An important line of future research is to identify the institutional characteristics that determine the composition of labor demand over the business cycle. For this investigation, one needs a model with endogenous firing. With a richer model of firing, one can study the impact of employment protection such as firing restriction on the importance of the intensive and extensive margins over the business cycle.

Another important line of future research is to consider differences in types of employment. While we assumed a single employment contract for all workers, the labor market in Japan is best understood as being polarized into regular and "non-regular" employment. The regular employment provides high job security and high wages, and the so-called non-regular employment, including part-time employment and temporary employment, offers less job security and low wages. Workers under the non-regular employment contract have been increasing, and they amount to 40% of total employees in Japan. It is therefore important to investigate a model similar to ours in which firms can create (and destroy) regular and non-regular jobs to evaluate its quantitative implications for employment fluctuations.

Appendix

A Proof of Proposition 1

Subtract (16) from (15) to obtain

$$J^E(S) - J^U(S) = W(S,h) - e(h) - b + [1 - \lambda - \theta q(\theta)] \left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right]. \tag{39}$$

Observe that, (9) and (17) imply

$$\frac{1-\beta}{\beta} \left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right] = \frac{\gamma}{q(\theta)}.$$

Use this to rewrite (39) as follows:

$$J^E(S) - J^U(S) = W(S,h) - e(h) - b + \frac{1 - \lambda - \theta q(\theta)}{1-\beta} \frac{\beta\gamma}{q(\theta)}. \tag{40}$$

Substitute (13) and (40) into (17) to obtain

$$\begin{aligned}
&\beta \left[\alpha A h^\alpha l^{\alpha-1} k^{1-\alpha} - W(S,h) - W_l(S,h) l + \frac{(1-\lambda)\gamma}{q(\theta)}\right] \\
&= (1-\beta) [W(S,h) - e(h) - b] + [1 - \lambda - \theta q(\theta)] \frac{\beta\gamma}{q(\theta)},
\end{aligned}$$

which reduces to $W(S,h) = \beta[\alpha A h^\alpha l^{\alpha-1} k^{1-\alpha} - W_l(S,h) l] + (1-\beta)[e(h) + b] + \beta\gamma\theta$, or

$$W_l(S,h) l + \frac{1}{\beta} W(S,h) = \alpha A h^\alpha l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta} [e(h) + b] + \gamma\theta. \tag{41}$$

This is a differential equation about the unknown earnings function. This equation satisfies for all $l \geq 0$, along with the condition that

$$W(S,h) l \leq A h^\alpha l^\alpha k^{1-\alpha}, \tag{42}$$

which requires that the total wage payment does not exceed the firm's revenue. It is useful to observe that

$$\begin{aligned}
\frac{\partial}{\partial l} \left[W(S,h) l^{\frac{1}{\beta}}\right] &= \left[W_l(S,h) l + \frac{1}{\beta} W(S,h)\right] l^{\frac{1}{\beta}-1} \\
&= \left[\alpha A h^\alpha l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta} [e(h) + b] + \gamma\theta\right] l^{\frac{1}{\beta}-1} \\
&= \alpha A h^\alpha l^{\alpha+\frac{1}{\beta}-2} k^{1-\alpha} + \left[\frac{1-\beta}{\beta} [e(h) + b] + \gamma\theta\right] l^{\frac{1}{\beta}-1}.
\end{aligned}$$

Since (42) implies $W(S,h)l^{\frac{1}{\beta}} \leq Ah^{\alpha}l^{\alpha+\frac{1}{\beta}-1}k^{1-\alpha}$, we have $\lim_{l\to 0} W(S,h)l^{\frac{1}{\beta}} = 0$. Thus, it follows that

$$\begin{aligned} W(S,h)l^{\frac{1}{\beta}} &= \int_0^l \left\{ \alpha Ah^{\alpha} i^{\alpha+\frac{1}{\beta}-2} k^{1-\alpha} + \left[\frac{1-\beta}{\beta} \left[e\left(h\right) + b \right] + \gamma\theta \right] i^{\frac{1}{\beta}-1} \right\} di \\ &= \frac{\alpha Ah^{\alpha}k^{1-\alpha}}{\alpha + \frac{1}{\beta} - 1} l^{\alpha+\frac{1}{\beta}-1} + \left[(1-\beta) \left[e\left(h\right) + b \right] + \beta\gamma\theta \right] l^{\frac{1}{\beta}}. \end{aligned}$$

Thus, we finally obtain

$$W(S,h) = \frac{\alpha Ah^{\alpha}k^{1-\alpha}}{\alpha + \frac{1}{\beta} - 1} l^{\alpha-1} + (1-\beta) \left[e\left(h\right) + b \right] + \beta\gamma\theta$$

as shown in the proposition.

B Entry and Exit

We follow Smith (1999) and Kudoh and Sasaki (2011) to assume that each entrant must create vacancies so that it operates with the steady-state level of employment l in the next period. This assumption rules out the size distribution of firms. Because the rate of filling a vacancy is $q(\theta)$, in order to achieve l_{+1} in the next period, the firm must create exactly $l_{+1}/q(\theta)$ vacancies in the current period. Thus, the value of entry is given by

$$J\left(0\right) = -\frac{\gamma l_{+1}}{q(\theta)} - k_{+1} + \delta \mathbb{E} J\left(S_{+1}\right), \tag{43}$$

Therefore, the number of firms, N_t, is determined by $J(0) = 0$, or

$$\frac{\gamma l_{+1}}{q(\theta)} + k_{+1} = \delta \mathbb{E} J\left(S_{+1}\right). \tag{44}$$

In any steady state, the firm's value of operation (without imposing (44)) is

$$\begin{aligned} (1-\delta)\, J\left(S\right) &= Ah^{\alpha}l^{\alpha}k^{1-\alpha} - W(S,h)l - \gamma v - x \\ &= \left[\frac{1-\beta}{\alpha\beta + 1 - \beta} \right] Ah^{\alpha}l^{\alpha}k^{1-\alpha} - (1-\beta)\left[e\left(h\right) + b \right] l - \beta\gamma\theta l - \gamma\frac{\lambda l}{q(\theta)} - \delta_k k \\ &= \frac{(1-\beta)\,(1-\alpha)}{\alpha\beta + 1 - \beta} Ah^{\alpha}l^{\alpha}k^{1-\alpha} + \frac{r\gamma}{q(\theta)} l - \delta_k k \\ &= r\frac{\gamma l}{q(\theta)} + rk, \end{aligned}$$

from which we obtain

$$\delta J(S) = \frac{\gamma l}{q(\theta)} + k.$$

Thus, in any steady state, the value of entry is

$$J(0) = -\frac{\gamma l}{q(\theta)} - k + \delta J(S) = 0.$$

Thus, firms are indifferent between entry and exit. As a result, the number of firms will be *indeterminate* in this economy in the sense that a free entry condition cannot pin down the number of firms. The same result holds in the standard neoclassical economy in which firms profits are zero. The result did not arise in Smith (1999) or Kudoh and Sasaki (2011) because in these models, the production technology exhibits decreasing returns to scale.

C Proof of Proposition 4

From the first-order conditions, we obtain $u'(c_t) = \delta u'(c_{t+1})[(1-\alpha)A_{t+1}h_{t+1}^{\alpha}l_{t+1}^{\alpha}k_{t+1}^{-\alpha} + 1 - \delta_k]$, $e'(h_t)l_t = u'(c_t)\alpha A_t h_t^{\alpha-1}l_t^{\alpha}k_t^{1-\alpha}$, and $g'(1-l_t) = e'(h_t)h_t - e(h_t) - b$. Thus, a steady state is given by a set of $\{h, l, K\}$ that satisfy

$$(1-\alpha)AK^{-\alpha} = r + \delta_k, \tag{45}$$

$$\alpha AK^{1-\alpha} = e'(h), \tag{46}$$

$$g'(1-l) = e'(h)h - e(h) - b, \tag{47}$$

where we have imposed $u'(c) = 1$ to be consistent with our model, in which there is no consumption smoothing motive. It is interesting to observe that, (30) and (31) and coincide with (45) and (46) when $\beta = 0$. With $\beta > 0$, the capital-labor ratio (K) of the economy with search frictions and wage bargaining is less than the one without, and hours of work per employee (h) for the economy with search frictions and wage bargaining is longer than the one without.

To facilitate comparison of the levels of employment for the two economies, we assume g to be log-linear in non-employment: $g(x) = e_1(\ln x - x)$, where e_1 is a positive constant. Since

$g'(x) = e_1(1-x)/x$, (47) reduces to

$$e_1 \frac{l}{1-l} = e'(h)\,h - e(h) - b. \tag{48}$$

This determines the relationship between h and l for the frictionless economy. In what follows we let $\beta = 0$. Eliminate θ from (32) and (33) to obtain

$$\frac{r+\lambda}{m_0}\gamma\left(\frac{\lambda}{m_0}\frac{l}{1-l}\right)^{\frac{\xi}{1-\xi}} = e'(h)\,h - e(h) - b,$$

which reduces to

$$\frac{r+\lambda}{m_0}\gamma\frac{\lambda}{m_0}\frac{l}{1-l} = e'(h)\,h - e(h) - b \tag{49}$$

when $\xi = 1/2$. It is now clear that the steady-state levels of employment l for the two economies coincide if

$$e_1 = \frac{r+\lambda}{m_0}\gamma\frac{\lambda}{m_0}.$$

(48) and (49) imply that if search frictions are sufficiently severe, then the level of employment in the economy with search frictions is below the one without frictions.

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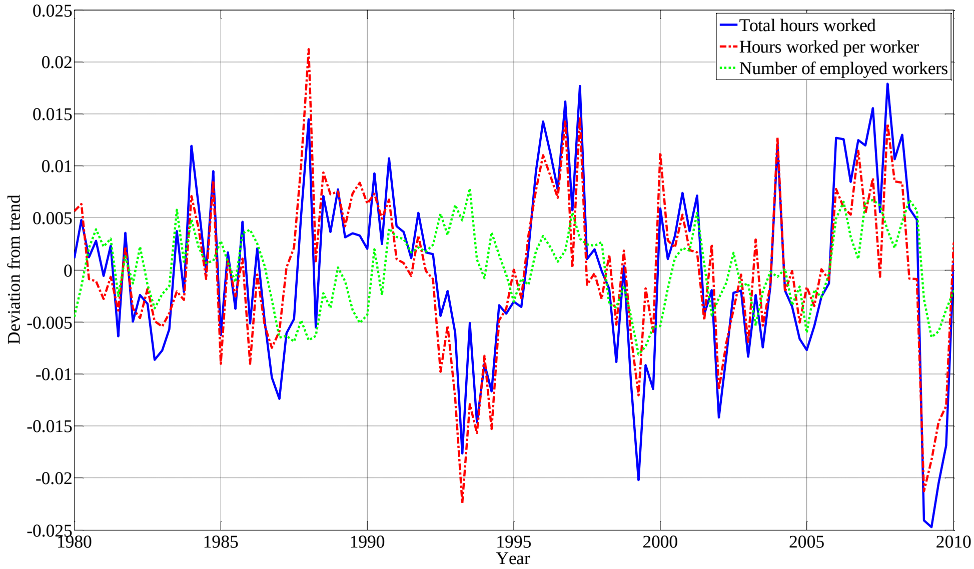

Figure 1: Total hours worked and its components over business cycles in Japan

Note: The solid line indicates the cyclical component of total hours worked. The dash-dotted line indicates the cyclical component of hours worked per worker. The dashed line indicates the cyclical components of employed workers. The cyclical components are obtained by using the HP filter with smoothing parameter 1600. Sample covers 1980Q2-2010Q4.

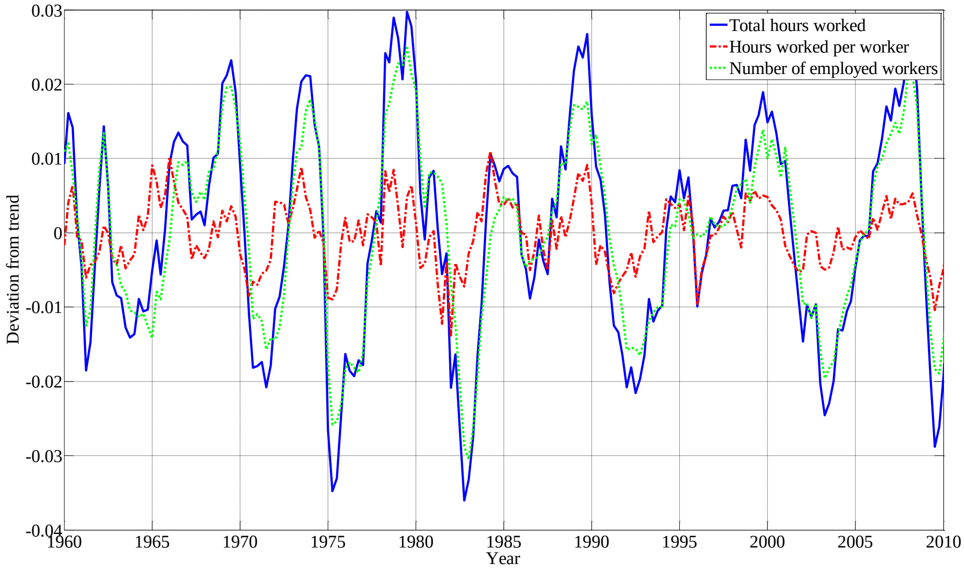

Figure 2: Total hours worked and its components over business cycles in the U.S.

Note: The solid line indicates the cyclical component of total hours worked. The dash-dotted line indicates the cyclical component of hours worked per worker. The dashed line indicates the cyclical components of employed workers. The cyclical components are obtained by using the HP filter with smoothing parameter 1600. Sample covers 1960Q1-2010Q4.

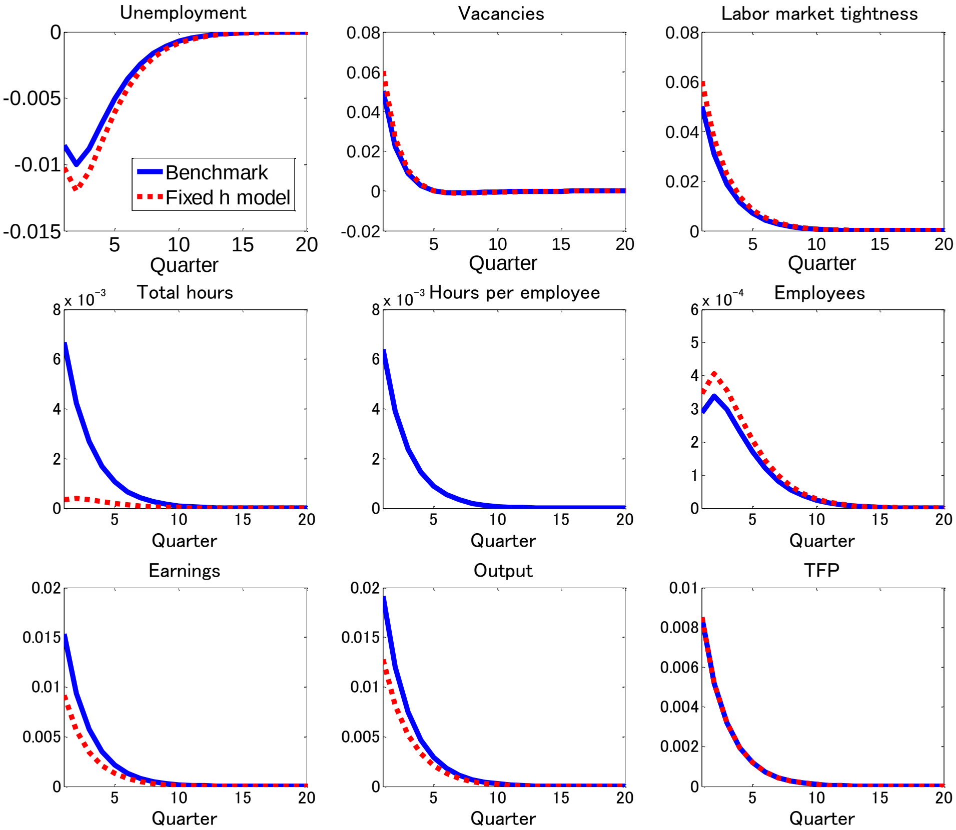

Figure 3: Impulse responses to a positive shock to TFP