# Incomplete Deposit Contracts, Banking Crises, and Monetary Policy*

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**Abstract**

Money plays important roles in modern financial systems. This study develops a banking model comprising monetary factors in order to investigate the relationship between money and banking crises. In the model, it is assumed that banking deposit contracts are not contingent on the state of nature. We show that under incomplete contracts, a banking crisis may occur when inflation is sufficiently low or high. The result appears to be consistent with empirical evidence. We also show that the zero-inflation policy can be optimal under either complete or incomplete banking contracts, despite the Friedman rule eliminating the possibility of crises.

## 1 Introduction

Banking crises have been cruel phenomena in many countries over many historical periods. The recent worldwide banking crisis in 2007–09

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was a stark reminder of the importance of liquidity, incomplete financial contracts, and macroeconomic policies.

In modern financial systems, money is crucial to liquidity, storage, transactions, payments, and so on. Since the central bank can create money without any costs and can control the inflation rate in the long run, monetary policies affect the portfolio choice of financial institutions, which in turn determine the likelihood that the entire banking system will run out of liquidity and experience a crisis. In addition, recent empirical studies show that banking crises and inflation are positively correlated (Demirguc-Kunt and Detragiache, 1998, 2005; Hardy and Pazarbasioglu, 1998; Boyd et al., 2002), which implies that inflation is arguably the main factor that helps to predict a banking crisis. Despite the apparent importance of monetary factors in financial systems, prior studies on monetary and banking crisis models have treated these topics in isolation, making it difficult to see the relationship between the two. In this study, we develop a banking model comprising monetary factors to examine how much money banks set aside for the future during a crisis, why banking crises tend to occur in high inflationary economies, and what the optimal rate of inflation is, given the importance of financial intermediaries.

More specifically, we extend the two-period-lived overlapping generation model of Champ et al. (1996) and Smith (2002), which creates an endogenous transaction role for money based on two factors: spatial separation and limited communication. First, spatial separation allows us to assume a set-up in which the agents in the economy are born in one of two symmetric islands, and a random fraction of agents on one island is relocated to the other at the end of each period. Furthermore, the stochastic relocations act as shocks to agents' liquidity preferences, which creates a role for banks to provide insurance against these shocks, as in Diamond and Dybvig (1983). Next, because of limited communication, we can assume that relocated agents must use fiat money because the privately issued liabilities are not accepted in the new location. Instead, the non-relocated agents can transact using checks or other credit instruments. This assumption allows money to be held even when dominated in the rate of return.

This study focuses on incomplete deposit contracts that result banking defaults, based on Allen and Gale (1998, 2004), because standard deposit contracts cannot be made contingent on any shocks in practice. As an alternative, Champ et al. (1996) and Smith (2002) consider only a situation in which banks offer complete deposit contracts to depositors. That is, the incentive constraint of non-relocated agents is always satisfied, so banks never default. This difference of the deposit contracts in each model leads to the difference of definitions of a banking crisis. Champ et al. (1996) and Smith (2002) regard a banking crisis as the case in which banks exhaust their cash reserves, liquidate their physical investments, or both. In this paper, we define a banking crisis as the case in which depositors withdraw their funds early to satisfy future consumption, which then leads to bank defaults.

Under incomplete contracts, banks must promise fixed repayments to early withdrawers independently of the state of nature. In this case, the banks must choose one of two types of contract: a default-preventing contract and a contract with defaults. Under the default-preventing contract, all banks hold enough reserves to self-insure themselves against liquidity shocks and promise low returns, which in turn generates solvency. Under the contract with defaults, banks find it optimal to reduce their reserve holdings and default with positive probability because avoiding default is costly. The banks compare the two types of contracts and choose one for depositors. We show that the latter contract may be adopted when the inflation rate is sufficiently low or high.

We also examine an optimal monetary policy. In the model with complete banking contracts, Smith (2002) shows that the Friedman rule (zero nominal interest rate) eliminates banking crises completely, but is never optimal. However, he does not specify an optimal rate of money growth. In contrast, we show that the zero-inflationary policy (fixed money supply) maximizes the steady-state welfare when banks can offer complete deposit contracts to depositors. In addition, we show that the zero-inflationary policy can be also optimal even when banks are forced to offer incomplete (non-contingent) deposit contracts. Our results imply the sub-optimality of the Friedman rule, which eliminates any types of crises under incomplete contracts. When the Friedman rule is implemented, the banks can offer the perfect insurance to agents and the risk-sharing term attains maximum. However, the level of intergenerational transfer is very low, which implies that the banks invest less in storage technology, and the level of consumption is also low. The zero-inflationary policy must balance these two opposing forces.

## 1.1 Related Literature

Our work is related to a number of important studies in banking literature. Bryant (1980) and Diamond and Dybvig (1983) are seminal studies that provide the first models of banking crises. Both papers have consumers with random liquidity demands and show that deposit contracts allowed this risk to be insured. Since then, banking models have been developed using two different approaches. The first is crises based on panics. The second is crises based on poor fundamentals arising from the business cycle. The panic-based approach is taken by Diamond and Dybvig (1983), Wallace (1988), Cooper and Ross (1998), Peck and Shell (2003), and Ennis and Keister (2009, 2010), among others, while the fundamental-based approach is taken by Bryant (1980), Chari and Jagannathan (1988), Champ et al. (1996), and Allen and Gale (1998, 2004), among others. There is a long-standing debate about the underlying causes of financial crises and how these events are best captured in economic models. While the current paper belongs to the second category of crises, most of the abovementioned studies do not focus on monetary factors.

An important contribution to the literature on money and banking crises is that of Champ et al. (1996). They construct overlapping generation economies where spatial separation and limited communication create a transaction role for money, and random liquidity shocks create a role for banks. They show that there will be a banking crisis if the shock is large enough to exhaust the banks' cash reserves, which is supported in empirical evidence from Canada and United States for the period 1880-1910. Smith (2002) considers a similar framework and shows that in an economy with high inflation, banks have an incentive to minimize cash reserves ex ante, which in turn increases banking fragility ex post. He also shows that the Friedman rule is suboptimal, despite the policy eliminating crises. The current paper develops this framework by considering incomplete contracts.

Other contributions on money and banking crises include Chang and Velasco (2000), Diamond and Rajan (2006), Skeie (2008), and Allen et al. (2014). Chang and Velasco (2000) introduce money as an argument in the utility function of the Diamond and Dybvig model, and develop a model of currency and banking crises. They show that a flexible exchange rate system achieves the social optimum if monetary policies are designed appropriately. Diamond and Rajan (2006) develop a banking model in which demand deposits are repayable in money. They show that price adjustments improve risk sharing because they introduce a form of state contingency to contracts, but that variations in the transaction value of money can lead to a banking crisis. Allen and Gale (1998) construct a banking model in which crises are caused by weak fundamentals and show that nominal contracts and an injection by the central bank achieve an incentive-efficient allocation. Allen et al. (2014) develop a variant of the Allen and Gale (1998) model, and show that with incomplete nominal deposit contracts, a decentralized equilibrium allocation can be efficient if the central bank accommodates the demands of the private sector for fiat money. Skeie (2008) extends the Diamond and Dybvig (1983) model and studies the effects of nominal contracts and monetary policy on interbank markets and banking fragility. However, these three-period models do not consider the effects of long-run inflation on financial systems.[1]

This study also bears a theoretical similarity to the work of Antinolfi et al. (2001), Boyd et al. (2004), Antinolfi and Keister (2006), and Matsuoka (2012).[2] Antinolfi et al. (2001) study the relationship be-

[1]Recently, banking models have been developed using a search-theoretic approach of money. See Berentsen et al. (2007), Williamson (2002), and Gu et al. (2013).

[2]The overlapping generations model with spatial separation and limited communication has become a workhorse for many areas of macroeconomics, including the analysis of business cycles, economic growth, financial development, and monetary policy. See, for example, Schreft and Smith (1997, 1998, 2000), Gomis-Porqueras and Smith (2003), Bhattacharya et al. (2009), Haslag and Martin (2007), Bhattacharya

tween various policies of the lender of last resort (LLR) and inflationary equilibria in a pure-exchange economy. They find that an LLR policy in which the central bank freely lends money at a zero nominal interest rate generates the Pareto optimal steady-state equilibrium and non-optimal inflationary equilibria. Antinolfi and Keister (2006) study a LLR policy and a monetary policy in a similar environment, and find that the policy combination achieves the market equilibrium that closely approximates the first-best allocation of resources. Their LLR policy plays a key role in mitigating communication friction, which generates a transaction role for money. Boyd et al. (2004) compare the situation with competitive versus monopolistic banking systems. They show that a monopolistic banking system faces a higher probability of banking crises when the inflation rate is below some threshold, while a competitive system is more fragile otherwise. Matsuoka (2012) considers a situation in which interbank markets are imperfect because of limited commitment, and shows that a proper combination of central bank loans and a monetary policy restore the constrained efficiency. Those analyses, however, consider only a situation in which banks can offer complete deposit contracts and do not go bankrupt. In this study, we focus on incomplete (non-state-contingent) deposit contracts, which are the most significant source of banking crises.

The remainder of the paper proceeds as follows. The next section lays out the basic elements of the model. Section 3 describes the equilibrium with complete banking contracts, based on Smith (2002). Section 4 describes the equilibrium with incomplete banking contracts. Section 5 illustrates equilibria with numerical examples. Finally, Section 6 concludes the paper.

## 2 The Environment

Periods are represented by $t = 0, 1, 2, \dots$. The world is divided into two spatially separated-symmetric locations, and each location is populated by an infinite sequence of two-period-lived overlapping generations.

and Singh (2008a, b), Ghossoub and Reed (2010), and Ghossoub (2012).

For simplicity, we consider a single-consumption-good economy with no population growth.

Young agents are ex ante identical and of unit mass. They are endowed with $w > 0$ units of goods when young, and none when old. They consume only when old and have the logarithmic utility function, $u(c_t) = \ln(c_t)$, where $c_t$ denotes the old-age consumption of agents born at period $t$.[3]

There are two types of assets: storage investment and money. One unit of the good invested in the storage investment at period $t$ yields $R > 1$ units of the good at period $t + 1$. The storage investments can be "scrapped." We assume that one unit of the storage investments scrapped at period $t$ yields $r < 1$ units of the good at the end of period $t$, where $1 - r$ represents a liquidation cost. One unit of the good invested in money at period $t$ yields $p_t/p_{t+1}$ units of the good at period $t + 1$, where $p_t$ denotes the price level at period $t$.

As in Townsend (1987), we introduce a transaction role for money by assuming that the two locations are spatially separated and that the communication between them is limited. The limited communication prevents privately issued liabilities from being verifiable in the other location. However, money is universally recognizable and noncounterfeitable, thus is accepted in both locations. In addition, during each period, agents can trade and communicate only with others in the same location.

At the beginning of period $t$, each young agent makes a bank deposit. After their deposits have been allocated between investments and money, a fraction $\pi_t$ of the agents in each location is relocated to the other location. These agents are called "movers." As in the Diamond and Dybvig model, relocation plays the role of a "liquidity preference shock," and it would be natural to assume that banks arise endogenously to insure agents against these shocks. It is also assumed that an agent's type is private information so that banks cannot tell whether the person withdrawing is a mover or a non-mover at the end of period $t$. The relocation probability $\pi_t$ is a random variable, which represents not only

[3] As in Champ et al. (1996) and others, this assumption of logarithmic utility allows us to solve the banks' problem analytically.

the fraction of all movers under a continuum of young agents, but also the aggregate liquidity shock. That is, a higher value of $\pi_t$ corresponds to a higher ratio of movers and demand for money. This is publicly observable, and identically distributed over time. Let $F$ represent the distribution function, which is assumed to be smooth and strictly increasing on $[0, 1]$, and $f$ the associated density function. The distribution $F$ is common knowledge.

Let $M_t$ denote the per capita stock of money outstanding at period $t$, where the initial money supply, $M_0$, is exogenously given and held by the old at the initial period 0. The government sets the money growth rate, $\sigma$, once and for all in the initial period under the following government constraint:

$$\tau_t = \frac{\sigma - 1}{\sigma} m_t, \tag{1}$$

where $m_t$ is the real money balance. The variable $\tau_t$ shows the real value of the lump-sum transfer (tax) under $\sigma < (>)1$. That is, if $\sigma < (>)1$, the monetary injections (withdrawls) are accomplished via the lump-sum transfer to (tax on) young agents.

# 3 Complete Banking Contracts

In this section, we consider the benchmark case in the sense that banks can offer complete deposit contracts in which the amount that can be withdrawn at each date is contingent on $\pi_t$. The model described here is based on Smith (2002).

As in Diamond and Dybvig (1983), the savings of all young agents will be intermediated. At period $t$, banks take deposits $w + \tau_t$ from young agents, and choose how much to invest in storage, $i_t$, and money balances, $m_t$. Movers must be given money or liquidated storage investments. The deposit contract can be represented by a pair of functions, $d_t^m(\pi_t)$ and $d_t^n(\pi_t)$, where the $m$ and $n$ represent movers and non-movers, giving the return of movers and non-movers conditional on $\pi_t$.

As is standard, we assume that the deposit market is competitive. Thus, banks behave as Nash competitors and announce return schedules $(d_t^m(\pi_t),\ d_t^n(\pi_t))$, taking the announced return schedules of other banks as given. Let $\alpha_t$ and $\beta_t$ denote the fraction of cash reserves bank pays out to movers and the fraction of storage investments liquidated at the end of period $t$, respectively. The bank's optimization problem is to maximize the expected utility of a representative agent:

$$\int_0^1 \big\{\pi_t \ln(d_t^m(\pi_t)(w+\tau_t)) + (1-\pi_t)\ln(d_t^n(\pi_t)(w+\tau_t))\big\} f(\pi_t)d\pi_t, \quad (2)$$

subject to the following constraints:

$$i_t + m_t = w + \tau_t, \quad (3)$$

$$\pi_t d_t^m(\pi_t)(w+\tau_t) = \alpha_t(\pi_t)\frac{p_t}{p_{t+1}}m_t + \beta_t(\pi_t) r i_t, \quad (4)$$

$$(1-\pi_t)d_t^n(\pi_t)(w+\tau_t) = (1-\alpha_t(\pi_t))\frac{p_t}{p_{t+1}}m_t + (1-\beta_t(\pi_t))R i_t, \quad (5)$$

$$d_t^m(\pi_t) \le d_t^n(\pi_t), \quad (6)$$

as well as $0 \le \alpha_t(\pi_t) \le 1$, $0 \le \beta_t(\pi_t) \le 1$, and the usual non-negativity constraints. Constraint (3) is the bank's balance sheet constraint. Constraint (4) states that movers must be given either money or the proceeds of liquidated investments. Constraint (5) states that the real payments to non-movers are equal to the value of the bank's remaining cash reserves plus the income from the non-liquidated investments. Constraint (6) is the incentive constraint that non-movers have no incentive to misrepresent their preferences. As seen below, the incentive constraint is always satisfied under complete contracts.

Defining $\gamma_t \equiv m_t/(w+\tau_t)$ as a reserve-deposit ratio, we can rewrite constraints (4) and (5) as:

$$\pi_t d_t^m(\pi_t) = \alpha_t(\pi_t)\frac{p_t}{p_{t+1}}\gamma_t + \beta_t(\pi_t)(1-\gamma_t)r,$$

$$(1-\pi_t)d_t^n(\pi_t) = (1-\alpha_t(\pi_t))\frac{p_t}{p_{t+1}}\gamma_t + (1-\beta_t(\pi_t))(1-\gamma_t)R.$$

Both $\alpha_t$ and $\beta_t$ are chosen after the realization of $\pi_t$, while $\gamma_t$ is chosen before the realization of $\pi_t$. Hence, the optimal values of $\alpha_t$ and $\beta_t$ can be chosen as the functions of $\gamma_t$ and $\pi_t$, as follows:

$$\max_{\alpha_t,\beta_t\in[0,1]} \pi_t \ln\left[\frac{\alpha_t\gamma_t}{\pi_t}\frac{p_t}{p_{t+1}} + \frac{\beta_t}{\pi}(1-\gamma_t)r\right] + (1-\pi_t)\ln\left[\frac{(1-\alpha_t)\gamma_t}{1-\pi_t}\frac{p_t}{p_{t+1}} + \frac{(1-\beta_t)(1-\gamma_t)}{1-\pi}R\right].$$

The solutions to the problem are:

$$\alpha_t(\pi_t) = \begin{cases} \frac{\pi_t}{\pi_t^*} & \text{if} \quad 0 \le \pi_t \le \pi_t^*, \\ 1 & \text{if} \quad \pi_t^* < \pi_t \le 1, \end{cases} \tag{7}$$

$$\beta_t(\pi_t) = \begin{cases} 0 & \text{if} \quad 0 \le \pi_t \le \pi_t^{**}, \\ \pi_t - \frac{R(1-\pi_t)}{rI_t}\frac{\gamma_t}{1-\gamma_t} & \text{if} \quad \pi_t^{**} < \pi_t \le 1, \end{cases} \tag{8}$$

where

$$I_t \equiv R\frac{p_{t+1}}{p_t}, \quad \pi_t^* \equiv \frac{\gamma_t}{\gamma_t + (1-\gamma_t)I_t}, \quad \text{and} \quad \pi_t^{**} \equiv \frac{\gamma_t}{\gamma_t + (1-\gamma_t)\frac{r}{R}I_t}.$$

Note that $\pi_t^* < \pi_t^{**}$ for any $I_t$. When the demand for liquidity is below the critical value, $\pi_t^*$, banks are able to meet the demand using their own cash reserves. Under such circumstances, it is optimal for them to pay out their remaining reserves to non-movers and not to liquidate their storage investments. When a liquidity shock between $\pi_t^*$ and $\pi_t^{**}$ occurs, all cash reserves of banks are paid out to movers, but they do not liquidate their investments because the opportunity cost of providing better insurance against the liquidity shock is sufficiently high. In this situation, the bank faces a "liquidity crisis." When the demand for liquidity is greater than $\pi_t^{**}$, banks pay out all their reserves to the movers and liquidate some of their investments. In this case, the benefit of providing better insurance outweighs its cost, indicating that the bank faces a "costly liquidity crisis."

Then, the returns to movers and non-movers can be represented by the

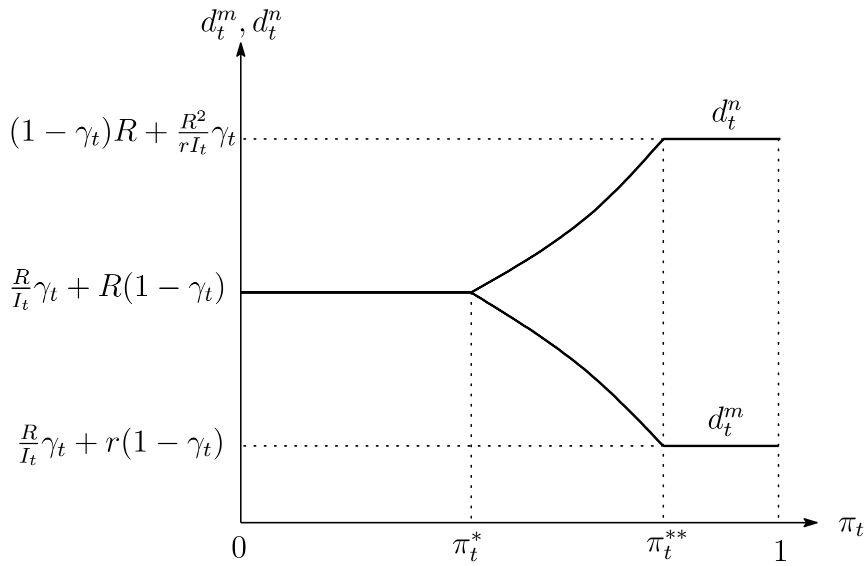

Figure 1: The deposit returns under complete contracts

following functions:

$$d_t^m(\pi_t) = \begin{cases} \frac{R}{I_t}\gamma_t + R(1-\gamma_t) & \text{if} \quad 0 \le \pi_t < \pi_t^*, \\ \frac{R}{I_t}\frac{\gamma_t}{\pi_t} & \text{if} \quad \pi_t^* \le \pi_t < \pi_t^{**}, \\ \frac{R}{I_t}\gamma_t + (1-\gamma_t)r & \text{if} \quad \pi_t^{**} \le \pi_t \le 1, \end{cases} \tag{9}$$

$$d_t^n(\pi_t) = \begin{cases} \frac{R}{I_t}\gamma_t + R(1-\gamma_t) & \text{if} \quad 0 \le \pi_t < \pi_t^*, \\ \frac{R(1-\gamma_t)}{1-\pi_t} & \text{if} \quad \pi_t^* \le \pi_t < \pi_t^{**}, \\ (1-\gamma_t)R + \frac{R^2}{rI_t}\gamma_t & \text{if} \quad \pi_t^{**} \le \pi_t \le 1. \end{cases} \tag{10}$$

Figure 1 illustrates the relationship between the deposit returns, $d_t^m(\pi_t)$ and $d_t^n(\pi_t)$, and $\pi_t$ given in (9) and (10), holding $R$ and $I_t$ constant. Note that for $\pi_t \in [0, \pi_t^*)$, money provides more consumption than is needed by the movers, and both returns are equalized. For $\pi_t \in [\pi_t^*, \pi_t^{**})$, investment liquidation does not occur, and the difference between the two returns is increasing in $\pi_t$. For $\pi_t \in [\pi_t^{**}, 1]$, banks liquidate their investments to prevent the gap from widening further. Note also that the incentive constraint (6) is always satisfied.

We now proceed to solve the optimal value of $\gamma_t$. Substituting the optimal values of $\alpha_t$ and $\beta_t$ into the banks' objective function, we obtain the optimal problem:

$$\max_{\gamma_t \in [0,1]} \int_0^1 \left\{ \pi_t \ln(d_t^m(\pi_t)) + (1-\pi_t) \ln(d_t^n(\pi_t)) \right\} f(\pi_t) d\pi_t, \tag{11}$$

subject to (9) and (10). The bank's objective function is a function of $\gamma_t$ and $I_t$. Inserting (9) and (10) into (11), the objectives can be rewritten as:

$$\begin{aligned} X(\gamma_t, I_t) \equiv & \max_{\gamma_t \in [0,1]} \int_0^{\pi_t^*} \ln\left( \frac{R}{I_t}\gamma_t + R(1-\gamma_t) \right) f(\pi_t) d\pi_t \\ & + \int_{\pi_t^*}^{\pi_t^{**}} \left\{ \pi_t \ln\left( \frac{R}{I_t}\frac{\gamma_t}{\pi_t} \right) + (1-\pi_t) \ln\left( \frac{R(1-\gamma_t)}{1-\pi_t} \right) \right\} f(\pi_t) d\pi_t \\ & + \int_{\pi_t^{**}}^1 \left\{ \pi_t \ln\left( \frac{R}{I_t}\gamma_t + (1-\gamma_t) r \right) + (1-\pi_t) \ln\left( (1-\gamma_t)R + \frac{R^2}{rI_t}\gamma_t \right) \right\} f(\pi_t) d\pi_t. \end{aligned}$$

The optimal value of $\gamma(I_t)$, which is defined as a function of $I_t$, is the solution to this problem and implicitly defined by:

$$(1-\gamma_t)\left(1 - \frac{r}{R}I_t\right)\pi_t^{**} = \int_{\pi_t^*}^{\pi_t^{**}} F(\pi_t) d\pi_t. \tag{12}$$

The optimal value of $\gamma(I_t)$ results from the trade-off between the two forces. First, since the return on cash is lower than on storage investments (i.e., $I_t \geq 1$), banks prefer to minimize cash reserves. At the same time, since they strive to provide insurance by equalizing the returns between movers and non-movers for all realizations of $\pi_t$, they must hold sufficient cash reserves. At the margin, the welfare gains from risk-sharing exactly offset the cost implied by the return dominance of storage investments over cash reserves.

As established in Smith (2002, Proposition 3), the optimal reserve-deposit ratio satisfies the following properties:

**Lemma 1** *(i)* $\gamma(1) = 1$*, (ii)* $\gamma(R/r) = 0$*, (iii)* $\gamma'(I_t) < 0$.

The proof of this is omitted. An increase in $I_t$ makes money costly, which gives an incentive for banks to hold less reserves. When $I_t = 1$, which is the Friedman rule, the return rates of both assets are equalized, and banks invest their entire deposits in cash reserves because cash has the advantage of insuring against the liquidity shock.

An equilibrium consists of sequences for prices $\{I_t\}$ and the decision rules of banks $\{\gamma_t, \alpha_t, \beta_t\}$, such that (i) given $\{I_t\}$, the decision rules solve the banks' problems in each period; (ii) the market clears when money is traded for goods at the beginning of each period; and (iii) the government budget constraint in equation (1) holds in each period.

The money market clears if

$$m_t = \gamma(I_t)(w + \tau_t). \tag{13}$$

Substituting the government budget constraint (1) into (13) yields

$$m_t = \frac{\gamma(I_t)w}{1 - \frac{\sigma-1}{\sigma}\gamma(I_t)}. \tag{14}$$

Combining (13) and (14), we obtain the after-tax/transfer income of young agents at $t$:

$$Y(I_t) \equiv w + \tau_t = \frac{w}{1 - \frac{\sigma-1}{\sigma}\gamma(I_t)}. \tag{15}$$

By definition, $I_t = Rp_{t+1}/p_t = \sigma R m_t/m_{t+1}$. Inserting (14) into this definition yields

$$I_t = \sigma R \frac{\gamma(I_t)}{\gamma(I_{t+1})} \frac{1 - \frac{\sigma-1}{\sigma}\gamma(I_{t+1})}{1 - \frac{\sigma-1}{\sigma}\gamma(I_t)}, \tag{16}$$

which describes the equilibrium evolution of the gross nominal interest rate, $\{I_t\}_{t=0}^{\infty}$.

We proceed to study the stationary behavior of the economy. From (16), the steady-state (gross) nominal interest rate is given by:

$$I = \sigma R. \tag{17}$$

Note that the money growth rate, $\sigma$, is equal to the steady-state inflation rate.

Let us consider the relationship between inflation and the probability of a liquidity crisis in the steady state. Using simple calculations, we find that both cut-off values, $\pi^*$ and $\pi^{**}$, are decreasing in $I$, implying that inflation increases the probability of a liquidity crisis. Higher inflation, which means higher opportunity costs of holding money, induces banks to economize, which in turn means that banks more often exhaust money, scrap productive investments, or both.

In the steady state, the expected utility of a representative young agent (2) is only defined as a function of $I$, as follows:[4]

$$W(I) \equiv \ln Y(I) + X[\gamma(I), I], \tag{18}$$

which states that the effects of the nominal interest rate (or equivalently, the inflation rate) on the steady-state welfare consist of two components. The first component represents an intergenerational transfer. Since any seigniorage collected is rebated to the young, inflation leads to a transfer of goods from the old money holders to the young agents. The second component represents risk-sharing between movers and non-movers. Since inflation decreases the rate of return on money and increases the probability of crises, it produces inequality between movers and non-movers.

The following proposition provides the optimal monetary policy that maximizes the steady-state welfare.[5]

**Proposition 1** *In an economy with complete banking contracts, the optimal monetary policy is to set the net money growth rate equal to zero (i.e., $I = R$).*

For the proof, see the Appendix. When the Friedman rule (i.e., $I = 1$) is implemented, the banks can offer the perfect insurance to agents, which implies that the liquidity crisis is completely eliminated and the risk-sharing term $X[\gamma(1), 1]$ attains a maximum. On the other hand, the level of the after-tax/transfer income, $Y(1)$, is very low, which implies that the banks invest less in storage technology and the level of con-

[4] We use the terms expected utility and welfare interchangeably.

[5] Smith (2002) shows that the Friedman rule (i.e., $I = 1$) is suboptimal in this economy with random liquidity shocks; however, he does not provide the optimal rate of inflation.

sumption is low. The zero inflation policy must balance the two effects.[6] Bhattacharya and Singh (2008a) show that the equilibrium allocation under $I = 1$ is identical to the constrained-efficient allocation in this environment.

In this section, we deliberately avoid using terminology such as "banking crisis" and instead use "liquidity crisis." This is because the incentive constraint (6) is always satisfied and the non-movers withdraw their funds when they are old. That is, the banks never default, which does not seem to capture some features of the reality observed in 2008. In the next section, we will consider a situation in which incomplete banking contracts may create a banking crisis.[7]

# 4 Incomplete Banking Contracts

In the benchmark model described in the previous section, banks can offer state-contingent deposit contracts. However, we do not observe such complex contracts in reality. In this section, we focus on incomplete banking contracts. That is, we assume that banks are forced to offer non-contingent deposit contracts to agents, as in Allen and Gale (1998, 2004a, b), where this type of contract can be justified by transaction costs, asymmetric information, and the nature of the legal system.

Under incomplete contracts, the return to movers, $d_t^m$, does not depend on the value of $\pi_t$. However, the return to non-movers, $d_t^n$, does depend on the value of $\pi_t$ because non-movers get whatever assets are left over. More specifically, without loss of generality, by setting $d_t^n = \infty$, we ensure that non-movers receive the residue of the bank's assets at next period. Thus, the deposit contract is characterized by the return to movers, $d_t^m \equiv d_t$ and the portfolio choice.

[6] Bhattacharya and Singh (2008b) consider a setting in which liquidity shocks are realized *before* the bank has chosen its portfolio and show that a negative net money growth rate is optimal.

[7] In existing literature, including Champ et al. (1996), Smith (2002), among others, a "banking crisis" is defined as a situation in which withdrawal demand is greater than total cash reserves and depositors in need of cash suffer consumption losses. In this study, such a situation is said to be a "liquidity crisis" rather than a "banking crisis."

As in the previous section, the bank faces the following constraints on the choices $i_t$, $m_t$, and $d_t$:

$$\pi_t d_t = \alpha_t \frac{p_t}{p_{t+1}} \gamma_t + \beta_t r(1-\gamma_t), \tag{19}$$

$$(1-\pi_t) d_t^n(\pi_t) = (1-\alpha_t) \frac{p_t}{p_{t+1}} \gamma_t + (1-\beta_t) R(1-\gamma_t), \tag{20}$$

$$d_t \le d_t^n(\pi_t). \tag{21}$$

The main difference between complete and incomplete contracts is that the return to movers, $d_t$, is fixed and chosen before the realization of $\pi_t$ because of the incompleteness of contracts. Since non-movers receive the residue of the bank's assets at $t+1$, claiming of these assets has to absorb the shocks to the movers.

To categorize the economy into three states according to the realization of $\pi_t$, we give the cut-off values of liquidity shock as follows:[8]

$$\tilde{\pi}_t^* = \frac{R}{I_t} \frac{\gamma_t}{d_t}, \quad \tilde{\pi}_t^{**} = \frac{Rr(1-\gamma_t) + \frac{R^2}{I_t}\gamma_t - r d_t}{(R-r) d_t},$$

where we use $\gamma_t = m_t/(w+\tau_t)$ and $I_t = R p_{t+1}/p_t$.

When $\pi_t \in [0, \tilde{\pi}_t^*)$, we say the economy is in the normal state. In the normal state, the bank can meet the cash demands of movers using only its reserves. Next, when $\pi_t \in [\tilde{\pi}_t^*, \tilde{\pi}_t^{**})$, which is classified as the liquidation state, the bank can satisfy the demands of movers only by liquidating some storage investment without violating the incentive constraint (21). In this case, the bank is said to be *illiquid*, but *solvent.* Finally, under $\pi_t \in [\tilde{\pi}_t^{**}, 1]$, the economy can be classified as being in a crisis state, which means that the bank cannot satisfy the demands of movers, even by liquidating all its assets, and hence defaults. In this case, the bank is *insolvent.* We discuss these three states in detail below.

Let us first consider the normal state. In this case, the realization of $\pi_t$ is below the critical value $\tilde{\pi}_t^*$, and banks can pay out only a fraction of its reserves to the movers. That is, the banks will not exhaust their

[8] We assume $\tilde{\pi}_t^* \le \tilde{\pi}_t^{**}$, which can be reduced to $d_t \le R(1-\gamma_t) + R\gamma_t/I_t$. This assumption ensures that banks will always find it optimal to liquidate the storage investments last. We will check later whether the assumption holds in equilibrium.

cash reserves and will not liquidate their investments (i.e., $\alpha_t < 1$ and $\beta_t = 0$). Since the movers receive the fixed return $d_t$, the remaining reserves ($\frac{R}{I_t}\gamma_t - \pi_t d_t$) go to the non-movers. The return schedule of movers and non-movers is given by:

$$(x_t^m(\pi_t), x_t^n(\pi_t)) = \left( d_t, \; \frac{R(1-\gamma_t) + \frac{R}{I_t}\gamma_t - \pi_t d_t}{1-\pi_t} \right), \tag{22}$$

where $x_t^m(\pi_t)$ and $x_t^n(\pi_t)$ denote the return to movers and non-movers, respectively, in the realization of $\pi_t$.

Next, consider the liquidation state in which banks liquidate part of their storage investments. Then, all cash reserves of banks are paid out to the movers and some investments are liquidated to meet the liquidity demands of movers, where the incentive constraint (21) is still satisfied (i.e., $\alpha_t = 1$ and $\beta_t > 0$). In this case, the banks are in "liquidity crises," but remain solvent. From equation (19), the proportion of the liquidated investments is

$$\beta_t = \frac{\pi_t d_t - \frac{R}{I_t}\gamma_t}{r(1-\gamma_t)}.$$

By substituting this into equation (20), we obtain the return to the non-movers. Then the return schedule is given by:

$$(x_t^m(\pi_t), x_t^n(\pi_t)) = \left( d_t, \; \frac{R(1-\gamma_t) - \frac{R}{r}(\pi_t d_t - \frac{R}{I_t}\gamma_t)}{1-\pi_t} \right). \tag{23}$$

Finally, we consider the crisis state in which all non-movers withdraw their funds at the end of the first period. In this case, banks exhaust their cash reserves and liquidate much of their storage investments and the incentive constraint is violated. That is, the non-movers misrepresent their preferences, announce that they are movers, and withdraw their funds from their banks. As a result, banks go bankrupt. In this case, the banks distribute their available resources to their depositors equally. The return schedule is given by

$$(x_t^m(\pi_t), x_t^n(\pi_t)) = \left( \frac{R}{I_t}\gamma_t + r(1-\gamma_t), \; \frac{R}{I_t}\gamma_t + r(1-\gamma_t) \right). \tag{24}$$

The deposit contract is characterized by the pair $(d_t, \gamma_t)$. Then we have the bank's problem as follows:

$$\max_{d_t,\gamma_t} \int_0^1 \left\{\pi_t \ln(x_t^m(\pi_t)) + (1-\pi_t)\ln(x_t^n(\pi_t))\right\} f(\pi_t) d\pi_t,$$

where

$$x_t^m(\pi_t) = \begin{cases} d_t & \text{if } 0 \le \pi_t < \tilde{\pi}_t^{**}, \\ \frac{R}{I_t}\gamma_t + r(1-\gamma_t) & \text{if } \tilde{\pi}_t^{**} \le \pi_t \le 1, \end{cases} \tag{25}$$

$$x_t^n(\pi_t) = \begin{cases} \frac{R(1-\gamma_t)+\frac{R}{I_t}\gamma_t - \pi_t d_t}{1-\pi_t} & \text{if } 0 \le \pi_t < \tilde{\pi}_t^{*}, \\ \frac{R(1-\gamma_t)-\frac{R}{r}(\pi_t d_t - \frac{R}{I_t}\gamma_t)}{1-\pi_t} & \text{if } \tilde{\pi}_t^{*} \le \pi_t < \tilde{\pi}_t^{**}, \\ \frac{R}{I_t}\gamma_t + r(1-\gamma_t) & \text{if } \tilde{\pi}_t^{**} \le \pi_t \le 1. \end{cases} \tag{26}$$

Since banking contracts cannot depend on the realization of $\pi_t$, the bank optimization problem must be solved at the initial stage. The next lemma establishes the property of the optimization problem.

**Lemma 2** *It is optimal for banks to set $d_t \ge \frac{R}{I_t}\gamma_t$ (i.e., $\tilde{\pi}_t^* \le 1$).*

The proof is given in the Appendix. This result states that a *liquidation-preventing contract*, in which banks choose not to liquidate their storage investments for any $\pi_t$ (i.e., $\tilde{\pi}_t^* > 1$), is never optimal. In other words, the banks will admit at least to scraping investments when they exhaust their cash reserves.

There are then two different deposit contracts that need to be considered: *default-preventing contracts* (Section 4.1) and *contracts with defaults* (Section 4.2).

## 4.1 Default-Preventing Contracts

Consider first default-preventing contracts (DPC) that do not have any equilibrium with banking defaults. Banks can avoid bank runs by offering a sufficiently low return for movers, having sufficiently large cash reserves, or both. That is, the banks' portfolio $(d_t, \gamma_t)$ must satisfy the following condition:

$$\frac{R}{I_t}\gamma_t \leq d_t \leq \frac{R}{I_t}\gamma_t + r(1-\gamma_t), \tag{27}$$

which is equivalent to $\tilde{\pi}_t^* \leq 1 \leq \tilde{\pi}_t^{**}$. Since the crisis state is not possible under DPC, the banks maximize the following expected utility:

$$\begin{aligned}\tilde{X}(d_t, \gamma_t, I_t) \equiv \max_{d_t,\gamma_t} \quad & \int_0^{\tilde{\pi}_t^*} \left\{ \pi_t \ln(d_t) + (1-\pi_t)\ln\left[\frac{R(1-\gamma_t) + \frac{R}{I_t}\gamma_t - \pi_t d_t}{1-\pi_t}\right]\right\} f(\pi_t)d\pi_t \\ & + \int_{\tilde{\pi}_t^*}^1 \left\{ \pi_t \ln(d_t) + (1-\pi_t)\ln\left[\frac{R(1-\gamma_t) - \frac{R}{r}(\pi_t d_t - \frac{R}{I_t}\gamma_t)}{1-\pi_t}\right]\right\} f(\pi_t)d\pi_t,\end{aligned} \tag{28}$$

subject to (27). The next lemma establishes the property of this solution.

**Lemma 3** *Under DPC, at the optimum, constraint (27) is binding (i.e.,* $d_t = \frac{R}{I_t}\gamma_t + r(1-\gamma_t)$ *or* $\tilde{\pi}_t^{**} = 1$*).*

The proof is shown in the Appendix. If $d_t < \frac{R}{I_t}\gamma_t + r(1-\gamma_t)$, it would be possible to increase the expected utility by holding $d_t$ constant and reducing $\gamma_t$, since $I_t \geq 1$. The lemma implies that the problem of contracts with defaults, which is discussed in the next subsection, has an interior solution.

Figure 2 illustrates the relationship between deposit returns and $\pi_t$, holding $R$ and $I_t$ constant. Note that for small values of $\pi_t$ (i.e., $0 \leq \pi_t \leq \tilde{\pi}_t^*$), money provides more consumption than is needed by the movers, so some money is stored and given to non-movers. The return inequality between movers and non-movers is increasing in $\pi_t$. For higher values of $\pi_t$ (i.e., $\tilde{\pi}_t^* < \pi_t \leq 1$), banks liquidate their investments to prevent the gap from widening further. In this case, the return to non-movers is given by $(1-\gamma_t)R + \frac{R^2}{rI_t}\gamma_t$, which is independent of $\pi_t$. Note also that the incentive constraint (21) is always satisfied.

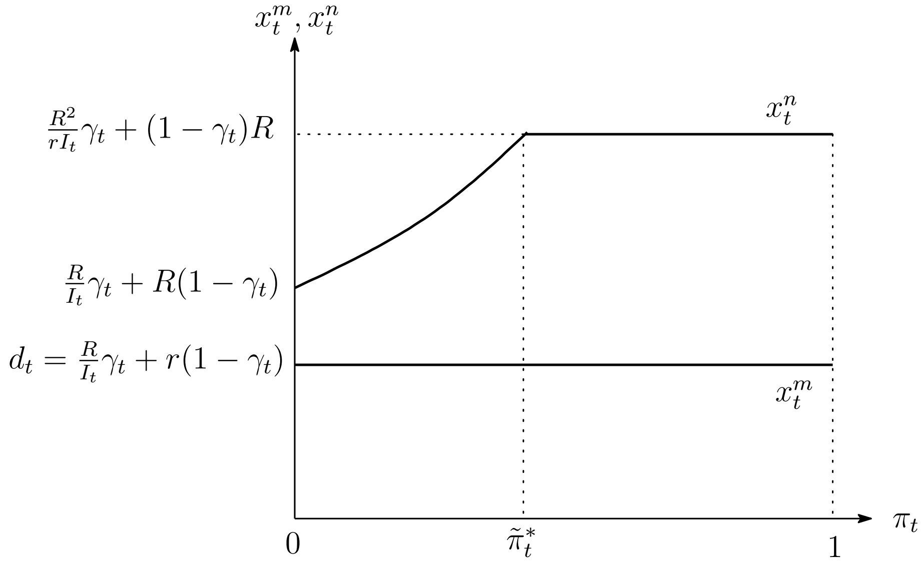


Figure 2: The deposit returns under DPC

The above problem can be rewritten as:

$$\max_{\gamma_t \in [0,1]} \int_0^1 \pi_t f(\pi_t) d\pi_t \ln\left[\frac{R}{I_t}\gamma_t + r(1-\gamma_t)\right] + \int_0^{\tilde{\pi}_t^*} (1-\pi_t) \ln\left[\frac{(R-\pi_t r)(1-\gamma_t) + (1-\pi_t)\frac{R}{I_t}\gamma_t}{1-\pi_t}\right] f(\pi_t) d\pi_t + \int_{\tilde{\pi}_t^*}^1 (1-\pi_t) \ln\left[R(1-\gamma_t) + \frac{R^2}{rI_t}\gamma_t\right] f(\pi_t) d\pi_t,$$

where $\tilde{\pi}_t^* = \frac{R}{I_t}\gamma_t / (\frac{R}{I_t}\gamma_t + r(1-\gamma_t))$. The first-order condition for this problem is:

$$\frac{\frac{R}{I_t} - r}{r(1-\gamma_t) + \frac{R}{I_t}\gamma_t}\left[\int_0^1 \pi_t f(\pi_t) d\pi_t + \int_{\tilde{\pi}_t^*}^1 (1-\pi_t) f(\pi_t) d\pi_t\right] + \int_0^{\tilde{\pi}_t^*} \frac{(1-\pi_t)[(1-\pi_t)\frac{R}{I_t} - (R-\pi_t r)]}{(R-\pi_t r)(1-\gamma_t) + (1-\pi_t)\frac{R}{I_t}\gamma_t} f(\pi_t) d\pi_t = 0. \quad (29)$$

Let $\tilde{\gamma}(I_t)$ denote the value of $\gamma_t$ that satisfies (29) and $\tilde{d}(I_t)$ the corresponding return to movers, expressed as a function of the (gross) nominal interest rate. The optimal reserve-deposit ratio, $\tilde{\gamma}(I_t)$, satisfies the following properties:

**Lemma 4** *(i)* $\tilde{\gamma}(1) = 1$, *(ii)* $\tilde{\gamma}(R/r) = 0$, *(iii)* $\tilde{\gamma}'(I_t) < 0$.

The proof of this is in the Appendix. The intuition of these results is similar to that under complete deposit contracts. From Lemma 4, we see that $\tilde{d}(1) = R$, $\tilde{d}(R/r) = r$, and $\tilde{d}'(I) < 0$. As in the previous section, the government budget constraint and the market-clearing condition are the same, and hence (1) and (13) continue to hold in equilibrium. Therefore, in the steady-state equilibrium, the after-tax/transfer income of young agents is given by:

$$\tilde{Y}(I) \equiv w + \tau = \frac{w}{1 - \frac{I-R}{I}\tilde{\gamma}(I)}. \tag{30}$$

In the steady state, the expected utility of a representative young agent (2) is only given by a function of $I$, as follows:

$$\tilde{W}(I) \equiv \ln \tilde{Y}(I) + \tilde{X}\left[\frac{R}{I}\tilde{\gamma}(I) + r(1 - \tilde{\gamma}(I)), \tilde{\gamma}(I), I\right]. \tag{31}$$

Having characterized the welfare, we provide the optimal monetary policy under DPC.

**Proposition 2** *Suppose that the banks provide DPC. Then the optimal monetary policy is to set the net money growth rate equal to zero (i.e.,* $I = R$*).*

For the proof, see the Appendix. The proposition confirms the robustness of the optimality of the zero-inflationary policy, despite the Friedman rule eliminating a costly liquidity crisis.[9]

## 4.2 Contracts with Defaults

Consider the optimal contract with banking defaults. Banks offer good returns to depositors in the normal state by taking the default risk. That is, banks will choose the portfolio $(d_t, \gamma_t)$ satisfying:

$$\frac{R}{I_t}\gamma_t + r(1 - \gamma_t) \leq d_t \leq \frac{R}{I_t}\gamma_t + R(1 - \gamma_t), \tag{32}$$

[9]Since $d \to R$ as $I \to 1$, we have $\tilde{\pi}^* \to 1$.

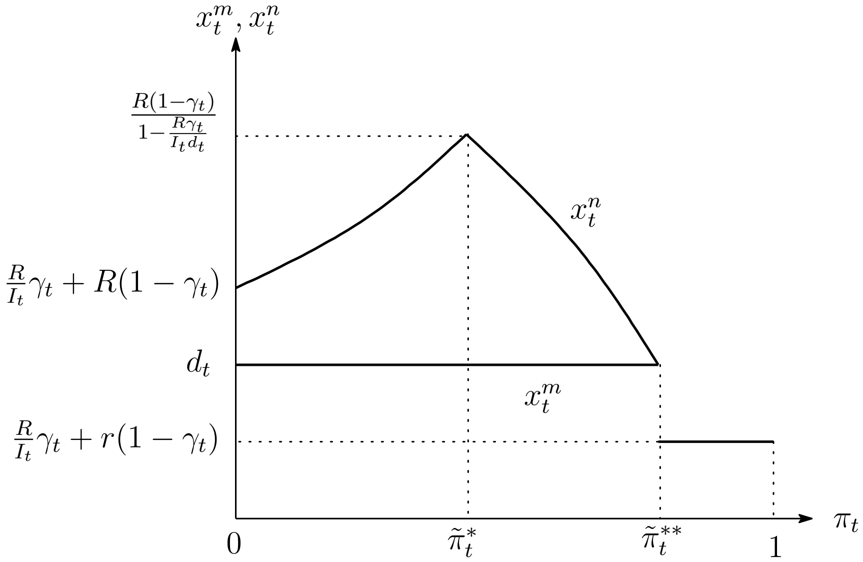


Figure 3: The deposit returns under the contract with defaults

which is equivalent to the combination of (A.1) and $\tilde{\pi}_t^{**} \leq 1$.

Figure 3 illustrates the relationship between the deposit returns and $\pi_t$, holding $R$ and $I_t$ constant. Note that for small values of $\pi_t$ (i.e., $0 \leq \pi_t < \tilde{\pi}_t^*$), money provides more consumption than the promised-constant repayments to the movers, and the difference of the two returns is increasing in $\pi_t$. For $\pi_t \in [\tilde{\pi}_t^*, \tilde{\pi}_t^{**})$, the banks scrap their investments to meet the liquidity demands of movers without violating the incentive constraint. The difference between the returns of movers and non-movers is decreasing in $\pi_t$. For high values of $\pi_t$ (i.e., $\pi_t \geq \tilde{\pi}_t^{**}$), it is impossible to pay the movers the fixed return $d_t$ without violating the incentive constraint, and a banking default inevitably ensues. Since the banks are identical, these widespread defaults can be interpreted as a "banking crisis."

Substituting (25) and (26) into the bank's objective function yields the problem:

$$\hat{X}(d_t,\gamma_t,I_t) \equiv \max_{d_t,\gamma_t} \int_0^{\tilde{\pi}_t^*} \left\{ \pi_t \ln(d_t) + (1-\pi_t)\ln\left[\frac{R(1-\gamma_t)+\frac{R}{I_t}\gamma_t - \pi_t d_t}{1-\pi_t}\right]\right\} f(\pi_t)d\pi_t$$
$$+\int_{\tilde{\pi}_t^*}^{\tilde{\pi}_t^{**}} \left\{ \pi_t \ln(d_t) + (1-\pi_t)\ln\left[\frac{R(1-\gamma_t)-\frac{R}{r}(\pi_t d_t - \frac{R}{I_t}\gamma_t)}{1-\pi_t}\right]\right\} f(\pi_t)d\pi_t$$
$$+\int_{\tilde{\pi}_t^{**}}^{1} \ln\left[\frac{R}{I_t}\gamma_t + r(1-\gamma_t)\right] f(\pi_t)d\pi_t. \quad (33)$$

Let $(\hat{d}(I_t),\hat{\gamma}(I_t))$ denote the solution to this problem, expressed as a function of the (gross) nominal interest rate. The properties of these values are given as follows:

**Lemma 5** *(i)* $\hat{\gamma}(1)=1$ *and* $\hat{d}(1)=R$*, (ii)* $\tilde{\gamma}(R/r)=0$ *and* $\hat{d}(R/r)=r$*.*

The proof is shown in the Appendix. These results are similar to those obtained in Lemma 1 and 4. Owing to the complexity, however, we are unable to show analytically that $\hat{\gamma}'(I_t)<0$ and $\hat{d}'(I_t)<0$. Numerical simulations in the next section confirm this result.

The after-tax/transfer income of young agents in the steady-state equilibrium is:

$$\hat{Y}(I) \equiv w+\tau = \frac{w}{1-\frac{I-R}{I}\hat{\gamma}(I)}. \quad (34)$$

In the steady state, the expected utility of a representative young agent is given as the function of $I$, as follows:

$$\hat{W}(I) \equiv \ln \hat{Y}(I) + \hat{X}\left[\hat{d}(I),\hat{\gamma}(I),I\right]. \quad (35)$$

Comparing $\hat{W}(I)$ and $\tilde{W}(I)$, we can see whether the banks should avoid bankruptcy or accept the risk of default with probability $1-F(\tilde{\pi}_t^{**})$. More precisely, it is better for the banks to take the risk of default if

$$\hat{W}(I) > \tilde{W}(I). \quad (36)$$

# 5 Numerical Examples

To clarify the concepts and results presented so far, we provide a sequence of numerical examples of equilibria with complete and incomplete banking contracts. In these examples, we successively decrease the scrap value $r$ to illustrate the properties of banking contracts. The equilibrium values (the reserve-deposit ratio, the cut-off values, and welfare) are illustrated for the case where $r = 0.8$ in Table 1, $r = 0.7$ in Table 2, and $r = 0.4$ in Table 3. In all our examples, $w = 1$ and $R = 1.2$, and $\pi_t$ is uniformly distributed so that $f(\pi) = 1$. In addition, the (gross) nominal interest rate, $I$, varies between 1.00 and 1.28. Although an interbank asset market has not been modeled here, the low scrap value captures a situation in which banks have to sell their long-term assets at "fire-sale" prices.

Table 1: $r = 0.8$

| | Complete Contracts | | | | DPC | | | Contract with Defaults | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I$ | $\gamma$ | $\pi^*$ | $\pi^{**}$ | $W$ | $\tilde{\gamma}$ | $\tilde{\pi}^*$ | $\tilde{W}$ | $\hat{\gamma}$ | $\hat{\pi}^*$ | $\hat{\pi}^{**}$ | $\hat{W}$ |
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 |
| 1.01 | 0.9653 | 0.9649 | 0.9763 | 0.0057 | 0.9829 | 0.9884 | 0.0028 | 0.9785 | 0.9824 | 0.9907 | 0.0035 |
| 1.02 | 0.9328 | 0.9316 | 0.9533 | 0.0108 | 0.9607 | 0.9729 | 0.0063 | 0.9549 | 0.9637 | 0.9838 | 0.0072 |
| 1.04 | 0.8737 | 0.8693 | 0.9089 | 0.0196 | 0.9105 | 0.9362 | 0.0136 | 0.9052 | 0.9244 | 0.9746 | 0.0145 |
| 1.08 | 0.7726 | 0.7588 | 0.8251 | 0.0323 | 0.8058 | 0.8521 | 0.0266 | 0.8046 | 0.8418 | 0.9672 | 0.0270 |
| 1.12 | 0.6869 | 0.6620 | 0.7461 | 0.0402 | 0.7073 | 0.7640 | 0.0355 | 0.7088 | 0.7571 | 0.9680 | 0.0357 |
| 1.15 | 0.6293 | 0.5962 | 0.6889 | 0.0438 | 0.6396 | 0.6983 | 0.0396 | 0.6418 | 0.6936 | 0.9711 | 0.0397 |
| 1.18 | 0.5756 | 0.5347 | 0.6329 | 0.0455 | 0.5766 | 0.6339 | 0.0416 | 0.5789 | 0.6308 | 0.9751 | 0.0418 |
| 1.20 | 0.5413 | 0.4958 | 0.5960 | 0.0459 | 0.5369 | 0.5917 | 0.0420 | 0.5391 | 0.5894 | 0.9779 | 0.0422 |
| 1.24 | 0.4750 | 0.4218 | 0.5225 | 0.0446 | 0.4617 | 0.5092 | 0.0406 | 0.4634 | 0.5081 | 0.9834 | 0.0408 |
| 1.28 | 0.4102 | 0.3520 | 0.4490 | 0.0411 | 0.3906 | 0.4289 | 0.0368 | 0.3917 | 0.4284 | 0.9883 | 0.0369 |

Table 2: $r = 0.7$

| | Complete Contract | | | | DPC | | | Contract with Defaults | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I$ | $\gamma$ | $\pi^*$ | $\pi^{**}$ | $W$ | $\tilde{\gamma}$ | $\tilde{\pi}^*$ | $\tilde{W}$ | $\hat{\gamma}$ | $\hat{\pi}^*$ | $\hat{\pi}^{**}$ | $\hat{W}$ |
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 |
| 1.01 | 0.9708 | 0.9705 | 0.9826 | 0.0048 | 0.9886 | 0.9932 | 0.0019 | 0.9838 | 0.9870 | 0.9917 | 0.0026 |
| 1.02 | 0.9435 | 0.9424 | 0.9656 | 0.0091 | 0.9733 | 0.9840 | 0.0042 | 0.9663 | 0.9736 | 0.9851 | 0.0054 |
| 1.04 | 0.8940 | 0.8902 | 0.9329 | 0.0164 | 0.9377 | 0.9613 | 0.0094 | 0.9294 | 0.9460 | 0.9751 | 0.0108 |
| 1.08 | 0.8102 | 0.7980 | 0.8714 | 0.0270 | 0.8588 | 0.9061 | 0.0191 | 0.8537 | 0.8890 | 0.9639 | 0.0202 |
| 1.12 | 0.7406 | 0.7182 | 0.8138 | 0.0334 | 0.7805 | 0.8448 | 0.0262 | 0.8034 | 0.8455 | 0.9537 | 0.0251 |
| 1.15 | 0.6948 | 0.6644 | 0.7724 | 0.0362 | 0.7253 | 0.7974 | 0.0295 | 0.7768 | 0.8194 | 0.9458 | 0.0273 |
| 1.18 | 0.6531 | 0.6147 | 0.7323 | 0.0377 | 0.6738 | 0.7501 | 0.0312 | 0.7538 | 0.7956 | 0.9387 | 0.0289 |
| 1.20 | 0.6270 | 0.5835 | 0.7060 | 0.0379 | 0.6415 | 0.7188 | 0.0315 | 0.7400 | 0.7806 | 0.9342 | 0.0296 |
| 1.24 | 0.5781 | 0.5249 | 0.6545 | 0.0370 | 0.5810 | 0.6572 | 0.0304 | 0.7154 | 0.7525 | 0.9258 | 0.0306 |
| 1.28 | 0.5323 | 0.4706 | 0.6038 | 0.0344 | 0.5254 | 0.5972 | 0.0273 | 0.6938 | 0.7264 | 0.9179 | 0.0310 |

Table 3: $r = 0.4$

| | Complete Contracts | | | | DPC | | | Contracts with Defaults | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I$ | $\gamma$ | $\pi^*$ | $\pi^{**}$ | $W$ | $\tilde{\gamma}$ | $\tilde{\pi}^*$ | $\tilde{W}$ | $\hat{\gamma}$ | $\tilde{\pi}^*$ | $\tilde{\pi}^{**}$ | $\hat{W}$ |
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 0.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 |
| 1.01 | 0.9783 | 0.9781 | 0.9926 | 0.0035 | 0.9955 | 0.9985 | 0.0007 | 0.9910 | 0.9929 | 0.9940 | 0.0015 |
| 1.02 | 0.9582 | 0.9574 | 0.9854 | 0.0067 | 0.9895 | 0.9964 | 0.0017 | 0.9815 | 0.9861 | 0.9886 | 0.0030 |
| 1.04 | 0.9219 | 0.9190 | 0.9715 | 0.0121 | 0.9750 | 0.9912 | 0.0038 | 0.9623 | 0.9730 | 0.9794 | 0.0058 |
| 1.08 | 0.8616 | 0.8521 | 0.9453 | 0.0197 | 0.9403 | 0.9777 | 0.0080 | 0.9324 | 0.9504 | 0.9628 | 0.0096 |
| 1.12 | 0.8131 | 0.7953 | 0.9210 | 0.0242 | 0.9019 | 0.9610 | 0.0114 | 0.9096 | 0.9309 | 0.9482 | 0.0120 |
| 1.15 | 0.7823 | 0.7576 | 0.9036 | 0.0261 | 0.8723 | 0.9469 | 0.0132 | 0.8950 | 0.9175 | 0.9381 | 0.0133 |
| 1.18 | 0.7552 | 0.7234 | 0.8869 | 0.0271 | 0.8429 | 0.9317 | 0.0141 | 0.8822 | 0.9049 | 0.9287 | 0.0142 |
| 1.20 | 0.7389 | 0.7022 | 0.8761 | 0.0272 | 0.8237 | 0.9211 | 0.0143 | 0.8744 | 0.8969 | 0.9227 | 0.0146 |
| 1.24 | 0.7094 | 0.6631 | 0.8552 | 0.0266 | 0.7864 | 0.8990 | 0.0136 | 0.8603 | 0.8817 | 0.9113 | 0.0150 |
| 1.28 | 0.6835 | 0.6278 | 0.8350 | 0.0250 | 0.7510 | 0.8761 | 0.0116 | 0.8480 | 0.8673 | 0.9005 | 0.0151 |

Note first that banks that must offer incomplete contracts to depositors hold more cash reserves (or equivalently, make fewer storage investments) than banks that can offer complete contracts, except in the case with sufficiently high $r$ and high $I$. In addition, as the inflation rate rises, which means that the opportunity cost of holding cash increases, the equilibrium reserve-deposit ratio, $\gamma$, declines under both complete and incomplete contracts, as indicated by Lemma 1 and 4. With an increasing inflation rate, the probabilities of a costly liquidity crisis, $1 - F(\pi^{**})$ and $1 - F(\tilde{\pi}^*)$, increase for both contracts. Alternatively, the probability of bankruptcy, $1 - F(\tilde{\pi}^{**})$, and inflation take a hump-shaped relationship when the contract with defaults is adopted. For sufficiently low and high inflation rates, the probability of bankruptcy becomes low, while it is high for mild inflation rates. The intuition is as follows. With an increase in the inflation rate, the banks not only hold low cash reserves (or equivalently, make more storage investments), but also promise low returns to depositors who withdraw early. This implies that the incentive constraint (21) is more likely to be satisfied. As a result, increases in the inflation rate put downward pressure on the cut-off value, $\tilde{\pi}^{**}$. At the same time, the number of depositors served liquidated investments increases (equivalently, a decline in $\tilde{\pi}^*$) and the returns to non-movers decrease, implying that the incentive constraint is less likely to be satisfied. As a result, increases in the inflation rate put upward pressure on the cut-off value, $\tilde{\pi}^{**}$. For high inflation rates, the latter effect dominates the former effect, and then $\tilde{\pi}^{**}$ is increasing in $I$. Tables 1–3 also show the range in which the probability of bankruptcy is increasing in inflation as the scrap value, $r$, declines.

Second, note that, as shown in Proposition 1 and 2, the Friedman rule ($I = 1.00$) eliminates any types of crises, but is never optimal for both banking contracts. At $I = 1.00$, the gain from the intergenerational transfer outweighs the loss of return inequality between movers and non-movers. The zero-inflationary policy ($I = 1.20$) balances the two effects and achieves the maximum welfare under complete contracts and DPC, but not under the contract with defaults. When the banks choose the contracts with defaults, the monetary authority may have an incentive to implement an inflationary policy when $r$ is sufficiently low, as illustrated in Tables 2 and 3.

Finally, we confirm the detailed welfare values. From all tables, we can see that the welfare under complete banking contracts are highest among them, which is reasonable because the incompleteness of contracts imposes a welfare cost on the depositors under the restriction on the consumption smoothing between states. Alternatively, our interests are to compare the levels of welfare under DPC with those under the contracts with defaults. When $r = 0.4$ and $0.8$, we can confirm that banks will choose the contract with defaults from $I = 1.00$ to $1.28$. Moreover, considering the concavity of the welfare function, even if the value of $I$ increases further, we can guess that this relationship is consistent. However, the consistent relationship does not hold under $r = 0.7$. That is, the DPC will be adopted when $1.12 \leq I < 1.24$. Instead, once the nominal interest rate falls below $I = 1.12$ or exceeds $I = 1.24$, the contract with defaults will be adopted. In other words, the banking defaults never occur under $1.12 \leq I < 1.24$. However, a banking crisis can occur with a positive probability under $I < 1.12$ or $1.24 \geq I$. It other words, a banking crisis occurs in either very low inflation (and deflation) environments or very high inflation environments, yielding a U-shaped relationship between a banking crisis and inflation.[10] Importantly, note that in any of the cases, a costly liquidity crisis can occur for any $I > 1$ under incomplete contracts, but its probability is lower than under complete contracts. Although there are few historical episodes of crises in very low inflation environments (e.g., the U.S. during the Great Depres-

[10] Jiang (2008) obtains similar results in a "dollarized" model economy.

sion and Japan in the 1990s), there are many episodes in inflationary environments. Our results are consistent with existing empirical literature reporting the positive relationship between a banking crisis and inflation.[11]

# 6 Conclusions

We have studied a monetary equilibrium economy in which banks provide a liquidity service to risk-averse depositors. In contrast to existing literature, we have considered a situation in which the banks must offer incomplete (non-state contingent) deposit contracts. We have shown that the equilibrium with incomplete contracts can result in serious banking crises. Banking crises tend to occur under relatively low or high inflation rates because the banks tend to choose the contract with defaults described above. Empirical evidence provide strong support for our results. In addition, we have shown that a zero-inflationary policy can be optimal under incomplete contracts and under complete contracts.

Our model can be extended in many ways. We discuss two possibilities here. The first is related to the policy analysis of the lender of last resort. One of the important roles of a lender of last resort is to provide elastic money in response to the liquidity demands of troubled banks. However, as many economists have pointed out, the policy can create a moral hazard that banks take excessive risks when choosing portfolios. Introducing a lender of last resort in our model, the moral hazard can be captured by changing the equilibrium contract types.[12] Another direction is related to the industrial organization of banks. Existing literature show that the competitive structure of the banking industry has significant impacts on financial outcomes.[13] It would be fruitful to compare the probability of a banking crisis and the optimal monetary policy across

[11] See Demirguc-Kunt and Detragiache (1998, 2005), Hardy and Pazarbasioglu (1998), and Boyd et al. (2002).

[12] Antinolfi et al. (2001), Antinolfi and Keister (2006), and others, abstract from the moral hazard problem associated with the presence of a lender of last resort in their models.

[13] See, for example, Boyd et al. (2004) and Ghossoub (2012).

different banking systems under incomplete contracts. We leave these important issues for future research.

# Appendix

**Proof of Proposition 1**: Because of the concavity of the utility function, the condition $W'(R) = 0$ is necessary and sufficient to obtain Proposition 1. From (15) and (17), we obtain the derivative of the first term of (18):

$$\frac{Y'(I)}{Y(I)} = \frac{I(I-R)\gamma'(I) + R\gamma(I)}{I[I-(I-R)\gamma(I)]}.$$

Substituting $I = R$ yields:

$$\frac{Y'(R)}{Y(R)} = \frac{\gamma(R)}{R}. \tag{a.1}$$

Next, we consider the derivative of the second term of (18). Applying the Envelope Theorem, we have:

$$\frac{\partial}{\partial I} X[\gamma(I), I] = X_2[\gamma(I), I],$$

where $X_2$ is the derivative of $X[\gamma, I]$ with respect to its second argument. With an elaborate calculation, we obtain:

$$X_2[\gamma(I), I] = \frac{1}{I}\left(\int_{\pi^*}^{\pi^{**}} F(\pi)d\pi - \pi^{**}\right). \tag{a.2}$$

Substituting the first-order condition (12) into equation (a.2) yields:

$$X_2[\gamma(I), I] = -\frac{\gamma(I)}{I},$$

which becomes $-\gamma(R)/R$ at $I = R$, completing the proof of the proposition. $\square$

**Proof of Lemma 2**: Suppose that $0 < d_t < \frac{R}{I_t}\gamma_t$, which is equivalent to $\tilde{\pi}_t^* > 1$. Then, the only possible state is the normal state. In this case, the bank's problem is:

$$\max_{d_t, \gamma_t} \int_0^1 \left\{ \pi_t \ln(d_t) + (1-\pi_t) \ln\left[\frac{R(1-\gamma_t) + \frac{R}{I_t}\gamma_t - \pi_t d_t}{1-\pi_t}\right]\right\} f(\pi_t)d\pi_t,$$

subject to $0 \leq \gamma_t \leq 1$. Since $I_t \geq 1$, $\gamma_t = 0$ maximizes the above objective function, contradicting our assumption. □

**Proof of Lemma 3**: Differentiating (28) with respect to $d_t$ yields

$$\frac{\partial}{\partial d_t}\tilde{X}(d_t,\gamma_t,I_t) = \frac{\int_0^1 \pi_t f(\pi_t)d\pi_t}{d_t} - \int_0^{\tilde{\pi}_t^*} \frac{\pi_t(1-\pi_t)}{R(1-\gamma_t)+\frac{R}{I_t}\gamma_t - \pi_t d_t} f(\pi_t)d\pi_t - \int_{\tilde{\pi}_t^*}^1 \frac{\pi_t(1-\pi_t)}{r(1-\gamma_t)+\frac{R}{I_t}\gamma_t - \pi_t d_t} f(\pi_t)d\pi_t.$$

Supposing $d_t = r(1-\gamma_t) + \frac{R}{I_t}\gamma_t$, we obtain:

$$\frac{\partial}{\partial d_t}\tilde{X}\left(r(1-\gamma_t)+\frac{R}{I_t}\gamma_t,\gamma_t,I_t\right) = \int_0^{\tilde{\pi}_t^*} \frac{\pi(R-r)(1-\gamma_t)f(\pi_t)d\pi_t}{\left[r(1-\gamma_t)+\frac{R}{I_t}\gamma_t\right]\left[(R-\pi_t r)(1-\gamma_t)+(1-\pi_t)\frac{R}{I_t}\gamma_t\right]} > 0,$$

which means that constraint (27) is binding. □

**Proof of Lemma 4**: Define $\bar{X}(\gamma_t, I_t) \equiv \tilde{X}(\frac{R}{I_t}\gamma_t + r(1-\gamma_t), \gamma_t, I_t)$. Evaluating the first derivative of $\bar{X}(\gamma_t, I_t)$ with respect to its first argument at $I_t = 1$ yields:

$$\begin{aligned}\bar{X}_1(\gamma_t,1) &= \frac{R-r}{r(1-\gamma_t)+R\gamma_t}\left[\int_0^1 \pi_t f(\pi_t)d\pi_t + \int_{\tilde{\pi}_t^*}^1 (1-\pi_t)f(\pi_t)d\pi_t\right] \\ &\quad - \int_0^{\tilde{\pi}_t^*} \frac{\pi_t(1-\pi_t)(R-r)}{(R-\pi_t r)(1-\gamma_t)+(1-\pi_t)R\gamma_t} f(\pi_t)d\pi_t \\ &= \frac{R-r}{r(1-\gamma_t)+R\gamma_t}\int_{\tilde{\pi}_t^*}^1 f(\pi_t)d\pi_t \\ &\quad + \int_0^{\tilde{\pi}_t^*} \frac{\pi_t(1-\gamma_t)(R-r)^2}{[r(1-\gamma_t)+R\gamma_t][(R-\pi_t r)(1-\gamma_t)+(1-\pi_t)R\gamma_t]} f(\pi_t)d\pi_t > 0,\end{aligned}$$

implying that the solution must be at the boundary point, $\gamma_t = 1$. This establishes part (i) of the lemma. In addition, we obtain

$$\bar{X}_1\left(\gamma_t, \frac{R}{r}\right) = -\int_0^{\tilde{\pi}_t^*} \frac{(1-\pi_t)(R-r)}{(R-\pi_t r)-\gamma_t(R-r)} f(\pi_t)d\pi_t < 0,$$

implying that the solution must be $\gamma_t = 0$, which establishes part (ii) of the lemma.

Since the optimal value of $\tilde{\gamma}(I_t)$ is determined by implicit differentiation of the identity:

$$\bar{X}_1(\tilde{\gamma}(I_t), I_t) \equiv 0,$$

we obtain:

$$\tilde{\gamma}'(I_t) = -\frac{\bar{X}_{12}(\gamma, I_t)}{\bar{X}_{11}(\gamma, I_t)}.$$

Differentiating $\bar{X}_1(\gamma_t, I_t)$ with respect to $I_t$ yields

$$\begin{aligned}
\bar{X}_{12}(\gamma, I_t) = -\frac{r(\tilde{\pi}_t^*)^2}{R\gamma_t^2}\left[\int_0^1 \pi_t f(\pi_t)d\pi_t + \int_{\tilde{\pi}_t^*}^1 (1-\pi_t)f(\pi_t)d\pi_t\right] \\
-\int_0^{\tilde{\pi}_t^*} \frac{(1-\pi_t)^2(R-\pi_t r)\frac{R}{I^2}}{[(R-\pi_t r)(1-\gamma_t)+(1-\pi_t)\frac{R}{I_t}\gamma_t]^2} f(\pi_t)d\pi_t \\
-\frac{R-r}{R}\frac{\tilde{\pi}_t^*(1-\tilde{\pi}_t^*)}{\gamma_t(1-\gamma_t)} f(\tilde{\pi}_t^*)\frac{\partial \tilde{\pi}_t^*}{\partial I_t} < 0.
\end{aligned}$$

In addition, by strict concavity, we have $\bar{X}_{11} < 0$. Therefore, we obtain

$$\tilde{\gamma}'(I_t) < 0.$$

This completes the proof of the lemma. $\square$

**Proof of Proposition 2**: As in the previous proof of Proposition 1, $\tilde{W}'(R) = 0$ allows us to achieve Proposition 2 because of the concavity of the utility function. From (30), we obtain the derivative of the first term of (31):

$$\frac{\tilde{Y}'(I)}{\tilde{Y}(I)} = \frac{I(I-R)\tilde{\gamma}'(I) + R\tilde{\gamma}(I)}{I[I-(I-R)\tilde{\gamma}(I)]}.$$

By evaluating this derivative at $I = R$, we obtain:

$$\frac{\tilde{Y}'(R)}{\tilde{Y}(R)} = \frac{\tilde{\gamma}(R)}{R}. \tag{a.3}$$

Next, we consider the derivative of the second term of (31). Define $\bar{X}(\gamma(I), I) \equiv \tilde{X}(\frac{R}{I}\tilde{\gamma}(I) + r(1-\tilde{\gamma}(I)), \tilde{\gamma}(I), I)$. Applying the Envelope

Theorem yields:

$$\frac{\partial}{\partial I}\bar{X}[\tilde{\gamma}(I), I] = \bar{X}_2[\tilde{\gamma}(I), I],$$

where $\bar{X}_2$ is the derivative of $\bar{X}[\tilde{\gamma}, I]$ with respect to its second argument. With an elaborate calculation, we obtain:

$$\bar{X}_2[\tilde{\gamma}(I), I] = -\frac{\tilde{\pi}^*}{I} + \frac{\tilde{\pi}^*}{I}\int_0^{\tilde{\pi}^*} \frac{(1-\pi)(R-r)(1-\tilde{\gamma}(I))}{(R-\pi r)(1-\tilde{\gamma}(I)) + (1-\pi)\frac{R}{I}\tilde{\gamma}(I)} f(\pi)d\pi. \tag{a.4}$$

With some long, but straightforward calculations, the first-order condition (29) can be reduced to

$$(1-r)(1-\tilde{\gamma}(I)) = \int_0^{\tilde{\pi}^*} \frac{(1-\pi)(R-r)(1-\tilde{\gamma}(I))}{(R-\pi r)(1-\tilde{\gamma}(I)) + (1-\pi)\frac{R}{I}\tilde{\gamma}(I)} f(\pi)d\pi. \tag{a.5}$$

Substituting (a.5) into (a.4) yields:

$$\bar{X}_2[\tilde{\gamma}(I), I] = -\frac{\tilde{\gamma}(I)}{I},$$

which becomes $-\tilde{\gamma}(R)/R$ at $I = R$. This completes the proof of the proposition. $\square$

**Proof of Lemma 5**: The solution $(\hat{d}(I_t), \hat{\gamma}(I_t))$ is derived from the following first-order conditions:

$$\begin{aligned}
\hat{X}_1(d_t, \gamma_t, I_t) = &\int_0^{\tilde{\pi}_t^*} \pi_t \left(\frac{1}{d_t} - \frac{1-\pi_t}{R(1-\gamma_t) + \frac{R}{I_t}\gamma_t - \pi_t d_t}\right) f(\pi_t)d\pi_t \\
&+ \int_{\tilde{\pi}_t^*}^{\tilde{\pi}_t^{**}} \pi_t \left(\frac{1}{d_t} - \frac{1-\pi_t}{r(1-\gamma_t) + \frac{R}{I_t}\gamma_t - \pi_t d_t}\right) f(\pi_t)d\pi_t \\
&+ \frac{\partial \tilde{\pi}_t^{**}}{\partial d_t} f(\tilde{\pi}_t^{**}) \ln\left(\frac{d_t}{r(1-\gamma_t) + \frac{R}{I_t}\gamma_t}\right) = 0,
\end{aligned}$$

$$
\begin{aligned}
\hat{X}_2(d_t,\gamma_t,I_t) = &-\int_0^{\tilde{\pi}_t^*} \frac{(1-\pi_t)\left(1-\frac{1}{I_t}\right)R}{R(1-\gamma_t)+\frac{R}{I_t}\gamma_t-\pi_t d_t} f(\pi_t)d\pi_t \\
&+\int_{\tilde{\pi}_t^*}^{\tilde{\pi}_t^{**}} \frac{(1-\pi_t)\left(\frac{R}{I_t}-r\right)}{r(1-\gamma_t)+\frac{R}{I_t}\gamma_t-\pi_t d_t} f(\pi_t)d\pi_t \\
&+\int_{\tilde{\pi}_t^{**}}^{1} \frac{\frac{R}{I_t}-r}{r(1-\gamma_t)+\frac{R}{I_t}\gamma_t} f(\pi_t)d\pi_t \\
&+\frac{\partial\tilde{\pi}_t^{**}}{\partial\gamma_t} f(\tilde{\pi}_t^{**})\ln\left(\frac{d_t}{r(1-\gamma_t)+\frac{R}{I_t}\gamma_t}\right) = 0.
\end{aligned}
$$

Evaluating $\hat{X}_2(d_t,\gamma_t,I_t)$ at $I_t = 1$ yields:

$$
\begin{aligned}
\hat{X}_2(d_t,\gamma_t,1) = &\int_{\tilde{\pi}_t^*}^{\tilde{\pi}_t^{**}} \frac{(1-\pi_t)(R-r)}{r(1-\gamma_t)+R\gamma_t-\pi_t d_t} f(\pi_t)d\pi_t + \int_{\tilde{\pi}_t^{**}}^{1} \frac{R-r}{r(1-\gamma_t)+R\gamma_t} f(\pi_t)d\pi_t \\
&+\frac{R}{d_t} f(\tilde{\pi}_t^{**})\ln\left(\frac{d_t}{r(1-\gamma_t)+R\gamma_t}\right) > 0,
\end{aligned}
$$

implying then the solution must be $\hat{\gamma} = 1$. Since the optimal value $\hat{d}(I_t)$ satisfies condition (32), by applying the Squeeze Theorem, we obtain $\lim_{I\to 1}\hat{d}(I_t) = R$, which establishes part (i) of the lemma.

Next, consider the case of $I_t = R/r$. Evaluating $\hat{X}_2(d_t,\gamma_t,I_t)$ at $I_t = R/r$ yields:

$$
\hat{X}_2\left(d_t,\gamma_t,\frac{R}{r}\right) = -\int_0^{\tilde{\pi}_t^*} \frac{(1-\pi_t)(R-r)}{R(1-\gamma_t)+r\gamma_t-\pi_t d_t} f(\pi_t)d\pi_t < 0,
$$

which means, the solution must be $\hat{\gamma} = 0$. By also evaluating $\hat{X}_1(d_t,\gamma_t,I_t)$ at $I_t = R/r$, we obtain:

$$
\hat{X}_1(d_t,0,R/r) = \int_0^{\tilde{\pi}_t^{**}} \pi_t\left(\frac{1}{d_t}-\frac{1-\pi_t}{r-\pi_t d_t}\right) f(\pi_t)d\pi_t - \frac{Rrf(\tilde{\pi}_t^{**})}{(R-r)d_t}\ln\left(\frac{d_t}{r}\right) < 0,
$$

implying the solution must be $\hat{d} = r$. As a result, we establish part (ii) of the lemma.

# References


[1] Allen, F. and D. Gale (1998) "Optimal financial crises." *Journal of Finance* 53(4): 1245-1284.

[2] Allen, F. and D. Gale (2004) "Financial Intermediaries and markets." *Econometrica* 72(4): 1023-1061.

[3] Allen, F., E. Carletti and D. Gale (2014) "Money, financial stability and efficiency." *Journal of Economic Theory* 149: 100-127.

[4] Antinolfi, G., E. Huybens and T. Keister (2001) "Monetary stability and liquidity crises: the role of the lender of last resort." *Journal of Economic Theory* 99: 187-219.

[5] Antinolfi, G. and T. Keister (2006) "Discount window policy, banking crises, and indeterminacy of equilibrium." *Macroeconomic Dynamics* 10: 1-19.

[6] Bhattacharya, J. and R. Singh (2008a) "Usefulness of the constrained planning problem in a model of money." *Macroeconomic Dynamics* 12: 503-525.

[7] Bhattacharya, J. and R. Singh (2008b) "Optimal choice of monetary instruments in an economy with real and liquidity shocks." *Journal of Economic Dynamics and Control* 32(4): 1273-1311.

[8] Boyd, J.H., P. Gomis, S. Kwak and B. Smith (2002) "A User's Guide to Banking Crises." Working Paper, University of Texas.

[9] Boyd, J.H., G. De Nicolo and B.D. Smith (2004) "Crises in competitive versus monopolistic banking systems." *Journal of Money, Credit, and Banking* 36: 487-506.

[10] Bryant, J. (1980) "A model of reserves, bank runs, and deposit Insurance." *Journal of Banking and Finance* 4: 335-344.

[11] Cooper, R. and T.W. Ross (1998) "Bank runs: liquidity costs and investment distortions." *Journal of Monetary Economy* 41(1): 27-38.

[12] Chari, V. and R. Jagannathan (1988) "Banking panics, information, and rational expectations equilibrium." *Journal of Finance* 43, 749-760.

[13] Champ, B., B.D. Smith, and S. Williamson (1996) "Currency elasticity and banking panics: Theory and evidence." *Canadian Journal of Economics* 29: 828-864.

[14] Diamond, D. and P. Dybvig (1983) "Bank runs, deposit insurance and liquidity." *Journal of Political Economy* 91: 401-419.

[15] Demirguc-Kunt, A. and E. Detragiache (1998) "The determinants of banking crises in developed and developing economies." IMF Staff Papers 45: 81-109.

[16] Demirguc-Kunt, A. and E. Detragiache (2005) "Cross-country empirical studies of systemic bank distress: a survey." IMF working paper 05/96, International Monetary Fund.

[17] Ennis, H.M. and T. Keister (2009) "Bank runs and institutions: the perils of intervention." *American Economic Review* 99(4): 1588-1607.

[18] Ennis, H.M. and T. Keister (2010) "Banking panics and policy responses." *Journal of Monetary Economy* 57: 404-419.

[19] Ghossoub, E.A. and R.R. Reed III (2010) "Liquidity risk, economic development, and the effects of monetary policy." *European Economic Review* 54: 252-268.

[20] Ghossoub, E.A. (2012) "Liquidity risk and financial competition: implications for asset prices and monetary policy." *European Economic Review* 56: 155-173.

[21] Gomis-Porqueras, P. and B.D. Smith (2003) "Seasonality and monetary policy." *Macroeconomic Dynamics* 7: 477-502.

[22] Matsuoka, T. (2012) "Imperfect interbank markets and the lender of last resort." *Journal of Economic Dynamics and Control* 36: 1673-1687.

[23] Schreft, S. and B. Smith (1997) "Money, banking and capital formation." *Journal of Economic Theory* 73: 157-182.

[24] Schreft, S. and B. Smith (1998) "The effects of open market operations in a model of intermediation and growth." *Review of Economic Studies* 65: 519-550.

[25] Schreft, S. and B. Smith (2000) "The evolution of cash transactions: Some implications for monetary policy." *Journal of Monetary Economics* 46: 97-120.

[26] Hardy, D.C. and C. Pazarbasioglu (1998) "Leading indicators of banking crises: was Asia different?" IMF Working Paper no. WP/98/91 (June).

[27] Haslag, J.H. and A. Martin (2007) "Optimality of the Friedman rule in an overlapping generations model with spatial separation." *Journal of Money, Credit, and Banking* 39: 1741-1758.

[28] Jiang, J.H., (2008) "Banking crises in monetary economies." *Canadian Journal of Economics* 41(1): 80-104.

[29] Peck, J. and K. Shell (2003) "Equilibrium bank runs." *Journal of Political Economy* 111(1): 103-123.

[30] Smith, B.D. (2002) "Monetary policy, banking crises, and the Friedman rule." *American Economic Review* 92: 128-134.

[31] Townsend, R. (1987) "Economic organization with limited communication." *American Economic Review*77: 954-971.

[32] Wallace, N. (1988) "Another attempt to explain an illiquid banking system: the Diamond and Dybvig model with sequential service taken seriously," *Federal Reserve Bank of Minneapolis Quarterly Review*, 12(4): 3-16.