Global Imbalances, Geography of Bubble Holding, and the Boom–Bust Cycle

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Abstract

We describe a simple model that explains global imbalances and the boom-bust cycle that have taken place over the past decade from the lens of rational bubble model. Four factors that we address are (i) the difference in a country's financial depth, (ii) the increase in savings in emerging countries, (iii) the emergence and bursting of asset bubbles, and (iv) the geography of the holding of bubbly assets. The developed model captures well the reality of the bubbly boom that has actually occurred. The global imbalance involves the emergence of bubbles, the boost to savings of emerging countries, high leverage and credit growth of advanced countries. The quantitative analysis shows that the geography of the holding of bubbly assets as well as the difference in the financial depth are necessary to explain global imbalances and the great recession in 2009.

Keywords: current account; rational bubbles; high savings; pledgeability

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1. Introduction

Imbalances in the current accounts have been accompanied by a great swing from a boom to a recession, particularly the great recession in advanced countries. Behind the imbalances, capital flows of savings glut emerging countries supposedly fueled credit growth and housing booms that were indeed unsustainable and ended in the financial crisis (e.g., Caballero and Krishnamurthy 2009). What we have witnessed is that on the onset of the crisis advanced countries were hit harder.

Two observations make the direct link between global imbalances and the financial crisis unclear. The large current account deficit of the US was not a cause of abrupt dollar depreciation unlike the prediction from the "intertemporal approach to the current account".[2] Next, the crisis hit deficit and surplus countries alike in the US and Europe, suggesting the working of some other force than global imbalances, such as the global flow of asset-backed securities (e.g., Obstfeld 2012). Whether the crisis can be directly attributed either to external factors such as high savings of Asian countries or internal factors such as the development of securitization is controversial.

The approach taken here is based on the idea that the systematic analysis of the boom and bust will uncover how the financial crisis is related to global imbalances. Figure 1 provides a clue to think about how related are the global boom and imbalances. Two upward graphes are measures of financial depth of advanced countries and the rest of the world(ROW), that are caluculated as the ratio of private credit by deposit banks and other financial institutions to GDP, as reported in Beck et al(2000). The external finance measure of advanced is shown to be higher than the other over time, implying

[2] In the face of 6 percent of the current account deficit per GDP, Obstfeld and Rogoff (2005a, 2005b), Blanchard *et al* (2005), and Edward (2005) argue against the current account sustainability, estimating the large deprecation of either real or nominal exchange rate is necessary to rebalance the current account.

the persistent difference in financial development. On the other hand, the downward graph, representing the output share of advanced in the world output is declining over time. The current account deficit of advanced is also worsening; it was almost zero in the 1990s and reached 1.5 perecnt as a ratio of GDP in 2007.

This figure seems to give an intuitive story on global imbalances. Serious borrowing constraints creates the demand shortage in emerging countries, and hence the imbalance of the current account with advanced countries. However, two questions emerge. The first concern is how to explain the fact that emerging countries with demand shortage grew faster than advanced countries given the difference in financial depth. The literature on international capital flows that stresses the role of the difference in a country's financial depth falls short of explaining convergence.[3] Indeed, emerging countries of current account surpluses have realized fast economic growth (e.g., Prasad et al. 2007).

The second concern is how to explain the fact that the crisis hit deficit and surplus countries alike in advanced countries. Although that literature can gives a foundation for explaining global imbalances, it also suggests that as sudden stops theoires predict (e.g., Mendoza 2010), economic damages from exogeneous shocks should be related to the country's external positions.

Asset backed commercial papers(ABCP) have played a central role in the early phase of the financial crisis. Acharya and Schnabl(2010) report that ABCP conduits set up by large commercial banks are concentrated on advanced countries. The geography

[3] This literature rather suggests divergence of income and wealth as a result of financial globalization. According to this explanation, low-income countries with immature financial markets should plunge into poverty through running capital outflows (e.g., Gertler and Rogoff 1991, Boyd and Smith 1997, Sakuragawa and Hamada 2001, Matsuyama 2004, and others).

of the holding of seemingly safe assets that turn out to be risky is one dimension to overcome this shortcoming.

In this paper we describe a simple model that explains global imbalances and the boom-bust cycle that have taken place over the past decade from the lens of rational bubble model. Four key factors we address are (i) the differences in a country's financial depth, (ii) the increase in savings in emerging countries, (iii) the emergence and bursting of asset bubbles, and (iv) the geography of the holding of bubbly assets.

Contoributions are twofold. The model captures well the reality of the bubbly boom that has actually occurred. The global imbalance involves the emergence of bubbles, the boost to savings of emerging countries, high leverage and credit growth of advanced countries. The imbalance is followed by the global boom that has been acconmapnied by the convergence of low-income countries toward high-income countries. In addition, bubbles are complementary to investments of both of the deficit and surplus countries.

The quantitative analysis shows that the geography of the holding of bubbly assets as well as the difference in the financial depth are necessary to explain global imbalances and the great recession in 2009. The model match data well given that 75 percent of bubbles are held by advanced countries that occupy 50 percent of output share in the world.

In this paper, we build on a number of earlier contributions to this important topic. To start, there is an extensive existing literature on current account imbalances. For instance, Caballero et al. (2008) and Mendoza et al. (2009) developed the idea that global imbalances resulted from differences in the financial depth of countries. However, while these differences in financial depth can help explain global imbalances, they are unable to explain either the recent global boom or the severe recessions in most

advanced countries. Our model also closely relates to a literature that addresses the allocation puzzle. For example, Buera and Shin (2010) explain this by combining increased domestic savings supported by the self-financing motive of financially constrained entrepreneurs with large-scale economic reforms inducing the reallocation of capital among firms with different productivities. Elsewhere, Song et al. (2011) explain the phenomenon as an outcome of a transition economy, where resources are reallocated to the sector that is more productive, but subject to a borrowing constraint.[5]

Our analysis is also related to an emerging literature that explains the transmission mechanism that generates the investment boom when bubbles emerge, including work by Farhi and Tirole (2012) and Martin and Ventura (2012), among others.[6] All these studies focus on bubbles in a single country model, the important shortcoming of which is that the bubbly boom is accompanied by a decrease in leverage. The multiple-country version of the bubbly economy can explain the investment boom of countries that benefit from the increase in leverage.

Finally, this paper is related to some literature relating asset bubbles to capital flows. That includes Caballero and Krishnamurthy (2006), and Hellwig and Lorenzoni (2009).

The remainder of the paper is organized as follows. In Section 2, we construct our model. Sections 3 describes the closed economy. Section 4 examines a two-country rational bubble model with financial globalization. In Section 5, we introduce the geography of the holding of bubbles into the model. In Section 6 we conduct simulations.

[5] Matsuyama (2014) provides a model that sheds a light on the allocation puzzle by developing the mechanism that links input-saving technological progress with endogenous pledgeability.
[6] There are excellent contributions that investigate the role of store of values in macroeconomics, including Tirole (1985), Woodford (1990), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1998).

2. The Basic Model

Consider an overlapping-generations economy over an infinite horizon. In each period, a unit mass of agents are born and live for three periods. Agents born in period *t* receive the labor income θe_t when young and $(1-\theta)e_{t+1}$ when middle-aged, with the growing factor $e_t = (1+g)^t e$. There is no population growth. We describe the preferences of agents by $\log c_{t-1}^y + \beta \log c_t^m + \beta^2 \log c_{t+1}^o$, where c_{t-1}^y is consumption when young, c_t^m in middle age, and c_{t+1}^0 in old age and $\beta(\leq 1)$ is the discount factor. Each generation is indexed by the period in which it is middle-aged. As an owner of the firm, each of middle-aged agents has access to one linear investment project that transforms one unit of a good into $R^f (>1+g)$ units of the good after one period. To motivate financial market imperfections, we assume that only part of the return, $R(< R^f)$, is able to be pledged to creditors. Limited liability protects the debtors. Assume that $R<1+g$, which is necessary for the borrowing constraint to be binding at the bubbly steady state, as will become obvious below.

3. The Closed Economy

Let us define young and middle-aged savings and investment as variables divided by the growth factor $(1+g)^t e$. Letting r_t denote the rate of return on assets that prevails between period *t*–1 and *t*, young agents born at period t–1 choose savings as

(1a) $$s_{t-1}^{y} = \frac{1}{(1+\beta)(1+r_t)}\{s_t^m - (1-\theta) + \beta\theta\frac{1+r_t}{1+g}\},$$

given the expected middle-aged savings s_t^m. Once they have become middle-aged, they choose savings as

(1b) $$s_t^m = \frac{\beta}{1+\beta}(\frac{1+r_t}{1+g}s_{t-1}^{y} + 1 - \theta).$$

Young and middle-aged savings are finnaly written as

(2a) $$s_t^{y} = \frac{\theta}{1+\beta+\beta^2}\{\beta(1+\beta) - \frac{(1+g)\omega}{1+r_{t+1}}\}, \text{ and}$$

(2b) $$s_t^m = \frac{\beta^2\theta}{1+\beta+\beta^2}(\frac{1+r_t}{1+g} + \omega),$$

where we set $\omega \equiv (1-\theta)/\theta$, capturing the labor income growth. Young savings are increasing in the expected interest rate and middle-aged savings are increasing in the current interest rate. Noteworthy, (2a) has an interpretation of also the saving rate that is increasing in the interest rate; even under the log utility, for a change in the interest rate the substitution effect is stronger than the income effect due to the labor income growth factor $(1+g)\omega$.[7]

A middle-aged agent begins the firm by investing i_t in the project. Given the interest rate prevailing between t and t+1, r_{t+1}, the agent is willing to start the firm if $R^f \geq 1+r_{t+1}$, which we call the *profitability constraint*. The amount of investment i_t is funded by the internal wealth s_t^m and the security issued by the firm, $(i_t - s_t^m)$, where the latter is limited to the present value of the pledged asset $Ri_t/(1+r_{t+1})$:

(3) $$i_t - s_t^m \leq Ri_t/(1+r_{t+1}).$$

[7] This formulation makes the analysis dramatically simple. One can alternatively use the CRRA utility function to derive the saving rate function that is increasing in the interest rate, but then the clear-cut results are hard to obtain.

Equation (3) states that the insurance of the security is limited to the pledged asset. We refer to this inequality as the *borrowing constraint*.

The investment of borrowing-constrained firms is written as a multiple of internal wealth s_t^m and leverage $1/\{1-R/(1+r_{t+1})\}$:

$$i_t = \frac{s_t^m}{1-R/(1+r_{t+1})}. \tag{4}$$

This equation reveals two distinctive routes through which a rising interest rate yields impacts on investment. On the one hand, a rise in the period t+1 interest rate decreases leverage, reducing investment financed by borrowing. On the other hand, a rise in the period t interest rate increases the net worth of the firm, stimulating investment financed by borrowing. We call the former *the leverage effect,* and the latter *the balance sheet effect*.

Market clearing in the capital market requires

$$s^y(r_{t+1}) + s^m(r_t) = i_t, \tag{5}$$

which states that young savings of generation *t*+1 and the middle-aged savings of generation *t* fund the investment of generation *t*.

We define the competitive equilibrium of a closed economy as a sequence $\{i_t, r_t\}_{t=0}^{\infty}$ that satisfies (1), (2), (4), and (5). We use these equations to describe finally the excess demand function as

$$ED(r_t, r_{t+1}) \equiv \{\beta(1+\beta) - \frac{(1+g)\omega}{1+r_{t+1}}\},, \tag{6}$$

where the first term represents the net demand for funds by middle-aged debtors and the second the supply of funds by young creditors. $ED(r,r)$ is strictly decreasing.

We focus on the case when the borrowing constraint is binding. As we prove in the Appendix, the borrowing constraint is binding in equilibrium if $ED(R^f - 1, R^f - 1) < 0$, or

(A) $$\frac{\beta^2 R}{R^f - R}\left(\frac{R^f}{1+g} + \omega\right) - \left\{\beta(1+\beta) - \frac{(1+g)\omega}{R^f}\right\} < 0$$

Otherwise, the profitability constraint should be binding with equality, where the interest rate is set to $1 + r_{t+1} = R^f$. Bubbles never arise due to $R^f > 1$. The investment realizes the first level: $i^{FB} \equiv s^y(R^f - 1) + s^m(R^f - 1)$.

Proposition 1: Suppose that (A) holds. A competitive equilibrium of a closed economy that satisfies the binding borrowing constraint then exists. The competitive equilibrium is dynamically stable. The steady-state interest rate denoted as $r(R)$ is increasing in the pledgeable return R and the labor income growth factor ω.

4. Bubbles, Global Imbalances, and the Global Boom

A widespread belief is that capital flows from savings-glut countries contribute in the US to the emergence of housing bubbles and the creation of liquidity. We describe this situation by introducing rational bubbles in the model. As is well known in the literature (e.g., Samuelson 1958, Diamond 1965), an interest rate that is lower than the growth rate creates a demand for liquidity. Rational bubbles can then emerge as liquidity when investors rationally anticipate that investors in the next generation will hold any bubble assets sold to them, even if overvalued relative to the underlying fundamentals, which are zero in this model.

Figure 3 illustrates several interest rates and the economic growth rate averaged over G7 countries (the US, the UK, Japan, Germany, France, Canada, and Italy).[8] Until

[8] Each of the interest rates and the economic growth rate presented is a simple average of G-7

the end of the 1990s, all the interest rates have been greater than the economic growth rate with the exception of the deposit rate, but this tendency changed around 2000, when some of interest rates were lower than the growth rates. Finally, with the exception of the long-term Treasury bond, all the interest rates have been lower than the growth rate from 2004 to 2008. Remarkably, in this interval the current account deficit is high; the deficit peaked at 6 percent in 2005–6.

The global economy consists of country E with immature financial markets and country A with developed financial markets, each of which has size of measure one. The variables for country E are denoted by the superscript "E", and those of country A by superscript "A". We identify the heterogeneity in each country's financial depth as the difference in the pledgeable return, i.e., $R^E < R^A$. Financial development is typically measured by the ratio of external finnce to GDP. For example, when external finance is measured by private credit by deposit banks and other financial institutions as reported in Beck et al(2000), the average of this ratio for the US during the 1990-2005 period is 1.74. The average for five emerging countries that run large current account surpluses, including China, Hong Kong, Korea, Singapore, Botswana, is 0.92 during the same interval.

Letting r_{t+1} denote the global interest rate, investment in each country is

(7a) $$i_t^E = \frac{s^m(r_t)}{1 - R^E/(1+r_{t+1})}, \text{ and}$$

countries (*source:* IFS). We exclude the 1991 German data. The money market rate, treasury bills, treasury bills of 3 years or longer, the deposit rate, and maximum overdraft reflect the call rate, the short-term rate on the government bond, the long-term rate on the government bond, the short-term deposit interest rate, and the loan interest rate, respectively.

(7b) $$i_t^A = \frac{s^m(r_t)}{1 - R^A/(1+r_{t+1})}.$$

Letting b_t denote global bubbles, bubbles evolve as

(8) $$(1+g)b_{t+1} = (1+r_{t+1})b_t.$$

Note that $b_t \geq 0$; that is, we exclude the possibility of negative bubbles.

Agents use savings to fund investments and asset bubbles. Bubbles trade internationally. The equilibrium in the global capital market then requires

(9) $$2s^y(r_{t+1}) + 2s^m(r_t) = i_t^E + i_t^A + b_t.$$

First of all, we define the competitive equilibrium of a bubbleless global economy that satisfies the binding borrowing constraint, as a sequence $\{i_t^E, i_t^A, r_t\}_{t=0}^{\infty}$ that satisfies (7a), (7b), and (9), with $b_t = 0$.

The bubbleless steady state, denoted $\{(i^E)^*, (i^A)^*, r^*\}$, is represented as

(10a) $$(i^E)^* = \frac{s^m(g)}{1 - R^E/(1+r^*)},$$ (10b) $$(i^A)^* = \frac{s^m(g)}{1 - R^A/(1+r^*)},$$ and

(10c) $$2s^y(r^*) + 2s^m(r^*) = (i^E)^* + (i^A)^*$$

We see from (10a) and (10b) that the difference in pledgeability leads to the difference in investment. A country with more mature financial markets realize a higher output $\theta R^f i^j/(1+g)^2 + 1$.

We turn to the question how financial globalization allocate investment across countries. The existing literature that stresses the role of the difference in the country's financial depth has argued that international capital flows gives reise to the divergence of investment and wealth and thus cannot explain the convergence. However, taking

into account the balance sheet effect through the change in the interest rate, this may be not the case. It is because whether investment rises or fall in each country depends on the relative strength of the leverage and balance sheet effects.

We investigate whether financial globalization promotes or represss investment of each country. The steady state investment of the closed economy is rewritten as $i^D = \frac{s^y(r^D) + s^m(\beta)\omega}{1 - Rs^m(\beta)/s^y(r^D)(1+g)}$, where note that $s^m(\beta) \equiv \beta^2/(1+\beta+\beta^2)$ is the middle-aged saving rate. On the other hand, the steady state investment of the global economy is rewritten as $i^* = \frac{s^m(\beta)(\frac{1+r^*}{1+g} + \omega)}{1 - R/(1+r^*)}$. We compare these two by

$$(11) \quad i^* - i^D = \frac{(1+g)\{s^m(\beta)\frac{1+r^*}{1+g} - s^y(r^D)\}\{(1+r^*-R)s^y(r^D) - Rs^m(\beta)(\frac{1+r^*}{1+g} + \omega)\}}{(1+r^*-R)\{(1+g)s^y(r^D) - Rs^m(\beta)\}}.$$

The following is useful for the analysis.

Result 1: The inequality $(1+r^*-R)s^y(r^D) > Rs^m(\beta)(\frac{1+r^*}{1+g} + \omega)$ holds if and only if $r(R) < r^*$.

Proof: We have $1 + r(R) = \frac{R\{s^y(r^D) + s^m(\beta)\omega\}}{s^y(r^D) - Rs^m(\beta)/(1+g)}$. If $r(R) < r^*$, rearranging the latter equation yields $(1+r^*-R)s^y(r^D) > Rs^m(\beta)(\frac{1+r^*}{1+g} + \omega)$. Conversely, $(1+r^*-R)s^y(r^D) < Rs^m(\beta)(\frac{1+r^*}{1+g} + \omega)$ requires $r^D > r^*$. Q.E.D.

The inequality $1 + r^* > R$ should hold in equilibria with binding borrowing constraints and in addition $(1+g)s^y(r^D) > Rs^m(\beta)$ should hold, guaranteeing the positively

valued investment. Thus it follows from (11) and Result 1 that when $r^D < r^*$, opening stimulates investment if and only if $s^m(\beta)(1+r^*) > (1+g)s^y(r^D)$ holds. As Figure 2 illustrates, opening the capital account stimulates the investment, increase savings even more, with being in surplus of the current account. Given this, we establish the following.

Proposition 2: Suppose that

(B) $$\beta^2\left(\frac{R^E}{1+g-R^E}+\frac{R^A}{1+g-R^A}\right)\left(\frac{R^f}{1+g}+\omega\right)-\left\{\beta(1+\beta)-\frac{(1+g)\omega}{R^f}\right\}<0$$

holds. When financial markets are globally integrated, the global interest rate r^* lies between the autarkic interest rates of the two countries, i.e., $r(R^E) < r^* < r(R^A)$. In addition, we have the following three patterns of investment allocation.

(I) Country E realizes smaller investment, and country A realizes larger investment if

$$\frac{(1+g)\omega}{1-\beta r^*} < \beta\{1+r(R^E)\}.$$

(II) Both countries E and A realize larger investment if

$$\beta\{1+r(R^E)\} < \frac{(1+g)\omega}{1-\beta r^*} < \beta\{1+r(R^A)\}.$$

(III) Country E realizes larger investment, and country A realizes smaller investment if

$$\beta\{1+r(R^A)\} < \frac{(1+g)\omega}{1-\beta r^*}.$$

Proof: The function $ED^j(r,r)$ is monotone decreasing, and satisfies $ED^E(r,r) < ED^A(r,r)$. Two autarkic interest rate satisfy $ED^E(r(R^E),r(R^E))=0$, and $ED^A(r(R^A),r(R^A))=0$, with $r(R^E) < r(R^A)$. The market clearing in the global market requires $ED^E(r,r)+ED^A(r,r)=0$. The equality holds only if $r(R^E) < r^* < r(R^A)$. The latter part is straightforward from the argument just before Proposition 2.

International capital flows give rise to three patterns of investment allocation, divergence, boom, and convergence. The case (I) corresponds to the case familiar in the literature that stresses the difference in the financial depth; international capital flows lead to the divergence of wealth across countries. In case (II), financial globalization stimulates investments of both the deficit and surplus countries through different channels in each country. The change in the interest rate driven by capital flows stimulates the investment in the surplus emerging country through the improvement of balance sheets, but stimulates the investment in the deficit advanced country through the increase in leverage. In case (III) convergence arises, but it does in the unfamiliar manner.

We turn to the bubbly economy. We define the competitive equilibrium of a bubbly global economy that satisfies the binding borrowing constraint, as a sequence $\{i_t^E, i_t^A, b_t, r_t\}_{t=0}^{\infty}$ that satisfies (7a), (7b), (8), and (9). The bubbly steady state, denoted $\{(i^E)^{**}, (i^A)^{**}, b^{**} r^{**}\}$, is represented as

(12a) $(i^E)^{**} = \dfrac{s^m(g)}{1 - R^E/(1+g)}$, (12b) $(i^A)^{**} = \dfrac{s^m(g)}{1 - R^A/(1+g)}$,

(12c) $b^{**} = 2\dfrac{\beta(1+\beta) - \omega}{1+\beta+\beta^2} - \dfrac{\beta^2}{1+\beta+\beta^2} s^m(g)\left(\dfrac{R^E}{1+g-R^E} + \dfrac{R^A}{1+g-R^A}\right)$, and

(12d) $r^{**} = g$.

Lettting $ED^j(r_t, r_{t+1})$ denote the excess demand function for funds in country $j(j = E, A)$, the condition for the existence of the bubbly equilibrium is written as $\sum_{j=E,A} ED^j(g, g) < 0$, or

(C) $$\beta^2\left(\frac{R^E}{1+g-R^E} + \frac{R^A}{1+g-R^A}\right) - 2\{\beta(1+\beta) - \omega\} < 0.$$

This condition is equivalent to the standard condition for the emergence of bubbles: that is, the steady state interest rate of the bubbleless global economy r^* is less than the economic growth rate g. Condition(C) requires $r(R^E) < g$, implying that saving rates of country E rise when bubbles emerge. A low pledgeability of either one country or both can lead to the emergence of asset bubbles, but high saving rates may contribute to the emergence of bubbles or may not, depending on pledgeability. Strictly we have

$$\frac{d(ED^E + ED^A)}{d\beta} = \{\beta\left(\frac{R^E}{1+g-R^E} + \frac{R^A}{1+g-R^A} - 1\right) - 1\}\frac{2}{1+\beta+\beta^2}.$$

A high β and thus high saving rates, combined with low pledgeability, are more likely to contribute to the emergence of bubbles, supporting the widespread view that high savings of Asian countries have contributed to the housing bubbles in the US.

In the presence of bubble assets, the current account of country $j(j = E, A)$ is expressed as $CA^j(r_t, r_{t+1}) \equiv (1+g)\{s^y(r_{t+1}) + s^m(r_t) - i_t^j - b_t^j\}$ $-\{s^y(r_t) + s^m(r_{t-1}) - i_{t-1}^j - b_{t-1}^j\}$, and at the steady state $CA^j = g\{s^y(r) + s^m(r) - i^j - b^j\}$, where b_t^j are held by agents in country j, and $b_t^E + b_t^A = b_t$. From Proposition 4 and the equality of savings across countries, in the absence of bubbles $CA^E > CA^A$ always holds.

By the indentity $CA^E + CA^A = 0$, a country with high leverage runs the current account deficit.

However, in the bubbly economy this may not be the case. Not only the leverage but also the geography of the holdings of bubbles affect the external position of a country. A country tends to run the larger current account deficit if the investors hold larger bubbles. It is important to to note that both of the deficit and surplus countries can hold bubbles.

Proposition 3: (a) In the competitive equilibrium of a bubbleless global economy, country A(E) with high(low) plefdgeability runs a current account deficit (surplus). (b) In the competitive equilibrium of a bubbly global economy, country A with high plefdgeability runs a current account deficit if at least a half of bubbles are held by the residence.

A more dispersion in pledgeability or a more geographically biased holding of bubbles can lead to the larger current account imbalance. The more mature financial markets a country has, the more bubbles the residence of that country has, the country runs the larger deficit, In addition, a boost to savings, combined with the dispersion in the leverage, can lead to the larger imbalance. The latter finding can support the widespread view that high savings of Asian countries contributed to the "global imbalances", behind which deficit advanced countries have high leverage, and surplus emerging countries have low leverage.

Condition (C) allows for the case of $r(R^E) < g < r(R^A)$ as well as $r(R^E) < r(R^A) < g$. The former case will be relevant to the reality because it explains why the bubbles

emerge in advanced countries with mature financial markets along with the development of financial globalization. The demand shortage in emerging countries, which arises due to serious borrowing constraints, leads to the imbalance in the current account with advanced countries, and along with the decline in the real interest rate the demand for liquidity, which provides a room for the emergence of asset bubbles.

One remarkable observation was that the leverage of financial intermediaries of the US was procyclical during the cycle from the boom to the recession (e.g., Adrian and Shin 2008).

Proposition 4: Suppose that Condition (C) holds and and the autarkic interest rate in country A is so high to satisfy $g < r(R^A)$. In the competitive equilibrium of a global bubbly economy, country A realizes the higher leverage than the bubbleless autarkic economy.

High savings and sufficiently low pledeagility in the other country supports this environment, in which a country with matured financial markets runs the increase in leverage and the holding of bubble assets. This finding is contrasted with the feature of the one-country version of the rational bubble model, in which the bubbly boom should be accompanied by a decrease in leverage (e.g., Farhi and Tirole 2012, Martin and Ventura 2012, and others).

The next conecen is when the financial globalization is followed by the emergence of bubbles, how international capital flows allocates the resource across countries, The global interest rate does not necessarily fall into between the two autarkic rate, and we

have more patterns of allocation than the bubbleless economy. We establish the following.

Proposition 5: Suppose that (C) holds, and that bubbles emerge when the global financial market is established.

(I) Country E realizes smaller investment, and country A realizes larger(smaller) investment if $g < (>) r(R^A)$ and

$$(1+g)\omega < \beta\{1+r(R^E)\}.$$

(II) Country E realizes larger investment, and country A realizes larger(smaller) investment if $g < (>) r(R^A)$ and

$$\beta\{1+r(R^E)\} < (1+g)\omega < \beta\{1+r(R^A)\}.$$

(III) Country E realizes larger investment, and country A realizes larger(smaller) investment if $r(R^A) < (>) g$ and

$$\beta\{1+r(R^A)\} < (1+g)\omega.$$

Qualitative properties are similar to Proposition 2 except for one respects. Three cases are distinguished in terms of paprameters, R^j, ω, and g. The case (I) is more likely to arise if both of pledgeability are high, the case (III) is more likely to arise if both are low, and the case (II) tends to arise if pledgeability of country A is high and that of country E is low.

Case (II) (when $g < r(R^A)$) seems to capture well the reality of the bubbly boom that has actually occurred. Global imbalances involve the emergence of bubbles, the boost to savings of emerging countries, high leverage and credit growth of advanced countries. Behind global imbalances, emerging countries run current account surpluses and realize fast growth in investment and output (e.g., Prasad et al. 2007). This observation has received attention in terms of the so-called "allocation puzzle"; among developing countries, countries with large current account surpluses have realized the fast growth in total factor productivity (e.g., Gourinchas and Jeanne 2014). This case can explain the global investment boom that has been acconmapnied by the convergence of low-income countries toward high-income countries given the difference in the financial depth.

The case (II) is conspicuous in that bubbles are complementary to investments of both of the deficit and surplus countries. This finding is contrasted with the recent several contributions that explain the crowding-in in the one-country version of the rational bubble model.

Before going to the next section, we summarize some dynamic properties. The bubbleless equilibrium is dynamically stable, and the bubbly equilibrium is saddle-path stable.

Proposition 6: Suppose that (B) holds. (a) There exists a competitive equilibrium of the global economy that satisfies the binding borrowing constraint. (b) The competitive equilibrium is dynamically stable. (c) The global investment defined by $i_t^E + i_t^A$ is dynamically stable, and increasing (decreasing) if $i_t^E + i_t^A < (>)(i^E)^* + (i^A)^*$.

Proposition 7: Suppose that Condition (C) holds. In the competitive equilibrium of a global bubbly economy, there exists a bubbly steady state that satisfies the binding borrowing constraint. There exists a unique competitive equilibrium that converges to the bubbly steady state, given the initial condition $i_{t-1}^E + i_{t-1}^A$.

Proof: See the Appendix.

5. Bursting of Bubbles and the Geography of Bubble Holding

We now consider the impacts of the bursting of asset bubbles on the global economy. The investor's portfolio between securities and bubbles is expected to influence the recession. However, investors are indifferent between holding these two assets, and intrinsically there is no mechanism that determines the portfolio in the model. Investors in both countries have the same preferences and endowments. As a benchmark we first examine when all investors hold the same asset portfolio.

It is useful to apply an alternative expression in order to describe the effects of the bursting of bubbles on investment dynamics. From (3) and (9), the middle-aged income of each country is given by $0.5(R^E i_{t-1}^E + R^A i_{t-1}^A)/(1+g) + 0.5b_t + \omega$ that consists of a claim to the security, bubbles, and the endowment. They save a constant frcation of that income:

$$s_t^m = \frac{\beta}{1+\beta}\{\frac{R^E i_{t-1}^E + R^A i_{t-1}^A}{2(1+g)} + \frac{b_t}{2} + \omega\}. \tag{13}$$

Combining (13) with (2) yields the demand function for the security issued by firms:

$$\frac{\beta+\beta^2}{1+\beta+\beta^2}(\frac{1+r_{t+1}}{1+g} + \omega) = \frac{R^E i_t^E + R^A i_t^A}{2(1+g)} + \frac{b_{t+1}}{2} + \omega. \tag{14}$$

The investment of firms is constrained by the security secured by the pledgeable asset $R^j i_t^j/(1+r_{t+1})$ $(j=E,A)$ and the internal wealth given by (13):

(15a) $$i_t^E = \frac{R^E i_t^E}{1+r_{t+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i_{t-1}^E + R^A i_{t-1}^A}{2(1+g)} + \frac{b_t}{2} + \omega\}, \text{ and}$$

(15b) $$i_t^A = \frac{R^A i_t^A}{1+r_{t+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i_{t-1}^E + R^A i_{t-1}^A}{2(1+g)} + \frac{b_t}{2} + \omega\}.$$

A set of equations (9), (14), (15a), and (15b) redefines the competitive equilibrium of the bubbly global economy.

Suppose that bubbles burst at period T for the timing when the middle-aged hold bubbles. Once bubbles burst, they lose part of their assets. Their savings are now $\frac{\beta}{1+\beta}\{\frac{R^E i_{T-1}^E + R^A i_{T-1}^A}{2(1+g)} + \omega\}$, which is less than (13). For periods after T, the demand and supply functions are written as

(16) $$\frac{\beta+\beta^2}{1+\beta+\beta^2}(\frac{1+r_{t+1}}{1+g} + \omega) = \frac{R^E i_t^E + R^A i_t^A}{2(1+g)} + \omega,$$

(17a) $$i_t^E = \frac{R^E i_t^E}{1+r_{t+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i_{t-1}^E + R^A i_{t-1}^A}{2(1+g)} + \omega\}, \text{ and}$$

(17b) $$i_t^A = \frac{R^A i_t^A}{1+r_{t+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i_{t-1}^E + R^A i_{t-1}^A}{2(1+g)} + \omega\}.$$

Equations (16), (17a), and (17b) fully describe the equilibrium for $t \geq T$ after the bubbles burst, given the initial condition $\{i_{T_c-1}^E, i_{T_c-1}^A, r_{T_c}\}$. This equilibrium is an alternative expression for the bubbleless global equilibrium.

We now examine which country is more severely affected by the bursting of bubbles.. We define the decline in investment by the ratio of investment just after when bubbles burst relative to the investment at the bubbly stetady state:

$$\frac{i_T^j}{(i^j)^{**}} = \underbrace{\frac{s^m(r_T)}{s^m(g)}}_{\textit{Balance sheet channel}} \times \underbrace{\frac{1-R^j/(1+g)}{1-R^j/(1+r_{T+1}^e)}}_{\textit{Leverage channel}}, \quad (j=E,A). \tag{18}$$

The change in investment arises through two channels. Looking at (14) and (16), we find the decline in the interest rate: $r_T < g$. In addition, when people rationally expect the future interest rate, we have $r_{T+1} = r_{T+1}^e < r_T$: the interest rate at period T+1 declines even more. The balance sheet effect works adversely, but the leverage effect works favorably. The balance sheet channel works commonly to all countries, but the leverage channel works more strongly for advanced countries. The following is established.

Proposition 8: Suppose that (C) holds. When all investors have the same asset portfolio, and rationally expect the future interest rate, the investment of country E declines more strongly than the investment of country A at the period when bubbles burst.

However, it is doubtful whether paniced people make a rational decision at the early stage of the crash; people will soon recognize the depreciation in their net worth, but it may take some time to foresee the future path of the interest rate. The following two step expectation formation seems to explain well the investment bust at the early stage of the crisis. At the instant of the bursting of bubbles, people respond by expecting myopically that the future interest rate remains the same; $r_{T+1}^e = g$, and at the later stage

they revise their expectation to fully incorporate the effect of the decline in the interest rate. The following is established.

Proposition 9: Suppose that (C) holds. When all investors have the same asset portfolio, and have the myopic expectation on the interest rate at the early stage of the crisis, the decline in investment is the same across all countries in terms of the growth rate during this interval of the period when bubbles burst.

Proposisitions 8 and 9 imply that countries with mature financial markets are hit less adversely to the extent that the leverage channel works. This finding falls short of explaining the reality in bust; what we have witnessed is that advanced countries have been affected more adversely.

To overcome this shortcoming, we introduce the geography of the holding of bubbles into the model. The decline in the interest rate driven by the global imbalance does not always create bubbles universally. The creation of bubbles requires also people's belief that may or may not be uniform; in reality the location of the emergence of bubbles and the holding of bubbles were concentrated on advanced countries.[11]

We focus on the geography of bubble holding by assuming that all agents hold bubbly assets in country A, but a proportion λ $(0 \leq \lambda \leq 1)$ of agents do not hold bubbly assets in country E. We later argue on the determinants of λ. We have already

[11] Acharya and Schnabl(2010) report that asset-backed commercial paper(ABCP) conduits set up by large commercial banks are concentrated on advanced countries that run either surplus or deficit of the current account. ABCP plays a central role in the early phase of the financial crisis, and is only one aspect of the financial crisis, but the location on ABCP conduits will be closely related to the geography of bubbly assets.

investigated the case for $\lambda = 0$ when all investors hold bubbles equally. In other polar case for $\lambda = 1$, investors in country A hold bubbles, but those in country E do not.

As long as bubbles persist, the asset portfolio has no any impact on the economy, , and (9), (14), (15a), and (15b) continue to express the allocation. However, once bubbles burst, the difference in the asset portfolio will affect the real allocation.

When bubbles burst at period T, the aggregate wealth of the middle-aged decline to $(R^E i^E_{T-1} + R^A i^A_{T-1})/(1+g) + 2\omega$. The demand function for the security remains (16). From the linear property of the supply function, the two supply functions are written finally as

(19a) $$i^E_T = \frac{R^E i^E_T}{1+r_{T+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i^E_{T-1} + R^A i^A_{T-1}}{2(1+g)} + \frac{\lambda b_T}{2(2-\lambda)} + \omega\}$$, and

(19b) $$i^A_T = \frac{R^A i^A_T}{1+r_{T+1}} + \frac{\beta}{1+\beta}\{\frac{R^E i^E_{T-1} + R^A i^A_{T-1}}{2(1+g)} - \frac{\lambda b_T}{2(2-\lambda)} + \omega\}$$.[12]

The difference of these equations from (17a) and (17b) is that bubble terms appear in the RHS's, with the opposite sign, implying that the size of bursted bubbles and its geography have significant impacts on the investment bust. The biased holding of bubbles results in a transfer of wealth from those who hold bubbles to those who do not, having impacts on investments in the opposite directions.

The bursting of bubbles can hit countries that hold more bubbles harder than countries holding fewer bubbles. This explains why the US with mature financial markets was hit hard; investors in the US held huge assets that resulted in being backed by bubbles.

[12] Note that i^E_T is now the investment per generation in country E.

6. Simulations

We use the developed model to conduct simulations. Table 1 depict some information around the Rehman crises for three regions, the US, advanced countries excluding the US, and the rest of the world(ROW). Countires selected as the advanced are listed in the table. Saving rates are different across regions; The saving rate of the US is lowest, and that of ROW is the highest. The current account deficit of the US is large, but the advanced excluding the US was not so large. Growth rates of GDP and investment in 2009 reveals the unbalanced bust across regions; advanced countries were hit harder.

Parameters selected are listed in Table 2. We take one period to be fifteen years. People work 45 years, and live 65 years, including 20 years for the childhood. We set the discout factors to match the saving rates. The discout factor of the US is lower than other regions, reflecting the US's low saving rate. We use the labor income growth factor ω as a free parameter to match data. We set $\omega = 0.45$, implying that middle-aged agents work the almost former half time as labour, and the latter half as entrepreneurs, establishing their own firms. The parameter choice for pledgeability needs some argument. We can rewrite (4), in the stock term, as

$$Asset_t = \frac{1}{1 - R/(1+r_t)} \times Internal \quad Wealth_t .$$

By definition, we have $Asset_t = Internal \quad Wealth_t \times External \quad Finance_t$. From these two we finally have

$$\frac{Asset_t}{GDP_t} = \frac{External \quad Finance_t}{GDP_t} \times \frac{1+r_t}{R} .$$

We find the value of the LHS from the familir relation: capital income share (α) = Return on capital $(R^f - 1)^{1/15} \times$ Capital. Since identically Capital=Wealth, we have

$Asset_t / GDP_t = 5.5$ by setting $\alpha = 0.33$. Our external finace measure is defined by the sum of private credit, stock market capitalization, and private bonds that are available in Beck et al(2000). Some values of market capitalization accrue to firm owners as inside equilty, and we assume that 70 % of them are accounted for as external finance We obtain the measure of each year by taking the weighted averages using the country's GDP share, and average them over the period 2000-2005. We exclude data for 2006-8 because stock market values are inflated by bubbles. The calculated external finace measure to GDP ratio of the US is the highest, 3.69, and that of ROW is the smallest, 0.90.[13] Given the interest rate $1+r_t(=1+g)$, we finally obtain the value for pledgeability. The size of regions is set to match the GDP share in 2008. This artifact is due to the model's limitation that the growth rate and the rate of physical return are the same for all.

We suppose that the world economy lies at the bubbly steady state in 2008 when the crisis occurs. Table 2 provides the results of some simulations. We define the investment bust as the decline in the investment from the investment at the bubbly steady state. We calculate the investment bust given the myopic expectation as in Proposition 9. We caluculate the output at the panic as the ratio of investment to output multiplied by the investment bust.

We show three simulations according to the distribution of bubble holding. The parenthesis (x, y, z) expresses that x percent of bubbles are held by the US, y percent are held by advanced(excluding the US), and z percent are held by ROW. In

[13] Countries used for constructing the external finace measure of ROW are Algeria, Argentina, Bangladesh, Brazil, Bulgaria, China, Colombia, Czech Republic, Egypt, Arab Rep., Hong Kong SAR, China, Hungary, India, Indonesia, Israel, Korea, Rep., Kuwait, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Poland, Romania, Russian Federation, Saudi Arabia, Singapore, Slovenia, Thailand, Turkey, Ukraine, and Vietnum. These countries occupy more than 95 percent of GDP in the total GDP of ROW.

setting the distribution paramters, we set the holding of the US to the total holding of advanced(including the US) to be 40 percent, that is, $x/(x+y)=0.4$, following Acharya and Schnabl (2010). They report that the US, as sponsors of ABCP, accounts for around 40 percent of the total outstanding of ABCP, that is perceived to have played an important role at the early stage of the great recession.

The case $(20,30,50)$ (Row 1) is the benchmark; people holds bubbles almost equally according to GDP, and the portfolio is not almost biased. The magnitudes of the investment and GDP busts are similar across regions, but do not match the data depicted in Row 4. The figures of the US and the advanced are underestimated, while the figure of ROW is overestimated. The current account fairly matchs data for the US, but not for advanced. "Globalization boom/bust" expresses how much the investment grows at the bubbly steady state relative to the bubbleless steady state under autarky. Put differently, this figure measures to what extent globalization followed by bubbles leads to the boom or bust. The results show that the US and ROW enjoy the bubbly boom, but the advanced runs the small bust. Given the large difference in pledgeability, financial globalization contributes to the convergence of low-income countries.

The case $(40,60,0)$ (Row 2) shows the case for most biased portfolio; 100 percent of bubble are held by the US and the advanced. The US and the advanced run the more than 20 percent of the investment bust, while ROW is shielded from the recession. The figures of the US and the advanced are overestimated, while the figure of ROW is underestimated. The current account deficit of the US is larger, reflecting larger holding of bubbles.

The case $(30,45,25)$ (Row 3) shows some intermediate case; 75 percent of bubbles are held by the US and the advanced. Fortunately, estimates for investment fairly match

the data. The cuurent account deficit of the US also matchs the data. Among 5.7 percent of the deficit, the saving less investment contributes by 3.7 percent and the holding of bubbles does by 2.0 percent. Developed financial markets and larger bubble holding contribute to the global imbalance. The low saving rate does not contribute to the deficit so much. The sign of the estimate in the advanced remains the opposite. Apart from the sign problem, the imbalance of the advanced is small relative to the US, but the magnitudes of the bust are almost equal, suggesting that the current account deficit and the magnitude of the recession are not directly related. If we divide the advanced into several groups, we may reproduce the case when the deficit and surplus countries are equaly hit hard.

We do some counterfactual experiments. The first experiment is the development of financial markets of ROW. Suppose that the pledgeability of ROW becomes twice from 0.22 to 0.44. There are two impacts. The investment bust become milder. The improvement in financial depth leads to the contraction of aggregate bubbles. The favorable effects are greater on the US and the advanced holding larger bubbles. Next, external positions improve for these two regions because they now hold smaller bubbles.

The second is the decline in the saving rates of ROW. Some Asian countries, including China, Hong Kong, Korea, Singapore, and Taiwan, has conrtibuited to high savings of ROW. The GDP-weighted average saving rate of these countries over 2005-2007 is 45.6 percent. Suppose that these countries have realized the same saving rate as other countries of ROW. Then the saving rate of ROW declines from 26.9 percent to 19.0 percent. The impacts are quatitatively similar to the first experiment except for that the boom of ROW becomes smaller.

The final experiment is the absence of convergence. The one of the model's features is that the investment bust is serious when the output share of countries of low pledgeability and high savings is high. Consider the case when financial globalization since 1990s did not lead to convergence. Suppose that the output share of the advanced is 65 perecent, the value in the 1980s. The impacts are similar to the first and second experiments.

All of the three factors, underdevelopment of financial markets of developing countries, saving glut, and the convergence, were sources of the great recession.

Two stories explaining he geography of the holding of bubbles

We provide two stories that explain the geography of the holding of bubbles. The first story stresses the role of the demand for bubbly assets by investors in advanced countries.

The occurrence of bubbles will depend on the people's belief toward the riskiness of bubbly assets. Bubbles will continue for some periods when people underestimate the riskiness of assets in question. The optimism to the financial innovation in the US may have contributed to the emergence and growth of bubbles. People may have had any groundless belief that securitization can eliminate the risk of asset price volatility to a significant level.

To motivate securitization, assume that bubbles emerge and crash at every period.[14] Formally, let δb^t denote bubbles that emerge at period t. At any period a fraction δ of existing bubbles crash so that at period t, $\delta(1-\delta)^k b^{t-k}$ units of bubbles survive among

[14] Martin and Ventura (2012) and Gali (2014) provide the formulation of the creation and crash of bubbles.

bubbles that emerged at period t-k. Newly born bubbles are attached to existing bubbles that did not crash. The aggregate bubbles at t are

$$b_t = \delta b^t + \delta(1-\delta)b^{t-1} + ... + \delta(1-\delta)^k b^{t-k} + ...$$

Each bubble is stochastic, but the aggregate bubbles are deterministic at any period. There is a room for financial intermediation to eliminate each idiosyncratic risk by pooling a large number of bubbles. A bank is viable as a risk sharing vehicle. This bank purchases bubbles from young agents with the same proportion as the aggregate distribution and issues the safe security backed by these bubbles to them. Additionally, suppose that all agents in country A have access to the monitoring technology to the bank's portfolio, but only a proportion λ $(0 \le \lambda \le 1)$ of agents do in country E. The attained bubbly equilibrium remains to be expressed by (7b), (8b), (8), and (9), with the feature of the biased holding of bubbles described in the previous section.[15]

We propose another story that focuses on the government's behavior of emerging countries. Capital flows from savings-glut countries supposedly contributed to the emergence of bubbles in the US, but these countries did not buy bubbly assets directly, such as repos and ABCPs created by US financial institutions; what they did buy were safe assets, such as US public debt (e.g., Bertaut et al. 2011). Governments of some emerging countries learned a lot from the Asian crisis; they invested the surpluses in US Treasuries to build up reserves to deal with possible sudden stops in capital flows (e.g., Aizennman and Marion 2003).

Tocapture this reality, we can assume that the government of country E prohibits the proportion λ of domestic agents from holding bubbly assets. Then the allocation

[15] Mendoza et al (2008) is an excellent work that considers the incompleteness of the insurance market and combine the financial depth with the asset portfolio.

becomes the same as the one in the first story. Here the parameter λ could be interpreted as the measure of excessive holding of safe assets.

From this interpretation, the pattern of the recession is history dependent; if there were no the Asian crisis, Asian countries would not have safeguarded against the possible crisis and we might have watched the different aftermath of the crisis.

This story stresses the strong demand for safe assets by emerging countries, while the former story does the strong demand for bubbles by advanced countries. Whatever the background story may be, it is possible to describe the geography of the holding of bubbles substantially within the same model.

Appendix

Proof of Proposition 1

We write the capital market clearing as

$$\text{(A1)} \qquad ED(r_t, r_{t+1}) \equiv \frac{\beta^2 R}{1+r_{t+1}-R}\left(\frac{1+r_t}{1+g}+\omega\right) - \left\{\beta(1+\beta) - \frac{(1+g)\omega}{1+r_{t+1}}\right\} = 0 .$$

We first demonstrate that there exists an interest rate that satisfies $ED(r,r)=0$ for $1+r \in \left(\max\left\{R, \frac{(1+g)\omega}{\beta(1+\beta)}\right\}, R^f\right)$, which is relevant for the binding borrowing constraint that consists of three inequalities: (i) $R<1+r$ (the binding borrowing constraint), (ii) $(1+g)\omega < \beta(1+\beta)(1+r)$ (the positive borrowing), and (iii) $1+r < R^f$ $ED(r,r)$ is decreasing for this region. If $R > \frac{(1+g)\omega}{\beta(1+\beta)}$, $\lim_{r \to R-1} ED(r,r) = +\infty$, while if $R < \frac{(1+g)\omega}{\beta(1+\beta)}$,

$ED(r,r)>0$ holds at $1+r=\frac{(1+g)\omega}{\beta(1+\beta)}$. Thus it is sufficient to show that $ED(R^f-1,R^f-1)<0$, which requires the parameter restriction:

(A) $$\frac{\beta^2 R}{R^f-R}(\frac{R^f}{1+g}+1)-\{\beta(1+\beta)-\frac{(1+g)\omega}{R^f}\}<0$$

We turn to the dynamics. We rewrite $ED(r_t,r_{t+1})=0$ as

(A1) $$\beta^2 R(\frac{1+r_t}{1+g}+\omega)=(1+r_{t+1}-R)\ \{\beta(1+\beta)-\frac{(1+g)\omega}{1+r_{t+1}}\}\equiv\Lambda(1+r_{t+1}).$$

The function $\Lambda(.)$ is increasing and convex for $1+r_t\in(\max\{R,\frac{(1+g)\omega}{\beta(1+\beta)}\},R^f)$. For $r_t<(>)r(R)$, $\Lambda(1+r_{t+1})=\beta^2 R(\frac{1+r_t}{1+g}+\omega)>(<)\ \Lambda(1+r_t)$, and r_t is increasing(decreasing). The interest rate is dynamically stable.

There exists two solutions, $1+r^-$ and $1+r^+$, that satisfy $\beta^2 R(\frac{1+r}{1+g}+\omega)=\Lambda(1+r)$, and satisfies $0<1+r^-<R<1+r^+$. Only $1+r^+$ satisfies the binding borrowing constraint. At $r=r^+$, $\partial r^+/\partial R>0$ holds. Q.E.D.

Proof of the effect of ω on the condition for crowding in/out

We prove by assuming g=0 without loss of generality. In order to prove (i), we use (A2) to solve the interest rate in terms of ω; $1+r(\omega)=\frac{B(\omega)+\sqrt{D(\omega)}}{2\beta\{1+\beta(1-R)\}}$, where

$B(\omega)\equiv R\beta(1+\beta)+\omega-R\beta^2\omega$ and $D(\omega)\equiv\{R\beta(1+\beta)+\omega(1-R\beta^2)\}^2-4R\omega\{1+\beta(1-R)\}$.

We define a new function, $1+\hat{r}(\omega)=\frac{R\beta(1+\beta)+\omega(1-R\beta^2)}{\beta\{1+\beta(1-R)\}}$, which is greater than $1+r(\omega)$ for any $\omega>0$ and equal only at $\omega=0$. $\beta\{1+\hat{r}(\omega)\}$ is increasing, with the slope being less than unity, and satisfies $\beta\{1+\hat{r}(0)\}=R(1+\beta)/\{1+\beta(1-R)\}$.

On the other hand, $\beta\{1+r(\omega)\}$ is also increasing and less than $\beta\{1+\hat{r}(\omega)\}$ for any $\omega>0$. There exists a threshold $\omega^*(>0)$ only below which $\omega<\beta\{1+r(\omega)\}$. This proves (i). (ii) is straightforward from the property that $r^D(.)$ is increasing in R. Q.E.D.

Proof of Proposition 6

The procedure of the proof is essentially the same as Proposition 1. The function $ED^E(r,r)+ED^A(r,r)$ is decreasing for $r\in(R^A-1,+\infty)$, and approaches infinity as $r\to R^A-1$. The nonbinding profitability constraint, $1+r<R^f$, requires (A). We rewrite (10) as

$$\beta^2 R(\frac{1+r_t}{1+g}+\omega)=\frac{2\{\beta(1+\beta)-\frac{(1+g)\omega}{1+r_{t+1}}\}}{\frac{R^E}{1+r_{t+1}-R^E}+\frac{R^A}{1+r_{t+1}-R^A}}\equiv\Lambda^*(1+r_{t+1}).$$

The function $\Lambda^*(.)$ is increasing and convex for $1+r_t\in(\max\{R^A,\frac{(1+g)\omega}{\beta(1+\beta)}\},R^f)$, where the borrowing constraint is binding. For $r_t<(>)r^*(R^E,R^A)$, $\Lambda^*(1+r_{t+1})=\beta^2R(\frac{1+r_t}{1+g}+1)>(<)\Lambda^*(1+r_t)$, and r_t is increasing (decreasing). The interest rate is dynamically stable.

Property (b) of Proposition 6 reveals some dynamic property of the global investment $i_t^E+i_t^A$. Equation (10) and the fact that r_{t+1} is positively related to r_t through Λ (in the proof of Proposition 1) imply that $i_t^E+i_t^A$ are positively related to r_t It follows directly from property (b) that $i_t^E+i_t^A$ is also dynamically stable, converging to the steady state. Q.E.D.

Proof of Proposition 7

From (2a), (2b), (8), and (9), we implicitly derive

(C1) $1+r_{t+1}=\Phi(r_t,b_t)$,

with $\Phi_r \equiv \frac{\partial \Phi}{\partial r_t} > 0$, and $\Phi_b \equiv \frac{\partial \Phi}{\partial b_t} > 0$. Incorporating (C1) into (8) yields

(C2) $(1+g)b_{t+1}=\Phi(r_t,b_t)b_t$.

These two equations constitute the two-dimensional dynamic system of (r_t,b_t). Letting ρ denote an eigenvalue and $M(\rho)=0$ denote the characteristic equation, we can write $M(\rho)=\rho^2-(\Phi_r+\Phi_b b+1)\rho+\Psi_r$. This function has the following properties; $M(1)=-b\Phi_b<0$, and $M(0)=\Psi_r>0$. Both eigenvalues are positive, and one is larger than unity and the other is less than unity. Therefore there exists the local stable manifold that is saddle-path stable. Furthermore, there exists a unique global stable manifold obtained through backward iteration of the local stable manifold that converges to the bubbly steady state. Q.E.D.

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Figure 1: External finance/GDP and Output Share

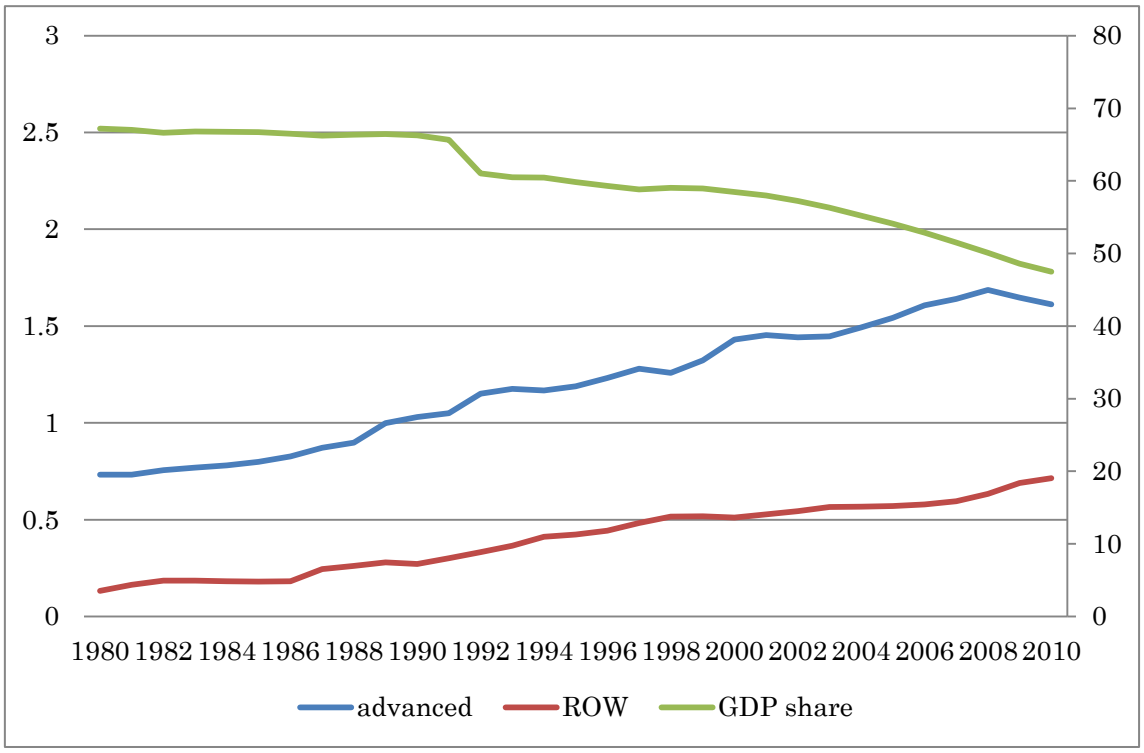

Figure 2: Current account surplus and investment boom

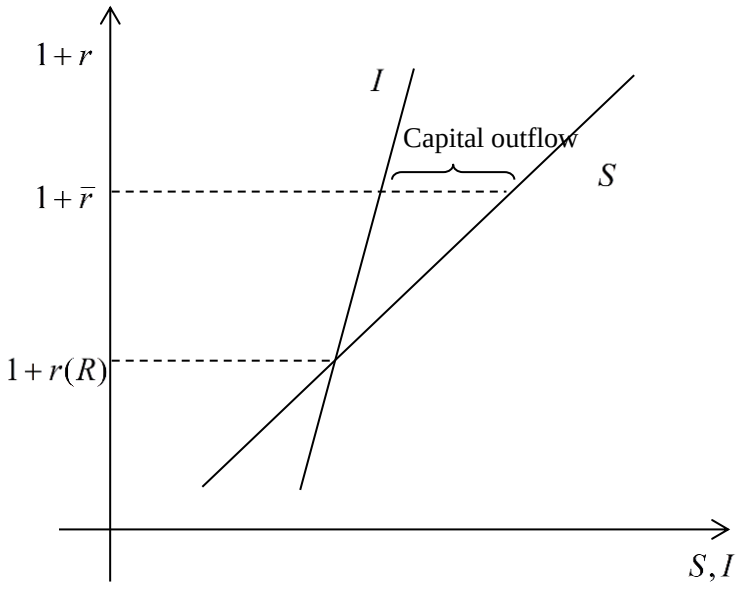

Figure 3: Interest Rates and the Growth Rate

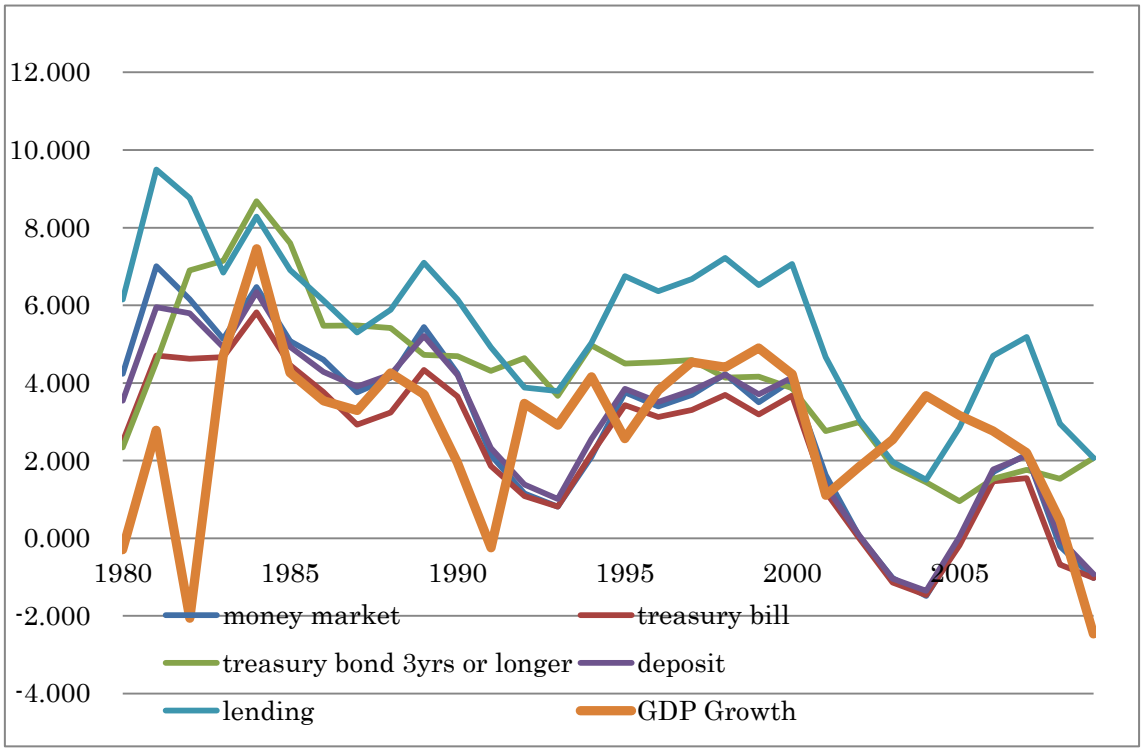

Table 1: Macroeconomic Data around Rehman Crises (unit: percent)

Savings rate (US)(2005-7, average)	15.0
Savings rate (advanced)(2005-7, average)	22.8
Savings rate (ROW)(2005-7, average)	26.9
Current Account/GDP(US) (2005-7, average)	-5.4
Current Account/GDP(advanced) (2005-7, average)	1.1
Current Account/GDP(US+advanced) (2005-7, average)	-1.6
GDP Growth rate (US)(2009)	-3.1
GDP Growth rate (advanced)(2009)	-4.0
GDP Growth rate (ROW)(2009)	0.6
Investment Growth Rate(US)(2009)	-15.7
Investment Growth Rate(advanced)(2009)	-14.1
Investment Growth Rate(ROW)(2009)	-8.2
GDP share(US)(2008)	20.4
GDP share(advanced)(2008)	30.1
GDP share(ROW)(2008)	49.5

Advanced countries include Austria, Australia, Belgium, Canada, Swaziland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, New Zealand, Portugal, Sweden, South Africa, and United States.

Table 2: Parameters

Discount factor(US)	$\beta^{US} = (0.955)^{15} \cong 0.5$
Discount factor(advanced)	$\beta^{A} = (0.96)^{15} \cong 0.54$
Discount factor(ROW)	$\beta^{ROA} = (0.965)^{15} \cong 0.58$
Economic Growth Rate	$1+g = (1.02)^{15} \cong 1.35$
Physical Return on Capital	$R^{f} = (1.06)^{15} \cong 2.4$
Labor Income Growth factor	$\omega = 0.45$
Pledgeability(US)	$R^{US} = 0.9$
Pledgeability(advanced)	$R^{A} = 0.54$
Pledgeability(ROW)	$R^{ROW} = 0.22$
Region size(US)	0.33
Region size(advanced)	0.55
Region size(ROW)	1

Table 3 Variables and the geography of bubble holding (unit: percent)

	Investment bust (*annual growth rate*)			Output bust (*annual growth rate*)		
bubble distribution	US	advanced	ROW	US	advanced	ROW
(20%, 30%, 50%)	-7.3	-8.7	-9.9	-1.7	-1.1	-0.8
(40%, 60%, 0%)	-20.9	-20.3	0.3	-7.1	-4.4	1.6
(30%, 45%, 25%)	-14.1	-14.5	-4.8	-4.4	-2.7	0.4
Data	-15.7	-14.1	-8.2	-3.1	-4.0	0.6

Current account (*ratio as of GDP*)		Globalization boom/bust			Savings rate (*ratio as of GDP*)		
US	advanced	US	advanced	ROW	US	advanced	ROW
-5.0	-0.4	23.5	-0.9	20.9	16.6	22.4	26.9
-6.3	-1.7	23.5	-0.9	20.9	16.6	22.4	26.9
-5.7	-1.1	23.5	-0.9	20.9	16.6	22.4	26.9
-5.4	1.1				15.0	22.8	26.9

Table 4 Counterfactual experiments

	Investment bust			Output bust		
	US	advanced	ROW	US	advanced	ROW
Financial depth	-5.1	-5.3	-1.0	-0.8	-0.1	1.2
Savings Glut	-5.9	-6.2	0	-1.1	-0.3	1.6
No Convergence	-4.2	-4.5	0.1	-0.5	0.1	1.5
data	-15.7	-14.1	-8.2	-3.1	-4	0.6

Current account		Globalization boom/bust			Savings rate		
US	advanced	US	advanced	ROW	US	advanced	ROW
-4.6	0.1	23.5	-0.9	1.2	16.6	22.4	25.5
-4.5	0.1	23.5	-0.9	13.9	16.6	22.4	19.0
-4.3	0.3	23.5	-0.9	20.9	16.6	22.4	26.9
-5.4	1.1				15	22.8	26.9