# Working Less and Bargain Hunting More: 

# Macro Implications of Sales during Japan's Lost Decades 

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#### Abstract

During the lost decades in the 1990s and 2000s, Japan witnessed the rising frequency of temporary sales and the declining hours worked. Motivated by this negative correlation, we construct a DSGE model with sales, wherein households go bargain hunting and firms determine its sale frequency endogenously. In the model, bargain hunting activity helps households substitute away from a relatively expensive brand item, but it is time consuming like labor supply. We show that the real effect of monetary policy continues to be present, but weakens because sales prices are frequently revised and endogenous bargain hunting enhances the strategic substitutability of sales. The real effect of technology shock, by contrast, strengthens. The model reveals that declines in hours worked during Japan's lost decades account for an actual rise in the frequency of sales and a rise in the fraction of bargain hunters.


Keywords: sales; monetary policy; lost decades; time use
JEL classification: E3, E5

[^0]
## 1 Introduction

The Japanese economy has been faced with so-called lost decades since the 1990s, experiencing very low rates of inflation (or deflation) and the GDP growth. Two remarkable observations during the Japan's lost decades are a change in firms' price setting strategy together with a change in households' working behaviors. Japanese firms increase the number of sales, that is, temporary cuts of good prices from regular prices, gradually but monotonically (its evidence is provided in the next section). Consequently, households purchase a greater number of goods at the sale price rather than at the regular price. During the same period, hours worked per worker displays a secular decline partly reflecting the statutory reduction in hours worked, jitan, and dampening output (Hayashi and Prescott (2002)).

In this paper, we ask whether the temporary sales play an important role in the variations of output and inflation in the Japanese economy, in relation with the households' time allocation decision between labor input, leisure, and searching for cheaper goods. This paper investigates macroeconomic implications of sales by extending the work of Guimaraes and Sheedy (2011, hereafter GS). GS develop a DSGE model in which the economy consists of two classes of consumers, price-insensitive customers called "loyal customers" and price-sensitive customers called "bargain hunters." It is shown that the firms' best pricing strategy is to hold periodic sales, since firms cannot tell them apart. The real effect of monetary policy hardly diminishs in the presence of sales, because sales are strategic substitutes. However, the model assumes that a proportion of the two classes of consumers is fixed.

In contrast to GS, the household in our model makes endogenous decision about the intensity by which it responds to a relative price differentials among items. Each household consists of an infinite number of shoppers, and it chooses a portion of shoppers acting as loyal customers and those acting as bargain hunters. With a sizable number of bargain hunters, the household can substitute away from a relatively expensive brand item. At the same time, searching activity for a lower price reduces the time available for labor input and leisure. This brings the household a cost when increasing bargain hunters.

We document new findings both on empirical data and theoretical models. Empirically, scanner data of Japan's supermarkets for as long as 20 years reveal that the
frequency of temporary sales has increased during Japan's lost decades. Retailers have conducted temporary price discounts more and more frequently, which has increased the importance of temporary sales for their revenues. During this period, both price elasticity of Japanese households and their time in shopping appear to have increased. The sale frequency is significantly correlated with labor market conditions. During Japan's lost decades, unemployment has risen and hours worked have declined. The VAR model shows that the frequency of sales moves in the opposite direction to hours worked in the business cycle frequency.

Theoretically, we reveal that macroeconomic implications are modified when both temporary sales and endogenous bargain hunting are incorporated in a model. The effect of an accommodative monetary policy shock on real economic activity weakens. The shock increases hours worked, which, in turn, increases (decreases) the fraction of loyal customers (bargain hunters). Observing this, firms lower their sale frequency. Because sale-priced goods are sold more than regular-priced goods in terms of quantity, those changes in households' and firms' actions yield a downward pressure on aggregate demand for goods. The real effect of monetary policy thus diminishs. ${ }^{1}$ According to simulation, the real effect of monetary policy continues to be present, but weakens by about $40 \%$ due to endogenous bargain hunting. Opposed to the monetary policy shock, the real effect of technology shock, by contrast, strengthens. In our model, a positive technology shock lowers hours worked. This decreases the marginal disutility of bargain hunting, and firms raise the sale frequency. Because sales-priced goods are sold more than regular-priced goods in terms of quantity, aggregate production increases further.

Regarding the Japanese economy, the model illustrates that Japan's declines in hours worked account for actual rises in the frequency of temporary sales during Japan's lost decades. In addition, our model suggests a downward (upward) trend in the fraction of loyal customers (bargain hunters). In other words, households have become increasingly price sensitive.

Our paper is relevant to the following three strands of research. On the retailer

[^1]side, the first strand examines whether the choice of temporary sales is orthogonal to changes in macroeconomic developments. Theoretically, this assumption plays a key role in yielding quantitatively important real effects of monetary policy. The GS model rests on this assumption. Empirical support is provided by Kehoe and Midrigan (2010), Eichenbaum et al. (2011), and Anderson et al. (2012). On the other hand, Klenow and Wills (2007) and Coibion, Gorodnichenko, and Hong (2012) as well as this paper provide evidence that the frequency of temporary sales is influenced by macroeconomic business cycles. ${ }^{2}$

The second concerns the interaction between hours worked and bargain hunting on the household side. Aguiar and Hurst (2007) use scanner data and time diaries to examine households' substitution between shopping and home production. They find that older households shop the most frequently and pay the lowest price. Lach (2007) analyzes store-level price data following the unexpected arrival of a large number of immigrants from the former Soviet union to Israel. He finds that the immigrants have a higher price elasticity and a lower search cost for goods than the native population. ${ }^{3}$

Third, regarding the models of sales, Varian (1980) shows firms' randomizing pricing strategy in the presence of informed and uninformed consumers. Salop and Stiglitz (1977) construct a model in which price dispersion arises across retailers, when consumers differ in the cost of gathering price information. Kehoe and Midrigan (2010) develop a DSGE model that incorporates not just menu cost associated with regular prices but also cost associated with deviations of sale prices from regular prices. ${ }^{4}$

The structure of this paper is as follows. Section 2 provides evidence for endogenous bargain hunting by looking at Japan's micro price data. Section 3 develops a model. Section 4 presents the model's impulse responses. Section 5 discusses Japan's lost decades. Section 6 concludes the paper.

[^2]
## 2 Evidence for Endogenous Bargain Hunting

This section documents various evidence to motivate and justify our modeling strategy regarding endogenous bargain hunting. First, from a goods-demand (and labor-supply) side, Japan's household survey on time use is used. It shows that working time moves in an opposite direction with shopping time. Second, from a goods-supply side, Japan's scanner, Point-of-Sales (POS), data are used. The data show that, during Japan's lost decades, temporary sales have played an increasingly important role in retailers' business. The frequency of sales is significantly correlated with the macroeconomic environment, in particular, indicators associated with the labor market. When hours worked is long, the frequency of sales tends to be low.

### 2.1 Survey on Time Use

Let us begin by looking at Survey on Time Use and Leisure Activities for Japan. The survey is conducted by the Statistical Bureau every five years. It asks around 200,000 people in 80,000 households about their daily patterns of time allocation. Questionnaire includes time use in working and shopping. In that respect, this survey helps us examine their relationship, which is the key to our model. ${ }^{5}$

Tables 1 and 2 show the summary results of households' time use in shopping and working (including commuting time for work and school), respectively. The sample is that of over 15 year old. Numbers in the tables indicate minutes per week. Two results are worth highlighting. First, at a cross-section level, shopping time is longer for those who are not working than those who are working. Also, female spends longer shopping time than male. Second, at a time-series level, shopping time steadily increased from 1986 to 2006, in particular for male. At the same time, hours worked continued to decline, although they picked up slightly in 2006. Those results appear to provide a support for our assumption that bargain hunting depends negatively on hours worked.

[^3]Table 1: Time Use in Shopping (minutes)

|  | Male <br> Working | Not working | Female <br> Working | Not working |
| :--- | :--- | :---: | :---: | :---: |
| 1986 | 6 | 9 | 27 | 37 |
| 1991 | 9 | 12 | 30 | 38 |
| 1996 | 11 | 15 | 30 | 39 |
| 2001 | 13 | 18 | 31 | 39 |
| 2006 | 14 | 20 | 31 | 39 |

Source: Statistical Bureau, "Survey on Time Use and Leisure Activities"

Table 2: Time Use in Working (including commuting time, minutes)

|  | Male <br>  <br>  <br> Working | Not working | Female <br> Working | Not working |
| :--- | :--- | :---: | :---: | :---: |
| 1986 | 493 | - | 371 | - |
| 1991 | 481 | - | 358 | - |
| 1996 | 469 | - | 345 | - |
| 2001 | 456 | - | 324 | - |
| 2006 | 470 | - | 335 | - |

Source: Statistical Bureau, "Survey on Time Use and Leisure Activities"

### 2.2 POS Data

From a goods-supply side, let us examine indicators of sales using scanner or Point of Sales (POS) data. ${ }^{6}$ The POS data are collected by Nikkei Digital Media from retail shops located in Japan. They are daily ranging from March 1, 1988 to February 28, 2013. The number of records amounts to 6 billion, where each record contains a number of units sold and sales in yen for a product $i$ at a shop $s$ on a date $t$. The cumulative number of products appearing during the sample period is 1.8 million. The data include processed food and domestic articles, and unlike CPI, does not include fresh food, recreational durable goods (TVs and PCs), and services (rent and utility). The coverage of the POS in CPI is 170 out of 588 items, which constitutes $17 \%$ of household's expenditure according to Family Income and Expenditure Survey. Each product $i$ is identified by the the Japanese Article Number (JAN) code.

Three advantages are noteworthy regarding our POS data. First, they include quantity information as well as price information. Second, the data frequency is daily, contrasting to the weakly US scanner data. Third, they have a long sample period, starting from 1988 up until now, which fully covers the period of lost decades.

Aggregation of variables takes the following four steps. First, at the lowest level of JAN codes, we collect a variable of interest, such as a price, for a product $i$ at a shop $s$ on a date $t$. Second, we aggregate it across shops with sales weights to derive weighted mean. Third, up to the 3 -digit code level, ${ }^{7}$ we aggregate it across products with sales weights to derive weighted mean. Last, we aggregate it across 3 -digit codes with sales weights to derive weighted mean. Weights are defined by the sales during the month in the previous year. If a date $t$ is January 1, 2012, for instance, we use the sales of January in 2011 as a weight.

### 2.2.1 Temporary Sales

From each record of the POS data, the price of a product is measured by its unit price, that is, revenues over the number of units sold for a product $i$ at a shop $s$ on a date $t$.

[^4]Recorded revenues exclude the contribution of consumption tax that was introduced in April 1989 and raised in April 1997.

The POS data are silent as to temporary sales. To isolate them from regular prices, we follow Eichenbaum, Jaimovich, and Rebelo (2011) and define the regular price of a good on a date by the most commonly observed price (mode price) during the 3 months centered on the date. ${ }^{8}$ Temporary sales are identified when the regular price differs from its posted price.

Figure 1 illustrates the time-series fluctuations of four variables associated with temporary sales: (i) the frequency of sales (\%), (ii) the magnitude of sale discount (\%), (iii) a ratio of quantities sold at the sale price to those at the regular price, and (iv) a ratio of revenue sold at the sale price to total revenue in a month (\%). Variable (iii) shows how much goods are sold at a sale price relative to a regular price, when a retailer chooses to do sales at a certain date. On the other hand, variable (iv) shows how much temporary sales contribute to a retailer's total revenue in a certain month, so it is subject to the frequency of sales. In the graphs, red dashed lines represent volatile raw series, while black solid lines represent those smoothed by the HP filter with $\lambda=14,400$.

This figure clearly shows that temporary sales have become increasingly important in households' expenditure activity during the two decades. The frequency of sales has risen from 15 to $25 \%$, indicating that temporary sale take places once in four days in current years. Parallel to the increase in the frequency, the magnitude of sale discount has shrunk from $20 \%$ to $14 \%$. This shrinking sale discount is dominated by the rise in the sale frequency: the revenue coming from temporary sales has reached $30 \%$ of total revenue during the 2000s, compared with $20 \%$ in the 1990s. The ratio of quantities sold at the sale price to those at the regular prices has been around 1.6 during the 2000s.

### 2.2.2 Price Elasticity

An advantage of using the POS data is the availability of both price and quantity series that enables us to investigate their relationships including price elasticity of goods demand. Figure 2 shows a scatter plot for daily quantity changes shown in vertical axis together with corresponding daily price changes shown in horizontal axis for the item of a cup noodle. The slope is clearly negative, which according to the standard theory

[^5]

Figure 1: Variables Associated with Temporary Sales
Note: Red dashed lines represent raw series, while black solid lines represent those smoothed by the HP filter with $\lambda=14,400$.


Figure 2: Price Changes vis-a-vis Quantity Changes for a Cup Noodle
that assumes supply slopes upward and demand slopes downward, suggests that supply shocks are prevalent in this goods market.

To examine how the demand elasticity of price has changed over time, we calculate the slope of quantity changes against price changes for each product and store and then construct the weighted median of slopes across products and stores. Figure 3 displays an annual time series of the aggregated slope from the mid-1990s up to the current years. The time series possesses an upward trend, indicating that households become increasingly price-sensitive in the current years. As GS assume, bargain hunters are considered to be more price sensitive than loyal customers. The upward trend of price elasticity suggests a fall in the fraction of loyal customers and a rise in that of bargain hunters, although they are assumed to be constant in the GS model. ${ }^{9}$

[^6]

Figure 3: Proxy for Price Elasticity

### 2.3 Relation between the Sale Frequency and Macroeconomic Indicators

### 2.3.1 Labor Market Indicators

Figure 4 illustrates historical movements of labor market indicators: the unemployment rate and hours worked. The graph shows shrinking labor input in the 1990s. Two forces are considered to be present. First, Japan was faced with the lost decades after the burst of the asset-price bubble in the early 1990s. That led to the prolonged contraction in the labor market. Second, although this story does not apply to the the rise in the unemployment rate, the statutory jitan contributed to the fall in hours worked. ${ }^{10}$ Jitan was gradually introduced by the government thorough revisions of the Labor Standards Law: 1988:1Q to 1993:4Q and 1997:2Q to 1998:4Q, while the extent of jitan varied across industries and establishment sizes. This figure suggests a positive correlation between labor input and temporary sales. The trend rises in the unemployment rate and trend declines in hours worked are accompanied by the trend rise in the frequency of sales.

[^7]


Figure 4: Frequency of Temporary Sales and Labor Market Indicators

### 2.3.2 Correlations between the Sale Frequency and Macroeconomic Indicators

The aforementioned relation can be observed not only their trend but also their businesscycle (medium-run) components. We isolate business-cycle components with a period of 1.5 to 8 years using the Baxter-King band pass filter and compute contemporaneous correlations between temporary sales and macroeconomic indicators. As for macroeconomic indicators, we focus on seven variables all expressed in logarithm: the unemployment rate, total hours worked, the number of employed persons, the index of industrial production, the monthly growth rate of CPI, the leading index, the coincident index, the lagging index (the last three are the components of Composite Indexes complied by Cabinet Office). Figure 5 depicts correlations with red circles. Blue dashed lines represent a $95 \%$ confidence interval for no correlation. ${ }^{11}$

The figure suggests that the frequency of sales is negatively correlated with hours worked at a $5 \%$ significant level. The frequency of sales rises, when hours worked decline. Albeit less significant, the frequency of sales rises when the unemployment rate

[^8]

Figure 5: Correlation between Price Components and the Macro Economy Note: Blue dashed lines represent a $95 \%$ confidence interval for two uncorrelated bandpass filtered series calculated by Monte Carlo simulation.
rises or the lagging index worsens. All these directions of changes appear to be plausible: households endogenously decrease their shopping intensity when they are busy in working. Correlations with the CPI inflation rate, the leading index, and the coincident index are insignificant.

### 2.3.3 Impulse Responses by VAR

To investigate interactions between the frequency of sales and macroeconomic indicators in a more structural way, we next estimate the vector auto-regression (VAR) model and calculate impulse responses to various shocks. Following Altig et al. (2011), we assume that the only shocks that affect productivity in the long run are innovations to neutral and capital-embodied technology. Monetary policy shocks have a contemporaneous impact on its policy instrument, but they do not have a contemporaneous impact on aggregate quantities and prices. Data are quarterly from 1989:1Q to 2013:1Q. The number of variables is 10: relative price of investment, labor productivity, a quarterly growth rate of CPI, hours worked multiplied by employment, the ratio of consumption to GDP, the ratio of investment to GDP, the frequency of temporary sales, the annualized short-term loan rate, the ratio of monetary base to nominal GDP, and the quarterly growth rate of Nikkei commodity price index. All variables are in logarithm, except for the short-term loan rate and the growth rates of CPI and commodity price. What is distinct is the


Figure 6: Response to Neutral Technology Shock
inclusion of the frequency of temporary sales in the VAR model. ${ }^{12}$
Figures 6 to 7 demonstrate impulse responses of aggregate quantities and prices including the frequency of sales to innovations to neutral and capital-embodied technology. ${ }^{13}$ A gray area indicates a $95 \%$ confidence interval. These graphs illustrate that the frequency of temporary sales move in an opposite direction to house worked. A positive technology shock, both that of neutral and capital-embodied technology, increases output and investment. The first shock increases consumption as well. Their impacts on hours worked differ between the two shocks. While the neutral technology shock decreases hours worked in the short-run, the capital-embodied technology shock increases them in the long-run. The frequency of sales increases to the former shock in the short-run, while it falls persistently to the latter shock. That is, to both types of the technology shocks, hours worked and the frequency of sales respond in an opposite direction.

[^9]
## Capital-embodied Technology Shock



Figure 7: Response to Capital-embodied Technology Shock

### 2.4 Difference from the United States

This section, so far, provided a number of evidence regarding the frequency of temporary sales and the intensity of shopping, together with their relationships with the macroeconomic environment. The bottom-line result is that the frequency of temporary sales moves in an opposite direction to hours worked. A retail shop is likely to conduct temporary sales, when households are less busy in working. This result is consistent with Aguiar and Hurst (2007) and Lach (2007), but makes a stark contrast with that existing literature on sale pricing in the United States such as Kehoe and Midrigan (2010), Eichenbaum et al. (2011), Anderson et al. (2012), who argue that the choice of temporary sales is orthogonal to macroeconomic circumstances. Most strikingly, Coibion, Gorodnichenko, and Hong (2012) report an opposite result: an increase in the unemployment rate decreases the frequency of temporary sales.

There are two reasons for the difference. First, Japan is not the United States. On a retailer side, the every-day-low-price strategy is not common in Japan. The key to the result of Coibion, Gorodnichenko, and Hong (2012) is the fact that every-day-low-price strategy is adopted mainly by lower-priced stores like Wal-Mart, while temporary sales are conducted mainly by higher-price stores. Suppose a rise in unemployment. Cus-
tomers switch from higher-priced stores to lower-priced stores to save their expenditure. Higher-priced stores give up such price-sensitive customers and concentrate on loyal customers. They thus reduce the frequency of temporary sales. In Japan, by contrast, even lower-priced stores conduct very frequent temporary sales. As Sudo, Ueda, and Watanabe (2014) point out, prices in Japan are revised ten times more frequently than those in the United States, due to frequent temporary sales. Although such a difference in retailers' strategies yields an opposite response of the frequency of sales between Coibion, Gorodnichenko, and Hong (2012) and ours, average prices paid by consumers (effective prices) move in the same direction. They decrease when the unemployment rate rises. On a consumer side, Japanese households are said to go shopping more frequently from Monday to Sunday; they walk to supermarkets nearby, purchase a small amount of products, and bring them back on foot; and US households go shopping mainly on weekends by car. According to Survey of Consumer Beahavior by Japanese Meat Information Service Center, the average shopping frequency is three to four times a week in the 2000s. ${ }^{14}$ These characteristics peculiar to Japan (or United States) may have caused the difference.

Second our POS data have long sample periods, from 1988 to 2013. This is well longer than the typical duration of business cycles, beneficial to investigate the relationships between sales and macroeconomic conditions. By contrast, widely used Dominick's data in the United States are relatively short, ranging from 1989 to 1997. If this second reason is dominant, our finding in Japan may hold globally: the frequency of temporary sales decreases, when hours worked rise.

## 3 Model

This section constructs the DSGE model of temporay sales by extending the GS model.

[^10]
### 3.1 Setup

Household We assume a cohort of consumers in a representative household who has the following lifetime utility function:

$$
\begin{equation*}
U_{t}=\sum_{j=0}^{\infty} \beta^{j} \mathrm{E}_{t}\left[v\left(C_{t+j}\right)-Z_{t+j}^{h} v\left(H_{t+j}+\phi_{L} H \frac{\left(1-L_{t+j}\right)^{\theta_{L}}}{(1-\lambda)^{\theta_{L}}}\right)\right], \tag{3.1}
\end{equation*}
$$

where $C_{t}$ is aggregate composite of differentiated consumption goods that is defined below, $H_{t}$ is hours worked ( $H$ is its steady state level), and $L_{t}$ is the share of shoppers that are chosen to be loyal customers in the cohort $\left(0<L_{t}<1\right)$. The share of loyal customers $L_{t}$ is endogenous, with its mean $\lambda$. Parameter $\beta$ is the subjective discount factor $(0<\beta<1)$, $\phi_{L}$ represents utility weight on being loyal customers, and $\theta_{L}>0$ represent the elasticity of utility from being loyal customers. $Z_{t}^{h}$ represents a stochastic shock to the utility weight of the labor supply with unit mean and its logarithm deviation denoted by $\varepsilon_{t}^{h}$. The positive shock works to decrease labor supply and increase the fraction of loyal customers. The function $v\left(C_{t}\right)$ is strictly increasing and strictly concave in $C_{t}$, and $v\left(X_{t}\right)$ is strictly increasing and convex in $X_{t}$.

The overall aggregator of consumption is given by

$$
\begin{equation*}
C_{t} \equiv\left[\int_{T}\left(\int_{B} c_{t}(\tau, b)^{\frac{\eta-1}{\eta}} d b\right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d \tau\right]^{\frac{\epsilon}{\epsilon-1}} \tag{3.2}
\end{equation*}
$$

where $c_{t}(\tau, b)$ is the household's consumption of brand $b \in B$ of product type $\tau \in$ $T$. GS give the example such that product types include beer and dessert and brand includes Corona beer and Ben \& Jerry's ice cream. As is assumed in GS, we set $\eta>\epsilon$, so that bargain hunters are more willing to substitute between different brands of a specific product type than consumers are to substitute between different product types. According to GS, this aggregator of consumption is that of bargain hunters.

The household faces the following condition when purchasing goods:

$$
\begin{equation*}
\left(\int_{\Lambda \in T} c_{t}(\tau, B)^{\frac{\epsilon-1}{\epsilon}} d \tau\right)^{\frac{\epsilon}{\epsilon-1}} \geq \bar{C}_{t} \tag{3.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{C}_{t}=\left(\frac{p_{B, t}(\tau)}{P_{t}}\right)^{-\epsilon} C_{t}^{*} \tag{3.4}
\end{equation*}
$$

and $\Lambda$ represents the set of product types to which consumers are loyal customers in the cohort. This measure is denoted by the aforementioned $L_{t}$. The left-hand side of equation (3.3) corresponds to the aggregator of consumption for loyal customers in GS. This condition states that, for the measure of $L_{t}$, a household has to purchase goods so as to maximize the aggregator of consumption for loyal customers. although they are actually bargain hunters as is expressed in equation (3.2). Equation (3.4) states that minimum amount is decreasing in the ratio of a price index for all brands of product type $\tau, p_{B, t}(\tau)$, to the average aggregate price level, $P_{t}$, with the elasticity of $\epsilon . C_{t}^{*}$ represents aggregate consumption spending.

Solving an expenditure minimization problem subject to equations (3.2) and (3.3) enables us to obtain the following demand function: ${ }^{15}$

$$
c_{t}(\tau, b)=\left\{\begin{array}{cc}
\left(\frac{p_{t}(\tau, b)}{p_{B, t}(\tau)}\right)^{-\eta}\left(\frac{p_{B, t}(\tau)}{P_{t}}\right)^{-\epsilon} C_{t}^{*} & \text { for } 1-L_{t} \text { population }  \tag{3.5}\\
\left(\frac{p_{t}(\tau, b)}{P_{t}}\right)^{-\epsilon} C_{t}^{*} & \text { for } L_{t} \text { population }
\end{array}\right.
$$

where $p(\tau, b)$ is the price of brand $b$ of product type $\tau$.
This demand function illustrates that bargain hunters who occupy $1-L_{t}$ population are agents that can freely substitute from a relatively expensive brand $b^{\prime}$ within a type $\tau$ goods. This form of demand is optimal for the household, in that it is derived from the optimization problem of equation (3.2) without the constraint of (3.4). By contrast, loyal customers who occupy $L_{t}$ population are agents that cannot make such substitutions. For the loyal customers, the relative price across $\tau$ matters but relative price across $b$ does not matter in determining their expenditure decision.

The household's budget constraint is given by

$$
\begin{equation*}
P_{t} C_{t}^{*}+E_{t}\left[Q_{t+1 \mid t} A_{t+1}\right] \leq W_{t} H_{t}+D_{t}+A_{t} \tag{3.6}
\end{equation*}
$$

where $W_{t}$ is the wage, $D_{t}$ is dividends received from firms, $Q_{t}$ is the asset pricing kernel, and $A_{t}$ is the household's portfolio of Arrow-Debreu securities.

The endogenous $L_{t}$ is the most important innovation made in this paper. In choosing the optimal $L_{t}$, the household confronts trade-off. On the one hand, an increase in $L_{t}$ raises one's utility. As equation (3.1) shows, it increases time for leisure by decreasing

[^11]the time for bargain hunting. On the other hand, the increase in $L_{t}$ decreases the benefit from bargain hunting. The household decreases one's utility by selecting the sub-optimal amount of demand, because the household is more strongly constrained by equation (3.4) and forced to purchase products according to the second line of the demand function (3.5). This second effect can be more formally illustrated by the relationship between utility-related consumption $C_{t}$ and spending-related consumption $C_{t}^{*}$. Appendix B shows that $C_{t}$ depends on not only $C_{t}^{*}$ but also the following consumption wedge $\digamma_{t}$ : ${ }^{16}$
\[

$$
\begin{equation*}
C_{t}=\digamma_{t} \cdot\left(\frac{P_{B, t}}{P_{t}}\right)^{-\epsilon} C_{t}^{*} \tag{3.7}
\end{equation*}
$$

\]

and that $\digamma_{t}<1$ and $d \digamma_{t} / d L_{t}<0$. As the household makes more bargain hunting, $L_{t}$ decreases and $\digamma_{t}$ increases. The household enjoys higher utility from the same amount of consumption spending $C_{t}^{*}$. If the household makes bargain hunting for all goods, that is, $L_{t}=0$, then we have $\digamma_{t}=1$.

Additionally, Calvo-type wage stickiness is introduced as in GS. Households supply differentiated labor inputs to firms. Wages can be adjusted at a probability of $1-\phi_{w}$.

Firms A good thing in our model is that firms' behavior is depicted in the exactly same way as GS. Firms in our model face the same demand function given by equation (3.5) as those in GS. It is thus optimal for firms to randomize their prices across shopping moments from a distribution with two prices. Firms set a normal high price $P_{N, t}$ with the frequency of $1-s_{t}$ and a low sale price $P_{S, t}$ with the frequency of $s_{t}$. We hereafter call the higher price a normal price following GS, instead a regular price. The only but important difference from GS is that firms optimize their pricing decisions by observing changes in the share of loyal customers $L_{t}$.

As GS argue, the strategic substitutability of sales plays a crucial role in firms' pricing. The more others have sales, the less an individual firm wants to have a sale. Suppose that other firms always have sales. If the individual firm stops a sale and sells its good at a normal price, its profit increases, because price-insensitive loyal customers tend to buy the good even at the normal price. As an opposite case, suppose that other firms have no sale. Because sales attract price-sensitive bargain hunters, the individual firm can

[^12]increase its profit by having sales. Such strategic substitutability makes firms randomize their prices between the high normal price and the low sale price.

Firms adjust their normal prices with Calvo-type price stickiness. In each period, firms have a probability of $1-\phi_{p}$ to reset their normal prices. Sale prices can be adjusted freely.

Wholesalers produce goods using a labor input:

$$
\begin{equation*}
Q_{t}=Z_{t}^{a} H_{t}^{\alpha} \tag{3.8}
\end{equation*}
$$

where $\alpha$ represents the elasticity of output with respect to hours and $Z_{t}^{a}$ represents a a stochastic shock to productivity, with unit mean and its logarithm deviation denoted by $\varepsilon_{t}^{a}$.

Monetary authority A monetary authority sets a nominal interest rate $i_{t}$ following the monetary policy rule of

$$
\begin{equation*}
i_{t}=\rho i_{t-1}+(1-\rho) \phi_{\pi} \pi_{t}^{N}+\varepsilon_{t}^{i} \tag{3.9}
\end{equation*}
$$

where $\pi_{t}^{N}$ represents a change in the normal price and $\rho$ represents a policy inertia. The official CPI excludes sales, so we assume that the monetary authority refers to the inflation rate based on the normal price with its degree $\phi_{\pi} . \varepsilon_{t}^{i}$ represents a shock to monetary policy, with its mean zero.

Resource constraint A resource constraint is given by

$$
\begin{equation*}
Y_{t}=C_{t}^{*}+Z_{t}^{g} \tag{3.10}
\end{equation*}
$$

where $Z_{t}^{a}$ represents a a stochastic shock to productivity. Its mean is zero and the logarithm deviation from the steady-state $Y$ is denoted by $\varepsilon_{t}^{g}$.

Exogenous shocks We consider four types of shocks. They are shocks to monetary policy, technology, government expenditure, and labor supply:

$$
\begin{align*}
\varepsilon_{t}^{i} & =\eta_{t}^{i}  \tag{3.11}\\
\varepsilon_{t}^{a} & =\rho_{a} \varepsilon_{t-1}^{a}+\eta_{t}^{a}  \tag{3.12}\\
\varepsilon_{t}^{g} & =\rho_{g} \varepsilon_{t-1}^{g}+\eta_{t}^{g}  \tag{3.13}\\
\varepsilon_{t}^{h} & =\rho_{h} \varepsilon_{t-1}^{h}+\eta_{t}^{h} . \tag{3.14}
\end{align*}
$$

As for the monetary policy shock, we do not assume an inertia, because the monetary policy rule has inertia on its own.

### 3.2 Log-Linearized Equations

Leaving detailed derivations to Appendix B, we summarize log-linearized equations. We denote log deviations of variables by small letters.

Sale Pricing It is optimal for a firm $j$ to adjust its sale price $p_{S, j, t}$ by one-for-one with a change in its nominal marginal cost $x_{t}+p_{t}$,

$$
\begin{equation*}
p_{S, j, t}=x_{t}+p_{t} \tag{3.15}
\end{equation*}
$$

where a real marginal cost is denoted by $x_{t}$.
The sale frequency is given by

$$
\begin{equation*}
s s_{t}=-\frac{1-\theta_{B}}{\varphi_{B}} \frac{1}{1-\psi} x_{t}-\left(\frac{1-\theta_{B}}{\varphi_{B}} \frac{A}{1-\psi}+\frac{1}{(\eta-\epsilon)(1-\lambda) \varphi_{B}}\right) l_{t} . \tag{3.16}
\end{equation*}
$$

The second term of the right-hand side is a new term from GS, indicating that as the share of loyal customers $l_{t}$ increases, firms decrease the sale frequency $s_{t}$. Like GS, an increase in the real marginal cost $x_{t}$ decreases the sale frequency. Because the sale price responds by one-for-one to the marginal cost, the sale price increases more than the normal price. That decreases relative demand for sales, thereby decreasing the sale frequency.

Fraction of Loyal Customers The fraction of loyal customers $l_{t}$ is given by:

$$
\begin{align*}
0 & =\left(\theta_{c}^{-1}+\frac{1}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha}-1\right) y_{t}-\frac{\delta}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha} w_{t} \\
& -\frac{\theta_{h}^{-1}}{\alpha} \varepsilon_{t}^{a}+\varepsilon_{t}^{h}-\left(\theta_{c}^{-1}-1\right) \varepsilon_{t}^{g} \\
& -\left(\frac{1}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha} B+\left(\theta_{L}-1\right) \frac{\lambda}{1-\lambda}+\theta_{h}^{-1} \phi_{L} \frac{\lambda}{1-\lambda}\right) l_{t} \\
& +\left(\theta_{c}^{-1}-1\right)\left\{f_{t}-\epsilon\left(x_{t}+\frac{1}{(\eta-\epsilon)(1-\lambda)} l_{t}\right)\right\} \\
& +\frac{\Xi}{1-\Xi} \xi_{t}+\frac{\eta-1}{\eta} f_{t} . \tag{3.17}
\end{align*}
$$

Two things are worth noting. First, the fraction of loyal customers $l_{t}$ increases with aggregate demand $y_{t}$, and consequently, hours worked $h_{t}$. As hours worked lengthen, the disutility from bargain hunting increases, and hence, the fraction of loyal customers increases.

Second, the fraction of loyal customers $l_{t}$ increases with the consumption wedge $f_{t}$. A rise in the consumption wedge means an improvement in utility from a given amount of consumption spending. As the wedge rises, the benefit from bargain hunting diminishes, raising the fraction of loyal customers.

We can also show that the consumption wedge increases with the ratio of the sale price to the normal price $\mu_{t}=P_{S, t} / P_{N, t}$ and decreases with the sale frequency $s_{t}$. In other words, as the sale price converges to the normal price or sales become less frequent, prices for differing goods become homogenous and the consumption wedge rises. Its detailed equation is given by

$$
\begin{equation*}
f_{t}=\frac{\eta}{\eta-1} \lambda \frac{\Xi \xi_{t}-(1-\Xi) l_{t}}{\lambda \Xi+(1-\lambda)} \tag{3.18}
\end{equation*}
$$

where

$$
\begin{align*}
\xi_{t} & =\frac{\left(\mu^{\epsilon \frac{1-\eta}{\eta}}-1\right)\left\{s \mu^{1-\eta}+(1-s)\right\}-\frac{\epsilon}{\eta}\left(\mu^{1-\eta}-1\right)\left\{s \mu^{\epsilon \frac{1-\eta}{\eta}}+(1-s)\right\}}{\left\{s \mu^{\epsilon \frac{1-\eta}{\eta}}+(1-s)\right\}\left\{s \mu^{1-\eta}+(1-s)\right\}} s s_{t} \\
& +\frac{\mu^{\epsilon \frac{1-\eta}{\eta}} \frac{1-\eta}{\eta}\left\{s \mu^{1-\eta}+(1-s)\right\}-\frac{\epsilon}{\eta} \mu^{1-\eta}(1-\eta)\left\{s \mu^{\frac{1-\eta}{\eta}}+(1-s)\right\}}{\left\{s \mu^{\epsilon \frac{1-\eta}{\eta}}+(1-s)\right\}\left\{s \mu^{1-\eta}+(1-s)\right\}} s \mu_{t} . \tag{3.19}
\end{align*}
$$

The ratio of the sale price to the normal price

$$
\begin{equation*}
\mu_{t}=\frac{1}{1-\psi}\left(x_{t}+A l_{t}\right) . \tag{3.20}
\end{equation*}
$$

Phillips curve with sales The New Keynesian Phillips curve with sales is given by

$$
\begin{align*}
& \pi_{t}=\beta E_{t} \pi_{t+1} \\
& +\frac{1}{1-\psi}\left\{\kappa x_{t}+\psi\left(\Delta x_{t}-\beta E_{t} \Delta x_{t+1}\right)+\kappa A l_{t}+A\left(\Delta l_{t}-\beta E_{t} \Delta l_{t+1}\right)\right\} \tag{3.21}
\end{align*}
$$

Compared with the standard New-Keynesian Phillips curve, the equation has two new terms. First, as in GS, changes in the real marginal cost, $\Delta x_{t}$, influence the inflation rate $\pi_{t}$. This is because the overall price depends on the sale price and the sale price is proportional to the real marginal cost as is shown in equation (3.15). Second, unlike GS, the fraction of loyal customers $l_{t}$ influences the inflation rate. As the fraction of loyal customers increases, the household substitutes less from a relatively expensive brand to cheaper brand. Observing this, firms lower their sale frequency, and hence, the overall price index increases.

The New Keynesian Phillips curve with respect to the normal price index is given by

$$
\begin{equation*}
\pi_{N, t}=\beta E_{t} \pi_{N, t+1}+\frac{\left(1-\phi_{p}\right)\left(1-\beta \phi_{p}\right)}{\phi_{p}}\left(x_{t}+p_{t}-p_{N, t}\right) . \tag{3.22}
\end{equation*}
$$

In other words, the curve for the normal price hardly changes in the presence of sales.
The real marginal cost $x_{t}$ is described as

$$
\begin{equation*}
x_{t}=\frac{1}{1+\gamma \delta} w_{t}+\frac{\gamma}{1+\gamma \delta}\left(y_{t}-B l_{t}\right) . \tag{3.23}
\end{equation*}
$$

As in the standard New-Keyensian model, the real marginal cost increases with the real wage $w_{t}$.

Wage Phillips curve We write down the wage Phillips curve:

$$
\begin{align*}
\pi_{W, t} & =\beta \pi_{W, t+1} \\
& +\frac{\left(1-\phi_{w}\right)\left(1-\beta \phi_{w}\right)}{\phi_{w}} \frac{1}{1+\varsigma \theta_{h}^{-1}} \\
& {\left[\left(\theta_{c}^{-1}+\frac{1}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha}\right) y_{t}-\left(1+\frac{\delta}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha}\right) w_{t}\right.} \\
& -\frac{\theta_{h}^{-1}}{\alpha} \varepsilon_{t}^{a}+\varepsilon_{t}^{h}-\theta_{c}^{-1} \varepsilon_{t}^{g} \\
& -\left(\frac{1}{1+\gamma \delta} \frac{\theta_{h}^{-1}}{\alpha} B+\theta_{h}^{-1} \theta_{L} \phi_{L} \frac{\lambda}{1-\lambda}\right) l_{t} \\
& \left.-\left(1-\theta_{c}^{-1}\right)\left\{f_{t}-\epsilon\left(x_{t}+\frac{1}{(\eta-\epsilon)(1-\lambda)} l_{t}\right)\right\}\right] . \tag{3.24}
\end{align*}
$$

where

$$
\begin{equation*}
\Delta w_{t}=\pi_{W, t}-\pi_{t} . \tag{3.25}
\end{equation*}
$$

The increase in the fraction of loyal customers decreases the real wage on both labordemand and labor-supply sides. On the labor-demand side, when the fraction of loyal customers increases, firms lower their sale frequency. Sales goods are generally sold more than normal goods in terms of quantity, and hence, total demand for the goods decreases. That diminishes the supply of the goods, and in turn, labor demand and the real wage. On the labor-supply side, as the fraction of loyal customers increases, the disutility from labor supply diminishes, which lowers the real wage. Moreover, the consumption wedge $f_{t}$ worsens, which increases labor supply and lowers the real wage due to the income effect.

## Euler equation

$$
\begin{align*}
y_{t} & =E_{t} y_{t+1}-\theta_{c}\left(i_{t}-E_{t} \pi_{t+1}\right)+\left(\varepsilon_{t}^{g}-E_{t} \varepsilon_{t+1}^{g}\right) \\
& +\left(1-\theta_{c}\right)\left\{\Delta f_{t+1}-\epsilon\left(\Delta x_{t+1}+\frac{1}{(\eta-\epsilon)(1-\lambda)} \Delta l_{t+1}\right)\right\} . \tag{3.26}
\end{align*}
$$

## Monetary policy rule

$$
\begin{equation*}
i_{t}=\rho i_{t-1}+(1-\rho) \phi_{\pi} \pi_{t}^{N}+e_{t}^{i} . \tag{3.27}
\end{equation*}
$$

Labor demand Production input is given by

$$
\begin{equation*}
h_{t}=\frac{1}{1+\gamma \delta} \frac{y_{t}-\delta w_{t}-B l_{t}}{\alpha}-\frac{1}{\alpha} \varepsilon_{t}^{a} . \tag{3.28}
\end{equation*}
$$

## 4 Impulse Response Functions

We simulate impulse response functions (IRFs) of economic variables to four types of shock. The first shock is an accommodative shock to the monetary policy rule. The second shock is a positive shock to wholesalers' production technology. The third shock is a government expenditure shock. The fourth shock is a labor supply shock.

### 4.1 Calibration

Most of the calibration of our model parameters is based on GS except for two things. First, a central bank obeys an interest rate monetary policy rule, setting $\rho=0.8$ and $\phi_{\pi}=1.5$, while a central bank in GS conducts a money growth rate rule. Second, we use parameters associated with sales so that they are consistent with Japan's POS data shown in Figure 1. The average of the frequency of sales $s$ is 0.18 ; the average ratio of quantities sold at the sale price to those at the regular prices $\chi$ is 1.7 ; and the average magnitude of sales discount is $17 \%$, which means $\mu=0.83$. The detailed settings are shown in Tables 3 and 4. As for the parameters associated with the fraction of loyal customers, we target a steady state level of the fraction of loyal customers to calibrate $\phi_{L}$ given $\theta_{L}$. We set two values for $\theta_{L}: 4$ or 100 . A lower $\theta_{L}$ implies a higher elasticity of the fraction of loyal customers. The value of $\theta_{L}=4$ is chosen to match the historical change in the frequency of sales as is shown below.

### 4.2 Comparison between the Standard New-Keynesian Model and the GS Model

To understand GS's results, let us begin with presenting IRFs in the GS model, in comparison with those in the standard New-Keynesian (NK) model. The GS model

Table 3: Model Parameters

| Parameters |  |  |
| :--- | :--- | :--- |
| $\beta$ | Discount factor | 0.9975 |
| $\theta_{c}$ | Elasticity of consumption | 0.333 |
| $\theta_{h}$ | Elasticity of labor supply | 0.7 |
| $\varsigma$ | Elasticity bw diffentiated labor | 20 |
| $\alpha$ | Elasticity of output to hours | 0.667 |
| $\gamma$ | Elasticity of marginal cost | 0.5 |
| $\phi_{p}$ | Calvo price stickiness | 0.889 |
| $\phi_{w}$ | Calvo wage stickiness | 0.889 |
| $\rho$ | Monetary policy rule inertia | 0.8 |
| $\phi_{\pi}$ | Monetary policy rule on inflation | 1.5 |


| Parameters |  |  |
| :--- | :--- | :--- |
| $\rho_{a}$ | Technology Shock | 0.85 |
| $\rho_{g}$ | Government Expenditure Shock | 0.85 |
| $\rho_{h}$ | Preference of Labor Supply shock | 0.85 |

Table 4: Parameters Related to Sales

| Target variables |  |  |
| :--- | :--- | :--- |
| $\mu$ | Price ratio of sales to normal | 0.83 |
| $\chi$ | Quantity ratio of sales to normal | 1.7 |
| $s$ | Sales frequency | 0.18 |
| Calibrated parameters |  |  |
| $\epsilon$ | Elasticity bw product types | 1.8969 |
| $\eta$ | Elasticity bw brands | 8.5529 |
| $\lambda$ | Fraction of loyal customers | 0.8877 |
| Calibrated parameters |  |  |
| $\phi_{L}$ | Utility weight | $1.310^{-4}$ |
|  | $\theta_{L}=100$ |  |
|  | on loyal customers | 0.0033 |$\theta_{L}=4$



Figure 8: IRFs under the GS Model and the Standard NK model
is the model discussed above without endogenous developments in the fraction of loyal customers. The standard NK model corresponds to the GS model without sales.

Figure 8 presents the IRFs of three economic variables: aggregate demand, inflation rates excluding sales (normal price changes), and nominal interest rates. The horizontal axis indicates time up to 12 quarters after a shock. The shock we give is an accommodative monetary policy shock by a unit size. Dashed and solid lines indicate IRFs in the GS model and in the standard NK model, respectively.

The left panel shows that, as GS argue, the real effect of monetary policy in the model with sales remains large, which is close to that in the model without sales. Real effects are similar, because sales are strategic substitutes.

### 4.3 Effects of Endogenous Bargain Hunting

IRFs in our model is presented, in comparison with those in the GS model. Graphs show the IRFs of 9 economic variables. Dotted lines indicate IRFs in the GS model. Thick and thin solid lines both indicate IRFs in the model with endogenous developments in the fraction of loyal customers, with differing elasticity parameter values $\theta_{L}=4$ and 100 .


Figure 9: IRFs to an Accommodative Monetary Policy Shock

### 4.3.1 Monetary policy shock

Figure 9 presents IRFs to the accommodative monetary policy shock, showing that temporary sales dampen its effect on aggregate demand by around $40 \%$. The mechanism runs as follows. In response to the monetary policy shock associated with lowering the nominal interest rate, aggregate demand increases. This raises hours worked. Since households spend more time in works, their disutility from bargain hunting increases. Under endogenous bargain hunting, the fraction of loyal customers (bargain hunters) increases (decreases). Observing this, firms lower their sale frequency. Since sales-priced goods are sold more than normal-priced goods in terms of quantity, the decrease in the sale frequency mitigates the increase in aggregate demand. The effect of monetary policy on demand diminishes, as $\theta_{L}$ is lower.

The attenuated real effect of monetary policy is also explained by intensified strategic substitutability of sales. Suppose that all firms but firm A raise their sale frequency. As in GS, it loses an incentive for firm A to raise its sale frequency, because its decreases the
marginal revenue from sales. In our model, additional channel emerges. When all firms but firm A raise their sale frequency, an aggregate price falls. This increases aggregate demand for goods, and in turn, aggregate demand for labor. Households supply more labor and lose time in bargain hunting. The fraction of loyal customers (bargain hunters) increases (decreases). By observing this, firm A lowers its sale frequency. Such intensified strategic substitutability of sales mitigates the real effect of monetary policy.

Inflation rates excluding sales (normal price changes) also fluctuate less in this model, compared with the GS model. As we explained in the previous section, the increase in the fraction of loyal customers functions to decrease the real wage and the real marginal cost. Although the increase in hours worked yields an upward pressure on the real marginal cost, the effect of the increase in the fraction of loyal customers functions dominates, when $\theta_{L}$ as low as 4 . In contrast, the model yields greater increases in inflation rates including sales than the GS model. This is because the aggregate price index increases with both the fraction of loyal customers and the sale frequency declining in response to the shock.

### 4.3.2 Technology shock

Figure 10 shows IRFs to the positive technology shock. Our model yields far greater effects on aggregate demand than the GS model and brings about inflation not deflation. In this type of sticky price model, the positive technology shock tends to decrease hours worked. This decreases (increases) the fraction of loyal customers (bargain hunters). Firms react to the shock by increasing their sale frequency. Because sales-priced goods are sold by a large amount, the increase in aggregate demand is magnified. The aggregate price falls, owing to the decrease in the fraction of loyal customers and the increase in the sale frequency. In contrast, the normal price increases. This results from increases in the real wage and the real marginal cost, brought about by the decrease in the fraction of loyal customers and the subsequent increase in the disutility of labor supply.

### 4.3.3 Government expenditure shock

As Figure 11 shows, economic responses to the positive government expenditure shock resemble those to the accommodative monetary policy shock. The real effect is attenuated.


Figure 10: IRFs to a Positive Technology Shock


Figure 11: IRFs to a Positive Government Expenditure Shock

### 4.3.4 Labor supply shock

Finally, we simulate IRFs to a shock to labor supply. This shock is formulated, being motivated by Hayashi and Prescott (2002). In analyzing Japan's lost decades and incorporating the effects of jitan, they introduce the following utility function:

$$
\log C_{t}-\alpha \frac{H_{t}}{40} E_{t}
$$

where $H_{t}$ and $E_{t}$ represent workweek length (hours) and the fraction of household members who work. For 1990 to 1992, they take $H_{t}$ as exogenous. In our model, as we showed in equation (3.1), we replace the exogenous $H_{t} / 40$ for the labor supply shock $Z_{t}^{h}$ and the endogenous $E_{t}$ for labor supply $H_{t}$ with $v\left(H_{t}+\phi_{L} \frac{\left(1-L_{t}\right)^{\theta_{L}}}{(1-\lambda)^{\theta_{L}}}\right) .{ }^{17}$

This labor supply shock is also related to an innovation in bargain hunting technology. Brown and Goolsbee (2002) argue that the internet lowers search cost for customers. In our model, $\phi_{L}$ in equation (3.1) serves as a candidate to capture bargain hunting technology, in that $\phi_{L}$ is interpreted as the degree of disutility from bargain hunting. A decrease in $\phi_{L}$ mitigates disutility from bargain hunting. If we interpret this as an innovation in bargain hunting technology, then the innovation encourages more bargain hunting (a fall in $\lambda$ ). ${ }^{18}$

Figure 12 demonstrates that this type of shock makes labor input and the fraction of loyal customers move in the opposite direction, unlike the above other types of shocks. The positive shock $\varepsilon_{t}^{h}$ makes labor supply costly, decreasing hours worked $h_{t}$. The decrease in $h_{t}$ functions to lower the fraction of loyal customers, while the positive labor supply shock directly functions to raise the disutility of bargain hunting, and in turn, the fraction of loyal customers. With the elasticity of labor supply below one, the former effect dominates the latter; the fraction of loyal customers decreases. In this respect, the labor supply shock is inconsistent with our actual observation for Japan. Although we do not show here, the fraction of loyal customers increases (unchanges), when the

[^13]

Figure 12: IRFs to a Labor Supply Shock
elasticity of labor supply exceeds (equals) one.

## 5 What Happened during Japan's Lost Decades

As we saw in Figure 1, the sale frequency $s$ continues to rise during Japan's lost decades. In the current section, based on our model, we argue that the working mechanism behind this observation may be attributed to the decline in hours worked coupled with the decline in employment. We discuss its implications to the macroeconomy.

### 5.1 Method

To support this argument, we conduct simulation by assuming that only the technology shock drives the economy. We draw the time-series path of the technology shock so as to account for the actual movements of hours worked in Japan. Although we do not
fully claim the validity of this assumption, we point out the following reasons. First, it is a natural step to regard the technology shock as a chief driving force of the economy, alongside the literature of Real Business Cycle theory. In addition, Hayashi and Prescott (2002) argue that the slowdown of the TFP contributes to Japan's lost decades. Second, when jitan shortened the workweek length, labor hoarding may have decreased. Resulting enhanced labor efficiency is regarded as a positive technology shock. As for hours worked, we use hours worked multiplied by one minus unemployment rate, to take account of total labor input.

Parameters associated with sales are calibrated to fit Japan's POS data as before. An estimated parameter is the persistence of the technology shock only. Our sample ranges from 1981:2Q to 2013:1Q. To match the data on hours worked, the time-series path of the technology shock is drawn. The model with this shock produces simulated time-series path of the sale frequency, and so on. For comparison, two models are used: the GS model and our model with endogenous developments in the fraction of loyal customers characterized by $\theta_{L}=4$, which is chosen to fit data. For simplicity, we neglect the zero lower bound on the nominal interest rate, which constrained the effectiveness of monetary policy during Japan's lost decades.

### 5.2 Simulation Results

Figure 13 demonstrates simulated variables associated with temporary sales: the frequency of sales $\left(\%, s_{t}\right)$, the magnitude of sale discount $\left(\%, 1-\mu_{t}\right)$, a ratio of quantities sold at the sale price to those at the regular price $\left(\chi_{t}\right)$, and a ratio of sales revenue sold at the sale price to total sales revenue in a quarter (\%). The figure illustrates that our model succeeds in explaining the increase in the frequency of sales $s_{t}$, in particular, its trend. The top-left panel plots the model-based and actual sale frequency. In terms of the direction of its trend and the size of its changes, the model-based sale frequency moves very closely to actual one. Both series show steady increases in the sale frequency in the 1990s and 2000s. In the 1980s, when the actual data are missing, our model suggests a slightly decreasing, but stable sale frequency. Around 2010, both series exhibit a dip. By contrast, the GS model predicts much attenuated changes in the sale frequency, which is almost flat in the graph. However, our model performs as poorly as the GS model in explaining the magnitude of sale discount $1-\mu_{t}$ and a ratio of quantities sold at the sale price to those at the regular price $\chi_{t}$, drawn in the top-right
and middle-left panels. These variables exhibit smaller fluctuations than actual ones. In sum, those simulation results suggest that our model improves the GS model in explaining the extensive margin of sales (sale frequency) but not the intensive margin (sales markup). As for a ratio of sales revenue sold at the sale price to total sales revenue, our model performs well. Note that the log-linearized deviation of this variable equals $1 /(1-s+s \chi) s_{t}+(1-s) /(1-s+s \chi) \chi_{t}$. Although our model fails to explain $\chi_{t}$, the success in explaining $s_{t}$ leads to the success in explaining this variable.

Our model reveals time-series changes in the fraction of loyal customers, which is unobservable. In the bottom-left panel, the fraction of loyal customers stays almost constant in the 1980s. During the lost decades, the 1990s and 2000s, then it exhibits a downward trend. Put differently, the fraction of bargain hunters increases. Obviously, in the GS model, the fraction remains constant. The rise in the fraction of bargain hunters is consistent with actual observations made in Table 1 and Figure 3. Because the price elasticity of loyal customers is considered to be lower than that of bargain hunters, the rise in the price elasticity is coherent to the rise in the fraction of bargain hunters. In the graph, the fraction of bargain hunters started to rise in the mid-1990s. This timing coincides with that in Figure 3.

### 5.3 Robustness

### 5.3.1 Other Explanations

Although we implemented simulation by assuming that the temporary technology shock drives the economy, this assumption is not necessarily guaranteed. Other types of shocks may be suitable to account for the actual decline in hours worked.

We consider two other types of shocks: a government expenditure shock and a labor supply shock. First, the government expenditure shock, in part, captures the idea that the statutory decline in hours worked is subsidized by fiscal policy, influencing governmental expenditure. The government expenditure shock is also categorized as a demand shock, opposed to the technology shock analyzed above. If Japan's lost decades are understood as a situation where demand was insufficient, negative demand shocks become a candidate for the driving force of the Japanese economy. For example, Sugo and Ueda (2008) estimate a sticky-price DSGE model and find that an investment adjustment cost


Figure 13: Model and Actual Paths of Sales-Related Variables
shock was a main driving force. Bayoumi (2001) and Caballero, Hoshi, and Kashyap (2008) emphasize a financial reason including a zombie lending as a cause of Japan's lost decades. Although the financial shock is not directly linked to the demand shock, the former is considered to influence demand for investment on the firm side. We confirmed that simulation results using the demand shock are not much different from that using the technology shock, although we do not show here.

Second, we conducted the similar exercise by assuming that the labor supply shock drives the actual changes in hours worked. This labor supply shock is motivated by Hayashi and Prescott (2002), as we explained in the previous section. We then found that our model predicts opposite movements. The sale frequency continues to fall, and the fraction of loyal customers and the inflation rate continue to rise. They are contrary to data and our aforementioned simulation results. Its reason is understood from Figure 12. With the labor supply elasticity below one, a decline in hours worked induces the rise in the the fraction of loyal customers, which lowers the frequency of sales.

Last, when all the three shocks are incorporated in the model, we confirmed that simulation results are almost the same as that shown in Figure 13 that uses only the technology shock.

### 5.3.2 Other Variables

In this paper, we dare not try to improve the goodness of fit for other macroeconomic variables such as the inflation rate and the GDP growth rate. This is because the focus of this paper is on hours worked, which serves as substitutes for bargain hunting, and in turn, influences firms' sale decision. Without constructing a richer model by incorporating capital investment with adjustment cost and the presence of the zero lower bound, for instance, it is too early to decompose developments in such macroeconomic variables. It would be nevertheless important to show how our simulation performs. Disappointingly, performance is very poor. Figure 14 shows that the simulated paths of the inflation rate and the GDP growth rate are very different from actual ones.


Figure 14: Model and Actual Paths of Macro Variables

## 6 Concluding Remarks

In this paper, we have examined macroeconomic implications of temporary sales. To this end, we have constructed a DSGE model with temporary sales and households' endogenous bargain hunting. The model has revealed that declines in hours worked during Japan's lost decades account for actual rises in the sale frequency and rises in the fraction of bargain hunters. Because sale prices are frequently revised and endogenous bargain hunting enhances the strategic substitutability of sales, the real effect of monetary policy weakens.

Future research needs to scrutinize the sources of business cycles. Moreover, further qualitative and quantitative evidence for endogenous bargain hunting needs to be presented.

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[^1]:    ${ }^{1}$ This result is also explained by intensified strategic substitutability of sales. Suppose that all firms but firm A raise their sale frequency. As in GS, it loses an incentive for firm A to raise its sale frequency, because its decreases the marginal revenue from sales. In our model, additional channel emerges. When all firms but firm A raise their sale frequency, an aggregate price falls. That increases aggregate demand for goods, and in turn, aggregate demand for labor. Households supply more labor and lose time in bargain hunting. The fraction of loyal customers (bargain hunters) increases (decreases). By observing this, firm A lowers its sale frequency. Such intensified strategic substitutability of sales mitigates the real effect of monetary policy.

[^2]:    ${ }^{2}$ Coibion, Gorodnichenko, and Hong (2012), however, report that the influence acts in an opposite and counter-intuitive direction. The frequency of sales falls when local unemployment rates rise. We discuss a reason for the difference in Section 2.
    ${ }^{3}$ See also Pashigian and Bowen (1991), Sorensen (2000), Brown and Goolsbee (2002), McKenzie and Schargrodsky (2004), Pashigian, Peltzman, and Sun (2003).
    ${ }^{4}$ Although they are not the model of sales, Benabou (1988), Watanabe (2008), and Coibion, Gorodnichenko, and Hong (2012) construct a model incorporating consumer search and price setting. Temporary sales are brought about by other reasons such as stock-out and implicit contracts with manufactuers, as well, but this paper chooses its base on the model of Varian (1980) and Guimaraes and Sheedy (2011).

[^3]:    ${ }^{5}$ For data liminations, we use only aggregated data.

[^4]:    ${ }^{6}$ See Abe and Tonogi (2010) and Sudo, Ueda, and Watanabe (2014) for details.
    ${ }^{7}$ Nikkei Digital Media defines a 3-digit code, from which we classify the types of products such as yogurt, beer, tobacco, and toothbrush.

[^5]:    ${ }^{8}$ They call it a reference price instead of a regular price. See Sudo, Ueda, and Watanabe (2014) for detailed discussions on the identification of sales.

[^6]:    ${ }^{9}$ In order to focus on the demand elasticity, we make two adjustments to the data sampling. First, we drop samples when realization of demand shock may be large by making use of the samples in the second and fourth quadrants in the scatter plot. Second, we employ the data of monthly changes for quantities and prices so as to eliminate effects stemming from temporary sales. Price changes below 3 yen are omitted. Because data are monthly, the number of sample for calculating a slope for a year is at most 12 . When the number of sample falls below 6 , we omit it. Even when we look at daily changes, we confirm a trend increase in price elasticity.

[^7]:    ${ }^{10}$ Before the revision, legal work hours were 48 per week. Legal work hours were gradually reduced to 40. Hours worked exceeding this legal limit should be compensated by at least a 25 -percent premium. See Kawaguchi, Naito, and Yokoyama (2008) for the analyses on jitan and Kuroda (2010) for the counter-argument asserting that hours worked hardly declined with demographic changes controlled.

[^8]:    ${ }^{11}$ Considering the possibility that the Baxter-King band pass filter yields spurious correlations, we calculate a $95 \%$ confidence interval by Monte Carlo simulation. That is, we replicate two independent time-series variables; draw their business-cycle components using the Baxter-King band pass filter; and calculate their correlation.

[^9]:    ${ }^{12}$ As Figure 4 showed, the frequency of temporary sales and hours worked exhibit non-zero trend. This may suggest using their changes in estimating the VAR model. We, however, used their levels, because theoretically they are $\mathrm{I}(0)$.
    ${ }^{13}$ Appendix A reports economic impacts of a monetary policy shock, which is insignificant. We decided not to show here, because the presence of the zero lower bound of nominal interest rate prevents us from isolating the monetary policy shock accurately.

[^10]:    ${ }^{14}$ However, to our surprise, the average shopping time of the US women is 352 minutes a week, according to American Time Use Survey in 2002. This is much longer than that of Japanese, depicted in Table 1.

[^11]:    ${ }^{15}$ See Appendix B. To be precise, we need to assume $c_{t}(\tau, b)^{-\frac{1}{\eta}} \ll c_{t}(\tau, b)^{-\frac{1}{\epsilon}}$. This is not a strict condition, because $\eta>\epsilon$.

[^12]:    ${ }^{16}$ The utility-related consumption $C$ also depends on the price ratio $P_{B} / P$, but that does not influence the household's decision of $L$ because the household is a price taker. As in GS, the price index for bargain hunters is the same for all product types that is, $P_{B}=p_{B}(\tau)$.

[^13]:    ${ }^{17}$ In our simulation, the elasticity of labor supply is 0.7 . In Hayashi and Prescott (2002), it equals one.
    ${ }^{18}$ To be more accurate, this labor supply shock differs from the shock to bargain hunting technology. The latter shock is considered to enter in the second term in the bracket in $v\left(H_{t}+\phi_{L} \frac{\left(1-L_{t}\right)^{\theta} L}{(1-\lambda)^{\theta} L}\right)$. Nevertheless, we confirmed that these two shocks yield the qualitatively same impulse responses.

