Asset Illiquidity and Dynamic Bank Capital Requirements

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Abstract

This paper introduces banks into a dynamic stochastic general equilibrium model by featuring asymmetric information as the underlying friction for banking. Asymmetric information about asset qualities causes a lemons problem in the asset market. In this environment, banks can issue liquid liabilities by pooling illiquid assets contaminated by asymmetric information. The liquidity transformation by banks results in a minimum value of common equity that banks must issue to avoid a run. This value increases with downside risk to the asset price and the expected degree of asset illiquidity. It rises during a boom if productivity shocks cause the business cycle.

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1 Introduction

Banking is a crucial part of the modern economy. This fact has been reconfirmed by the recent financial crisis. Yet, an effort to integrate banks into macroeconomic models is still ongoing in the literature. Recent work in this strand of literature includes Chen (2001), Gertler and Karadi (2011), He and Krishnamurthy (forthcoming), Rampini and Viswanathan (2010), Brunnermeier and Sannikov (2011), and Gertler, Kiyotaki, and Queralto (2011). These papers highlight various roles of banks, such as loan monitoring, debt enforcement, and asset management. In this paper, I focus on yet a different aspect of banks—the suppliers of liquid assets. I introduce this aspect of banks into a dynamic general equilibrium model by featuring asymmetric information about asset qualities as the underlying friction. The model shows that banks supplying liquid assets must satisfy a minimum value of common equity to avoid a bank run. This value is determined by macroeconomic conditions. Thus, a threat of a bank run makes banks show macroprudential behavior endogenously, if banks are rational and do not have any agency problem with depositors or equity holders as assumed in the model.

The model is a version of AK model. The economy grows through investments of goods into some real assets, which in turn generates goods in each period. The opportunity to invest into real assets arrives randomly at each investor in each period. Those who receive investment opportunities must finance their investments by selling their existing assets because of borrowing constraints. The secondary market for real assets, however, is contaminated by a lemons problem: each unit of real assets depreciates at an i.i.d. rate in each period and the rate is the private information of the seller. As a result, investors withhold real assets with low depreciation rates because of undervaluation in the market. I call this non-traded fraction of real assets illiquid.

I introduce banks into this environment. Banks are public companies issuing deposits and common equity. This assumption is based on the fact that banks are public companies in practice. Using the funds raised, banks buy real assets in the secondary market, and pool them. Through asset pooling, the idiosyncratic depreciation rates of each bank's real assets average out. As a result, each bank's total revenue becomes public information. Investors can, therefore, resell bank deposits and equity backed by banks' revenues without a lemons problem. They are willing to hold these securities as liquid assets.

Banks can pool real assets because, unlike investors, they do not have private information about the depreciation rates of their real assets. If they had private information, they would keep only real assets with low depreciation rates, reselling the other fraction of real assets in the market. Thus, pooled real assets would be unbundled in this case. The assumption that investors have better information than banks reflects firms' superior knowledge about their own production and trading partners in practice. For example, interpret investments into real assets in the model as including the provision of trade credit by firms to enhance their suppliers' production and, hence, their own production. Trade credit is normally illiquid as outsiders cannot easily assess its quality. The result of the model provides an explanation as to why banks can still discount trade credit to various firms.[1]

The model implies a minimum bank equity requirement based on Value-at-Risk. Due to the illiquidity of real assets, the present discounted value of future revenues from a bank's assets is greater than the market value of the assets. As part of multiple equilibria, a bank suffers a self-fulfilling bank run if the face value of its deposits exceeds the market value of its assets. To eliminate any possibility of a run in this case, a bank must limit the issuance of deposits to the worst possible market value of its assets in the next period. Thus, the rest of the present discounted value of its assets must be financed through common equity. This

[1] This function of banking is different from that of mutual funds. Mutual funds bundle securities that are already tradable in the securities market. The benefit of using mutual funds is to delegate portfolio adjustments to professional asset managers and to save transaction costs in the market. I do not analyze this effect of bundling here.

value is the minimum common equity value that a bank must satisfy to avoid a run.

The minimum common equity value can be decomposed into two factors:

$$
\begin{aligned}
&\text{Minimum common equity value}\\
= \;& \text{Expected discounted value of future revenues from the bank's assets}\\
&\quad - \text{Discounted worst possible market value of the assets in the next period}\\
= \;& (\text{Expected discounted value of future revenues from the assets}\\
&\qquad - \text{Expected discounted market value of the assets in the next period})\\
&+ (\text{Expected discounted market value of the assets in the next period}\\
&\quad - \text{Discounted worst possible market value of the assets in the next period})\\
= \;& \text{Expected illiquidity of the assets (the first parenthesis)}\\
&+ \text{Downside risk to the market value of the assets (the second parenthesis).}
\end{aligned}
$$

Both factors on the right-hand side fluctuate endogenously over the business cycle. Through a calibration exercise, I show that the minimum common equity value is pro-cyclical if aggregate productivity shocks cause the business cycle. In this case, banks need to raise more equity during a boom than a recession. Given rationality and no agency problem with depositors or equity holders, banks voluntarily satisfy this minimum equity requirement to avoid a run, if the probability of the worst state in the next period is sufficiently high.

The cyclicality of the minimum common equity value is consistent with countercyclical capital buffer recently introduced by Basel III. This result provides an interpretation of Basel III such that Basel III imposes on actual banks the behavior of rational banks with no agency problem, in case there is some irrationality or moral hazard, such as risk shifting, at actual banks. Also, the model implies that common equity (i.e., bank capital) is unnecessary in an equilibrium in which depositors never run to banks. In light of this result, Basel III can be interpreted as preventing over-optimistic behavior of banks. If banks believe that a no bank-run equilibrium will hold, then they have no incentive to maintain bank capital. In case such bank expectations are over-optimistic, policy makers impose a bank capital requirement that is robust even if a self-fulfilling bank run can occur.

1.1 Related literature

Besides the aforementioned papers, this paper is related to several other strands of literature.

Kiyotaki and Moore (2012) incorporate asset illiquidity into a dynamic stochastic general equilibrium model by introducing resaleability constraints. These constraints limit the fraction of assets that each agent can resell per period. Tomura (2012) endogenizes the resaleability constraints by introducing asymmetric information into a set-up similar to Kiyotaki and Moore's model. The way to endogenize asset illiquidity in a competitive market follows Banerjee and Maskin (1996) and Eisfeldt (2004). In this paper, I introduce public companies functioning as banks into the model developed by Tomura (2012).

Williamson (1988) and Gorton and Pennacchi (1990) consider asymmetric information as the underlying friction for banking. They model a bank as a coalition of agents to overcome adverse selection in the market. In contrast to their cooperative game-theoretic approach, I introduce banks into a competitive equilibrium model, in which agents take as given the competitive market prices of bank securities when they decide whether to fund banks.

A self-fulfilling bank run due to asset illiquidity in this paper is similar to the one analyzed by Diamond and Dybvig (1983). A crucial difference, however, is in that asset illiquidity is endogenous in this paper. As a result, the degree of asset illiquidity fluctuates over the business cycle. This feature of the model leads to the finding that endogenous fluctuations in asset illiquidity result in a dynamic minimum equity requirement for banks. Also, I find that downside risk to the market value of bank assets is another determinant of a minimum equity requirement. This finding is similar to the papers by Diamond and Rajan (2000, 2001). In this paper, I derive the two factors for a minimum equity requirement in a unified framework.

Finally, Covas and Fujita (2010) analyze the effects of Basel I and II on real economic activity by featuring banks as the suppliers of credit lines. Their model is based on Kato's (2006) model, which extends Holmström and Tirole's (1998) model to a dynamic stochastic general equilibrium model. This paper adds to their work by discussing the model's implications for Basel III.

2 Model of asset illiquidity

I start from presenting a basic model without banks to illustrate endogenous asset illiquidity due to asymmetric information. This model is based on Tomura (2012).

Time is discrete. There is a $[0, 1]$ continuum of infinite-lived agents. Each agent maximizes the expected discounted utility of consumption of goods:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{i,t}, \tag{1}$$

where β ($\in (0, 1)$) is the time discount factor, i ($\in [0, 1]$) is the index for each agent, t ($= 0, 1, 2, ...$) is the time period, and $c_{i,t}$ is the consumption of goods in period t. Agents generate goods from capital stock at the beginning of each period through a linear function:

$$y_{i,t} = \alpha_t k_{i,t-1}, \tag{2}$$

where $y_{i,t}$ is output, $k_{i,t-1}$ is the quantity of capital held at the beginning of period t, and α_t is the aggregate productivity shock. This shock is the structural shock in this paper.

Each infinitesimal unit of capital depreciates at an i.i.d. rate after production. The distribution of depreciation rates in each period is uniform over $[\bar{\delta} - \Delta,\ \bar{\delta} + \Delta]$, where $\bar{\delta}$ ($\in (0, 1)$) denotes the mean and Δ ($\in [0, \min\{\bar{\delta}, 1-\bar{\delta}\}]$) determines the range of idiosyncratic depreciation rates.[2] Thus, each agent holds capital with uniformly distributed depreciation

[2] The range of Δ in its definition ensures that the depreciation rates of capital are non-negative and not more than unity.

rates after production in each period.

The current depreciation rate of each unit of capital is the private information of the agent who uses the capital for production in the period. This assumption is motivated by heterogeneity in asset quality in reality and the fact that the quality of an asset is often the private information of the owner.[3] I simplify the information dynamics by assuming that depreciation rates in each period become public information at the beginning of the next period.

Agents can trade depreciated capital in a competitive secondary market. Because of the private information, every agent has a common price of capital, Q_t, regardless of the depreciation rate of each unit of capital sold. I assume that the price is independent of the quantity of capital traded in each transaction, since a linear pricing follows immediately from arbitrage if agents can make any number of transactions in the market in each period.[4] The realized average depreciation rate of capital bought by an agent equals the average depreciation rate of capital sold in the market, which is a standard assumption in the literature (e.g., Eisfeldt 2004).

At the end of each period, agents can produce new capital from goods. If an agent invests an amount $x_{i,t}$ of goods, then the agent obtains an amount $\phi_{i,t}x_{i,t}$ of new capital at the beginning of the next period, given the agent's investment efficiency, $\phi_{i,t}$. I assume that $\phi_{i,t} \in \{0, \phi\}$ ($\phi > 0$), and that the probability that $\phi_{i,t} = \phi$ is ρ ($\in (0, 1)$) for all i and

[3] The qualities of consumer durables such as a car and a house are easily observable examples of private information. For empirical analysis of business capital, see Eisfeldt and Rampini (2006). Also see Ashcraft and Schuermann (2008) and Downing, Jaffee, and Wallace (2009) on the presence of asymmetric information in the mortgage-backed securities market.

[4] This point is made by Banerjee and Maskin (1996, footnote 16). Suppose that there are two prices of capital, Q and Q', for two different quantities of capital gross of depreciation, X and X', respectively. Given that $Q > Q'$ without loss of generality, each agent can sell and buy the price-quantity pairs (Q, X) and (Q', X'), respectively, infinitely many times to earn an arbitrarily large profit. Note that the private information about the depreciation rates of capital does not matter here, since arbitraging agents do not have to take a net position in capital gross of depreciation. The linear pricing may not hold if an agent can participate in only one transaction in each period. Gale (1992) assumes such limited participation in a competitive market with adverse selection and derives a quantity-contingent pricing in a separating equilibrium.

t. Thus, only agents with $\phi_{i,t} = \phi$ can invest in new capital. I call agents with $\phi_{i,t} = \phi$ "productive" and those with $\phi_{i,t} = 0$ "unproductive". Each agent learns the value of $\phi_{i,t}$ at the beginning of period t.

I assume that each agent cannot borrow against their investments in new capital. Even though prohibiting any borrowing is not essential for the main result of the model as long as borrowing constraints on productive agents bind, the basic model without banks has an analytical solution with this assumption.

2.1 Utility maximization problem of each agent

The following maximization problem summarizes the environment for each agent:

$$\max_{\{c_{i,t},\, x_{i,t},\, h_{i,t},\, l_{i,\delta,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{i,t} \tag{3}$$

$$\text{s.t. } c_{i,t} + x_{i,t} + Q_t h_{i,t} = \alpha_t k_{i,t-1} + Q_t \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t}\, d\delta, \tag{4}$$

$$k_{i,t} = \phi_{i,t} x_{i,t} + (1-\hat{\delta}_t) h_{i,t} + \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} (1-\delta) \left(\frac{k_{i,t-1}}{2\Delta} - l_{i,\delta,t} \right) d\delta, \tag{5}$$

$$l_{i,\delta,t} \in \left[0,\ \frac{k_{i,t-1}}{2\Delta}\right],\ c_{i,t} \geq 0,\ x_{i,t} \geq 0,\ h_{i,t} \geq 0, \tag{6}$$

where $h_{i,t}$ is the quantity of capital gross of depreciation that the agent buys in the secondary capital market, $l_{i,\delta,t}$ is the density of capital with depreciation rate δ that the agent sells in the market, and $\hat{\delta}_t$ is the average depreciation rate of capital sold in the market. Eqs. (4) and (5) are the flow-of-funds constraint and the law of motion for capital net of depreciation (i.e., $k_{i,t}$), respectively. Eq. (6) contains the short-sale constraint on capital and non-negativity constraints on the choice variables. Note that $k_{i,t-1}/(2\Delta)$ is the uniform density of the agent's capital with each depreciation rate over $[\bar{\delta}-\Delta,\ \bar{\delta}+\Delta]$. Each agent takes as given the probability distribution of $\{Q_t,\ \hat{\delta}_t,\ \alpha_t,\ \phi_{i,t}\}_{t=0}^{\infty}$.[5]

[5]All variables except α_t and $\phi_{i,t}$ are state-contingent. The notation of state contingency is omitted here.

In Eq. (5), the quantity of capital net of depreciation, $k_{i,t}$, is the sufficient state variable for the agent's capital at the beginning of the next period, because the current depreciation rates of capital will be public information then. Also, the realized average depreciation rate of capital bought by the agent equals $\hat{\delta}_t$, as assumed above.

2.2 Definition of an equilibrium

The secondary market price of capital, Q_t, and the average depreciation rate of capital sold in the secondary market, $\hat{\delta}_t$, are determined by the following conditions:

$$\hat{\delta}_t = \frac{\int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} \delta \, l_{i,\delta,t} \, d\delta \, di}{\int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t} \, d\delta \, di}, \tag{7}$$

$$\int h_{i,t} \, di = \int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t} \, d\delta \, di. \tag{8}$$

The numerator of the fraction on the right-hand side of the first equation is the total depreciation of capital sold, and the denominator is the total quantity of capital sold. The second equation is a standard market clearing condition for the secondary capital market. Given the value of $k_{i,-1}$ for each i and the probability distribution of $\{\alpha_t, \phi_{i,t}\}_{t=0}^{\infty}$, an equilibrium is the solution to the maximization problem for each i defined by Eqs. (3)-(6) and $(Q_t, \hat{\delta}_t)$ satisfying Eqs. (7)-(8) for all t.

In the basic model, I assume only that the aggregate productivity shock, α_t, follows a stochastic process implying a well-defined expectation operator, E_0, in Eqs. (3)-(6) in an equilibrium. Because of the log utility function, the functional form of equilibrium conditions is invariant to the specification of the stochastic process.

2.3 Endogenous asset illiquidity

I briefly summarize the results in the basic model.[6]

In the equilibrium, productive agents invest in new capital if and only if their investment efficiency, ϕ, is so high that $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$. In this case, they are willing to obtain goods from unproductive agents to enhance their investments in new capital. The only channel for the transfer of goods from unproductive agents to productive agents, however, is the secondary capital market because of borrowing constraints. Thus, agents sell and buy capital in the secondary capital market when they become productive and unproductive, respectively.

The secondary market price of capital, Q_t, is identical for every unit of capital sold because the depreciation rates of capital are the private information of the sellers, as assumed above. As a result, agents withhold capital with low depreciation rates to avoid the undervaluation of the capital in the market:

Proposition 1 *If $\Delta > 0$, then productive and unproductive agents sell only the fractions of capital with depreciation rates greater than $\min\{1 - Q_t\phi,\ \hat{\delta}_t\}$ and $\hat{\delta}_t$, respectively.*

Proof. See Appendix A. ■

I call the non-traded fraction of capital illiquid. The existence of illiquid capital reduces the amount of goods that productive agents obtain for their investments in new capital by selling their existing capital:

Proposition 2 *There exists unique equilibrium in the basic model. Denote aggregate investment in new capital, $\int x_{i,t}\,di$, by X_t and aggregate output, $\int y_{i,t}\,di$, by Y_t. In the equilibrium, $X_t/Y_t > 0$ if and only if $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$. In this case, the value of X_t/Y_t is higher with $\Delta = 0$ than with $\Delta > 0$.*

[6] See Tomura (2012) for more detailed analysis of the basic model.

Proof. See Appendix B. ■

Note that there is no asymmetric information if $\Delta = 0$. Thus, an increase in Δ from zero to a positive number introduces asymmetric information into the economy.

In the next section, I introduce banks into this environment. I will show that the illiquidity of capital leads to agents' demand for liquid securities issued by banks. Banks, however, are subject to a self-fulfilling bank run if their equity values fall below a certain threshold.

3 Model of banking

3.1 Banks

In addition to the agents described above, there are a continuum of homogeneous banks. Banks are public companies issuing two types of securities: deposits and common equity. Agents can buy these securities in a securities market. Those buying common equity direct the behavior of banks as the owners. Banks can spend the funds raised on buying capital in the secondary capital market.

When buying capital, banks cannot know the depreciation rate of each unit of capital sold in the secondary market. This assumption is as same as the one for agents. Also, banks cannot know the current depreciation rate of each unit of their own capital after production, until the rate becomes public information at the beginning of the next period. Thus, banks have less information than agents. This assumption reflects firms' superior knowledge about their own production and trading partners in practice. For example, interpret investments into capital in the model as including the provision of trade credit by firms to enhance their suppliers' production and, hence, their own production. Trade credit is normally illiquid as outsiders cannot easily assess its quality. Investments into capital in the model can also include other types of information-intensive assets held by firms, such as direct investments into non-public, small firms.[7]

Banks can produce goods from their capital through the same linear function as Eq. (2). After production, each infinitesimal unit of capital held by each bank depreciates at an i.i.d. rate. The distribution of the depreciation rates is as same as the uniform distribution for each agent defined above. Banks do not have ability to invest in new capital.

3.2 Utility maximization problem of an agent

With bank deposits and equity, the flow-of-funds constraint for each agent is modified to:

$$c_{i,t} + x_{i,t} + Q_t h_{i,t} + b_{i,t} + (1+\zeta)V_t s_{i,t} = \\ \alpha_t k_{i,t-1} + Q_t \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t}\, d\delta + \tilde{R}_t b_{i,t-1} + (D_t + V_t)s_{i,t-1}, \quad (9)$$

where: $b_{i,t}$ and $s_{i,t}$ are the value of bank deposits and the number of units of bank equity, respectively, that the agent holds at the end of period t; $\tilde{R}_t$ is the ex-post gross deposit interest rate defined below; and V_t is the price of bank equity. Also, ζ (> 0) is an equity transaction cost per value of bank equity. This cost reflects the fact that it is more costly to manage equity than deposits. The values of $b_{i,t}$ and $s_{i,t}$ must be non-negative because agents cannot short-sell bank securities by assumption.

Agents take as given the value of V_t and the probability distribution of $\tilde{R}_{t+1}$ in the next period. The utility maximization problem of each agent implies the following Euler equations

[7]Given no private information held by banks, I can exclude the case in which banks transfer their private information to their equity holders, i.e., their owners. Hence, each agent has an identical value of the secondary market price of capital, Q_t, even if some banks sell their capital in the market.

for V_t and $\tilde{R}_{t+1}$:

$$(1+\zeta)V_t = E_t\left[\Lambda_{V,t+1}(D_{t+1}+V_{t+1})\right], \tag{10}$$

$$1 = E_t\left[\Lambda_{R,t+1}\tilde{R}_{t+1}\right], \tag{11}$$

where $\Lambda_{V,t+1}$ and $\Lambda_{R,t+1}$ denote the stochastic discount factors, $\beta c_{i,t}/c_{i,t+1}$, for agents buying bank equity and deposits, respectively, in period t.[8]

3.3 Flow of funds for a bank

The flow-of-funds constraint for each bank can be written as:

$$D_t S_{B,t-1} + \tilde{R}_t B_{B,t-1} + Q_t(H_{B,t} - L_{B,t}) = \alpha_t K_{B,t-1} + B_{B,t} + V_t(S_{B,t} - S_{B,t-1}), \tag{12}$$

where: D_t is the dividends per unit of bank equity; $S_{B,t-1}$ is the units of bank equity outstanding at the beginning of period t; $B_{B,t-1}$ is the value of deposits outstanding at the beginning of period t; $H_{B,t}$ and $L_{B,t}$ are the amounts of capital gross of depreciation that a bank buys and sells in the secondary capital market, respectively; and $K_{B,t-1}$ is the quantity of capital held by a bank at the beginning of period t.[9] The left-hand side and the right-hand side of Eq. (12) are expenditure and income, respectively. The last term on the right-hand side is the revenue from newly issued equity if it is positive, and the expenditure on equity repurchases if it is negative.

[8] Eqs. (10)-(11) are the first-order conditions with respect to $b_{i,t}$ and $s_{i,t}$ for those buying bank equity and deposits, respectively. Eqs. (10)-(11) hold even if banks choose not to supply deposits or equity, because Eqs. (10)-(11) only require some agents to be indifferent to holding bank equity and deposits, respectively.

[9] Throughout this paper, the index for each bank is omitted from the notation of the variables because banks are homogeneous.

3.4 Bank run

The ex-post gross interest rate on bank deposits, $\tilde{R}_t$, equals the contracted gross deposit rate set in the previous period, $\bar{R}_{t-1}$, if the bank does not default. A bank suffers a run, however, if the face value of bank deposits, $\bar{R}_{t-1}B_{B,t-1}$, exceeds the market value of bank assets at the beginning of the period, $(\alpha_t + Q_t)K_{B,t-1}$. In this case, the bank must sell its capital immediately in the secondary market to maximize the repayment to depositors. Hence:

$$\tilde{R}_t = \begin{cases} \bar{R}_{t-1}, & \text{if } \bar{R}_{t-1}B_{B,t-1} \leq (\alpha_t + Q_t)K_{B,t-1}, \\ \frac{(\alpha_t+Q_t)K_{B,t-1}}{B_{B,t-1}}, & \text{if } \bar{R}_{t-1}B_{B,t-1} > (\alpha_t + Q_t)K_{B,t-1}, \end{cases} \tag{13}$$

$$L_{B,t} = K_{B,t-1}, H_{B,t} = V_t = D_t = 0, \text{ if } \bar{R}_{t-1}B_{B,t-1} > (\alpha_t + Q_t)K_{B,t-1}. \tag{14}$$

The deposit recovery rate in the second line of Eq. (13), $(\alpha_t + Q_t)K_{B,t-1}/B_{B,t-1}$, is less than the contracted rate, $\bar{R}_{t-1}$. Thus, a bank defaults if hit by a run, as expected by depositors running to the bank. The deposit recovery rate is determined by the flow-of-funds constraint (12), given Eq. (14). Agents running to a bank take as given the secondary market price of capital, Q_t, because each bank is so infinitesimal that the failure of one bank does not affect Q_t.

3.5 Profit maximization problem of a bank

Being a public company, each bank maximizes the cum-dividend value of equity, $(D_t + V_t)S_{B,t-1}$, for its existing equity holders in each period. In doing so, each bank internalizes Eqs. (10)-(11), given the joint probability distribution of $\Lambda_{V,t+1}$ and $\Lambda_{R,t+1}$. Thus, each bank takes into account the responses of its equity price, V_t, and the contracted gross deposit rate for its deposits, $\bar{R}_t$, to its behavior.[10] This assumption on the behavior of a public company

[10]Each bank takes as given the equity prices and the deposit rates of the other banks. The values of V_t and $\bar{R}_t$ are identical for every bank in equilibrium because banks are homogeneous.

is standard in the literature.[11]

Substituting Eq. (10) into the flow-of-funds constraint (12) yields the following recursive form for the cum-dividend value of equity, $(D_t + V_t)S_{B,t-1}$:

$$
\begin{aligned}
(D_t + V_t)S_{B,t-1} = \Omega_t(K_{B,t-1}, B_{B,t-1}, \bar{R}_{t-1}) \equiv \\
\max_{\{H_{B,t}, L_{B,t}, B_{B,t}\}} \; & \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + B_{B,t} \\
& + E_t \left[\frac{\Lambda_{V,t+1}\Omega_{t+1}(K_{B,t}, B_{B,t}, \bar{R}_t)}{1+\zeta} \right], \qquad (15) \\
\text{s.t. Eqs. } & (11), (13), \text{ and } (14), \\
& K_{B,t} = (1 - \hat{\delta}_t)H_{B,t} + (1 - \bar{\delta})(K_{B,t-1} - L_{B,t}), \qquad (16) \\
& L_{B,t} \in [0, K_{B,t-1}], \; H_{B,t} \geq 0, \; B_{B,t} \geq 0. \qquad (17)
\end{aligned}
$$

In the profit maximization problem, a bank chooses how much capital to buy and sell in the secondary capital market ($H_{B,t}$ and $L_{B,t}$) and the amount of deposits to raise ($B_{B,t}$). Once the values of these choice variables are determined, the amount of dividends per equity (D_t) and the value of equity issuance or repurchase ($S_{B,t} - S_{B,t-1}$) can be induced from Eqs. (10), (12), and (15).

The constraint set includes Eqs. (13)-(14), because a bank takes into account the risk of a bank run. A bank also internalizes the determination of $\bar{R}_t$ through Eqs. (11) and (13), as assumed above. Eq. (16) is the law of motion for capital held by a bank. In this equation, the realized average depreciation rate of capital bought by a bank equals the average depreciation rate of capital sold in the market, $\hat{\delta}_t$, as assumed for agents. Also, the value of $L_{B,t}$ does not depend on the depreciation rate of each unit of capital sold by a bank,

[11] For example, see Woodford (2003) for the profit maximization problem of an intermediate-good producing firm in a standard New-Keynesian model. In this type of model, each firm maximizes the present discounted value of current and future profit with the same stochastic discount factor as households', given that households own the equity of the firm. The present discounted value of current and future profit equals the cum-dividend equity price if a competitive equity market is introduced into the model.

because a bank can sell its capital only randomly without knowing the current depreciation rate of each unit of its capital. Hence, the average depreciation rate of capital sold by a bank equals the average depreciation rate of capital held by the bank (i.e., $\bar{\delta}$) by the law of large numbers. Eq. (17) contains the short-sale constraint in the secondary capital market and non-negativity constraints on a bank's choice variables. A bank takes as given the probability distribution of $\{Q_t,\ \hat{\delta}_t,\ \alpha_t,\ \Lambda_{V,t},\ \Lambda_{R,t}\}_{t=0}^{\infty}$.[12]

3.6 Shock process

Hereafter, I assume that the aggregate productivity shock, α_t, follows a two-state Markov process: $\alpha_t \in \{\bar{\alpha}, \underline{\alpha}\}$; and the transition probability function denoted by P is such that $P(\alpha_{t+1} = \bar{\alpha} \mid \alpha_t = \bar{\alpha}) = \bar{\eta}_\alpha$ and $P(\alpha_{t+1} = \underline{\alpha} \mid \alpha_t = \underline{\alpha}) = \underline{\eta}_\alpha$ for all t.

3.7 Definition of an equilibrium

Given $(k_{i,-1}, s_{i,-1}, b_{i,-1})$ for each i, $(K_{B,-1}, B_{B,-1}, \bar{R}_{-1})$, and the probability distribution of $\{\alpha_t,\ \phi_{i,t}\}_{t=0}^{\infty}$, an equilibrium consists of: the solutions to the maximization problems for agents and banks; the secondary market price of capital, Q_t, that clears the market; the average depreciation rate of capital sold in the secondary market, $\hat{\delta}_t$, that satisfies its definition; and the equity price and the contracted gross deposit interest rate, $(V_t, \bar{R}_t)$, that satisfy Eqs. (10)-(11), given the stochastic discount factors for the agents holding bank equity and deposits, $(\Lambda_{V,t}, \Lambda_{R,t})$. See Appendix C for an analytical expression of equilibrium conditions.

[12] If there is no existing equity holder for a bank (i.e., $S_{B,t-1} = 0$), then the bank maximizes the net profit from the initial public offering and consumes the profit right away. Because the net profit equals the value of Ω_t, this case is covered by the maximization problem (15). The net profit becomes zero in the equilibrium.

4 Equilibrium dynamics

4.1 Parameter specification

Lacking the closed-form solution for the equilibrium conditions, I solve the model numerically. For the benchmark parameter values, I set ($\bar{\delta}$, Δ, ϕ, β, ζ, ρ) = (0.1, 0.09, 4.75, 0.99, 0.02, 0.45), and $\bar{\alpha} = \underline{\alpha} = 0.03$. With these values, the model approximately replicates the following sample averages of annual data on the balanced growth path: the real GDP growth rate (3.4%), the real interest rate on three-month Treasury bills (3.9%), and the ratio of commercial bank credit to aggregate fixed assets (15.0%) in the U.S. over 1948-2007; the capital-to-asset ratio of banks required by Basel Committee (8%); the annual depreciation rate of capital (10%); and the equity premium for S&P 500 over 1948-1992 as reported by Rouwenhoust (1995) (1.99%).[13]

For dynamic analysis, I consider the following stochastic process of the aggregate productivity shock: ($\bar{\alpha}$, $\underline{\alpha}$) = (0.0306, 0.0294) and $\bar{\eta}_\alpha = \underline{\eta}_\alpha = 0.75$. The other parameters are fixed to the benchmark values specified above. The two possible values of the aggregate productivity shock represent a boom and a recession, each of which lasts for four years on average, given the annual frequency of the model. The conditional average of the output growth rate ($(Y_t - Y_{t-1})/Y_{t-1}$ where Y_t denotes aggregate output) is 4.36% when $\alpha_t = \bar{\alpha}$, and 2.01% when $\alpha_t = \underline{\alpha}$. See Appendices D and E for the complete set of the equilibrium laws of motion for aggregate variables that hold with this set of parameter values. To compute a stochastic dynamic equilibrium, I use a projection method to approximate the state-space solution for the model non-linearly. See Appendix F for the numerical solution method.

[13]The sample averages are matched by: $(Y_t - Y_{t-1})/Y_{t-1}$ where Y_t denotes aggregate output; $\bar{R}_t - 1$; $K_{B,t}/(K_{B,t} + \int k_{i,t}\, di)$; $V_t S_{B,t}/(B_{B,t} + V_t S_{B,t})$; $\bar{\delta}$; and $(D_t + V_{t+1})/V_t - \bar{R}_t$, in order. The first three sample averages are from the BEA and the Federal Reserve Board. The value of $\bar{\alpha}$ is arbitrary, because the model can be normalized by $\bar{\alpha}$.

4.2 Illiquidity of capital

Hereafter, I sketch the feature of the equilibrium with the parameter values specified above. The following inequalities imply that the investment efficiency of productive agents, ϕ, is so high that they invest only in new capital:

$$\phi^{-1} < Q_t(1-\hat{\delta}_t)^{-1}, \tag{18}$$

$$1 > E_t\left[\frac{\beta c_{i,t}\tilde{R}_{t+1}}{c_{i,t+1}} \,\middle|\, \phi_{i,t}=\phi\right], \tag{19}$$

$$(1+\zeta)V_t > E_t\left[\frac{\beta c_{i,t}(D_{t+1}+V_{t+1})}{c_{i,t+1}} \,\middle|\, \phi_{i,t}=\phi\right]. \tag{20}$$

Accordingly, productive agents sell their existing capital and bank securities (i.e., deposits and equity) to obtain goods to invest in new capital.[14]

Proposition 1 holds for productive agents as in the basic model without banks. Thus, a lemons problem in the secondary capital market makes productive agents to sell only a fraction of capital with high depreciation rates.[15] As a result, the average depreciation rate of capital sold in the secondary capital market, $\hat{\delta}_t$, exceeds the unconditional average depreciation rate of capital, $\bar{\delta}$:

$$\hat{\delta}_t > \bar{\delta}. \tag{21}$$

[14]Eq. (18) indicates that productive agents invest in new capital, rather than buy capital in the secondary market. The left-hand sides of Eqs. (19) and (20) are the marginal costs of buying bank deposits and equity, respectively. The right-hand sides are the expected discounted returns on bank deposits and equity for productive agents. The costs exceed the expected discounted returns because a high return on investment in new capital lowers the stochastic discount factor for productive agents. Thus, productive agents do not buy bank deposits or equity.

[15]Proposition 1 does not exactly hold for unproductive agents in the model with banks. The availability of bank securities increases unproductive agents' gains from obtaining goods by selling their capital, because they can buy bank securities with the obtained goods. As a result, the threshold for the depreciation rate of capital sold by unproductive agents falls below $\hat{\delta}_t$. The threshold remains above $\hat{\delta}_t - \Delta$ for both productive and unproductive agents with the parameter values specified above.

In contrast, bank deposits and equity are free of a lemons problem. Given Eq. (21), the law of motion for capital (16) implies that a bank loses capital net of depreciation (i.e., $K_{B,t}$) if it buys and sells capital in the secondary market simultaneously (i.e., $H_{B,t} > 0$ and $L_{B,t} > 0$). This result holds because a bank must sell its capital randomly without knowing the depreciation rate of each unit of its capital. Thus, a bank can commit to pooling its entire capital as long as it buys capital in the secondary market (i.e., $H_{B,t} > 0$).

The idiosyncratic depreciation rates of the entire capital held by each bank average out to the unconditional average depreciation rate of capital, $\bar{\delta}$. Thus, the total revenue from each bank's assets becomes public information. As a result, the market value of bank securities, which are backed by each bank's revenue, becomes free of a lemons problem:[16]

Proposition 3 *Suppose Eqs.* (18)-(20) *hold in an equilibrium. If* $H_{B,t} > 0$ *and no bank run occurs in period* t*, then:*

$$\bar{R}_{t-1} B_{B,t-1} + (D_t + V_t) S_{B,t-1} = \left[\alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t} \right] K_{B,t-1}. \tag{22}$$

Proof. See Appendix D. ∎

Proposition 3 implies that the total market value of bank securities equals the present discounted value of current and future income from bank assets, given no bank run.[17] Ac-

[16]Proposition 3 does not require $H_{B,t} > 0$ or no bank run for all t. It holds even if there is a positive probability of a bank run in the future.

[17]Note that $Q_t(1-\hat{\delta}_t)^{-1}$ on the right-hand side of Eq. (22) is the marginal acquisition cost of capital net of depreciation in the secondary market. In equilibrium, this cost equals the present discounted value of future marginal income from capital net of depreciation. If the cost is more than the present discounted value of future marginal income from capital net of depreciation, then a bank would buy no capital in the secondary market, which contradicts $H_{B,t} > 0$. If it is less, then a bank would issue an arbitrary large amount of securities to buy an arbitrarily large amount of capital, which violates the market clearing condition for the secondary capital market. Hence, the right-hand side of Eq. (22) is the present discounted value of current and future income from capital at the beginning of each period t.

cordingly, the market value of bank securities exceeds the market value of bank assets, $(\alpha_t + Q_t)\, K_{B,t-1}$, at the beginning of each period:

$$\left[\alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t}\right] K_{B,t-1} > (\alpha_t + Q_t)\, K_{B,t-1}, \tag{23}$$

in which the strict inequality holds given Eq. (21). Thus, productive agents can obtain more goods to invest in new capital if they hold bank securities rather than capital.

Ex-ante, unproductive agents buy bank securities in case they become productive in the next period.[18] Also, they do not buy capital in the secondary market, but sell a fraction of their capital with sufficiently high depreciation rates to raise funds to buy bank securities. They keep the other fraction of capital to avoid the undervaluation of the capital in the secondary capital market.[19]

4.4 Banks' incentive for liquidity transformation

It is optimal for banks to meet the demand for bank securities, given no bank run. In this case, banks arbitrage between the market prices of bank securities and the secondary market price of capital, Q_t. Thus, banks issue deposits and equity to buy capital in the secondary market in each period (i.e., $H_{B,t} > 0$ for all t). The purchase of capital in each period makes it credible for banks to pool capital, as described above. In the equilibrium, the marginal acquisition cost of capital net of depreciation, $Q_t(1-\hat{\delta}_t)^{-1}$, rises to the present discounted value of future marginal income from capital net of depreciation, so that the arbitrage free condition holds.[20] Hence, banks earn no rent.

[18]Thus, the stochastic discount factors for the holders of bank equity and deposits, $\Lambda_{V,t}$ are $\Lambda_{R,t}$, respectively, equal the one for unproductive agents. Every unproductive agent has an identical stochastic discount factor in each period due to the log utility function.

[19]This behaviour implies that every agent holds some amount of capital to sell after any history. Given Eq. (18), a productive agent invests in new capital, which becomes existing capital in the next period. If a productive agent becomes unproductive afterward, then the agent sells only part of the agent's capital in each period.

[20]This arbitrage free condition is incorporated by Eq. (22). See footnote 17 for more details.

4.5 Minimum equity-to-asset ratio based on Value-at-Risk

The number of possible values of the aggregate productivity shock in the next period, α_{t+1}, is two in each period, as assumed above. I denote by $\underline{\omega}_{t+1}$ the smaller market value of each unit of bank assets in the next period, $\alpha_{t+1} + Q_{t+1}$, given the state of period t. Given a sufficiently high probability of the low state as implied by the values of $\bar{\eta}_\alpha$ and $\underline{\eta}_\alpha$ specified above, each bank limits the issuance of deposits to satisfy:

$$\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}, \tag{24}$$

for all t, so that no bank run will occur in the next period.[21]

This result holds because a bank suffering a run has to immediately sell its entire capital in the secondary market despite a lemons problem. Thus, eliminating the possibility of a run in the next period increases the value of equity, $V_t S_{B,t-1}$, for current bank equity holders in each period t. Also, banks minimize equity issuance as long as they remain free of a run in the next period. Banks prefer deposits to equity as a funding source, because the equity transaction cost, ζ, makes agents require a higher rate of return on equity than deposits.[22]

The right-hand side of Eq. (24) is the Value-at-Risk of bank assets, that is, the market value of the assets in the worst-case scenario. The limit on deposits based on Value-at-Risk

[21] This result is similar to endogenous borrowing constraints considered by Geanakoplos (2009). The difference is in that the cost of default arises from asymmetric information in this paper.

[22] To see this result, compare Eqs. (10) and (11).

implies a minimum equity-to-asset ratio that each bank satisfies to avoid a bank run:[23]

$$\begin{aligned}\frac{V_t S_{B,t}}{B_{B,t}+V_t S_{B,t}} &= \frac{E_t\left\{\Lambda_{V,t+1}(1+\zeta)^{-1}\left[\alpha_{t+1}+Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}(1-\bar{\delta})-\underline{\omega}_{t+1}\right]K_{B,t}\right\}}{Q_t(1-\hat{\delta}_t)^{-1}K_{B,t}} \\ &= \frac{E_t\left(\Lambda_{V,t+1}\left\{\left[Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}(1-\bar{\delta})-Q_{t+1}\right]+(\alpha_{t+1}+Q_{t+1}-\underline{\omega}_{t+1})\right\}\right)}{(1+\zeta)Q_t(1-\hat{\delta}_t)^{-1}}.\end{aligned} \tag{25}$$

The numerator is the amount of equity that a bank must issue. This term equals the expected discounted difference between the present discounted value of future revenues from bank assets and the limit on deposits.[24] This term is discounted by $\Lambda_{V,t+1}(1+\zeta)^{-1}$, which is the pricing kernel for equity in Eq. (10). The denominator is the total size of the balance sheet at the end of the period, which equals the sum of deposits and equity value, $B_{B,t}+V_t S_{B,t}$.

The second line of Eq. (25) decomposes the numerator into two factors: the expected illiquidity of bank assets and the downside risk to the market value of bank assets. In Eq. (25), the degree of illiquidity of bank assets in the next period is represented by $Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}(1-\bar{\delta})-Q_{t+1}$. Note that $Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}$ equals the present discounted value of future marginal income from capital net of depreciation, while Q_{t+1} is the secondary market price of capital. The downside risk to the market value of bank assets is represented by $\alpha_{t+1}+Q_{t+1}-\underline{\omega}_{t+1}$, where $\underline{\omega}_{t+1}$ denotes the lowest possible value of $\alpha_{t+1}+Q_{t+1}$ in the next period given the state of period t, as defined above.

[23]Eq. (25) is derived from $L_{B,t}=0$ and Eqs. (10), (12), (16), (22), and (24). Substituting $L_{B,t}=0$ and Eqs. (16) and (22) into Eq. (12) yields $B_{B,t}+V_t S_{B,t}=Q_t(1-\hat{\delta}_t)^{-1}K_{B,t}$. To confirm the first equality in Eq. (25), substitute Eq. (22) into $D_{t+1}+V_{t+1}$ in Eq. (10) and replace $\bar{R}_t B_{B,t}$ with $\underline{\omega}_{t+1}K_{B,t}$ in the equation, given Eq. (24).

[24]Note that $Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}$ equals the present discounted value of future marginal income from capital net of depreciation in period $t+1$. Thus, $[\alpha_{t+1}+Q_{t+1}(1-\hat{\delta}_{t+1})^{-1}(1-\bar{\delta})]K_{B,t}$ is the present discounted value of current and future revenues from bank assets at the beginning of the next period.

4.6 Numerical example of equilibrium dynamics

The minimum equity-to-asset ratio is dynamic because both the expected illiquidity of bank assets and the downside risk to the market value of bank assets fluctuate over the business cycle. To illustrate this result, Figure 1 shows a sample path of the stochastic dynamic equilibrium with the parameter values specified above. The sample path is generated after the aggregate productivity shock, α_t, switches between the two values (i.e., a boom and a recession) every four periods (i.e., years) for sufficiently many periods. This sample path features a regular business cycle.

Figure 1 indicates that the minimum equity-to-asset ratio, $V_t S_{B,t}/(B_{B,t} + V_t S_{B,t})$, co-moves with output. This result is due to the downside risk to the market value of bank assets. An increase in α_t raises the secondary market price of capital, Q_t, through a rise in aggregate current income.[25] At the same time, it pushes up the economic growth rate, Y_t/Y_{t-1}. Thus, the cum-dividend market price of capital, $\alpha_t + Q_t$, becomes higher during a boom than a recession. The expected value of $\alpha_{t+1} + Q_{t+1}$ increases with α_t, because the level of α_t is persistent given the stochastic process of α_t specified above.[26] Accordingly, the downside risk to $\alpha_{t+1} + Q_{t+1}$ becomes higher during a boom. This effect raises the minimum equity-to-asset ratio during a boom.

The expected illiquidity of bank assets also fluctuates over the business cycle.[27] During a boom, a rise in Q_t induces productive agents to sell capital with lower depreciation rates.[28] As

[25]With the log utility function, the effect of an increase in the expected future aggregate productivity, α_{t+1}, on Q_t is completely offset by the effect of an expected rise in future consumption, $c_{i,t+1}$, on the stochastic discount factor, $\beta c_{i,t}/c_{i,t+1}$.

[26]More specifically, the persistence is implied by the values of $\bar{\eta}_\alpha$ and $\underline{\eta}_\alpha$.

[27]This feature of the model contrasts with Kiyotaki and Moore's (2012) model, which takes shocks to asset illiquidity as exogenous.

[28]The value of $\hat{\delta}_t$ remains below the upper bound, $\bar{\delta} + \Delta$, for all t, because productive agents sell some capital with depreciation rates lower than $\hat{\delta}_t$ to obtain goods to invest in new capital. See Proposition 1 and Eq. (18) to confirm this behaviour of productive agents. The sales of high-quality capital by productive agents ensure a positive trading volume in the secondary capital market by attracting buyers. Thus, $\hat{\delta}_{t+1}$ fluctuates between $\bar{\delta}$ and $\bar{\delta} + \Delta$, given Eq. (21).

a result, the average depreciation rate of capital sold in the secondary market, $\hat{\delta}_t$, drops. This effect reduces the expected illiquidity of bank assets during a boom. This effect, however, is dominated by the downside risk to the market value of bank assets, because $\hat{\delta}_{t+1}$ does not fluctuate enough when α_{t+1} affects $\hat{\delta}_{t+1}$ only indirectly through Q_{t+1}.

5 Implications for the dynamic bank capital rule in Basel III

Basel Committee announced a new regulatory bank capital standard, Basel III, in December 2010 (Basel Committee on Banking Supervision 2010). One of the new features of the standard is a dynamic bank capital rule called countercyclical capital buffer. Under this rule, the national authority in each country can require banks in its jurisdiction to increase bank capital by 2.5% when excessive credit growth is observed. Basel III emphasizes common equity as the core part of bank capital.

The cyclicality of the minimum equity-to-asset ratio shown above is consistent with countercyclical capital buffer. Figure 1 shows that the banks' share of aggregate capital, $K_{B,t}/(K_{B,t} + \int k_{i,t}\, di)$, increases during a boom. This result holds because an increase in the secondary market price of capital, Q_t, during a boom induces productive agents to sell more capital in the secondary market. Banks absorb the capital sold, given that unproductive agents do not buy capital in the secondary market. Hence, the minimum equity-to-asset ratio, $V_t S_{B,t}/(B_{B,t} + V_t S_{B,t})$, rises with an expansion in financial intermediation during a boom.

Banks in the model voluntarily satisfy the minimum equity-to-asset ratio to avoid a bank run, given rationality and no agency problem with depositors or equity holders as assumed above. This result provides an interpretation of Basel III such that Basel III imposes on actual banks the behavior of rational banks with no agency problem, in case there is some irrationality or moral hazard, such as risk shifting, at actual banks.

This interpretation is consistent with the representative hypothesis proposed by Dewatripont and Tirole (1994). In this hypothesis, they regard a regulator as an implicit agent of depositors. Even though they do not model a regulator explicitly in their model, they interpret the optimal contract between a bank and depositors as the prudential regulation that an implicit regulator should impose on banks on behalf of depositors. In this paper, the minimum equity-to-asset ratio can be interpreted as a regulation imposed by an implicit regulator acting on behalf of many, small bank equity holders and depositors.

Also, a bank run in the equilibrium described above is self-fulfilling. As part of multiple equilibria, there exists another equilibrium in which depositors never run to banks. This feature of the model is the same as Diamond and Dybvig's (1983) model. In the latter equilibrium, banks do not have to issue common equity. In light of this result, countercyclical capital buffer in Basel III can be interpreted as preventing over-optimistic behavior of banks. If banks believe that a no bank-run equilibrium will hold, then they have no incentive to maintain bank capital. In case such bank expectations are over-optimistic, policy makers impose a bank capital requirement that is robust even if a self-fulfilling bank run can occur.

6 Conclusions

I have introduced banks into a dynamic stochastic general equilibrium model by featuring asymmetric information as the underlying friction for banking. Banks are public companies as in practice. In this environment, banks can issue liquid securities by pooling illiquid assets. Banks, however, suffers a run if they fail to satisfy an endogenous minimum value for their common equity. The minimum equity-to-asset ratio is increasing in the expected illiquidity of bank assets as well as the downside risk to the market value of bank assets. It rises with a credit expansion during a boom, if aggregate productivity shocks cause the business cycle. This result is consistent with countercyclical capital buffer introduced by Basel III.

In this paper, banks can make bank securities free of asymmetric information through asset pooling. A question remains regarding how asymmetric information about the qualities of bank securities affects the economy. Also, the calibration of the model implies a smaller increase in the minimum equity-to-asset ratio during a boom than countercyclical capital buffer. This result is based on the focus of the paper on a regular business cycle. It remains a question if the quantitative implication of the model sustains in the presence of an asset bubble, given the fact that a housing bubble in the U.S. led to the introduction of Basel III after the recent financial crisis. Addressing these issues is left for future research.

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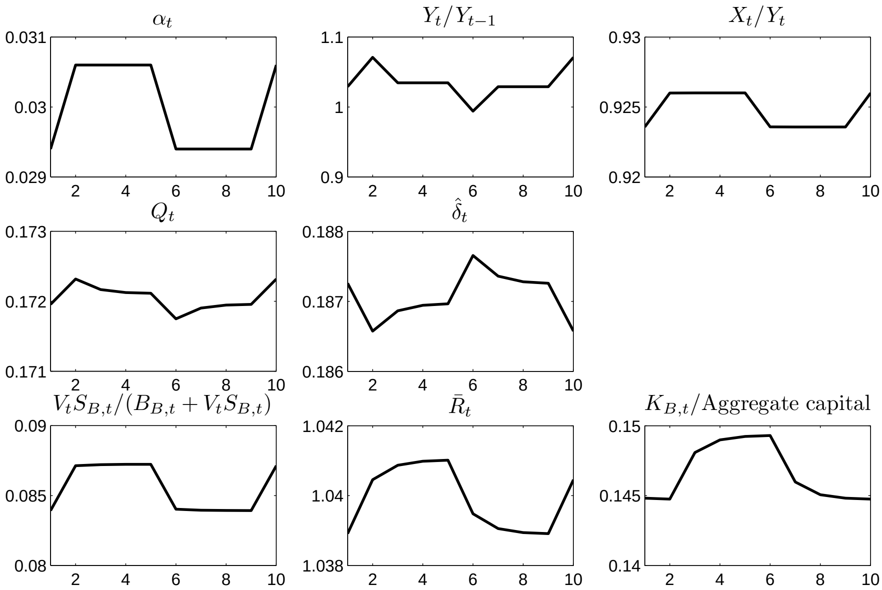
Figure 1: Dynamic equilibrium with banks: the business cycle driven by α_t



Notes: "$K_{B,t}$/Aggregate capital" denotes $K_{B,t}/(K_{B,t} + \int k_{i,t}\, di)$. Parameter values are $(\bar{\delta},\ \Delta,\ \phi,\ \beta,\ \zeta,\ \rho) = (0.1,\ 0.09,\ 4.75,\ 0.99,\ 0.02,\ 0.45)$, $(\bar{\alpha},\ \underline{\alpha}) = (0.0306,\ 0.0294)$, and $\bar{\eta}_\alpha = \underline{\eta}_\alpha = 0.75$. The figure shows a sample path after α_t keeps changing its value every 4 periods for a sufficiently long time.

Appendices (not for publication)

A Proof of Proposition 1

The first-order condition with respect to $l_{i,\delta,t}$ implies that each agent sells only the fraction of the agent's capital whose shadow value is exceeded by the market price, Q_t:

$$l_{i,\delta,t} = \begin{cases} k_{i,t-1}(2\Delta)^{-1}, & \text{if } Q_t > \lambda_{i,t}(1-\delta), \\ 0, & \text{otherwise}, \end{cases} \tag{A.1}$$

where:

$$\lambda_{i,t} = \begin{cases} \min\{\phi^{-1}, Q_t(1-\hat{\delta}_t)^{-1}\}, & \text{if } \phi_{i,t} = \phi, \\ Q_t(1-\hat{\delta}_t)^{-1}, & \text{if } \phi_{i,t} = 0. \end{cases} \tag{A.2}$$

The variable $\lambda_{i,t}$ denotes the shadow value of capital net of depreciation at the end of period t (i.e., $k_{i,t}$), so that $\lambda_{i,t}(1-\delta)$ is the shadow value of capital with depreciation rate δ. The envelop theorem implies that $\lambda_{i,t}$ equals current consumption, $c_{i,t}$, multiplied by the Lagrange multiplier for Eq. (5). Also, given the envelop theorem, the first-order condition with respect to $k_{i,t}$ yields Eq. (A.2), which implies that the value of $\lambda_{i,t}$ equals the marginal acquisition cost of capital net of depreciation for each agent. The minimum operator in Eq. (A.2) reflects that productive agents can choose the cheaper way to obtain capital net of depreciation between investing in new capital and buying existing capital in the secondary capital market. The first and the second options, respectively, cost the amounts ϕ^{-1} and $Q_t(1-\hat{\delta}_t)^{-1}$ of goods per capital net of depreciation. Unproductive agents buy capital in the secondary market for storing their wealth as they cannot invest in new capital.

B Proof of Proposition 2

Proposition 2 is a corollary of the following two propositions:

Proposition B.1 *There exists unique equilibrium in the basic model. The values of Q_t and μ_t in equilibrium are such that:*

$$\begin{cases} \mu_t = \bar{\delta} + \Delta \ \ \textit{and} \ \ 1 - Q_t\phi \geq \mu_t, & \textit{if } \phi\beta\alpha_t \leq (1-\beta)(1-\bar{\delta}), \\ \mu_t = \bar{\delta} \ \textit{and} \ \ 1 - Q_t\phi = \bar{\delta}, & \textit{if } \phi\beta\alpha_t \in ((1-\beta)(1-\bar{\delta}), \Lambda) \ \textit{and} \ \Delta = 0, \\ \mu_t \in \left(\Xi, \bar{\delta} + \Delta\right) \ \textit{and} \ \ 1 - Q_t\phi \in (\bar{\delta} - \Delta, \mu_t), & \textit{if } \phi\beta\alpha_t \in ((1-\beta)(1-\bar{\delta}), \Lambda) \ \textit{and} \ \Delta > 0, \\ \mu_t = \Xi \ \ \textit{and} \ \ 1 - Q_t\phi \leq \bar{\delta} - \Delta, & \textit{if } \phi\beta\alpha_t \geq \Lambda, \end{cases} \tag{B.1}$$

where $\Xi \equiv \bar{\delta} + (1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta$ and:

$$\Lambda \equiv \frac{1-\bar{\delta}+\Delta}{1-\Xi}\left[\frac{1-\bar{\delta}}{1-\rho} - \beta\left(\int_{\bar{\delta}-\Delta}^{\Xi} \frac{1-\delta}{2\Delta}d\delta + \int_{\Xi}^{\bar{\delta}+\Delta} \frac{1-\Xi}{2\Delta}d\delta\right)\right] > (1-\beta)(1-\bar{\delta}). \tag{B.2}$$

Proof. Eqs. (A.1) and (A.2) imply that each agent sells the fraction of capital whose depreciation rate is greater than $\delta_{i,t}$, which is defined as:

$$\delta_{i,t} = \begin{cases} 1 - Q_t\phi, & \text{if } \phi_{i,t} = \phi, \\ \hat{\delta}_t, & \text{if } \phi_{i,t} = 0. \end{cases} \tag{B.3}$$

Substituting this into Eq. (7) yields:

$$\hat{\delta}_t = \frac{\rho \int_{\max\{\bar{\delta}-\Delta,\, \min\{1-Q_t\phi,\, \hat{\delta}_t\}\}}^{\bar{\delta}+\Delta} \delta(2\Delta)^{-1}\, d\delta + (1-\rho)\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \delta(2\Delta)^{-1}\, d\delta}{\rho \int_{\max\{\bar{\delta}-\Delta,\, \min\{1-Q_t\phi,\, \hat{\delta}_t\}\}}^{\bar{\delta}+\Delta} (2\Delta)^{-1}\, d\delta + (1-\rho)\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} (2\Delta)^{-1}\, d\delta}. \tag{B.4}$$

Due to the log utility function, each agent consumes a fraction $1-\beta$ of net worth and saves the rest in each period:

$$c_{i,t} = (1-\beta)\left(\alpha_t + \int_{\bar{\delta}-\Delta}^{\delta_{i,t}} \frac{\lambda_{i,t}(1-\delta)}{2\Delta}\, d\delta + \int_{\delta_{i,t}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta\right) k_{i,t-1}, \tag{B.5}$$

$$\lambda_{i,t}k_{i,t} = \beta\left(\alpha_t + \int_{\bar{\delta}-\Delta}^{\delta_{i,t}} \frac{\lambda_{i,t}(1-\delta)}{2\Delta}\, d\delta + \int_{\delta_{i,t}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta\right) k_{i,t-1}. \tag{B.6}$$

In these equations, capital that the agent keeps is evaluated by its shadow value, $\lambda_{i,t}$, and capital that the agent sells is evaluated by the secondary market price, Q_t. Combining Eqs. (5), (8), (A.1) and (B.6) and normalizing the combined equation by unproductive agents' capital at the beginning of the period, $\int_{\{i|\phi_{i,t}=0\}} k_{i,t-1}\, di$, I can obtain:

$$\begin{aligned} &\frac{Q_t}{1-\hat{\delta}_t}\left\{\frac{1-\bar{\delta}}{1-\rho} - \frac{\rho}{1-\rho}\int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\, \min\{1-Q_t\phi,\, \hat{\delta}_t\}\}} \frac{1-\delta}{2\Delta}\, d\delta\right\} \\ &\qquad = \beta\left(\alpha_t + \frac{Q_t}{1-\hat{\delta}_t}\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t} \frac{1-\delta}{2\Delta}\, d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta\right). \end{aligned} \tag{B.7}$$

Given the normalization, the left-hand side is the shadow value of capital net of depreciation that must be held by unproductive agents at the end of the period for a given amount of capital sold by productive agents, and the right-hand side is the fraction of unproductive agents' net-worth that is spent on capital net of depreciation for saving. The market clearing condition (Eq. 8) requires both sides to be equal. Overall, Eqs. (B.4) and (B.7) jointly determine the equilibrium values of Q_t and $\hat{\delta}_t$ given the aggregate productive shock, α_t.

Rewrite Eqs. (B.4) and (B.7) as:

$$0 = \Psi(Q_t, \hat{\delta}_t) \equiv (1-\rho)(\hat{\delta}_t)^2 - 2\hat{\delta}_t(\bar{\delta} + \Delta - \rho \max\{\bar{\delta} - \Delta,\ \min\{1 - Q_t\phi,\ \hat{\delta}_t\}\}) \\ + (\bar{\delta} + \Delta)^2 - \rho(\max\{\bar{\delta} - \Delta,\ \min\{1 - Q_t\phi,\ \hat{\delta}_t\}\})^2, \tag{B.8}$$

$$0 = \Gamma(Q_t, \hat{\delta}_t) \equiv 1 - \bar{\delta} - \rho \int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\ \min\{1-Q_t\phi,\ \hat{\delta}_t\}\}} \frac{1-\delta}{2\Delta}\, d\delta \\ - \beta(1-\rho)\left[\frac{(1-\hat{\delta}_t)\alpha_t}{Q_t} + \int_{\bar{\delta}-\Delta}^{\hat{\delta}_t} \frac{1-\delta}{2\Delta}\, d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta\right]. \tag{B.9}$$

Hereafter a "root" means a root for $\Psi(Q_t, \hat{\delta}_t) = 0$ and $\Gamma(Q_t, \hat{\delta}_t) = 0$ unless mentioned otherwise.

Given $\max\{\bar{\delta} - \Delta,\ \min\{1 - Q_t\phi,\ \hat{\delta}_t\}\}$ in Eqs. (B.8) and (B.9), it is convenient to split the domain of Q_t into three regions, $[0, (1-\bar{\delta}-\Delta)\phi^{-1})$, $[(1-\bar{\delta}-\Delta)\phi^{-1}, (1-\bar{\delta}+\Delta)\phi^{-1}]$, and $((1-\bar{\delta}+\Delta)\phi^{-1}, \infty)$. For each of the three regions, I will derive the necessary and sufficient conditions under which $\Gamma(Q_t, \hat{\delta}_t) = 0$ and $\Psi(Q_t, \hat{\delta}_t) = 0$ have a root in the region, given $\hat{\delta}_t \in [\bar{\delta} - \Delta, \bar{\delta} + \Delta]$.

First, suppose $Q_t \in ((1-\bar{\delta}+\Delta)\phi^{-1}, \infty)$. In this case $\max\{\bar{\delta} - \Delta,\ \min\{1 - Q_t\phi,\ \hat{\delta}_t\}\} = \bar{\delta} - \Delta$ as $\hat{\delta}_t \geq \bar{\delta} - \Delta > 1 - Q_t\phi$. Then Eq. (B.8) yields that $\hat{\delta}_t = \bar{\delta} + (1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta$. Note that $\partial\Gamma(Q_t, \hat{\delta}_t)/\partial Q_t > 0$ given $\max\{\bar{\delta} - \Delta,\ \min\{1 - Q_t\phi,\ \hat{\delta}_t\}\} = \bar{\delta} - \Delta$. Thus, there exists unique root that satisfies $Q_t \in ((1-\bar{\delta}+\Delta)\phi^{-1}, \infty)$ and $\hat{\delta}_t \in [\bar{\delta} - \Delta, \bar{\delta} + \Delta]$ if and only if $\Gamma((1-\bar{\delta}+\Delta)\phi^{-1}, \bar{\delta} + (1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta) < 0$, which is equivalent to:

$$\phi\beta\alpha_t > \frac{1-\bar{\delta}+\Delta}{1-\Xi}\left[\frac{1-\bar{\delta}}{1-\rho} - \beta\left(\int_{\bar{\delta}-\Delta}^{\Xi} \frac{1-\delta}{2\Delta} d\delta + \int_{\Xi}^{\bar{\delta}+\Delta} \frac{1-\Xi}{2\Delta} d\delta\right)\right], \tag{B.10}$$

where $\Xi \equiv \bar{\delta} + (1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta$.

Second, suppose $Q_t \in [0, (1-\bar{\delta}-\Delta)\phi^{-1})$. In this case $\max\{\bar{\delta}-\Delta,\ \min\{1-Q_t\phi,\ \hat{\delta}_t\}\} = \hat{\delta}_t$ as $\hat{\delta}_t \leq \bar{\delta} + \Delta < 1 - Q_t\phi$. Then Eq. (B.8) implies that $\hat{\delta}_t = \bar{\delta} + \Delta$. Eq. (B.9) in turn implies that $Q_t = (1-\bar{\delta}-\Delta)\beta\alpha_t(1-\beta)^{-1}(1-\bar{\delta})^{-1}$. Thus, there exists unique root that satisfies $Q_t \in [0,(1-\bar{\delta}-\Delta)\phi^{-1})$ and $\hat{\delta}_t \in [\bar{\delta}-\Delta, \bar{\delta}+\Delta]$ if and only if $(1-\bar{\delta}-\Delta)\beta\alpha_t(1-\beta)^{-1}(1-\bar{\delta})^{-1} < (1-\bar{\delta}-\Delta)\phi^{-1}$, which is equivalent to $\phi\beta\alpha_t < (1-\beta)(1-\bar{\delta})$ given the assumption that $\Delta \in [0, \min\{\bar{\delta}, 1-\bar{\delta}\})$. It can be shown that $\Gamma((1-\bar{\delta}+\Delta)\phi^{-1}, \bar{\delta}+(1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta) < 0$ and $\phi\beta\alpha_t < (1-\beta)(1-\bar{\delta})$ are mutually exclusive, given the assumption that $\rho < 1$.

Third, suppose $Q_t \in [(1-\bar{\delta}-\Delta)\phi^{-1}, (1-\bar{\delta}+\Delta)\phi^{-1}]$. In this case, $\max\{\bar{\delta}-\Delta,\ \min\{1-Q_t\phi,\ \hat{\delta}_t\}\} = \min\{1-Q_t\phi,\ \hat{\delta}_t\}$ as $1-Q_t\phi \geq \bar{\delta}-\Delta$ and $\hat{\delta}_t \geq \bar{\delta}-\Delta$. Moreover, $\min\{1-Q_t\phi,\ \hat{\delta}_t\} = 1-Q_t\phi$ because, given Eq. (B.8), $1-Q_t\phi > \hat{\delta}_t$ would imply $\hat{\delta}_t = \bar{\delta}+\Delta$ as shown in the second case above, which contradicts $1-Q_t\phi \leq \bar{\delta}+\Delta$. Given $\max\{\bar{\delta}-\Delta,\ \min\{1-Q_t\phi,\ \hat{\delta}_t\}\} = 1-Q_t\phi$, Eq. (B.8) implies that $1-Q_t\phi = [1+(\sqrt{\rho})^{-1}]\hat{\delta}_t - (\sqrt{\rho})^{-1}(\bar{\delta}+\Delta)$. Substituting this equation in Eq. (B.9), denote $\Gamma(\phi^{-1}\{1-[1+(\sqrt{\rho})^{-1}]\hat{\delta}_t+(\sqrt{\rho})^{-1}(\bar{\delta}+\Delta)\}, \hat{\delta}_t)$ by $\Theta(\hat{\delta}_t)$. It is possible to show that $d\Theta(\hat{\delta}_t)/d\hat{\delta}_t < 0$, given the assumption that $\bar{\delta}+\Delta < 1$. Note that $Q_t \in [(1-\bar{\delta}-\Delta)\phi^{-1}, (1-\bar{\delta}+\Delta)\phi^{-1}]$ is equivalent to $\hat{\delta}_t \in [\bar{\delta}+(1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta, \bar{\delta}+\Delta]$ given $1-Q_t\phi = [1+(\sqrt{\rho})^{-1}]\hat{\delta}_t - (\sqrt{\rho})^{-1}(\bar{\delta}+\Delta)$. Thus there exists unique root that satisfies $Q_t \in [(1-\bar{\delta}-\Delta)\phi^{-1}, (1-\bar{\delta}+\Delta)\phi^{-1}]$ and $\hat{\delta}_t \in [\bar{\delta}-\Delta, \bar{\delta}+\Delta]$ if and only if $\Theta(\bar{\delta}+(1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta) \geq 0$ and $\Theta(\bar{\delta}+\Delta) \leq 0$, which are equivalent to $\Gamma((1-\bar{\delta}+\Delta)\phi^{-1}, \bar{\delta}+(1-\sqrt{\rho})(1+\sqrt{\rho})^{-1}\Delta) \geq 0$ and $\phi\beta\alpha_t \geq (1-\beta)(1-\bar{\delta})$.

Hence combining the three cases for the value of Q_t described above proves uniqueness of equilibrium. ■

Proposition B.2 *Denote $\int x_{i,t}di$ by X_t and $\int y_{i,t}di$ by Y_t. Then, in equilibrium in the basic* *model:*

$$\frac{X_t}{Y_t}\begin{cases} = 0, & \textit{if } \phi\beta\alpha_t(1-\bar{\delta})^{-1} \le 1-\beta, \\ < \beta - \dfrac{(1-\beta)(1-\bar{\delta})}{\phi\alpha_t}, & \textit{if } \Delta > 0 \textit{ and } \phi\beta\alpha_t(1-\bar{\delta})^{-1} > 1-\beta, \\ < \dfrac{\beta\rho}{1-\beta(1-\rho)}, & \textit{if } \Delta > 0 \textit{ and } \phi\beta\alpha_t(1-\bar{\delta})^{-1} \ge (1-\rho)^{-1}-\beta, \\ = \beta - \dfrac{(1-\beta)(1-\bar{\delta})}{\phi\alpha_t}, & \textit{if } \Delta = 0 \textit{ and } \phi\beta\alpha_t(1-\bar{\delta})^{-1} \in (1-\beta,(1-\rho)^{-1}-\beta), \\ = \dfrac{\beta\rho}{1-\beta(1-\rho)}, & \textit{if } \Delta = 0 \textit{ and } \phi\beta\alpha_t(1-\bar{\delta})^{-1} \ge (1-\rho)^{-1}-\beta. \end{cases} \tag{B.11}$$

Proof. First, suppose $\phi\beta\alpha_t \le (1-\beta)(1-\bar{\delta})$. The part of the proof of Proposition B.1 for this case shows $\hat{\delta}_t = \bar{\delta} + \Delta \le 1 - Q_t\phi$ in this case. Similarly to Eq. (A.2), it can be shown that $x_{i,t} = 0$ for agents with $\phi_{i,t} = \phi$, if $\phi^{-1} > Q_t(1-\hat{\delta}_t)^{-1}$. Thus $X_t/Y_t = 0$.

Second, suppose $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$ and $\Delta > 0$. The part of the proof of Proposition B.1 for this case shows $\phi^{-1} < Q_t(1-\hat{\delta}_t)^{-1}$ in this case. Then, the equilibrium value of X_t/Y_t is determined by:

$$\frac{X_t}{Y_t} = \frac{\rho}{\alpha_t}\left[\beta\alpha_t + \beta\int_{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}^{\bar{\delta}+\Delta}\frac{Q_t}{2\Delta}\,d\delta - (1-\beta)\int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}\frac{1-\delta}{2\Delta\phi}\,d\delta\right], \tag{B.12}$$

which is derived from Eqs. (5) and (B.6) for productive agents and $Y_t = \alpha_t\int k_{i,t-1}di$, given $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$.

Substituting Eq. (B.12) in the second line of Eq. (B.11), it can be shown that:

$$
\begin{aligned}
&\phi\alpha_t \left[\beta - \frac{(1-\beta)(1-\bar{\delta})}{\phi\alpha_t} - \frac{X_t}{Y_t}\right] \\
&= \phi\beta\alpha_t - (1-\beta)(1-\bar{\delta}) \\
&\qquad - \rho \left[\phi\beta\left(\alpha_t + \int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta\right) - (1-\beta)\int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}} \frac{1-\delta}{2\Delta}\, d\delta\right] \\
&= (1-\rho)\phi\beta\alpha_t - \int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{\rho\phi\beta Q_t}{2\Delta}\, d\delta \\
&\qquad - (1-\beta)\left[\rho\int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{1-\delta}{2\Delta}\, d\delta + (1-\rho)\left(\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\delta}{2\Delta}\, d\delta + \int_{\bar{\delta}-\Delta}^{\hat{\delta}_t} \frac{1-\delta}{2\Delta}\, d\delta\right)\right] \\
&> (1-\rho)\left[\phi\beta\alpha_t - (1-\beta)\left(\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t} \frac{1-\delta}{2\Delta}\, d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta\right)\right] - \int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{\rho\phi Q_t}{2\Delta}\, d\delta.
\end{aligned}
\tag{B.13}
$$

The last inequality is obtained by substituting $\phi^{-1} < Q_t(1-\hat{\delta}_t)^{-1}$ and Eq. (B.4), which implies:

$$
\begin{aligned}
&\rho\int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta + (1-\rho)\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta \\
&\qquad = \rho\int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{1-\delta}{2\Delta}\, d\delta + (1-\rho)\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\delta}{2\Delta}\, d\delta.
\end{aligned}
\tag{B.14}
$$

Substituting Eq. (B.14) into Eq. (B.7) yields that:

$$
\begin{aligned}
&\beta\alpha_t = \\
&\frac{Q_t}{1-\hat{\delta}_t}\left[\frac{\rho}{1-\rho}\int_{\max\{\bar{\delta}-\Delta,\, 1-Q_t\phi\}}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta + (1-\beta)\left(\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t} \frac{1-\delta}{2\Delta}\, d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta} \frac{1-\hat{\delta}_t}{2\Delta}\, d\delta\right)\right].
\end{aligned}
\tag{B.15}
$$

Then substituting this into the right-hand side of Inequality (B.13) implies that the right-hand side of Inequality (B.13) equals:

$$(1-\rho)(1-\beta)\left(\frac{\phi Q_t}{1-\hat{\delta}_t}-1\right)\left[\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t}\frac{1-\delta}{2\Delta}\,d\delta+\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta}\frac{1-\hat{\delta}_t}{2\Delta}\,d\delta\right], \tag{B.16}$$

which is positive given $\phi^{-1} < Q_t(1-\hat{\delta}_t)^{-1}$. Thus $\beta-(1-\beta)(1-\bar{\delta})(\phi\alpha_t)^{-1} > X_t/Y_t$ if $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$ and $\Delta > 0$.

Third, suppose $\phi\beta\alpha_t \geq [(1-\rho)^{-1}-\beta](1-\bar{\delta})$ and $\Delta > 0$. Eq. (B.12) implies that:

$$\begin{aligned}&\frac{\phi\alpha_t}{\rho}\left[\frac{\beta\rho}{1-\beta(1-\rho)}-\frac{X_t}{Y_t}\right]\\&=\frac{(1-\rho)\beta^2\phi\alpha_t}{1-\beta(1-\rho)}\\&\quad-\left[\phi\beta\left(\alpha_t+\int_{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}^{\bar{\delta}+\Delta}\frac{Q_t}{2\Delta}\,d\delta\right)-(1-\beta)\int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}\frac{1-\delta}{2\Delta}\,d\delta\right].\end{aligned} \tag{B.17}$$

Solving Eq. (B.15) for $Q_t(1-\hat{\delta}_t)^{-1}$ and substituting it in the right-hand side of Eq. (B.17), it can be shown that $\beta\rho[1-\beta(1-\rho)]^{-1}-X_t/Y_t > 0$ if and only if:

$$\begin{aligned}&\frac{\beta^2\phi\alpha_t(1-\rho)}{1-\beta(1-\rho)}\left(\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t}\frac{1-\delta}{2\Delta}d\delta-\int_{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}^{\hat{\delta}_t}\frac{1-\hat{\delta}_t}{2\Delta}d\delta\right)\\&\qquad+\int_{\bar{\delta}-\Delta}^{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}\frac{1-\delta}{2\Delta}d\delta\left[\frac{\rho}{1-\rho}\int_{\max\{\bar{\delta}-\Delta,\,1-Q_t\phi\}}^{\bar{\delta}+\Delta}\frac{1-\hat{\delta}_t}{2\Delta}\,d\delta\right.\\&\qquad\qquad\left.+(1-\beta)\left(\int_{\bar{\delta}-\Delta}^{\hat{\delta}_t}\frac{1-\delta}{2\Delta}\,d\delta+\int_{\hat{\delta}_t}^{\bar{\delta}+\Delta}\frac{1-\hat{\delta}_t}{2\Delta}\,d\delta\right)\right]>0.\end{aligned} \tag{B.18}$$

This strictly inequality holds, as the part of the proof of Proposition B.1 for $\phi\beta\alpha_t \geq [(1-\rho)^{-1}-\beta](1-\bar{\delta})$ and $\Delta > 0$ shows that $\hat{\delta}_t > \bar{\delta}-\Delta$, given $\Delta > 0$.

Forth, suppose $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$ and $\Delta = 0$. The part of the proof of Proposition B.1 for this case shows $\hat{\delta}_t = \bar{\delta} \geq 1-Q_t\phi$ in this case. Aggregating Eqs. (5) and (B.6) for each type of agent yields that:

$$\phi^{-1} \int_{\{i|\phi_{i,t}=\phi\}} k_{i,t} di = \beta(\alpha_t + Q_t)\rho \int k_{i,t-1} di, \tag{B.19}$$

$$Q_t(1-\bar{\delta})^{-1} \int_{\{i|\phi_{i,t}=0\}} k_{i,t} di = \beta(\alpha_t + Q_t)(1-\rho) \int k_{i,t-1} di, \tag{B.20}$$

$$\int k_{i,t} di = \phi X_t + (1-\bar{\delta}) \int k_{i,t-1} di, \tag{B.21}$$

$$\int_{\{i|\phi_{i,t}=0\}} k_{i,t} di \leq (1-\bar{\delta}) \int k_{i,t-1} di. \tag{B.22}$$

It is straightforward to show that, if $\bar{\delta} = 1 - Q_t\phi$, then $X_t/Y_t = \beta - (1-\beta)(1-\bar{\delta})(\phi\alpha_t)^{-1}$. Inequality (B.22) implies that $\phi\beta\alpha_t \leq [(1-\rho)^{-1} - \beta](1-\bar{\delta})$ must hold. (Note that $X_t > 0$ is satisfied given $\phi\beta\alpha_t > (1-\beta)(1-\bar{\delta})$.) On the other hand, if $\bar{\delta} > 1 - Q_t\phi$, then productive agents sell all of their capital, given Eq. (A.1). Thus Inequality (B.22) must hold in equality, which implies $X_t/Y_t = \beta\rho[1-\beta(1-\rho)]^{-1}$. In this case, $\bar{\delta} > 1 - Q_t\phi$ is equivalent to $\phi\beta\alpha_t > [(1-\rho)^{-1} - \beta](1-\bar{\delta})$. ■

C The market clearing condition for Q_t and the definitions of $\hat{\delta}_t$, $\Lambda_{V,t}$, and $\Lambda_{R,t}$ in the model of banking

The definition of the average depreciation rate of capital sold in the secondary market, $\hat{\delta}_t$, is:

$$\hat{\delta}_t = \frac{\int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} \delta \; l_{i,\delta,t} \; d\delta \; di + \bar{\delta} L_{B,t}}{\int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t} \; d\delta \; di + L_{B,t}}. \tag{C.1}$$

Note that the average depreciation rate of capital sold by each bank is the unconditional average, $\bar{\delta}$. Thus, $\bar{\delta} L_{B,t}$ is the total depreciation of capital sold by banks.

The market clearing condition for the secondary market price of capital, Q_t, is:

$$\int h_{i,t}\, di + H_{B,t} = \int \int_{\bar{\delta}-\Delta}^{\bar{\delta}+\Delta} l_{i,\delta,t}\, d\delta\, di + L_{B,t}, \tag{C.2}$$

which adds the sale and the purchase of capital by banks to Eq. (8) in the basic model.

The definitions of the stochastic discount factors for the agents holding bank equity and deposits, $\Lambda_{V,t+1}$ and $\Lambda_{R,t+1}$, are:

$$\Lambda_{V,t+1} \equiv \frac{\beta c_{i^*,t}}{c_{i^*,t+1}}, \quad i^* \equiv \underset{i\in[0,1]}{\operatorname{argmax}} E_t \left[\frac{\beta c_{i,t}(D_{t+1} + V_{t+1})}{(1+\zeta)c_{i,t+1}} \right], \tag{C.3}$$

$$\Lambda_{R,t+1} \equiv \frac{\beta c_{i^{**},t}}{c_{i^{**},t+1}}, \quad i^{**} \equiv \underset{i\in[0,1]}{\operatorname{argmax}} E_t \left[\frac{\beta c_{i,t}\tilde{R}_{t+1}}{c_{i,t+1}} \right], \tag{C.4}$$

respectively. These definitions imply that the buyers of bank deposits and equity are those who value them most. Otherwise, there would exist some agents whose first-order conditions with respect to $s_{i,t}$ or $b_{i,t}$ do not hold with equality. In such a case, the optimum condition for the agents' utility maximization problem would be violated.

D Proof of Proposition 3

I solve the banks' profit maximization problem (15) to prove Proposition 3. As assumed in the main text, the number of exogenous states is two in each period. Given the values of period-t variables, denote the smaller value of $\alpha_{t+1} + Q_{t+1}$ by $\underline{\omega}_{t+1}$ and the larger value by $\bar{\omega}_{t+1}$. The conditional probability that $\alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1}$ is denoted by $P_t(\bar{\omega}_{t+1})$, and the one for $\alpha_{t+1} + Q_{t+1} = \underline{\omega}_{t+1}$ is denoted by $P_t(\underline{\omega}_{t+1})$.

I start from proving the following lemma:

Lemma 1 *Suppose that Ω_{t+1} satisfies Eqs.* (D.9)-(D.11) *for period $t+1$. Split the constraint set of the maximization problem* (15) *into three regions: $\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1} K_{B,t}$; $\bar{R}_t B_{B,t} \in$*

$(\underline{\omega}_{t+1}K_{B,t}, \bar{\omega}_{t+1}K_{B,t}]$; and $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1}K_{B,t}$. Then, in equilibrium, $\bar{R}_t B_{B,t}$ equals $\underline{\omega}_{t+1}K_{B,t}$ at optimum in the first region and $\bar{\omega}_{t+1}K_{B,t}$ at optimum in the second region.

Proof. Use the Lagrange method to solve the maximization problem in the first and the second region. For the second region, solve the maximization problem in the closure of the region and suppose that Ω_{t+1} takes the limit value when $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1}K_{B,t}$. This makes the function Ω_{t+1} differentiable in each region. This expansion of the second region does not affect the solution to the maximization problem, since it will be shown that $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1}K_{B,t}$ at optimum in the second region.

In the first region, $\bar{R}_t$ is determined solely by Eq. (11) and can be taken as exogenous for a bank. Eq. (11) implies that $\bar{R}_t > 0$, since agents never choose zero consumption with the time-separable log utility function in equilibrium. The first-order condition with respect to $B_{B,t}$ is:

$$1 - \frac{1}{1+\zeta} E_t \left[\frac{\beta c_{i,t} \bar{R}_t}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right] - \bar{\theta}_{rgn1,t} \bar{R}_{t+1} = 0, \tag{D.5}$$

where $\bar{\theta}_{rgn1,t}$ is the Lagrange multiplier for the upper bound of the first region ($\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1}K_{B,t}$). Thus, $\bar{\theta}_{rgn1,t} = \zeta(1+\zeta)^{-1}(\bar{R}_t)^{-1} > 0$, given $\zeta > 0$ and $\bar{R}_t > 0$. Hence, $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1}K_{B,t}$ at optimum in the first region.

For the second region, if $K_{B,t} = 0$, then the claim in the lemma is automatically satisfied since Eq. (11) implies that $B_{B,t}$ must be 0, given that $K_{B,t}(B_{B,t})^{-1}$ in the equation is replaced with infinity if $B_{B,t} = 0$. Hereafter suppose $K_{B,t} > 0$ in the second region. In equilibrium, Q_t is alway positive and thus $\underline{\omega}_t > 0$ for all t, since otherwise each agent would demand an infinite amount of capital in the secondary market, which would violate the market clearing condition for the secondary capital market. In the second region, $K_{B,t} > 0$ and $\underline{\omega}_{t+1} > 0$ imply that $B_{B,t} > 0$ and $\bar{R}_t > 0$, since $B_{B,t}$ must be non-negative by the non-negativity constraint. The first-order conditions with respect to $B_{B,t}$ and $\bar{R}_t$ in the second region are respectively:

$$
\begin{aligned}
1-\frac{P_t(\bar{\omega}_{t+1})}{1+\zeta}E_t\left[\frac{\beta c_{i,t}\bar{R}_t}{c_{i,t+1}}\middle|\,\phi_{i,t}=0,\ \alpha_{t+1}+Q_{t+1}=\bar{\omega}_{t+1}\right]& \\
+(\underline{\theta}_{rgn2,t}-\bar{\theta}_{rgn2,t})\bar{R}_t& \\
-\theta_{PC,t}P_t(\underline{\omega}_{t+1})E_t\left[\frac{\beta c_{i,t}\underline{\omega}_{t+1}K_{B,t}}{c_{i,t+1}(B_{B,t})^2}\middle|\,\phi_{i,t}=0,\ \alpha_{t+1}+Q_{t+1}=\underline{\omega}_{t+1}\right]&=0, \quad \text{(D.6)}
\end{aligned}
$$

$$
\begin{aligned}
-\frac{P_t(\bar{\omega}_{t+1})}{1+\zeta}E_t\left[\frac{\beta c_{i,t}B_{B,t}}{c_{i,t+1}}\middle|\,\phi_{i,t}=0,\ \alpha_{t+1}+Q_{t+1}=\bar{\omega}_{t+1}\right]& \\
+(\underline{\theta}_{rgn2,t}-\bar{\theta}_{rgn2,t})B_{B,t}& \\
+\theta_{PC,t}P_t(\bar{\omega}_{t+1})E_t\left[\frac{\beta c_{i,t}}{c_{i,t+1}}\middle|\,\phi_{i,t}=0,\ \alpha_{t+1}+Q_{t+1}=\bar{\omega}_{t+1}\right]&=0, \quad \text{(D.7)}
\end{aligned}
$$

where $\bar{\theta}_{rgn2,t}$ is the Lagrange multiplier for the upper bound of the closure of the second region ($\bar{R}_tB_{B,t} \leq \bar{\omega}_{t+1}K_{B,t}$), $\underline{\theta}_{rgn2,t}$ is the Lagrange multiplier for the lower bound of the closure of the second region ($\bar{R}_tB_{B,t} \geq \underline{\omega}_{t+1}K_{B,t}$), and $\bar{\theta}_{PC,t}$ is the Lagrange multiplier for Eq. (11). Eqs. (D.6) and (D.7) imply that $\theta_{PC,t} = B_{B,t}$. Substituting this into Eq. (D.6) leads to:

$$
(\bar{\theta}_{rgn2,t}-\underline{\theta}_{rgn2,t})\bar{R}_t=\frac{\zeta P_t(\bar{\omega}_{t+1})}{1+\zeta}E_t\left[\frac{\beta c_{i,t}\bar{R}_t}{c_{i,t+1}}\middle|\,\phi_{i,t}=0,\ \alpha_{t+1}+Q_{t+1}=\bar{\omega}_{t+1}\right], \quad \text{(D.8)}
$$

which in turn indicates that $\bar{\theta}_{rgn2,t} > 0$ and $\underline{\theta}_{rgn2,t} = 0$, given $\zeta > 0$ and $\bar{R}_t > 0$. Thus, $\bar{R}_tB_{B,t} = \bar{\omega}_{t+1}K_{B,t}$ at optimum in the second region. ■

Given this lemma, the following proposition holds:

Proposition D.3 *Suppose Eqs.* (18)-(20) *hold in equilibrium. Then:*

$$\tilde{R}_t B_{B,t-1} + (D_t + V_t) S_{B,t-1} = \begin{cases} \left[\alpha_t + \lambda_{B,t}(1-\bar{\delta})\right] K_{B,t-1}, & \text{if } \bar{R}_{t-1} B_{B,t-1} \leq (\alpha_t + Q_t) K_{B,t-1}, \\ (\alpha_t + Q_t) K_{B,t-1}, & \text{if } \bar{R}_{t-1} B_{B,t-1} > (\alpha_t + Q_t) K_{B,t-1}, \end{cases} \tag{D.9}$$

where $\lambda_{B,t} \equiv \max\{\lambda'_{B,t},\ \lambda''_{B,t}\}$ *and:*

$$\lambda'_{B,t} \equiv E_t \left\{ \frac{\beta c_{i,t} \left[\alpha_{t+1} + \lambda_{B,t+1}(1-\bar{\delta}) - \underline{\omega}_{t+1}\right]}{(1+\zeta) c_{i,t+1}} + \frac{\beta c_{i,t}\ \underline{\omega}_{t+1}}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right\}, \tag{D.10}$$

$$\begin{aligned} \lambda''_{B,t} \equiv \ & P_t(\bar{\omega}_{t+1}) E_t \left\{ \frac{\beta c_{i,t} \left[\alpha_{t+1} + \lambda_{B,t+1}(1-\bar{\delta}) - \bar{\omega}_{t+1}\right]}{(1+\zeta) c_{i,t+1}} \middle| \begin{array}{c} \phi_{i,t} = 0 \\ \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \end{array} \right\} \\ & + E_t \left[\frac{\beta c_{i,t} \left(\alpha_{t+1} + Q_{t+1}\right)}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right], \end{aligned} \tag{D.11}$$

$$\bar{R}_t B_{B,t} = \begin{cases} \underline{\omega}_{t+1} K_{B,t}, & \text{if } \lambda'_{B,t} > \lambda''_{B,t}, \\ \bar{\omega}_{t+1} K_{B,t}, & \text{if } \lambda'_{B,t} < \lambda''_{B,t}. \end{cases} \tag{D.12}$$

Also:

$$\lambda_{B,t} = \frac{Q_t}{1-\hat{\delta}_t}, \qquad \text{if } H_{B,t} > 0, \tag{D.13}$$

$$L_{B,t} = 0, \qquad \text{if } \hat{\delta}_t > \bar{\delta} \text{ and } H_{B,t} > 0. \tag{D.14}$$

Proof. Suppose that Ω_{t+1} satisfies Eqs. (D.9)-(D.11) for period $t+1$. Note that Eq. (D.9) satisfies Eqs. (13) and (14).

To verify Eq. (D.9), split the constraint set of the maximization problem (15) into three regions: $\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1} K_{B,t}$; $\bar{R}_t B_{B,t} \in (\underline{\omega}_{t+1} K_{B,t}, \bar{\omega}_{t+1} K_{B,t}]$; and $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1} K_{B,t}$. First of all, any point in the third region, $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1} K_{B,t}$, is weakly dominated by $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$, since the feasible set of the choice variables is identical and the value of Ω_{t+1} is always 0 in the third region while it can be positive with $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$. Thus, the third region can be ignored.

By Lemma 1, $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$ and $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$ at optimum in the first and the second region, respectively. Denote the maximum values of the objective function of the maximization problem (15) in the first and the second region by Ω'_t and Ω''_t, respectively. Given that Ω_{t+1} satisfies Eqs. (D.9)-(D.11) for period $t+1$, substituting the optimal values of $\bar{R}_t B_{B,t}$ in the first and the second region and Eqs. (11), (13), and (14) into the objective function of the maximization problem (15) yields:

$$\Omega'_t = \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda'_{B,t} K_{B,t}, \tag{D.15}$$

$$\Omega''_t = \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda''_{B,t} K_{B,t}. \tag{D.16}$$

The global solution to the maximization problem (15) can be obtained by maximizing the values of Ω'_t and Ω''_t with satisfying Eq. (16), $L_{B,t} \in [0, K_{B,t-1}]$, and $H_{B,t} \geq 0$. Since the first and the second region have the same feasible set of $H_{B,t}$ and $L_{B,t}$, $\Omega_t = \Omega'_t$ if $\lambda'_{B,t} \geq \lambda''_{B,t}$ and $\Omega_t = \Omega''_t$ if $\lambda'_{B,t} \leq \lambda''_{B,t}$. This result proves Eqs. (D.10)-(D.12).

Given this result, now prove Eqs. (D.13) and (D.14). The maximization problem (15) can be rewritten as:

$$\Omega_t = \max_{\{H_{B,t}, L_{B,t}\}} \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda_{B,t} K_{B,t},$$
$$\text{s.t. Eqs. (13), (14) and (16)}, \ L_{B,t} \in [0, K_{B,t-1}], \ H_{B,t} \geq 0, \tag{D.17}$$

where $\lambda_{B,t} = \max\{\lambda'_{B,t}, \ \lambda''_{B,t}\}$. Note that Eq. (11) is already incorporated by the definitions of $\lambda'_{B,t}$ and $\lambda''_{B,t}$. The maximization problem (D.17) implies that the equilibrium value of $\lambda_{B,t}$ satisfies:

$$\lambda_{B,t} \begin{cases} = Q_t(1-\hat{\delta}_t)^{-1}, & \text{if } H_{B,t} > 0, \\ = Q_t(1-\bar{\delta})^{-1}, & \text{if } L_{B,t} \in (0, K_{B,t-1}), \\ \leq Q_t(1-\bar{\delta})^{-1}, & \text{if } L_{B,t} = K_{B,t-1}, \\ \in [Q_t(1-\bar{\delta})^{-1},\ Q_t(1-\hat{\delta}_t)^{-1}], & \text{if } H_{B,t} = 0 \text{ and } L_{B,t} = 0. \end{cases} \tag{D.18}$$

When $\hat{\delta}_t > \bar{\delta}$, Eq. (D.18) implies that $L_{B,t} = 0$ if $H_{B,t} > 0$ and that $H_{B,t} = 0$ if $L_{B,t} > 0$. Thus Eqs. (D.13) and (D.14) are proved. Substituting Eqs. (16) and (D.18) into the objection function in the maximization problem (D.17) proves Eq. (D.9). ■

This proposition is sufficient to prove Proposition 3. In this proposition, $\lambda'_{B,t}$ and $\lambda''_{B,t}$ denote the presented discounted values of marginal income from capital net of depreciation to a bank, when $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$ and $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$, respectively. The proposition implies that a bank chooses the face value of bank deposits, $\bar{R}_t B_{B,t}$, to maximize the present discounted value of its future income. Also, the total market value of bank securities, $\tilde{R}_t B_{B,t-1} + (D_t + V_t) S_{B,t-1}$, equals the present discounted value of the current and future income from the bank's capital, $[\alpha_t + \lambda_{B,t}(1-\bar{\delta})] K_{B,t-1}$, given no bank run in the current period. Note that a bank maximizes bank deposits given the risk of a bank run that the bank chooses to take, because the bank equity holding cost, ζ, makes equity financing costly.

Also, Eq. (D.13) is a standard arbitrage free condition for a bank, in which the right-hand side of the equation is the marginal acquisition cost of capital net of depreciation in the secondary capital market. If the equality were violated, then the quantity of capital bought by each bank in the market, $H_{B,t}$, would be either infinity or 0, which would violate the market clearing condition for the secondary capital market, or would contradict $H_{B,t} > 0$.

Finally, Eq. (D.14) implies that a bank is worse off by selling its capital randomly without knowing the depreciation rate of each unit of its capital, if it buys capital in the secondary capital market with an higher average depreciation rate ($\hat{\delta}_t$) than the average depreciation rate of its own capital ($\bar{\delta}$).

E The equilibrium laws of motion for aggregate variables in the model of banking

I show the equilibrium laws of motion in the model of banking. Suppose Eqs. (18)-(20) hold in equilibrium.

E.1 The fraction of capital sold by each agent in the secondary capital market

Eq. (A.1) for the basic model holds in the model of banking. Thus:

$$\lambda_{i,t} = \begin{cases} \phi^{-1}, & \text{if } \phi_{i,t} = \phi, \\ \lambda_{U,t}, & \text{if } \phi_{i,t} = 0, \end{cases} \tag{E.1}$$

where $\lambda_{U,t}$ denotes the common value of $\lambda_{i,t}$ for unproductive agents, which satisfies:

$$\begin{cases} \lambda_{U,t} = Q_t(1-\hat{\delta}_t)^{-1}, & \text{if } h_{i,t} > 0 \text{ for all } i \text{ s.t. } \phi_{i,t} = 0, \\ h_{i,t} = 0 \text{ for all } i \text{ s.t. } \phi_{i,t} = 0, & \text{if } \lambda_{U,t} < Q_t(1-\hat{\delta}_t)^{-1}. \end{cases} \tag{E.2}$$

Substituting the value of $\lambda_{i,t}$ for each type of agent into Eq. (A.1) yields the lower bound of the depreciation rates of capital sold by each agent, $\delta_{i,t}$:

$$\delta_{i,t} = \begin{cases} \delta_{P,t} \equiv \max\left\{\bar{\delta} - \Delta,\ 1 - \phi Q_t\right\}, & \text{if } \phi_{i,t} = \phi, \\ \delta_{U,t} \equiv \max\left\{\bar{\delta} - \Delta,\ 1 - Q_t(\lambda_{U,t})^{-1}\right\}, & \text{if } \phi_{i,t} = 0. \end{cases} \tag{E.3}$$

The maximum operator ensures that the value of $\delta_{i,t}$ is within the range of the distribution of depreciation rates. Eqs. (18) and (E.3) indicate that $\delta_{P,t} < \hat{\delta}_t$ and $\delta_{U,t} \le \hat{\delta}_t$. Thus, $\delta_{i,t} \le \bar{\delta} + \Delta$ for all i. Also, substituting Eq. (18) into Eq. (E.3) yields $\delta_{P,t} < \hat{\delta}_t$, which implies $\hat{\delta}_t < \bar{\delta} + \Delta$ given Eq. (C.1).

E.2 Consumption and saving by each agent

Given the log utility function, each agent consumes a fraction $1 - \beta$ of net worth and saves the rest in each period:

$$c_{i,t} = (1 - \beta) w_{i,t}, \tag{E.4}$$

$$\lambda_{i,t} k_{i,t} + b_{i,t} + (1 + \zeta) V_t s_{i,t} = \beta w_{i,t}, \tag{E.5}$$

where $w_{i,t}$ is the agent's net worth defined by:

$$w_{i,t} \equiv \left(\alpha_t + \int_{\bar{\delta}-\Delta}^{\delta_{i,t}} \frac{\lambda_{i,t}(1 - \delta)}{2\Delta} \, d\delta + \int_{\delta_{i,t}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta} \, d\delta \right) k_{i,t-1} + \tilde{R}_t b_{i,t-1} + (D_t + V_t) s_{i,t-1}. \tag{E.6}$$

In Eqs. (E.5) and (E.6), the fractions of capital kept and sold by the agent are evaluated by the shadow value of capital net of depreciation for the agent, $\lambda_{i,t}$, and the secondary market price of capital, Q_t, respectively.

E.3 The equilibrium laws of motion for aggregate variables

Given Eqs. (18)-(20):

$$x_{i,t} > 0, h_{i,t} = b_{i,t} = s_{i,t} = 0, \text{ if } \phi_{i,t} = \phi, \tag{E.7}$$

as described in Section 4.2. Also, in Proposition D.3, suppose that $\lambda'_{B,t} > \lambda''_{B,t}$, $H_{B,t} > 0$, and $\lambda_{U,t} < Q_t/(1-\hat{\delta}_t)$ for all t. Note that $\lambda'_{B,t} > \lambda''_{B,t}$ implies $\hat{\delta}_t > \bar{\delta}$. Thus, $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$, $L_{B,t} = 0$, and $\lambda_{B,t} = Q_t/(1-\hat{\delta}_t)$. Also, $h_{i,t} = 0$ and $b_{i,t} + s_{i,t} > 0$ if $\phi_{i,t} = 0$, given Eq. (E.2).

Now aggregate Eq. (E.5) for productive and unproductive agents separately. Then substitute the equalities described in the previous paragraph and Eqs. (22), (E.1), (E.2), (E.3) and (E.7) into Eqs. (5), (16), (C.1) and (C.2) and Eq. (E.5) after aggregating these equations for each type of agent. It holds that:

$$\frac{K_{P,t}}{\phi} = \beta\rho \left\{ \left(\alpha_t + \int_{\bar{\delta}-\Delta}^{\delta_{P,t}} \frac{1-\delta}{\phi \cdot 2\Delta}\, d\delta + \int_{\delta_{P,t}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta \right) (K_{P,t-1} + K_{U,t-1}) + \left[\alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t} \right] K_{B,t-1} \right\}, \tag{E.8}$$

$$\lambda_{U,t} K_{U,t} + \left[\frac{(1+\zeta)Q_t}{1-\hat{\delta}_t} - \frac{\zeta \underline{\omega}_{t+1}}{\bar{R}_t} \right] K_{B,t} = \beta(1-\rho) \left\{ \left[\alpha_t + \int_{\bar{\delta}-\Delta}^{\delta_{U,t}} \frac{\lambda_{U,t}(1-\delta)}{2\Delta}\, d\delta + \int_{\delta_{U,t}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\, d\delta \right] (K_{P,t-1} + K_{U,t-1}) + \left[\alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t} \right] K_{B,t-1} \right\}, \tag{E.9}$$

$$K_{P,t} = \phi X_t + \rho \left(K_{P,t-1} + K_{U,t-1} \right) \int_{\bar{\delta}-\Delta}^{\delta_{P,t}} \frac{1-\delta}{2\Delta} d\delta, \tag{E.10}$$

$$K_{U,t} = (1-\rho) \left(K_{P,t-1} + K_{U,t-1} \right) \int_{\bar{\delta}-\Delta}^{\delta_{U,t}} \frac{1-\delta}{2\Delta} d\delta, \tag{E.11}$$

$$K_{B,t} = (1-\hat{\delta}_t) H_{B,t} + (1-\bar{\delta}) K_{B,t-1}, \tag{E.12}$$

$$K_{P,t} + K_{U,t} + K_{B,t} = \phi X_t + (1-\bar{\delta})(K_{P,t-1} + K_{U,t-1} + K_{B,t-1}), \tag{E.13}$$

$$\hat{\delta}_t = \frac{\rho \int_{\delta_{P,t}}^{\bar{\delta}+\Delta} \delta\, d\delta \; + (1-\rho) \int_{\delta_{U,t}}^{\bar{\delta}+\Delta} \delta\, d\delta}{\rho(\bar{\delta}+\Delta-\delta_{P,t}) + (1-\rho)(\bar{\delta}+\Delta-\delta_{U,t})}, \tag{E.14}$$

$$\frac{Q_t}{1-\hat{\delta}_t} = E_t \left\{ \left. \frac{\beta c_{i,t} \left[\alpha_{t+1} + \frac{Q_{t+1}}{1-\hat{\delta}_{t+1}}(1-\bar{\delta}) - \underline{\omega}_{t+1} \right]}{(1+\zeta) c_{i,t+1}} + \frac{\beta c_{i,t}\, \underline{\omega}_{t+1}}{c_{i,t+1}} \right| \phi_{i,t} = 0 \right\}, \tag{E.15}$$

where $K_{P,t} \equiv \int_{\{i|\phi_{i,t}=\phi\}} k_{i,t}\ di$ and $K_{U,t} \equiv \int_{\{i|\phi_{i,t}=0\}} k_{i,t}\ di$, and $B_{U,t}$ and $S_{U,t}$ are similarly defined. Also, given the value of α_{t+1}, Eqs. (E.4)-(E.6) imply:

$$\frac{\beta c_{i,t}}{c_{i,t+1}} = \frac{\lambda_{U,t}K_{U,t} + \left[\frac{(1+\zeta)Q_t}{1-\hat{\delta}_t} - \frac{\zeta\underline{\omega}_{t+1}}{\bar{R}_t}\right]K_{B,t}}{(\alpha_{t+1}+\Psi_{t+1})K_{U,t} + \left[\alpha_{t+1} + \frac{Q_{t+1}(1-\bar{\delta})}{1-\hat{\delta}_{t+1}}\right]K_{B,t}}, \tag{E.16}$$

where:

$$\Psi_{t+1} \equiv \begin{cases} \int_{\bar{\delta}-\Delta}^{\delta_{P,t+1}} \frac{1-\delta}{\phi\cdot 2\Delta}\ d\delta + \int_{\delta_{P,t+1}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\ d\delta, & \text{if } \phi_{i,t+1}=\phi, \\ \int_{\bar{\delta}-\Delta}^{\delta_{U,t+1}} \frac{\lambda_{U,t+1}(1-\delta)}{2\Delta}\ d\delta + \int_{\delta_{U,t+1}}^{\bar{\delta}+\Delta} \frac{Q_t}{2\Delta}\ d\delta, & \text{if } \phi_{i,t+1}=0. \end{cases} \tag{E.17}$$

Given Eqs. (11) and (C.4) for $\bar{R}_t$, Eq. (E.3) for $\delta_{P,t}$ and $\delta_{U,t}$, and the definition of $\underline{\omega}_{t+1}$, Eqs. (E.8)-(E.17) determine the equilibrium dynamics of ($K_{P,t}$, $K_{U,t}$, $K_{B,t}$, $H_{B,t}$, X_t, $\delta_{P,t}$, $\delta_{U,t}$, $\hat{\delta}_t$, Q_t, $\underline{\omega}_{t+1}$, $\bar{R}_t$, $\lambda_{U,t}$) recursively. Once the dynamics is obtained, the values of $\lambda'_{B,t}$, $\lambda''_{B,t}$ and $V_tS_{B,t}/(B_{B,t}+V_tS_{B,t})$ can be derived from Eqs. (D.10), (D.11), and (25), in order.

F The numerical solution method for dynamic equilibrium in the model of banking

I solve the dynamic equilibrium with the set of parameter values specified in Section 4 by approximating the equilibrium laws of motion, Eqs. (E.8)-(E.15), by the following projection method:

Step 0. Because the equilibrium laws of motion are homogeneous of degree 1 with respect to $K_{P,t-1}$, $K_{U,t-1}$ and $K_{B,t-1}$, set grid points on the state space for $K_{P,t-1}$, $K_{U,t-1}$, and the aggregate productivity shock, α_t. The value of $K_{B,t-1}$ is set to $1-K_{P,t-1}-K_{U,t-1}$ on each grid point. Guess the equilibrium values of endogenous variables on each grid point, including $\bar{\omega}_{t+1}$ and $\underline{\omega}_{t+1}$. Call this correspondence between state variables and endogenous variables as a "candidate array".

Step 1. Suppose the candidate array returns the correct equilibrium values for each state of $K_{P,t}$, $K_{U,t}$, $K_{B,t}$, and the aggregate productivity shock in the next period. The points between the grid points in the state space are approximated by linear interpolation. Given this, derive another candidate array for the current period that satisfies the equilibrium laws of motion.

Step 2. Compare the candidate array for the current period and the one for the next period. If the ratio of each element between the two arrays becomes sufficiently close to 1, then take the candidate array as the equilibrium correspondence. Otherwise, update the candidate array by a linear combination of the two arrays and go back to Step 1.

In the numerical examples in this paper, I set grid points in the $\pm$ 5% range of the deterministic steady state values of $K_{P,t-1}$ and $K_{U,t-1}$. The number of grid points are 20 for each of the two variables. The convergence criterion in Step 2 is 1e-03. In updating the candidate array in Step 2, the weight on the candidate array for the current period is 0.001. The initial guess in Step 0 is obtained through homotopy starting from the set of parameter values with which the deterministic steady state provides a successful initial guess of the candidate array that leads to convergence.

The equilibrium laws of motion, Eqs. (E.8)-(E.15), are valid if Eqs. (18)-(20), $\lambda'_{B,t} > \lambda''_{B,t}$, $H_{B,t} > 0$, and $\lambda_{U,t} < Q_t/(1-\hat{\delta}_t)$, as described in Appendix E, and also if all variables are non-negative. These inequalities are checked for each element of the converged candidate array. Starting from the deterministic steady state, I run random simulations of the dynamics for 5000 periods to confirm that the equilibrium dynamics move within the grid points satisfying the inequalities.