Financial Frictions, Misallocation and Plant-Size Distribution*

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Abstract

We quantify the effects of financial frictions on the loss of aggregate productivity and plant-size distribution through resource misallocation. To this aim, we first measure the distortions (or wedges) on capital and output by applying the static monopolistic competition model to a rich plant-level data set of manufacturers in Japan. Next, we develop a dynamic model of monopolistic competition in which entrepreneurs are subject to productivity shocks and borrowing constraints, but can accumulate savings.

Calibrating the dynamic model to match the plant-size distribution of manufacturers in Japan, we find that aggregate total factor productivity (TFP) would be higher by 11.3% if there were no borrowing constraint, which accounts for 48.1% of measured TFP losses caused by capital distortions.

Our counterfactual experiments show that as borrowing constraint is relaxed, output becomes more dispersed, which is consistent with the hypothetical plant-size distribution that would be realized without distortions on capital. However, relaxing borrowing constraint would decrease the share of very large plants (top 1%), which is inconsistent with our result from removing distortions on capital.

Our results suggest that financial constraint is a significant, but not the sole dominant factor of aggregate productivity loss and plant-size distribution.

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1 Introduction

Financial frictions can result in inefficient allocation of capital and lower aggregate productivity. Productive firms cannot obtain sufficient outside funds to expand, while unproductive firms may not shrink thanks to abundant and cheap funds available. If capital and other resources were to move from unproductive to productive firms, aggregate productivity would be higher. The problem is to what extent they lower aggregate productivity. Is the impact of financial frictions on aggregate productivity economically significant? Specifically, do they account for a major share of total productivity losses caused by resource misallocation? The last question is relevant given that misallocation can be caused by various factors besides capital market frictions, including tax and subsidy, entry restriction and regulation, anti-trust policy, mismatches of skill between labor demand and supply, and size-dependent policy.

Recent studies have tried to answer at least part of these questions. Some quantitative studies attribute low aggregate productivity in developing countries largely to financial market imperfections (Jeong and Townsend (21), among others). However, even if entrepreneurs cannot raise capital in the market, they might accumulate it out of savings. The question is, therefore, whether productive entrepreneurs can accumulate sufficient savings to escape from external financing constraints while they are productive. To address this question, therefore, plant- or firm-level data is essential to calibrate the idiosyncratic productivity process. Using plant-level data, some recent studies reach quite different conclusions. Buera et al. (5) show that financial frictions can explain TFP losses up to 40%. Moll (26) also find a significant effect of financial frictions, showing that financial frictions in two emerging market economies (Chile and Colombia) can explain aggregate productivity losses up to 20% relative to the US. On the other hand, Midrigan and Xu (25) estimate total factor productivity (TFP) losses from misallocation reaches at most 5-7% even in an economy with no external finance.

The aim of this paper is to provide new evidence about TFP losses caused by financial market frictions and to evaluate them in total resource misallocation. To this aim, we first measure plant-level distortions and resultant aggregate TFP losses using Japanese manufacturing plant-level data. Specifically, we measure capital distortions (taxes on rental cost of capital) and output distortions (taxes on output), and calculate TFP losses and hypothetical plant size distribution that would be realized with no distortions. Next, we construct a dynamic general equilibrium model in which entrepreneurs are subject to productivity shocks and borrowing constraints in capital expenditure. This model specification is motivated by our finding that a proxy for external finance dependence is positively correlated with measured capital distortions. Since our dynamic model reduces to a static model with only capital distortions (given endogenously determined entrepreneurs' savings), we calibrate it to match the plant-size distribution that would be achieved with no output distortions (but with capital distortions).

We find that TFP would rise by 11.3% if there were no borrowing constraint, which accounts for 48.1% of measured TFP losses caused by capital distortions, and 23.9% of total TFP losses caused by both capital and output distortions. Our counterfactual experiments show that as borrowing constraint is relaxed, output becomes more dispersed, which is consistent with the hypothetical plant-size distribution that would be realized without distortions on capital. However, relaxing borrowing constraint would decrease the share of very large plants (top 1%), which is inconsistent with our result from removing distortions on capital. Our results suggest that financial constraint is a significant, but not the sole dominant factor of aggregate productivity loss and plant-size distribution.

We contribute to the recent quantitative studies on the role of financial frictions on misallocation (Midrigan and Xu (25), Moll (26), and Buera Kabosiki and Shin (5)) mainly in three ways ways.

First, we evaluate the role of financial frictions in total resource mislloactaion. To this aim, we measure total resource misallocation using the methodology developed by Hsieh and Klenow (20) (HK, hereafter). Our evaluation of the role of financial frictions is precise in the sense that we use the same data set in a consistent way when we measure total resource misallocation and misallocation caused by financial frictions. Given that evidences on total resource misallocation are still rare except for HK and Restuccia and Rogerson (30), our results on total misallocation might be of its own interest. On the other hand, Burea, Kaboski and Shin (5), Midrigan and Xu (25) and Moll (26) all examine the role of financial frictions in a different context: they investigate to what extent financial frictions can account for cross-country differences in aggregate productivity.

Second, we use a rich establishment-level data set that spans about three decades. Using long-period data, we can accurately estimate the persistency and variability of plant-level productivity, which are critically important to whether productive entrepreneurs can accumulate sufficient wealth while they are productive.

Finally, we exploit the implications of financial frictions on plan-size distribution. The link between financial frictions and firm- or plant-size distribution is theoretically shown by Cabral and Mata (7). Their insight motivates us to calibrate the borrowing constraint parameter to match plant size distribution. In addition, by comparing the effects of borrowing constraint and capital distortions on plant-size distribution, we can examine whether borrowing constraint is a dominant factor for capital distortions. To my knowledge, few preceding quantitative studies explore how the degree of borrowing constraint alters plant-size distribution. A notable exception is Buera, Kaboski and Shin (5), who investigate the dispersion of firm size. In addition to the dispersion, we pay attention to the share of very large plants (e.g., top 1%) as well, which we find sensitive to the degree of financial frictions. [1]

There are some other strands of literature that is closely related to this paper. One is studies on finance and firm dynamics. Cooley and Quadrini (11), among

[1]Midrigan and Xu (25) use the information on plant-size distribution just to obtain the model parameters. Arellano et al. (2) study how the variation in financial development affects the relationship among firm size, leverage and growth. However, they do not explore the effect of financial development on firm size distribution per se.

others, find that borrowing constraints make young and small firms grow faster than mature and large firms. Arellano et al. (2) have recently provided cross-country evidence on the relationship between financial development and firm dynamics, and developed a quantitative model where financial frictions drive firm growth and debt financing through the availability of credit and default risk. Though their model is similar to ours in that firms face financial frictions and idiosyncratic productivity, they focus on firm dynamics and not on macroeconomic consequences. For example, they set safe interest rate exogenously while we derive it endogenously in a general equilibrium setting. On the other hand, they derive financial contracts endogenously, while we simply impose collateral constraints.[2]

Another related literature is empirical studies on finance and economic growth (Rajan and Zingales (29), among others). We investigate the relationship between plant-level capital distortions and a proxy for external finance dependence that we construct to identify demand for external funds based on Rajan and Zingales' idea. To our best knowledge, this paper is the first that relates the plant-level distortions to a measure of external finance dependence. Our findings that distortions (or wedges) on capital is positively correlated with a measure of external finance dependence motivates us to model the borrowing constraint on capital expenditures.

Finally, this paper is complementary to the literature on misdirected credit or "zombie" lending by Japanese banks. Peek and Rosengren (27) find that Japanese banks allocated credit in an inefficient manner when they were in distress in the 1990s. Caballero et al. (6) dubbed firms that were insolvent but kept afloat thanks to subsidized bank credit "zombies" and found that zombie-dominated industries exhibited lower productivity.[3] Unfortunately, our data set is not suitable to investigate the effect of zombie lending because it covers only manufacturing plants and not plants in construction, real estate, wholesale and retail, and other non-manufacturing industries, where most of the zombies resided (Caballero et al. (6)). In fact, long-run plant-level data, which is essential to estimate productivity persistency, was available only for manufactures. In addition, we analyze the long-run impacts of borrowing constraint and do not focus on the banking crisis period of the 1990s, when zombie lending was prevalent.[4] Caballero et al. (6) assume that as the number of zombies in a industry increases, the profitability of a firm in that

[2]Firm dynamics has been recently studied from various viewpoints. Rossi-Hansberg and Wright (31), for example, investigates the effects of human capital accumulation on establishment size distribution and net exit rates.

[3]Caballero et al. (6) do not quantify the aggregate effects of zombie lending. Kwon et al. (22) apply the methodology developed by Restuccia and Rogerson (30) to Japanese manufacturing data set, *Census of Manufactures* to quantify the effect of zombie lending on aggregate productivity growth. They found that 35% of the decline in aggregate productivity growth due to inefficient labor reallocation in the 1990s was attributable to "zombie lending." Note that they treat distortions on capital exogenously, while we derive them endogenously as an outcome of borrowing constraint.

[4]Fukuda and Nakamura (15) document that a majority of "zombies" substantially recovered during the first half of the 2000s. Akiyoshi and Kobayashi (1) empirically reveal the effect of the Japanese banking crisis on firm-level productivity. For another other quantitative study concerning the effect of financial crises on aggregate TFP, see Pratap and Urrutia (28).

industry decreases due to congestion in factor markets (higher factor prices) or competition in product market (lower product prices), preventing productive firms from operating. In our model, as borrowing constraint tightens, demand for capital decreases and hence factor prices decrease, allowing less productive firms to expand. The difference is whether unproductive firms are "subsidized" (through cheap bank credit) or productive firms are "taxed" (in terms of shadow costs arising from borrowing constraint).

The rest of the paper proceeds as follows. In Section 2, by applying the methodology developed by HK to Japan's manufacturing plant-level data set, we measure plant-level distortions and aggregate TFP losses. In Section 3, we relate the plant-level distortions to our measure of external finance dependence controlling for various plant- and industry- characteristics. Motivated by the evidence in Section 3, we proceed to Section 4 to construct a dynamic model with financial frictions. In Section 5, we calibrate the dynamic model to the plant-size distribution and quantify the role of financial frictions in aggregate TFP and plant-size distribution. Section 6 concludes.

2 Measurement of Distortions and Aggregate TFP Losses

In this section, we describe how we measure the plant-level distortions on capital and output, and the effect of distortions on aggregate productivity and plant size distribution. To this aim, we simply assume that producers are subject to exogenous distortions and do not explore the sources of the distortions. In the dynamic model elaborated in sections 4 and 5, we focus on one source of capital distortions: external financing constraint.

2.1 Static Model

We follow HK and set up a static, partial equilibrium model of monopolistic competition. There are a representative final good producer, representative sectoral good producers, and entrepreneurs who produce differentiated goods in each sector (industry). The final good producer and the sectoral good producers operate in a perfectly competitive market, while entrepreneurs operate in a monopolistic market.

The final good producer combines the output Y_s of industry $s \in \{1,,,S\}$ and produces output Y using a Cobb-Douglas production technology

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \quad where \sum_{s=1}^{S} \theta_s = 1. \tag{2.1}$$

Denoting the price of industry output Y_s by P_s, cost minimization and competitive market imply

$$P_s Y_s = \theta_s P Y \tag{2.2}$$

, where $P = \prod \left(\frac{P_s}{\theta_s}\right)^{\theta_s}$ represents the final good price and equals to the marginal cost. We choose the final good as a numeraire so that $P = 1$.

The sectoral good producer s combines differentiated product $si \in \{1, , , M_s\}$ to produce industry output Y_s using a CES production technology

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 \tag{2.3}$$

Denoting the price of differentiated good si by P_{si}, cost minimization and competitive market imply

$$P_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}} \tag{2.4}$$

σ represents the price elasticity of demand for differentiated good si.

An entrepreneur si produces a differentiated good si from capital K_{si} and labor L_{si} using a constant-returns-to scale Cobb-Douglas production technology with idiosyncratic TFP, A_{si},

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s} \tag{2.5}$$

We consider two kinds of distortions that divert marginal revenue from marginal cost. One is output distortion, τ_{Ysi}, and the other capital distortion, τ_{Ksi}. Alternatively, we can consider labor and capital distortions, but anyway, we can identify two of the three because there are two factors of production.[5] τ_Y may be high for firms that government impose strict regulations on production, while it may be low for firms that gain subsidies. τ_K may be high for firms that depend on costly external finance, and low for firms that have access to subsidized credit.

The entrepreneur's objective is to maximize profits subject to the demand (2.4) and techonology (2.5) ,

$$\Pi_{si} = (1 - \tau_{Ysi}) P_{si} Y_{si} - w L_{si} - (1 + \tau_{Ksi}) R K_{si}, \tag{2.6}$$

where w is the wage and R is the rental cost of capital.

2.2 Method for Measuring Plant-level Distortions and TFPQ

In this subsection, we describe how we retrieve entrepreneur-specific distortions from observable data. From the entrepreneur's optimal decision and demand function (2.4),

$$1 + \tau_{Ksi} = \frac{\alpha_s}{1 - \alpha_s} \frac{w L_{si}}{R K_{si}} \tag{2.7}$$

[5] Suppose alternatively that the entrepreneur faces labor distortion τ^*_{Lsi} and capital distortion τ^*_{Ksi}. Then, comparing the first order conditions, we see that $1 - \tau_{Ysi} = \frac{1}{1+\tau^*_{Lsi}}$ and $1 + \tau_{Ksi} = \frac{1+\tau^*_{Ksi}}{1+\tau^*_{Lsi}}$. We may say that output and capital distortions in the text are standardized by labor distortion. Which specification you choose does not affect our measures of TFP efficiency defined below (TFPGAIN or TFPGAP).

$$1 - \tau_{Ysi} = \frac{\sigma}{\sigma - 1} \frac{wL_{si}}{(1 - \alpha_s) P_{si} Y_{si}}, \tag{2.8}$$

and

$$A_{si} = \kappa_s \frac{(P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}, \quad where\ \kappa_s = (P_s^{\sigma} Y_s)^{\frac{-1}{\sigma-1}}. \tag{2.9}$$

We can observe plant-level wage compensation wL_{si} and nominal output $P_{si}Ysi$. By setting the rental cost R at a plausible value, we can retrieveτ_{Ksi} and τ_{Ysi} from (2.7) and (2.8). Though we cannot observe κ_s, it does not affect A_{si} relative to TFP_s. So, we set $\kappa_s = 1$. Then, we can obtain A_{si} from observable nominal output $P_{si}Y_{si}$ according to (2.9) even though plant-level price P_{si} and output Y_{si} are not separately observable. To derive Y_{si}, we raise $P_{si}Y_{si}$ to the power of $\frac{\sigma}{\sigma-1}$ based on demand function (2.4). We call A_{si} "TFPQ" hereafter following Foster et al. (14) and HK.

2.3 Sectoral and Aggregate TFP

Once we measure plant-level distortions and TFPQ, it is straightforward to derive sectoral and aggregate TFP as follows.

First, we derive a firm's revenue productivity (TFPR) (see Foster, Haltiwanger and Syverson (14)) from the first-order conditions as

$$TFPR_{si} \equiv P_{si} A_{si} = \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{R}{\alpha_s}\right)^{\alpha_s} \left(\frac{w}{1 - \alpha_s}\right)^{1-\alpha_s} \frac{(1 + \tau_{Ksi})^{\alpha_s}}{1 - \tau_{Ysi}} \tag{2.10}$$

Eq. (2.10) suggests that without distortions, TFPR would be equalized across firms. With distortions, however, TFPR is larger for the firm that constrains its size due to higher distortions.

Next, we define a sectoral average of TFPR as follows:

$$\overline{TFPR_s} \equiv \frac{\sigma}{\sigma - 1} \left(\frac{R}{\alpha_s \sum_{i=1}^{M_s} \frac{(1-\tau_{Ysi})}{1+\tau_{Ksi}} \frac{P_{si}Y_{si}}{P_sY_s}}\right)^{\alpha_s} \left(\frac{w}{(1 - \alpha_s) \sum_{i=1}^{M_s} (1 - \tau_{Ysi}) \frac{P_{si}Y_{si}}{P_sY_s}}\right)^{1-\alpha_s} \tag{2.11}$$

Using these definitions, we can express sectoral TFP as[6]

$$TFP_s \equiv \frac{Y_s}{K_s^{\alpha_s} L_s^{1-\alpha_s}} = \left[\sum_{i=1}^{Ms} \left(A_{si} \frac{\overline{TFPR_s}}{TFPR_{si}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \tag{2.12}$$

Eq. (2.12) shows how the distribution of τ_{Ysi} and τ_{Ksi} affects TFP_s. Suppose that τ_{Ysi} and τ_{Ksi} are equal across firms so that $TFPR_{si}$ is also equal across firms. Then, efficient allocation is achieved even if the common values of τ_{Ysi} and τ_{Ksi} are not zero. Allocation is inefficient to the extent that τ_{Ysi} and τ_{Ksi}, and hence $TFPR_{si}$ are dispersed across firms.

Finally, we can aggregate sectoral outputs as

$$Y = \prod_{s=1}^{S} \left(TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s} \right)^{\theta_s} \tag{2.13}$$

If there were no distortion on output or capital, that is, if $\tau_{Ksi} = \tau_{Ysi} = 0$ for all i , then $TFPR_{si} = TFPR_s$ for all i and TFP_s would be $\overline{A}_s \equiv \left(\sum_{i=1}^{Ms} A_{si}^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$. Given sectoral capital K_s and labor L_s, aggregate output would be $Y_{efficient} \equiv \prod_{s=1}^{S} \left(\overline{A}_s K_s^{\alpha_s} L_s^{1-\alpha_s} \right)^{\theta_s}$.[7]

We define TFPGAP as the ratio of actual aggregate output to the efficient aggregate output that would be achieved without distortions while keeping sectoral capital and labor at actual levels:

(2.14)

$$TFPGAP \equiv \frac{Y}{Y_{efficient}} = \prod_{s=1}^{S} \left(\frac{TFP_s}{\overline{A}_s} \right)^{\theta_s} = \prod_{s=1}^{S} \left[\sum_{i}^{Ms} \left(\frac{A_{si}}{\overline{A}_s} \frac{\overline{TFPR_s}}{TFPR_{si}} \right)^{\sigma-1} \right]^{\frac{\theta_s}{\sigma-1}}$$

[6]To derive (2.12), define firm-level marginal revenue products of capital and labor as $MRPL_{si} \equiv \frac{\partial(P_{si}Y_{si})}{\partial L_{si}} = \left(\frac{\sigma-1}{\sigma}\right)(1-\alpha_s)\frac{P_{si}Y_{si}}{L_{si}} = \frac{w}{(1-\tau_{Ysi})}$, and $MRPK_{si} \equiv \frac{\partial(P_{si}Y_{si})}{\partial K_{si}} = \left(\frac{\sigma-1}{\sigma}\right)\alpha_s\frac{P_{si}Y_{si}}{K_{si}} = \frac{(1+\tau_{Ksi})R}{1-\tau_{Ysi}}$, and sectoral marginal revenue product of capital and labor as $\overline{MRPL_s} \equiv \frac{w}{\sum_{i=1}^{M_s}(1-\tau_{Ysi})\frac{P_{si}Y_{si}}{P_sY_s}}$, and $\overline{MRPK_s} \equiv \frac{R}{\sum_{i=1}^{M_s}\frac{(1-\tau_{Ysi})}{1+\tau_{Ksi}}\frac{P_{si}Y_{si}}{P_sY_s}}$. Then, equilibrium allocation of labor and capital across sectors are

$$L_s = \sum_{i=1}^{M_s} L_{si} = L \frac{(1-\alpha_s)\theta_s/\overline{MRPL_s}}{\sum_{s'=1}^{S}(1-\alpha_{s'})\theta_{s'}/\overline{MRPL_{s'}}}$$

and

$$K_s = \sum_{i=1}^{M_s} K_{si} = K \frac{\alpha_s\theta_s/\overline{MRPK_s}}{\sum_{s'=1}^{S}\alpha_{s'}\theta_{s'}/\overline{MRPK_{s'}}}.$$

Substitute L_s and K_s into the definition of TFP_s to obtain (2.12).

[7]Even if τ_{Ksi} and τ_{Ysi} are not equal to zero, but only if they are all equalized across entrepreneurs, $TFPR_{si} = TFPR_s$ for all i and TFP_s would be $\overline{A}_s$. "No distortion" is just a special case to the efficient allocation, though we use the term "no distortion" or "without distortions" to refer to the efficient allocation below.

We also report TFPGAIN that measures the gains of TFP from removing all distortions:

$$TFPGAIN \equiv \frac{1}{TFPGAP} - 1 \tag{2.15}$$

Similarly, let $\hat{A}_s$ denote sectoral productivity achieved if there were no distortion on output, that is, if $\tau_{Ysi} = 0$ for all i, but τ_{Ksi} remains at the actual level . A bit of algebra yields

$$\hat{A}_s = \frac{\left[\sum_{si=1}^{Ms} \left(\frac{A_{si}}{(1+\tau_{Ksi})^{\alpha}}\right)^{\sigma-1}\right]^{\alpha+\frac{1}{\sigma-1}}}{\left[\sum_{si=1}^{Ms} \left(\frac{1}{1+\tau_{Ksi}}\right)\left(\frac{A_{si}}{(1+\tau_{Ksi})^{\alpha}}\right)^{\sigma-1}\right]^{\alpha}} \tag{2.16}$$

Then, we can define $\widehat{TFPGAP}$ as the ratio of actual aggregate output to aggregate output that would be achieved without output distortions while keeping sectoral capital and labor at actual levels:

$$\widehat{TFPGAP} = \prod_{s=1}^{S} \left(\frac{TFP_s}{\hat{A}_s}\right)^{\theta_s} = \prod_{s=1}^{S} \left[\sum_{i}^{Ms} \left(\frac{A_{si}}{\hat{A}_s} \frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}} \tag{2.17}$$

Now it would be useful to clarify the notion of TFPGAP (or TFPGAIN). First, TFPGAP is an efficiency measure relative to the highest TFP achievable without distortions given the market structure of monopolistic competition. It is not an efficiency measure relative to the highest TFP achievable without any distortions in a competitive market, where the most productive firm would take all the market share. This is clear from the definition of our measure of efficient sectoral TFP, $\overline{A}_s$, which depends on the price elasticity of demand, σ. Second, TFPGAP reflects the variation in distortions and not the average of distortions. In this point, measurement of misallocation is in contrast with business cycle accounting (e.g., Chari et al., (9)), which focuses on the aggregate level of distortions across time. This point is also important in comparing TFPGAP across time. Suppose, for example, that labor and capital market regulations are relaxed and that some firms can now enjoy lower labor and capital costs. Though these changes would certainly reduce the average distortions, if they widen the differences in factor prices each firm faces, TFPGAP may not shrink but even rise. What is important here is who can benefit from deregulations. If less productive firms benefits from deregulations but more productive firms do not, TFPGAP may well rise. Thirdly, TFPGAP is an aggregate measure of intra-industry misallocation, and not of interindustry misallocation. It is relative to the highest TFP achievable if TFPR is equalized within each industry. We cannot account for interindustry misallocation since, as (2.7) and (2.8) show, we cannot distinguish industry-specific distortions from industry-specific technological parameter α_s. Finally, TFPGAP is a measure of allocation efficiency given total resources. A rise in the average capital distortions, for example, may reduce capital accumulation and hence output. However, it does not necessarily lead to inefficient allocation given total amounts of capital.

2.4 Distortions and Plant-Size Distribution

In this subsection, we derive some implications of distortions on plant-size distribution. The first-order conditions of the entrepreneur yield

$$(2.18) \quad log(Y_{si}) = \sigma log(1 - \tau_{Ysi}) - \alpha\sigma log(1 + \tau_{Ksi}) + \sigma log(A_{si}) + const,$$

where *const* is a term that does not depend on the firm si. Eq. (2.18) suggests that size distribution is informative about the distribution of distortions. Specifically, $log(Y_{si})$ tends to be more dispersed if either type of distortions are more dispersed, if either type of distortions are less positively (or more negatively) correlated with TFPQ, or if the two distortions are more positively (or less negatively) correlated with each other.

2.5 Data

For plant-level data, we use the *Census of Manufacturers* of Japan conducted by Ministry of Economy, Trade, and Industry. The Census covers all establishments located in Japan (excluding those belonging to the government)[8], and falling under manufacturing industry in years with the last digit of 0,3,5 and 8. In other years, the Census covers establishments with four or more employees. To construct plant-level productivity, we need to exclude those plants whose fixed tangible asset data is not available. We are left with the plants with 10 employees or more from 1981-2000 and 2005, and those with 30 employees or more from 2001-2004 and 2006-2008. The greatest merit of the Census is its long time horizon and the widest coverage of establishments in manufacturing industries. We can exploit the long-period data to estimate the persistency of plant-level TFP, which is a key variable in quantifying the effect of financial constraint on aggregate productivity. An obvious shortcoming of the Census is that it excludes establishments in non-manufacturing industries. [9]

We use the Census data from 1981 to 2008.The information we use from the *Census of Manufacturers* is the plant's industry (at the four-digit level), labor compensation (excluding non-wage compensation), value-added, export-revenues, and

[8]Though the data is at the plant level and not the firm level, most of the plants are owned by single-plant firms. In 2008, for example, 84.4% of the plants (222,145 out of 263,061 plants) are owned by single-plant firms.

[9]In Japan, *Financial Statements Statistics of Corporations by Industry (FSSC)* published by Ministry of Finance covers firms in both manufacturing and non-manufacturing industries. However, *FSSC* is a firm-level statistics and not an establishment-level one. In addition, firms with equity capital less than 600 million yen (about 7.5 million dollars) are randomly sampled in *FSSC*, which makes it difficult to construct a panel data set.

capital stock. We reclassify the plants into 51 industries based on JIP Database, published by Research Institute of Economy, Technology and Industry (RIETI).

To measure plant-level distortions and TFP, we adjust the quality of workers and hours worked based on the assumption that the quality and hours worked is reflected by plant-level wage relative to the industry average.

We set the rental price of capital to $R = 0.1$. We have in mind a 4% interest rate and a 6% depreciation rate. Another reason for setting the value of R is that we intend to compare our estimate of Japan with the US estimate by HK, who also set $R = 0.1$. We set the elasticity of substitution between plant value-added to $\sigma = 3$, again the same value used by HK. TFP gap is increasing in σ, so we chose it conservatively; estimates of substitutability of competing manufactures rage from three to ten [10]

We set α_s as one minus industry-level labor share. We implicitly assume that rents from marksups are divided into payments to labor and capital pro rata in each industry. The data source of labor share is JIP Database.

To exclude outliers, we trim the 1% tails of $log(MRPK_{si}/\overline{MRPK}_s)$, $log(MRPL_{si}/\overline{MRPL_s})$ and $log(A_{si}/\bar{A}_s)$ across industries and recalculate the industry-level variables: wL_s, K_s, $PsYs$, $\overline{TFPR}_s$, θ_s, $\bar{A}_s$, and $\hat{A}_s$. The number of plants vary from 39,981 to 170,789. Total plant-year observations amounts to 3,565,341.

See Data Appendix for the details of our data construction.

2.6 TFP Gap and Plant-Size Distribution

Figure 1 depicts the distribution of measured plant-level log of TFPQ, $log(A_{si})$, output distortion, $log(1+\tau_{Ysi})$, and capital distortion, $log(1+\tau_{Ksi})$. Table 1 shows the descriptive statistics. The standard deviation of the deviations of $log(A_{si})$ from its industry mean is 0.98, which is slightly larger than the US manufacturing counterpart estimated by HK (0.83 on average of the three sample years).[11] Median values of τ_{Ysi} and τ_{Ksi} are -0.17 and 0.73, respectively, suggesting that a typical firm obtains "subsidies" on output and pays "taxes" on capital.

Table 2 shows that TFPGAP from both types of distortions is 0.679 on average during the sample years, suggesting that without any distortions aggregate TFP

[10] Broda and Weinstein (4) report that mean value of elasticity of substitution ranges from 4.0 to 17.3 and its median from 2.2 to 3.7, depending on different aggregation levels and time periods. Cooper and Ejarque (12) conducted structural estimation of investment and reported the estimated elasticity of profit to capital was around 0.7, which, together with the capital share of 1/3, implies a markup of about 15%, or equivalently, σ of about 8. Using Japan's plant-level data set from *Census of Manufacturers* over the 1981-2000 period, Kwon et al. (22) estimated the production functions by industry and found the average degree of returns to scale, which is tantamount to $\frac{\sigma-1}{\sigma}$, from 0.461 to 1.038, suggesting that the lower bound of σ is 1.855.

[11] The standard deviations of TFPQ estimated by us and by HK are much larger than those estimated by Foster et al. (14) (0.22). HK suggests one of the reasons for the difference is that TFPQ estimated by HK (and us) should reflect a quality and variety of a plant's product, not just its physical productivity. Another reason is that HK's results (and ours) cover all industries, where as Foster et al. analyze a dozen industries whose products are deemed to be homogeneous.

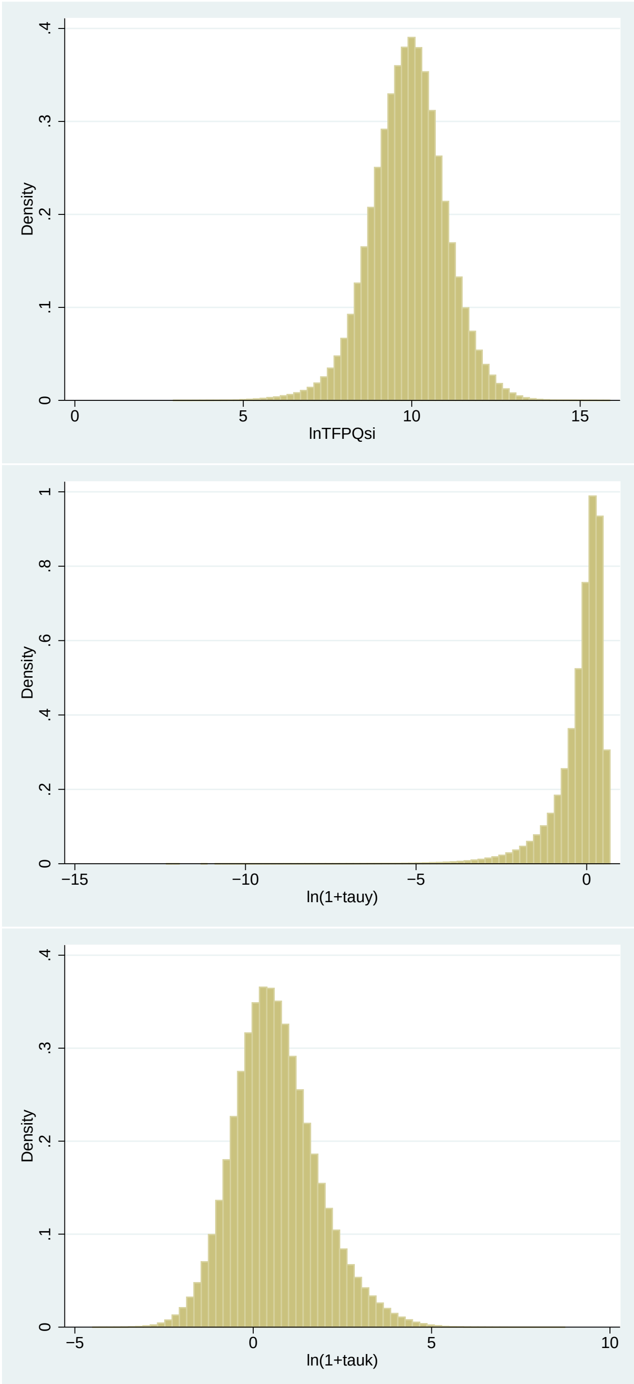

Figure 1: **Density of** $log(A)$**,** $log(1 + \tau_Y)$**, and** $log(1 + \tau_K)$

Table 1. Descriptive Statistics for TFPQ, Output Distortion and Capital Distortion

	lnTFPQ $\ln(A_{si})$	Output distortion τ_{Ysi}	Capital distortion τ_{Ksi}
Mean	9.90	-0.62	-0.59
Median	9.92	-0.17	-0.17
Minimum	2.91	-135.36	-0.99
Max	15.88	0.98	6285.31
Std. Dev.	1.11	1.86	1.63
Std. Dev.2	0.98		
Interquartile Range	1.27		
90 percentile - 10 percentile	2.48		
Skewness	-0.19	-12.82	-7.33
Kurtosis	3.82	536.71	157.64
Obs	3565341	3565341	3565341

Note: Std. Dev.2, Interquartile Range, and 90 percentile -10 perncentile show each of ln(TFPQ/TFPQ_industry mean).

would increase by 47.2%. Though we do not account for measurement error or model misspecifications, the magnitude of $TFPGAIN$ for the Japanese manufacturers can be compared to the US counterparts measured by HK. If capital and labor are hypothetically reallocated to equalize marginal products to the extent observed in the US, manufacturing TFP gains of 7.7%. If we compare Japan's average to the US worst year level, the relative gain is 3.0%, which is much smaller than HK's estimates for China (30%-50%) and India (40%-60%). Table 2 also reports that TFPGAP in case of $\tau_Y = 0$ is 0.810, suggesting that distortions on capital lower aggregate TFP by 23.4%. About half of the TFP losses from misallocation is cuased by distortions on capital.

Table 2. TFP Gap and TFP Gains

	Output distortoin and Capital distortion=0	Output distortion=0	Average in the United States(1977, 1987 and 1997) from Hsieh and Klenow(2009)
TFPGAP	0.679	0.810	0.733
TFPGAIN	47.18%	23.40%	36.60%

Figure 2 depicts $TFPGAP$ over time, which shows a slightly declining trend, suggesting that misallocation tended to slightly worsen over the last three decades. Though Japan has gradually relaxed regulations in various fields over the last three decades, including regulations on issuing uncollateralized corporate bonds (Hoshi and Kashyap (18)), the declining trend of GDP growth rate might have hindered smooth reallocation of resources across plants. $TFPGAP$ also exhibits weak procyclicality[12]: it temporarily fell in years 1998-99, 2005 and 2007-08. The dip in 1998-99 coincides with the severe banking crisis and recession in Japan, though $TFPGAP$ dropped only by about 1 percentage point even in this crisis period. Figure 2 also depicts $\widehat{TFPGAP}$, a loss only from capital distortions. It shows a trend and cyclical pattern similar to $TFPGAP$. Since we are interested in the long-run or steady-state misallocation rather than its short-run movements, we do not delve into the factors that cause movements of TFPGAP over time.

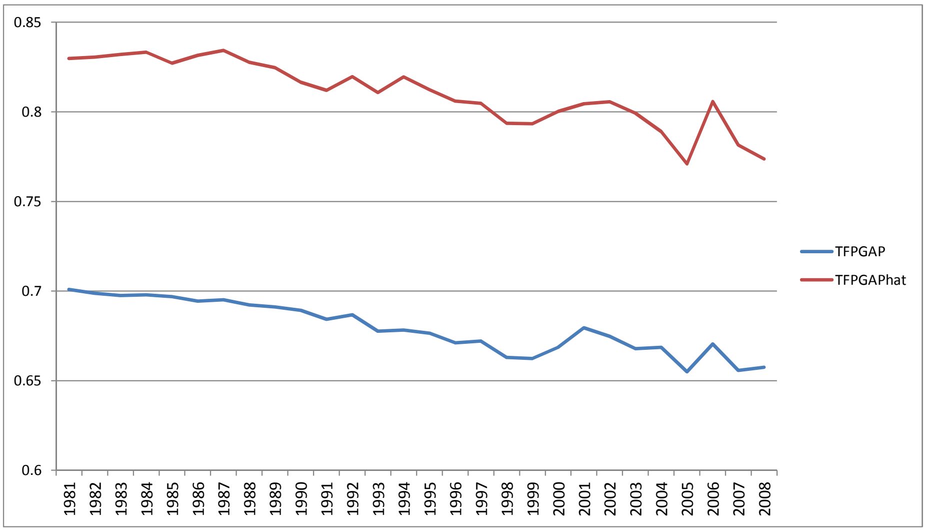

Figure 2: $\widehat{TFPGAP}(\tau_{Ysi} = 0)$ **and** $TFPGAP(\tau_{Ysi} = \tau_{Ksi} = 0)$

We calculate "efficient" output distribution that would be achieved without any distortions and compare it with the actual output distribution. Figure 3 plots the efficient vs. actual size distributions of plants. Size is measured as plant value-added. Columns 1 and 2 in Table 3 show the actual and hypothetical (efficient) size distributions, respectively. The hypothetical efficient distribution is more dispersed than the actual one in terms of interquartile range (the difference between the upper and lower quartiles). As (2.18) suggests, in theory, removing distortions may or may not reduce output dispersion depending on the variance of distortions and their correlations with TFPQ and their correlations with each other. Our result suggests that the latter two effects more than offset the first effect. Our result is also consistent with the US evidence (HK).

In Appendix 1, we show the actual and efficient plant size distributions by decades, suggesting that the efficient distribution is more dispersed than the actual

[12]the correlation of TFP gap and the growth rates in market-sector GDP and manufacturing GDP are 0.55 and 0.31, respectively.

one irrespectively of the sample periods.

Looking into details of size distribution, we find that without distortions, there should be a larger share of largest plants (top 0.01% to top 20%) and a smaller share of smallest plants (bottom 20%).

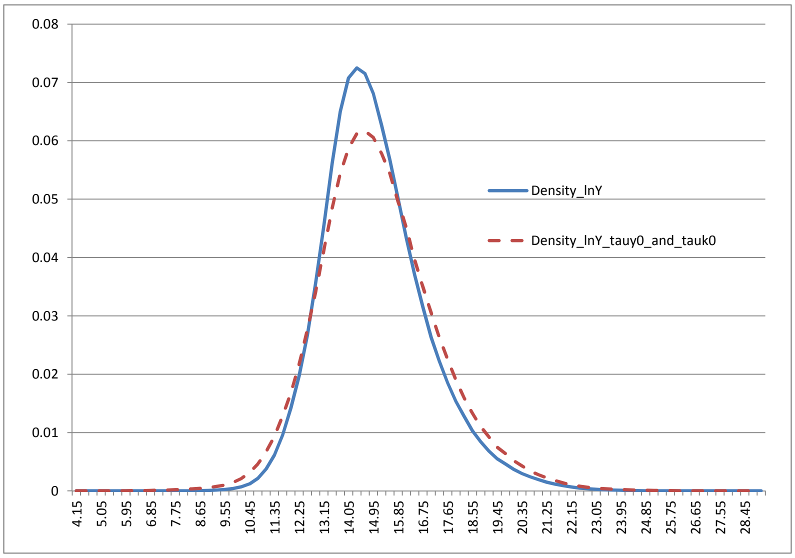

Figure 3: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} in case of $\tau_{Ysi} = \tau_{Ksi} = 0$) (red line).**

Column 3 shows the hypothetical output shares without distortions on output (but with distortions on capital) and Figure 4 compares them with the actual distribution. We find a size distribution similar to that of no distortion case (Column 2), though the change from actual distribution is smaller.

Comparing Columns 2 and 3, we see that removing capital distortions would increase the interquartile range of output, increase the share of largest plants, and decrease the share of smallest plants.

2.7 Robustness

We now provide some robustness checks on our baseline calculations of hypothetical efficiency gains shown by Table 2.

First, we change the elasticity of substitution within industries, σ, from 3 to 6. We find that TFP gains from removing all distortions ($\tau_Y = \tau_K = 0$) amount to 50.7%, slightly higher than 47.2% the case of $\sigma = 3$. On the other hand, TFP gains from removing only output distortions ($\tau_K = 0$) decreases from 23.4% to 18.6%, suggesting that the effects of capital distortions are sensitive to this elasticity.

Next, we change the criterion of excluding outliers from the 1% tails of TFPR ($log(MRPK_{si}/\overline{MRPK}_s)$ and $log(MRPL_{si}/\overline{MRPL_s})$) and TFPQ ($log(A_{si}/\bar{A}_s)$)

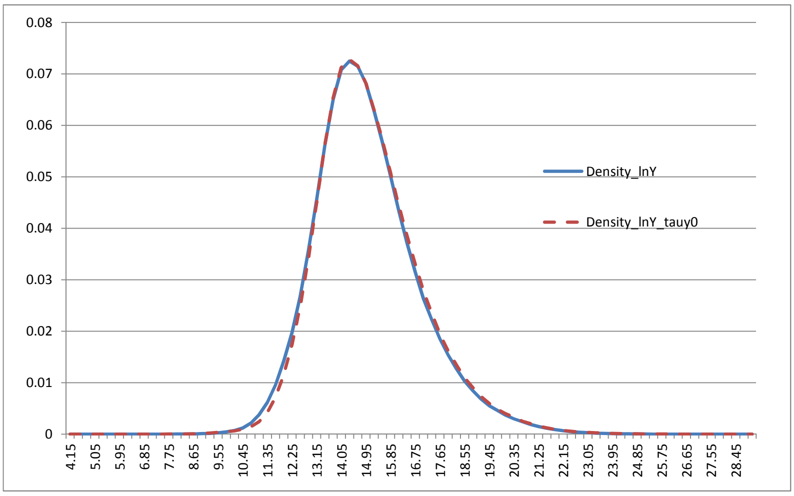

Figure 4: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} in case of $\tau_{Ysi} = 0$) (red line)**

Table 3. Size Distribution

	Actual output	Output distortoin and Capital distortion=0	Output distortion=0
interquartile range	2.334	2.698	2.349
25th percentile	13.430	13.389	13.501
75th percentile	15.764	16.088	15.850
Fraction of Y largest 0.01%	13.77%	27.67%	20.28%
Fraction of Y largest 0.1%	37.22%	55.58%	45.95%
Fraction of Y largest 1%	69.14%	80.58%	74.42%
Fraction of Y largest 5%	87.04%	92.72%	89.30%
Fraction of Y largest 10%	92.59%	95.87%	93.49%
Fraction of Y largest 20%	96.30%	98.06%	96.74%
Fraction of Y smallest 1%	0.00%	0.00%	0.00%
Fraction of Y smallest 5%	0.01%	0.00%	0.01%
Fraction of Y smallest 10%	0.03%	0.01%	0.03%
Fraction of Y smallest 20%	0.12%	0.04%	0.10%

to their 2% tails. We find that the hypothetical TFP gains from removing all distortions fall from 47.2% to 35.1%, and the TFP gains from removing only output distortions fall from 23.4% to 15.2%. Thus measurement error in the remaining 1% may be important, but there still remains large gains from removing distortions. It is also notable that HK reports that TFP gains in China and India also fall when they trim 2% tails (from 87% to 69% for China, and from 128% to 106% for India), suggesting that efficiency gains for Japan relative to those countries do not virtually change.

In Appendix 2, we show plant size distributions in the cases of $\sigma = 6$ and $\pm 2\%$ trimming. In both cases, the hypothetical efficient distribution is more dispersed than the actual one, as in the baseline case.

3 Distortions and External Finance Dependence

To bridge the measurement of distortions in the previous section and model-based analysis in the following sections, this section examines whether plant-level distortions are positively correlated with a measure of external finance dependence after controlling for plant- and industry-level characteristics. If the supply of external finance is limited due to informational or contractual frictions, a plant that needs more external finance is subject to a higher distortion. We test this hypothesis here. In addition, if our measures of distortions are systematically correlated with plant- or industry- specific variables, we can confirm that our measure of distortions is not just a figment of measurement errors, though we cannot say that our measure is free from measurement errors.

3.1 Proxy for External Finance Dependence

As a proxy for external finance dependence, we use the industry-level median of external finance dependence using a sample of Japanese listed firms over the period of 1981-2007. Data on the actual use of external financing by our sample plants is not available. But even if it were, it would not be useable because it would reflect the equilibrium between the demand for external funds and its supply. Since the latter is precisely what we are testing for, this information is contaminated. Demand for external finance depends on industry-specific technological factors, including the initial project scale, the gestation period, the cash harvest period, and the requirement for continuing investment. Based on this idea, Rajan and Zingales (29) conducted cross-country study using the industry-level median of US publicly listed firms, which are least likely to be subject to external financing constraint. Based on the same idea, we use the industry-level median of Japanese publicly listed firms. Financial statements of publicly listed firms are less susceptible to measurement errors than unlisted firms, which constitutes another reason for using financial statement data of listed firms.

Following Rajan and Zingales (29), we measure external finance dependence by the difference between capital expenditures and cash flow from operations divided by capital expenditures. Cash flow from operations is the sum of current profit, depreciation, increases in notes payable, and increases in reserves for possible loan losses minus the sum of increases in notes receivable, corporate taxes, and increases in inventory. We use the JIP classification of 51 industries. Table 4 shows the external dependence ratios across industries.[13]

3.2 Regression Results

We choose three dependent variables. One is $TFPRGAP_{si} \equiv \frac{(1+\tau_{Ksi})^{\alpha_s}}{1-\tau_{Ysi}}$, which is the ratio of actual $TFPR_{si}$ to efficient $TFPR_s$ (Eq. (2.10)), and measures the total distortions. The other two are τ_{Ysi} and τ_{Ksi}.

As explanatory variables, we add some control variables and year dummies to the external finance dependence. We conduct the following plant-level regression using weighted least squares with industry value-added shares as weights:

$$Distortion_{sit} = \beta Fin_s + \gamma X_{sit} + \alpha Year_t + \epsilon_{sit}, \tag{3.1}$$

where $Distortion_{sit}$ is either $TFPRGAP_{sit}$, τ_{Ysit} or τ_{Ksit}, Fin_s is our measure of external finance dependence, X_{sit} is plant- or industry-specific control variables, $Year_t$ is a year dummy, and ϵ_{sit} is a disturbance. For control variables X_{sit}, we include variables that may affect distortions: export dummy that takes one if the plant exports, plant size measured by log of number of employees, organization dummies: corporation and self-employed company dummies (with a benchmark of cooperative dummy), industry-level regulation index complied by JIP Database, industry-level average employee age dummies (20s-60s, with a benchmark of 10s), and industry-level average share of part-timers. We expect external finance dependence is positively correlated with τ_{Ksit} and possibly with $TFPRGAP_{si}$. If exporters face higher costs for exports or various taxes and transportation costs, then the coefficient on export dummy will be positive in the regression of τ_{Ysit}, and possibly of $TFPRGAP_{si}$. Regulation index is expected to take a positive coefficient in all the regressions. As for labor market variables, we expect that regular workers and elderly workers are more protected from losing jobs and receive higher wages relative to their productivity than part-timers and young workers. If this is the case, the coefficient on the share of part-timers will take a negative coefficient and higher age dummies will take positive coefficients in the regression of τ_{Ysit}, and possibly of $TFPRGAP_{si}$.

The number of observations reduces to about 120000 due to limited data availability. Table 5 shows the estimation results. We report standard errors adjusted

[13] Regulations on corporate bond issuance had been gradually relaxed from the 1980s and completely lifted in 1996 in Japan (Hoshi and Kashyap (19). However, a large part of the post-deregulation period (1996-2008) is coincident with the Japanese banking crisis period (1995-2005). This is why we did not restrict our sample period to the post-deregulation period to obtain industry-level external finance dependence ratio. For a similar reason, we excluded year 2008, when the global financial crisis affected Japanese financial markets.

Table 4. External dependence by industry in Japan

	JIP industry classification	External dependence(1981-2007)
8	Livestock products	0.546
9	Seafood products	-0.064
10	Flour and grain mill products	0.337
11	Miscellaneous foods and related products	0.460
12	Prepared animal foods and organic fertilizers	0.525
13	Beverages	0.581
15	Textile products	0.450
16	Lumber and wood products	0.486
17	Furniture and fixtures	0.166
18	Pulp, paper, and coated and glazed paper	0.656
19	Paper products	0.348
20	Printing, plate making for printing and bookbinding	0.300
21	Leather and leather products	0.587
22	Rubber products	0.510
23	Chemical fertilizers	0.548
24	Basic inorganic chemicals	0.616
25	Basic organic chemicals	0.577
26	Organic chemicals	0.568
27	Chemical fibers	0.477
28	Miscellaneous chemical products	0.468
29	Pharmaceutical products	0.324
30	Petroleum products	0.643
31	Coal products	0.679
32	Glass and its products	0.571
33	Cement and its products	0.595
34	Pottery	0.468
35	Miscellaneous ceramic, stone and clay products	0.480
36	Pig iron and crude steel	0.516
37	Miscellaneous iron and steel	0.542
38	Smelting and refining of non-ferrous metals	0.705
39	Non-ferrous metal products	0.596
40	Fabricated constructional and architectural metal products	0.343
41	Miscellaneous fabricated metal products	0.264
42	General industry machinery	0.394
43	Special industry machinery	0.536
44	Miscellaneous machinery	0.351
45	Office and service industry machines	0.322
46	Electrical generating, transmission, distribution and industrial apparatus	0.534
47	Household electric appliances	0.399
48	Electronic data processing machines, digital and analog computer equipment and accessories	0.565
49	Communication equipment	0.527
50	Electronic equipment and electric measuring instruments	0.131
51	Semiconductor devices and integrated circuits	0.582
53	Miscellaneous electrical machinery equipment	0.435
54	Motor vehicles	0.511
55	Motor vehicle parts and accessories	0.431
56	Other transportation equipment	0.558
57	Precision machinery & equipment	0.380
58	Plastic products	0.449
59	Miscellaneous manufacturing industries	0.248
92	Publishing	-0.134

Note: External Dependence= (Capital Expenditures- Cash Flow from Operations)/Capital Expenditures for clustering at the industry level. The first to the third columns show the regression results of $TFPRGAP_{sit}$, $\tau_{Y sit}$, and $\tau_{K sit}$, respectively. In the regressions of $TFPRGAP_{sit}$ and $\tau_{K sit}$, the coefficients on Fin_s are positive and significant. On the other hand, in the regression of $\tau_{Y sit}$, the coefficients on Fin_s is not significant. In the regression of $\tau_{K sit}$, we add the log of firm age and its intersection with Fin_s to the explanatory variables. Theoretical and empirical studies suggest that young firms are more likely to be financially constrained (e.g., Diamond (13) and Sakai et al. (32)). The fourth and fifth columns of Table 5 are consistent with the preceding studies: $log(age)$ in the fourth column and its intersection with Fin_s in the fifth column are both significant and negative.

Table 5. Estimation Results

	TFPRGAPs		tauysi		tauksi		tauksi		tauksi	
External dependence(1981-2007)	0.025		-0.913		4.162	***	4.133	***	9.159	***
ln(age)							-1.091	***	-0.326	
ln(age)*External dependence(1981-2007)									-1.884	***
Regulation Index	0.157		-0.185		-0.225		-0.217		-0.215	
Export dummy	0.165	***	0.210	***	-0.327	*	-0.355	**	-0.353	**
ln(L)	0.062	***	0.130	***	-0.883	***	-0.648	***	-0.643	***
Corporation dummy	0.149	***	0.083		0.206		0.214		0.215	
Cooperative dummy	(omitted)		(omitted)		(omitted)		(omitted)		(omitted)	
Self emoloyed conmany dummy	-0.300	***	-1.202	***	3.658	***	3.500	***	3.500	***
Workers aged 20-29 ratio	18.711	***	33.607	*	14.242		18.004		19.521	
Workers aged 30-39 ratio	15.262	***	29.209	**	-4.823		-2.581		-1.382	
Workers aged 40-49 ratio	17.714	***	36.760	*	2.139		4.513		5.892	
Workers aged 50-59 ratio	18.361	***	36.141	**	-17.474		-13.326		-11.965	
Workers aged 60+ ratio	17.211	***	33.196	**	0.962		3.764		4.942	
Part-time workers ratio	0.506		0.830		5.718		5.526		5.490	
Constant	-16.071	***	-33.586	**	5.366		4.317		0.959	
Year dummy	yes		yes		yes		yes		yes	
Number of obs	3481312		3481312		3481312		3481312		3481312	
R-squared	0.060		0.124		0.016		0.019		0.019	
Root MSE	0.801		1.400		13.612		13.593		13.592	

Notes: *** and ** show sstatistical significance at the 1% and 5% level.
Standart errors are adjusted for clustering at the Industry level.

Among the control variables, regulation index is not significant. Export dummy takes positive and significant coefficients in the regressions of $TFPRGAP_{sit}$ and $\tau_{Y sit}$, as is expected, while it takes a negative coefficient in the regression of $\tau_{K sit}$. Size (log of number of employees) takes significantly positive coefficients in the regressions of $TFPRGAP_{sit}$ and $\tau_{Y sit}$, while it takes a negative coefficient in the regression of $\tau_{K sit}$. The latter result suggests that a smaller firm tends to incur a higher distortion on capital. This empirical result is also consistent with many preceding studies, though we should be careful that plant-size is potentially an endogenous variable affected by the degree of financial frictions. Self-employed company dummy takes significant coefficients with opposite signs to those of size. Corporation dummy takes a positive coefficient in the regression of $TFPRGAP_{sit}$. All worker age group shares above 20s take positive and significant coefficients in the regressions of $TFPRGAP_{sit}$ and $\tau_{Y sit}$, suggesting that the share of teenagers have negative effects on output distortions, as is expected.Finally, the share of part-timers does not take a significant coefficient in any regressions.

In sum, most of the control variables take coefficients with expected signs, suggesting that our measure of distortions is not just a figment of measurement errors. Importantly, our results strongly suggest that external finance dependence tends to increase capital distortions.

4 Dynamic Model

This section provides a dynamic monopolistic competition model of heterogeneous entrepreneurs that are subject to idiosyncratic productivity shocks and collateral constraint, but can accumulate savings.

4.1 Setup

The economy is populated by a continuum of final good producers, entrepreneurs who produce differentiated goods, and workers.

4.1.1 Final good producers

Final good produces are born at the beginning of each period and die at the end of each period. The final good producer operates in a perfectly competitive good market and produces a single final good Y using a constant-returns-to-scale technology from intermediated goods. Let $i \in [0,1]$ be an index of an intermediated good, as well as the entrepreneur producing it. The production function of the final good is

$$Y_t = \left(\int Y_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1 \tag{4.1}$$

4.1.2 Entrepreneurs

Entrepreneurs live indefinitely. Each entrepreneur i has an objective given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma} \tag{4.2}$$

, where C_{it} is consumption of the final good. Each entrepreneur owns a private firm which uses K_{it} units of capital and L_{it} units of labor to produce

$$Y_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha} \tag{4.3}$$

units of the intermediate good, where $\alpha \in (0.1)$. Each entrepreneur operates in a monopolistic market and sets the price given the demand from the final good producer. The log of productivity, $a_{it} = log(A_{it})$, follows a continuous-state Markov process with transition density

$$Pr(a_{it+1} = a' | a_{it} = a) = \phi(a'|a) \tag{4.4}$$

We assume that agents cannot access to intertemporal borrowing or state-contingent claims. Agents can store final goods across periods and can rent capital in a rental market each period. Let B_{it} denote the entrepreneur's assets at the beginning of period t, expressed in terms of the final good. The entrepreneur rents K_{it} before production takes place. We assume that the amount the entrepreneur can rent is limited by a collateral constraint:

$$K_{it} \leq \lambda B_{it}, \qquad \lambda \geq 1 \tag{4.5}$$

This constraint is analytically convenient and naturally arises from a limited enforcement problem[14]. If $\lambda = 1$, the entrepreneur cannot borrow externally, while if $\lambda = \infty$, the entrepreneur faces no intratemporal borrowing constraint.

The entrepreneur decides how much to consume C_{it} and to save B_{it+1} subject to its budget constraint:

$$C_{it} + B_{it+1} = P_{it}Y_{it} - RK_{it} - wL_{it} + (1 + r)B_{it} \tag{4.6}$$

where R is rental rate of capital and r is net return to savings. Let δ denote the depreciation rate. Competitive rental market implies that $R = r + \delta$. The amount available for consumption and saving is equal to profit (that is, revenue net of wage bills and capital rents) plus return to savings[15]. Since we focus on the steady state equilibrium where aggregate variables are constant across time, we do not index aggregate prices, w, R or r by t here and below. We summarize the time line.

t ———————————————————————————— ▶ $t + 1$

Own savings	Rent capital	Produce	Pay rents and wage	Consume	Save
B_{it}	K_{it}	Y_{it}	$RK_{it} + wL_{it}$	C_{it}	B_{it+1}

4.1.3 Workers

There is a unit mass of workers and each worker provides L efficiency units of labor inelastically. Following Moll (2011), we assume that workers cannot save so that they immediately consume their labor income.[16]

[14]The collateral constraint arises when the borrower can steal a fraction $1/\lambda$ of rented capital K_{it}, but would lose his asset B_{it} as a punishment if he did so. See Moll (26) and Buera et al. (5)

[15]The entrepreneur supplies B_{it} and demands K_{it} at the rental market. Since B_{it} depreciates at the rate of δ, the beginning-of-period asset B_{it} evolves to $(1 + R - \delta)B_{it} = (1 + r)B_{it}$ at the end of period.

[16]We can alternatively assume that workers can save but cannot borrow. In that case, whenever the interest rate is lower than their subjective discount rate, workers continue to dissave until they have zero asset. In the long-run equilibrium, workers are in effect hand-to-mouth consumers as we assume in the text (see, e.g., Moll (2011).

4.2 Individual Behavior

4.2.1 Final good producers

Final good producers' cost minimization implies

$$P_i = PY^{\frac{1}{\sigma}} Y_i^{\frac{-1}{\sigma}}, \tag{4.7}$$

where P is the marginal cost, and, in a competitive market, equals to the final good price, which we normalize to one (as a numeraire):

$$P = \left(\int P_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = 1. \tag{4.8}$$

4.2.2 Entrepreneurs

Since debt is intratemporal, we can decompose the entrepreneur's problem into 1) the static profit maximization problem to choose K_{it} and L_{it} given B_{it}, and 2) the dynamic problem to choose C_{it} and B_{it+1}.

The profit maximization problem reduces to

$$\Pi(B_{it}, A_{it}) = \max_{K_{it}, L_{it}} \; P_{it}Y_{it} - (r+\delta)K_{it} - wL_{it} \tag{4.9}$$

subject to the production function (4.3), the demand function (4.7), and collateral constraint (4.5).

The demand function implies that σ is the price elasticity and that the mark-up ratio is $\frac{\sigma}{\sigma-1}$. Let μ_{it} denote the Lagrange multiplier on the collateral constraint. The first order conditions lead to

$$MRPL_{it} = P_{it} \left(1 - \frac{1}{\sigma}\right) F_L = w \tag{4.10}$$

and

$$MRPK_{it} = P_{it} \left(1 - \frac{1}{\sigma}\right) F_K = \delta + r + \mu_{it}, \tag{4.11}$$

where $MRPL_{si} \equiv \frac{\partial(P_{si}Y_{si})}{\partial L_{si}}$ and $MRPK_{si} \equiv \frac{\partial(P_{si}Y_{si})}{\partial K_{si}}$ denote marginal revenue products of capital and labor, respectively, and F_L and F_K denote marginal products of capital and labor, respectively. At the optimum, $MRPL$ is equal to wage, while $MRPK$ is eqaul to rental cost plus the shadow cost μ_{it}. Defining the capital wedge as $\tau_{Kit} \equiv \frac{\mu_{it}}{r+\delta}$,

$$\tau_{it} = \begin{cases} 0 & if \quad \lambda b_{it} \geq \bar{b}(A_{it}) \\ \frac{\alpha}{r+\delta} \left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\frac{w}{1-\alpha}\right)^{(1-\alpha)(1-\sigma)} (\lambda b_i)^{-1} A_{it}^{\sigma-1} \right]^{\frac{1}{1-\alpha(1-\sigma)}} - 1 & otherwise. \end{cases} \tag{4.12}$$

, where $b_{it} = \frac{B_{it}}{Y}$ and

$$\bar{b}(A_{it}) = A_{it}^{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\frac{\delta+r}{\alpha}\right)^{\alpha(1-\sigma)-1} \left(\frac{w}{1-\alpha}\right)^{(1-\alpha)(1-\sigma)} \tag{4.13}$$

Panel (a) in Figure 5 shows the optimal capital k against the initial savings b, both of which are standardized by aggregate output, for some values of A. The static optimization policy suggests that optimal capital level depends on the beginning-of-period savings for firms with higher productivity and low savings. For these firms, the collateral constraint is likely to bind and the capital distortion tends to be larger.

Due to the presence of capital wedge τ_{Kit}, total factor revenue productivity, $TFPR_{it} \equiv P_{si}A_{si}$ differs across plants.

$$TFPR_{it} = \frac{\sigma}{\sigma-1}\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}(1+\tau_{Kit})^{\alpha} \tag{4.14}$$

We turn to the entrepreneur's dynamic problem, which can be written recursively as

$$\mathbf{V}(B,a) = max\frac{C^{1-\gamma}}{1-\gamma} + \beta\int \mathbf{V}(B',a')\phi(a'|a)da' \tag{4.15}$$

subject to the budget constraint

$$C = (1+r)B + \Pi(B,a) - B' \tag{4.16}$$

Using the lower case as a variable standardized by Y (say, $c = \frac{C}{Y}$), we can rewrite this problem as the standardized problem:

$$V(b,a) = max\frac{c^{1-\gamma}}{1-\gamma} + \beta\int V(b',a')\phi(a'|a)da' \tag{4.17}$$

subject to the budget constraint

$$c = (1+r)b + \pi(b,a) - b' \tag{4.18}$$

The optimal savings decision satisfies

$$c^{-\gamma} = \beta\int c'^{-\gamma}\left(1+r+\lambda\mu(b',a')\right)\phi(a'|a)da' \tag{4.19}$$

, where we used the envelope condition of the static problem, $\pi_1(b,a) = \lambda\mu(b,a)$. If $\beta < \frac{1}{1+r}$ (, which turns out to be the case in the steady state equilibrium), the entrepreneur wants to consume more and save less. But then, the collateral constraint may bind in the future ($\mu' > 0$). The entrepreneur decides the optimal saving taking into consideration this trade-off. Panel (b) in Figure 5 shows the optimal b'/b against b for some values of a. As the current productivity (and hence expected future productivity) is higher and as the current savings is lower, the entrepreneur is willing to save more.

4.3 Steady State Equilibrium

An equilibrium is the aggregate prices (P_t, r_t, w_t), the distribution of P_{it}, and the corresponding quantities such that i) all agents maximize their objectives subject to the relevant constraints given the aggregate prices, ii) P_t is consistent with the aggregation of entrepreneurs' price setting P_{it}, and $P_t = 1$, and iii) the capital and labor markets clear.

The capital and labor market clearing conditions are, respectively,

$$\int K_{it}di = \int B_{it}di \quad for\ all\ t \tag{4.20}$$

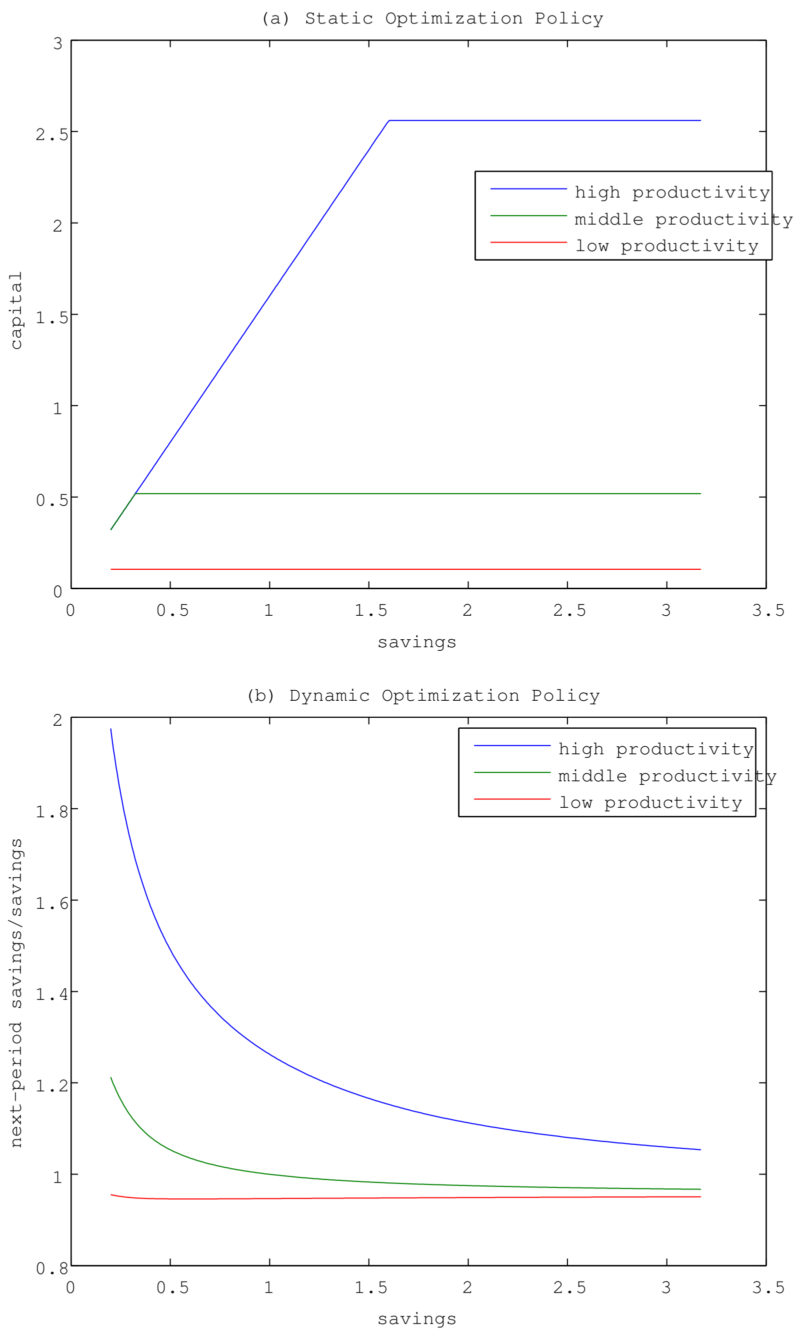

Figure 5: **Entrepreneur's Optimal Policy**

and

$$\int L_{it} di = L \quad for\ all\ t \tag{4.21}$$

We focus on the steady state equilibrium in which all aggregate prices and quantities are constant across periods.

We can obtain the relationships among aggregate prices and quantities. First, aggregating the entrepreneur's factor demands (4.10) and (4.11), and using the aggregate price equation (4.8) and the labor market clearing conditions, (4.21), we obtain the steady-state equilibrium output and wage:

$$Y = \left(\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\alpha}{r+\delta}\right)\right)^{\frac{\alpha}{1-\alpha}} \left(\int \left(\frac{A_i}{(1+\tau_{Ki})^{\alpha}}\right)^{\sigma-1} di\right)^{\frac{1}{(1-\alpha)(\sigma-1)}} L \tag{4.22}$$

and

(4.23)

$$w = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) \left(\int \left(\frac{A_i}{(1+\tau_{Ki})^{\alpha}}\right)^{\sigma-1} di\right)^{\frac{1}{(1-\alpha)(\sigma-1)}},$$

, given the steady-state equilibrium value of r and the distribution of τ_{Ki}, which, in turn, is determined by the distribution of b_i derived from the dynamic savings decision (4.19).

We can also obtain aggregate capital,

(4.24)

$$K = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\alpha}} \left(\frac{\rho+\delta}{\alpha}\right)^{\frac{-1}{1-\alpha}} \left(\int \left(\frac{A_i}{(1+\tau_{Ki})^{\alpha}}\right)^{\sigma-1} di\right)^{\frac{1}{(1-\alpha)(\sigma-1)}-1} \left(\int \frac{A_i^{\sigma-1}}{(1+\tau_{Ki})^{\alpha(\sigma-1)+1}} di\right) L$$

In our model, Y and K are linear in L, reflecting the constant-returns-to-scale technology.

Aggregate TFP and TFP gap can be derived by the same procedure in the static model developed in Section 1. The only difference is that $\tau_{Yi} = 0$ here.

$$TFP = \frac{\left[\int \left(\frac{A_i}{(1+\tau_{Ki})^{\alpha}}\right)^{\sigma-1} di\right]^{\alpha+\frac{1}{\sigma-1}}}{\left[\int \left(\frac{A_i^{\sigma-1}}{(1+\tau_{Ki})^{\alpha(\sigma-1)+1}}\right) di\right]^{\alpha}} \tag{4.25}$$

and

$$TFPGAP \equiv \frac{TFP}{\overline{A}}, \tag{4.26}$$

where $\overline{A} = \left(\int A_i^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}$ is the efficient TFP level.

5 Quantitative Analysis

In this section we study a quantitative version of the dynamic model parameterized to fit the plant-level facts.

5.1 Parameterization

5.1.1 Assigned Parameters

The period is one year. We set the intertemporal elasticity of substitution, γ, equal to 1. We set the standard production function parameters: the depreciation rate of capital is set at $\delta = 0.06$ and the share of capital equal to $\alpha = 0.33$. The elasticity of substitution between a pair of differentiated goods is set at $\sigma = 3$, following HK. Later we see the sensitivity of our results to the choice of σ. We set $L = 1$ as normalization.

5.1.2 Productivity Parameters

We assume that $a_{it} = log(A_{it})$ is the sum of two components,

$$a_{it} = Z_i + \tilde{a}_{it} \tag{5.1}$$

where Z_i is a permanent productivity component and $\tilde{a}_{it}$ is a transitory productivity shock. $exp(Z_i)$ is distributed according to a Pareto with an upper bound H and a lower bound of unity as a normalization. Denoting the shape parameter by μ, CDF is

$$F_{expZ}(x) = \frac{1 - x^{-\mu}}{1 - H^{-\mu}} \tag{5.2}$$

The transitory productivity component $\tilde{a}_{it}$ is an AR(1) process,

$$\tilde{a}_{it} = \rho_{\tilde{a}}\tilde{a}_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma^2) \tag{5.3}$$

Combining (5.1) and (5.3), we obtain

$$a_{it} = \rho_{\tilde{a}} a_{it-1} + (1 - \rho_{\tilde{a}})Z_i + \epsilon_{it} \tag{5.4}$$

, which suggests that a_{it} follows an AR(1) process with a plant-level fixed effect. So, we estimate $\rho_{\tilde{a}}$ using a dynamic panel GMM estimator of Arellano and Bond (1991). We use the first-step estimator using a_{it-s}, where $s \geq 2$, as instruments. We confirmed that our instruments are valid using the test for autocorrelation and Sargan test of overidentifying restrictions. To construct the estimate of Z_i, we note that $E[a_{it}] = Z_i$ from (5.4), where $E[\cdot]$ denotes the expectation over t. A natural estimate of Z_i is the average of a_{it} over time. Using the estimates of $\rho_{\tilde{a}}$ and Z_i, we construct the estimates of ϵ from (5.4) and use its standard deviation as the estimate of σ.

Figure 6 shows the distribution of the estimated Z_i, suggesting that Z_i is distributed Pareto beyond some threshold.[17] As we focus below on the distribution of plants with a relatively large size, we do not specify the distribution Z_i for very low levels. Instead, to estimate μ, we trim the low values of $\hat{Z}_i$ and apply the Hill (maximum likelihood) estimator (Hill, (17)) to the trimmed sample set. We set the threshold at a relatively low level to be conservative; with a low threshold, the $\hat{\mu}$ becomes lower, and TFP loss turns out to be lower. Finally, we calculate $H = exp(max(Z_i) - min(Z_i))$ for the trimmed sample set.

GMM estimator yields $\rho_{\tilde{a}} = 0.691$. [18] We trimmed the bottom 1% ($Z_i < 7.028$) and obtained $mu = 3.106$, $\sigma = 0.479$, and $H = exp(15.777 - 7.028)$. [19]

[17] Watanabe et al. (34) also document that total factor productivity of Japanese firms is distributed Pareto beyond some threshold, though they do not distinguish a permanent component from a transitory one.

[18] We confirmed that this value does not virtually change whether we use the whole sample set or the trimmed sample set.

[19] Simulated TFPGAP is not sensitive to the exact proportion of Z_i we trim. If we alternatively

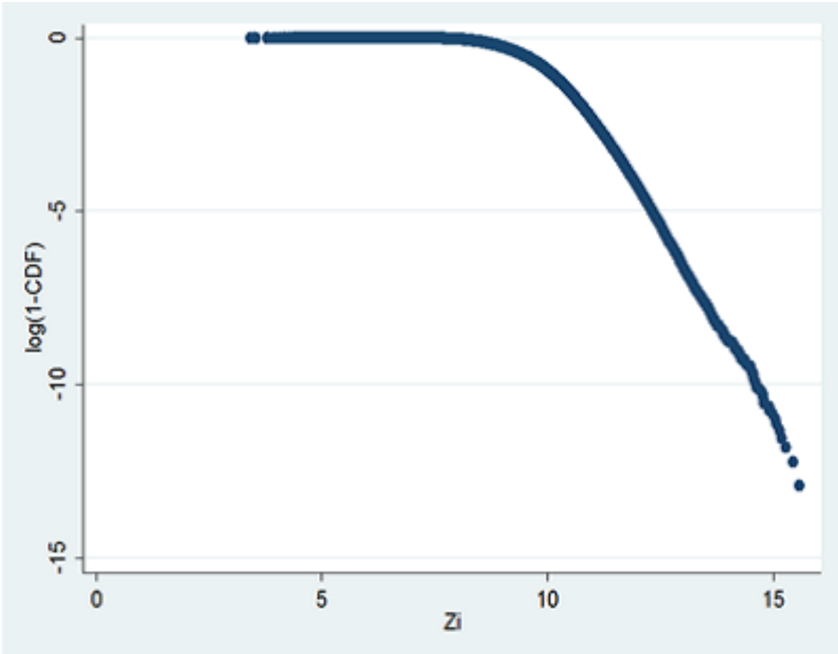

Figure 6: **Distribution of Z_i**

5.1.3 Discount Factor β and Borrowing Constraint Parameter λ

We set the discount factor β so that $r = 4\%$ in the steady-state equilibrium. Actually, in equilibrium, interest rate depends crucially on the borrowing constraint λ as well as β. As is explained below, we set λ to replicate as closely as the actual plant-size distribution. So, we set the combination of β and λ that achieve $r = 4\%$ and replicate the actual plant-size distribution as closely as they can. We find that $\beta = 0.862$ satisfies those conditions.[20]

To set the collateral constraint parameter λ, we use the information on plant size distribution. Since our dynamic model does not account for output distortions, we set λ to match the plant size distribution that would be achieved without output distortions (Table 3). Specifically, we choose λ to match output shares of very large firms, top 0.01%, top 0.1%, and top 1%, since we find that they are sensitive to λ. Moll (26) sets the borrowing constraint parameter to match aggregate debt-to-GDP ratio, while Midrigan and Xu (25) use both aggregate debt-to-GDP ratio and plant size distribution. However, we think that aggregate debt is not usable to pin down the borrowing constraint parameter for several

trim the bottom 2% ($Z_i < 7.812$), we obtain $\mu = 4.434$ and $H = exp(15.777 - 7.812)$. Resetting β and λ at 0.86 and 1.5, respectively, following the procedure described in the next subsection, we find that TFPGAIN is 0.104, close to the baseline TFPGAP, 0.113. On the other hand, simulated plant-size distribution is sensitive to the specific proportion of Z_i we trim. In the case of the bottom 2% trimming, RMSE defined by (5.5) is 0.726, suggesting a much worse fit than the baseline case of 0.051. Some recent studies propose methods to estimate the lower bound (Clauset et al.(10) and Malevregne et al.(24)). We leave the application of these methods to our data set to the future work.

[20]Though we do not explicitly model the entrepreneur's death or the firm's failure, we may interpret a relatively low β as the subjective discount factor multiplied by a survival rate.

reasons. First, aggregate debt includes loans to households as well as loans to firms. Second, though our model can account for only net debt, firms usually hold financial assets as well as debt. Finally, though firms often issue equity to raise external funds, we consider only a simple debt, and hence it is difficult to match the external finance in our model to actual debt data.

Let Γ_i^d represent the fraction of output of the largest $X(i)\%$, where $X(1) = 0.01\%$, $X(2) = 0.1\%$, and $X(3) = 1\%$, respectively. Let $\Gamma_i(\lambda)$ denote the simulation counterparts for a value of λ. We choose λ so that the following root mean square error is minimized,

$$RMSE = \left(\frac{1}{3}\sum_{i=1}^{3}\left(ln(\Gamma_i^d) - ln(\Gamma_i(\lambda))\right)^2\right)^{\frac{1}{2}} \tag{5.5}$$

We find that RMSE is minimized at $\lambda = 1.3$.

Table 6 summarizes the parameters we use in the benchmark simulation.

Table 6. Parameters

beta	Discount factor	0.862
gamma	Relative risk aversion	1
alpha	Capital share	0.33
sigma	Elasticity of substitution between differentiated goods	3
delta	Depriciation rate	0.06
lambda	Borrowing constraint	1.3
rho_atilda	Serial correlation of productivity	0.691
sigma_atilda	Standard deviation in AR(1) process of productivity	0.479
mu	Shape parameter of permanet productivity	3.106
H	Upper bound of permanent productivity	exp(8.749)

5.2 Solution Method

From the standardized static problem, (4.12), we see that the distribution of τ_{Ki} depends on the distribution of b_i and factor prices r and w. On the other hand, from the standardized dynamic problem (4.19), we see that the distribution of b_i depends on τ_{Ki} (or μ_{it}). We adopt the following steps to solve for the equilibrium.[21]

1. Set the initial (r, w) at arbitrary values.

2. Solve for τ_{Ki} at any values of b_i from the normalized static problem and then solve for the distribution of b_i from the dynamic problem.

3. Calculate the right hand side of (4.23) and check whether they are equal to the initially given w. Aggregate K_i and B_i and check whether the good market condition (4.20) is satisfied.

[21] To solve for the dynamic optimization, we use the method of endogenous grid points described by Caroll (8). We constructed a discrete approximation to the Pareto distribution of Z_i by setting 51 equiprobable grid points. To construct a discrete approximation to AR(1) process of $\tilde{a}_{it}$, we followed Tauchen and Hussey (33) and set 11 grid points. We simulated for 1000 firms and 300 periods. We discarded the initial 200 periods and averaged over the last 100 periods to obtain the steady-state aggregate variables.

4. If either of the labor or goods market clearing conditions is not satisfied, reset (r, w) and repeat steps 2 and 3 until both conditions are satisfied.

5.3 Productivity and Size Distribution in the Benchmark Economy

Table 7 shows the benchmark simulation results. RMSE for the shares of output of the largest 0.01%, 0.1%, and 1% is 5.1%, suggesting a good fit to the hypothetical data for $\tau_{Ksi} = 0$. The shares of output of the largest 5%, 10%, and 20 are also close to the hypothetical data, though we do not try to match those moments. RMSE for the size distribution of the largest 0.01% thorough 20% is 4.7%.

TFP gap is 0.899, meaning that TFP would increase by 11.3% if borrowing constraint were removed. This TFP gain accounts for 48.1% of measured TFP losses caused by capital distortions, and 23.9% of measured TFP losses caused by both capital and output distortions. The proportion of entrepreneurs who are subject to binding borrowing constraints is 62.6%. Median value of the premium or the shadow cost of capital (μ_{it} in (4.11)) among constrained entrepreneurs is 9.6%.

5.4 Counterfactual Experiments

We conduct counterfactual experiments in which we vary the collateral constraint parameter λ and hold all other parameters constant: no-external finance economy ($\lambda = 1$), an economy with fairly developed financial markets ($\lambda = 10$), and an economy with virtually no financial constraint ($\lambda = 100$).

If no external finance were allowed ($\lambda = 1$), TFP would decrease by 3.5% from the benchmark case. Labor productivity would decrease more, by 5.6%. As the external finance constraint tightens, demand for capital decreases, which, in turn, decreases interest rate, savings, and capital per labor. By endogenizing the interest rate, we can capture significant effect of financial constraint on capital accumulation and labor productivity through interest rate. [22]

Looking at the other extreme counterfactual experiment, an economy with virtually no borrowing constraint ($\lambda = 100$), we see that TFP and labor productivity would increase by 11.2% and by 19.3%, respectively, from the benchmark economy.

The economy with $\lambda = 10$ is close to the extreme case with $\lambda = 100$: TFP and labor productivity would increase by 10.2% and 17.8%, respectively, from the benchmark economy.

Figure 7 shows the distributions of output in Panel A and τ_{Ki} in Panel B for the benchmark and no-borrowing-constraint cases.

As we relax the borrowing constraint (i.e., increase λ) from the benchmark economy, output becomes more dispersed in terms of interquartile range (i.e., the difference between the upper and lower quartiles): it monotonically increases from 2.568 in case of no external finance ($\lambda = 1$) to 3.091 in the no-constraint case ($\lambda = 100$). [23] This suggests that the effect of the decrease in the correlation between capital distortions and TFPQ outweighs the effect of the decrease in the variance of capital distortions (see section 2.4). The former effect is intuitive because borrowing constraint is likely to bind productive firms and thus

[22] Midrigan and Xu (25) analyzes a small open economy with an exogenous interest rate.

[23] We cannot directly compare the interquartile ranges of the benchmark economy with the actual data (or the hypothetical data with $\tau_Y = 0$), since we use some normalization in the simulation.

Table 7. Simulation Results

	Hypothetical Data	Simulation Data
	tau_Y=0	Benchmark
lambda		1.3
Factor Prices		
Interest rate		0.04
Wage		1.020
Size distribution		
fraction of y largest 0.01%	0.203	0.207
fraction of y largest 0.1%	0.460	0.443
fraction of y largest 1%	0.744	0.688
fraction of Y largest 5%	0.893	0.842
fraction of Y largest 10%	0.935	0.899
fraction of Y largest 20%	0.967	0.948
fraction of y smallest 20%	0.0010	0.0010
RMSE(0.01%-0.1%-1%)		0.051
RMSE(0.01%-0.1%-1%-5%_10%_20%)		0.047
Aggregate Productivity		
TFPGAP	0.810	0.899
TFP GAIN	0.234	0.113
Ratio to Hypothetical Data (tau_Y=0)	(1.000)	0.481
Labor Productiviy		3.439
External Finance		
Fraction of Financially Constrained Plants		0.626
Median premium if constrained		0.096

Table 8. Counterfactual Experiments

	Simulation Data			
	Benchmark	No external Finance	Developed Financial	Virtually no Constraint
lambda	1.3	1	10	100
Factor Prices				
Interest rate	0.040	0.022	0.087	0.093
Wage	1.020	0.980	1.137	1.146
Size distribution				
Interquartile range of log(Y)	2.707	2.568	3.049	3.091
Fraction of y largest 0.01%	0.207	0.239	0.132	0.133
Fraction of y largest 0.1%	0.443	0.501	0.364	0.367
Fraction of y largest 1%	0.688	0.715	0.650	0.656
Fraction of Y largest 5%	0.842	0.854	0.843	0.846
Fraction of Y largest 10%	0.899	0.906	0.909	0.912
Fraction of Y largest 20%	0.948	0.950	0.958	0.960
Fraction of y smallest 20%	0.0010	0.0011	0.0006	0.0005
RMSE(0.01%-0.1%-1%)	0.051	0.109	0.292	0.284
RMSE(0.01%_0.1%_1%_5%_10%_20%)	0.047	0.080	0.208	0.202
Aggregate Productivity				
TFPGAP	0.899	0.868	0.990	0.999
% Change from Benchmark	0.0%	-3.5%	10.2%	11.2%
TFP GAIN	0.113	0.153	0.010	0.001
Ratio to Hypothetical Data (tau_Y=0)	0.481	0.652	0.043	0.003
% Change from Benchmark	0.0%	35.5%	-91.1%	-99.4%
Labor Productiviy	3.439	3.245	4.053	4.103
% Change from Benchmark	0.0%	-5.6%	17.8%	19.3%
External Finance				
Fraction of Financially Constrained Plants	0.626	0.792	0.055	0.001
Median premium if constrained	0.096	0.117	0.063	0.095

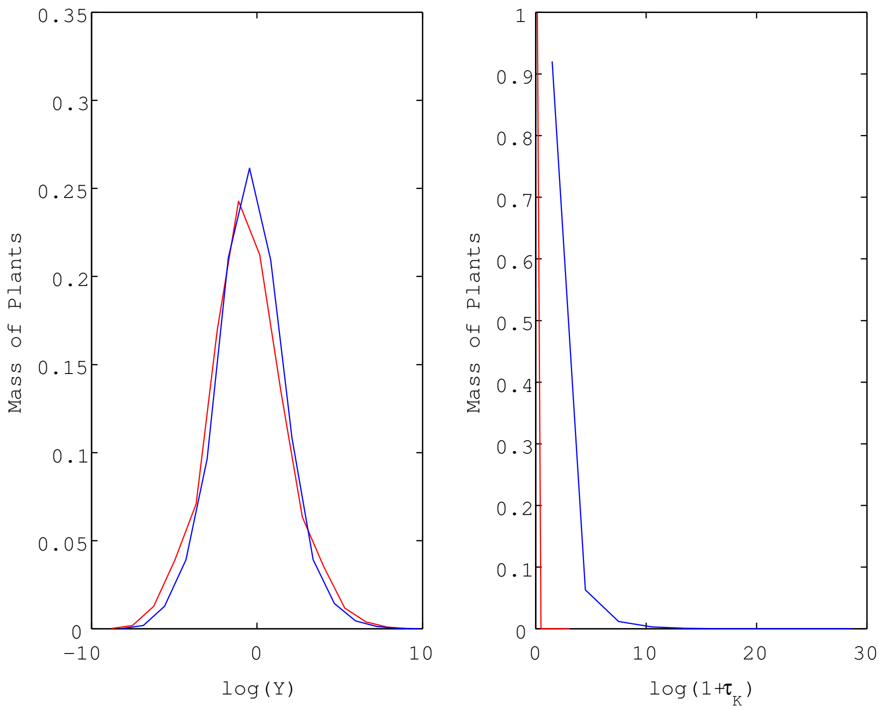

Figure 7: **Distribution of $log(Y_i)$ and $log(1 + \tau_{Ki})$ for simulated data. The blue line is the benchmark economy ($\lambda = 1.3$) and the red line is the no-financial-constraint economy ($\lambda = 100$).**

to increase the correlation between firm-level TFP and distortions on capital, resulting in a lower dispersion of output.[24]

However, looking the details of output distribution, we find that as borrowing constraint is relaxed, the output shares of largest plants (top 0.01%, 0.1%, and 1%) and smallest plants (bottom 20%) decrease. This result is not consistent with our finding in Section 2 that removing capital distortion would decrease the share of smallest plants but unambiguously increase the share of largest plants up to top 20%. This result suggests that borrowing constraint is a significant, but not the sole dominant factor for the distortion of the plant-size distribution.

5.5 Quantitative Comparison with Preceding Studies

Midrigan and Xu (25) report a very small effect of borrowing constraint on aggregate TFP, while Moll (26) argue that financial frictions have a substantial adverse impact on aggregate TFP. For example, in an economy with no external finance, Midrigan and Xu reports TFP losses of 5-7%, while Moll reports 20%. Our results lie between the two, 15.3%.

One of the key factors that cause quantitative differences lies in the stochastic process of idiosyncratic productivity shocks. As the productivity is more persistent (with a high $tildea$), productive entrepreneurs can grow out of financial constraint by accumulating savings while they remain productive. Similarly, given the unconditional variance of technology shocks ($\frac{\sigma^2}{(1-\rho_{\tilde{a}})^2}$) a lower standard deviation of changes in productivity enables productive entrepreneurs to accumulate savings and to self-finance.[25]

To qualitatively evaluate how the persistency of productivity shocks and affect TFPLOSS, we set $\rho_{\tilde{a}} = 0.92$, the value used by Midrigan and Xu, and $\sigma = 0.26$ to keep the unconditional variance of $\tilde{a}$. Resetting β to make the equilibrium interest rate 0.04 ($\beta = 0.9086$) while keeping λ at the baseline case ($\lambda = 1.3$), we find that TFPGAIN is 4.1%, less than half of the baseline case (11.3%). Thus, how to accurately set parameters of technology shocks is essential in quantifying the effects of financial frictions. We have exploited a rich data set of manufacturers that enable us to estimate the persistency and standard deviation of changes in productivity.

Model specification also affects the quantitative results. Midrigan and Xu assumes a small open economy where interest rate is exogenous. In contrast, this paper, like Moll, endogenizes the interest rate from capital market clearing. As the borrowing constraint becomes more stringent, the equilibrium interest rate tends to be lower, resulting in the expansion of less productive firms and hence large TFP losses.

To quantify the general equilibrium effect through factor prices, we compare a small open economy with the closed economy in the case of no external finance. Specifically, we set all the parameters at the no-external-finance case ($\lambda = 1$) and conduct a simulation

[24]Buera, Kaboski and Shin (5) show that financial frictions *increase* the dispersion of the establishment-level productivity and size because talented-but-poor individuals delay entry and incompetent-but-rich entrepreneurs remain in business for longer, which is in contrast with our model experiments. Though their result is not consistent with our hypothetical data without capital distortions, we need to see how the implications for size distribution change by incorporating entry/exit margins into our model.

[25]Midrigan and Xu set the persistency parameter ($\rho_{\tilde{a}}$) at a relatively high value (0.92), while Moll set it at a relatively low value (0.8), and we set it at an even lower value of 0.691 than Moll. Though Moll's specification of technology is different from ours, he seems to set the variability of changes in productivity at a high value relative to our settings. Foster et al. (14) estimated the US firm-level persistency parameter to be 0.86.

while keeping the wage and interest rate at the baseline closed economy values ($w = 1.02$ and $r = 0.04$). We find that TFPGAIN is 13.0 %, which is smaller than the TFPGAIN in the no-external-finance closed economy (15.3%).

5.6 Sensitivity Analysis

5.6.1 Product Substitutability

As we discussed above, our choice of the product substitutability parameter $\sigma = 3$ might be low. So, to check the sensitivity of our result to this parameter, here we set it to a rather large value, $\sigma = 6$. We reset the productivity parameters $\rho_{\tilde{a}}$, σ, μ and H, discount factor β, and the collateral constraint parameter λ using the newly estimated A_{si}[26], and the hypothetical plant-size distribution when $\tau_Y = 0$.

The new parameters and simulation results are shown by Panels A and B of Table 9. We find that in this case, since product substitutability is high, the shares of largest plants in the simulation data become very large relative to the hypothetical share without output distortions at any value of the borrowing constraint parameter. Even without external finance ($\lambda = 1$), the share of top 0.01%, for example, is 39.8%, which is far from the counterpart of the hypothetical data (14.6%). If we set λ at a larger value, the plant-size distribution becomes more different from the hypothetical data. Since the share of largest firms is high, the demand for capital is also large. To keep the equilibrium interest rate at a moderate 4%, we have to assume a high discount factor, $\beta = 0.962$. As a result, the proportion of financially constrained firms becomes as low as 4.2% and TFP gain from removing borrowing constraint is now as small as 2.6%.

We consider this simulation to be unrealistic given the very different plant-size distributions of the simulation data from the hypothetical data without output distortions.

Table 9. Simulation Parameters and Results when product substitutability is high.

A. Parameters

beta	Discount factor	0.962
gamma	Relative risk aversion	1
alpha	Capital share	0.33
sigma	Elasticity of substitution between different	6
delta	Depriciation rate	0.06
lambda	Borrowing constraint	1
rho_atilda	Serial correlation of productivity	0.577
sigma_atil	Standard deviation in AR(1) process of pr(	0.457
mu	Shape parameter of permanet productivity	3.241
H	Upper bound of permanent productivity	exp(10.706)

[26] In Table 9, we report the simulation results when we set μ and H using the sample whose bottom 2% is trimmed because we find that Z_i seems to be distributed Pareto for a relatively large value. But the results do not change virtually if we trim the bottom 1% of the sample.

Table 9. Simulation Parameters and Results when product substitutability is high.
B. Simulation Ressults

	Hypothetical Data	Simulation Data
	tau_Y=0	
lambda		1
Factor Prices		
Interest rate		0.04
Wage		5.21
Size distribution		
fraction of y largest 0.01%	0.146	0.398
fraction of y largest 0.1%	0.353	0.862
fraction of y largest 1%	0.624	0.988
fraction of Y largest 5%	0.797	0.998
fraction of Y largest 10%	0.864	0.999
fraction of Y largest 20%	0.919	1.000
fraction of y smallest 20%	0.004	0.000
RMSE(0.01%-0.1%-1%)		0.818
RMSE(0.01%-0.1%-1%-5%_10%_20%)		0.590
Aggregate Productivity		
TFPGAP	0.844	0.975
TFP GAIN	0.185	0.026
Ratio to Hypothetical Data (tau_Y=0)		0.140
Labor Productiviy		13.507
External Finance		
Fraction of Financially Constrained Plants		0.042
Median premium if constrained		0.043

6 Conclusion

We have evaluated the quantitative effects of borrowing constraint on aggregate TFP and plant-size distribution using a dynamic model of monopolistic competition. The model, when parameterized to account for plant-size distribution, predicts that aggregate TFP would be higher by 11.3% if there were no borrowing constraint, which accounts for 48.1% of measured TFP losses caused by capital distortions, and 23.9% of measured TFP losses caused by both capital and output distortions. Labor productivity would be even higher, by 19.3%, without borrowing constraint.

There are, however, still some work to be done to understand the full effects of financial frictions on misallocation. A potentially important factor that our model is missing is the entry/exit margins. Buera, Kaboski and Shin (5) show that financial frictions significantly distort the entry and exit of entrepreneurs and thus lower aggregate TFP while Midrigan and Xu (25) find that allowing for entry/exit margins do not significantly lower TFP if new entrants receive an endowment that depends on their ability. We leave quantifying the effects of endogenous entry/exit margins using our long-run data set to a future work.

Our counterfactual experiments show that relaxing borrowing constraint makes firm-size distribution more dispersed, which is consistent with our hypothetical data with no capital distortions. Looking into more details, however, we find that relaxing borrowing constraint *decreases* the shares of largest and smallest plants while simply removing capital distortions decreases the share of smallest plants but *increases* the share of largest plants. This is a puzzling result, suggesting that other sources of distortions are dominant factors determining the distortion of the size distribution.

We have analyzed a particular type of friction: borrowing constraint. Subsidized credit, including credit guarantee and loans at low interest rates provided by government-affiliated financial institutions to small and medium-sized firms, is a natural candidate of capital distortions that distort the plant-size distribution[27]. We leave the effect of this policy-induced distortion to a future research.

Finally, we allow for only one kind of capital that can be pledged as collateral. In reality, however, only tangible capital may be be pledged as collateral while intangible capital may not. Considering a significant contribution of intangible capital to production (e.g., McGrattan and Prescot (23)), allowing for two kinds of capital of which different proportions can be pledged as collateral may improve the accuracy of our results.

[27] Other types of regulations and subsidies that depend on establishment size may also quantitatively important to account for the size distribution (Guner et al. (16))

Data Appendix. Measurement of Plant-Level Output and Input

The data source is the Census of Manufactures conducted by the Ministry of Economy, Trade and Industry (METI). We used data for the period of 1981-2008. To measure TFP at the plant level, we construct the data of output and factor inputs as follows.

Gross Output is measured as the sum of shipments, revenues from repairing and fixing services, and revenues from performing subcontracted work. Gross output is deflated by the output deflator taken from the Japan Industrial Productivity Database (JIP) 2010 and converted to values in constant prices of 2000.

Intermediate Input is defined as the sum of raw materials, fuel, electricity and subcontracting expenses for consigned production used by the plant. Using Corporate Good Price Index (CGPI) published by Bank of Japan, intermediate input is converted to values in constant prices of 2000.

Value Added $(P_{si}Y_{si})$ is defined as the difference between gross output and intermediate input.

Capital Input (K_{si}) is measured as real capital stock, defined as follows:

Capital Input (K_{si}) = The nominal book value of tangible fixed assets from the Census of Manufactures x The book-to-market value ratio for each industry (γ_{st}).

The book-to-market value ratio for each industry(γ_{st}) is calculated using the industry-level data of real capital stock(K_{st}^{JIP}) and real value added(Y_{st}^{JIP}) taken from JIP database as follows,

$$\frac{Y_{st}^{JIP}}{K_{st}^{JIP}} = \frac{\sum_{i\in s} Y_{sit}^{CM}}{\sum_{i\in s} BVK_{sit}^{CM} \times \gamma_{st}}.$$

$\sum_{i\in s} Y_{sit}^{CM}$ is the sum of plant's value added and $\sum_{i\in s} BVK_{sit}^{CM}$ is the sum of the nominal book value of tangible fixed assets of industry s in the Census of Manufactures.

Labor Input (L_{si}) is defined as follows,

$$L_{si} = \frac{\tilde{w}_{si}\tilde{L}_{si}}{w_s}.$$

$\tilde{w}_{si}$ is the plant-level wage compensation divided by the number of workers $\tilde{L}_{si}$, and multiplied by non-wage compensation ratio. Non-wage compensation ratio is aggregate non-wage compensation divided by aggregate wage compensation obtained from System of National Accounts (SNA) of Japan. $w_s = \frac{\sum_i \tilde{w}_{si}\tilde{L}_{si}}{\sum_i \tilde{L}_{si}}$ is the industry-level average of w_{si}.

Appendix 1. Plant Size Distributions by decades

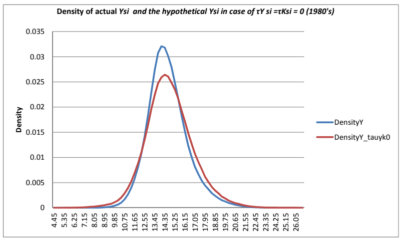

Figure 8: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} (red line) in the 1980s (1981-1989).**

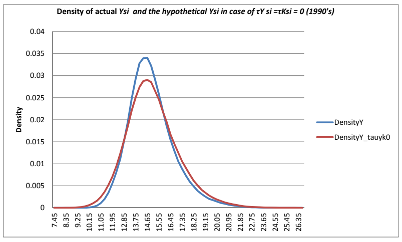

Figure 9: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} (red line) in the 1990s (1990-1999).**

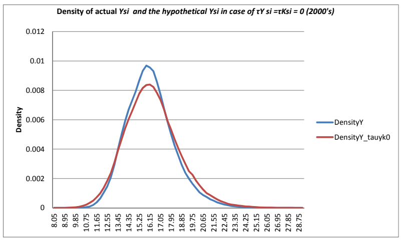

Figure 10: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} (red line) in the 2000s (2000-2008).**

Appendix 2. Plant Size Distributions in the cases of $\sigma = 6$ and $\pm 2\%$ trimming

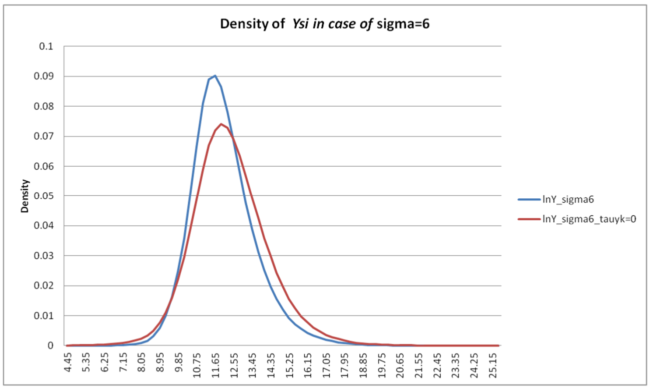

Figure 11: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} in case of $\tau_{Ysi} = \tau_{Ksi} = 0)$ when $\sigma = 6$ (red line).**

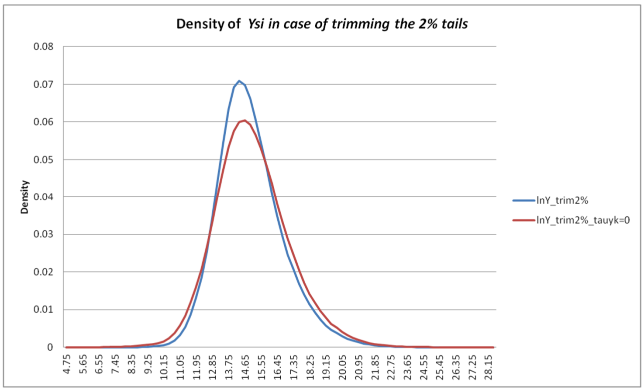

Figure 12: **Density of actual Y_{si} (blue line) and the hypothetical Y_{si} in case of $\tau_{Ysi} = 0)$ (red line) when trimming $\pm 2\%$**

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