Are DSGE Approximating Models Invariant to Shifts in Policy?: A Credit Channel Experiment

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Abstract

This paper discusses whether the parameter invariance problem, as stated in the Lucas (1976), applies to the standard new Keynesian DSGE model if the model misspecification occurs in the credit channel. I simulate a monetary policy shift driven by a change in the target inflation and examine whether the approximating model can serve as a useful guide for policymakers even if the parameters fail to remain strictly invariant. First, this paper finds that the parameter invariance problem is large in magnitude under the misspecification considered. Second, it finds that the additional welfare loss arising from the parameter change is small. Third, the potential gain from using a better approximating model with credit channel is found to be large and hence credit channel should not be omitted from future model development.

Keywords: DSGE approximating models, Lucas Critique, Bayesian estimation, Financial Accelerator model

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1 Introduction

Since the late-1990's, the analytical tool known as the Dynamic Stochastic General Equilibrium (DSGE) model has become the workhorse for analyzing the effect of monetary policy. Unlike the previous generation of models, its behavioral equations characterizing model variables are derived from the optimizing principle, and further pinned down by the parameters that describe preferences and technology of agents ("private sector parameters"). Many researchers now regard DSGE models as structural in the sense of Lucas (1976). For the purpose of monetary policy analysis, a time-dependent pricing (TDP) model, a la Calvo (1983) and Yun (1996), has become popular due to its tractability and ease of interpretation.

Until recent, few people have questioned the TDP model during a financial turmoil as in the 2007-08 credit crisis. Most DSGE models used in central banks omit "credit channel" (i.e. monetary policy transmission process through the supply of funds) in their core part.[1] In light of the recent experience, this practice is now being reviewed. However, it is not that obvious how the lack of credit channel would lead to an imminent problem as often claimed by experts. When the work of Lucas (1976) is taken literally, it also implies that if a policymaker uses an estimating model with even a slightest misspecification, then regardless of how structural the estimating model might seem, *any* policy shift would trigger the estimated private sector parameters to vary. In other words, strict parameter invariance is never attainable. Thus a meaningful question would be to ask whether a monetary policymaker is able to choose policy wisely despite the potential model misspecification. As the Fed still continues its massive expansionary policy (a.k.a. "quantitative easing") after many years since the beginning of the credit crisis, this topic should draw attention from a broad audience.

This paper explores the question of whether credit channel should be incorporated in

[1] For example, the Bank of England includes credit in the non-core part of the model, in which rather arbitrary dynamic equations serve as "proxies for short-run effects such as credit constraints, house price effects, confidence and accelerator effects" (Harrison *et al.*, 2005). For examples used in other institutions see Bayoumi *et al.* (2004), Erceg *et al.* (2006), Coenen *et al.* (2007).

the TDP model, by combining several simulation and estimation methods. It replicates two phases of the crisis that actually happened after 2007, the first phase immediately after the crisis broke out (e.g. from the summer of 2007 until the end of that year) and the second phase when the Fed shifted its policy to combat the crisis (e.g. 2008 and thereafter). The primary focus is in the second phase; the result of the first phase is used as a benchmark when assessing model prediction.

I chose a pair of relatively small-scaled DSGE models, one of which is used as the data generating model (DGM) that reflects the reality and the other as the approximating model (AM) used by the central bank. Except that the former features a credit channel, the two models share the same mechanisms of the standard TDP model. For the credit channel model, Financial Accelerator model of Bernanke, Gertler and Gilchrist (1999) was chosen (FA-DGM hereafter). The private sector parameters were calibrated from the recent U.S. data. These were kept fixed throughout the entire exercise, so that parameter instability can be fully attributed to the misspecification in the AM. The TDP-AM was used to estimate the parameters from the generated data and to find the optimal monetary policy. In estimation, Bayesian technique was used to facilitate computation.

In this model, policy shift involves changes in both target inflation rate and the monetary policy rule coefficients. Target inflation is introduced to make the policy shift more realistic and relevant to the current policy debate. This modification adds an interesting interaction between the Phillips curve and the financial accelerator mechanism, which has been left unnoticed until now in the literature. Monetary policy rule is specified as the Taylor rule with inflation and output gap components. Our policymaker sequentially chooses the target inflation and the optimal Taylor rule coefficients based on the welfare loss criterion that is detailed in section 3. When calculating the welfare loss, the estimates of the private sector parameters are used as inputs.

There are three key results in this paper. The first result is that the estimated parameters change significantly before and after the policy shift. The size of instability is comparable to what happens before and after the credit crisis. Among the parameters, risk aversion, labor elasticity, and government expenditure shock exhibit non-trivial changes. It demonstrates that a small misspecification in the credit channel can lead to a large parameter instability. This result suggests that many empirical works that report instability in the private sector parameters should be viewed with some caution.[2]

Second, the welfare consequence of the misspecification is relatively minor. If the central bank raises the target inflation from 2 to 5%, the result suggests that the parameter instability leads to 2% higher welfare loss. This is only a fraction of the 8% welfare loss that was anticipated from allowing such higher inflation. From a central banker's perspective, the result demonstrates robustness of the TDP model. The model also leads to the correct welfare ordering among alternative policy options.

Third, the potential benefit of putting financial accelerator mechanism into the TDP-AM is large. If we assume that the central banker *had known* the true DGM and used it as the AM, their policy could have reduced the welfare loss by an additional 10%. This is encouraging for the practitioners of DSGE model who are working on the extension of the credit channel model.

Even though the theoretical importance of credit channel seemed almost too obvious to the academics' eye since the time of Irving Fisher (e.g. Fisher, 1933), none of the existing literature have explicitly examined the role of credit channel from the parameter invariance problem's perspective.[3] The parameter invariance problem itself has been widely studied in the empirical literature, mostly comparing the forward-looking structural model with the backward-looking or static empirical model.[4] This approach often leads to a problem of comparing two very different modeling strategies, therefore not yielding any practical direction of future model development. This paper compares the forward-looking models for both structural and empirical model, highlighting the role of the credit channel as the only difference. Even though its approach might go beyond the Lucas' (1976) original intention, it still closely follows its spirit. A similar approach was used by two other papers. Cogley and Yagihashi (2010) study the effect of a monetary policy shift when the

[2]See for example, Smets and Wouters (2003) and Canova (2009).

[3]For recent credit channel models, see for example Iacoviello (2005), Goodfriend and McCallum (2007), Christiano *et al.*(2007), De Fiore and Tristani (2007), Gerali *et al.* (2008), Curdia and Woodford (2009)

[4]See for example Leeper and Sims (1994), Rudebusch (2005), and Lubik and Surico (2010)

misspecification occurs in the Phillips Curve. The policy shift is specifically designed to replicate the Volcker disinflation during the early 1980's. Chang, Kim and Schorfheide (2010) study the effect of a fiscal policy shift when the misspecification occurs in both asset and labor market. Their policy shift mimics the U.S. tax policy since the 1960's. Both papers reach opposite conclusions with regard to the importance of model misspecification, which proves that the nature of the parameter invariance problem is context-driven and hard to generalize.

The next section describes how the model misspecification in the credit channel affects the strict invariance of the estimated parameters in theory. The third section explains the full model in detail and how the parameters used in the DGM are calibrated. The fourth section shows the main result of the parameter invariance problem. The fifth section examines how robust the TDP-optimal policy is in the context of monetary policy analysis. The sixth section examines whether the main finding holds under the alternative parameterization of the credit market friction. The last section concludes.

2 Misspecification of the Credit Channel

In order to assess the approximate invariance of the TDP-AM, I use the conceptual framework developed in Cogley and Yagihashi (2010). This experiment puts two models on the table, a relatively complex one that features the credit channel and a simpler TDP-AM that abstracts from it. Under the assumption that policymakers do not know the DGM and that they use the AM to interpret data emanating from the DGM, this paper simulates data from the DGM and fit the AM to those data. To focus on the parameter invariance problem arising from the misspecification, sufficiently long subsamples are generated corresponding to different credit channel conditions and the monetary policies, then fit the approximating model to each subsample and examined the extent to which estimates of structural parameters change.[5]

More formally, the estimation can be stated as solving the following minimization

[5] In reality policymaker faces many other issues such as estimation uncertainty and identification problem. See An and Schorfheide (2007) and Canova and Sala (2009).

problem

$$\arg\min KLIC = \int \log\left(\frac{p_{DGM}(Y|\theta_{DGM})}{p_{AM}(Y|\theta_{AM})}\right) p_{DGM}(Y|\theta_{DGM})dY,$$

where $KLIC$ stands for the distance metric known as the Kullback-Leibler Information Criterion. $p_i(Y|\theta_i)$ represents the likelihood function for both models $i = (DGM, AM)$, the vector Y represents endogenous variables common to both models, and θ_i represents a vector of model-specific parameters which can be partitioned into policy parameters (θ_i^{pol}) and private sector parameters (θ_i^{priv}). The Bayesian consistency theorem assures that given our long sample period the estimates converge in probability to what is called the "pseudo-true value" estimates of the private sector parameters ($\widehat{\theta}_{AM}^{priv}$) while treating policy parameters as known constants.[6] Due to the misspecification in the TDP-AM, there will always be an asymptotic bias i.e. $\widehat{\theta}_{AM}^{priv} \neq \theta_{DGM}^{priv}$. The policymaker's best hope is that $\widehat{\theta}_{AM}^{priv}$ remains approximately invariant when there is a policy shift.

In the context of the credit channel experiment, the main misspecification is captured in the following two behavioral equations

$$E_t R_{t+1}^k = E_t\left[\frac{MPK_{t+1} + (1-\delta)\, Q_{t+1}}{Q_t}\right], \tag{1}$$

$$E_t R_{t+1}^k = E_t\left[\left(\frac{Q_t K_{t+1}}{N_t}\right)^v R_{t+1}\right]. \tag{2}$$

Equation (1) represents the demand for loanable funds. $E_t R_{t+1}^k$ is the ex ante return on capital which consists of the marginal product of capital (MPK) and the net gain of the capital price (Q). Equation (2) represents the supply for loanable funds. The household saving is collected by the representative financial intermediary and lent out to the "entrepreneurs". This lending rate has to cover monitoring cost in case the entrepreneur defaults, in addition to the the risk-free rate (R) that is promised to pay to the household. The ratio R^k/R is defined as the risk premium that is modelled as a monotonically increasing function of the economywide leverage ratio (QK/N) and the credit market friction (v). In the TDP-AM, credit market friction is zero and the return on capital always equals the risk-free rate. Thus, FA-DGM nests the TDP-AM as its special case.

[6] For more details, see the appendix in Gelman *et al.* (2000)

Another source of misspecification arises from different steady state values. In the FA-DGM the return on capital is higher than that of the TDP-AM because of the nonzero risk premium. Consequently the FA-DGM has a lower capital output ratio and a lower investment share of output compared to the TDP-AM. These differences in the implied steady state values will affect the dynamics of the variables.

3 The Models

The Financial Accelerator model used in this paper closely follows Bernanke *et al.* (1999). This model has a wide support among researchers and policymakers in the field.[7] The fact that it nests the TDP approximating model (TDP-AM hereafter) as its limiting case makes it easy to identify the source of misspecification while avoiding unnecessary complication in both simulation and estimation process.

In order to make the policy shift more realistic and relevant to the current policy debate, the original model is modified slightly. First, I introduce positive trend inflation in both the FA and the TDP models. This involves a log-linearization of the new Keynesian Phillips curve around the positive trend inflation, as shown in Ascari (2004) and Ascari and Ropele (2007). Thus in equilibrium, changes in target inflation rate result in the shift of the Phillips curve. Second, I let the central bank compute the optimal Taylor rule coefficients. To introduce a policy trade-off between inflation and output stabilization objective, a cost push shock is added to the model. Subsections 3.1-3.7 specify the components of the FA / TDP models. Subsection 3.8-3.10 explain how the structural parameters are calibrated and how the equilibrium is computed.

3.1 Households

The representative household chooses consumption C_t, labor supply L_t and bond holding D_{t+1} to maximize lifetime utility subject to a budget constraint. Its optimization problem

[7] See Mishkin (2008).

can be expressed as,

$$\max E_t \sum_{j=0}^{\infty} \beta^i \left[\frac{1}{1-\sigma} C_{t+j}^{1-\sigma} - \frac{\zeta}{1+\frac{1}{\tau}} L_{t+j}^{1+\frac{1}{\tau}} \right], \tag{3}$$

$$s.t.\ C_t + \frac{D_{t+1}}{P_t} = W_t L_t + \frac{R_t^n}{P_t} D_t + profit_t.$$

D_t is defined as the household's one-period nominal coupon bond maturing at time t and pays a gross nominal interest rate R_t^n. W_t is the real wage that is determined in a competitive factor market. $profit$ is the sum of real profits from various types of firms. P_t is the economywide price level index. β, σ, τ are the preference parameters of the household that describe the future discount factor, risk aversion, and elasticity of labor supply, respectively. The first order conditions of (3) yield standard Euler equation and intratemporal efficiency conditions:

$$C_t^{-\sigma} = \beta R_{t+1} E_t C_{t+1}^{-\sigma},$$

$$\frac{\zeta L_t^{1/\tau}}{C_t^{-\sigma}} = W_t,$$

where I define $R_{t+1} \equiv \frac{R_{t+1}^n}{\Pi_{t+1}}$ as the risk-free real interest rate and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ as gross inflation.

3.2 Capital producers

There are three types of firms involved in the production process of the economy. They are called capital producers, intermediate goods producers and final goods producers. The competitive capital producers purchase raw output as a input I_t and combine it with currently available capital K_t to produce new capital goods K_{t+1} that will be used in the next period's production process. It is assumed that capital producers are subject to the quadratic capital adjustment costs. The adjustment costs X_t are specified as

$$X_t = \frac{\chi}{2} \left(\frac{I_t - \delta K_t}{K_t} \right)^2 K_t,$$

where the adjustment cost parameter χ is chosen so that it satisfies $X(I_t)' = \chi \left(\frac{I_t}{K_t} - \delta \right) > 0$ and $X(I_t)'' = \chi \frac{1}{K_t} > 0$. δ is the depreciation rate that applies to all the capital that exists in a given period. These newly produced capital goods are then sold to the financial intermediary within the same time period at the price Q_t. Since perfect competition is assumed in this industry, the maximization problem of capital producers is

$$\max_{I_t} Q_t \left[I_t - \frac{\chi}{2} \left(\frac{I_t - \delta K_t}{K_t} \right)^2 K_t \right] - I_t.$$

The associated first order condition is

$$Q_t = \left[1 - \chi \left(\frac{I_t}{K_t} - \delta \right) \right]^{-1},$$

where capital price Q_t is positively related to the investment demand relative to the capital stock.

3.3 Final good producers

Final good producers simply aggregate the intermediate goods that each firm z has produced into one final good. Their production function can be written as

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

Taking prices as given, profit maximization of the final good producers subject to (4) implies the downward sloping demand curve,

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t, \forall\ z\ .$$

Final output is absorbed as either consumption C, investment I, government expenditure G or entrepreneur's consumption C^e that is explained in section 3.5. Therefore the economywide resource constraint is

$$Y_t = C_t + I_t + G_t + C_t^e.$$

3.4 Intermediate goods producers

An intermediate goods producer z, indexed in the unit interval, faces monopolistic competition in the market. It hires labor and capital from an economywide competitive market and transforms it into output via the usual Cobb-Douglas production function.

$$Y_t(z) = A_t K_t^{\alpha}(z) L_t^{1-\alpha}(z),$$

where the aggregate technology shock is modeled as

$$\begin{aligned} A_t &= (A_{t-1})^{\rho_A} \exp(e_{A,t}), \\ e_{A,t} &\sim N(0, \sigma_A^2), \end{aligned}$$

Closely following the assumptions in the standard New Keynesian DSGE model originally developed by Calvo (1983) and Yun (1996), firms are facing a stochastic probability ρ of not being able to adjust their price in a given time period.[8] Firms have to take into account that they may not be able to adjust their prices in the future. Hence the firm z's maximization problem is given as

$$\begin{aligned} &\max_{p_t(z)} E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} \left[p_t(z) - MC_{t+j}(z) \right] Y_{t+j}(z), \\ &s.t. Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t, \end{aligned}$$

where $p_t(z) \equiv P_t(z)/P_t$ is the relative price for each firm, $\Delta_{j,t+j}$ is the relevant discount factor[9] and MC is the marginal cost calculated as the minimized real unit cost of production. Rearranging the first order condition yields the optimal price equation

$$p_t^* \equiv \frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} MC_{t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon-1} Y_{t+j}},$$

which states that the optimal relative price will be a function of relevant discount rate, future marginal cost, and output. With a nonzero target inflation $\overline{\Pi} \neq 0$, log-linearizing this equation around the steady state yields the following equations

$$\begin{aligned} \widehat{\Pi}_t &= \beta \overline{\Pi} E_t \widehat{\Pi}_{t+1} + \kappa_1 \widehat{MC}_t + \kappa_2 \left(\sigma \widehat{C}_t - \widehat{Y}_t \right) \\ &\quad + \kappa_2 \left[(\epsilon - 1) E_t \widehat{\Pi}_{t+1} + E_t \widehat{\Phi}_{t+1} \right] + e_{cp,t}, \end{aligned} \tag{5}$$

[8] This implies that the longer the spell of nonadjustment for a firm, the higher the probability for that firm to be able to adjust the price in the next period, hence its name "time-dependent" pricing (TDP).

[9] Formally it is defined as $\Delta_{j,t+j} \equiv \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \times \beta \left(\frac{C_{t+2}}{C_{t+1}} \right)^{-\sigma} \times ... \times \beta \left(\frac{C_{t+j}}{C_{t+j-1}} \right)^{-\sigma} \right] = \beta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma}$

$$\widehat{\Phi}_t = \left(1 - \rho\beta\overline{\Pi}^{\epsilon-1}\right)\left(\widehat{Y}_t - \sigma\widehat{C}_t\right) + \rho\beta\overline{\Pi}^{\epsilon-1}\left(\left(\epsilon - 1\right)E_t\widehat{\Pi}_{t+1} + E_t\widehat{\Phi}_{t+1}\right), \tag{6}$$

where the auxiliary variable Φ is defined as $\Phi_t = E_t \sum_{j=0}^{\infty} \rho^j \Delta_{j,t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1} Y_{t+j}$ as in Ascari and Ropele (2007), letting a hat represent the deviation of the variables from their steady state values. The "slope" parameters are given as $\kappa_1 = \frac{\left(1-\rho\overline{\Pi}^{\epsilon-1}\right)\left(1-\rho\beta\overline{\Pi}^{\epsilon}\right)}{\rho\overline{\Pi}^{\epsilon-1}}$, $\kappa_2 = \beta\left(\overline{\Pi} - 1\right)\left[1 - \rho\overline{\Pi}^{\epsilon-1}\right]$, respectively. These two parameters describe the trade-off between nominal and real variables in this economy. Unlike in the traditional New Keynesian Phillips Curve, these slope parameters also capture the effect of target inflation $\overline{\Pi}$.[10] To introduce a policy trade-off between inflation and output stabilization objective, I add cost push shock $e_{cp,t} \sim N(0, \sigma_{cp}^2)$ to the equation (5).

3.5 Financial Intermediary and "Entrepreneur"

In the FA-DGM, there is imperfection in the capital market that generates risk premium between the return on capital (R^k) and the risk-free interest rate (R). A risk-neutral financial intermediary collects funds from the representative household, then lends out to the individual entrepreneurs. Entrepreneurs are subject to the risk of bankruptcy through the idiosyncratic productivity disturbance ω. The disturbance is assumed to have a standard log-normal distribution $\ln\omega \sim N\left(-0.5\sigma_e^2, \sigma_e^2\right)$ with the normalized mean of unity. A threshold value ω^* is specified, that makes entrepreneur solvent

$$\omega^* R^k QK = Z(QK - N),$$

where I define R^k as the ex post average return on capital, Z as the gross loan rate and N as the net worth that provides internal financing for the entrepreneur. If the entrepreneur j faces an adverse productivity disturbance $\omega^j < \omega^*$, then that entrepreneur becomes insolvent and declares bankruptcy. Therefore ω^* determines the overall default rate $F(\omega^*)$ that is expressed as the cumulative distribution function of the idiosyncratic

[10] For more discussion on how the trend inflation affects the New Keynesian model, see for example Sahuc (2006) and Bakhshi *et al.* (2007).

shock, i.e.

$$F(\omega^*) = \int_0^{\omega^*} f(\omega) d\omega$$

where $f(\omega)$ takes the lognormal distribution.

The financial intermediary does not know how entrepreneurs will fare before the shock hits them individually. The financial intermediary can only know each entrepreneur's state through the repayment of its loan. In case the entrepreneur becomes insolvent, the financial intermediary collects all that is left of the realized gross payoff to the entrepreneur's capital (QK) by paying a proportional unit cost μ. The total cost for the financial intermediary is then defined as

$$\mu \int_0^{\omega^*} \omega f(\omega) \, d\omega * R^k QK \equiv \mu G(\omega^*) R^k QK, \tag{7}$$

where the integral part represents the average value of ω when entrepreneur becomes insolvent. The expected gross profit of the financial intermediary is the sum of revenues raised from both solvent and insolvent entrepreneurs, that is

$$\left[\int_0^{\omega^*} \omega f(\omega) \, d\omega + \int_{\omega^*}^{\infty} \omega^* f(\omega) \, d\omega \right] R^k QK \equiv \Gamma(\omega^*) R^k QK. \tag{8}$$

The expected net profit is the gross profit in equation (8) minus the agency cost in equation (7).

The optimal contract problem can be formulated as the entrepreneur's profit maximization problem subject to a financial intermediary's participation (zero profit) condition

$$\max \left(1 - \Gamma(\omega^*)\right) R^k QK,$$

$$s.t. \left(\Gamma(\omega^*) - \mu G(\omega^*)\right) R^k QK = R(QK - N).$$

Solving this leads to equation (2), an expression for the supply of loanable funds in a dynamic form

$$E_t R_{t+1}^k = \left(\frac{Q_t K_{t+1}}{N_{t+1}} \right)^v R_{t+1},$$

where the credit market friction parameter v represents the elasticity of the risk premium R^k/R with respect to the economywide leverage ratio QK/N. The larger the capital, the more leveraged entrepreneurs become in financing their investment demand. The financial intermediary asks for higher premium to cover the additional cost of insolvency.[11]

Upon receiving the fund, entrepreneurs purchase physical capital from capital producers and provide capital service to the intermediate goods producers. In determining the demand for capital, entrepreneurs consider the marginal product of capital (MPK) as well as the expected capital gain from holding it over time.[12] This leads to equation (1), an expression for the demand for loanable funds equation

$$E_t R_{t+1}^k = E_t \left[\frac{MPK_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right],$$

where δ is the depreciation rate of capital. When credit market friction is positive, in steady state we have $R^k = MPK + 1 - \delta > R$, which implies that the marginal product of capital is higher and capital output ratio is lower compared to the zero friction case.

Finally, the net worth accumulates based on the profit made by entrepreneurs plus income of supplying entrepreneurial labor ($W^e \times L^e$). Entrepreneurial labor L^e is introduced to assure that the entrepreneur enters the economy with positive wealth.[13] I further assume that each entrepreneur has a constant probability γ of surviving to the next period. This is to avoid entrepreneurs from accumulating net worth until they self-finance the entire project. These assumptions yield the net worth accumulation process.[14]

$$N_{t+1} = \gamma \left[R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\omega^*} \omega f(\omega) \, d\omega * R^k Q K}{Q_{t-1} K_t - N_t} \right) (Q_{t-1} K_t - N_t) \right] + W^e. \quad (9)$$

[11] Note that if $v = 0$, financial intermediary does not require any compensation ($R^k = R$) and the risk premium is zero.

[12] Entrepreneurs are forced to sell back the entire capital to capital producers after they are used for production and re-purchase a new level of capital in the next period.

[13] L^e is normalized to unity for all period.

[14] If entrepreneurs fail to stay in the economy, they will consume the remaining wealth (C^e) and exit the market.

3.6 Government

I assume that the role of fiscal policy is to provide additional disturbance to the economy which inhibits persistence. The law of motion for government expenditure is given as

$$\begin{aligned} G_t &= (G_{t-1})^{\rho_G} \exp(e_{G,t}), \\ e_{G,t} &\sim N(0, \sigma_G^2). \end{aligned}$$

The monetary policy rule takes the form of Taylor rule (Taylor, 1993)

$$\begin{aligned} R_{t+1}^n &= (\Pi_t)^{\rho_\pi} (Y_t)^{\rho_Y} \exp(e_{m,t}) \\ e_{m,t} &\sim N(0, \sigma_m^2) \end{aligned}$$

where R^n is the nominal interest rate. e_m is a white noise policy shock. The central bank calculates the optimal Taylor rule coefficients given a particular target inflation rate and the estimated structural parameters. In particular, the policymaker chooses the Taylor rule parameters ρ_Π and ρ_Y in order to minimize the expected welfare loss. This can be expressed as

$$\min_{\rho_\Pi, \rho_Y} WL_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[var(\Pi_t) + var(Y_t) \right], \tag{10}$$

$$s.t. \rho_\Pi, \rho_Y \geq 0,$$

and subject to the structural equations in the TDP-AM that are described in the next section.

3.7 The TDP approximating model

The approximating model shares many of the structural equations with the FA-DGM, including the Phillips Curve with the positive trend inflation. However the model suffers from the misspecification in the credit channel that was fully specified for the FA-DGM in the section 3.5.

Due to this misspecification, the central bank is not aware of the existence of the supply of loanable funds equation (2) and the law of motion for net worth (9). Since risk premium is zero for all period, the expected return on capital equals the risk-free interest rate and the demand for capital equation (1) is replaced by

$$R_{t+1} = E_t \left[\frac{MPK_{t+1} + (1-\delta)\, Q_{t+1}}{Q_t} \right].$$

For these reasons, the optimal policy that is derived from using the TDP-AM ("TDP-optimal" rule) will be different from the optimal policy that is derived from using the FA-DGM ("FA-optimal" rule), which is infeasible to the policymaker.

A special case arises when we impose the restriction that the credit market friction is zero. Prior to the credit crisis, equation (2) is nonexistent and equation (9) is redundant to the overall model dynamics. In this special case, the two models, FA-DGM and TDP-AM, become isomorphic. As a result, the TDP-optimal Taylor rule coincides with the FA-optimal Taylor rule. In the literature, this "no-FA case" is often used as a benchmark scenario when examining the role of financial accelerator. Here, the no-FA case is used to calculate how much the parameters vary due to the credit market friction alone. This will be contrasted to the parameter invariance problem arising from the policy shift, which is the main theme of this paper.

3.8 Calibration of Private Sector Parameters

The model parameters, which are summarized in table 1 and 2, largely follow the literature: the discount factor β is chosen to imply an annual real interest rate of 4.3%. The elasticity of demand ϵ implies a markup of 10% in the steady state. The labor supply elasticity τ is calibrated using aggregate data. The price stickiness parameter ρ implies average nonadjustment of 6.7 months. The government share of output $\overline{G}/\overline{Y}$ is set to match the historical average.

I updated the FA parameters used in Bernanke *et al.* (1999) so that it reflects the recent data. To calibrate the credit market friction parameter, first a time series of risk premium, default rate, and leverage ratio are constructed as in Yagihashi (2011). Second, the averages of each series during 1988Q1-2009Q2 were calculated, which resulted in the risk premium of 4.5%, default rate of 6.3% and leverage ratio of 1.8. Finally, I chose the standard deviation of the idiosyncratic productivity disturbance σ_e and the bankruptcy cost μ so that the implied steady state risk premium, default rate and leverage ratio matches those of the data. Solving the optimal contract problem detailed in Bernanke *et al.* (1999) results in the credit market friction parameter $\nu = 0.073$, which is slightly higher compared to that of Bernanke *et al.*'s 0.05.

Table 1: Estimated Parameters

Parameter name	Values
Risk aversion σ	1
Labor supply elasticity τ	1
Capital adjustment cost χ	0.25
Price stickiness ρ	0.55
Persistence of tech. shock ρ_A	0.9
Persistence of gov. shock ρ_G	0.9
Size of tech. shock σ_A	0.01
Size of gov. shock σ_G	0.01
Size of cost push shock σ_{cp}	0.01
Size of mon. policy shock σ_m	0.01

Table 2: Other Structural Parameters

Parameter name	Values
Discount factor $\beta = 1/\overline{R}$	0.99
Capital share α	0.33
Elasticity of demand ϵ	11
Government spending share of output $\overline{G}/\overline{Y}$	0.21
Entrepreneur's share of output $\overline{C^e}/\overline{Y}$	0.01
Depreciation rate δ	0.025
credit market friction υ	0.073
Survival rate of entrepreneur γ	0.973

3.9 Calibration of Policy Parameters

I consider two sets of policy parameters, each of them consisting of the different central bank's long-run target for inflation (Π^{SS}) and the resulting feedback coefficients in the

Taylor rule (ρ_Π, ρ_Y). Following Lucas (1976), policy shifts that take the form of a one-time, permanent, unanticipated shift in these parameters are examined. This means that a learning process is suppressed and immediate convergence to a new rational-expectations equilibrium is assumed. These sets of policy parameters are summarized in table 3.

Table 3: Policy Parameters

	(1) Low inflation policy	(2) Reflationary policy
(Annual) Target inflation $\Pi^{SS} \equiv \overline{\Pi}^4 - 1$	2%	5%
Taylor rule coefficient: inflation ρ_Π	4.05	2.24
Taylor rule coefficient: output gap ρ_Y	0.37	0
cf. Credit market friction	0	0.073

The low inflation policy parameters are derived under 2% target inflation and zero credit market friction. This resembles the Fed's policy before the credit crisis broke out in the summer of 2007. A 2% target inflation rate is specified, following the widely accepted value among practitioners.[15] The Taylor rule parameters $\rho_\Pi = 4.05, \rho_Y = 0.37$, reported in the column (1) of table 3, are obtained by minimizing the welfare criterion in equation (10). These values are similar to the empirical estimates during the Volcker-Greenspan era that reported large reaction coefficient to inflation and relatively muted reaction to output gap.[16] Since $\nu = 0$, the policymaker has no problem estimating the structural parameters in DGM and the TDP-optimal policy coincides with the FA-optimal policy.[17]

The reflationary policy parameters are derived under a 5% target inflation and positive credit market friction $\nu = 0.073$. The 5% inflation value is taken from the Fernandez-Villaverde and Rubio-Ramirez (2007)'s estimation, matching the temporally spike in the target inflation rate during the S&L crisis that occurred at the end of the 1980's. In this experiment the events unfold as follows. First, the positive credit market friction alters the data generated from the FA model. Second, using the newly gathered data, the

[15] This value is close to the annualized quarter to quarter PCE inflation during the past two decades.

[16] For example, Fernandez-Villaverde and Rubio-Ramirez (2007) report that the ρ_Π was in the range of 3.5 to 4.5 during 1980-2000 in the United States.

[17] The actual estimates will be reported in table 7 in the estimation section.

policymaker re-estimates the structural parameters. Third, the policymaker reoptimizes the Taylor rule coefficients under the higher target inflation and the newly estimated parameters. This results in $\rho_\Pi = 2.24, \rho_Y = 0$ as reported in the column (2) of table 3.

To understand how the target inflation rate is tied to the Taylor rule coefficients in this model, table 4 summarizes the effect of higher target inflation on Taylor rule coefficients and the associated welfare losses. Here a benchmark case with $\nu = 0$ (no-FA case) is assumed.

Table 4: Welfare Losses in AM, no-FA Case

(Annual) Target inflation $\Pi^{ss} \equiv \overline{\Pi}^4 - 1$	2%	3%	5%	7%
Taylor rule coefficient: inflation ρ_Π	4.05	3.19	2.03	1.88
Taylor rule coefficient: output gap ρ_Y	0.37	0.19	0	0
Welfare loss WL	7.6673	7.8592	8.2762	8.7843
Δ% from Π^{ss} =2%	-	2.5%	7.9%	14.6%

Credit market friction is set to zero. The unit of welfare loss is 1E-4 = 0.0001

As shown in the table, reflationary policy yields a much muted Taylor rule coefficient for both inflation and output gap. The welfare loss defined in equation (10) increases exponentially as the target inflation rises. When the target inflation is raised from 2% to 5%, welfare loss rises by 7.9%, from 7.6673E-4 to 8.2762E-4. This increase in the welfare loss can be regarded as the long-run cost of the reflationary policy that the policymaker has to accept.

3.10 Solving for the Equilibrium

The rational expectations equilibrium is characterized in three steps. First, I solve for the model's deterministic steady state. Second, the model equations are log-linearized around the steady state values of each variables and stacked into a system of linear expectational difference equations. Lastly, that system is solved to find the approximate equilibrium law of motion.

Table 5 summarizes key properties of the steady state. Using the parameterization in the previous sections, I find that in the FA-DGM the annual capital output ratio is 1.6, investment spending share of output is 0.16, and consumption spending share of output is 0.64. In the TDP-AM these numbers are 2.1, 0.21,and 0.59 respectively. The larger capital output ratio in the TDP-AM reflects the effects of zero risk premium, which in turn lowers the marginal product of capital for a given natural rate of interest $\overline{R}$ and depreciation rate δ. The difference in capital output ratios leads to the differences in the expenditure shares.

Table 5: Steady-state Values

	FA	TDP
credit market friction parameter ν	0.073	0
Consumption spending share of output $\overline{C}/\overline{Y}$	0.64	0.59
Investment spending share of output $\overline{I}/\overline{Y}$	0.16	0.21
(Annual) Capital output ratio $\overline{K}/4\overline{Y}$	1.6	2.1
(Annual) Ex ante real return on capital $(\overline{R}^k)^4$	1.088	1.043
(Annual) Risk premium $(\overline{R}^k)^4 - \overline{R}^4$	4.5%	0%
(Annual) Default rate $F(\omega^*)$	6.3%	0%
Leverage ratio $\overline{QK}/\overline{N}$	1.8	1

Next, I illustrate how the financial accelerator mechanism works under different monetary policies. Impulse response functions are drawn for the 1% expansionary monetary policy shock. Dotted lines in figure 1 represent impulse response associated with the low inflation policy and zero credit market friction. Dash-dotted lines represent impulse response associated with the same low inflation policy but with a positive credit market friction. Finally, solid lines represent impulse response associated with the higher target inflation.[18]

The comparison between the dotted line and dash-dotted line illustrates the basic financial accelerator mechanism. The expansionary monetary policy shock lowers the real interest rate and the risk premium, stimulating investment and the economywide output. The lower real interest rate raises the net worth, which keeps the risk premium low in

[18]For the third case, I keep the Taylor rule parameters the same as in other cases in order to isolate the effect of target inflation.

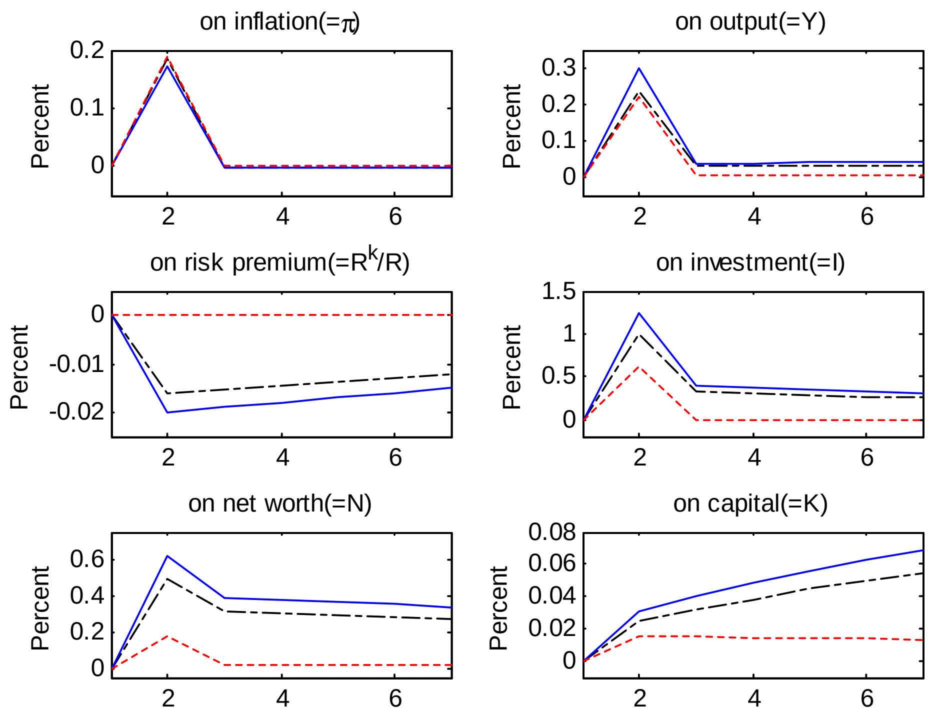

Figure 1: Impulse Responses to an 1% Expansionary Monetary Policy Shocks: No-FA with 2% inflation (Dotted line), FA with 2% inflation (Dash-dotted line) and FA with 5% inflation (Solid line)

the preceding periods as well. Through the persistent dynamics of net worth, the initial expansionary effect will be propagated further into the future, keeping the real interest low and investment high.

The comparison between the dash-dotted line and solid line shows how the financial accelerator mechanism is enhanced through the higher target inflation rate. Note that in this model, target inflation enters into the Phillips curve equation (5) and the auxiliary equation (6) which captures the dynamics of stochastic discount factor. These two equations provide the additional expansionary effect to the economy through amplifying the initial monetary policy shock. Among the six variables shown in the figure 1, output in the upper right corner gets the most significant expansionary effect from the reflationary policy.

4 Strict and Approximate Parameter Invariance

4.1 Data Generation and Prior Choice

In conducting the simulation, 550,000 quarterly observations were generated for output, inflation, nominal interest rate and consumption, and the first 50,000 observations were discarded to ensure that initial conditions have worn off. The large sample size assure that the asymptotic standard errors are tiny, allowing us to focus on how point estimates change across policies. The number of observables matches the number of structural shocks in the TDP-AM to avoid stochastic singularity.

Next, the structural parameters were estimated using the Bayesian DSGE estimation method that maximizes the posterior kernel density. The priors introduce curvature in the optimization and facilitates computation.[19] The standard deviations of the priors are set so that the estimates can quickly converge to the pseudo-true values. The distributions of the priors are chosen so that they respect the domain of the parameter.[20] Table 6 summarizes the prior choice of the individual parameters.

[19] See An and Schorfheide (2007) for more discussion on the use of Bayesian method.

[20] In particular, the persistence parameters ρ, ρ_A, ρ_G have beta priors, positive parameters σ, τ, χ have gamma priors and standard deviations of the shocks have inverse-gamma priors.

Table 6: Prior Choice

	Distribution	Mode	95% credible set
Risk aversion σ	Gamma	1	[0.523, 2.087]
Labor supply elasticity τ	Gamma	1	[0.523, 2.087]
Capital adjustment cost χ	Gamma	0.25	[0.133, 0.438]
Price stickiness ρ	Beta	0.55	[0.197, 0.851]
Persistence of tech. shock ρ_A	Beta	0.9	[0.635, 0.967]
Persistence of gov. shock ρ_G	Beta	0.9	[0.635, 0.967]
Size of tech. shock σ_A	Inv-Gamma	0.01	[0.006, 0.084]
Size of gov. shock σ_G	Inv-Gamma	0.01	[0.006, 0.084]
Size of cost push shock σ_{cp}	Inv-Gamma	0.01	[0.006, 0.084]
Size of mon. policy shock σ_m	Inv-Gamma	0.01	[0.006, 0.084]

95% credible set is calculated as the 5% HPDI from the posterior draws.

4.2 Pseudo-True Values in the Low Inflation Policy

The estimated pseudo-true values under the low inflation policy are shown in the columns (1) and (2) of table 7, with asymptotic standard errors in parenthesis.

Table 7: Pseudo-true Values

	DGM values	Estimates (1) "old"	(2) "new"	(3) "final"
Risk aversion σ	1	1.00 (0.001)	1.17 (0.002)	1.14 (0.001)
Labor supply elasticity τ	1	1.00 (0.066)	1.63 (0.061)	2.78 (0.109)
Capital adjustment cost χ	0.25	0.25 (0.000)	0.24 (0.000)	0.24 (0.000)
Price stickiness ρ	0.55	0.55 (0.005)	0.52 (0.003)	0.49 (0.002)
Persistence of tech. shock ρ_A	0.9	0.90 (0.000)	0.93 (0.000)	0.93 (0.000)
Persistence of gov. shock ρ_G	0.9	0.90 (0.001)	0.95 (0.0001)	0.95 (0.001)
Size of tech. shock σ_A	0.01	0.010 (0.000)	0.010 (0.0000)	0.010 (0.000)
Size of gov. shock σ_G	0.01	0.010 (0.000)	0.014 (0.000)	0.015 (0.000)
Size of cost push shock σ_{cp}	0.01	0.010 (0.001)	0.010 (0.000)	0.010 (0.000)
Size of mon. policy shock σ_m	0.01	0.010 (0.000)	0.010 (0.000)	0.010 (0.000)
cf. credit market friction in DGM ν	-	0	0.073	0.073

Note: Asymptotic standard errors are shown in parentheses.

Almost all posterior standard errors are driven down to near zero due to the long time series, which guarantees that our subsequent findings on parameter invariance is a consequence of model misspecification. An exception is the standard errors on the labor supply elasticity. This is because of the inherent identification problem associated with the baseline TDP-AM. Cogley and Yagihashi (2010) conduct a separate simulation exercise that shows this weak identification does not affect the result on our parameter invariance problem.

In the column (1) of the table (the "old" estimates), the parameters are estimated using the low inflation policy and zero credit market friction. As expected, the estimates remain almost identical to the DGM counterparts.[21] In the column (2) of the table (the "new" estimates), the parameters are estimated by using the same low inflation policy but with a positive credit market friction. Several parameters now show large asymptotic biases, most notably the labor elasticity that is 1 in the "old" estimates and 1.63 in the "new" estimates. Estimate for risk aversion σ also increases significantly from 1 to 1.17.[22] Standard deviation of the government expenditure shock σ_G is estimated to be 40% larger than the DGM value. Estimate for price stickiness ρ decreases from 0.55 to 0.52, which translates into half a month shorter average duration between price changes. Quantitatively, this does not seem to be large given the wide range of estimates reported in the empirical literature. [23] Finally, both of the persistence parameters ρ_A, ρ_G increase slightly towards a unit root.

This result shows that most of the parameters fail to remain invariant, in particular the three aggregate demand parameters σ, τ, σ_G which are closely related to the function of the credit channel. Recall that in the DGM these parameters as well as the policy parameters were kept fixed and only the credit market friction parameter were allowed to change. Canova (2009) has estimated the structural parameters of the United States

[21] This also holds for the labor elasticity parameter τ, which was relatively weakly identified in the approximating model.

[22] According to Lucas (1987), welfare cost of business cycle is proportional to the constant relative risk aversion parameter. Under the "new" estimates, the household perceives 17% higher welfare cost, given the same level of consumption fluctuation.

[23] See for example Klenow and Kryvtsof (2008).

using the new Keynesian model and found that both private and policy parameters have changed during the 1980's.[24] The experiment in this paper shows that a misspecified model can generate the same result without resorting to any structural changes in the estimated private sector parameters.

4.3 Pseudo-true Values after the Reflationary Policy is Implemented

Let's examine whether the parameters remain invariant after the reflationary policy shift has occurred. As discussed earlier, when policymakers shift the policy guided by a misspecified approximating model, estimates of structural parameters need not remain invariant. To examine this point, first the FA-DGM was simulated under the reflationary policy with higher target inflation. Taylor rule parameters were reoptimized, using the 5% target inflation rate and the "new" estimates reported in the column (2) of table 7. Next, the structural parameters are estimated with the data generated from the FA-DGM. These "final" estimates are reported in the column (3) of the same table. The *changes* from the "new" estimates to the "final" estimates are then compared to the changes from the "old" estimates to the "new" estimates. The latter provides a benchmark for evaluating whether the parameter invariance problem associated with the policy shift is significant in size. Recall that the changes from "old" to "new" estimate were caused by the credit market friction parameter changing from zero to 0.073. Since zero credit market friction would be a strong assumption in reality, the change in the parameters observed in the benchmark case is likely to be exaggerated. This put the parameter invariance problem to a stringent test.

Most notably, the labor elasticity parameter τ changes from 1.63 to 2.78. The size of the change exceeds the change from the previous experiment, both in level as well as in percentage change. Next, price stickiness decreases from 0.52 to 0.49, which is equivalent to ten days shorter average duration between price changes. Even though this change looks minor, recall that the size is similar to the change from the benchmark case. The

[24] Similar results were reported by Smets and Wouters (2003, 2005).

standard deviation of the government shock increases from 0.014 to 0.015, which is less than half of the change from the previous experiment. Risk aversion decreases from 1.17 to 1.14, partially masking the asymptotic bias caused by the credit channel misspecification. Finally, the capital adjustment cost parameter and the other shock related parameters stay virtually unchanged in this experiment.

Comparing this paper's result with the other empirical studies provides further insight into the significance of the parameter invariance problem reported in this paper. Smets and Wouters (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007) both provide estimates of the structural parameters for the United States for the sample period that involves policy shift. Smets and Wouters (2005) estimate the structural parameters for two separate sample periods, one that covers the entire period which include the high inflation period in the 70's (1974-2002) and another that only covers the low inflation period (1982-2002). They report that labor supply elasticity fell from 0.69 to 0.53, price stickiness rose from 0.87 to 0.91, and the standard deviation of the government shock fell from 0.55 to 0.43. Fernandez-Villaverde and Rubio-Ramirez (2007) estimate the parameter drift using a longer time period (1955-2000) and find that the price stickiness is negatively correlated with the inflation rate. This paper's results are qualitatively in line with their findings, once the direction of the policy shift is controlled for. However, their finding on the parameter drift could be driven by the lack of credit channel in their models, triggered by the policy shift that occurred in the early 80's.

Cogley and Yagihashi (2010) and Chang, Kim and Schorfheide (2010) isolate the effect of the policy shift from the effect of the structural shift in the economy. Cogley and Yagihashi (2010) allow the misspecification to occur in the Phillips Curve in the AM and subject the economy to a disinflationary policy shift that mimics the Volcker disinflation in the early 1980's. They also find that many of the parameters fail to remain invariant across a policy shift, but the affected parameters are quite different from those in this paper. The most affected parameters include the risk aversion parameter and the persistence of technology shock, in addition to the labor supply elasticity found in this paper. If the policy direction were reversed from disinflation to reflation, their risk aversion would go up and the labor supply elasticity would go down, which would provide a totally opposite picture from the one outlined in this paper.

Chang, Kim and Schorfheide (2010) allow two misspecifications to occur simultaneously in their AM, one in the asset market and the other in the labor market. The economy is then subjected to several alternative tax policies, such as “labor tax cut” and “more transfers”. They also find that three of the four estimated parameters, including labor supply elasticity, fail to remain invariant after the policy shifts. However, in their case, the parameter invariance problem almost disappears when one of the two misspecifications is accounted for in the AM.

5 Welfare Consequence of the TDP-optimal Policy

Even though individual parameters may change by a significant amount, in sum they may not affect the conduct of policymaking from the central banker’s perspective. In this section, the welfare consequence of the reflationary policy, derived from the “new” estimates that involved misspecification, is evaluated. First, the reflationary policy is compared to another policy that is derived from the “old” estimates. Second, the same reflationary policy is compared to the policy derived by treating the FA-DGM as known.

5.1 TDP-optimal Policy and the Central Bank’s Belief

In the previous experiment, the policymaker chooses the reflationary policy using the “new” estimates and higher target inflation. We have seen that some of the parameter estimates fail to remain invariant under the reflationary policy, but for the “final” estimates to become available, policymakers would have to wait for another long time period while the new data are collected. Let us assume that sufficient time has passed for the policymaker to obtain the “final” estimates caused by their reflationary policy. How would the belief of the policymaker regarding its policy outcome be altered?

For the following experiments, the reflationary policy is renamed as the “TDP-optimal” policy in order to distinguish with the alternative policies. This policy (ρ_{Π}, ρ_Y)=$(2.24, 0)$

is optimal in the sense that it minimizes the welfare loss criterion in the TDP-AM, when treating the "new" estimates as structural. The welfare losses are then calculated under the "new" and "final" estimates while keeping the policy fixed. This will provide information on how much the losses perceived by the policymaker change across the two set of estimates. For comparison, another policy in which the "old" estimates are used in the optimizing process is added. This yields (ρ_{Π}, ρ_Y)=$(2.03, 0)$, which is slightly more accommodative to inflation than the TDP-optimal policy. I call this the "TDP-suboptimal" policy because it is necessarily suboptimal under the "new" estimates. Table 8 summarizes the result.

Table 8: Welfare Losses in AM, FA case

	TDP-optimal	TDP-suboptimal
Taylor rule coefficient: inflation ρ_{Π}	2.24	2.03
Taylor rule coefficient: output gap ρ_Y	0	0
(a) WL under "new" estimates	11.0674	11.0730
(b) WL under "final" estimates	11.2539	11.3355
Δ% from (a) to (b)	+1.68%	+2.37%

Target inflation is set to 5%. The unit of welfare loss is 1E-4 = 0.0001

As shown in the bottom row of table 8, the TDP-optimal policy suffers from 1.68% higher welfare loss caused by the parameter change. Our policymaker would simply regard this as the loss caused by another round of shifts in the structural parameters rather by its own reflationary policy. In terms of magnitude, this number 1.68% does not seem particularly large. For example, table 4 showed that even a modest reflationary policy from 2% to 3% is associated with 2.5% higher welfare loss. When we consider reflation from 2% to 5%, the implied welfare loss jumps up by 7.9%, exceeding 1.68% by a significant amount.

If the policymaker chose the suboptimal policy instead, the parameter shift from "new" to "final" would have resulted in 2.37% higher welfare loss as shown in the bottom row of table 8. When compared to the 1.68% in the optimal policy case, this shows that by not updating the estimates policymakers will suffer from a larger welfare deterioration.

However, this magnitude looks rather subdued for the same reason mentioned above. Most of the potential gain is already achieved by following the optimizing process and the parameter shifts play little role in altering the policymakers' belief. This experiment shows that the TDP-optimal policy is surprisingly robust to parameter invariance problem under the misspecification in the credit channel.

5.2 TDP-Optimal Policy in the FA-DGM

Next, the TDP-optimal policy is put to a more stringent test, namely comparing it with the "infeasible" FA-optimal policy. The FA-optimal policy is obtained by minimizing the welfare loss criterion using the FA-DGM as the underlying model. The welfare losses in the DGM are then compared under different policies. The parameters are initialized at the calibrated parameters in table 1. The TDP-optimal policy is necessarily suboptimal in the DGM since it uses the misspecified model in its optimization process. Table 9 shows the result.

Table 9: Welfare Losses in DGM

	(1) FA-optimal	(2) TDP-optimal	(3) TDP-reoptimized
Taylor rule coefficient: inflation ρ_Π	8.61	2.24	2.20
Taylor rule coefficient: output gap ρ_Y	3.33	0	0
Welfare loss WL	8.5121	9.3921	9.3859
$\Delta\%$ from FA-optimal case	-	+10.34%	+10.27%

Target inflation is set to 5%. The unit of welfare loss is 1E-4 = 0.0001

The FA-optimal policy in column (1) has large Taylor rule coefficients for both inflation and output gap, because the existence of the financial accelerator mechanism makes monetary policy with strong reaction coefficients desirable to stabilize the economy. TDP-optimal policy in column (2) adds 10.34% of welfare loss to the infeasible FA-optimal policy, which is significantly higher even if we take into account the loss from the higher target inflation rate. This can be regarded as the absolute cost of using the misspecified model, never recognized by the policymaker.

Another important question is whether the adoption of TDP-AM would lead to an adverse policy outcome in the DGM. Chang, Kim and Schorfheide (2010) report that the parameter invariance problem would lead to an incorrect welfare ordering among the policies, given that their AM is misspecified. In this paper we let the policymaker reoptimize its policy using the "final" estimates and see whether it leads to such a deterioration of welfare in the DGM. The "TDP-reoptimized" policy in column (3) of table 9 represents this case. The Taylor rule coefficient ρ_Π falls slightly from 2.24 to 2.20, reflecting the underlying parameter changes. The welfare improves slightly from 9.3921E-4 to 9.3859E-4, meaning that the TDP-AM suggests the policy change in the right direction. On the other hand, the reoptimized policy still suffers from 10.27% higher welfare loss compared to the FA-optimal policy.

Figure 2 allows us to visualize the relative performance of TDP-optimal policy in the FA-DGM, using an isoloss contour map in (ρ_Π, ρ_Y) space. The numbers in the figure are normalized so that the FA-optimal rule produces a relative loss of 1 and the contour lines represent gross deviations from the FA-optimal policy.

In the figure 2, FA-optimal policy is located in the center of the contour map and TDP-optimal policy is at the lower left corner. Note that the TDP-optimal policy quickly improves its welfare outcome by increasing both the Taylor rule coefficients. For example, a policy $(\rho_\Pi, \rho_Y) = (2.7, 0.6)$ would achieve a welfare loss that is only 2% higher than the FA-optimal policy. However, according to the same figure, increasing only the ρ_Π would increase the welfare loss. Increasing only the ρ_Y would initially reduce the welfare loss, but soon the loss will start rising again and eventually induce indeterminacy of equilibrium in the model. With only the TDP-AM as a guide, it would be extremely difficult for the policymaker to close the 10.34% welfare gap. To achieve a better welfare outcome the policymaker would have to develop a better approximating model.

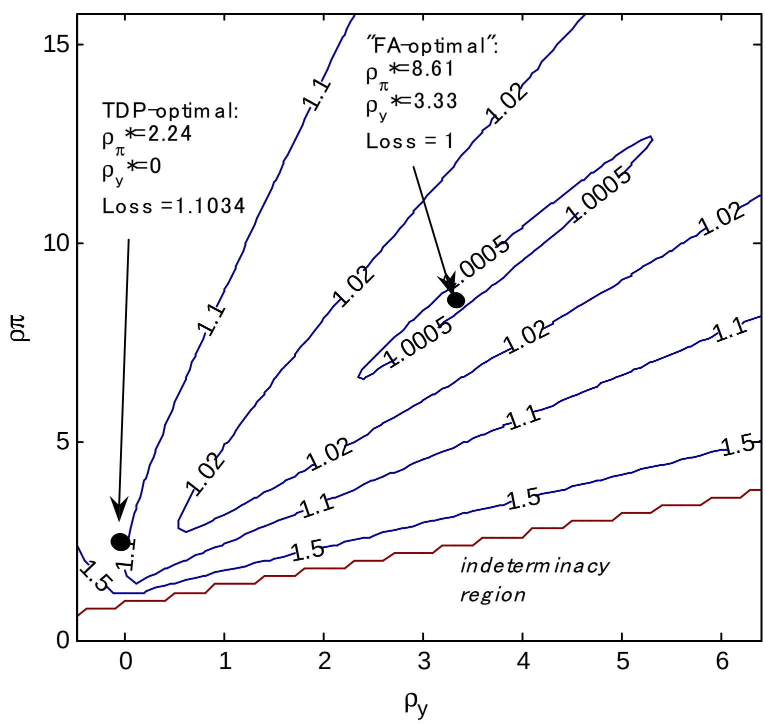

Figure 2: Contour map showing welfare loss under different policies

6 The Role of Credit Market Friction

The benchmark FA-DGM used a specific set of credit channel parameters that were calibrated from the historical data of the United States. This section examines cases with different degrees of credit market friction. Two scenarios are added, "modest" and "crisis" scenarios, which implies different values for steady-state risk premium, default rate, and leverage ratio. The parameter values for individual scenarios are obtained by solving the entrepreneur's profit maximization problem as described in Bernanke *et al* (1999). Table 10 summarizes the results.

Table 10: Steady-state Values for Different credit market frictions

	Modest	Base	Crisis
credit market friction parameter ν	0.056	0.073	0.127
Consumption spending share of output $\frac{\overline{C}}{\overline{Y}}$	0.63	0.64	0.66
Investment spending share of output $\frac{\overline{I}}{\overline{Y}}$	0.17	0.16	0.14
(Annual) Capital output ratio $\frac{\overline{K}}{4\overline{Y}}$	1.7	1.6	1.4
(Annual) Ex ante real return on capital $(\overline{R}^k)^4$	1.077	1.088	1.116
(Annual) Risk premium $(\overline{R}^k)^4 - \overline{R}^4$	3.5%	4.5%	7.4%
(Annual) Default rate $F(\omega^*)$	5.8%	6.3%	7.7%
Leverage ratio $\frac{\overline{QK}}{\overline{N}}$	1.9	1.8	1.6

The case with baseline parameterization is named the "base" scenario.

Note that under our parameterization higher credit market friction is associated with higher risk premium, default rate and lower leverage ratio. The modest scenario is similar to that of Bernanke *et al*'s baseline parameterization.

Next figure 3 illustrates how the financial accelerator mechanism works under different credit market friction. Dotted lines represent impulse response associated with the modest credit market friction. Dash-dotted lines represent impulse response associated with the baseline credit market friction. Finally, solid lines represent impulse response associated with the crisis level credit market friction. For all cases, the target inflation is 2% and the Taylor rule parameters represent the low inflation policy.

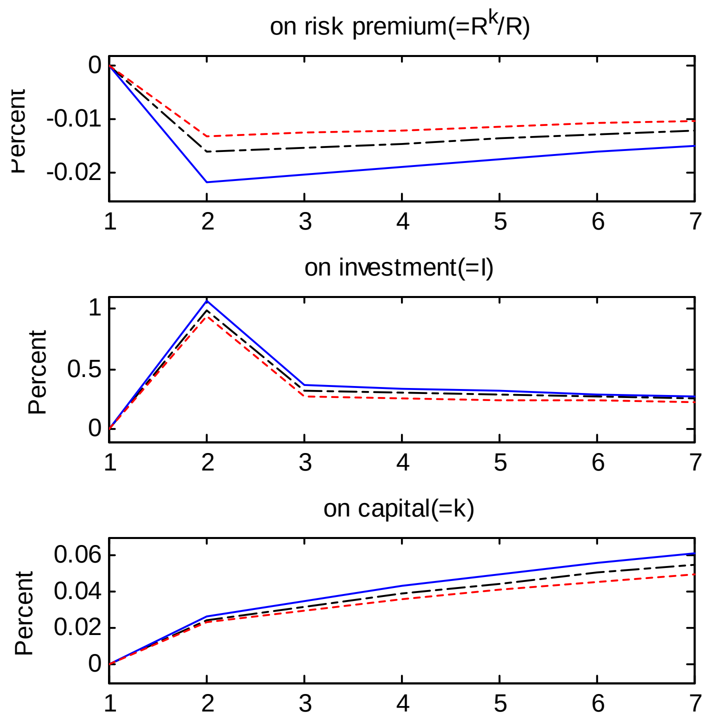

Figure 3: Impulse Responses to an 1% Expansionary Monetary Policy Shocks: Modest friction (Dotted line), Baseline friction (Dash-dotted line) and Crisis level friction (Solid line)

Figure 3 confirms that larger credit market friction produces larger amplification in the investment dynamics through the behavior of risk premium. The initial difference is propagated through the net worth dynamics and eventually translate into different capital accumulation.

Table 11 summarizes the change in the pseudo-true values between the "new" estimates and the "final" estimates.

Table 11: Change in the Pseudo-true Values

	Modest	Base	Crisis
Risk aversion σ	-0.015	-0.024	-0.028
Labor supply elasticity τ	-0.006	$+1.150$	$+0.472$
Capital adjustment cost χ	-0.003	-0.004	-0.005
Price stickiness ρ	$+0.001$	-0.027	-0.054
Persistence of tech. shock ρ_A	± 0.000	$+0.004$	$+0.004$
Persistence of gov. shock ρ_G	± 0.000	-0.001	-0.005
Size of tech. shock σ_A	-0.0001	-0.0002	-0.0001
Size of gov. shock σ_G	$+0.0013$	$+0.0015$	$+0.0015$
Size of cost push shock σ_{cp}	± 0.0000	$+0.0001$	$+0.0001$
Size of mon. policy shock σ_m	± 0.0000	± 0.0000	± 0.0000
cf. credit market friction in DGM ν	0.056	0.073	0.127

Each number represent change from "new" estimates to "final" estimates

Overall, the magnitude of the individual parameter change is closely associated with the degree of credit market friction ν presented in the bottom row. For example, the change in the risk aversion parameter σ is -0.015 when $\nu = 0.056$, but that magnitude increases to -0.024 and -0.028 as ν increases to 0.073 and 0.127 respectively. Similarly, price stickiness ρ is almost unaffected in the modest case (+0.001) but falls substantially in the crisis case (-0.054).[25] An interesting case is the labor elasticity τ, which has the largest change in the baseline case (+1.150) compared to the other two cases (-0.006, +0.472). The same can be observed for the standard deviation for the technology shock σ_A , although with a lesser magnitude. This shows that parameter invariance problem is not simply monotonically related to the credit market friction, even though in most of the other parameters it appears that way.

[25] In terms of implied duration between price changes, the latter falls by roughly one month.

Next, I examine how the central bank's belief regarding their policy outcome is affected by the parameter invariance problem. This can be measured through the change in the welfare loss in the TDP-AM. Table 12 summarizes this result.

Table 12: Welfare Losses in AM with Different credit market frictions

	Modest	Base	Crisis
Taylor rule coefficient: inflation ρ_Π	2.2440	2.2441	2.1415
Taylor rule coefficient: output gap ρ_Y	0	0	0
(a) WL under "new" estimates	10.9502	11.0674	11.1453
(b) WL under "final" estimates	10.9376	11.2539	11.2255
Δ% from (a) to (b)	-0.12%	+1.69%	+0.72%
cf. credit market friction in DGM ν	0.056	0.073	0.127

Target inflation is set to 5%.The unit of welfare loss is 1E-4 = 0.0001

As in the baseline case, the welfare losses do not vary much under different credit market frictions. Under the modest credit market friction in the second column, the welfare improves by 0.12% from the first estimate to the second, unlike in other cases. This demonstrates that the parameter invariance problem is a statistical property rather than grounded on the economic insight. Under the crisis case in the last column, the parameter invariance problem causes less change in the welfare loss (+0.72%) compared to the baseline case in the third column (+1.69%). This could potentially mislead the policymaker to believe that the their approximating model is performing well, when it is actually performing poorly.

This section provides two additional insights to the main result. First, for the majority of the parameters, the degree of parameter shift is closely related to the degree of credit market friction, but there are also a few exceptions, such as labor elasticity. Second, the parameter invariance problem does not seem to play much of a role in altering the central banker's belief, but it also shows that the resulting welfare loss cannot be simply attributed to the degree of credit market friction in a straightforward manner. This might in practice hinder central bankers from developing a model that helps minimize the potential welfare loss.

7 Conclusion

This paper examines the parameter invariance problem of the workhorse DSGE model, when the misspecification occurs in the credit channel. A financial accelerator (FA) model was chosen to represent the complicated reality and a simpler time-dependent pricing (TDP) model was chosen as the approximating model. Next, a reflationary monetary policy that involves an increase in the target inflation together with a re-optimized Taylor rule was simulated, given the estimated structural parameters. After such policy shift, some of the parameters significantly fail to remain invariant due to the model misspecification. In the unknown DGM, the TDP-optimal policy performs significantly worse compared to the infeasible FA-optimal policy that treats the complicated reality as known. However the same TDP-optimal policy shows overall robustness to the parameter invariance problem, justifying its use even during the financial turmoil. Finally, the paper examined how the alternative parameterization of the credit market friction would affect the main result. It finds that parameter invariance problem cannot be simply attributed to the credit market friction that reflects the degree of misspecification in the approximating model.

One possible limitation to this paper is that the model uses the presumably *ad hoc* welfare loss criterion when computing the Taylor rule coefficients. Even though this is a straightforward approach for this type of welfare problem and used by many other researchers, for some it would be more convincing to apply the model-consistent welfare criterion with the focus on the household sector. Alternatively one could do a robust control analysis using the same objective function, a method that has become increasingly popular among the central banking community. These are left as interesting future extensions.

Another possible limitation is that the models was kept relatively small in favor of parsimonicity. Some might prefer to repeat the same exercise in a richer model setting such as Smets and Wouters (2003)'s model. However, making the model large could mask the true cost of omitting the credit channel because of the additional parameters that needs to be estimated. Therefore expansion of the model should be treated with much care.

It is important to note that simply adding financial accelerator mechanism into the model would not immediately reduce the model uncertainty that policymakers face. Using the Bayesian language there is no single "true" model that encompasses the whole reality. Hence misspecification that we consider is only relative to other misspecified models. In other words, the experiment in this paper is useful only within the model space that are considered at a given point in time. Future credit channel models can be added to the pool of candidates and compared to the existing ones.

Recently, many new generation credit channel models have become available to the policymaker. It is becoming easier for policymakers to build a large-scale DSGE model which looks reasonable at a first sight. However, as mentioned above, large models involve many hidden costs, not just in terms of resources used to build and maintain, but also through making the policy decision process less intuitive and hard to follow for everyone else. Therefore, the gain of having credit channel structure in the workhorse model should be examined carefully by considering the type of policy options that is available to the policymaker.

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