

Frictional Wage Dispersion in Search Models: A Quantitative Assessment

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Abstract

In a large class of search models we derive a tight prediction for a measure of frictional wage dispersion—the mean-min wage ratio—that depends on statistics of labor market turnover (unemployment inflow and outflow rates, job-to-job transitions) and preference parameters (discount rate, value of non-market time), but is independent of the wage offer distribution. For plausible parameterizations of preferences, the observed magnitude of worker flows implies that in the basic search model, and in most of its extensions, frictional wage dispersion is very small. Notable exceptions are some of the most recent models of on-the-job search, where sizeable frictional wage dispersion can coexist with observed labor market transitions. Because of its broad applicability, our new metric allows us to rationalize the diverse empirical findings in the large literature estimating structural labor-market models with frictions. Our findings are also relevant for business-cycle studies, as they reveal that search models face a trade-off between frictional wage dispersion in the cross-section and cyclical unemployment fluctuations in the time-series.

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1 Introduction

Does the law of one price hold in the labor market, i.e., are identical workers paid the same wage? We use the term *frictional wage dispersion* for any departures from the law of one price, and the goal in this paper is to assess its quantitative magnitude. Since the labor market is the main source of income for most individuals, the amount of frictional wage inequality should be informative for anyone interested in efficiency, equity, and the provision of social insurance.¹

Our approach is to use observed worker choice, along with search theory (as in McCall, 1970, Mortensen, 1970, and countless follow-up studies), to infer the frictional component of wage dispersion.² The specific observations that we exploit are worker flow data from the labor market (such as unemployment duration and separation rates). Moreover, we place quantitative restrictions on the preference parameters that appear in our search models (discount rates and the value of non-market time).

A central methodological finding in our paper is that for the wide range of search models we consider, one can derive a measure of frictional wage dispersion without any knowledge of the wage-offer distribution the worker is drawing from. This measure is the “mean-min wage ratio”, i.e., the ratio of the average accepted wage to the lowest accepted, or reservation, wage. It turns out that the mean-min ratio in all these search models can be related, in simple closed form, to the preference parameters and the flow statistics; in the very most basic search model, for example, the mean-min ratio is an easily interpretable function of the discount rate, value of non-market time, separation rate, and unemployment duration. This tight relation between the mean-min ratio and a small set of model parameters is merely an implication of optimal job search behavior, and it is independent of the particular equilibrium mechanism underlying the wage-offer distribution.

Thus, we argue that it is a misconception to think that search models can generate any amount of wage differentials as long as the wage-offer distribution is sufficiently dispersed, because there is a causal link—dictated by optimal job search behavior—between the wage

¹A similar view is expressed in Mortensen (2005, page 2).

²Direct measurement, using cross-sectional data on individual wages, is an alternative strategy. If one could precisely measure the empirical distribution of wages for similar workers employed in the same labor market, this strategy would be viable. In practice, however, individual survey data in the U.S. offer very limited information on worker and job characteristics. All one can hope for is an estimate of “residual” wage dispersion, e.g., from a Mincerian wage regression, which contains its frictional component, as well as components that are unrelated to frictions (e.g., unobserved worker heterogeneity, compensating differentials, measurement error). In Hornstein et al. (2007a), we use different data sources and methodologies to arrive at estimates of residual wage dispersion.

offer dispersion and worker flows, on which there is reliable survey data. Put differently, for given preference parameters, the amount of frictional wage dispersion in search models is constrained by the observed size of the transition rates of workers.

When calibrating the baseline search model, for plausible values of preference parameters, the observed labor-market transition rates imply a mean-min ratio of a mere 1.05, i.e., the average wage can only be five percent above the lowest wage paid. Through the lenses of this particular class of search models, deviations from the law of one price are thus negligible. The key reason for this finding is the short duration of unemployment spells in the data. Intuitively, given that unemployed workers choose to take jobs quickly, they must not perceive a high option value of waiting for better job offers. In the basic model, this option value is determined precisely by wage dispersion. Taking workers' flow data at face value, one can escape the conclusion of a very low mean-min ratio only if workers are implausibly impatient or have an implausibly low (indeed negative) value of non-market time.

In the second part of the paper, we ask what type of changes in the basic model would allow greater frictional wage dispersion to coexist with empirical observations on worker flows. The basic search model relies on four assumptions that we relax one by one: (i) perfect correlation between job values and initial wage; (ii) risk neutrality; (iii) random search; and (iv) no on-the-job search. For all of these extensions, we can derive closed-form expressions for the mean-min ratio that are easy to interpret and evaluate quantitatively. The generality of our analysis builds on the fact that a solution for the mean-min ratio always arises very naturally from the reservation wage equation, the cornerstone of optimal job search.

The three first extensions—where we allow for imperfect correlation between job value and wages, for risk aversion, and for directed search—do not lead to anything but very minor adjustments of the benchmark mean-min ratio. Thus, our basic quantitative finding that frictional wage dispersion is small holds in a wider range of search models.

The fourth extension, allowing on-the-job search, has much richer implications. Intuitively, if workers can also search while employed, they may optimally choose to exit quickly from unemployment even in presence of a very dispersed distribution of wage offers, since they do not completely give up the option value of search. The mean-min wage ratio derived from Burdett's (1978) job ladder model shows that the higher the arrival rate of offers on the job, the larger is frictional wage dispersion in the model. Therefore, in models of on-the-job search, it is the frequency of job-to-job transitions, besides unemployment duration, that constrains the magnitude of frictional wage dispersion. Under

the same calibration for preferences as in the baseline model, now the observed labor market flows can coexist with wage dispersion up to five times as large, i.e., the mean-min wage ratio is around 1.25. Therefore, through the lenses of models with on-the-job search, deviations from the law of one price are more significant, albeit still fairly modest in absolute size.

We continue our study of models with on-the-job search by considering environments where search effort is endogenous (Christensen et al., 2005), and where employers have the ability to make counteroffers (Postel-Vinay and Robin, 2002a, 2002b; Dey and Flinn, 2005; Cahuc, Postel-Vinay and Robin, 2006) or to offer wage-tenure contracts (Stevens, 1999; Burdett and Coles, 2000). In these versions of the model, our analysis based on the mean-min ratio reveals that much greater frictional wage dispersion can be attained, i.e., mean-min ratios above 2, without any severe conflict with available flow data.³

Our result that the size of frictional wage dispersion is constrained by observed worker flows is useful as an organizing tool for interpreting the findings of the empirical literature where search models are structurally estimated by jointly fitting cross-sectional data on worker flows and the whole wage distribution (see Eckstein and Van den Berg, 2007, for a recent survey). Unlike our approach, which does not use wage data, this literature takes the view that worker characteristics can be either proxied by observables, or estimated. Typically, it finds very low (negative and large) estimates of the value of non-market time, extremely high estimates of the interest rate, substantial estimates of unobserved worker heterogeneity, or very large estimates of measurement error.⁴

The relevance of the value of non-market time in our analysis time does suggest a potential link to the recent debate on the ability of search models to replicate the business-cycle volatility of unemployment and vacancies (see, e.g., Hall, 2005; Shimer, 2005; Hagedorn and Manovskii, 2008): while high values of non-market time are needed for large unemployment volatility in the time-series, low, often negative, values are required for large wage dispersion in the cross-section. An important remark is that, as elaborated upon above, our analysis exploits the idea that labor market transitions, such as unemployment outflow rates, reflect an optimal response to the perceived wage dispersion. Unemployment, however, can of course be caused more directly by frictions, such

³Of course, there are other implications that still need to be confronted in order to assess these settings fully—such as details of search effort costs, existence of entry fees into jobs, ability of firms to commit, and the determination of compensation when workers move across jobs—which are beyond the scope of this paper.

⁴When a successful fit is achieved without the need for sizeable measurement error or heterogeneity, often the consequences of the parameter estimates for the model-implied value of non-market time and/or the discount rate are not fully pursued. See Section 8 for details.

as in the canonical single-productivity matching (Pissarides, 1985) or mismatch (Shimer, 2007a) models, and thus our analysis does not have any implications for these settings.

Finally, we note that there are few other attempts in the literature to directly quantify the extent of actual search frictions. While we investigate wage dispersion induced by frictions, Gautier and Teulings (2006) measure the output loss from misallocation when adding random search to an assignment model of the labor market with heterogeneous workers and firms, and Van den Berg and Van Vuuren (2008) estimate how average wages are affected by search frictions proxied by the number of job offers per unit of time.

The rest of the paper is organized as follows. Section 2 derives the expression for the mean-min ratio in the canonical search model. Section 3 quantifies its implications. Sections 4, 5, 6, and 7 outline the four significant generalizations of the canonical model and evaluate them quantitatively. Section 8 discusses the empirical search literature from the perspective of our findings. Section 9 concludes the paper. Several of the theoretical propositions in the present paper are proved in a separate Technical Appendix: Hornstein, Krusell, and Violante (2009).

2 A new measure of wage dispersion

We begin with the canonical search model of the labor market, the sequential search model developed by McCall (1970) and Mortensen (1970). We formulate this model, and all the others we study, in continuous time. We show that one can easily derive an analytical expression for a particular measure of frictional wage dispersion: the mean-min ratio, i.e., the ratio between the average wage and the lowest wage paid in the labor market to an employed worker. Next, we argue that allowing the wage offer distribution, exogenous in the basic search model, to be determined in equilibrium has no impact on the result.

2.1 The sequential search model

Consider an economy populated by ex-ante equal, risk-neutral, infinitely lived individuals who discount the future at rate r . Unemployed agents receive job offers at the instantaneous rate λ_u . Conditionally on receiving an offer, the wage is drawn from a well-behaved distribution function $F(w)$ with upper support w^{\max} . Draws are i.i.d. over time and across agents. If a job offer w is accepted, the worker is paid a wage w until the job is exogenously destroyed. Separations occur at rate σ . While unemployed, the worker receives a utility flow b which includes unemployment benefits and a value of leisure and home production, net of search costs and of the direct disutility of being jobless. In what follows, we refer

to b as the net value of non-market time. Thus, we have the Bellman equations

$$rW(w) = w - \sigma [W(w) - U] \quad (1)$$

$$rU = b + \lambda_u \int_{w^*}^{w^{\max}} [W(w) - U] dF(w), \quad (2)$$

where $rW(w)$ is the flow (per period) value of employment at wage w , and rU is the flow value of unemployment. In writing the latter, we have used the fact that the optimal search behavior of the worker is a reservation-wage strategy: the unemployed worker accepts all wage offers w above $w^* = rU$, at a capital gain $W(w) - U$. Solving equation (1) for $W(w)$ and substituting in (2) yields the reservation-wage equation

$$w^* = b + \frac{\lambda_u}{r + \sigma} \int_{w^*}^{w^{\max}} [w - w^*] dF(w).$$

Without loss of generality, let $b = \rho \bar{w}$, where $\bar{w} = \mathbb{E}[w | w \geq w^*]$. Then,

$$\begin{aligned} w^* &= \rho \bar{w} + \frac{\lambda_u [1 - F(w^*)]}{r + \sigma} \int_{w^*}^{w^{\max}} [w - w^*] \frac{dF(w)}{1 - F(w^*)} \\ &= \rho \bar{w} + \frac{\lambda_u^*}{r + \sigma} [\bar{w} - w^*], \end{aligned} \quad (3)$$

where $\lambda_u^* \equiv \lambda_u [1 - F(w^*)]$ is the job-finding rate. Equation (3) relates the lowest wage paid (the reservation wage) to the average wage paid in the economy through a small set of model parameters.

If we now define the mean-min wage ratio as $Mm \equiv \bar{w}/w^*$ and rearrange terms in (3), we arrive at

$$Mm = \frac{\frac{\lambda_u^*}{r + \sigma} + 1}{\frac{\lambda_u^*}{r + \sigma} + \rho}. \quad (4)$$

The mean-min ratio Mm is our new metric for frictional wage dispersion, i.e., wage differentials entirely determined by luck in the search process. This measure has one important property: it does not depend *directly* on the wage offer distribution F . Put differently, the theory allows predictions on the magnitude of frictional wage dispersion, measured by Mm , without requiring any information on F . The reason is that all that is relevant to know about F , i.e., its probability mass below w^* , is already contained in the job finding rate λ_u^* , which we can measure directly through labor market transitions, and treat as a parameter.

The model's mean-min ratio can thus be written as a function of a four-parameter vector, $(r, \sigma, \rho, \lambda_u^*)$. Thus, looking at this relation, if we measure the discount rate r to

be high (high impatience), for given estimates of σ , ρ , and λ_u^* , an increased Mm must follow. Similarly, a higher measure of the separation rate σ increases Mm (it reduces job durations and thus decreases the value of waiting for a better job opportunity). A lower estimate of the value of non-market time ρ would also increase Mm (agents are then induced to accept worse matches). Finally, a lower contact rate λ_u pushes Mm up, too (it makes the option value of search less attractive).⁵

2.2 Equilibrium determination of the wage offer distribution

The fact that in the sequential search model the wage offer distribution $F(w)$ is exogenously given is not at all restrictive. To make this argument more formally, consider the two standard equilibrium versions of the canonical search model, the island model of Lucas and Prescott (1974), and the matching model of Pissarides (1985), in its version with heterogeneous match productivities.

Island model Consider the stylized version of the island model described in Rogerson, Shimer, and Wright (2006). The economy has a continuum of islands. Each island is indexed by its productivity level p , distributed as $F(p)$. On each island there is a large number of firms operating a linear technology in labor $y = pn$, where n is the number of workers employed. In every period, there is a perfectly competitive spot market for labor on each island. An employed worker is subject to exogenous separations at rate σ . Upon separation, she enters the unemployment pool. Unemployed workers search for employment and at rate λ_u they run into an island drawn randomly from $F(p)$.

Competition among firms drives profits to zero, and thus in equilibrium $w = p$. It is immediate that one obtains exactly the same set of equations (1)-(2) for the worker. As a consequence, the search island model yields the same reservation wage equation (3), and the same expression for Mm as in (4).

Matching model The equilibrium matching model of Pissarides (1985) –extended to allow for heterogeneous match productivities– has three key additional features relative to the sequential search problem described in Section (2.1). First, there is free entry of vacant firms (or jobs). Second, the flow of contacts between vacant jobs and unemployed workers is governed by an aggregate matching technology. Third, workers and firms are

⁵In Hornstein et al. (2007a), we compare the Mm ratio to other commonly used dispersion measures. We point out that in the context of the literature on “ideal” inequality indexes (Cowell, 2000), the Mm ratio has the same properties as quantile ratios, a class of indexes that is routinely used in the empirical inequality literature. We also show, for example, that if the wage distribution belongs to the Gamma family with shape parameter γ , then $cv = \frac{1}{\sqrt{\gamma}} \left[\frac{Mm-1}{Mm} \right]$ where cv is the coefficient of variation.

ex-ante equal, but upon meeting they jointly draw a value p , distributed according to $F(p)$, which determines flow output of their potential match. Once p is realized, they bargain over the match surplus in a Nash fashion and determine the wage $w(p)$.

In the Technical Appendix, we prove that this matching model implies exactly the same reservation wage equation (3), and the same expression for Mm (4) as in the canonical model. Intuitively, free entry and the matching function only play the role of determining endogenously the value for λ_u . Nash bargaining provides a mapping between the exogenous distribution of match productivities $F(p)$ and the wage offer distribution $F(w)$.⁶

Similarly, suppose that efficiency-wage theory lies behind F : identical workers may be offered different wages because different employers have different assessments of what wage is most appropriate in their firm to elicit effort. From the worker's perspective, the end result is still a wage distribution F from which they must sample. In conclusion, the equation based on the Mm ratio appears to be of remarkably general use in order to understand frictional wage dispersion. It is independent of the particular equilibrium mechanism underlying the wage offer distribution F . It is merely an implication of optimal search behavior of homogeneous workers –all facing the same set of labor parameters $(r, \sigma, \rho, \lambda_u, F)$ – in any model with 1) perfect correlation between job values and initial wages, 2) risk neutrality, 3) random search, 4) no on-the-job search.

3 Quantitative implications for the mean-min ratio

In this section, we explore the quantitative implications of the class of search models defined in Section 2 for our statistic of frictional wage dispersion, the mean-min ratio Mm . We ask the question: how large is the frictional component of wage dispersion implied by these models, when plausibly calibrated?

U.S. baseline calibration We set the period to one month.⁷ An interest rate of 5% per year implies $r = 0.0041$. Ours is a two-state model of the labor market with ex-ante equal workers. Under these same assumptions, Shimer (2007b) computes, for the period 1994:Q1-2007:Q1, an average monthly separation rate σ of 3% and a monthly job finding probability of 43%. These two numbers imply a mean unemployment duration of 2.3

⁶It is immediate to see that the same argument applies to the large class of search models with vintage capital, where the productivity of a job is linked to the technological vintage embodied in the job (e.g., Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Hornstein et al., 2007b).

⁷The Mm ratio has the desirable property of being invariant to the length of the time interval. A change in the length of the period affects the numerator and denominator of the ratio $\lambda_u^*/(r + \sigma)$ proportionately, leaving the ratio unchanged. The parameter ρ is unaffected by the period length.

months, and an average unemployment rate of 6.5%.⁸

Shimer (2005) sets ρ to 41% mainly based on average unemployment insurance replacement rates. As discussed by Hagedorn and Manovskii (2008), this is likely to be a lower bound. For example, taxes increase the value of ρ since leisure and home production activities are not taxed. Given that higher values for b will reinforce our argument, we proceed conservatively and set $\rho = 0.4$. Below, we discuss the implications of setting ρ to higher and lower values.

This choice of parameters implies $Mm = 1.046$: the model predicts a 4.6% “frictional” differential between the average wage and the lowest wage paid in the labor market. This number appears very small. What explains the inability of the most basic search model to generate quantitatively significant frictional wage dispersion, once plausibly parameterized? We answer in two ways. First, just mechanically, note that

$$Mm = \frac{\frac{\lambda_u^*}{r+\sigma} + 1}{\frac{\lambda_u^*}{r+\sigma} + \rho} = \frac{\frac{0.43}{0.0341} + 1}{\frac{0.43}{0.0341} + 0.4} = \frac{12.6 + 1}{12.6 + 0.4} = 1.046. \quad (5)$$

This “unpleasant search arithmetic” illustrates that what accounts quantitatively for our finding is that the job finding rate λ_u^* is an order of magnitude larger than the separation rate σ ; hence, the term $\lambda_u^*/(\sigma + r)$ dominates both 1 and ρ , the other two terms of the expression in (5).

The second, more intuitive, interpretation of our finding is that in the search models discussed in Section 2, workers remain unemployed if the option value of search is high. The latter, in turn, is determined by the dispersion of wage opportunities.⁹ The short unemployment durations, as in the U.S. data, thus *reveal* that agents do not find it worthwhile to wait because frictional wage inequality is tiny. The message of search theory is that “good things come to those who wait,” so if the wait is short, it must be that good things are not likely to happen.

A “European” calibration It is well known that in Europe unemployment spells last much longer, up to five or ten times more for some countries (Machin and Manning, 1999, Table 5). Does this observation mean that the search model would predict much higher frictional wage dispersion for European labor markets? Not necessarily, because what matters for the Mm ratio is unemployment durations relative to job durations

⁸We focus on the post 1994 period because estimates of job-to-job flows that we use in Section 7 refer to this period.

⁹As explained in the Introduction, in the original Pissarides (1985) model there is no productivity (or wage) heterogeneity. Equilibrium unemployment exists because of the frictions generated by the aggregate matching function, and thus unemployment duration is not linked to wage dispersion. Therefore this argument does not apply to that model.

(λ_u^*/σ) and in European data they are both longer than in the U.S. labor market. To see this, recall that steady-state unemployment is $u = \sigma / (\sigma + \lambda_u^*)$. Using this formula in expression (4) allows one to rewrite the Mm ratio as

$$Mm = \frac{\frac{\sigma}{r+\sigma} \frac{1-u}{u} + 1}{\frac{\sigma}{r+\sigma} \frac{1-u}{u} + \rho} \simeq \frac{\frac{1-u}{u} + 1}{\frac{1-u}{u} + \rho},$$

where the “approximately equal” sign is obtained by setting $r = 0$, a step justified by the fact that r is of second order compared to the other parameters in that expression. Even setting the unemployment rate to 15%, an upper bound for Europe, with $\rho = 0.4$ one obtains $Mm = 1.099$. This is a twofold improvement over the baseline, but in absolute terms it remains a small number.¹⁰

Sensitivity analysis To calibrate the pair (λ_u^*, σ) , we used the definitions of job finding and separation rates which are consistent with the two-state model of the labor market implicit in the canonical search framework. When adding an explicit “inactivity state”, Shimer (2007b) estimates the monthly employment-unemployment transition rate to be 2% and the monthly unemployment-employment transition rate to be 32% (1994:Q1-2007:Q1). Note that both λ_u^* and σ decrease, and what matters for frictional wage dispersion is their ratio. When using these new estimates together with values of r and ρ as in the baseline calibration, we obtain $Mm = 1.044$. We conclude that the precise definition of “unemployment” does not affect our findings.

With respect to the interest rate r , we have used a standard value, but it is possible that unemployed workers, especially the long-term unemployed, face a higher interest rate if they want to access unsecured credit. However, even by setting $r = 0.035$, i.e., an interest rate of over 50% per year, undoubtedly an upper bound, the implied Mm ratio is only 1.085. In Section 5 we develop this point further.

3.1 The value of non-market time

Much less is known about the value of non-market time as a fraction of the mean wage, i.e., ρ . The baseline calibration of $\rho = 0.4$ based on Shimer (2005) does not take into account direct search costs, or the psychological cost of unemployment. With a lower net value of non-market time, the observed unemployment durations would be consistent with larger frictional wage dispersion.

¹⁰Moreover, social benefits for the unemployed are much more generous in Europe than in the US. For example, Hansen (1998) calculates benefits replacement ratios with respect to the average wage up to 75% in some European countries. A larger ρ reduces the mean-min ratio.

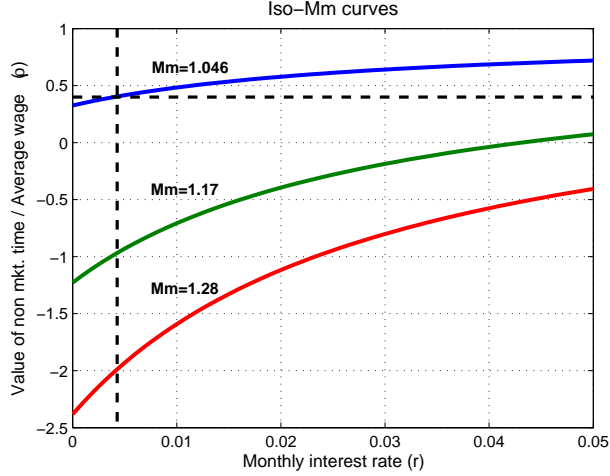


Figure 1: Sensitivity of the Mm ratio with respect to ρ and r

Search costs In the Technical Appendix, we solve a version of the baseline search model with endogenous search effort choice.¹¹ The mean-min ratio is given by

$$Mm = \frac{\frac{\lambda_u^*}{r+\sigma} \frac{1}{1+\gamma} + 1}{\frac{\lambda_u^*}{r+\sigma} \frac{1}{1+\gamma} + \rho}, \quad (6)$$

where $1/\gamma$ is the elasticity of marginal return to search with respect to the search effort level. This expression highlights the role of search costs. The larger is γ , the less sensitive are returns to search to effort. Optimal effort must be high to affect the job offer rate, and search costs increase making unemployment less attractive. In turn, this lowers the reservation wage and raises the Mm ratio. Christensen et al. (2005) estimate that γ is around one, which in the baseline calibration implies $Mm = 1.088$. Therefore, explicitly incorporating search costs of plausible magnitude goes in the right direction, but does not affect our conclusion.

“Psychological” costs of unemployment Perhaps the psychological cost of being unemployed, over and beyond search effort, is truly very large.¹² Figure 1 plots various iso- Mm curves in the (r, ρ) space, when all other parameters are set to their baseline

¹¹In the model, the unemployed agent optimally chooses λ_u by trading-off the direct disutility of search $c_u(\lambda_u)$, and the returns from choosing a higher contact rate. We follow Christensen et al. (2005) in selecting the isoelastic functional form for the search cost $c_u(\lambda_u) = \kappa_u \lambda_u^{1+1/\gamma}$ where κ_u is a scaling constant. In Section 7 we extend our analysis to a version of this model with on-the-job search.

¹²A number of authors in the health and social behavioral sciences have argued that unemployment can lead to stress-related illnesses due, e.g., to financial insecurity or to a loss of self-esteem. This psychological cost would imply an additional negative component in b . Economists have argued that this empirical literature has not convincingly solved the serious endogeneity problem underlying the relationship between employment and health status and even have, at times, reached the opposite conclusion, i.e., that there is a positive association between time spent in non-market activity and health status (e.g., Ruhm 2003).

value. The results are quite striking. Even for annual interest rates around 25% per year ($r = 0.02$), generating a “frictional” difference between the average and the minimum wage around 30% requires values of ρ below minus one. To be concrete, consider that, for a worker earning \$500 per week, $\rho = -1$ means the following: in order to avoid unemployment, she would be willing to *work for free* for a week, *pay* \$500, and at the end of the week draw a job offer from the *same* distribution she would face if she had remained unemployed. This scenario appears economically implausible.

Insisting that the disutility of unemployment is a large negative number would also have far-fetched implications for business cycle analysis, and for the rest of macroeconomics. Hall (2005), Shimer (2005), and Hagedorn and Manovskii (2008) pointed to the difficulty of frictional labor market models to generate sizeable time-series fluctuations in aggregate unemployment and vacancies. Even in presence of real wage rigidity, ρ needs to be high and positive for these models to hold any chance of producing realistic movements in both aggregate variables. More in general, this view entails that unemployment would need to be added to consumption and leisure as an explicit argument of utility functions, thus radically altering our current understanding of aggregate labor supply.

3.2 Taking stock

When plausibly calibrated to match labor market flows the baseline search model implies that wage differentials arising among similar workers purely because of search frictions are very small. Intuitively, frictional wage dispersion in search models is constrained by the size of worker flows. Our sensitivity analysis establishes the robustness of this finding. Only with large and negative values of non market time, sizeable Mm ratios can coexist with the observed unemployment duration. Even though direct knowledge of this parameter is scant, we argued that negative values of ρ are economically implausible.

The baseline model relies on 1) perfect correlation between job values and initial wages, 2) risk-neutrality, 3) random search, 4) no on-the-job search. In the rest of the paper, we relax, one by one, these four key assumptions. These extensions allows us to inspect some of the most recent, and exciting, contributions to the search literature.

4 Imperfect correlation between job values and wages

There are three main reasons why the initial wage drawn by the unemployed worker may not map one for one into the corresponding job value.

Compensating differentials Wages are only one component of total compensation.

In a search model where a job offer is a bundle of a wage and other amenities, short unemployment duration can coexist with large wage dispersion, as long as non-wage job attributes are *negatively correlated* with wages so that the dispersion of total job values is indeed small.

This hypothesis, which combines the theory of compensating differentials with search theory, does not show too much promise. First, it is well known that certain key non-wage benefits such as health insurance tend to be positively correlated with the salary, e.g., through firm size.¹³ Similarly, layoff rates are larger for low-paid jobs (see Pinheiro and Vissers, 2006 for a survey of the evidence). Second, illness or injury risks are very occupation-specific and consumption deflators are very location-specific, while our implications for frictional wage dispersion apply to labor markets narrowly defined by occupational and geographical boundaries. Third, differences in work shifts and part-time penalties are quantitatively small. Kostiuk (1990) shows that genuine compensating differentials between day and night shifts can explain at most 9% of wage gaps in selected occupations. Manning and Petrongolo (2005) calculate that part time penalties for observationally similar workers are around 3%.¹⁴

Stochastic wages If wages fluctuate randomly on the job, the initial wage offer is not a good predictor of the continuation value of employment. Since employment maintains an option value, unemployed workers will be more willing to accept initially low wage offers, which reduces the reservation wage w^* and increases frictional dispersion.

In the Technical Appendix we develop an extension of the baseline model where unemployed workers draw wage offers from the distribution $F(w)$ at rate λ_u , but now these wage offers are not permanent. At rate δ , the employed worker draws again from $F(w)$. Draws are i.i.d. over time. Separations are now endogenous and will occur at rate $\sigma^* \equiv \delta F(w^*)$.¹⁵ In this model, the Mm ratio becomes

$$Mm = \frac{\frac{\lambda_u^* - \delta + \sigma^*}{r + \delta} + 1}{\frac{\lambda_u^* - \delta + \sigma^*}{r + \delta} + \rho}. \quad (7)$$

¹³For example, the mean wage in jobs offering health insurance coverage is 15%-20% higher than in those not offering it; see Dey and Flinn (2008).

¹⁴Recently, Bonhomme and Jolivet (2009) have estimated a search model where a job has several non-wage attributes (commuting, working times, job security, etc...) beyond its monetary compensation and find that they command insignificant compensating differentials, which are often found to be of the wrong sign.

¹⁵The setup of Mortensen and Pissarides (1994) is similar to that described here, with one difference: upon employment, all workers start with the highest wage, w^{\max} , and thus they only sample from $F(w)$ while employed. In the Technical Appendix we show that the resulting Mm ratio for the Mortensen-Pissarides model is strictly bounded above by that in equation (7) below.

As $\delta \rightarrow 0$, the Mm ratio converges to equation (4) with $\sigma^* = 0$, since without any shock during employment every job lasts forever. As $\delta \rightarrow \lambda_u$, unemployed workers accept every offer above b since being on the job has an option value equal to being unemployed.

The parameter δ maps into the degree of persistence of the wage process during a job spell. In particular, in a discrete time model where $\delta \in (0, 1)$ the autocorrelation coefficient of the wage process is $1 - \delta$.¹⁶ Longitudinal data offer overwhelming evidence that wages are very persistent, indeed near a random walk at annual frequency, so plausible values of δ are close to zero.¹⁷ In conclusion, adding wage randomness with plausible persistence has negligible impact on the size of frictional dispersion in search models.¹⁸

Returns to experience If workers accumulate experience during employment, then the value of the job has a dynamic component untied to the current payoff. An unemployed worker is willing to lower her reservation wage in exchange for such long-term gains, i.e. she is willing to pay for work experience. This environment can generate a larger Mm ratio. Consider a version of the baseline search model where, during employment, experience h is expected to grow at the instantaneous rate γ . Earnings for the worker are wh , i.e. the wage offered by hiring firms is per unit of experience. In order to bound the growth of experience, we must assume that individuals quit the labor force at rate ϕ .

In the Technical Appendix, we show that in this economy the mean-min ratio of wages per unit of experience (hence among similar workers) is given by

$$Mm = \frac{\frac{\lambda_u^*}{r+\phi+\sigma-\gamma} + \frac{1}{1-\gamma/(r+\phi)}}{\frac{\lambda_u^*}{r+\phi+\sigma-\gamma} + \rho}, \quad (8)$$

an expression that includes (4) as a special case when $\phi = \gamma = 0$. The Mm rises with returns to experience γ , as expected.

We set the average duration of working life to 40 years, and assume that wages grow by a factor of 2 over the working life because of accumulated experience. Both values are fairly uncontroversial. At a monthly frequency, this parameterization implies $\phi = 0.0021$ and $\gamma = 0.0017$. Plugging these two values, together with those in Section 3, in equation (8) yields $Mm = 1.076$. Even though frictional wage dispersion almost doubles, it remains small in absolute value.¹⁹

¹⁶In the Technical Appendix we also show that a discrete time version of this model leads exactly to equation (7).

¹⁷Even for an autocorrelation of 0.5 at annual frequency –undoubtedly a lower bound for persistence– the monthly value for δ would be 0.056 implying a Mm ratio of 1.084 (in the baseline, $Mm = 1.046$).

¹⁸Recently, Alvarez and Shimer (2009) encounter this problem in a Lucas-Prescott style environment where each island/industry has very persistent wage process. In order to match the observed industry wage differentials in the cross section, they are forced to assume huge search costs (i.e., ρ negative).

¹⁹This expression is related that uncovered by Burdett, Carrillo-Tudela and Coles (2009). They incor-

5 Risk aversion

Risk-averse workers particularly dislike states with low consumption, like unemployment. Compared to risk-neutral workers, *ceteris paribus*, they lower their reservation wage in order to exit unemployment rapidly, thus allowing Mm to increase.

No storage Let $u(c)$ be the utility of consumption, with $u' > 0$, and $u'' < 0$. To make progress analytically, we assume that workers have no access to storage, i.e., $c = w$ when employed, and $c = b$ when unemployed. It is clear that this model will give an *upper bound* for the role of risk aversion: with any access to storage, self-insurance or borrowing, agents can better smooth consumption, thus becoming effectively less risk-averse.

To obtain the reservation-wage equation with risk aversion, observe that in the Bellman equations for the value of employment and unemployment, the monetary flow values of work and leisure are simply replaced by their corresponding utility values. The reservation-wage equation (3) then becomes

$$u(w^*) = u(\rho\bar{w}) + \frac{\lambda_u^*}{r + \sigma} [\mathbb{E}_{w^*} [u(w)] - u(w^*)], \quad (9)$$

with $\mathbb{E}_{w^*} [u(w)] = \mathbb{E} [u(w) | w \geq w^*]$. A second-order Taylor expansion of $u(w)$ around the conditional mean \bar{w} yields

$$u(w) \simeq u(\bar{w}) + u'(\bar{w})(w - \bar{w}) + \frac{1}{2}u''(\bar{w})(w - \bar{w})^2.$$

Take the conditional expectation of both sides of the above equation and arrive at

$$\mathbb{E}_{w^*} [u(w)] \simeq u(\bar{w}) + \frac{1}{2}u''(\bar{w}) \text{var}(w) \quad (10)$$

where $\text{var}(w)$ denotes the wage variance. Let $u(w)$ belong to the CRRA family, with θ representing the coefficient of relative risk aversion.²⁰ Then, using (10) in (9), and rearranging, we obtain

$$Mm \simeq \left[\frac{\frac{\lambda_u^*}{r + \sigma} \left(1 + \frac{1}{2}(\theta - 1)\theta \text{cv}^2 \right) + \rho^{1-\theta}}{\frac{\lambda_u^*}{r + \sigma} + 1} \right]^{\frac{1}{\theta-1}}. \quad (11)$$

porate these same considerations in a version of the Burdett and Mortensen (1998) model of on-the-job search. They derive a closed-form expression for the Mm ratio which, as ours, clearly shows that qualitatively returns to experience can increase frictional wage dispersion.

²⁰It is easy to derive third- and fourth-order approximations of the reservation-wage equation involving the coefficients of skewness and kurtosis. In Hornstein et al. (2007a) we show that our conclusions remain extremely robust to higher-order approximations.

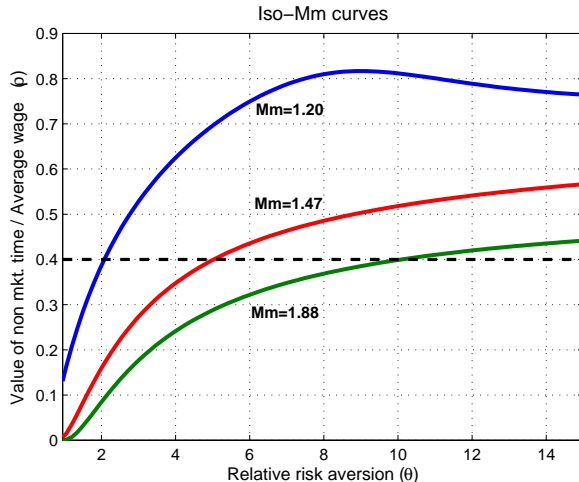


Figure 2: Sensitivity of the Mm ratio with respect to ρ and risk aversion

It is immediate to see that, for $\theta = 0$, the risk-neutrality case, the expression above equals that in equation (4).

To assess the quantitative role of risk aversion, we start with the baseline parameterization for (λ_u^*, σ, r) described on Section 3. Based on the evidence provided in Hornstein et al. (2007a), we set the coefficient of variation cv to 0.30. Figure 2 plots iso- Mm curves in the (θ, ρ) space. For $\rho = 0.4$, the baseline model with risk aversion predicts a Mm ratio around 1.2 when $\theta = 2$, a conventional risk aversion coefficient, and a Mm ratio of 1.88 for $\theta = 10$. Under this extreme view of agents' insurance possibilities and with high values for risk aversion, search models can be consistent with sizeable frictional dispersion.

Self-insurance Recall the upper-bound nature of our previous experiment: plausibly calibrated models of risk-averse individuals who have access to a risk-free asset for self-insurance purposes are much closer to full insurance than to an environment with no storage (see, e.g., Aiyagari, 1994).²¹ In such models, workers choose their precautionary savings so that their marginal utility in equilibrium becomes near constant, and hence wage (and other) outcomes are close to those in a model with linear utility. This observation severely limits risk aversion as a source of large frictional wage dispersion.

For example, Krusell, Mukoyama, and Sahin (2009) embed the Pissarides (1985) matching model into an economy where individual unemployment risk is uninsurable. Households can save through a risk-free asset, but borrowing is precluded. Differences in outside options related to cross-sectional wealth inequality induce wage dispersion among ex-ante equal workers through Nash bargaining. However, wage dispersion introduced

²¹For example, it is well known that as $r \rightarrow 0$, the bond economy with “natural debt limits” converges to complete markets (Levine and Zame, 2002).

through this channel remains extremely small for reasonable calibrations.²²

6 Directed search

The directed search model (Moen, 1997) provides an important alternative view to random search. The model assumes that firms post wages, unemployed workers observe all the wages offered and direct their search towards the most attractive firms. A higher wage attracts more applicants to the job. In turn, more applicants means a lower contact rate for the unemployed. Therefore, in directing their search, workers trade-off higher wages with longer expected unemployment durations and, in equilibrium, they are indifferent about where to apply.

As in all other derivations, to characterize the Mm ratio in this model we do not need to make any assumption on firms' wage posting behavior. It suffices to focus on workers' search behavior. Consider the value for an unemployed worker directing her search to firm i , $rU_i = b + \lambda_i(W_i - U_i)$, where λ_i is the worker's contact rate. If hired by firm i , the value of employment for this worker is $rW_i = w_i - \sigma(W_i - U_i)$, where w_i is the wage promised by the hiring firm. Combining these two equations yields

$$rU_i = \frac{b(r + \sigma) + \lambda_i w_i}{r + \sigma + \lambda_i} = rU,$$

where the second equality is the equilibrium condition requiring unemployed workers to be indifferent about where to direct their search.

Consider the condition stating workers' indifference between directing their search to the firm posting the lowest wage (w_{\min}) and to the firm posting the average wage (\bar{w}):

$$\frac{b(r + \sigma) + \lambda_{\min} w_{\min}}{r + \sigma + \lambda_{\min}} = \frac{b(r + \sigma) + \bar{\lambda} \bar{w}}{r + \sigma + \bar{\lambda}}, \quad (12)$$

where $w_{\min} < \bar{w}$ and $\lambda_{\min} > \bar{\lambda}$. In the Technical Appendix, we show that equation (12) implies the following expression for the Mm ratio in the model with directed search:

$$Mm = \frac{\left(1 + \frac{\lambda_{\min}}{r + \sigma}\right) \frac{\bar{\lambda}}{\lambda_{\min} - \bar{\lambda}} + 1}{\left(1 + \frac{\lambda_{\min}}{r + \sigma}\right) \frac{\bar{\lambda}}{\lambda_{\min} - \bar{\lambda}} + \rho} \leq \frac{\frac{\bar{\lambda}}{r + \sigma} + 1}{\frac{\bar{\lambda}}{r + \sigma} + \rho}, \quad (13)$$

where the weak inequality is obtained by letting $\lambda_{\min} \rightarrow \infty$, an upper bound (recall that λ_{\min} is the highest arrival rate in the economy). The only difference between this

²²In the Technical Appendix, we study a sequential search environment where workers with CARA preferences can borrow and save through a riskless asset. The CARA assumption allows to derive the Mm ratio in closed form. The quantitative predictions of this model for frictional wage dispersion are very similar to the risk-neutral case, even for values of relative risk aversion around ten.

upper bound for the Mm ratio and its expression for the baseline model in equation (4) is that here $\bar{\lambda}$ is not the average job finding rate, but the job finding rate in the “submarket” of firms offering the average wage. Intuitively, the idea of directed search is that a longer queue length (i.e., unemployment duration) is associated to a greater wage gain, conditional on being hired. This association between duration of search and wage gain is evocative of the mechanics of random search.

Unfortunately, $\bar{\lambda}$ is not easily observable. One of the very few data sets containing information on the number of job offers firms make, and the wage they pay is the Employment Opportunities Pilot Projects. Wolthoff (2009), who uses these data, finds almost no difference between the average job offer rate and the job offer rate of firms paying the average wage²³

7 On-the-job search

If new job offers arrive also during employment, workers are willing to leave unemployment faster since they do not entirely forego the option value of search. This property breaks the link between duration of unemployment and wage dispersion which was at the heart of our analysis so far. In this section, we extend our investigation based on the Mm ratio to several leading models of on-the-job search.

7.1 The job ladder model

We generalize the model of Section 2 and turn it into the canonical job ladder model outlined by Burdett (1978). A worker employed with wage \hat{w} encounters new job opportunities w at rate λ_e . These opportunities are randomly drawn from the wage offer distribution $F(w)$ and they are accepted if $w > \hat{w}$. A large class of equilibrium wage posting models with random search, starting from the seminal work by Burdett and Mortensen (1998), derives the optimal wage policy of the firms and the implied equilibrium wage offer distribution as a function of structural parameters. As for our analysis in Section 2, it is not necessary, at any point in our derivations, to specify what F looks like. Our expression for Mm will hold for any F as long as every worker (employed or unemployed) faces the same wage offer distribution. Without loss of generality, to simplify the algebra, we posit that no firm would offer a wage below the reservation wage w^* ; thus, $F(w^*) = 0$.

²³We thank Ronald Wolthoff for providing these numbers. They are the outcome of an estimation not contained in the paper, where $\rho = 0.40$. Interestingly, even in his richer model with multiple applications—which we have not studied here—frictional wage dispersion remains tiny ($Mm = 1.03$).

The flow values of employment and unemployment are:

$$\begin{aligned} rW(w) &= w + \lambda_e \int_w^{w^{\max}} [W(z) - W(w)] dF(z) - \sigma [W(w) - U] \\ rU &= b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z), \end{aligned}$$

and the reservation-wage equation becomes

$$w^* = b + (\lambda_u - \lambda_e) \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \lambda_e [1 - F(z)]} dz. \quad (14)$$

It is easy to see that, in steady state, the cross-sectional wage distribution among employed workers is

$$G(w) = \frac{\sigma F(w)}{\sigma + \lambda_e [1 - F(w)]}. \quad (15)$$

Using this relation between $G(w)$ and $F(w)$ in the reservation-wage equation (14), and exploiting the fact that the average wage is

$$\bar{w} = w^* + \int_{w^*}^{w^{\max}} [1 - G(z)] dz, \quad (16)$$

we arrive at the new expression for the Mm ratio,

$$Mm \simeq \frac{\frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} + 1}{\frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} + \rho}, \quad (17)$$

in the model with on-the-job search. The details of this derivation are in the Technical Appendix.²⁴

Since the Mm ratio is increasing in λ_e , it should be clear that the model will have a good chance of generating a large Mm ratio for high values of λ_e . In the extreme case where the search technology is the same in both employment states and $\lambda_e = \lambda_u$, the reservation wage will be equal to b , since searching when unemployed gives no advantage in terms of arrival rate of new job offers, and $Mm = 1/\rho$.

The crucial new parameter of this model is the arrival rate of offers on the job λ_e . To pin down λ_e , note that the average employment-to-employment (EE) transition rate χ in the model is given by the expression

$$\chi = \lambda_e \int_{w^*} [1 - F(w)] dG(w) = \frac{\sigma (\lambda_e + \sigma) \log \left(\frac{\sigma + \lambda_e}{\sigma} \right)}{\lambda_e} - \sigma. \quad (18)$$

²⁴In particular, the ‘‘approximately equal’’ sign originates from one step of the derivation where we have set

$$\frac{r + \sigma}{r + \sigma + \lambda_e [1 - F(z)]} \approx \frac{\sigma}{\sigma + \lambda_e [1 - F(z)]},$$

a valid approximation since, for plausible calibrations, r is negligible compared to σ .

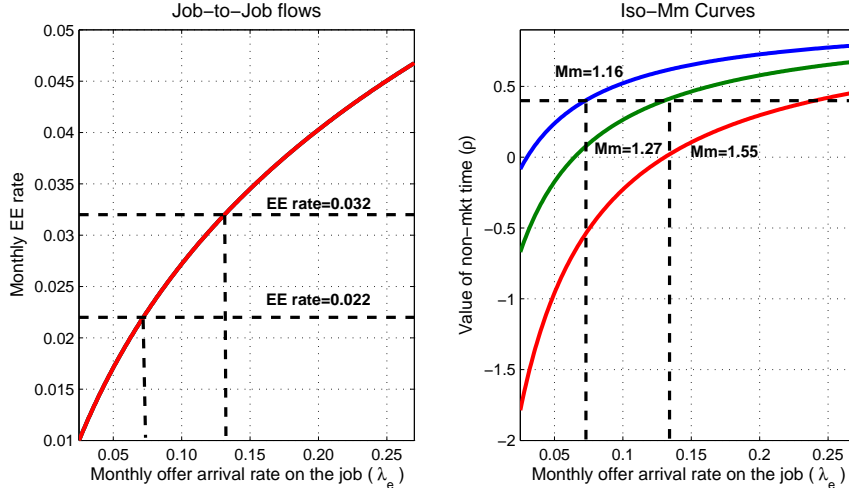


Figure 3: Sensitivity of the Mm ratio with respect to ρ and λ_e

For a given a value of the employment-to-unemployment (EU) flow rate σ , there is a one to one mapping between χ and λ_e . The most recent empirical evidence sets monthly job-to-job flows χ between 2.2% and 3.2% of employment. From SIPP data, Nagypal (2008, Table 6) sets χ to 2.2%. Based on monthly CPS data, Fallick and Fleischman (2004) estimate $\chi = 2.7\%$, and Nagypal (2008, Table 4) estimates $\chi = 2.9\%$. Moscarini and Vella (2008) argue that a different treatment of missing records in the monthly CPS leads to an upward revision of χ up to 3.2%.²⁵

In what follows, we ask how much frictional wage inequality the canonical on-the-job search model can generate, in terms of Mm ratio, while at the same time being consistent with the labor market transitions between employment and unemployment discussed in Section 3 and, in addition, a monthly job-to-job transition rate between 2.2% and 3.2%.

Figure 3 illustrates that the model with on-the-job search predicts a significantly larger frictional component of wage dispersion. Once λ_e is chosen so that the model can generate transitions in the range of U.S. data, the model produces Mm ratios between 1.16 and 1.27 when ρ is set to 0.40. Recall, from Figure 1, that the baseline model would be consistent with Mm ratios around the same values only for $\rho = -1$ and $\rho = -2$, respectively. In its most favorable, but extreme, parameterization –one where the EE rate is 3.2% and $\rho = 0$ – the model with on-the-job search is capable of generating a Mm ratio of 1.55, more than a tenfold increase with respect to the benchmark.²⁶

²⁵Maintaining that $\sigma = 0.03$, SIPP and CPS imply a total monthly separation rate between 5.2% and 6.2%. Estimates of worker flows from JOLTS are roughly consistent with this range. Davis et al. (2008) develop a method to improve statistics of worker flows based on raw survey data from JOLTS, and arrive at a monthly separation rate around 5% in 2000.

²⁶In Hornstein et al. (2007a) we derive the Mm ratio for a version of the job ladder model with stochas-

7.2 Endogenous search effort

Recently, Christensen et al. (2005, CLMNW hereafter) have extended Burdett's job ladder model allowing for the optimal choice of the offer arrival rates, both off and on the job.²⁷

Let $c_i(\lambda_i) = \kappa_i \lambda_i^{1+1/\gamma}$ be the search effort cost, as a function of the chosen arrival rate, in employment state $i \in \{u, e\}$. During unemployment, the optimal effort choice is a scalar λ_u^o . During employment, the search cost is independent of current earnings w , whereas the return to search is declining in these earnings. Thus, the optimal effort choice, $\lambda_e^o(w)$, is decreasing in w . Therefore the contact rate at the reservation wage is higher than the average contact rate in the economy, and the latter, in turn, is higher than $\lambda_e^o(w^{\max})$ where w^{\max} is the upper bound of the earnings distribution. Indeed, optimality requires $\lambda_e^o(w^{\max}) = 0$.

In the Technical Appendix we use these inequalities to derive bounds on the Mm ratio implied by the model.²⁸ In particular, if we let $\lambda_u \equiv \lambda_u^o$ and $\lambda_{w^*} \equiv \lambda_e^o(w^*)$ for notational simplicity, we have

$$\frac{\frac{\lambda_u - \lambda_{w^*}}{r + \sigma} \frac{1}{1 + \gamma} + 1}{\frac{\lambda_u - \lambda_{w^*}}{r + \sigma} \frac{1}{1 + \gamma} + \rho} \leq Mm \leq \frac{\frac{\lambda_u - \lambda_{w^*}}{r + \sigma + \lambda_{w^*}} \frac{1}{1 + \gamma} + 1}{\frac{\lambda_u - \lambda_{w^*}}{r + \sigma + \lambda_{w^*}} \frac{1}{1 + \gamma} + \rho}. \quad (19)$$

CLMNW estimate this model on Danish data. As mentioned in Section 3.1, they estimate γ to be around one. They also obtain a tight estimate for the job finding rate of workers at the reservation wage, $\lambda_{w^*} = 0.07$, that is substantially lower than the job finding rate for the unemployed, $\lambda_u = 0.11$, reported by Rosholm and Svarer (1999) on the same data.

Suppose that in the U.S. labor market λ_{w^*} is also roughly 64% of λ_u . Then, an estimate for the U.S. would set λ_{w^*} at 0.27. Using this value for λ_{w^*} in (19), together with the values for the other parameters already discussed in the paper, we obtain a lower bound of 1.22 and an upper bound of 1.91 for Mm . Therefore, it appears that the model could be consistent with a large amount of frictional wage dispersion.

But how large is the value of non-market time, *net of search costs*, implied by this parameterization? Recall from Section 3.1 that search costs, which make unemployment unattractive relative to employment, can be thought of as lowering ρ . The bounds on the Mm ratio, together with the first-order condition for optimal search effort during unemployment, allow us to construct bounds for the search cost during unemployment

tic wages. Similarly to what we argued in Section 4, the ability of this model to predict larger frictional wage dispersion than the simple on-the-job search model with constant wages is severely constrained by the observation that within an employment relationship wages are very persistent over time.

²⁷In Section 3.1, we presented a version of this same model without on-the-job search.

²⁸We thank Dale Mortensen for providing us with these derivations.

$c_u(\lambda_u)$ as a fraction of the average wage \bar{w} . In the Technical Appendix, we show that

$$\frac{\lambda_u}{r + \sigma + \lambda_{w^*}} \frac{\gamma}{1 + \gamma} \left(1 - \frac{1}{Mm}\right) \leq \frac{c_u(\lambda_u)}{\bar{w}} \leq \frac{\lambda_u}{r + \sigma} \frac{\gamma}{1 + \gamma} \left(1 - \frac{1}{Mm}\right). \quad (20)$$

Assuming an Mm ratio of 1.7, the average of the two bounds in (19), the inequalities in (20) yield a lower bound of 0.29 and an upper bound of 2.6 for the normalized search cost during unemployment. With $\rho = 0.4$, the implied net-of-search-cost value of non-market time $(\rho - c_u(\lambda_u)) / \bar{w}$ is then close to zero or, most likely, largely negative.

This calculation echoes one of the central observations of our paper: even with on-the-job search, sizeable frictional dispersion hinges upon a large disutility of unemployment.

7.3 Sequential auctions and wage-tenure contracts

In a series of recent papers, Postel-Vinay and Robin (2002a, 2002b), Dey and Flinn (2005), and Cahuc et al. (2006) have developed a new search model where firms are allowed to make counteroffers when one of their employees is contacted by an outside firm. The competition between the two employers may result either in a job-to-job move or in a salary increase on the current job, depending on the productivity gap between the two competing firms. Through the second channel, the wage distribution can fan out even without a separation occurring. Therefore, this model contains a much weaker link between wage dispersion and job-to-job flows, which is what prevents the standard on-the-job search environment from generating large frictional dispersion.²⁹

Consider a simple version of the sequential auction model where all firms have equal productivity p . The wage determination mechanism is based on Dey and Flinn (2005) and Cahuc et al. (2006). An unemployed worker extracts a fraction β of the the surplus from the firm, but an employed worker who is contacted by an outside firm extracts all the surplus from the current employer and receives wage $w = p$. It is easy to see that the reservation-wage equation (derived in the Technical Appendix) is

$$w^* = b + \frac{\beta(r + \sigma + \lambda_u) - (1 - \beta)\lambda_e}{r + \sigma + \beta\lambda_u} (p - b). \quad (21)$$

This expression implies that, for low values of workers' bargaining power β , one can achieve $w^* < b$. This is, for example, the case in the Postel-Vinay and Robin (2002a, 2002b) version of the model, where $\beta = 0$. As long as $\lambda_e > 0$, the worker expects to earn

²⁹The argument developed in our paper suggests that what would further discipline the amount of frictional wage dispersion in this model is data on the frequency of job offers matched by the current employer. Such data, at the moment, is not available.

a higher wage after she has been hired, and thus she is willing to accept a low entry-wage. As a result, $Mm > 1/\rho$.³⁰

Stevens (1999) and Burdett and Coles (2001) generalize the wage-posting model of Burdett and Mortensen (1998) by allowing employers to offer long-term wage-tenure contracts on a take-it-or-leave-it basis. This long-term wage tenure contract is observationally equivalent to the wage path arising in the counteroffer model with $\beta = 0$. Consider the same simple economy with homogenous firms, and restrict attention to a family of contracts (w^*, w_T, λ_T) such that the entrant worker is paid w^* (optimally determined by the firm in equilibrium) and then, at rate λ_T , i.e., after an expected tenure length of $1/\lambda_T$, her wage jumps up to w_T . Clearly, an optimal contract that minimizes costly turnover for the firm sets $w_T = p$. This wage contract is isomorphic to the wage in the counteroffer model if $\lambda_T = \lambda_e$. This equivalence means that in search environments with wage-tenure contracts, frictional wage dispersion can be large, even allowing $Mm > 1/\rho$.

Although the sequential auction model is undoubtedly a good representation for certain high-skill occupations (e.g., academic jobs), it does not appear to be a widespread mechanism for wage setting in the labor market at large.³¹ An additional limit of this model is that, for high values of λ_e , the entry wage w^* may be negative.³² The version with wage-tenure contracts avoids this problem by restricting contracts so that $w^* = \omega \in (0, b)$ with equation (21) determining the expected tenure $1/\lambda_T$ at which the worker sees his salary increasing to p . A possible drawback of the Stevens-Burdett-Coles model is its heavy reliance on the assumption that firms can commit to long-term contracts. In practice, it appears that renegotiation is frequent.³³

Conditional on these shortcomings, whose analysis is beyond the scope of our paper, this new set of search environments seems to admit sizeable frictional wage dispersion while, at the same time, matching observed labor market flows.

³⁰Papp (2009) develops a general equilibrium version of this model with heterogeneous firms and shows that, once parsimoniously parameterized, it can generate Mm ratios beyond 2.0 for $\rho = 0.4$ and r set at 4% per year, while at the same time being consistent with the empirically observed size of labor market flows.

³¹One reason is that in many labor markets asymmetric information may prevent the firm from being able to verify the outside offer. See Mortensen (2005) for a discussion of why counteroffers are uncommon in actual labor markets.

³²In the Technical Appendix, we show that if $\beta = 0$, our baseline calibration would indeed imply that $w^* < 0$, independently of p .

³³Kiyotaki and Lagos (2006) develop an equilibrium model of two-sided, on-the-job search with no commitment (i.e., with continuous renegotiation) where the worker, even when facing only one employer, always has the chance of making a take-it-or-leave-it offer with a fixed probability.

8 Relation to structural estimation of search models

Since the pioneering effort of Flinn and Heckman (1983), a rather vast literature on structural estimation of search models has developed (see Eckstein and van den Berg, 2007, for a recent survey). These contributions have generated many valuable insights on the functioning of labor markets and on policy analysis. From our perspective though, it is important to highlight the difficulty of these models to simultaneously match wage dispersion and labor market flows with plausible parameterization, and without resorting to measurement error or unobserved skill heterogeneity to soak up the large wage residuals in the data. We now proceed to discussing a number of examples from the literature.

In one of the first attempts at a full structural estimation, Eckstein and Wolpin (1990) estimate the Albrecht and Axell (1984) search model with worker heterogeneity in the value of non-market productivity and conclude that their model cannot generate any significant wage dispersion, and that almost all of the observed wage dispersion is explained through measurement error. Eckstein and Wolpin (1995) reach a far better match between model and data, by introducing a five-point distribution of unobserved worker heterogeneity within each race/education group (8 groups in total). In spite of such heterogeneity, however, for many of the groups the estimates of b remain extremely small or negative (see their Table 7, page 284). In this work, thus, wage dispersion is for the most part accounted for by heterogeneity in observable and unobservable characteristics. In our view, this procedure, which is quite frequent in this literature, can perhaps be categorized more as part of the human-capital theory of wages: it delivers wage inequality, but this inequality is not frictional in nature.³⁴

Negative estimates of the net value of non market time are quite common. The survey paper by Bunzel et al. (2001) estimates several models with on-the-job search on Danish data. When firms are assumed to be homogeneous, the point estimate for ρ is -2 . With heterogeneity in firms' productivity it increases to just about zero. Only the model with measurement error produces a large and positive estimate of ρ .³⁵ Flinn (2006) estimates a Pissarides-style matching model of the labor market, without on-the-job search, to evaluate the impact of the minimum wage on employment and welfare. In his model, as

³⁴A theoretical argument has also been raised against this kind of model of frictional wage dispersion. Gaumont et al. (2006) demonstrate that wage dispersion in an Albrecht and Axell (1984) model with worker heterogeneity in the value of leisure is fragile. As soon as an arbitrarily small search cost is introduced, the equilibrium unravels and we are back to the "Diamond paradox", i.e., to an equilibrium with a unique wage.

³⁵These values for ρ are obtained from Bunzel et al. (2001) by dividing the estimates of b for the entire sample, in Tables II and V, by the average wage from Table I.

is typical in estimation exercises, the pair (ρ, r) is not separately identified. Setting r to 5% annually in his model implies roughly $\rho = -4$.³⁶

An example of extreme parameter estimates can even be found in Postel-Vinay and Robin (2002a)—one of the most successful models in generating large frictional dispersion. Under risk neutrality, their estimates of the discount rate r always exceed 30% per year in every occupational group, reaching 55% for unskilled workers, where they find no role for unobserved heterogeneity. Recall, from our analysis of Section 3 that a negative value for ρ and a high value for r are two sides of the same coin.

Whenever authors restrict (r, ρ) to plausible values ex-ante, not surprisingly in light of our results, they end up finding that frictions play a minor role. For example, Van den Berg and Ridder (1998) estimate the Burdett-Mortensen model on Dutch data allowing for measurement error and observed worker heterogeneity (58 groups defined by education, age and occupation). They set r to zero and b to equal unemployment benefits for each group, i.e., roughly 60% of the average wage. They conclude that observed heterogeneity and measurement error account for over 80% of the empirical wage variation. Moscarini (2003) develops an equilibrium search model where workers learn about their match values, based on Jovanovic (1979). When the model is calibrated, r is set to 5% annually and ρ to 0.6. His model generates a Mm ratio of just 1.16 (Moscarini, 2005, Table 2).

A number of papers in the literature claim that the (on-the-job search) model is successful in simultaneously matching both the wage distribution and labor market transition data (see, e.g., Bontemps et al., 2000; Jolivet et al., 2005). These claims of success need to be properly reinterpreted in light of our findings. The typical strategy in these papers is, first, to estimate the employment wage distribution $G(w)$ non-parametrically without using the search model. Next, the model is used to predict the wage-offer distribution $F(w)$ through a steady-state relationship like (15), where the structural parameters of the relationship $(\sigma, \lambda_u, \lambda_e)$ are estimated by matching transition data. Success is then expressed as a good fit (in some specific metric). However, the exercise cannot be fully accomplished because it neglects the implications of the joint estimates of $F(w)$ and of the transition parameters for the relative value of non-market time ρ (or, similarly, for the interest rate r). The key additional “test” that we are advocating would thus entail using the estimated $F(w)$ in the reservation-wage equation (14) and, given an estimate of w^* (for example, the bottom-percentile wage observed), backing out the implied value for ρ . In light of our results, we maintain that ρ would be often negative or close to zero.

In conclusion, while we recognize substantial progress in this literature, the success is

³⁶Calculations are available upon request.

often only partial when it relies on “free parameters”. In short, important parameters, such as the value of non-market time and the discount rate, respectively, are considered free parameters, i.e., values that are far from what we view as plausible are routinely “accepted” in the estimation. Alternatively, unobserved heterogeneity or measurement error must be introduced, with amounts that are also free parameters, in order to match the data. Our contribution in this context is to show that, through the lenses of the mean-min ratio, one can organize many seemingly puzzling and unrelated findings in the literature on structural estimation of search models in a unified way.

9 Conclusions

Search theory maintains that similar workers looking for jobs in the same labor market may end up earning different wages according to their luck in the search process. The resulting wage dispersion has a “frictional” nature. An important question in macro and labor economics is: how large is, empirically, this component of wage inequality? This paper has proposed a simple, but widely applicable, structural method for quantifying the frictional component of wage dispersion predicted by search models. The strategy is based on a particular measure of wage differentials, the mean-min ratio, that arises very naturally, in closed form, from the reservation-wage equation, the cornerstone of a vast class of search models. A key property of our proposed metric is that it does not require any knowledge about the wage offer distribution, and its derivation is independent of the specific equilibrium mechanism underlying the wage offer distribution.

We begun by proving that, when plausibly calibrated to match labor market transition data, the textbook search model (perfect correlation between wage and job value, risk neutrality, random search, no on-the-job search) would imply that frictions play virtually no role in determining wage inequality among ex-ante similar workers: the mean-min wage ratio is less than 1.05. Throughout the paper, we relax the key assumptions of the canonical model, one by one. While most of these generalized models predict larger frictional wage dispersion, in absolute terms its size remains modest. However, the most recent developments of on-the-job-search models, including those with endogenous search effort, sequential auctions among competing employers, and firm posting of wage-tenure contracts, seem to accommodate more easily sizeable frictional wage dispersion with labor market flows of the observed magnitude.

Besides being applicable in many search environments, our analysis based on the mean-min ratio is very transparent. This transparency has allowed us to interpret in a unified

way a number of arguably enigmatic and unrelated findings within the literature that structurally estimates search models –notably, the necessity to tolerate large measurement error, sizeable unobserved workers’ heterogeneity, or implausible parameter values in order to jointly account for both transition and wage data.

A far-reaching, and general, implication of our findings is that the smaller the value of non-market time (denoted ρ above, expressing this benefit as a *fraction* of the average wage), the larger the component of cross-sectional wage dispersion that is attributable to frictions. Recently Hall (2005), Shimer (2005) and Hagedorn and Manovskii (2008) fueled the debate on the ability of the canonical matching model (Pissarides, 2000) to generate enough time-series fluctuations in aggregate unemployment and vacancies. There, it is pointed out that the model, without introducing significant real-wage rigidity, requires a very high ρ in order to produce sharp movements in vacancy and unemployment rates. Therefore, the time-series facts necessitate a value of ρ close to one to explain the data, and cross-sectional facts demand a value of ρ below zero.³⁷ It is paramount, in future work, to keep this tension between time series and cross-section in mind while developing and using frictional theories of the labor market.

³⁷Building on our insight, Bilts, Chang and Kim (1994) explore this trade-off in the Mortensen-Pissarides (1994) version of the matching model with aggregate and idiosyncratic shocks, and find that it jeopardizes its performance: the model cannot produce both realistic cross-sectional wage dispersion and realistic cyclical fluctuations in unemployment.

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