Liquidity Constraints in a Monetary Economy*

Leo Ferraris† Makoto Watanabe‡

October 8, 2008

Abstract

This paper presents a microfounded model of money which has a consumption and an investment market. We consider an economy in which only part of the returns of investment can be pledged. A liquidity constraint arises when the pledgeable part of the returns are not enough to pay for investment costs. We show that when the liquidity constraint is binding, agents may make a cash downpayment and money can perform two roles – as a provider of liquidity services and exchange services. The liquidity constraint constitutes a channel though which under-investment occurs even at low inflation rates. Our main contribution is to provide a simple framework using a standard monetary-search approach that allows us to study the issue of liquidity and its effect on investment.

Keywords: Liquidity, Money, Search

JEL: E40

*We are grateful to Nobu Kiyotaki and Randy Wright for their insightful comments and suggestions. Thanks are also due to participants at the 2008 Money, Banking and Payments Conference in Chicago Federal Reserve Bank for useful discussions. Financial support from the Spanish government in the form of research grant, SEJ 2006-11665-C02, and research fellowship, Juan de la Cierva, is gratefully acknowledged.

†Corresponding author. Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe Madrid, SPAIN. Email: lferrari@eco.uc3m.es, Tel.: +34-91-624-9619, Fax: +34-91624-9329.

‡Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe Madrid, SPAIN. Email: mwatanab@eco.uc3m.es, Tel.: +34-91624-9331, Fax: +34-91624-9329.

1 Introduction

Money is the medium used to transfer resources on the spot, while liquidity refers to the availability of a medium to transfer resources over time. The monetary search literature initiated by Kiyotaki and Wright (1989) has been successful in providing a solid micro-foundation based on trade frictions for the emergence of money as a medium of exchange. On the other hand, a recent growing literature emphasizes the importance of financial frictions and liquidity constraints for the emergence of a medium to transfer resources over time. In particular, Kiyotaki and Moore (2001b) study the effect of limited supply of liquid assets on investment. Although, intuitively, money and liquidity would seem to be linked, these two approaches take them as separate issues.

The objective of the present paper is to explore a simple framework using a standard monetary search approach that allows us to study the issue of liquidity and its effect on investment. We are particularly interested in the relationship between money as a medium of spot trade and a medium of trade over time. Following Kiyotaki and Wright (1989), we assume that there exist frictions in spot trade. We introduce the notion of pledgeability and consider a possibility that the fundamental impediment arising in spot trade seeps into the credit market and hinders trade over time. In such an economy, agents may use money as a means of financing investment and money can perform two roles, as a provider of exchange and liquidity services.

Specifically, we consider a version of divisible money model developed by Lagos and Wright (2005) which has a consumption and an investment market. Trading on the consumption market is subject to randomness and is not observable, thereby money is used to lubricate the exchange of consumption goods. Trading on the investment market is instead frictionless. However, part of the investment returns accrue randomly to agents while they are trading on the opaque consumption market, and these returns cannot be pledged to outside investors to pay for investment costs. Thus, liquidity constraints may ensue.

Within this setup, we show that when the average productivity of the returns is large enough to cover investment costs, the investment project is self-financing and money is used on the consumption but not on the investment market. Money is used as a medium of exchange but not as a provider of liquidity. In this case, equilibrium displays a dichotomous nature: agents make an investment decision independently of liquidity concerns and the equilibrium investment is at the optimal level from purely productive point of view for all inflation rates. Thus, inflation generates distortions only in terms of consumption. On the contrary, when the average productivity of the returns is relatively small, liquidity constraints arise in equilibrium and agents make an under-investment. In this case, agents use money both to relax the liquidity constraint and to finance consumption and thus inflation generates distortions both in terms of investment and consumption – a relationship which turns out to be complementary. However, for sufficiently high inflation rates, real money balance of agents is sufficiently low and money is relatively useless as a provider of liquidity service. Therefore, agents do not use money to finance investment for high rates of inflation even if the liquidity constraint is binding.

Our paper shares features with Kiyotaki and Wright (1989) and Kiyotaki and Moore (2001a), placing – so to speak – the former at the heart of the latter. The definition of liquidity we adopt in this paper is related to that used in a non-monetary model of Holmstrom and Tirole (1998), where moral hazard is responsible for the limited pledgeability of returns. In our model, liquidity issues are linked to the role of money as a medium of exchange. Our paper is also related to Duffie, Garleanu and Pedersen (2005) and Lagos and Rocheteau (2007) who study liquidity issues in a framework where trades are mediated by specialists. A set of empirical implications of our model, among others, negative effects of inflation on investment and the existence of a threshold inflation rate above which investment is insensitive to inflation, are consistent with evidence found by Boyd, Levine and Smith (2001).

The rest of the paper unfolds as follows. Section 2 presents the model, Section 3 discusses our results and Section 4 concludes. All proofs are contained in the Appendix.

2 The Model

2.1 The environment

We use a competitive version of the divisible money model developed by Lagos and Wright (2005). Time is discrete and continues for ever. At the start of each period the economy is inhabited by a $[0, 1]$ continuum of homogeneous entrepreneurs and a $[0, 1]$ continuum of homogeneous investors. Each period is divided into three sub-periods: morning, afternoon and evening. Agents discount future payoffs at a rate $\beta \in (0, 1)$ across periods, but there is no discounting between the three sub-periods. A market is open in each sub-period. The marginal costs of all the production are measured in terms of utility, and we normalize all the marginal costs to be one. Economic activities in each sub-period are as follows.

Morning. At the beginning of each morning, each investor produces a good which we shall refer to as *an investment good*. During the morning each entrepreneur is randomly matched to one investor. An entrepreneur offers a contract to an investor in order to buy the investment good. We will be more specific about the terms of contracts below. The investment good is worth zero in the hands of the investor, but once in the hands of an entrepreneur it can generate a perishable output. An investment good q_1 yields an output with a technology, denoted by $g(q_1)$, at the end of both morning and afternoon within a given period. The function $g(\cdot)$ is twice continuously differentiable and strictly increasing and concave in its argument. It satisfies $g(0) = 0$, $g'(0) = \infty$ and $g'(\infty) = 0$, where the dash "$'$" stands for the first derivative. In what follows we shall refer to the morning output as *an early return* and the afternoon output as *a late return* of investment goods. The late return is stochastic and we will describe it shortly below. The investment is a one-period event and the investment good fully decays at the end of the afternoon. The entrepreneurs have a linear preference whenever they have an opportunity to consume these outputs.

Afternoon. After the day market has closed, another market opens during the afternoon. In this market entrepreneurs can exchange among each other a perishable good, referred to as a *consumption good*. There exists also an intrinsically worthless good, which is perfectly divisible and storable, called fiat money. The trades in the afternoon market are subject to frictions and we model the frictions following the spirit of the monetary search model of Kiyotaki and Wright (1989). There are two main ingredients. First, the trades in the afternoon market are anonymous, and so the trading histories of agents are private knowledge. This implies, among other things, that investors cannot observe the activities of individual entrepreneurs during the afternoon. Second, entrepreneurs face randomness in their preferences and production possibilities. At the beginning of each afternoon, an entrepreneur is selected to be either a buyer or a seller. The former event happens with probability $\delta \in (0, 1)$ and the latter happens with probability $1 - \delta$. Once a seller, the entrepreneur does not wish to consume the consumption goods but are able to produce and sell them on the market. At the same time, the seller's production ability implies an access to the technology $g(\cdot)$ as well, hence the seller has an opportunity to consume the late return of investment. Once a buyer, the entrepreneur does not have the production technologies but wishes to consume the consumption goods. We denote by $u(q_2)$ the utility function of the consumption goods q_2. The function $u(\cdot)$ is twice continuously differentiable, strictly increasing and concave in its argument, and satisfies $u'(0) = \infty$ and $u'(\infty) = 0$. With no ability to access the technology $g(\cdot)$, the buyer does not have an opportunity to consume the late return of the investment and the investment good he has bought during the morning decays. Finally we assume the afternoon market is competitive and so agents take the market price, denoted by p, as given.

Evening. During the evening there is another opportunity for production. Agents can produce an output with non-contractible effort. The evening market is walrasian and the output is traded at a per unit price normalized to unity. Fiat money can be traded for the output on this market at a price, denoted by ϕ, per unit.

Money. The assumptions described above, i.e, the random buyer/seller division and the anonymity of transactions, are sufficient to ensure an essential role of money as a medium of exchange in the afternoon market: the sellers must receive money for immediate compensation of their products (i.e., consumption goods). The supply of fiat money is controlled by the government so that $M = \pi M_{-1}$, where M denotes the money stock at a given period and π denotes the gross growth rate of the money supply which we assume to be constant. Subscript -1 (or $+1$) stands for the previous (or next) period. New money is injected, or withdrawn, at the end of each period in the form of lump-sum transfers or taxes by an amount denoted by τ. All agents receive transfers or are taxed equally.

2.2 Efficiency

Given the production and consumption opportunities described above, the planner treats agents symmetrically and selects an amount of investment goods q_1 and consumption goods q_2, so as to maximize the average expected utility per period,

$$[g(q_1) - q_1] + [(1-\delta)g(q_1) + \delta(u(q_2) - q_2)] .$$

The planner faces the identical objective function across all the periods. The first term in the objective function represents the total net expected utility during the morning and the second term represents the total net expected utility during the afternoon. Notice that while all entrepreneurs can consume the early return of the investment goods q_1, it is only a proportion $1-\delta$ (or δ) of entrepreneurs who can consume the late return (or the consumption goods q_2) during the afternoon.

A unique solution to the planner's problem, denoted by q_1^*, q_2^*, exists and satisfies the first-order conditions

$$(2-\delta)g'(q_1^*) = 1 \quad (1)$$

$$u'(q_2^*) = 1. \quad (2)$$

(1) equates the total expected marginal returns, measured in terms of utility, of the investment goods ($= (2-\delta)g'(\cdot)$) to its total marginal costs ($= 1$), and (2) equates the total marginal utility of the consumption goods to its total marginal costs.

2.3 Steady state monetary equilibrium

In what follows, we construct steady state monetary equilibria where agents of identical type take identical strategies, all real variables are constant over time and money is valued (i.e. $\phi > 0$).

At the start of each period, entrepreneurs are randomly assigned to competitive investors, and offer a contract which involves a payment out of future resources in exchange for an amount of investment goods. There are two important characteristics of the contracts we will describe below. First, in our environment long term contracts are not available because of the random matching process in a large economy: there is no chance for an entrepreneur and an investor who are matched in any given period to meet with each other again at any other future period. Second, the presence of informational frictions in the afternoon market implies that the late return of investment goods cannot be pledged to outside investors. This is because the outcome of the afternoon market accrues privately to individual entrepreneurs and investors cannot observe it. Thus, an entrepreneur who enters such a market can always claim without fear of repercussions that he has spent all his money holdings and consumed the entire returns, and holds no resources to pay out to the investor. This monetary nature of trades further implies that investors and entrepreneurs loose track of each other at the end of the afternoon, thereby no financial claims on the evening output, as well as on the afternoon output, can be written.

We assume that the morning output of entrepreneurs is fully pledgeable and that contracts between the entrepreneur and the investor can be made contingent on the early return of investment. Given the non-pledgeability described above, the payments must happen at the end of the morning before the afternoon market opens. A contract between an entrepreneur and an investor specifies the amount q_1 of investment goods that the entrepreneur buys from the investor, generating output with technology $g(q_1)$, and its payment - the entrepreneur pays out an amount z of the morning output (i.e. early return) and a fraction θ of his money holdings. Formally, z, θ must satisfy:

$$
\begin{aligned}
z + \theta\phi m &= q_1; & (3)\\
z &\leq g\left(q_1\right); & (4)\\
\theta &\in [0,1]. & (5)
\end{aligned}
$$

Condition (3) is the participation constraint of the investor where the L.H.S. represents the total payment of the entrepreneur and the R.H.S. is production costs of the investor. The entrepreneur makes an offer so that the investor is indifferent between producing or not. The amount ϕm represents the entrepreneur's real money holdings at the start of a given period. Condition (4) states that the payment with output cannot exceed the early return that accrues during the morning, and condition (5) states that the payment via money cannot exceed the money holdings of the entrepreneur at the start of the period. As the condition (4) represents a constraint on the liquidation possibility of the investment returns we shall refer to it as *the liquidity constraint*. Given values of q_1 and $g(q_1)$, observe that a larger amount of money pledged $\theta\phi m$ implies a smaller amount of outputs that the entrepreneur has to pay out of its returns when the liquidity constraint (4) is binding. Thus, the use of money as a payment can mitigate the binding liquidity constraint.

Below, we describe the value function only for the entrepreneurs given that the investors will not carry any money from one period to the next. Because there is no reason for the investors to carry money into the future, one can assume without loss of generality that they will spend it all in the evening of the same period.

The evening: walrasian market. We work backward and start with the evening market. During the evening, agents trade, consume and produce an output. At the start of any given evening, the expected value of an entrepreneur who holds $\hat{m}$ money and enters the evening market, denoted by $W(\hat{m})$, satisfies

$$\begin{aligned} W(\hat{m}) &= \max_{x,e,m_{+1}\geq 0} x - e + \beta V(m_{+1}) \\ & \quad \text{s.t.} \quad x - e = \phi(\hat{m} - m_{+1}) + \tau \end{aligned}$$

where $V(m_{+1})$ denotes the expected value of entering into the next morning market with holdings m_{+1} of money. The nominal price in the evening market is normalized to 1, and so ϕ represents the relative price of money. Given these prices, the initial money holding $\hat{m}$ and the government tax or transfer τ, the agent chooses an amount of consumption x, effort e and the future money holdings m_{+1}. Note that the initial money holding $\hat{m}$ at the start of a given evening depends on the agent's activities during the morning and afternoon of the same period. If an entrepreneur has started the morning with m money, paid θ money to the investor, and sold q_2^s (or bought q_2) units on the afternoon market at a price p, then his initial money holding at the start of the evening is given by $\hat{m} = (1-\theta)m + pq_2^s$ (or $\hat{m} = (1-\theta)m - pq_2$).

Substituting out the term $x - e$ in the value function using the constraint, we obtain the first order condition

$$\beta V'(m_{+1}) = \phi. \tag{6}$$

Observe that m_{+1} is determined independently of $\hat{m}$ (and of m), and hence all entrepreneurs hold the same amount of money at the beginning of any given morning market.

The afternoon: consumption market. After the repayment has happened at the end of morning, entrepreneurs either buy and consume consumption goods, or produce and sell them on the market during the afternoon. At the start of any given afternoon, the expected value of an entrepreneur who holds q_1 investment goods and $(1-\theta)m$ money, and enters the afternoon market, denoted by $Z(q_1,(1-\theta)m)$, satisfies

$$\begin{aligned} Z(q_1,(1-\theta)m) &= \delta \left\{ \begin{array}{c} \max_{\ q_2 \geq 0} \ u(q_2) + W((1-\theta)m - pq_2) \\ \text{s.t. } pq_2 \leq (1-\theta)m \end{array} \right\} \\ &+ (1-\delta)\left\{ \max_{q_2^s \geq 0} \ g(q_1) - q_2^s + W((1-\theta)m + pq_2^s). \right\} \end{aligned}$$

If the entrepreneur turns out to be a buyer, which happens with probability δ, then he can buy and consume the consumption goods q_2 up to his money holdings $(1-\theta)m$ at the market price p. He then carries $(1-\theta)m - pq_2$ money to the evening. $W((1-\theta)m - pq_2)$ is his continuation value specified before. If the entrepreneur turns out to be a seller, which happens with probability $1-\delta$, then he can produce an amount of the consumption goods, denoted by q_2^s, with unit marginal costs and sell it at the market price p. The seller who has invested an amount q_1 in the morning obtains and consumes the late returns of the investment, $g(q_1)$, during the afternoon. The seller's continuation value is given by $W((1-\theta)m + pq_2^s)$.

The first order conditions are

$$u'(q_2) = (\rho + \phi)p \tag{7}$$

$$1 = \phi p. \tag{8}$$

where $\rho \geq 0$ denotes the multiplier of the buyer's budget constraint $pq_2 \leq (1-\theta)m$. To derive these conditions, we use the envelope conditions, $\partial W(\cdot)/\partial q_2 = -\phi p$ for the buyer and $\partial W(\cdot)/\partial q_2^s = \phi p$ for the seller. (7) states that the buyer consumes the amount of q_2 so that its marginal utility ($= u'(\cdot)$) equals the unit price measured in the real term ($= \phi p$) plus the cost of tightening the budget constraint ($= \rho p$). (8) states that the seller produces up to the point where the marginal production costs ($= 1$) equal the real market price. Since the seller's problem is linear, we make a tie-breaking assumption that the seller chooses to produce if indifferent to doing so. Finally, the complementary slackness condition is

$$\rho((1-\theta)m - pq_2) = 0. \tag{9}$$

The morning: investment market. At the start of each period, each entrepreneur is randomly matched to an investor. Entrepreneurs offer investors the contract, described above, which specifies a payment z, θ out of their future resources in exchange for an amount of investment goods q_1. The repayment happens at the end of the morning. An entrepreneur who holds m money at the start of any given morning has the expected value, denoted by $V(m)$, satisfying

$$V(m) \quad = \quad \max_{q_1, z, \theta \geq 0} [g(q_1) - z + Z(q_{1,}, (1-\theta)m)]$$

subject to the constraints (3)-(5). After paying out z outputs and $\theta\phi m$ money in real term to the investor, the entrepreneur can consume the remaining $g(q_1) - z$ morning outputs (i.e., an early return of the investment net of the output-payment) and carry the remaining $(1-\theta)m$ money to the afternoon. $Z(\cdot)$ is the continuation value described before.

Solving (3) for z and applying this solution to the value function and (4), we can reduce the programme to the following form:

$$\begin{aligned} V(m) \quad = \quad & \max_{q_1, \theta \geq 0} [g(q_1) - q_1 - \theta\phi m + Z(q_{1,}, (1-\theta m)] \\ & \text{s.t.} \qquad q_1 - \theta\phi m \leq g(q_1) \\ & \qquad\qquad \theta \in [0, 1]. \end{aligned}$$

Using this expression and denoting by $\mu \geq 0$ the multiplier of the liquidity constraint (4) and $\gamma \geq 0$ the multiplier for a constraint $\theta \geq 0$ in (5), we obtain the first order conditions:

$$(2-\delta)g'(q_1) \quad = \quad 1 + \mu(1 - g'(q_1)); \tag{10}$$

$$\mu + \frac{\gamma}{\phi m} \quad = \quad \frac{\delta\rho}{\phi}. \tag{11}$$

Note that $\theta = 1$ cannot be the solution because of the Inada condition $u'(0) = \infty$, hence the other constraint in (5), $\theta \leq 1$, can be ignored. To derive these conditions we use the envelope conditions, $\partial Z(\cdot)/\partial q_1 = g'(q_1)$ and $\partial Z(\cdot)/\partial\theta = -(\delta\rho + \phi)m$. The L.H.S. of (10) represents the total expected marginal returns of the investment q_1 accruing in the morning and afternoon, while the R.H.S. represents the marginal production costs ($= 1$) plus the marginal cost of relaxing the liquidity constraint ($= \mu(1 - g'(\cdot))$). Observe that if the liquidity constraint (4) is slack $\mu = 0$ then the investment decision is made independently of the liquidity concerns and the outcome is efficient, $q_1 = q_1^*$, while if the liquidity constraint is binding $\mu > 0$ then the entrepreneur makes an under-investment, $q_1 < q_1^*$.

Using (7) and (8), the condition (11) can be written as

$$\mu + \frac{\gamma}{\phi m} = \delta \left(u'(q_2) - 1 \right). \tag{12}$$

The L.H.S. of this equation represents the marginal benefit of increasing an extra share of monetary payment θ, which is the marginal benefit of relaxing the constraints (4) and (5). The R.H.S. represents the marginal opportunity cost of increasing θ, which is the marginal opportunity cost of reducing an extra unit of money holdings measured by the net marginal utility as a buyer, $u'(q_2) - 1$, during the afternoon.

The complementary slackness conditions are

$$\mu \left(g\left(q_1\right) - q_1 + \theta\phi m \right) = 0 \tag{13}$$

$$\gamma\theta = 0. \tag{14}$$

For $\mu > 0$, (12) - (14) imply $\mu = \delta(u'(q_2) - 1)$, $q_1 = g(q_1) + \theta\phi m$ and $\gamma = 0$ if $\theta > 0$, while $\mu < \delta(u'(q_2) - 1)$, $q_1 = g(q_1)$ and $\theta = 0$ if $\gamma > 0$. Therefore, factors influencing the marginal utility of consumption and the money holdings, can affect the binding liquidity constraint $\mu > 0$ and investment decisions if and only when a positive fraction of money holdings are pledged $\theta > 0$.

Finally, the envelope condition for m is

$$V'(m) = \phi \left[(1 - \theta) \left(\frac{\delta\rho}{\phi} + 1 \right) + \mu\theta \right]. \tag{15}$$

Euler equation. We now derive the Euler equation. Plugging (15) into (6) with an updating and rearranging it using (7) and (8), we obtain the Euler equation for money holdings m:

$$\phi = \beta\phi_+ \left[(1-\theta)\left(\delta u'(q_2) + 1 - \delta\right) + \mu\theta \right]. \tag{16}$$

In the above equation, the marginal cost of obtaining an extra unit of money today ($= \phi$) equals the discounted value of its expected marginal benefit obtained tomorrow. The marginal value has two components. First, an extra unit of money allows for further consumption: the entrepreneur can consume an extra unit during afternoon as a buyer yielding a marginal utility $u'(\cdot)$ and during night as a seller yielding a marginal utility 1. This return of money accrues from its role as a medium of exchange and is captured in the first term. Since a fraction θ of the money has to be repayed before the consumption can occur during the afternoon and evening, this term is multiplied by $1-\theta$. Second, an extra unit of money reduces the need to pledge output for the payment. This return of money accrues its role as enhancer of liquidity and is captured in the second term $\mu\theta$. It is important to observe that this second role of money is absent when $\theta = 0$ and/or $\mu = 0$.

Existence, uniqueness and characterization of equilibrium. So far we have described the optimality conditions of individual entrepreneurs taking the market prices p, ϕ as given. We now describe the market clearing conditions. These are the final equilibrium requirements in our economy. Market clearing in the morning is guaranteed by bilateral meetings, while market clearing in the afternoon requires

$$\delta q_2 = (1-\delta) q_2^s. \tag{17}$$

Money market clearing implies

$$\frac{\phi_{+1}}{\phi} = \frac{1}{\pi} \tag{18}$$

while market clearing in the evening can be ignored by virtue of Walras Law.

Below we consider policies where $\pi \geq \beta$ and when $\pi = \beta$ (which is the Friedman rule) we only consider the limiting equilibrium as $\pi \to \beta$. This implies:

Lemma 1 *For $\pi > \beta$, the budget constraint of buyers must be binding, i.e. $(1-\theta)m = pq_2$.*

Applying the binding budget constraint $(1-\theta)m = pq_2$ and (8) to the complementary slackness condition (13), we obtain

$$\mu \left[g(q_1) - q_1 + \frac{\theta}{1-\theta} q_2 \right] = 0. \tag{19}$$

Applying this expression to (7), (8), (12) and (14) we obtain:

$$\mu + \frac{\gamma}{q_2} = \delta \left(u'(q_2) - 1 \right). \tag{20}$$

Using (18) and (20), the Euler equation (16) can be simplified to

$$\frac{\pi}{\beta} - 1 = \delta \left(u'(q_2) - 1 \right). \tag{21}$$

Definition 1 *A symmetric steady state monetary equilibrium is a set of quantities $q_1, q_2, q_2^s \in (0, \infty)$, prices $p, \phi \in (0, \infty)$, multipliers $\mu, \gamma \in [0, \infty)$, and a share of monetary payment $\theta \in [0, 1)$ satisfying the first order conditions (8), (10), (20), the Euler equation (21), the market clearing conditions, (17), (18), and the complementary slackness conditions (14), (19).*

Observe that (14), (20)-(21) imply that it is impossible to have the case $\mu = 0$ and $\theta > 0$ for $\pi > \beta$, i.e., the case in which the liquidity constraint is not binding but a positive amount of money is pledged. This implies that, in our model, the only role of money can play in the morning is to relax the liquidity constraint. Hence, the possible cases are: [1] the liquidity constraint is not binding, $\mu = 0$, and no money is pledged, $\theta = 0$; [2] the liquidity constraint is binding, $\mu > 0$, and no money is pledged, $\theta = 0$; [3] the liquidity constraint is binding, $\mu > 0$, and a positive amount of money is pledged, $\theta > 0$. Below we show either case can emerge.

Proposition 1 *Suppose* $g(q_1^*)/q_1^* \geq 1$*. Then, a unique equilibrium exists for all* $\pi > \beta$ *in which the liquidity constraint is not binding,* $\mu = 0$*, and no money is pledged,* $\theta = 0$*. Further, it satisfies:* $q_1 = q_1^*$ *for all* $\pi > \beta$*;* $q_2 \in (0, q_2^*)$ *is strictly decreasing in* $\pi \in (\beta, \infty)$*;* $q_1 \to q_1^*$*,* $q_2 \to q_2^*$ *as* $\pi \to \beta$*.*

Proposition 2 *Suppose* $g(q_1^*)/q_1^* < 1$*. Then, a unique equilibrium exists for all* $\pi > \beta$ *in which the liquidity constraint is binding,* $\mu > 0$*. It satisfies:* $q_2 \in (0, q_2^*)$ *is strictly decreasing in* $\pi \in (\beta, \infty)$*;* $q_1 \to q_1^*$*,* $q_2 \to q_2^*$ *as* $\pi \to \beta$*. Further, there exists a unique* $\hat{\pi} \in (\beta, \infty)$ *such that* $q_1 = \hat{q}_1 \in (0, q_1^*)$ *at* $\pi = \hat{\pi}$ *and:*

1. $\theta > 0$ *for* $\pi \in (\beta, \hat{\pi})$ *and* $\theta = 0$ *for* $\pi \in [\hat{\pi}, \infty)$*;*

2. $q_1 \in (\hat{q}_1, q_1^*)$ *is strictly decreasing in* $\pi \in (\beta, \hat{\pi})$ *and* $q_1 = \hat{q}_1$ *for all* $\pi \in [\hat{\pi}, \infty)$*.*

Comparison of Proposition 1 and 2 uncovers an important effect of the liquidity constraint on the investment decision of entrepreneurs. Proposition 1 shows that the constraint is never binding $\mu = 0$ for all $\pi > \beta$ if an average return of investment is sufficiently high, $g(q_1^*)/q_1^* \geq 1$, while Proposition 2 shows that the constraint is binding $\mu > 0$ for all $\pi > \beta$ otherwise, $g(q_1^*)/q_1^* < 1$. In the former case, the liquidity constraint is irrelevant for the investment decision of entrepreneurs and equilibrium displays a dichotomous nature: the amount of entrepreneurs' investment is at the efficient level $q_1 = q_1^*$ for all $\pi > \beta$ and the investment market is insulated from monetary factors. In the latter case, the binding liquidity constraint implies costs to the investment of entrepreneurs and leads to an under-investment $q_1 < q_1^*$ for all $\pi > \beta$. In any case the Friedman rule implements the efficient outcome both in terms of investment and consumption in our economy: $q_1 \to q_1^*$, $q_2 \to q_2^*$ as $\pi \to \beta$.

Proposition 2 identifies the role of money to mitigate the liquidity constraint. When the average return of investment is relatively low, there is a relatively tight bound on the amount of output that can be pledged. This induces entrepreneurs to pledge some money to relax the liquidity constraint. Indeed, a positive fraction of money holdings are used for the payment of investment for low inflation rates, i.e., $\theta > 0$ for $\pi \in (\beta, \hat{\pi})$. Within this region, money provides liquidity service and the binding constraint can cause the monetary factor to seep into the investment market: As inflation grows, the shadow cost of relaxing the liquidity constraint is increased, thereby the investment level decreases with inflation, i.e., $\mu > 0$ is increasing and q_1 is decreasing in $\pi \in (\beta, \hat{\pi})$. However, a lower investment level q_1 implies a higher average return of the investment $g(q_1)/q_1$ that can relax the liquidity constraint. Thus, money becomes relatively less useful as a provider of liquidity services as the rate of inflation increases. Therefore, for sufficiently high rates of inflation, no money is used for the payment, i.e., $\theta = 0$ for $\pi \in [\hat{\pi}, \infty)$. Within this region, money plays no role as an enhancer of liquidity, thereby both $\mu > 0$ and q_1 are constant for all $\pi \in [\hat{\pi}, \infty)$. Nevertheless, money still serves as a medium of exchange and so the amount of consumption decreases as the money holdings become more costly, i.e., q_2 decreases with $\pi \in (\beta, \infty)$ irrespective of the productivity parameter.

3 Discussion

The notion of inability to pledge the entire returns of a project is at the heart of a series of papers by Nobuhiro Kiyotaki and John H. Moore. In their view, an entrepreneur can issue claims to an investor only up to a certain fraction of his future returns, for instance because of moral hazard reasons. Could the investor spend such claims with a third party, extraneous to the initial deal, instead of having to hold on to them till the project pays off, he would be keener to lend in the first place. A circulating, money-like instrument which is held for its transaction value rather than its maturity value, may emerge in a world where individuals cannot trust each other to keep their promises.

In the present paper, we have followed from the monetary search approach and taken the point of view that money is used to overcome the difficulties associated with the exchange process. Money already exists in society as a medium of spot trade. In such a world, we argue, the frictions – such as randomness and opacity of transactions – hindering trade may affect the ability to pledge returns to outsiders. In other words, a different sort of evil is the root of all money or maybe, as it were, "money is the root of all evil" (The Bible, 1 Timothy 6:10).

Our model has the following implications. A society with relatively unorganized and opaque markets in some sectors of the economy may be more likely to face trouble in other relatively more organized sectors such as the investment markets. Even when markets do not function smoothly and contracts are poorly enforced, though, a sufficient level of technological sophistication of the productive sector may allow the economy to avoid major disruptions to investment.[1] In such a case, steady state inflation turns out not to affect investment. When the economy is technologically less developed though, inflation will have an adverse effect on investment, adverse effect which thins out as inflation increases until it eventually – for high enough inflation – dies out altogether. Our model would thus predict a differential impact of inflation on investment according to the stage of development of a country as represented both by its market frictions and its level of technological sophistication. As regards the negative but decreasing effect of steady state inflation on investment, Boyd, Levine and Smith (2001) found that anticipated inflation negatively affects various measures of financial depth which in turn affect investment, and that the effect is smaller for higher inflation, until it vanishes at high inflation rates.

4 Conclusion

We presented a monetary model where a liquidity constraint can arise in equilibrium. This in turn creates a role for money as a provider of liquidity services as well as exchange services.

[1]A non-monetary model by Gertler and Rogoff (1990) also features investment market imperfections which arise endogenously and depend on a country level of development.

References

[1] Duffie, D., Garleanu,N., Pedersen, L.H., 2005. Over-the-counter markets. Econometrica 73, 1815-1847.

[2] Boyd, J.H., Levine, R., Smith, B.D., 2001. The impact of inflation on financial sector performance. Journal of Monetary Economics 47, 221-248.

[3] Gertler, M., Rogoff, K., 1990. North-south lending and endogenous domestic capital market inefficiencies. Journal of Monetary Economics 26, 245-266.

[4] Holmstom, B., Tirole, J., 1998. Private and public supply of liquidity. Journal of Political Economy 106, 1-40.

[5] Kiyotaki, N., Moore, J., 2001a. Evil is the root of all money. American Economic Review, Papers and Proceedings 85, 62-66.

[6] Kiyotaki, N., Moore, J., 2001b. Liquidity, business cycles and monetary policy. mimeo, Princeton University.

[7] Kiyotaki, N., Wright, R., 1989. On money as a medium of exchange. Journal of Political Economy 97, 927-954.

[8] Lagos, R., Rocheteau, G., 2007. Search in asset markets: market structure, liquidity and welfare. American Economic Review, Papers and Proceedings 97, 198-202.

[9] Lagos, R., Wright, R., 2005. A unified framework for monetary theory and policy analysis. Journal of Political Economy 113, 463-484.

5 Appendix

Proof of Lemma 1.

Suppose $\rho = 0$. Then, (7) and (8) imply that

$$u'(q_2) = 1.$$

This, however, contradicts (21) for $\pi > \beta$, hence if a solution exists for $\pi > \beta$ then we must have $\rho > 0$ leading to $(1-\theta)m = pq_2$ by (9). ■

Proof of Proposition 1.

Before proceeding observe that solving (10) for $\mu \geq 0$ yields

$$\mu = \max\left\{0, \frac{(2-\delta)g'(q_1)-1}{1-g'(q_1)}\right\}. \tag{22}$$

The problem is now reduced to finding a solution $q_1, q_2, \mu, \gamma, \theta$ that satisfies (14), (19)-(22). In what follows we use the following properties: as $g(\cdot)$ is a strictly concave function and satisfies $g(0) = 0$, it holds that

$$\frac{g(q_1)}{q_1} > g'(q_1) \tag{23}$$

for all $q_1 \in (0, \infty)$ and $g(q_1)/q_1$ is strictly decreasing in $q_1 \in (0, \infty)$. Observe also that whenever an equilibrium exists, (21) implies a unique solution $q_2 = q_2(\pi)$, which is strictly decreasing in $\pi > \beta$ and satisfies $q_2(\pi) \to q_2^*$ as $\pi \to \beta$ and $q_2(\pi) \to 0$ as $\pi \to \infty$.

Given $g(q_1^*)/q_1^* \geq 1$ the proof of Proposition 1 proceeds with the following steps: Using (14), (19), (20) and (22), Step 1 shows $\mu, \theta > 0$ cannot be a solution; Using (19) and (22), Step 2 shows $\mu > 0$, $\theta = 0$ cannot be a solution. By Step 1 and 2, since $\mu = 0, \theta > 0$ are not possible, the only possible case is $\mu = \theta = 0$, implying $\gamma > 0$. In this case, (22) with $\mu = 0$ identifies a unique solution $q_1 = q_1^*$, which is independent of π. With $q_2 \in (0, q_2^*)$ satisfying (21) this solution in turn satisfies (14), (19)-(22) and so it is a unique equilibrium.

<u>**Step 1**</u> If $g(q_1^*)/q_1^* \geq 1$, then $\mu, \theta > 0$ cannot be a solution.

Proof of Step 1. Suppose $\mu > 0$ and $\theta > 0$. $\theta > 0$ implies $\gamma = 0$ by (14). Applying $\gamma = 0$ to (20) and substituting out $\mu > 0$ from (20) and (22) , we get

$$\frac{\pi}{\beta} - 1 = \frac{(2-\delta)g'(q_1)-1}{1-g'(q_1)}.$$

The R.H.S. of this equation is strictly decreasing in $q_1 \in (g'^{-1}(1), q_1^*)$. This equation has a unique solution $q_1 = q_1(\pi)$, which is strictly decreasing in $\pi > \beta$ and satisfies $q_1(\pi) \to q_1^* \equiv g'^{-1}(1/(2-\delta))$ as $\pi \to \beta$ and $q_1(\pi) \to g'^{-1}(1)$ as $\pi \to \infty$. This further implies $q_1 < q_1^*$ for all $\pi > \beta$. As $g(q_1)/q_1$ is strictly decreasing in $q_1 \in (0, \infty)$, we must have

$$\frac{g(q_1)}{q_1} > \frac{g(q_1^*)}{q_1^*} \geq 1$$

for all $q_1 < q_1^*$ and hence for all $\pi > \beta$. However, (19) and $\mu > 0$ require

$$g(q_1) - q_1 = -\frac{\theta}{1-\theta} q_2 \tag{24}$$

given $q_2 > 0$ satisfying (21) and so $g(q_1)/q_1 > 1$ contradicts $\theta > 0$. This completes the proof of Step 1.

Step 2 If $g(q_1^*)/q_1^* \geq 1$, then $\mu > 0$, $\theta = 0$ cannot be a solution.

Proof of Step 2. Suppose $\mu > 0$ and $\theta = 0$. $\mu > 0$ and $\theta = 0$ imply $g(q_1) = q_1$ by (19). For $g(q_1^*)/q_1^* \geq 1$ this is possible only when $g(q_1^*) = q_1^*$ and $q_1 = q_1^*$. However, this contradicts $\mu > 0$ because applying $q_1 = q_1^*$ to (22) yields

$$\mu = \frac{(2-\delta)g'(q_1^*) - 1}{1 - g'(q_1^*)} = 0.$$

This completes the proof of Step 2. ■

Proof of Proposition 2.

Given $g(q_1^*)/q_1^* < 1$ the proof of Proposition 2 proceeds with the following steps. Using (19) and (22), Step 1 shows $\mu = \theta = 0$ cannot be a solution. As $\mu = 0, \theta > 0$ is not possible, this implies the only possible cases are either $\mu, \theta > 0$ or $\mu > 0$, $\theta = 0$. Using (14), (19), (20) and (22), Step 2 then shows that there exists a unique $\hat{\pi} \in (\beta, \infty)$ such that $\theta > 0$ for $\pi \in (\beta, \hat{\pi})$ and $\theta = 0$ for $\pi \in [\hat{\pi}, \infty)$. In the former region, we have $\gamma = 0$, (20) and (22) identify a unique $q_1 = q_1(\pi)$, which is strictly decreasing in $\pi \in (\beta, \hat{\pi})$, and (19) identifies a unique $\theta \in (0, 1)$. In the latter region, we have $\gamma \geq 0$ satisfying (20), and (19) identifies a unique q_1, which is independent of $\pi \in [\hat{\pi}, \infty)$. With $q_2 \in (0, q_2^*)$ satisfying (21) and $\mu > 0$ satisfying (22), this solution in turn satisfies (14), (19)-(22) and so it is a unique equilibrium.

Step 1 If $g(q_1^*)/q_1^* < 1$, then $\mu = \theta = 0$ cannot be a solution.

Proof of Step 1. Suppose $\mu = \theta = 0$. $\mu = 0$ implies $q_1 = q_1^*$ by (22). Further, (19) requires that $g(q_1) \geq q_1$ and so $g(q_1^*) \geq q_1^*$, which contradicts $g(q_1^*)/q_1^* < 1$. This completes the proof of Step 1.

Step 2 If $g(q_1^*)/q_1^* < 1$, then there exists a unique $\hat{\pi} \in (\beta, \infty)$ such that $\theta > 0$ for $\pi \in (\beta, \hat{\pi})$ and $\theta = 0$ for $\pi \in [\hat{\pi}, \infty)$.

Proof of Step 2. Suppose $\mu > 0$ and $\theta > 0$. Then as shown in the Step 1 in the proof of Proposition 1, there exists a unique solution $q_1 = q_1(\pi)$ $(< q_1^*)$ to (20) and (22), which is strictly decreasing in $\pi > \beta$ and satisfies $q_1(\pi) \to q_1^*$ as $\pi \to \beta$ and $q_1(\pi) \to g'^{-1}(1)$ as $\pi \to \infty$. Observe (23) implies at $q_1 = g'^{-1}(1)$ we have

$$\frac{g\left(g'^{-1}(1)\right)}{g'^{-1}(1)} > 1 = g'\left(g'^{-1}(1)\right).$$

As $g(q_1)/q_1$ is strictly decreasing in $q_1 \in (0, \infty)$, $q_1 = q_1(\pi) \in (g'^{-1}(1), q_1^*)$ is strictly decreasing in $\pi \in (\beta, \infty)$ and $g(q_1^*)/q_1^* < 1$, this implies that there exists a unique $\hat{\pi} \in (\beta, \infty)$ such that $\hat{q}_1 = q_1(\hat{\pi}) \in (g'^{-1}(1), q_1^*)$ and

$$\frac{g(\hat{q}_1)}{\hat{q}_1} = 1.$$

This further implies $g(q_1) - q_1 < 0$ for $\pi \in (\beta, \hat{\pi})$ and $g(q_1) - q_1 \geq 0$ for $\pi \in [\hat{\pi}, \infty)$. Therefore, given $q_2 > 0$ satisfying (21), it follows that (24) (which was constructed by (19) with $\mu > 0$) identifies a unique $\theta \in (0, 1)$ for $\pi \in (\beta, \hat{\pi})$. For $\pi \in [\hat{\pi}, \infty)$ the only remaining possibility is the case $\mu > 0$ and $\theta = 0$.

Suppose now that $\mu > 0$ and $\theta = 0$. Then, (19) determines a unique $q_1 = \hat{q}_1$ ($< q_1^*$) which is independent of π. On the other hand, (20), (21) and (22) imply

$$\gamma = q_2 \left(\frac{\pi}{\beta} - 1 - \frac{(2 - \delta) g'(\hat{q}_1) - 1}{1 - g'(\hat{q}_1)} \right).$$

This expression shows, given $q_2 > 0$ satisfying (21) it holds that $\gamma > 0$, implying $\theta = 0$, if and only if $\pi \in (\hat{\pi}, \infty)$. At $\pi = \hat{\pi}$ we must have $\gamma = \theta = 0$. This completes the proof of Step 2. ■