

Product Innovation, Stock Price, and Business Cycle*

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Abstract

This paper studies the interaction between product innovation, stock price, and aggregate output. I propose an endogenous variety business cycle model in which existing firm's expertise is essential for innovation of new products. In the model, the firm value reflects not only the product the firm is manufacturing today but also the firm's ability to introduce new products in the future. I study how the model responds to various shocks, with special attention to the lead and lag structure between the stock price and output. The stock price leads GDP, which is a well known empirical regularity that most standard models have difficulty generating.

1 Introduction

This paper studies the interaction between product innovation, stock price, and aggregate output. In the literature product innovation is often modeled as an independent entrepreneur's rent seeking activity that does not require any skill or knowledge. But in this paper I propose a model in which existing firm's expertise is essential for innovation of new products.

The model is motivated by the empirical evidence of recent studies using product-level micro data. For example, Broda and Weinstein (2007) find that 92% of product creation occurs within an existing firm. Similarly, Bernard, Redding and Schott (2006) find that 94% of product addition occurs within an existing facility. These observations suggest that there is something that gives incumbents an edge in introducing new products. This paper proposes a

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hypothesis—it is expertise that gives incumbents the edge—and investigates its implication for the joint dynamics of the stock price and output against various shocks.

The model economy has two sectors: the goods producing sector and the R&D sector. The goods producing sector has competitive final goods producing firms and monopolistic intermediate goods producing firms. New intermediate goods are introduced in the R&D sector. An intermediate goods producing firm acquires expertise about a product by manufacturing the product. To fix the idea, imagine a key is generated as a byproduct. The key is an essential input for innovation of new products. The key is perishable, but is produced every time the product is manufactured. In this setup, the product value reflects not only the usual monopoly rent but also the value as a key generator.

I study how the economy responds to a shock that makes product innovation easier, which I call a favorable shock in the R&D sector. Development of internet infrastructure in the late 80's is an excellent example of such a shock.¹ The shock increases output but only with a lag because the shock itself does not improve goods producing productivity, but a massive introduction of new products does. The aggregate equity value, however, increases immediately because the value of expertise increases.

This response is different from the one observed in the standard setup in which independent entrepreneurs without expertise can introduce products. In such a model, a favorable shock in the R&D sector causes a stock market crash. Each existing product immediately loses its value because some of its future demand is seized by newly introduced products. The advent of the new products pushes up the aggregate equity value, but the process takes time. Therefore the stock market crashes on impact.

Equity prices are a well-known leading indicator of GDP. But constructing a macroeconomic model in which the equity price is a leading indicator has been a challenge because a standard productivity shock drives the equity price and the output simultaneously in the same direction.² In my model, a rise in the aggregate equity value signals future economic boom because when a favorable shock in the R&D sector hits the economy the equity price leads GDP. I assess the quantitative implication using a calibrated model; the model generates empirically reasonable cross-correlations between the aggregate firm value and GDP.

A growing literature explores implications of technological innovation on the stock market.³ In Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001), technological innovations cause a stock market crash. They argue that there is a time lag between the arrival of the news and the actual advent of the

¹For a brief history of the internet, see Leiner, Cerf, Clark, Kahn, Kleinrock, Lynch, Postel, Roberts and Wolf (1999).

²Backus, Roulledge and Zin (2007) is a recent contribution. They assume a recursive utility and a predictable component of productivity.

³See Jovanovic and MacDonald (1994), Laitner and Stolyarov (2003), Manuelli (2003), and Pastor and Veronesi (2008).

new technology, but the old technology immediately loses its value because of expectation of obsolescence. In their model, the old technology has nothing to do with introducing the new technology. In my model, expertise about the current products helps in innovation of new products.

A recent literature known as the expectation driven business cycle explores implications of news that contains information about future productivity.⁴ In a basic RBC model, a future productivity gain causes recession today because it is optimal to enjoy leisure until the productivity gain is realized. This literature modifies the basic model to have an economic boom in response to optimistic news. In my model, a favorable shock in the R&D sector similarly predicts future productivity improvement. Output decreases for a few periods, but an economic boom follows soon because the R&D boom quickly improves productivity.

Bilbiie, Ghironi and Melitz (2007) and McGrattan and Prescott (2007) are closely related papers. Bilbiie et al. (2007) study business cycle implications of an endogenous variety model. They investigate joint dynamics of firm entry and output, and in their model independent entrepreneurs without expertise can start a new firm with a sunk entry cost. McGrattan and Prescott (2007) study a two sector competitive economy with intangible capital.⁵ They focus on the U.S. economy in the 90's, and analyze the trend with perfect foresight. My focus in this paper is on the economy's fluctuations.

2 Planner's Problem

This section presents the planner's problem to show technologies and core mechanisms in a clean environment. The only difference from the market economy is the absence of the monopoly distortion. Readers are invited to start with this section, but can safely skip it.

2.1 Technology

The planner maximizes the welfare

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{[C_{t+j} (1 - L_{t+j})^\varphi]^{1-\sigma}}{1 - \sigma} \right].$$

⁴See Beaudry and Portier (2004), Jaimovich and Rebelo (2006), Jaimovich and Rebelo (2007), and Christiano, Ilut, Motto and Rostagno (2007).

⁵The intangible capital draws much attention. Examples are Hall (2001a), Hall (2001b), Basu, Fernald, Oulton and Srinivasan (2003), Corrado, Hulten and Sichel (2005), and Corrado, Hulten and Sichel (2006). The intangible investment in the literature tends to be a broad concept that includes not only R&D but also spending in the computerized information, advertisement, and firm specific training.

C_t is consumption of final goods; L_t is labor supply. The production function of the final goods is

$$Z_t = \left[\int_0^{N_t} (A_t L_{i,t})^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}.$$

A_t is a productivity shock. $L_{i,t}$ is the labor input in production of intermediate goods of index i . θ is the elasticity of substitution between differentiated intermediate goods. N_t is the measure of intermediate goods variety that is available in period t . The production function of the new variety is

$$N_{E,t}(N_t, R_t) = S_t N_t^{1-\nu} R_t^\nu \quad (1)$$

with $0 \leq \nu \leq 1$. S_t is an exogenous shock; R_t is final goods input for the variety creation. A fraction δ_N of the total varieties becomes unavailable at the end of every period; N_t evolves as

$$N_{t+1} = (1 - \delta_N)(N_t + N_{E,t}).$$

The resource constraints are

$$\begin{aligned} L_t &= \int_0^{N_t} L_{i,t} di, \\ Z_t &= C_t + R_t. \end{aligned}$$

2.2 Variety and Productivity

It is clear that the planner should use the same amount of labor input across different intermediate goods; $L_{i,t} = L_{i',t}$ for all $i, i' \in [0, N_t]$, which then implies that

$$L_{i,t} = \frac{L_t}{N_t}$$

for all $i \in [0, N_t]$. Substituting it into the production function,

$$Z_t = A_t N_t^{\frac{1}{\theta-1}} L_t.$$

Notice that the variety makes the goods production more efficient.

2.3 Efficiency Conditions

The planner's problem can be simplified to

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{[C_{t+j} (1 - L_{t+j})^\varphi]^{1-\sigma}}{1 - \sigma} \right].$$

s.t.

$$C_t + R_t = A_t N_t^{\frac{1}{\theta-1}} L_t, \quad (2)$$

$$N_{t+1} = (1 - \delta_N) (N_t + S_t N_t^{1-\nu} R_t^\nu). \quad (3)$$

Let λ_t^C and λ_t^N be the Lagrangian multipliers of the first and the second constraints. The first order conditions are

$$C_t^{-\sigma} (1 - L_t)^{\varphi(1-\sigma)} - \lambda_t^C = 0, \quad (4)$$

$$-C_t^{-\sigma} (1 - L_t)^{\varphi(1-\sigma)} \frac{\varphi C_t}{1 - L_t} + \lambda_t^C A_t N_t^{\frac{1}{\theta-1}} = 0, \quad (5)$$

$$-\lambda_t^C + \lambda_t^N (1 - \delta_N) \left(\nu \frac{N_{E,t}}{R_t} \right) = 0, \quad (6)$$

$$-\lambda_t^N + \beta E_t \left[\lambda_{t+1}^C \frac{Z_{t+1}}{(\theta-1) N_{t+1}} + \lambda_{t+1}^N (1 - \delta_N) \left(1 + (1 - \nu) \frac{N_{E,t+1}}{N_{t+1}} \right) \right] = 0. \quad (7)$$

Substituting (4) into (5),

$$A_t N_t^{\frac{1}{\theta-1}} = \varphi \frac{C_t}{1 - L_t}. \quad (8)$$

This condition says that the utility gain from marginal labor input should be equal to marginal labor disutility.

From (6) and (7),

$$\frac{1}{\nu \frac{N_{E,t}}{R_t}} = E_t \left[\beta \frac{\lambda_{t+1}^C}{\lambda_t^C} (1 - \delta_N) \left(\frac{Z_{t+1}}{(\theta-1) N_{t+1}} + \left((1 - \nu) \frac{N_{E,t+1}}{N_{t+1}} \right) \frac{1}{\nu \frac{N_{E,t+1}}{R_{t+1}}} + \frac{1}{\nu \frac{N_{E,t+1}}{R_{t+1}}} \right) \right]. \quad (9)$$

(9) is a standard pricing equation. The left hand side is equal to $(1 - \delta_N) \lambda_t^N / \lambda_t^C$, the shadow price of the variety N_t in terms of the final goods. In the right hand side, contributions of the marginal variety increase are collected and multiplied by the stochastic discount factor. The contributions come in three different forms. First, the additional variety contributes to the production of the final goods in period $t + 1$, and the first term captures this effect. Second, the additional variety contributes to the variety creation in period $t + 1$, and the second term captures this effect. Hence, the first two terms together can be thought of the dividend of the additional variety. Lastly, the third term captures the capital gain. (2), (3), (8), (9) together with (1) and (4) solve the planner's problem.

3 Market Economy

3.1 Goods Producing Sector

The goods producing sector has the representative competitive final goods producing firm and the monopolistic intermediate goods producing firms. The final goods producing firm purchases the differentiated intermediate goods and

assembles the final goods. Assembling process does not require any resource other than the intermediate goods. The production function is

$$Z_t = \left[\int_0^{N_t} Z_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $Z_{i,t}$ is the intermediate goods of index i . N_t is the measure of the intermediate goods variety on the market in period t which is represented by the interval $[0, N_t]$ without loss of generality. Competition ensures an equilibrium price of the final goods P_t equal to

$$P_t = \left[\int_0^{N_t} P_{i,t}^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (10)$$

where $P_{i,t}$ is the price of the intermediate goods of index i . The final goods firm demands

$$Z_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Z_t.$$

The intermediate goods producing firms are monopolists. A multi-product firm is not allowed; i.e., a single intermediate goods producing firm can produce up to a single differentiated product for the technological reason.⁶ Therefore, N_t represents both the measure of the intermediate goods producing firm and the measure of the available intermediate goods variety.

The firm's production function is linear in labor;

$$Z_{i,t} = A_t L_{i,t}.$$

A_t is exogenous random variable which I call a productivity shock. The firm chooses the price to maximize the real profit;

$$\pi_{i,t}^Z = \max_{P_{i,t}} \left\{ \left[\frac{P_{i,t}}{P_t} - \frac{W_t}{A_t} \right] \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Z_t \right\} \quad (11)$$

where W_t is the real wage. The optimal price is

$$\frac{P_{i,t}}{P_t} = \frac{\theta}{\theta-1} \frac{W_t}{A_t}. \quad (12)$$

⁶It is possible to introduce a multi-product firm. The result does not change as long as the intermediate goods firms do not have a positive mass of varieties (otherwise monopolistic competition breaks) and the variety contributes to the R&D in a linear way (i.e., a firm that has two products is twice more productive in R&D than a firm that has a single product).

3.2 R&D Sector

The measure of the intermediate goods producing firms is endogenously determined. A fraction δ_N of the intermediate goods producing firms dies at the end of every period. New intermediate goods firms are produced by R&D. Let $N_{E,t}$ be the aggregate measure of newly created intermediate goods producing firms. The measure of firms evolves as

$$N_{t+1} = (1 - \delta_N)(N_t + N_{E,t}).$$

Importantly, I assume that only the existing intermediate goods producing firms have an ability to produce new firms. Namely, the existing firm acquires expertise about the product she is currently manufacturing and the expertise is essential input to innovate new products. R&D is risky at individual level. If an intermediate goods producing firm spends $R_{i,t} \geq 0$ of the final goods as R&D input, she produces n new intermediate goods producing firms with probability

$$\Pr(n|R_{i,t}, S_t).$$

S_t is an exogenous random variable which I call the R&D shock. Its expectation is

$$\sum_{n=0}^{\infty} n \Pr(n|R_{i,t}, S_t) = S_t R_{i,t}^{\nu}. \quad (13)$$

When an intermediate goods producing firm succeeds to produce new intermediate goods firms, she sells the ownership of the new firms to the household. The new intermediate goods firm born in period t starts her business in period $t + 1$. A firm's ownership is traded at ex-dividend market value V_t . An intermediate goods producing firm i chooses the R&D input to maximize the expected profit from R&D;

$$\pi_{i,t}^{R\&D} = \max_{R_{i,t} \geq 0} \{S_t R_{i,t}^{\nu} V_t - R_{i,t}\}. \quad (14)$$

The first order condition is

$$\nu S_t R_{i,t}^{\nu-1} V_t = 1. \quad (15)$$

The aggregate firm entry $N_{E,t}$ and the aggregate R&D input R_t are defined as

$$N_{E,t} = \int_0^{N_t} S_t R_{i,t}^{\nu} di, \quad (16)$$

$$R_t = \int_0^{N_t} R_{i,t} di. \quad (17)$$

3.3 Household

The representative household maximizes the expected lifetime utility flow

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{[C_{t+j} (1 - L_{t+j})^\varphi]^{1-\sigma}}{1 - \sigma} \right] \quad (18)$$

subject to the flow budget constraint

$$C_t + V_t x_t = W_t L_t + (D_t + V_t) (1 - \delta_N) x_{t-1}. \quad (19)$$

The left-hand side is the uses of funds; the household either consumes or purchases the ownership of the intermediate goods producing firms. $x_t \in [0, 1]$ is the measure of the intermediate goods producing firms held by the household and V_t is the ex-dividend market value of a single intermediate goods producing firm. The right-hand side is the sources of funds. W_t is the real wage and L_t is the labor supply. Because a fraction δ_N of the firms disappeared at the end of the period $t - 1$, the household holds $(1 - \delta_N) x_{t-1}$ of the intermediate goods producing firms at the beginning of period t . D_t is an average dividend each of the firms pays.

First order conditions are

$$W_t = \varphi \frac{C_t}{1 - L_t}. \quad (20)$$

$$V_t = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi(1-\sigma)} (1 - \delta_N) (D_{t+1} + V_{t+1}) \right], \quad (21)$$

3.4 Equilibrium

The market clearing conditions are

$$L_t = \int_0^{N_t} L_{i,t} di,$$

$$x_t = N_t + N_{E,t},$$

$$Z_t = C_t + R_t.$$

The symmetric equilibrium is defined as a sequence of prices and quantities that solves all the agents' problems and satisfies the market clearing conditions at every period.

3.5 Firm Value

An intermediate goods producing firm earns the profit π_t^Z in the goods producing sector and the expected profit $\pi_t^{R\&D}$ in the R&D sector. The firm pays the total

profit as dividend. From (21), the ex-dividend market value of the intermediate goods firm is

$$V_t = E_t \left[\beta \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi(1-\sigma)} (1 - \delta_N) (\pi_{t+1}^Z + \pi_{t+1}^{R\&D} + V_{t+1}) \right]. \quad (22)$$

We can find an analytical expressions of π_t^Z and $\pi_t^{R\&D}$ in the symmetric equilibrium. Because the intermediate goods producing firms are symmetric, (10) implies the equilibrium relative price is

$$\frac{P_{i,t}}{P_t} = N_t^{\frac{1}{\theta-1}}. \quad (23)$$

From (23) and (10), the equilibrium real wage is

$$W_t = \frac{\theta - 1}{\theta} A_t N_t^{\frac{1}{\theta-1}}. \quad (24)$$

Substituting (23) and (24) into (11), we find the equilibrium profit in the goods producing sector is

$$\pi_t^Z = \frac{Z_t}{\theta N_t}.$$

Now let's find $\pi_t^{R\&D}$. Because of the symmetry, (17) implies

$$R_{i,t} = \frac{R_t}{N_t}. \quad (25)$$

Substituting (25) into (17), we find the aggregate variety production function;

$$N_{E,t} = S_t N_t^{1-\nu} R_t^\nu. \quad (26)$$

Substituting (25) into (15),

$$\nu \left(\frac{N_{E,t}}{R_t} \right) V_t = 1. \quad (27)$$

The equilibrium profit in the R&D sector is

$$\pi_t^{R\&D} = (1 - \nu) \frac{N_{E,t} V_t}{N_t}.$$

Substituting these expressions into (22), we find

$$V_t = E_t \left[\beta \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi(1-\sigma)} (1 - \delta_N) \left(\frac{Z_{t+1}}{\theta N_{t+1}} + (1 - \nu) \frac{N_{E,t+1} V_{t+1}}{N_{t+1}} + V_{t+1} \right) \right]. \quad (28)$$

3.6 National Account

GDP in my model economy is defined as the final goods production minus the aggregate R&D spending;

$$Y_t = Z_t - R_t.$$

The reason I subtract the R&D spending is that under the current national income accounting system, the R&D spending is not treated as investment but treated as the business expense.

We can derive an accounting identity from the household budget constraint and market clearing conditions;

$$\begin{aligned} Y_t &= Z_t - R_t \quad (\text{production account}) \\ &= C_t \quad (\text{consumption account}) \\ &= W_t L_t + D_t N_t - V_t N_{E,t}. \quad (\text{income account}) \end{aligned}$$

3.7 Steady State

The non-stochastic steady state is defined as the symmetric equilibrium in which the productivity shock and the R&D shock are always constant. I log-linearize the economy around the steady state, and study the fluctuations driven by the productivity shock and the R&D shock. The linearized system appears in the appendix.

4 Calibration

4.1 Parameter

The time unit is a quarter. The discount rate β is set to be .99. I set the depreciation rate of the variety δ_N to be .06, which is based on the finding of Broda and Weinstein (2007) that the annual product exit rate is 24%.⁷ The elasticity of substitution among different specialized intermediate varieties θ has an implication on the steady state labor share to GDP. I set θ to make the steady state labor share to be 68.4%, which implies $\theta = 3.0$.⁸ Following

⁷The value of the depreciation rate is also consistent with the rate of patent obsolescence estimated by Pakes and Schankerman (1984) using patent renewal data; their point estimate is 25% annual depreciation.

⁸The steady state labor share is calculated as follows. The data source is the “Gross National Income by Type of Income” (Table 1.10 of NIPA tables). Proprietor’s income, taxes on production and imports, subsidies, and the business current transfer payments are difficult to be allocated to labor income or profit. Following Cooley and Prescott (1995), I assume that the labor share in those uncertain components is the same as the labor share in the rest. I calculate the labor share by first subtracting those components from the gross national income and then calculating the fraction of the compensation of employees in it. The historical average of the labor share is 68.4%.

β	δ_N	θ	ν	σ	φ
.99	.06	3.0	.015	.16	2.74

Table 1: Calibrated Values

Rotemberg and Woodford (1997), I set the constant relative risk aversion σ to be .16.⁹ I set φ to make the steady state hours $L = .2$.

The Bureau of Economic Analysis recently released a research and development satellite account. The ratio of nominal R&D spending to nominal GDP is stable around 2.8%. I set ν to make the steady state value of R/Y in my model economy 2.8%, which implies $\nu = .015$.

4.2 Shock Processes

I estimate the stochastic processes of the productivity shock A_t and the R&D shock S_t with the maximum likelihood. I first impose the parametric assumption that $\log A_t$ and $\log S_t$ follow AR(1) processes

$$\begin{aligned}\log A_t &= \rho_a \log A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, \sigma_a^2) \\ \log S_t &= \rho_s \log S_{t-1} + \varepsilon_{s,t}, \quad \varepsilon_{s,t} \sim N(0, \sigma_s^2)\end{aligned}$$

where $-1 < \rho_a, \rho_s < 1$, and $\varepsilon_{a,t}$ and $\varepsilon_{s,t}$ are serially uncorrelated. The unconditional mean of A_t and the unconditional mean of S_t are normalized to one because they are irrelevant to the log-linearized system. I assume that the innovation of the productivity shock $\varepsilon_{a,t}$ and the innovation of the R&D shock $\varepsilon_{s,t}$ are uncorrelated. This is because they represent very different types of shocks. The productivity shock A_t governs how much physical goods is produced given certain labor input; weather is a good real world counterpart of $\varepsilon_{a,t}$. R&D shock S_t governs how much new products is innovated given a certain R&D input; the R&D performance is not much affected by weather but is more affected by the state of the basic scientific knowledge. Weather and breakthroughs in the basic scientific research are reasonably assumed as uncorrelated series.

I use GDP and R&D data in my estimation. I take these data from the Bureau of Economic Analysis. Both series are divided by the population and expressed in logarithm. I first apply the band-pass filter of Christiano and Fitzgerald (2003) to extract the stationary component of periodicity less than 32 years. The annual R&D series is then interpolated to quarterly series. The sample period is 1959:Q1 to 2004:Q4. The detrended actual GDP and R&D are matched with GDP and R&D in the log-linearized model. The parameter values that maximizes the likelihood are $\rho_a = .962$, $\rho_s = .959$, $\sigma_a = .0088$, and $\sigma_s = .0109$.

⁹The implied intertemporal elasticity of substitution is larger than empirical findings such as Yogo (2004). But as Rotemberg and Woodford (1997) argue, high intertemporal elasticity of substitution is reasonable in a model without physical capital.

I do not use the stock price data in my estimation because so far this version of the model cannot create anything like the volatility we see in the actual data together with empirically reasonable volatility of output.¹⁰ I extract the information about the R&D shock from the R&D data, and use the model to make a contribution to a qualitative puzzle with respect to timing.

The periodicity I allow to pass the band-pass filter is substantially lower than 8 years, which is the usual business cycle frequency. This is because I expect that the R&D shock has implication in the lower frequency as major technological breakthroughs in history had long lasting effect on the economy. Lower frequency cycles are important but less explored subject, while Comin and Gertler (2006) is an important exception. This paper is a contribution to the subject.

5 Results

5.1 Model's Performance

I am interested in the joint dynamics of output Y_t and the total firm value V_t^{total} defined as

$$V_t^{total} = [N_t + N_{E,t}] V_t.$$

The left column of Figure 1 shows the impulse responses of the output and the total firm value to a positive productivity shock. A positive productivity shock increases output and the total firm value immediately and persistently. The output increases because the productivity shock makes the goods production more efficient.

The total firm value increases for two reasons. First, a positive productivity shock allows each monopolistic firm to produce his differentiated goods at cheaper cost, which increases the firm's profit in the goods producing sector and hence raises the individual firm value V_t . Second, the increase of the individual firm value attracts more R&D investment because it makes R&D more profitable. The R&D boom then increases the firm entry $N_{E,t}$. The total firm value increases because of the intensive margin V_t and the extensive margin $N_{E,t}$.

A positive R&D shock decreases output on impact although the decrease is so subtle that it is not easy to see from the figure. The initial decrease is caused because the economy shifts more resource from consumption to R&D. The output gradually increases in subsequent periods because the goods production becomes more efficient as the available variety increases.

¹⁰The problem is not unique to my model. To my best knowledge, there is no macro-economic model that successfully generates the volatility of the stock price together with reasonable key business cycle statistics. The root of the problem is the same as that of the long-lasting problem—why the actual stock price is so volatile compared to dividends—that dates back to Shiller (1981).

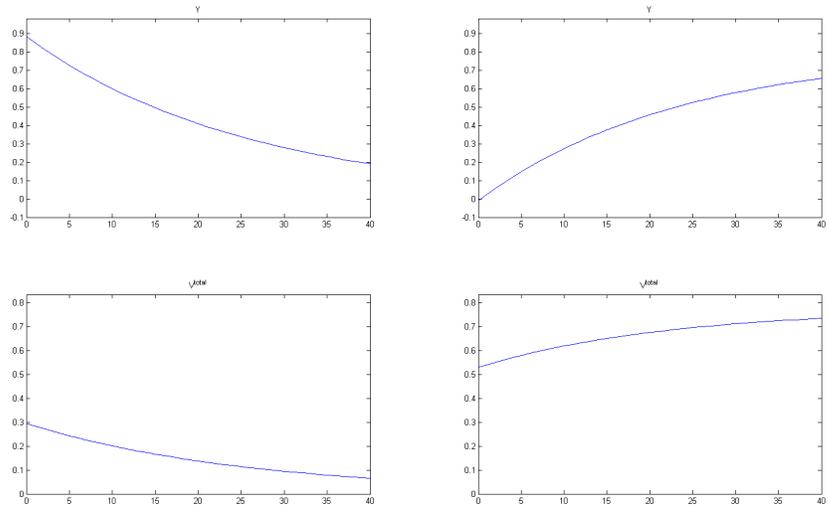


Figure 1: Impulse response functions of output and the total firm value to the productivity shock (first column) and to the R&D shock (second column). The sizes of the impulses are one-standard deviation of respective shocks.

The total firm value increases immediately because both the individual firm value and the firm entry increase. The firm entry increases because the R&D shock makes the firm creation more efficient. The increase of the individual firm value is less obvious; the mechanism is worth a closer look because it is at the heart of this paper.

Figure 2 shows responses of individual firm's profit in goods producing sector, expected profit in the R&D sector, expected dividend, and the individual firm value to a positive R&D shock. As shown in the upper left panel of Figure 2, a positive R&D shock is bad news for the profit in the goods producing sector. This is because the demand for each firm decreases as rival firms increase; remember each of the firms produces a specialized intermediate variety that is partially substitutable with other varieties.

The R&D shock, however, is good news for the profit in the R&D sector as the upper right panel of Figure 2 shows. The existing intermediate goods producing firm has an ability to produce new intermediate goods producing firms and optimally uses the ability to make a profit. Because the R&D shock makes the firm creation more efficient, the profit in the R&D sector increases.

As the lower left panel of Figure 2 shows, the expected dividend increases because on balance the positive effect in the R&D sector dominates the negative effect in the goods producing sector. Because the individual firm value is a present discounted value of the future profit stream, the individual firm value increases as the lower right panel of Figure 2 shows.

In an alternative model specification in which existing firms do not have expertise but independent entrepreneurs can innovate products without any skill or knowledge, a positive R&D shock decreases the individual firm value. In the appendix, I show this version of the model and responses in the model.

Our two-sector two-shock model allows to study the relative importance of each shock at different frequencies. Figure 3 shows the forecast error variance decomposition, i.e., the share of the forecast error variance attributed to the R&D shock at different forecast horizons.

The top panel shows the variance decomposition of the output. The productivity shock plays dominant role to explain output fluctuation in the short horizon. The R&D shock becomes more important as the forecast horizon extends. The lower panel shows the variance decomposition of the total firm value. The R&D shock is the major source of the total firm value fluctuation even in the short run. In the medium run, from 33 quarters to 128 quarters, almost all of the total firm value fluctuation is explained by the R&D shock.

5.2 Cross Correlation

In the model economy, a rise in the total firm value today signals an economic boom in the future. This is because a positive R&D shock increases the total firm value immediately and increases output with a delay, while a positive productivity shock increases the total firm value and the output contemporaneously. In this sense, the total firm value is a leading indicator of GDP.

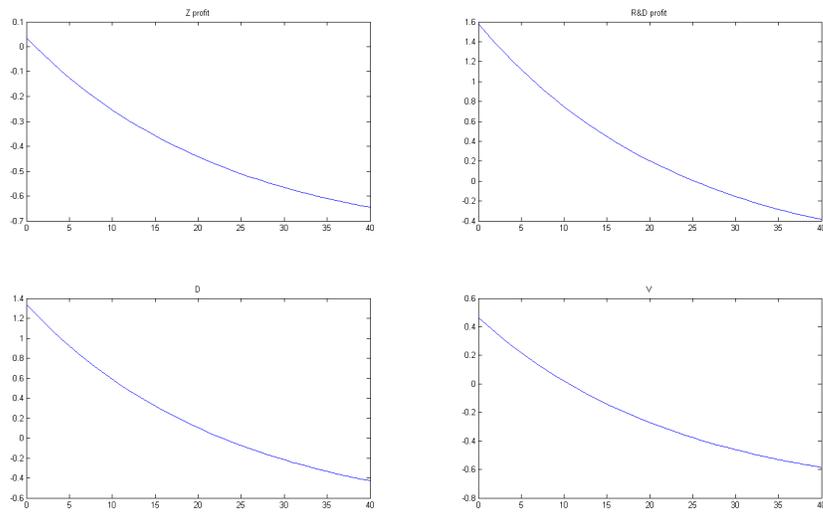


Figure 2: Responses to a positive R&D shock: the individual firm's profit in the goods producing sector (the upper left panel), the individual firm's expected profit in the R&D sector (the upper right panel), the individual firm's expected dividend (the lower left panel), and the individual firm value (the lower right panel).

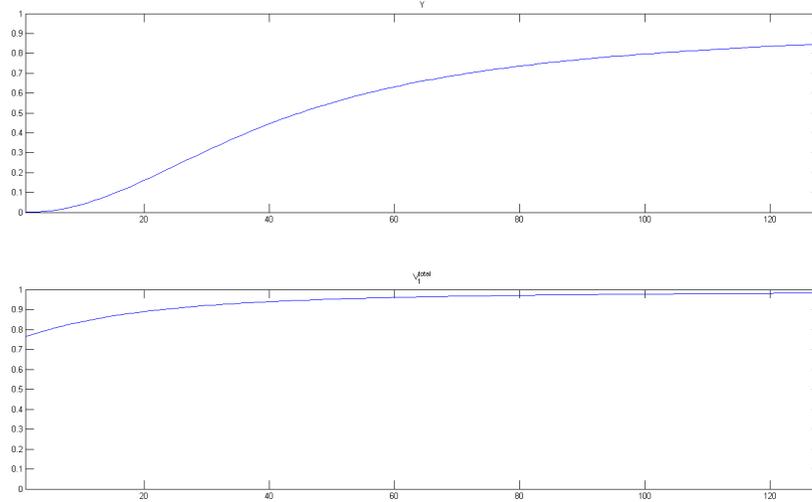


Figure 3: The forecast error variance decomposition. The fraction of the forecast error attributed to R&D shock at different horizons to 128 quarters.

I calculate cross correlations of the total firm value and GDP. The cross correlation function is defined as

$$r(k) = Corr(V_t^{total}, Y_{t-k}).$$

Negative values of k correspond to correlations of the current total firm value with future GDPs, and vice versa. I simulate 1,000 artificial data sets of 199 data points, and calculate the statistics for each set.

The result is plotted in the left panel of Figure 4. The cross correlation function is not only positive, but also asymmetric with stronger correlations in negative values of k . Namely, the current total firm value is more strongly correlated with the future output than with the past output. This asymmetry is a characteristic of a leading indicator.

The right panel of Figure 4 plots the cross correlation of the corresponding actual data. I use quarterly GDP and the broadly defined total firm value calculated by Hall (2001a).¹¹ The sample period is 1950:1Q to 1999:3Q. The

¹¹Hall (2001a) measures the total firm value as the market value of outstanding equities plus the imputed market value of bonds plus the reported value of other financial liabilities less financial assets for all nonfarm, nonfinancial corporations. Hall (2001a) ignores the residual assets reported in the Flow of Funds accounts because his interest is the link between the asset value and the asset return but no information about the residual assets' return is available. Because I am interested in the firm value itself, I add the residual assets in my definition of the total firm value.

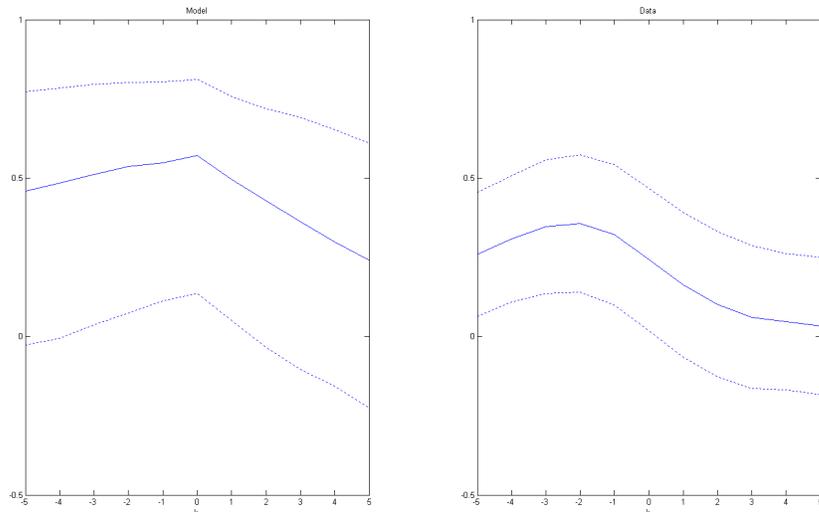


Figure 4: Cross-correlation of the total firm value in period t and GDP in period $t - k$. The left panel plots the cross-correlation function of artificially generated data; the solid line shows the median, the dotted band shows the 2.5% and 97.5% quantiles of 1,000 simulations. The right panel plots the cross-correlation in the actual data; the solid line is the sample cross-correlation and the dotted band is 95% confidence interval. Both data are detrended with the band-pass filter that passes the stationary component of periodicity less than 32 years.

cross correlations for the actual data are similar to those for the simulated data. Importantly, the procyclicality and the asymmetry are observed.

5.3 Vector Autoregression

In this subsection, I compare the model's impulse responses to the impulse responses obtained in the structural Vector Autoregression (SVAR) model. This is much more challenging exercise than comparing cross-correlations because the impulse response function in VAR contains much richer information.

The empirical framework is a bi-variate VAR model;

$$\begin{bmatrix} Y_t \\ V_t^{total} \end{bmatrix} = c + \sum_{j=1}^5 B_j \begin{bmatrix} Y_{t-j} \\ V_{t-j}^{total} \end{bmatrix} + u_t, \quad u_t \sim \text{i.i.d. } N(0, \Sigma). \quad (29)$$

I assume that this system is driven by the productivity innovation $\varepsilon_{a,t}$ and the R&D innovation $\varepsilon_{s,t}$. The fundamental innovations ε_t and the VAR shock u_t

are related with an impact matrix Γ ;

$$u_t = \Gamma \begin{bmatrix} \varepsilon_{a,t}/\sigma_a \\ \varepsilon_{s,t}/\sigma_s \end{bmatrix}.$$

The identification problem arises because the moment restriction

$$\Gamma'\Gamma = \Sigma$$

imposes only three restrictions while there are four unknowns in Γ . We need additional restriction to identify the fundamental innovations from the data. Popular zero restrictions are not available because the model economy does not imply a binding zero restriction either on impact or in the long-run. But the model economy does have a fairly robust implication: a positive productivity shock increases output and the total equity value immediately.

Relying on this implication, I identify the two fundamental innovations by the pure-sign restriction of Uhlig (2005). My sign restriction is that a positive productivity innovation increases the output and the total firm value on impact. Because the restriction is too weak to uniquely pin down the impact matrix Γ , I treat all the possible Γ that satisfies the sign restriction equally likely. Hence, the resulting error bands of impulse responses contain not only the uncertainty in the VAR parameters $(c, B_1, \dots, B_5, \Sigma)$ but also the uncertainty in identifying the impact matrix Γ . A technical discussion appears in the appendix.

The first column of Figure 5 plots responses to the identified productivity shock. By construction, the identified productivity shock increases GDP and the total firm value on impact. The productivity shock has persistent positive effects on both variables. These responses are consistent with those observed in the model economy.

The second column of Figure 5 plots responses to the identified R&D shock. Remember I do not impose any prior restriction on these responses. The identified R&D shock increases the total firm value immediately and persistently, but it decreases GDP initially and then gradually increases GDP. These responses are consistent with those observed in the model economy.

6 Conclusion

This paper studies an endogenous variety model in which existing firm's expertise is essential input for innovation of new products. Because the expertise is acquired through manufacturing an existing product, the existing product's value reflects not only the usual monopoly rent but also the value as an expertise generator. A favorable shock in the R&D sector increases the total firm value immediately because the value of expertise increases immediately. The same shock increases output sluggishly because the goods producing productivity increases after new products are introduced. Because of these asymmetric responses, the stock price becomes a leading indicator in the model.

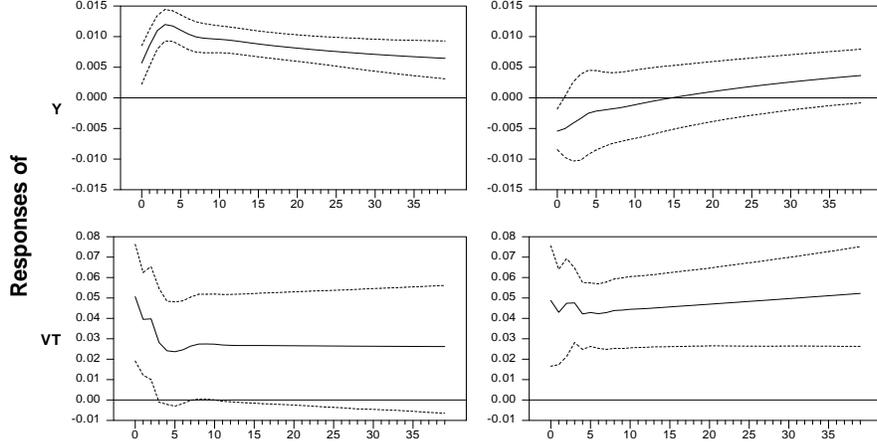


Figure 5: Impulse response functions of GDP and the total firm value in the structural VAR model. The first column is the responses to the productivity shock. The second column is the responses to the R&D shock. Dotted bands are 68% error bands.

A Log-linearized Model

A.1 System of Equations

The following equations summarize the equilibrium.

$$A_t N_t^{\frac{1}{\theta-1}} = \frac{\varphi \theta}{\theta - 1} \frac{Y_t}{1 - L_t}, \quad (30)$$

$$V_t = E_t \left[\beta (1 - \delta_N) \left(\frac{Y_t}{Y_{t+1}} \right)^\sigma \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi(1-\sigma)} \left(\frac{Z_{t+1}}{\theta N_{t+1}} + (1 - \mu) \frac{V_{t+1} N_{E,t+1}}{N_{t+1}} + V_{t+1} \right) \right], \quad (31)$$

$$R_t = \mu V_t N_{E,t}, \quad (32)$$

$$Z_t = A_t N_t^{\frac{1}{\theta-1}} L_t, \quad (33)$$

$$Y_t + R_t = Z_t, \quad (34)$$

$$N_{t+1} = (1 - \delta_N) (N_t + N_{E,t}), \quad (35)$$

$$N_{E,t} = S_t N_t^{1-\nu} R_t^\nu, \quad (36)$$

$$V_t^{total} \equiv V_t [N_t + N_{E,t}]. \quad (37)$$

A.2 Steady State Relations

From (35),

$$\begin{aligned}\delta_N N &= (1 - \delta_N) N_E, \\ \frac{N_E}{N} &= \frac{\delta_N}{1 - \delta_N}.\end{aligned}$$

From (34),

$$\frac{Z}{Y} = 1 + \frac{R}{Y}. \quad (38)$$

From (24),

$$\frac{WL}{Y} = \frac{\theta - 1}{\theta} \frac{Z}{Y}. \quad (39)$$

From (38) and (39)

$$\theta = \frac{1}{1 - \frac{\frac{WL}{Y}}{1 + \frac{R}{Y}}}.$$

From (31),

$$\frac{VN_E}{Z} = \frac{\beta \delta_N}{\theta [1 - \beta (1 - \mu \delta_N)]}.$$

From (32)

$$\frac{R}{Z} = \frac{\beta \mu \delta_N}{\theta [1 - \beta (1 - \mu \delta_N)]}. \quad (40)$$

ν is found from (40),

$$\mu = \frac{\theta \frac{R}{Z}}{\delta_N [1 - \theta \frac{R}{Z}]} \frac{1 - \beta}{\beta}.$$

Combining (30) and (33),

$$\varphi = \frac{\theta - 1}{\theta} \frac{1 - L}{L} \frac{Z}{Y}.$$

A.3 Log-linearize the system

From (30),

$$\log A_t + \frac{1}{\theta - 1} \hat{N}_t = \hat{Y}_t + \frac{L}{1 - L} \hat{L}_t.$$

From (31),

$$\hat{V}_t = E_t \left[\begin{array}{l} \sigma (\hat{Y}_t - \hat{Y}_{t+1}) - \varphi (1 - \sigma) \frac{L}{1 - L} (\hat{L}_{t+1} - \hat{L}_t) \\ + (1 - \beta (1 - \mu \delta_N)) (\hat{Z}_{t+1} - \hat{N}_{t+1}) + \beta (1 - \mu) \delta_N (\hat{R}_{t+1} - \hat{N}_{t+1}) \\ + \beta (1 - \delta_N) \hat{V}_{t+1} \end{array} \right].$$

From (32),

$$\hat{R}_t = \hat{V}_t + \hat{N}_{E,t}.$$

From (33),

$$\hat{Z}_t = \log A_t + \frac{1}{\theta - 1} \hat{N}_t + \hat{L}_t.$$

From (34),

$$\frac{Y}{Z} \hat{Y}_t + \frac{R}{Z} \hat{R}_t = \hat{Z}_t.$$

From (35),

$$\hat{N}_{t+1} = (1 - \delta_N) \hat{N}_t + \delta_N \hat{N}_{E,t}.$$

From (36),

$$\hat{N}_{E,t} = \log S_t + (1 - \nu) \hat{N}_t + \nu \hat{R}_t.$$

From (37),

$$\hat{V}_t^{total} = \hat{V}_t + (1 - \delta_N) \hat{N}_t + \delta_N \hat{N}_{E,t}.$$

B Alternative Model Specification

I present a version of the model in which independent entrepreneurs can introduce products without any skill or knowledge. The aggregate production function of the new intermediate goods producing firm is the same as before;

$$N_{E,t} = S_t N_t^{1-\nu} R_t^\nu. \quad (41)$$

The measure of the existing products N_t enters into the production function (41) as pure externality. Entrepreneurs are price takers. If an entrepreneur spends r_t of final goods as R&D input, she can produce, on average,

$$\left(\frac{N_{E,t}}{R_t} \right) r_t$$

of new intermediate goods producing firms. The entrepreneur maximizes the expected profit;

$$\max_{r_t} \left[\left(\frac{N_{E,t}}{R_t} \right) V_t - 1 \right] r_t.$$

Because becoming entrepreneurs does not require any skill or knowledge, free entry and competition assures that the zero profit condition holds in equilibrium;

$$\left(\frac{N_{E,t}}{R_t} \right) V_t = 1. \quad (42)$$

Because the variety's contribution in the R&D sector is pure externality, existing intermediate goods firms cannot claim any rent in the R&D sector. The individual firm value is determined by

$$V_t = E_t \left[\beta \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi(1-\sigma)} (1 - \delta_N) \left(\frac{Z_{t+1}}{\theta N_{t+1}} + V_{t+1} \right) \right]. \quad (43)$$

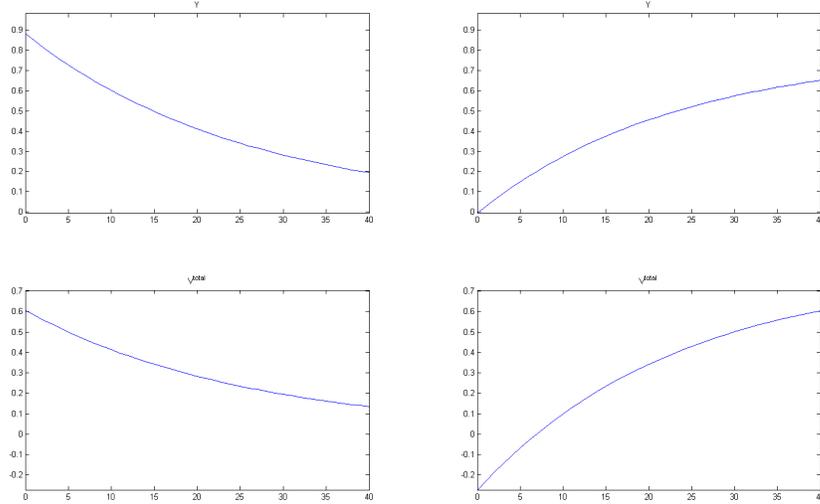


Figure 6: Impulse response functions in the model in which innovation of new products does not require expertise. The first column plots responses to the productivity shock. The second panel plots responses to the R&D shock.

Replacing (27) and (28) in the benchmark model with (42) and (43) gives the alternative model.

Figure 6 plots the impulse responses in this version of the model. The lower right panel plots the impulse response function of the total firm value to a positive R&D shock. The total firm value crashes when a favorable R&D shock hits the economy.

C Sign Restriction

A technical detail of the pure sign restriction is presented. The restriction implemented in this paper is a special case of Uhlig (2005). The empirical framework is

$$\begin{bmatrix} Y_t \\ V_t^{total} \end{bmatrix} = c + \sum_{j=1}^5 B_j \begin{bmatrix} Y_{t-j} \\ V_{t-j}^{total} \end{bmatrix} + u_t, \quad u_t \sim \text{i.i.d. } N(0, \Sigma). \quad (44)$$

How to estimate the VAR parameters $(B, \Sigma) = ((c, B_1, \dots, B_5), \Sigma)$ is extensively discussed in the literature. A Bayesian estimation is particularly convenient because the posterior distribution for (B, Σ) given a Normal-Wishart prior in (B, Σ) is known. See Sims and Zha (1999) for the discussion.

To give a structural interpretation, I assume that the system is driven by the productivity innovation $\varepsilon_{a,t}$ and the R&D innovation $\varepsilon_{s,t}$ which are related with u_t through an impact matrix Γ ;

$$u_t = \Gamma \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{s,t} \end{bmatrix}. \quad (45)$$

We want to identify the productivity innovation and the R&D innovation, or equivalently, identify the impact matrix Γ .

It is convenient to express the impact matrix Γ in the polar coordinate;

$$\Gamma = \begin{bmatrix} r_1 \cos \theta_1 & r_1 \sin \theta_1 \\ r_2 \cos \theta_2 & r_2 \sin \theta_2 \end{bmatrix} \quad (46)$$

with $r_1, r_2, \theta_1, \theta_2 \geq 0$. The moment restriction

$$\Gamma' \Gamma = \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

imposes three restrictions

$$\begin{aligned} r_1^2 &= \sigma_1^2 \\ r_2^2 &= \sigma_2^2 \\ r_1 r_2 \cos(\theta_1 - \theta_2) &= \sigma_{12}. \end{aligned}$$

Solving the system gives

$$\begin{aligned} r_1 &= \sigma_1 \\ r_2 &= \sigma_2 \\ \theta_1 - \theta_2 &= \arccos\left(\frac{\sigma_{12}}{\sigma_1 \sigma_2}\right) + 2\pi k \end{aligned}$$

where \arccos is a function from $[-1, 1]$ to $[0, \pi]$ that satisfies $\cos(\arccos(x)) = x$ and k is a generic integer. Substitute them back into (46),

$$\Gamma(\theta_2|\Sigma) = \begin{bmatrix} \sigma_1 \cos\left(\theta_2 + \arccos\left(\frac{\sigma_{12}}{\sigma_1 \sigma_2}\right)\right) & \sigma_1 \sin\left(\theta_2 + \arccos\left(\frac{\sigma_{12}}{\sigma_1 \sigma_2}\right)\right) \\ \sigma_2 \cos \theta_2 & \sigma_2 \sin \theta_2 \end{bmatrix}. \quad (47)$$

I introduce the notation $\Gamma(\theta_2|\Sigma)$ to stress its dependence on θ_2 and Σ . This is a way to present the identification problem; the moment restriction $\Gamma \Gamma' = \Sigma$ restricts the structure of Γ up to (47), but it is not unique yet. Actually, $\Gamma(\theta_2|\Sigma) \Gamma(\theta_2|\Sigma)' = \Sigma$ is satisfied for any value of $\theta_2 \in [0, 2\pi]$.

A widely used identification technique is assuming that Γ is lower triangular; that is equivalent to assume $\theta_2 = -\arccos\left(\frac{\sigma_{12}}{\sigma_1 \sigma_2}\right)$ in our notation. Another popular strategy is assuming some of the innovations have permanent effect while the others have only transitory effect (see Blanchard and Quah (1989)).

The zero restrictions, however, are not appropriate for my study because they are inconsistent with my model economy. The restriction I impose is

consistent with my model economy but probably the least controversial one; that is, a positive productivity innovation has immediate positive effects on the output and the total firm value. That is, I choose $\Gamma(\theta|\Sigma)$ such that the two elements in its first column are positive. Obviously, the restriction is too weak to uniquely pin down the impact matrix Γ but multiple possibilities remain. That problem is dealt with by putting a prior that treats all the possibilities satisfying the sign restriction equally likely.

Formally, the identification assumption is written as follows. The parameters (B, Σ, θ) are drawn jointly from a prior on $R^{5 \times 2 \times 2} \times P_2 \times [0, 2\pi]$ where P_2 is the space of positive definite 2×2 matrices. The prior is proportional to a Normal-Wishart in (B, Σ) whenever both $\Gamma(\theta|\Sigma)_{11} > 0$ and $\Gamma(\theta|\Sigma)_{21} > 0$ are satisfied, and zero otherwise.

The posterior distribution of (B, Σ, θ) is given by the usual Normal-Wishart posterior for (B, Σ) times the indicator function on $\{\Gamma(\theta|\Sigma)_{11} > 0 \text{ and } \Gamma(\theta|\Sigma)_{21} > 0\}$. To draw from this posterior, I take a joint draw from both the posterior for the unrestricted Normal-Wishart posterior for the VAR parameters (B, Σ) as well as a uniform distribution over $\theta \in [0, 2\pi]$. If both $\Gamma(\theta|\Sigma)_{11} > 0$ and $\Gamma(\theta|\Sigma)_{21} > 0$ are satisfied, I keep the draw. Otherwise, I discard it. I repeat sufficiently often and construct error bands based on the accepted draws.

D General Model (Preliminary)

This section shows a general version of the model. I introduce the physical capital and trade. The main results hold in this general environment, and the model has as good simulated business cycle moments as a standard business cycle model.

D.1 Goods Producing Sector

The goods producing sector has the representative competitive final goods producing firm and the monopolistic intermediate goods producing firms. The final goods producing firm purchases the differentiated intermediate goods and assembles the final goods. Assembling process does not require any resource other than the intermediate goods. The production function is

$$Z_t = \left[\int_0^{N_t} Z_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $Z_{i,t}$ is the intermediate goods of index i . N_t is the measure of the intermediate goods variety on the market in period t which is represented by the interval $[0, N_t]$ without loss of generality. Competition ensures an equilibrium price of the final goods P_t equal to

$$P_t = \left[\int_0^{N_t} P_{i,t}^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (48)$$

where $P_{i,t}$ is the price of the intermediate goods of index i . The final goods firm demands

$$Z_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Z_t$$

The intermediate goods producing firms are monopolists. A multi-product firm is not allowed; i.e., a single intermediate goods producing firm can produce up to a single differentiated product for the technological reason. Therefore, the measure of the intermediate goods producing firm and the measure of the available intermediate goods variety are identical.

The firm's production function is

$$Z_{i,t} = K_{i,t}^\alpha (A_t L_{i,t})^{1-\alpha} \quad (49)$$

A_t is exogenous random variable that I call a productivity shock. The firm chooses the price to maximize the real profit;

$$\pi_{i,t}^Z = \max_{P_{i,t}} \left\{ \left[\frac{P_{i,t}}{P_t} - J_t \right] \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Z_t \right\} \quad (50)$$

where J_t is the unit cost to produce one unit of $Z_{i,t}$;

$$J_t = \frac{r_{K,t}^\alpha (W_t/A_t)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}.$$

D.2 R&D Sector

The measure of the intermediate goods producing firms is endogenously determined. A fraction δ_N of the intermediate goods producing firms dies at the end of every period, and new intermediate goods firms are produced by R&D. Let $N_{E,t}$ be the aggregate measure of newly created intermediate goods producing firms. The measure of firms evolves as

$$N_{t+1} = (1 - \delta_N) (N_t + N_{E,t}).$$

Importantly, I assume that only the existing intermediate goods firms have an ability to produce new firms because the detailed knowledge about an existing product is essential to create new varieties, and only the firm that is currently producing the product can acquire it. R&D is risky at individual level. If an intermediate goods producing firm spends $R_{i,t} \geq 0$ of the final goods as R&D input, she produces n new intermediate goods producing firms with probability

$$\Pr(n|R_{i,t}, S_t).$$

Its expectation is

$$\sum_{n=0}^{\infty} n \Pr(n|R_{i,t}, S_t) = S_t R_{i,t}^\nu. \quad (51)$$

S_t is an exogenous random variable that I call the R&D shock.

When an intermediate goods producing firm succeeds to produce new intermediate goods firms, she sells the ownership of the new firms to the household. The new intermediate goods firm born in period t starts her business in period $t + 1$; its ownership is traded at ex-dividend market value V_t . An intermediate goods producing firm i chooses the R&D input optimally to maximize the expected profit $\pi_{i,t}^{R\&D}$;

$$\pi_{i,t}^{R\&D} = \max_{R_{i,t} \geq 0} \{S_t R_{i,t}^\nu V_t - R_{i,t}\}. \quad (52)$$

The first order condition is

$$\nu S_t R_{i,t}^{\nu-1} V_t = 1. \quad (53)$$

The aggregate firm entry $N_{E,t}$ and the aggregate R&D spending R_t are defined as

$$N_{E,t} = \int_0^{N_t} S_t R_{i,t}^\nu di, \quad (54)$$

$$R_t = \int_0^{N_t} R_{i,t} di. \quad (55)$$

D.3 Capital Firm

A competitive representative capital firm owns and rents the physical capital K_t . The firm's problem is

$$V_{K,t}(K_t) = \max_{I_t} \left\{ r_{K,t} K_t - I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta_K \right)^2 K_t + E_t [\lambda_{t,t+1} V_{K,t+1}(K_{t+1})] \right\} \quad (56)$$

with transition rule

$$K_{t+1} = (1 - \delta_K) K_t + I_t.$$

The first order conditions are

$$q_t = 1 + \phi \left(\frac{I_t}{K_t} - \delta_K \right). \quad (57)$$

$$q_t = E_t \left[\lambda_{t,t+1} \left(r_{K,t+1} + \frac{\phi}{2} \left[\left(\frac{I_{t+1}}{K_{t+1}} \right)^2 - \delta_K^2 \right] + q_{t+1} (1 - \delta_K) \right) \right], \quad (58)$$

λ_t is a Lagrangian multiplier.

D.4 Household

D.4.1 Individual Household

The household is modeled following Blanchard (1985). A continuum of households lives on a unit mass. Each atomistic household throughout his life faces a constant probability of death $\omega \in [0, 1]$; because the same number of new households are born every period, the population is constant.

The financial market is complete for aggregate uncertainty. About the idiosyncratic death shock, a competitive life-insurance company offers a contract with which a household pays $\frac{1-\omega}{\omega}$ contingent on his death and receives 1 contingent on his survival. Because households do not have a bequest motive and they are prohibited to leave negative bequests, households will contract to have all of their wealth return to the life insurance company.

The problem of the household born in period $s \leq t$ is

$$\max E_t \left[\sum_{j=0}^{\infty} [\beta (1 - \omega)]^j (\log c_{t+j}(s) + \varphi \log (1 - l_{t+j}(s))) \right]$$

subject to

$$c_t(s) + E_t [\Lambda_{t,t+1} x_t(s)] = \frac{1}{1 - \omega} x_{t-1}(s) + W_t l_t(s), \quad (59)$$

$$\lim_{n \rightarrow \infty} E_t [\Lambda_{t,t+n+1} x_{t+n}(s)] \geq 0. \quad (60)$$

$E_t[\cdot]$ takes expectation with respect to aggregate uncertainty; $x_t(s)$ is the state-contingent claim; $\Lambda_{t,t+1}$ is the stochastic discount factor; W_t is the real wage. With (59), (60), and a transversarity condition, we can derive a single intertemporal budget constraint

$$\frac{1}{1 - \omega} x_{t-1}(s) = E_t \left[\sum_{j=0}^{\infty} \Lambda_{t,t+j} (1 - \omega)^j (c_{t+j}(s) - W_{t+j} l_{t+j}(s)) \right]. \quad (61)$$

The first order conditions are

$$\frac{W_t}{c_t(s)} = \frac{\varphi}{1 - l_t(s)}, \quad (62)$$

$$\frac{1}{c_t(s)} = \frac{\beta^j}{\Lambda_{t,t+j}} \frac{1}{c_{t+j}(s)}. \quad (63)$$

Substitute (62) and (63) into (61), we find an individual consumption rule;

$$c_t(s) = \frac{1 - \beta(1 - \omega)}{1 + \varphi} \left(\frac{1}{1 - \omega} q_{t-1}(s) + H_t \right), \quad (64)$$

where H_t satisfies

$$H_t = W_t + E_t [\Lambda_{t,t+1} (1 - \omega) H_{t+1}]. \quad (65)$$

D.4.2 Aggregation

The aggregate consumption C_t , the aggregate labor supply L_t , and the aggregate state contingent claim X_t are defined as

$$\begin{aligned} C_t &= \sum_{j=0}^{\infty} \omega (1 - \omega)^j c_t(t - j), \\ L_t &= \sum_{j=0}^{\infty} \omega (1 - \omega)^j l_t(t - j), \\ X_t &= \sum_{j=0}^{\infty} \omega (1 - \omega)^j x_t(t - j). \end{aligned}$$

From (59) and $x_{t-1}(t) = 0$,

$$C_t + E_t[\Lambda_{t,t+1} X_t] = X_{t-1} + W_t L_t. \quad (66)$$

From (62),

$$W_t = \frac{\varphi C_t}{1 - L_t}. \quad (67)$$

From (63), (64), and $x_t(t+1) = 0$,

$$\Lambda_{t,t+1} = \frac{\beta(1 - \omega) C_t}{C_{t+1} - \omega \frac{1 - \beta(1 - \omega)}{1 + \varphi} H_{t+1}}. \quad (68)$$

D.5 Equilibrium

The resource constraint is

$$Z_t = C_t + I_t + R_t + B_t - (1 + r) B_{t-1}.$$

B_t is the net bond holdings; the economy is a small open economy that can borrow or lend freely at the constant real interest rate r . Market clearing conditions for the asset market, the physical capital market, and the labor market are

$$X_{t-1} = (1 + r) B_{t-1} + V_{K,t}(K_t) + (D_t + V_t) N_t \quad (69)$$

$$K_t = \int_0^{N_t} K_{i,t} di \quad (70)$$

$$L_t = \int_0^{N_t} L_{i,t} di. \quad (71)$$

In (69), I am assuming that the capital firm and the intermediate goods firms are owned by the domestic households. The symmetric equilibrium is defined as a sequence of prices and quantities that solves the agent's problems and satisfies the market clearing conditions at every period.

α	β	δ_K	δ_N	θ	ν	φ	ϕ	ω	r
.2	.9905	.025	.06	6	.033	3.12	1	.005	.01

Table 2: Calibrated Values

D.6 Parameters

The time unit is a quarter. I set the depreciation rate of the physical capital δ_K to be .025. The depreciation rate of the variety δ_N is set to be .06 as before. The capital adjustment cost ϕ is set to be 1, corresponding to .25 in the annual model. The value of ϕ is motivated by Hall (2004) who uses the annual data and finds that the adjustment parameter is not much above zero. The elasticity of substitution among different specialized intermediate varieties θ is set to be 6, a commonly used value in the literature. α is set to make the steady state labor share to be 68.4%. I set the probability of household death ω to be .005, which implies the household life expectancy is 50 years. I set the real interest rate of risk free bond r to be .01. I set ν so that the steady state value of R/Y in my model economy is 2.8%, which implies $\nu = .033$. The coefficient on the leisure φ and the subjective discount rate β are set to make the steady state hours L to be .2 and the steady state net foreign bond holdings B to be 0.

D.7 Shock Processes

I estimate the stochastic processes of the productivity shock A_t and the R&D shock S_t with the maximum likelihood. I first impose the parametric assumption that $\log A_t$ and $\log S_t$ follow AR(1) processes

$$\begin{aligned}\log A_t &= \rho_a \log A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, \sigma_a^2) \\ \log S_t &= \rho_s \log S_{t-1} + \varepsilon_{s,t}, \quad \varepsilon_{s,t} \sim N(0, \sigma_s^2)\end{aligned}$$

where $-1 < \rho_a, \rho_s < 1$, and $\varepsilon_{a,t}$ and $\varepsilon_{s,t}$ are serially uncorrelated. The unconditional mean of A_t and the unconditional mean of S_t are normalized to one because they are irrelevant to the log-linearized system. I assume that the innovation of the productivity shock $\varepsilon_{a,t}$ and the innovation of the R&D shock $\varepsilon_{s,t}$ are uncorrelated.

I take GDP and R&D data from the Bureau of Economic Analysis. Both series are divided by the population and expressed in logarithm. I first apply the band-pass filter of Christiano and Fitzgerald (2003) to extract the stationary component of periodicity less than 32 years. The annual R&D series is then interpolated to quarterly series. The sample period is 1959:Q1 to 2004:Q4. The detrended actual GDP and R&D are matched with GDP and R&D in the log-linearized model. The parameter values that maximizes the likelihood are $\rho_a = .998$, $\rho_s = .943$, $\sigma_a = .0107$, and $\sigma_s = .0111$.

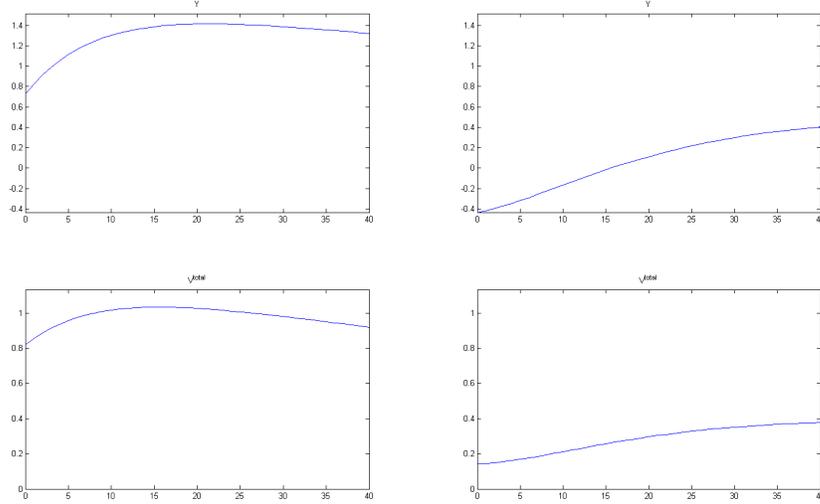


Figure 7: Impulse response functions in the general model. The first column plots responses of output and the total firm value to the productivity shock, and the second column plots responses to the R&D shock. The sizes of the impulses are one-standard deviation of respective shocks.

D.8 Results

D.8.1 Impulse Response Function

I am interested in the joint dynamics of output Y_t and the total firm value V_t^{total} defined as

$$Y_t = Z_t - R_t.$$

$$V_t^{total} = [N_t + N_{E,t}] V_t + K_{t+1} q_t.$$

The left column of Figure 7 shows the impulse responses of the output and the total firm value to a positive productivity shock. A positive productivity shock increases output and the total firm value immediately and persistently. The right column of Figure 7 shows the impulse responses of the output and the total firm value to a positive R&D shock. Output decreases on impact, and then gradually recovers. The total firm value increases immediately and persistently. These responses are consistent with responses observed in the SVAR model (Figure 5).

Figure 8 plots impulse responses of key macro variables to a positive productivity shock. Figure 9 plots impulse responses of key macro variables to a positive R&D shock.

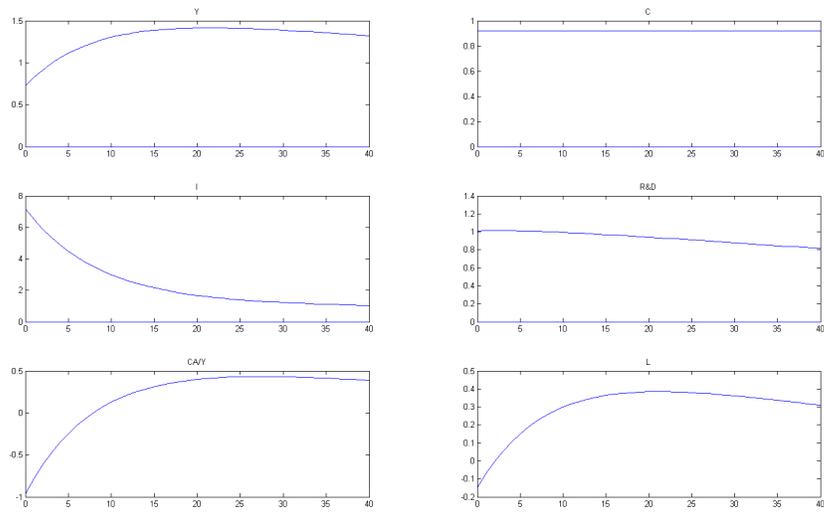


Figure 8: Impulse responses to a positive productivity shock: GDP (upper left panel), consumption (upper right panel), physical investment (middle left panel), R&D spending (middle right panel), current account as a fraction of GDP (lower left panel), and hours worked (lower right panel).

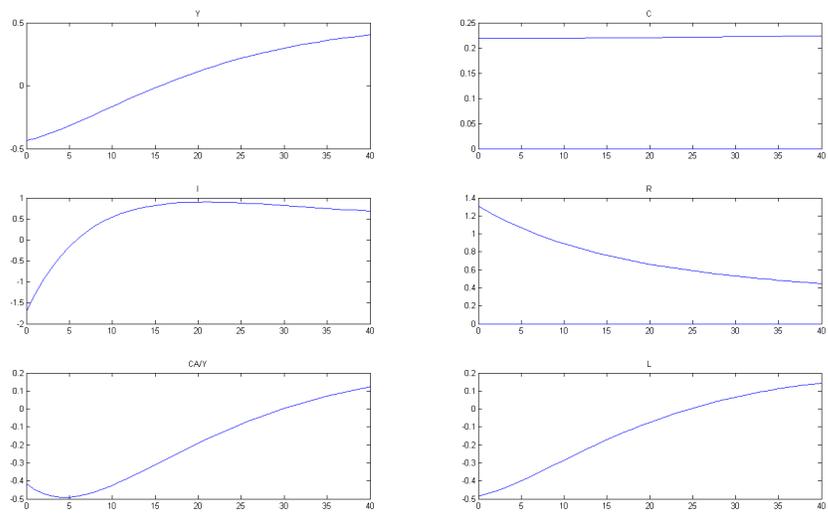


Figure 9: Impulse responses to a positive R&D shock: GDP (upper left panel), consumption (upper right panel), physical investment (middle left panel), R&D spending (middle right panel), current account as a fraction of GDP (lower left panel), and hours worked (lower right panel).

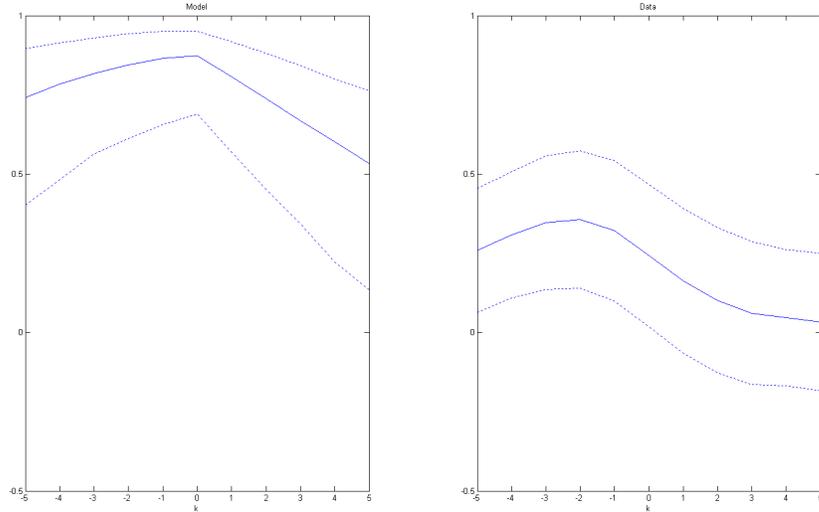


Figure 10: Cross-correlation of the total firm value in period t and GDP in period $t - k$. The left panel plots the cross-correlation function of artificially generated data from the general model; the solid line shows the median, the dotted band shows the 2.5% and 97.5% quantiles of 1,000 simulations. The right panel plots the cross-correlation in the actual data; the solid line is the sample cross-correlation and the dotted band is 95% confidence interval. Both data are detrended with the band-pass filter that passes the stationary component of periodicity less than 32 years.

D.8.2 Cross Correlation

Figure 10 plots the cross-correlation function is defined as

$$r(k) = \text{Corr}(V_t^{\text{total}}, Y_{t-k})$$

for the artificially generated data and the actual data. The procyclicality and the asymmetry are well generated from the general model.

D.8.3 Business Cycle Moments

Table 3 shows the business cycle moments of the data, artificially generated data from the general model, and those reported in King and Rebelo (2000).¹²

¹²Sample period is 1950:1Q to 2007:3Q. Data for GDP Y , consumption C , investment I , current account CA are taken from NIPA. Labor hours are taken from Bureau of Labor Statistics; it is defined as the number of nonfarm employees (CES0000000001) time average weekly hours of production workers (EEU00500005). All data are divided by the population and expressed in logarithm.

	σ_X			$Corr(X_t, X_{t-1})$			$Corr(X_t, Y_t)$		
	Data	Model	KR	Data	Model	KR	Data	Model	KR
Y	1.63	1.26 [1.05 1.49]	1.39	0.84	0.79 [0.72 0.85]	0.72	1.00	1.00 [1.00 1.00]	1.00
C	1.25	1.20 [1.04 1.39]	0.61	0.80	0.71 [0.62 0.78]	0.79	0.80	0.69 [0.56 0.77]	0.94
I	7.21	9.19 [7.92 10.47]	4.09	0.79	0.68 [0.59 0.75]	0.71	0.84	0.77 [0.71 0.81]	0.99
L	1.76	0.74 [0.63 0.86]	0.48	0.81	0.76 [0.67 0.82]	0.71	0.85	0.45 [0.26 0.60]	0.97
CA/Y	0.35	1.42 [1.23 1.64]	0	0.83	0.71 [0.63 0.78]	N/A	-0.28	-0.23 [-0.39 -0.06]	0

Table 3: Business Cycle Moments (HP filtered)

The actual data and artificially generated data are both detrended with the HP filter. Figures reported in the middle columns are median, 5% quantile, and 95% quantile of 1,000 sets of artificially generated data. The general model generates reasonable business cycle moments.

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