Nonlinear Income Variance Profile and Consumption
Inequality over the Life Cycle*

Naohito Abe† Tomoaki Yamada‡
Hitotsubashi University Rissho University

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Abstract

In an economy with a seniority wage system, elderly workers are subject to
greater income risks when they lose their jobs than young workers are. This pa-
per investigates: (1) whether we can observe the age dependence of idiosyncratic
income risks; and (2) the importance of age dependence for the evolution of in-
equalities in consumption using Japanese micro data. Our estimation of the income

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†Corresponding Author. The Institute of Economic Research, Hitotsubashi University. E-mail:
nabe@ier.hit-u.ac.jp. Phone: +81-425-80-8347.
‡Faculty of Economics, Rissho University. E-mail: tyamada@ris.ac.jp. Phone: +81-3-5487-3239
process demonstrates a strong age dependence of income risks; at the age of 48, the variance of permanent income shocks begins to increase, which creates a non-linear age–variance profile of income. This paper also uses structural estimation of a precautionary savings life cycle model to demonstrate that the nonlinearity in the income process is crucial for understanding the evolution of the consumption inequalities over age.

**JEL Classification:** D12, D31, D52, E21

**Key Words:** Income Risk, Buffer Stock Savings, Consumption Inequality, Method of Simulated Moments.

## 1 Introduction

It is widely recognized that idiosyncratic labor market uncertainty is one of the most important determinants of household consumption and savings when the market is incomplete. If uninsured income risks contain a permanent component, as Deaton and Paxson (1994) demonstrated, both income and consumption variances grow as people get older. The gap between the two variances, given age, depends on various characteristics of the economy such as household preferences, the degree of market incompleteness, and the magnitude of the income risks. An increasing number of papers regard the gap as a precious source of information for investigating the economy with an incomplete capital market. Blundell and Preston (1998) was one of the first studies that investigated the relationship between the evolutionary patterns of consumption and income variances over age to identify permanent income shocks. With a calibrated model, Storesletten, Telmer and Yaron (2004b) demonstrated the importance of a social security system to replicate the observed pattern of consumption and income inequalities in the United
In many papers of calibration or structural estimation of household consumption, the specification of income process is crucial, and the variance of permanent shocks is especially important. Quite naturally, extensive attempts have been conducted to estimate the household income process in detail.\textsuperscript{1} Banks, Blundell and Brugiavini (2001) estimated the income process of households in Great Britain using an ARCH structure. Applying the Panel Study of Income Dynamics (PSID), Meghir and Pistaferri (2004) estimated a more general ARCH process. Both studies revealed significant changes in income volatilities over the years. In addition, the latter paper considered the age effects in permanent income shocks and demonstrated that age in the USA does not influence the variance of permanent components in the income process.

The results of Meghir and Pistaferri (2004) suggest that, other things being equal, elderly workers confront the same degree of permanent income shocks as younger workers. Storesletten, Telmer and Yaron (2001) reported that inequalities in hours of work are approximately constant across age, suggesting that it is not necessary to treat elderly workers differently from younger workers in the calibrated model.\textsuperscript{2}

Many empirical papers in labor economics propose different stories for the income process. After the seminal work by Topel (1991), many attempts have been conducted to investigate the relationship between the accumulation of firm specific skills and the wage level. Recent careful investigation by Abowd and Kang (2002) demonstrated that a worker with 10 years of tenure is paid about 6.9 percent more than a worker with

\textsuperscript{1}Lilliard and Weiss (1979), MaCurdy (1982), and Abowd and Card (1989) are particularly influential works in the field. A recent study by Gouvenen (2005) allows for various heterogeneity in estimating the income process.

\textsuperscript{2}Except for the difference as a result of different life stages.
one-year tenure. If tenure is an important factor for the wage level, the risk of job loss will be more costly for elderly than young workers. Ohtake and Kawaguchi (2005) found that elderly workers in Japan confront higher probabilities of wage cuts and lower probabilities in finding new jobs, which suggests that elderly workers face greater risks in permanent income. If the variance of permanent shocks in income depends on age, the age-variance profile of income and consumption takes a nonlinear form.\(^3\) Deaton and Paxson (1994) and Ohtake and Saito (1998) demonstrated that the age-variance profile of income is linear in the United States, but is not linear in other economies such as Japan and Taiwan.

This paper investigates: (1) whether the idiosyncratic income risks depends on age; and (2) the importance of age dependence for evolution of inequalities in consumption. We take advantage of a large Japanese micro data set that covers the years 1984-1999. As the data set contains variables before and after the bubble economy, we can also analyze the effects of drastic macroeconomic changes to the household income process.\(^4\) Following a procedure similar to that of Storesletten, Telmer and Yaron (2004a), we estimate the income process of Japanese households. The procedure adopted in this paper is different from previous studies in that it allows for the age dependence of permanent income shocks. Our estimation of the income process of Japanese households demonstrated strong age dependence of income risks. As suggested by Ohtake and Saito (1998), at the age of 48, the variance of permanent income shocks begins to increase,

\(^3\)As Deaton and Paxson (1994) demonstrated, the slope of the age-variance profile is the variance of the permanent shocks. If the variance of the permanent component is constant across age, the slope is constant.

\(^4\)While the average growth rate of real GDP during 1984-1990 was 4.7 percent, during the so called “lost decade”, 1990-1999, real GDP growth rate was reduced to 1.2 percent.
which creates a nonlinear age-variance income profile.

Following the results of the estimations of the income process, we investigate whether the age dependence of the permanent income shocks is important for explaining the evolutionary pattern of consumption inequalities in Japan. A structural estimation of consumers’ dynamic behavior is conducted for this purpose. We build a life cycle saving model with an incomplete capital market that is similar to the buffer stock saving model by Carroll (1997), Gourinchas and Parker (2002) and Storesletten et al. (2004b). The structural estimation reveals that the model can replicate the changes in variances of consumption over age in Japan, under plausible parameter values only when we consider the age dependence of income shocks. Hence, consideration of nonlinearity in age-variance profile of income is crucial for understanding the consumption behavior of Japanese households.

In addition to the importance of age dependence of income risks, this paper contributes to the literature in several ways. First, while Storesletten et al. (2004b) found that consideration of social security system is necessary to produce the observed level of consumption inequality in the US, our structural estimation showed that without considering such policies, we can obtain similar level of consumption inequalities from the standard precautionary saving life cycle model. Second, we use simulated second moments to conduct statistical inferences of the model. Inferences based on second moments have several advantages over first moments or calibrations without inferences. As Kydland and Prescott (1982) argued, second moments are expected to be less vulnerable to measurement errors than first moments. The level of consumption likely includes age-specific factors that are absent from our model such as health, education, and housing-related expenditures. Although measurement errors have a direct effect on
the level of consumption, we expect their impacts on second moments to be less serious than on first moments. In addition, observed consumption contains many durable or semi-durable goods. By concentrating on second moments of non-durable and service expenditure, we can avoid complicated issues such as treatments of semi-durable goods like clothes in the model. Third, we conduct statistical inferences on several parameter values such as the degree of risk aversions. Household’s attitudes toward risk are crucial for explaining consumption smoothing and inequalities. By conducting inferences over the parameter, we can introduce “metrics” to evaluate the sensitivity of the estimated value of the risk aversion in our empirical analysis.

The paper is organized as follows. Section 2 discusses our data and briefly explains some characteristics of Japanese households. Section 3 determines the estimates of the income process. Section 4 introduces a dynamic model of consumption with an incomplete capital market. Section 5 presents the procedures of our structural estimation. Section 6 discusses the estimation results, and the final section concludes.

2 Data

The data we employ in this paper are microdata from the National Survey of Family Income and Expenditure (NSFIE), conducted by the Japanese government every five years. The four surveys conducted in 1984, 1989, 1994, and 1999 are used for this project. The surveys contain information on the income earned in the previous year and the three-month average expenditure between September and November of each surveyed year. The total sample size for each survey of the NSFIE is about 50,000, making it one of the largest household surveys in the world with detailed information on income and
consumption. The details regarding the survey, such as the sampling procedures and definitions of terms, are described in volumes published for each survey by the Statistics Bureau, Ministry of Public Management, Home Affairs, Posts and Telecommunications. The information is available in both Japanese and English.\(^5\)

We selected a sample of households according to the following criteria: (1) households with two or more members;\(^6\) (2) where the household head is a man aged between 25 and 70;\(^7\) (3) with a household head who is not engaged in the agricultural sector; (4) with a head who is not self-employed or a company director; (5) with fewer than nine family members;\(^8\) and (6) without any missing data. We remove self-employed or company directors from our sample because their incomes reflect corporate performances rather than as being an employee, which substantially increases their income variances. The number of observations we use were 39,030 (1984), 41,558 (1989), 44,077 (1994), and 38,721 (1999).

Our empirical analysis is divided into three steps. First, we create a series of variances of consumption and income for every age and year, which provides us with cohort level

\(^5\)Hayashi, Ando and Ferris (1988) and Hayashi (1995) describe in detail the features of NSFIE that make the survey suitable for analysis of consumption and saving behaviors. Several shortcomings, however, are noticeable. First, the NSFIE does not provide panel data; therefore, we cannot control individual fixed effects directly. Second, the NSFIE does not contain information on educational background. Finally, the data in 1984 do not include information on the size of the firms in which individuals are working.

\(^6\)Single households are surveyed only in September. As other households report the average consumption between September and November, we cannot treat singles and others alike. In this paper, we have removed singles from our sample.

\(^7\)We eliminate observations whose household head is younger than 25 or older than 70 because the number of observations for such younger or older households is small.

\(^8\)There are fewer than 100 households that have more than nine members each year.
To control for observable household characteristics such as household composition and location, we regress logged consumption and income on several variables. The variances of the residuals are used for the remaining steps. Second, following Storesletten et al. (2004a), we estimate the income process using the estimated variances from the first step. Finally, we estimate a dynamic model of consumption based on the variances of consumption obtained from the first step and the income process estimated in the second step.

To obtain the series of consumption and income variances at cohort level, we regress logged household income and consumption on (1) constant; (2) dummy variables for the number of family members; and (3) prefecture and large city dummies. In a regression of household consumption, we add the number of family members who are younger than 15 to the regressors. Figure 1 reports the variances of residuals obtained from the income regressions. The age-variance profile is upward sloping, which suggests the existence of permanent income shocks. Another noteworthy feature is the shape of the profile. Up to the age of 60, the profile looks convex, that is, the slope is increasing. Since the shapes of the four curves are very similar to each other, the convexity is hardly a by-product of cohort effects. After the age of 60, the variance ceases to increase and remains at a high level. Figure 2 exhibits the age-variance profile of consumption. The convexity of the profile is much clearer in consumption than in income. In all the years

\[^9\]Because we use four NSFIE surveys, the number of regressions is 184 (= (70 – 25 + 1) × 4).

\[^{10}\]When we calculate the logged consumption, we subtract education and medical expenses because the model we use in a later section does not contain medical risks or offspring-related expenditure.

\[^{11}\]Since the survey is conducted every five years and we have only four surveys, controlling cohort effects as in Deaton and Paxson (1994) by regressing cohort dummies would be difficult in this paper. Rather, to eliminate cohort effects, we take differences of income variances at each cohort in the next section.
surveyed, the consumption variance decreases up to the age of 40, after which it begins to increase rapidly at an accelerating rate.\textsuperscript{12}

Because people can receive a public pension after the age of 60, for the remainder of the paper we further restrict our sample to households with the head aged between 25-60, which provides us with 34,991 (1984), 35,663 (1989), 36,084 (1994), and 29,675 (1999) observations.

3 The Income Process

Following Storesletten et al. (2004a), suppose that an individual income can be decomposed into several components. Denoting the natural logarithms of income for the $i$th household of age $h$ at time $t$ as $y_{ih}^t$, and observable components such as location and gender as $X_{it}^h \beta^h_t$, and the unobserved components $u_{it}^h$, the income process admits following decomposition,

$$y_{it}^h = X_{it}^h \beta^h_t + u_{it}^h. \quad (1)$$

When $X_{it}^h$ includes employment status, the residual does not directly reflect the income risks caused by unemployment. In this section, we investigate Equation (1) under several different specifications. A process for $u_{it}^h$ is specified as:

$$u_{it}^h = \alpha_{it} + \varepsilon_{it} + z_{it}^h. \quad (2)$$

$$z_{it}^h = \rho z_{it-1}^h + \eta_{it}. \quad (3)$$

\textsuperscript{12}This paper is not the first to point out the convexity in both income and age-variance profiles in Japan. Deaton and Paxson (1994) found that households in Taiwan have a similar shape. Ohtake and Saito (1998) suggested that Japanese labor customs such as frequent promotions and layoffs after age of 40 are one cause of this convexity.
Where $\alpha_{it}$ is the time invariable household’s fixed effect, $\varepsilon_{it}$ is the i.i.d. transitory shock, and $z_{it}^h$ is the permanent component. We assume $\alpha_{it}$, $\varepsilon_{it}$, and $\eta_{it}^h$ are independent of each other, and $\alpha_{it} \sim N(0, \sigma^2_\alpha)$, $\varepsilon_{it} \sim N(0, \sigma^2_\varepsilon)$, $\eta_{it}^h \sim N(0, \sigma^2_{\eta,h,t})$. $\rho$ is a scalar that determines the persistency of the income shock through $z_{it}^h$. We also assume $z_{it}^h = 0$ where we set $h = 24$. That is, at the age of 25, people enter the labor market and begin to be exposed to income risks.

Because the NSFIE does not trace the same households over different surveys, we cannot use the first difference of income within a household to control for household fixed effects as in other works such as Blundell and Preston (1998). We demonstrate that, with some assumptions, $\rho$ and several variances such as $\sigma^2_{\eta,h,t}$ can be identified from the information on the dynamics of variances at cohort level. First, we take the variance of (2) and (3),

$$\text{var}(u_{it}^h) = \sigma^2_\alpha + \sigma^2_\varepsilon + \text{var}(z_{it}^h),$$

$$\text{var}(z_{it}^h) = \rho^2 \text{var}(z_{it-1}^h) + \sigma^2_{\eta,h,t}.$$

Combining the above two equations, we can develop the following expression,

$$\text{var}(u_{it}^h) - \rho^2 \text{var}(u_{it-1}^h) - \sigma^2_{\eta,h,t} - (1 - \rho^2)(\sigma^2_\alpha + \sigma^2_\varepsilon) = 0.$$  

If $\rho = 1$, as in Deaton and Paxson (1994), the above equation becomes simple,

$$\text{var}(u_{it}^h) - \text{var}(u_{it-1}^h) = \sigma^2_{\eta,h,t}. \quad (4)$$

The equation implies that in each cohort, the variance of income increases by the variance of permanent shock, $\sigma^2_{\eta,h,t}$, each year.

If $\sigma^2_{\eta,h,t}$ is constant over time and age, the income variance grows at a constant rate. Hence, the age-variance profile becomes a straight line. Many empirical analyses such as
Meghir and Pistaferri (2004) demonstrated that the linear profile is a good approximation for the US household.\textsuperscript{13} However, as previously discussed, under some labor contract regimes such as the seniority wage and lifetime employment systems, the profile becomes nonlinear. Obviously, Figure 1 demonstrates for Japan a convex relationship consistent with findings by Deaton and Paxson (1994) and Ohtake and Saito (1998).

Several procedures capture the nonlinear relationship between age and income variance. The first is to allow $\rho$ to be different from unity. If $\rho > 1$, the age-variance profile has an increasing slope. The shape in Figure 2, however, suggests a more complicated nonlinear income process in Japan because the nonlinearity in consumption variances is greater than that of income. Ohtake and Saito (1998) provided several possible explanations for the nonlinearity, for example, a special promotion system within a firm. Although many empirical and theoretical investigations have been conducted on Japanese employment customs, there is no consensus on the mechanism that characterizes the Japanese labor market.\textsuperscript{14} Thus, we have decided that, rather than depending on specific behavioral models in labor markets, we attempt to model the data described by Ohtake and Saito (1998). Consequently, as the second model, we assume $\rho = 1$ and the constant variance, $\sigma^2_{\eta,h,t} = \sigma^2_{\eta}$, until a certain “Turning Age” and allow for growth in variances, $\sigma^2_{\eta,h,t}$, after this age.

We estimate the following two models.

[Model1]

$$\text{var}(u^h_{it}) - \text{var}(u^{h-1}_{it}) - \sigma^2_{\eta,h,t} = 0,$$

\textsuperscript{13}Storesletten, Telmer and Yarons’ (2004a,b) estimation of $\rho$ with US data is 0.9989.  
\textsuperscript{14}Hashimoto and Raisian (1985) and Aoki (1988) are very influential works on this issue.
\[ \sigma^2_{\eta,h,t} = \sigma^2_{\eta} + \delta_11994\text{dummy} + \delta_21999\text{dummy}, \text{ if } h \leq \text{Turning Age}, \]

\[ \sigma^2_{\eta,h,t} = \left( \sigma^2_{\eta} + \delta_11994\text{dummy} + \delta_21999\text{dummy} \right)^{(h-\text{Turning Age})g}, \text{ if } h > \text{Turning Age}, \]

[Model 2]

\[ \text{var}(u^h_{it}) - \rho^2 \text{var}(u^{h-1}_{it}) - \sigma^2_{\eta,h,t} - (1 - \rho^2)(\sigma^2_{\alpha} + \sigma^2_{\varepsilon}) = 0, \]

\[ \sigma^2_{\eta,h,t} = \sigma^2_{\eta} + 1994\text{dummy} + 1999\text{dummy} . \]

We use the maximum likelihood method to estimate parameters, \((\sigma^2_{\eta}, \delta_1, \delta_2, g, \text{Turning Age})\) for Model 1, and \((\sigma^2_{\eta}, \delta_1, \delta_2, g, \text{Turning Age})\) for Model 2. In Model 2, we need information of \(\sigma^2_{\alpha} + \sigma^2_{\varepsilon}\) to conduct our estimation. Because the NSFIE is cross-sectional, we cannot identify \(\sigma^2_{\alpha}\) or \(\sigma^2_{\varepsilon}\), directly. We can estimate the variance of permanent shock, however, as long as the variances of fixed effects, \(\sigma^2_{\alpha}\), and temporary shocks, \(\sigma^2_{\varepsilon}\), are constant over time. Assuming \(z^2_{it} = 0\), it is possible to derive \(\sigma^2_{\alpha} + \sigma^2_{\varepsilon} = \text{var}(u^2_{it})\), which enables us to estimate Model 2 from our data set. Dummies representing years are included to capture the year effects on permanent income shocks.\(^{15}\) 1994\text{dummy} and 1999\text{dummy} take unity if the data are in the 1994 and 1999 surveys, otherwise they are zero.\(^{16}\) As the variance, \(\text{var}(u^h_{it})\), we use the variances of the residuals of (1) for each

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\(^{15}\) Although we need parameter values for annual data for later sections, the NSFIE is available only with five year intervals. In addition, we do not know the true variance of \(u^h_{it}\) and \(\sigma^2_{\alpha} + \sigma^2_{\varepsilon}\). Therefore, in the estimation, we use \(\bar{\text{var}}(u^h_{it}) - \bar{\text{var}}(u^{h-5}_{it}) - 5\sigma^2_{n,h,t} = \epsilon_1\) for Model 1, and \(\bar{\text{var}}(u^h_{it}) - \rho^{10}\bar{\text{var}}(u^{h-5}_{it}) - \sigma^2_{n,h,t} - (1 - (\rho^8 + \rho^6 + \rho^4 + \rho^2))(\sigma^2_{\alpha} + \sigma^2_{\varepsilon}) = \epsilon_2\), for Model 2 where \(\bar{\text{var}}(u^h_{it})\) and \(\sigma^2_{\alpha} + \sigma^2_{\varepsilon}\) are the sample variances, \(\epsilon_1\) and \(\epsilon_2\) are the error terms that represent small sample biases and other specification errors. The error terms are assumed to be i.i.d. and follow the Normal distributions.

\(^{16}\) Because we utilize the information on the changes in variances in the same cohort during the five year interval between the surveys, 1994\text{dummy} and 1999\text{dummy} capture the changes in variances between 1989-1994, and 1994-1999.
surveyed year and age. We construct eight different types of residuals by using different controls for (1). Table 1 contains a detailed explanation for each specification.

Tables 2 and 3 report the estimation results for Model 1 and Model 2, respectively. Except for Spec 1, we can reach an interior optimum for the Turning Age at around age 48. The growth rate of the variance after the turning age is significantly positive for all the specifications. The significance of the growth rate, $g$, implies that we can reject the hypothesis that the income process is linear, such as $\sigma_{n,h,t}^2 = constant$ for all ages. The year effects for 1999 are extremely small and insignificant for most cases. We can observe an increase in permanent shocks in income only in Spec 7 and Spec 8. This implies we can observe an increase in volatility of permanent income shock only within the same industry groups, or the same types of jobs. Meanwhile, 1994 effects are generally positive and significant except for Spec 3.

According to Table 3, the AR1 coefficient of the lagged variance, $\rho$, exceeds unity. This is a natural consequence from the convex age-variance profile. The 1994-1999 dummy is significantly positive only for Specs 7 and 8. Considering the results in Tables 2 and 3 together, we conclude that an increase in inequality in permanent income arises only within industries or job types. To evaluate the relative explanatory power of Models 1 and 2, we conduct Vuong’s test for non-nested models. The result is reported in Table 4. As we expect from the differences in likelihood values, Model 1 is always selected.

Spec 4 to Spec 8 control for employment status of the household head, which implies that the variances obtained from these specifications do not reflect income risks due to employment status. To capture the income risks from the labor markets, we use Spec 1 to Spec 3. Spec 3 controls for family composition and location of the households. Although these are not completely exogenous, they are not likely to be the main cause
of the income risks. We consider them as a part of the family fixed effects. Therefore, hereafter, we use the results from Spec 3, in which the year effects are not significant. Table 5 reports the results of Model 1 estimation without the year effects. There are no substantial differences between Tables 2 and 5 in the Turning Age or $g$.

4 The Household Model

In this section, to explain the nonlinearity of the age-variance profile of consumption depicted in Figure 1, we introduce a consumption/saving model developed by Carroll (1997) and extended by Gourinchas and Parker (2002) and Cagetti (2003). In the model, because of missing insurance markets, agents confront idiosyncratic earnings risks and accumulate savings as a buffer stock for precautionary motives when they are workers. If a negative income shock is realized, the agent decumulates his/her assets for the purpose of smoothing the consumption path over the life cycle. This is why the model is known as the “buffer-stock saving model.” Carroll (2004) carefully explains the theoretical properties of the model.

Consider a household’s lifecycle consumption/saving problem. Although households live for at most $T$ periods, they face survival probabilities $\{s_h\}_{h=1}^{T}$ for each age $h$. Consequently, death is uncertain and a household may die with an accidental bequest. The household supplies labor with an inelastic supply curve for $R < T$ and obtains labor

\[17^{17}\text{For empirical studies on the market incompleteness in the United States, see Cochrane (1990) and Mace (1991); in the case of Japan, see Kohara, Ohtake, and Saito (2002). Storesletten et al. (2001) investigated the market incompleteness from a different point of view.}\]
earnings, and retires after $R$ years. The household maximizes its expected utility:

$$U\left(\{C_h\}_{h=1}^T\right) = E\sum_{h=1}^{T} \beta^{h-1} \frac{C_h^{1-\gamma}}{1-\gamma} S_h,$$

where $\beta > 0$ is a discount factor, $s_j$ is a death probability between $j$ and $j+1$, and $S_h$ is a cumulative death probability for age $h$. We assume that the instantaneous utility function is of the constant relative risk aversion form, i.e., $u(C_h) = C_h^{1-\gamma}/(1-\gamma)$. At the beginning of age $h$, each household has some cash on hand (net worth plus current income), $X_h$. We also assume they have some financial wealth when they enter the economy. The cash on hand is, then, allocated between consumption $C_h$ and savings:

$$C_h = X_h - W_h,$$

where $W_h$ represents the financial wealth at the end of age $h$. The wealth yields interest in the next period, and the gross interest rate is given by $(1 + r)$. We assume that the interest rate is constant over time and for each cohort. We also assume all households face liquidity constraints, i.e., $W_h \geq 0$. The next period’s cash on hand is given by:

$$X_{h+1} = (1 + r)W_h + Y_{h+1},$$

where, during working age, the household receives labor earnings, $Y_{h+1}$. As in Section 3, all households face idiosyncratic labor income risks, which can be decomposed into permanent income component, $\psi_{t+1}$, and transitory shocks, $\xi_{h+1}$. The permanent income component evolves according to a deterministic growth rate, $G_h$, and a permanent shock, $\phi_h$. As shown in the previous section, as the variances of consumption in Japan are convex over age, we have to consider age-specific permanent income shocks; this

\textit{18}If there is a non-zero probability that an agent’s income becomes zero, the no-Ponzi game condition prohibits the household from borrowing.
differentiates our model from previous research such as Gourinchas and Parker (2002) and Cagetti (2003).

The average income growth rates can be easily obtained by the NSFIE. We assume that the transitory and permanent shocks follow the log-normal distribution with means \((-\sigma_{\xi,t}^2/2, -\sigma_{\phi,h,t}^2/2)\) and variances \((\sigma_{\xi,t}^2, \sigma_{\phi,h,t}^2)\). By increasing the number of simulations, the average income in the model approaches the mean of the data-generating process. Note that the means and variances of the shocks depend upon not only age \(h\) but also year \(t\).

\[
Y_{h+1} = \psi_{h+1} \xi_{h+1}, \quad \text{if } h + 1 \leq R \tag{5}
\]

\[
\psi_{h+1} = G_{h+1} \phi_{h+1} \psi_h, \tag{6}
\]

\[
\ln \phi_h \sim N \left(-\sigma_{\phi,h,t}^2/2, \sigma_{\phi,h,t}^2\right), \quad \ln \xi_h \sim N \left(-\sigma_{\xi,t}^2/2, \sigma_{\xi,t}^2\right),
\]

where initial permanent component \(\psi_1\) is determined by the “fixed effect,” \(\alpha\), which cannot be shared.

A retiree draws a social security payment from the government. The amount of the pension is determined by the permanent income level at the time of retirement multiplied by \(b\):

\[
Y_{h+1} = b \psi_{h+1}, \quad \text{if } h + 1 \geq R + 1 \tag{7}
\]

where \(b\) is determined by the replacement rate observed in Japan.

Defining the value function of the problem as \(V_h(X_h, \psi_h)\), where the cash on hand, \(X_h\), and the permanent income, \(\psi_h\), are state variables, we define a household’s opti-
mization problem as a dynamic programming of the following form:\(^{19}\)

\[
V_h(X_h, \psi_h) = \max \left\{ \frac{C_h^{1-\gamma}}{1 - \gamma} + s_h \beta E_h V_h(X_{h+1}, \psi_{h+1}) \right\}
\]

subject to

\[
X_{h+1} = (1 + r)[X_h - C_h] + Y_{h+1}, \ (5), \ (6) \text{ and } (7).
\]

To minimize the number of state variables, by using homogeneity of the value function, we divide both sides of the Bellman equation by \(\psi_h\). Then, the model becomes a simple form such as\(^{20}\):

\[
v_h(x_h) = \max \left\{ \frac{c_h^{1-\gamma}}{1 - \gamma} + s_h \beta \Gamma_{h+1} v_h(x_{h+1}) \right\}
\]

subject to

\[
x_{h+1} = \begin{cases} 
(1 + \tau) \frac{[x_h - c_h]}{\Gamma_{h+1}} + \xi_{h+1}, & \text{if } h \leq R, \\
(1 + \tau) \frac{[x_h - c_h]}{\Gamma_{h+1}} + b, & \text{if } h > R,
\end{cases}
\]

where \(\Gamma_{h+1} \equiv G_{h+1} \phi_{h+1}\), and the lower cases are the normalized variables (e.g., \(c_h \equiv C_h/\psi_h, x_h \equiv X_h/\psi_h\)). The corresponding policy functions are denoted by \(c_h = c_h(x_h)\).

The Euler equation is, then, given by:

\[
c_t^{-\gamma} \geq s_h \beta (1 + r) E_h (\Gamma_{h+1} c_{h+1})^{-\gamma}.
\]

It is well known that the problem has no analytically tractable solution. Therefore, we need to resort to a numerical method. We apply "the endogenous gridpoints method" to compute the policy function from the Euler equation (12). For detailed procedures, see Carroll (2005).

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\(^{19}\)We have omitted time subscript \(t\) because it creates no confusion.

\(^{20}\)For normalization of the model, see Carroll (2004) or the appendix in Abe and Yamada (2006).
5 The Method of Simulated Moment

Following from the previous section, given the estimated income process obtained in Section 3, we examine how well the model can explain the variances profile of age-consumption. Note that our objectives in this paper are not to estimate the fundamental parameters of the Japanese economy exactly, but to investigate the consistent parameters with regard to income and consumption inequalities. In other words, we investigate the differences between the consumption and income inequalities by using the buffer stock saving model under plausible parameters.

Our estimation procedures basically follow previous research such as Gourinchas and Parker (2002) and Cagetti (2003). The method is called “the method of simulated moments (hereafter, MSM)” and was employed in the aforementioned studies by proceeding with the two following steps. First, given a set of fundamental parameters, they solve the model numerically and generate several simulated data. Second, the studies seek to find the parameters that match the simulated data with those of the real economy in the US. By using the MSM, the studies estimate the fundamental parameters \((\beta, \gamma)\) by matching the averages of age-consumption or age-wealth profiles, which implies that Gourinchas and Parker (2002) and Cagetti (2003) used the first moments of the microdata. In our estimations we use the second moments of the cross-sectional consumption distribution (i.e., the variances) instead of the first moments. We estimate the fundamental parameter of our model, \(\gamma\), using the second-order moments.

Since it is practically impossible to solely estimate all parameters of the economy using the method of simulated moments, we conduct both estimation and calibration for our fundamental parameters. Following Gourinchas and Parker (2002), we take two
steps for the estimation. As a first-stage estimation, to solve the model numerically, we calibrate and estimate some important parameters from the NSFIE. As a second step, we estimate the relative risk aversion by using those parameters.

5.1 First Stage Estimation

For computing the policy function of the household’s problem, we have to specify several parameters such as the growth rate of average income \( \{G_{h,t}\}_{h=1}^{T} \), the stochastic income process \( (\sigma_{\xi,t}^{2}, \sigma_{\phi,h,t}^{2}) \), the average and variances of initial financial wealth \( (\bar{x}_{25,t}, \sigma_{\bar{x}_{25,t}}^{2}) \), the discount factor \( \beta \), the survival probabilities \( \{s_h\}_{h=1}^{T} \), the replacement rate \( b \), and the fraction of the fixed effect \( \sigma_{\alpha}^{2} \).

a. Growth rate of income: Computing the growth rates \( \{G_{h,t}\}_{h=1}^{T} \) involves the average age-income profiles (i.e., \( G_{h+1,t} = \bar{Y}_{h+1,t}/\bar{Y}_{h,t} \)) for each year. For each age and each year, we have computed the average income, \( \bar{Y}_{h,t} \), by controlling the same variables as in the previous section. Afterwards, using fourth order polynomials, we smooth out the age-income data and obtain the profiles. For details, see Figure 4 in Abe and Yamada (2006).

b. Stochastic income process: The idiosyncratic income process in the model is specified entirely the same as in Section 3. Our formulation of the stochastic income process (5) and (6) can be easily transformed into the equation (1), (2), and (3) defined in Section 3. In the MSM estimation, we use estimated results of only the case of \( \rho = 1 \) reported in Table 5, because of “the curse of dimensionality.” That is, we need to solve a two-dimensional Bellman’s equation even after the normalization if \( \rho \) is not equal to

\[21\]

\[\text{For approximation of the distribution of } \phi \text{ and } \xi, \text{ we use the Gauss-Hermite quadrature method, and take 32 and 40 gridpoints respectively. For details of the quadrature method, see Judd (1998)}\]
unity, which is costly to solve numerically.

c. **Initial financial wealth:** Households have some financial assets when they enter the economy. We assume that all agents start their economic activity at 25 years old. Because financial wealth at \( h = 1 \) must be determined outside the model, we have to estimate and calibrate the distribution of the wealth for a 25-year-old, i.e., \( x_{1,t} = X_{1,t}/\psi_{1,t} \). It is not easy, however, to directly associate the wealth derived from our model with that observed from the data. We have introduced a liquidity constraint in our model, which implies that the financial wealth must be non-negative. In the NSFIE, there are many households with negative assets; one possible way to eliminate them is to remove them. Such a procedure, however, overestimates wealth at 25 years old. Therefore, first, we estimate averages and variances of the normalized cash on hand with negative assets, as in Table 6. Second, in the simulation process, agents with negative assets are replaced with zero assets.

d. **Discount factor:** Note that the discount factor \( \beta \) is a fundamental parameter, and is estimated in other studies. We do not estimate \( \beta \) and \( \gamma \) simultaneously because, as increasing both variables changes the consumption variances in the same direction (the variances decrease), it is difficult to correctly estimate both parameters. Moreover, the discount factor is closely related to the interest rate. We set \( \beta \) as equal to 0.95 and test the model with several interest rates; \( r = \{3\%, 4\%, 5\%\} \). From a theoretical point of view, the interest rate must be lower than the subjective discount rate if agents live forever and the market is incomplete, even though it is not well known if agents live a finite period.

e. **Survival probabilities:** The maximum living period is set to be \( T = 61 \), which implies that an agent lives at most 85 years. Following the Japanese tradition, the
retirement age is exogenously determined to be 60 years old (i.e., \( R = 36 \)). The survival rates are taken from the National Institute of Population and Social Security Research (2002).

**f. Replacement rate:** Retirees finance their expenditure with savings and the public pension. From the average income profiles obtained above, the parameter \( b \) is set so that the replacement rate of social security is 59.3% for each year, i.e., \( b_t \in \{0.54, 0.59, 0.59, 0.54\} \). Such values make the replacement rate (average income after retirement/average income over life cycle) the model case of the Ministry of Health, Labour and Welfare in Japan, namely 59.3%.

**g. Fixed effect:** It is impossible to distinguish the ratio of the fixed effect and the transitory shocks because of data limitation.\(^{22}\) Thus, to investigate quantitative consequences of the fixed effects, we estimate the model parameters under different values for the ratio of the fixed effects in the total income variance at initial age, \( \{0.4, 0.5, 0.6\} \). The fixed effects plus transitory shocks for each year are calculated from the averages of the 25–29-years-olds.\(^{23}\)

### 5.2 Second Stage Estimation: Details of the MSM Procedures

We use the method of simulated moments developed by Pakes and Pollard (1989), Duffie and Singleton (1993), and applied by Gourinchas and Parker (2002) and Laibson, Repetto, and Tabacman (2004), who estimated the consumption function. The

\(^{22}\)Recall that the NSFIE is constituted as a repeated cross section: there is no information about the same households’ income over several years. Moreover, the NSFIE does not contain data on educational background.

\(^{23}\)On the effect of education for lifetime earnings, see Keane and Wolpin (1997).
information we utilize for estimation is the second moments of consumption,

\[ g_{t,h}(\gamma) = \text{var}(\hat{C}_{t,h}) - \text{var}(\tilde{C}_{t,h}(\gamma)), \]

(13)

or

\[ g_{t,h}(\gamma) = \left[ \text{var}(\hat{C}_{t,5,h+5}) - \text{var}(\hat{C}_{t,h}(\gamma)) \right] - \left[ \text{var}(\tilde{C}_{t,5,h+5}) - \text{var}(\tilde{C}_{t,h}(\gamma)) \right], \]

(14)

where \( \text{var}(\hat{C}_{t,h}) \) is the sample variance of the logarithms of consumption for the Japanese economy in year \( t \) at \( h \) years old, and \( \text{var}(\tilde{C}_{t,h}(\gamma)) \) is the sample variance of logarithms of simulated consumption given \( \gamma \), the relative risk aversion. The discrepancies between \( \text{var}(\hat{C}_{t,h}) \) and \( \text{var}(\tilde{C}_{t,h}(\gamma)) \) stem from various sources such as small sample biases, numerical errors, specification errors. We assume the errors are i.i.d. and follow the normal distribution.

Suppose that a variance parameter of the log-likelihood function is \( \chi \), then, we can write the log likelihood as

\[ L = -\frac{R}{2} \ln (2\pi) - \frac{R}{2} \ln \chi^2 - \frac{1}{2} \sum_{t} \sum_{h=1}^{R} \left( \frac{1}{\chi^2} g_{t,h}(\gamma)^2 \right), \quad t \in \{1999, 1994, 1989, 1984\}. \]

To compute the likelihood function, we first compute the approximated policy functions for each age based upon the Euler equation (12). Following that, by generating idiosyncratic income processes, we simulate for many agents the variances of age-consumption profiles. The number of agents simulated is set to be 10,000. From the

\[ ^{24}\text{Education and medical expenditures are excluded from household consumption in our definition of consumption. While, the consumption data are created as the income data in the previous section, in the first stage, we add the number of children to the regressors to control for the effects of having children on the household consumption.} \]
simulated data set, we can easily obtain the variances of logarithms of consumption and compare them to the consumption data.

The basic idea behind the estimation is the same as in Gourinchas and Parker (2002). Meanwhile, because we use the variances of consumption of cross-section distribution, there are two possible ways of estimation. The first approach is using the level information of the logged consumption variances, (13). The other method is, as in the previous section, by using slopes from within the cohort variances as in (14).

The variances of the estimated parameters are obtained from the inverted Hessian such as:

\[
\{I(\Theta)\}^{-1}, \{I(\Theta)\} = -E \left[ \frac{\partial^2 L(\Theta)}{\partial \Theta \partial \Theta'} \right],
\]

where \( \Theta = (\gamma, \chi) \) is the vector of structural parameters.

From those procedures, we estimate the fundamental parameter \( \gamma \), and find that the model with nonlinear permanent shocks can explain Japanese consumption inequality very well.

6 Estimated Results

The objectives for conducting the estimations are not to search for the true values of the parameters, but rather to investigate whether the estimated parameters give us the best fit with the consumption inequality within the plausible range. If our estimated parameters remain within the reasonable range, and if the overall fit is acceptable, we can fairly say that our structural model can explain the spread of the consumption inequality over the life cycle in Japan.

We establish that the parameters obtained in the MSM are actually plausible. The
simulated consumption profile generated by the buffer stock saving model with plausible parameters can accurately explain the observed consumption inequalities over the life cycle only when we take into account the nonlinearity of the stochastic income process. In other words, if the agents confront the constant permanent income risks over age measured by the variances of income, then the generated consumption path does not account for the consumption profile with the plausible parameters.

6.1 Estimation Using the Variances Level of Consumption for Each Cohort

As a starting point, we discuss our estimated results by considering the estimation using the variance level information of each cohort, i.e. the estimation based upon equation (13). This procedure uses all four years variances of age-consumption from 1984 to 1999, and compares them with the data. Each household in 1984, 1989, 1994, and 1999, confronts different earnings profiles, idiosyncratic transitory shocks and fixed effects, and financial wealth at 25 years old. Therefore, we generate four simulated consumption paths and obtain four variance profiles of age-consumption.

As in Figure 3, our simulated results can well explain the variances of the Japanese consumption profile.25 When young, the variances of consumption seem almost flat around 0.10, which is very close to the corresponding variances of income. At middle age, the consumption inequality sharply rises, and at retirement age, it appears to remain constant. Those characteristics are accurately replicated by the consumption path generated by the policy function and simulation with plausible parameters.

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25For depicting Figure 3, we set the interest rate to be 4 percent and the fraction of the fixed effect as a benchmark is 0.5.
Table 7 reports the estimated results using four years of pooled data and the levels of the variances of age-consumption. The fraction of the fixed effect is determined from 0.4 to 0.6 and the interest rate is set as \( r \in \{0.03, 0.04, 0.05\} \). Moreover, we estimate the fraction of the fixed effect by including the fixed effect parameter in the simulated process. It is apparent that the estimated relative risk aversion is very sensitive to the fixed effects and the interest rate, and it takes values from 0.12 to 2.83. Although the estimated risk aversion is within a wide range, we can positively say that those values are plausible and acceptable as demonstrated in the literature. Even though the fixed effects cannot be estimated directly from the NSFIE, the values estimated by the MSM appear to be plausible (about 0.6).\(^{26}\) If the fraction of the fixed effect is high, the amount of risks households can hedge is small.

What makes the estimated values so different? The answer is found in the amount of savings, which is accumulated for hedging the income risks. As risk aversion increases, households are inclined to hold greater savings. If the interest rate is high, households are willing to save more at the same risk aversion. Therefore, given the relationship between the variances of consumption and income, as the interest rate increases, the risk aversion is estimated to be low, as revealed in Table 7. This suggests that if the relationship between the variances of consumption and income are close to each other (and the discount rates are constant over the four years), then the recent low interest rate in Japan implies that the relative risk aversion might be high.\(^{27}\)

\(^{26}\)Note that the standard deviation of the risk aversion increases when we include the fraction of the fixed effect in the estimation.

\(^{27}\)As shown in Figures 1 and 2, the relationship between the income risks and the consumption inequality seems to have ergodicity. Thus, we have estimated the relative risk aversion by the MSM using purely cross-section data, and obtained the results that the relative risk aversions are estimated to be
6.2 Estimation Using Within Cohort Effect

One possible objection about the estimation is the use of the “level” of variances as opposed to the slopes of the variances, as in Section 3. By using slopes of the variances of consumption, we can focus on the pure effects of each cohort. Because controlling the variable results in a large specification error for the level of the variances, it is preferable to use the slopes of variances. Therefore, we estimated the risk aversion based upon Equation (14), and reported in Table 8. Although the risk aversions in Table 8 are relatively high compared to those in Table 7, the qualitative properties of the estimation are the same as those given above. Figure 4a and Figure 4b demonstrates that the simulated consumption variances for the four years replicate the data correctly for each cohort.

6.3 The Importance of Nonlinearity for Japanese Economy

Research by Storesletten et al. (2004b) quite successfully explains the US data by adopting the precautionary savings model and linear income process estimated by the PSID. However, as demonstrated in the previous section or by Ohtake and Saito (1998), the variances of Japanese or Taiwanese consumption grow increasingly in the middle age. In the buffer stock saving model with linear permanent shock process, as in Figure 5, the corresponding variances profile of age-consumption becomes a concave function over the age.\textsuperscript{28,29} Therefore, Storesletten et al. (2004b)’s estimation is likely to perform poorly.

\textsuperscript{28}Since it is easy to compute the linear regression parameters, they are omitted. Notice that the fixed effects plus transitory shocks have also changed.

\textsuperscript{29}For detailed discussion of the concavity of the variances profile of consumption, see Storesletten et al. (2004b).
if we apply the same estimation to the Japanese economy. Although they explain the shape of the age-consumption inequality in the United States, they fail to explain the “level” of variances of logarithms of consumption; as a result Storesletten et al. (2004b) concluded that the social security system is essential for understanding the level fit.

In Storesletten et al. (2004b), the relative risk aversion is fixed to be 2. Our approach exhibits a sharp contrast. Using a partial equilibrium model, we calibrated the replacement rate from the public pension system in Japan, and estimated the relative risk aversion that is consistent with the Japanese data’s shape and level. Using the nonlinear stochastic income process, we are able to replicate the consumption inequality with a plausible risk aversion. Meanwhile, if we assume the variances of permanent shocks to be constant over age, the risk aversion that maximizes the log-likelihood has a value that is unacceptably large (Table 9). Moreover, at least for the Japanese economy, Vuong’s test reveals that the nonlinear income process model is superior to the model with linear regression.

7 Concluding Remarks

In this paper, we have investigated the determinants of consumption inequalities in Japan between 1984 and 1999. There are two major findings: (1) Japanese elderly households confront greater permanent income risks than do younger households; and (2) the age dependence of the income risks is crucial for understanding the consumption inequalities in Japan. These findings suggest that when we consider Japanese household behavior toward income risks, we need to take into account labor market customs such as lifetime employment and the seniority wage system.
In addition, we have studied whether the permanent income shock is increasing. Our results demonstrate that, in general, we cannot confirm significant changes in income risks during the sample period, 1984–1999. Significant increases can be observed only within the same industry or the same type of job.

There are several remaining issues to be examined. First, although income inequalities are basically constant over the sample periods for the same age groups, inequalities in consumption are rising particularly among the young generation. We suspect that this is due to an increase in amounts of housing loans. Because of the Bank of Japan’s zero interest rate policy and several special tax provisions for housing loans, the amount of debt among young households has increased dramatically, which has likely made young households more vulnerable to income risks and has increased inequalities in consumption even though the income volatilities are unchanged. In addition, other risks such as health risks have to be incorporated into our model. As the number of elderly people increases due to the rapid aging of the Japanese population, the importance of risks related to health and illness is likely to increase for the whole economy. Finally, a more detailed consideration of the labor market is necessary. In this paper, we did not use a specific model for describing age dependence of income risks. Although the difficulty in computation is formidable, we have to incorporate this aspect in detail to understand the household behavior toward income risks.

References


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<tr>
<th>Spec1</th>
<th>Constant</th>
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</thead>
<tbody>
<tr>
<td>Spec2</td>
<td>Spec1 + the Number of Household Member Dummies</td>
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<tr>
<td>Spec3</td>
<td>Spec2 + Area Information</td>
</tr>
<tr>
<td>Spec4</td>
<td>Spec3 + Employment Status Dummy of Household Head</td>
</tr>
<tr>
<td>Spec5</td>
<td>Spec4 + the Number of Employed Household Member Dummies</td>
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<tr>
<td>Spec6</td>
<td>Spec5 + Industrial Dummy of Household Head</td>
</tr>
<tr>
<td>Spec7</td>
<td>Spec4 + Type of Job Dummy of Household Head</td>
</tr>
<tr>
<td>Spec8</td>
<td>Spec6 + Spec7</td>
</tr>
</tbody>
</table>

### Area Information
- Dummy for the Major Three Metropolitan Areas: 1 or 0
- Dummies for the City Groups (the size of population):
  - Five Ranked, Four dummies
  - Dummies for 10 Districts: Nine Dummies
  - Dummies for the Large Four Cities: Three Dummies
  - Dummy for the Largest Three Cities: 1 or 0
- Dummy for the location of Prefectural Administration Center: 1 or 0

### Industrial Dummies
- Agricultural, Forestry, and Fisheries Workers*
- Mining
- Construction
- Manufacturing
- Electricity, Gas, Water, and Heat
- Retail, Wholesale, and Hospitality
- Financial Institutions
- Real Estate
- Service
- Public Officer
- Other (including Unemployed)

### Job Type Dummies
- Full Time Nonoffice Workers
- Part Time Nonoffice Workers
- Office Workers
- Individual Proprietors*
- Corporate Administrators*
- Agricultural, Forestry, and Fisheries Workers*
- Professional Service
- Other Occupation
- No Occupation

*See The NSFIE Reports published by Statistical Bureau, Ministry of Public Management, Home Affairs, Posts and Telecommunications, Japan for more detailed explanation for each variable. Variables with * are not included in our regressions since we exclude households that correspond with the categories.
<table>
<thead>
<tr>
<th></th>
<th>Spec1</th>
<th>Spec2</th>
<th>Spec3</th>
<th>Spec4</th>
<th>Spec5</th>
<th>Spec6</th>
<th>Spec7</th>
<th>Spec8</th>
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<td><strong>after Turning Age</strong></td>
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<td>(21.33)</td>
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<td><strong>0.0732457</strong></td>
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#: We are unable to obtain interior value for turning age in this specification. 31 is the minimum possible value in our estimation.

*, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.

Under all specifications, we drop observations to households that: (1) have a female head of household;
(2) are single households; (3) work in the agricultural sector; (4) are self-employed; and (5) are company directors or firm managers
Spec1 uses the variances of the raw data.
Spec2 uses the variances of the residuals after controlling for the number of household members.
Spec3: Controlled for the number of household members and the area information.
Spec4: Controlled as in Spec3 and employment status of the family head.
Spec5: Controlled as in Spec4 and the number of employed family members.
Spec6: Controlled as in Spec5 and industry the household head is working in
Spec7: Controlled as in Spec5 and household head’s type of job
Spec8: Controlled as in Spec7 and the industry the head is working for.
Table 3
Estimation of Permanent Income Shocks with Auto Correlation

<table>
<thead>
<tr>
<th></th>
<th>Spec1</th>
<th>Spec2</th>
<th>Spec3</th>
<th>Spec4</th>
<th>Spec5</th>
<th>Spec6</th>
<th>Spec7</th>
<th>Spec8</th>
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<td>1.0383***</td>
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<td>(171.10)</td>
<td>(186.65)</td>
<td>(181.57)</td>
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<td>0.00081543</td>
<td>0.0007063</td>
<td>0.00098472</td>
<td>0.0022628**</td>
<td>0.0023226**</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(-0.99)</td>
<td>(-0.18)</td>
<td>(1.06)</td>
<td>(0.61)</td>
<td>(0.92)</td>
<td>(2.01)</td>
<td>(2.12)</td>
</tr>
<tr>
<td>σ</td>
<td>0.034716</td>
<td>0.031614</td>
<td>0.028338</td>
<td>0.024676</td>
<td>0.024934</td>
<td>0.024174</td>
<td>0.023661</td>
<td>0.023409</td>
</tr>
<tr>
<td>log-likelihood</td>
<td>180.57</td>
<td>189.28</td>
<td>199.45</td>
<td>212.32</td>
<td>211.35</td>
<td>214.23</td>
<td>216.23</td>
<td>217.22</td>
</tr>
<tr>
<td>Fixed Effects &amp; Transitivity Shocks</td>
<td>0.130568</td>
<td>0.119139375</td>
<td>0.1014679</td>
<td>0.10083625</td>
<td>0.0838144</td>
<td>0.07842955</td>
<td>0.07807105</td>
<td>0.0732457</td>
</tr>
</tbody>
</table>

#: We are unable to obtain interior value for turning age in this specification. 31 is the minimum possible value in our estimation.
*, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.
Under all specifications, we drop observations to households that: (1) have a female head of household; (2) are single households; (3) work in the agricultural sector; (4) are self-employed; and (5) are company directors or firm managers
Spec1 uses the variances of the raw data.
Spec2 uses the variances of the residuals after controlling for the number of household members.
Spec3: Controlled for the number of household members and the area information.
Spec4: Controlled as in Spec3 and employment status of the family head.
Spec5: Controlled as in Spec4 and the number of employed family members.
Spec6: Controlled as in Spec5 and industry the household head is working in
Spec7: Controlled as in Spec5 and household head’s type of job
Spec8: Controlled as in Spec7 and the industry the head is working for.
### Table 4: Vuong's Nonnested Tests

<table>
<thead>
<tr>
<th>Spec</th>
<th>Vuong's Statistics</th>
<th>P-Values</th>
<th>Chosen Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spec1</td>
<td>-3.16</td>
<td>0.0008</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec2</td>
<td>-2.68</td>
<td>0.0037</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec3</td>
<td>-2.76</td>
<td>0.0029</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec4</td>
<td>-2.14</td>
<td>0.016</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec5</td>
<td>-2.12</td>
<td>0.017</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec6</td>
<td>-2.67</td>
<td>0.0038</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec7</td>
<td>-2.97</td>
<td>0.0015</td>
<td>Model 1</td>
</tr>
<tr>
<td>Spec8</td>
<td>-3.61</td>
<td>0.00016</td>
<td>Model 1</td>
</tr>
</tbody>
</table>

Vuong's Statistics show the value for AR1 Model versus Nonlinear Model.
Spec1 uses the variances of the raw data.
Spec2 uses the variances of the residuals after controlling for the number of household members.
Spec3: Controlled for the number of household members and the area information.
Spec4: Controlled as in Spec3 and employment status of the family head.
Spec5: Controlled as in Spec4 and the number of employed family members.
Spec6: Controlled as in Spec5 and industry the household head is working in
Spec7: Controlled as in Spec5 and household head’s type of job
Spec8: Controlled as in Spec7 and the industry the head is working for.
Table 5
Estimation of Permanent Income Shocks without Year Effects

<table>
<thead>
<tr>
<th>Spec1</th>
<th>Spec2</th>
<th>Spec3</th>
<th>Spec4</th>
<th>Spec5</th>
<th>Spec6</th>
<th>Spec7</th>
<th>Spec8</th>
</tr>
</thead>
</table>
| Variances of Permanent
Shocks | 0.0014483*** | 0.0018296*** | 0.002027*** | 0.0020211*** | 0.0025257*** | 0.0022108*** | 0.0013699*** | 0.0014516*** |
|       | (9.80) | (13.26) | (5.30) | (4.82) | (5.86) | (5.78) | (4.04) | (12.15) |
| Variance Growth Rate after
Turning Age | 0.20003*** | 0.20993*** | 0.19170*** | 0.16612*** | 0.19546*** | 0.21894*** | 0.30676*** | 0.29797*** |
|       | (58.24) | (110.43) | (9.19) | (7.10) | (7.29) | (8.21) | (7.11) | (312.77) |
| Turning Age | 45 | 48 | 48 | 48 | 51 | 51 | 52 | 52 |
| σ | 0.025331 | 0.024551 | 0.021466 | 0.020015 | 0.020335 | 0.018628 | 0.0181 | 0.017965 |
| log-likelihood | 209.88 | 212.79 | 225.28 | 231.79 | 230.31 | 238.47 | 241.14 | 241.84 |
| Fixed Effects & Transitivity
Shocks | 0.130568 | 0.119139375 | 0.1014679 | 0.1000830625 | 0.0838144 | 0.07842955 | 0.07807105 | 0.0732457 |

#
We are unable to obtain interior value for turning age in this specification. 31 is the minimum possible value in our estimation.
*, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.
Under all specifications, we drop observations to households that: (1) have a female head of household;
(2) are single households; (3) work in the agricultural sector; (4) are self-employed; and (5) are company directors or firm managers
Spec1 uses the variances of the raw data.
Spec2 uses the variances of the residuals after controlling for the number of household members.
Spec3: Controlled for the number of household members and the area information.
Spec4: Controlled as in Spec3 and employment status of the family head.
Spec5: Controlled as in Spec4 and the number of employed family members.
Spec6: Controlled as in Spec5 and industry the household head is working in
Spec7: Controlled as in Spec5 and household head’s type of job
Spec8: Controlled as in Spec7 and the industry the head is working for.
Table 6: Normalized Financial Wealth at 25 Years Old

<table>
<thead>
<tr>
<th>Year</th>
<th>99</th>
<th>94</th>
<th>89</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Financial Wealth</td>
<td>0.4726</td>
<td>0.4176</td>
<td>0.4480</td>
<td>0.4786</td>
</tr>
<tr>
<td>at 25 Years Old</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance of Financial</td>
<td>0.5937</td>
<td>0.4154</td>
<td>0.3284</td>
<td>0.3170</td>
</tr>
<tr>
<td>Wealth at 25 Years Old</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Table 7

**Method of Simulated Moments Estimation using the Variance Level**

<table>
<thead>
<tr>
<th>Interest Rate:</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Fixed Effect</td>
<td>estimated</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Estimated $\gamma$</td>
<td>2.8326***</td>
<td>0.4533***</td>
<td>1.1496***</td>
</tr>
<tr>
<td></td>
<td>(2.33)</td>
<td>(12.12)</td>
<td>(6.66)</td>
</tr>
<tr>
<td>Estimated $\chi$</td>
<td>0.0170***</td>
<td>0.018***</td>
<td>0.0182***</td>
</tr>
<tr>
<td></td>
<td>(19.07)</td>
<td>(16.10)</td>
<td>(17.80)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>382.06</td>
<td>373.39</td>
<td>372.58</td>
</tr>
<tr>
<td>Estimated Fraction of Fixed Effect</td>
<td>0.6023</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Std. of Fixed Effect</td>
<td>0.0320</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: t value in parenthesis.

- Fixed effects plus transitory shocks are 0.10541(99), 0.10193(94), 0.10923(89) and 0.10617(84).
- On the left column for each interest rate, the fraction of fixed effect is estimated by the maximum likelihood.
- *, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.
<table>
<thead>
<tr>
<th>Interest Rate:</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Fixed Effect</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Estimated $\gamma$</td>
<td>4.2328***</td>
<td>4.4449**</td>
<td>5.0712**</td>
</tr>
<tr>
<td>Estimated $\chi$</td>
<td>(2.69)</td>
<td>(2.48)</td>
<td>(2.33)</td>
</tr>
<tr>
<td>Estimated $\chi$</td>
<td>0.0171***</td>
<td>0.0180***</td>
<td>0.0190***</td>
</tr>
<tr>
<td>Estimated $\chi$</td>
<td>(17.56)</td>
<td>(18.31)</td>
<td>(17.12)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>246.27</td>
<td>241.66</td>
<td>236.74</td>
</tr>
</tbody>
</table>

Note: t value in parenthesis.

Fixed effects plus transitory shocks are 0.10541(99), 0.10193(94), 0.10923(89) and 0.10617(84).

*, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.
Table 9  
Method of Simulated Moments Estimation using the Variance Level (Linear Regression)

<table>
<thead>
<tr>
<th>Interest Rate:</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Fixed Effect</td>
<td>0.4</td>
</tr>
<tr>
<td>Estimated $\gamma$</td>
<td>2.8585***</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
</tr>
<tr>
<td>Estimated $\chi$</td>
<td>-0.0271***</td>
</tr>
<tr>
<td></td>
<td>(-12.34)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>315.18</td>
</tr>
<tr>
<td>Std. of Fixed Effect</td>
<td>-</td>
</tr>
<tr>
<td>Vuong's Statistics</td>
<td>3.54</td>
</tr>
<tr>
<td>Chosen Model</td>
<td>Nonlinear</td>
</tr>
</tbody>
</table>

Note: t value in parenthesis.
*: Fixed effects plus transitory shocks are 0.06743(99), 0.07085(94), 0.07594(89) and 0.07925(84).
**: *, **, and *** represent significance at 10%, 5%, and 1% level, respectively. t value in parenthesis.
Figure 2: Age Log-Consumption Variance Profile

The graph shows the variance of the logarithm of consumption over different ages and years. The x-axis represents age, ranging from 25 to 70, and the y-axis represents the variance of the logarithm of consumption, ranging from 0.00 to 0.25. The graph includes data points for different years: 1984, 1989, 1994, and 1999, each indicated by different symbols. The trend indicates an increase in variance as age increases, with slight variations across different years.
Figure 3: The Variances of the Logarithms of Consumption (Data and Simulation)
Figure 4a: The Variances of the Logarithms of Simulated Consumption for Each Cohort

The variances are shown for different ages, with each age group represented by a different marker. The ages considered are:

- Age = 26 (in 1984)
- Age = 28 (in 1984)
- Age = 30 (in 1984)
- Age = 32 (in 1984)
- Age = 34 (in 1984)
- Age = 36 (in 1984)
- Age = 38 (in 1984)
- Age = 40 (in 1984)
- Age = 42 (in 1984)
- Age = 44 (in 1984)
- Age = 46 (in 1984)
- Age = 48 (in 1984)
- Age = 50 (in 1984)
Figure 4b: The Variances of the Logarithms of Consumption for Each Cohort

- age=26 (in 1984)
- age=28 (in 1984)
- age=30 (in 1984)
- age=32 (in 1984)
- age=34 (in 1984)
- age=36 (in 1984)
- age=38 (in 1984)
- age=40 (in 1984)
- age=42 (in 1984)
- age=44 (in 1984)
- age=46 (in 1984)
- age=48 (in 1984)
- age=50 (in 1984)
Figure 5: Linear Regression and the Variances of Consumption

(γ=10.45, Fraction of the Fixed Effect=0.5)