Japanese Saving Rate*

Kaiji Chen  Aysê İmrohoroğlu  Selahattin İmrohoroğlu

Department of Finance and Business Economics
Marshall School of Business
University of Southern California
Los Angeles, CA 90089-1427

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Abstract

In this paper, we use two different models to study the time path of the saving rate in Japan between 1961-1998. Our results indicate that both an infinite horizon, complete markets setup and an overlapping generations model with incomplete markets are able to generate saving rates that are remarkably similar to the data during this period. We conduct counterfactual experiments to isolate the impact of several factors such as the social security system, fiscal policy, and total factor productivity growth on saving behavior. Our results identify changes in the growth rate of total factor productivity and the low initial capital stock as the main factors generating the time series behavior of the net national saving rate in Japan. In other words, it seems that there is nothing peculiar about the Japanese saving behavior.

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1 Introduction

There have been substantial differences between the saving rates of Japan and U.S. in the last forty years that have motivated extensive research in this area. Figure 1 presents net national saving rates for the two countries between 1961 and 1998.  

![Figure 1: Data](image)

Attempts to explain the relatively high saving rate in Japan have focused on several factors. Discussions ranged from economic factors to preferences peculiar to Japan as well as the relevance of life-cycle versus dynastic models. Ando, Yamashita, and Murayama (1986) mention that one of the reasons for the high saving rate in Japan during 1970s is the high growth rate during those years. Hayashi (1986) argues that low initial capital stock is the main reason behind the high saving rate in the late 1960s and the early 1970s. Christiano (1989) examines if the low initial capital stock can lead to the observed behavior of the saving rate in a one-sector, infinite horizon, representative agent model. He assumes that in 1946 the Japanese capital-output ratio is at 12% of its steady-state level, and computes the transition path to a steady-state which coincides with that of the U.S. economy. His results indicate that the standard neoclassical growth model cannot generate anything close to the actual

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The data are obtained from Hayashi (1989) who provides a comprehensive data set that corrects for differences in accounting and measurement standards between the two countries. According to Hayashi (1989) there are two major differences between the accounting standards of the two countries. First, Japanese National Income and Product accounts report depreciation based on historical cost as opposed to the replacement cost as in the U.S. Second, government investment is explicitly accounted for in Japan, where as in the U.S. all government purchases are classified as government consumption. While Dekle and Summers (1991) argue that the implied depreciation rates in Japan are implausibly high with these adjustments, Hayashi (1991) argues that they are justified due to the high depreciation rate for owner-occupied housing and the treatment of equipment capital.
time path of the saving rate.\(^2\) Dekle (1986) and Hayashi (1986) emphasize the importance of a bequest motive in understanding Japanese savings. Hayashi, Ito, and Slemrod (1987) present a life-cycle simulation analysis which includes housing purchase decisions. Their results indicate that the contribution of the down payment requirement seems to be too small to explain the large differential in the saving rates of the two economies. They are able to generate high saving rates only when they introduce a bequest motive. They also show the tax deductibility of mortgage interest payments and the tax exempt status of interest income to have a small impact on the aggregate saving rate. Horioka, Yamashita, Nishikawa, and Iwamoto (2002) argue that bequest motives are weak in Japan both absolutely and relative to the U.S. and suggest that the life-cycle model is the dominant model of household behavior in Japan.

The difficulties faced in explaining the differences between the U.S. and Japanese saving rates by using standard models seem to have spurred research that focuses on many other factors that may be peculiar to Japan, such as the bonus system that exists in Japan, high housing prices, high educational costs and high marriage costs.\(^3\) There does not appear to be a consensus on the importance of many of these factors.

In this paper, we ask the following question. ‘What features of the Japanese economy are critical in generating the observed saving behavior between 1961 and 1998?’ Our approach is in line with the recent use of the one-sector growth model to explain ‘Great Depressions’. In particular, we follow the methodology of Cole and Ohanian (1999) and Kehoe and Prescott (2002) in using an applied general equilibrium setup to account for the actual time path of Japanese saving behavior.\(^4\) We use two calibrated general equilibrium models to evaluate the extent to which either model economy can generate the year-to-year fluctuations in the Japanese saving rate and capital-output ratio. The first model is a standard infinite horizon model with complete markets, and the second model is an overlapping generations setup with incomplete markets. The former has been the workhorse of macroeconomics, and the latter model is helpful in assessing the role of public institutions such as the unfunded social security system in Japan given that the size of the retirement benefits has changed significantly over time. In both models, we incorporate the actual time path of total factor productivity (TFP) and government fiscal policy parameters.

\(^2\) Christiano then introduces a subsistence level of consumption in the period utility function that makes the marginal utility of the distance between consumption and its subsistence level very small as consumption falls (and saving rises rapidly in the first few years of convergence) and therefore dampens the desire to save. The resulting (smooth) path of ‘slow convergence’ does better in terms of mimicking the Japanese saving rate.

\(^3\) See Horioka (1990) for a survey.

We start by repeating Christiano’s (1989) exercise in the standard one-sector, neoclassical growth model with an infinitely-lived representative agent facing complete markets. Then, we employ an overlapping generations (OLG) model populated by 80-period-lived individuals facing mortality risk and borrowing constraints. Private annuity markets and credit markets are closed by assumption. Until the mandatory retirement age, agents in this economy work an exogenously given number of hours and accumulate assets to self-insure and to provide for old-age consumption. After retirement agents receive social security benefits that are financed by a payroll tax. In both models, the return on asset holdings and the wage rate are at least in part determined by the profit maximizing behavior of a firm with a constant returns to scale technology.

We calibrate both model economies to Japanese data for the 1961-1998 period. We use the average population growth rate, the average tax rates on capital income, labor income, and consumption for both models, and observed social security replacement rate, and conditional survival probabilities that prevailed in that time period for the OLG model. We conduct deterministic simulations, as in Hayashi and Prescott (2003), and perform an ‘accounting exercise’ to evaluate several factors that may explain the differences in saving rates between Japan and the U.S. The simulations take the actual capital stock in 1960 as the initial condition and use the actual time path of TFP. This exercise not only allows us to identify the role of TFP growth in a standard infinite horizon model, but it also facilitates a quantitative exercise to understand the relative importance of certain fiscal policy parameters that can only be present in an OLG setup.

Our quantitative findings indicate that both the standard infinite horizon, and the OLG models are capable of generating saving rates that mimic the data remarkably well once simulations take into account the time series behavior of TFP. Given this result we continue with the OLG framework and conduct several counterfactual experiments to quantify the role of different factors in understanding the time series behavior of saving rates in Japan between 1961-1998. Our numerical results indicate that two factors alone can account for most of the differences between the saving rates in Japan and the U.S. in this time period. These are the TFP growth rate and the low level of the initial capital stock. We show that if Japan were to be faced with the U.S. TFP during this period as well as a relatively high initial capital stock, the time path of the saving rate in Japan would have looked very similar to that of U.S. These findings suggest that the impact of factors such as the social security system or the aging of the population on the Japanese saving rate in this time period are quantitatively small. In other words, it seems that there is nothing peculiar about the Japanese saving behavior.
2 Two Models

We start this section by describing the standard infinite horizon model and the OLG model we use to understand the factors behind the time series behavior of the saving rate in Japan.

2.1 The Standard Neoclassical Model

As our benchmark model we use an exogenous labor version of the model used in Hayashi and Prescott (2003). In this framework households solve

$$\max \sum_{t=0}^{\infty} \beta^t \log C_t$$

subject to

$$C_t + X_t \leq w_t E_t + r_t K_t - \tau (r_t - \delta) K_t - \pi_t,$$

where $C_t$ is consumption and $\beta$ is the subjective discount factor, $E_t$ is the measure of labor input, $\tau$ is the tax rate on capital income, $w_t$ is the real wage, $\pi_t$ is a lump sum tax and $r_t$ is the rental rate of capital. Households are assumed to own the capital, $K_t$, and rent it to businesses. Aggregate output $Y_t$ is divided between consumption, $C_t$, investment $X_t$, and government purchases of goods and services, $G_t$.

$$C_t + X_t + G_t = Y_t.$$

The law of motion for the capital stock is given by $K_{t+1} = (1 - \delta) K_t + X_t$ where $\delta$ is the depreciation rate.$^5$

The aggregate production function is given by

$$Y_t = A_t K_t^\theta E_t^{1-\theta},$$

where $\theta$ is the income share of capital and $A_t$ is total factor productivity which grows exogenously.

2.2 OLG Model

2.2.1 The Environment and Demographics

The question we want to study requires the computation of a transition path to a steady state. However, it is easier to start the description of the model with a stationary overlapping generations setup similar to that in Auerbach and Kotlikoff (1987) but with a different market structure.

$^5$In some of our experiments we will let $\delta$ be a function of time since Hayashi (1989) argues that there were large changes in the depreciation rate in Japan over time.
At each date $s$, a new generation of individuals is born. We denote the population growth rate by $\eta\%$. Individuals face long but random lives, work until the mandatory retirement age of $j_R$, and might live through maximum possible age $J$. Life-span uncertainty is described by $\psi_j$, the conditional survival probability from age $j$ to $j+1$. We assume a stationary population by making the survival probabilities $\{\psi_j\}_{j=1}^J$ and the population growth rate $\eta$ time-invariant. We also assume $\psi_J = 0$. The cohort shares, $\{\mu_j\}_{j=1}^J$, are given by

$$\mu_j = \frac{\psi_{j-1}}{(1 + \eta)} \mu_{j-1}, \text{ where } \sum_{j=1}^J \mu_j = 1. \quad (1)$$

Although the notation above assumes a stationary population, it is easy to allow the conditional survival probabilities and the population growth rate to vary over time. This would allow us to capture the impact of the aging of the population along the transition path to the eventual balanced growth path. For the time being, we abstract from demographic dynamics.\(^6\)

2.2.2 Technology

There is a representative firm with access to a constant returns to scale Cobb-Douglas production function with deterministic total factor productivity $A_s$:

$$Y_s = A_s K_s^\theta H_s^{1-\theta}, \quad (2)$$

where $K_s$ and $H_s$ are aggregate capital and labor inputs, respectively, $\theta$ is capital’s output share, and TFP grows at the rate $g_s \sqrt[1/(1-\theta)]{0} > 0$. We assume that $H_s = \tilde{h} N_s$, and $N_s$ grows at the rate $\eta$.

The aggregate capital stock evolves according to the law of motion:

$$K_{s+1} = (1 - \delta_s)K_s + X_s,$$

where $X_s$ is aggregate gross investment and $\delta_s$ is the rate of depreciation of capital at time $s$.

The stand-in firm rents capital and labor from the households in competitive spot markets at the rates $r_s$ and $w_s$, respectively, and maximizes its profits. Factor prices equal their marginal productivities:

$$r_s = \theta A_s \left(\frac{K_s}{H_s}\right)^{\theta-1},$$

$$w_s = (1 - \theta) A_s \left(\frac{K_s}{H_s}\right)^\theta.$$

\(^6\)Braun, Ikeda, and Joines (2004) study the impact of the aging of the population on the Japanese saving rate in the 1990s and beyond.
2.2.3 Households

A household who is $i$ years old at time $t$ solves the following problem:

$$\max \sum_{j=i}^{J} \beta^{j-i} \left[ \prod_{k=i}^{j} \psi_k \right] u(c_{j,s})$$

subject to a sequence of budget constraints over the remaining lifetime:

$$(1 + \tau_c)c_{j,s} + a_{j+1,s+1} = R_s a_{j,s} + (1 - \tau_{h,s} - \tau_{n,s}) w_s \varepsilon_j h + b_{j,s} + \ell_s,$$  \hspace{1cm} (4)

where $\beta$ is the subjective discount factor and $c_{j,s}$ is consumption of an age-$j$ household at time $s = t + j - i$. Asset holdings at the beginning of age $j$ at time $s$ are given by $a_{j,s}$. They earn the gross interest rate (net of taxes and depreciation) $R_s = [1 + (1 - \tau_{a,s})(r_s - \delta_s)]$. The age-efficiency profile is denoted by $\{\varepsilon_j\}_{j=1}^{J}$ and $h$ indicates the exogenous number of hours worked in a week. The tax rates on consumption, capital income, and labor income are denoted by $\tau_c$, $\tau_{a,s}$, and $\tau_{h,s}$, respectively. $b_{j,s}$ denotes social security benefits received by an age-$j$ household at time $s$, to be described later, and $\tau_{n,s}$ is the payroll tax for social security at time $s$. Retirement benefits $b_{j,s}$ are a fraction $\lambda_s$ of average lifetime earnings. Each household receives a lump-sum amount $\ell_s$ which is the sum of a government transfer (to clear its budget) and the redistribution of accidental bequests. Note that we allow for some of the tax rates and the rate of depreciation $\delta_s$ to vary over time. We do not allow for annuity markets and assume that there is no borrowing:

$$a_{j,s} \geq 0, \text{ all } j, s,$$  \hspace{1cm} (5)

with $a_{1,s} = 0$ for all $s$. Since death is certain beyond $J$, households choose $a_{J+1,s} = 0$.

The above notation allows for some transitional generations that will have to re-solve their remaining lifetime optimization problem in response to an unanticipated change in their environment, starting from an initial balanced growth path or given initial conditions. For a newborn at time $t$, the objective function simplifies to

$$\sum_{j=1}^{J} \beta^{j-1} \left[ \prod_{k=1}^{j} \psi_k \right] u(c_{j,s}),$$  \hspace{1cm} (6)

and $s = t + j - 1$.

We use recursive tools to solve the individual’s perfect foresight decision problem. Let $V_{j,s}(a_{j,s})$ denote the value function of an age-$j$ individual at time $s = t + j - 1$. We compute the value functions for $j = 1, 2, \ldots, J$, and all $s$, using

$$V_{j,s}(a_{j,s}) = \max_{\{c_{j,s}, a_{j+1,s+1}\}} \left\{ u(c_{j,s}) + \beta \psi_j V_{j+1,s+1}(a_{j+1,s+1}) \right\}$$  \hspace{1cm} (7)

subject to (4) and (5).\footnote{We discretize the state space and numerically obtain the value functions and the accompanying decision rules. See for example Imrohoroglu, Imrohoroglu, and Joines (1999).}
2.2.4 Social Security

Social security benefits are given by

\[ b_{j,s} = \begin{cases} 
0 & \text{for } j = 1, 2, \cdots, j_R - 1, \\
b_{j_R, t+} & \text{for } j = j_R, j_R + 1, \cdots, J.
\end{cases} \]

where the pension received by a new retiree at time \( t + j_R - i \) is given by

\[ b_{j_R, t+} = \lambda_s \frac{1}{j_R - 1} \sum_{j=1}^{j_R-1} w_{t+j-i} \varepsilon_j (1 + g)^{j_R-j}. \]

Note that the retirement benefits received by an individual are constant throughout the individual’s lifetime, although successive cohorts receive successively larger benefits at the rate of TFP growth.

We assume that the system is unfunded so that the payroll tax is selected to equate the total benefits to total taxes collected for each time period. Total benefits paid at time \( t + j_R - i \) are equal to \( \sum_{j=j_R}^{J} \mu_j b_{j, t+} = b_{j_R, t+} \sum_{j=j_R}^{J} \mu_j (1 + g)^{j_R-j} \). The social security tax rate is then given by

\[ \tau_{n,s} = \frac{b_{j_R, t+} \sum_{j=j_R}^{J} \mu_j (1 + g)^{j_R-j}}{w_{t+j_R-i} \sum_{j=1}^{j_R-1} \mu_j \varepsilon_j}. \]  

2.2.5 Government

In addition to managing the unfunded social security system, the government needs to finance its per capita purchases \( G_s \) by taxing consumption, labor and capital income, and confiscating unintended bequests. We require period-by-period budget balance which necessitates a (per capita) lump-sum transfer \( \ell_s \).

\[ \tau_c \sum_{j=1}^{J} \mu_j c_{j,s} + \tau_h \sum_{j=1}^{J} \mu_j w_s \varepsilon_j h + \tau_{a,s} \sum_{j=1}^{J} \mu_j (r_s - \delta_s) a_{j,s} + \sum_{j=1}^{J-1} (1 - \psi_j) a_{j+1,s} / (1 + \eta) = G_s + \ell_s. \]  

2.2.6 Aggregation

Aggregate variables are computed in the usual way by obtaining the weighted average of different cohorts’ decision rules, using the population weights determined by our demographic assumptions. For example, (per capita) aggregate capital and labor inputs, and consumption are given by:

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\(^8\)Whether accidental bequests are given directly to survivors, or to survivors of a particular age group does not seem to matter quantitatively.
\begin{align*}
K_s/N_s &= \sum_{j=1}^{J} \mu_j a_{j,s}, \\
H_s/N_s &= \sum_{j=1}^{J} \mu_j h_{j,s}, \\
C_s/N_s &= \sum_{j=1}^{J} \mu_j c_{j,s}.
\end{align*}

2.2.7 Recursive Competitive Equilibrium

A government policy consists of \( \{G_s, \tau_c, \tau_a, \tau_h, \tau_n, \lambda_s, \ell_s\}_{s=s_1}^{s_2} \), where \( s_1 \) and \( s_2 \) are some initial and final dates. An allocation is given by a sequence of decision rules
\( \{A_{j+1,s+1}(a), C_{j,s}(a)\}_{j=1}^{J} \) over \([s_1,s_2]\). A price system is a sequence of pairs \( \{w_s, r_s\}_{s=s_1}^{s_2} \). For a given government policy, a Recursive Competitive Equilibrium is an allocation and price system such that

- the allocation solves the dynamic program (7) for all individuals, given the price system and government policy,
- the allocation maximizes firms’ profit by satisfying (3),
- the allocation and government policy satisfy the government’s budget constraint (9) given the price system,
- the social security system is unfunded, that is (8) satisfied, and,
- the commodity market clears

\[ C_s + X_s + G_s = Y_s. \]

3 Data and Calibration

We calibrate the model economies to the 1961-1998 Japanese economy using data provided by Hayashi and Prescott (2003).\(^9\) The capital share parameter, \( \theta \) is set to its average value over 1961-1998. The subjective discount factor and the risk aversion parameter are set so that the capital output ratio is 2 at the final steady state. We set \( h \) and \( E \) so that the average

\(^9\)TFP data provided by Hayashi and Prescott (2003) start in 1960. However, since the saving rate in Japan between 1955 and 1960 shows dramatic changes, we calculate Japanese TFP and report the results on the saving rate for that time period as well in the sensitivity analysis.
labor input in the model matches the labor input used in growth accounting to generate the level of TFP for Japan.\textsuperscript{10} The period utility function is taken as:

\[ u(c_{j,s}) = \frac{c_{j,s}^{1-\sigma} - 1}{1 - \sigma}, \]

where \( \sigma \) is the coefficient of relative risk aversion.

For the steady state calculations we set the values for the share of government purchases, \( G_s/Y_s \), the depreciation rate \( \delta_s \), tax rates on capital income, \( \tau_{a,s} \), labor income, \( \tau_{h,s} \), and consumption, \( \tau_c \), equal to their average values over 1961-1998.\textsuperscript{11} The resulting values used for the steady state are \( G/Y = 15\% \), \( \delta = 10\% \), \( \tau_a = 35\% \), \( \tau_h = 10\% \), \( \tau_c = 5.6\% \).\textsuperscript{12} We take the TFP factor as 0.087 at the final steady state which is assumed to be achieved in 1999. The growth rate of TFP is set to its 1961-1998 average value of 1.9\%, the growth rate of the population is taken as 1.2\%.

Since our main question is to examine the determinants of the saving rate in Japan between 1961-1998, our simulations take the actual capital output ratio in 1960 as the initial condition. More precisely, we set the initial level of capital to 32\% of its level in 1990.\textsuperscript{13} We use the data for actual TFP, \( A_s \), during this time period. In our benchmark experiment we use the average value for the depreciation rate, \( \delta_s \), share of government purchases, \( G_s/Y_s \), and the tax rate on capital income, \( \tau_{a,s} \), which are the same as the steady state values mentioned above. Since these variables display significant variation over this time period we check the sensitivity of our results by conducting additional experiments where the actual time series values for these variables are used in the simulations.

The calibration goals we have specified, such as obtaining a capital output ratio of 2 at the steady state, necessitate differences between the two models in terms of some of the parameters chosen. The following table lists the calibrated parameters:

\begin{table}
\begin{tabular}{|c|c|}
\hline
Parameter & Value \tabularnewline
\hline
\hline
\end{tabular}
\end{table}

\textsuperscript{10} For the OLG model this procedure results in \( h = 40 \), and for the infinite horizon model \( E = 35 \).

\textsuperscript{11} The data for these variables are provided in Table 1 in Appendix A. The same table also includes data on the capital output ratio, net national saving and the after tax interest rate in Japan. This information will be used to examine if the saving rate, capital output ratio and the interest rate generated by the models mimic the data.

\textsuperscript{12} The labor income tax rate in the model does not include the payroll tax for social security since that tax rate is computed endogenously to clear the social security balance for the government.

\textsuperscript{13} We also need to assume an initial distribution of assets among age groups in the OLG model in order to start out our simulations. In our benchmark results we use a uniform distribution of assets. We also experiment with a hump-shaped distribution that is generated endogenously at the final steady state. The results are not sensitive to this feature of the model.
Table 1: Calibration

<table>
<thead>
<tr>
<th></th>
<th>OLG</th>
<th>INF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>capital share</td>
<td>0.363</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.999</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>risk aversion</td>
<td>1.5</td>
</tr>
<tr>
<td>${\varepsilon_j}_{j=1}^J$</td>
<td>efficiency</td>
<td>Hayashi, Ito, and Slemrod (1987)</td>
</tr>
<tr>
<td>${\psi_j}_{j=1}^J$</td>
<td>survival prob.</td>
<td>Japanese Life Tables</td>
</tr>
</tbody>
</table>

For the OLG model we approximate the replacement rate for social security following Oshio and Yashiro (1997) who indicate that it was roughly equal to 17% for 1961-1976 and 40% afterwards. We take the age-specific efficiency profiles from Hayashi, Ito, and Slemrod (1987). In addition, we take the survival probabilities from the Japanese Life Tables for 1970.\(^{14}\)

To summarize, our benchmark experiment uses the actual time series values for $A_s$ for the period 1961-1998 and assume long-run averages for all the exogenous variables for the periods after 1998.

4 Results

In examining the time series behavior of the saving rate in Japan, we follow Hayashi and Prescott (2003) and conduct deterministic simulations. We do not argue that the entire path of TFP during 1961-1998 could have been perfectly foreseen by agents even though we treat it as if it were. We start by examining the net national saving rate (net national saving divided by NNP) generated by a standard infinite horizon model. Then we examine the saving rate generated by the overlapping generations model and perform counterfactual experiments to isolate the factors that impact the behavior of the saving rate in Japan.

4.1 Infinite Horizon Model

We start this section by repeating the exercise in Christiano (1989) for the time period he had studied. This involves simulating the infinite horizon model with a constant TFP growth rate of 3% starting from 1956. Figure 2 displays the results of this experiment. The actual data on the saving rate for Japan in this time period displays two humps, one in the early sixties and one in the late sixties. However, the simulated saving rate approaches its steady state value by late sixties. These results had led Christiano (1989) to conclude that the standard model failed in generating saving rates for Japan that resemble the data.

\(^{14}\)“Abridged Life Tables For Japan 2002”, Statistics and Information Department, Ministry of Health, Labour and Welfare.
As we have mentioned in the calibration section, the time series data for TFP start from 1960. In the following experiment we introduce the actual time series for TFP into the infinite horizon model and examine the saving rate generated by the model.\textsuperscript{15}

Figure 3 shows that the standard model performs fairly well in capturing the major movements in the saving rate in this time period. The only difference between Figure 2 and Figure 3 is the introduction of the actual time series for TFP in Figure 3.

\textsuperscript{15}In the sensitivity analysis we extend the simulations to start in 1955 by calculating $A_t$ for the period 1955-1960.
4.2 OLG Model

In the next figure we present the saving rate in the data and the one generated by the OLG model where the time series sequence on TFP and the social security replacement rate are the only exogenous time series information that are included in the simulations. The rest of the exogenous variables are set to their long term averages. The average saving rate for the period is 13.6 in the data and 13.1 in the model.

![Figure 4: Saving Rate: Data and the OLG Model](image)

As can be observed from Figure 4, the model economy generates fluctuations in the saving rate that resemble the data remarkably well for most of the periods. The results are very similar to the ones obtained in the infinite horizon model.

The main discrepancies between the simulated data and the actual data for both models are in the late 1960s and early 1970s, and mid 1980s. In both time periods the saving rate generated by the model is smaller than the data.

In the next experiment, we include time series data for the capital income tax, depreciation rate and the government’s share in GNP into the model. Figure A2 in the appendix shows the significant time variation in the deprecation rate and the capital income tax rate in this time period, with the deprecation rate declining and the capital income tax rate increasing.

This experiment, which is displayed in Figure 5, moves the two series closer to each other in the mid 1960s and early 1970s.\(^\text{16}\)

\(^{16}\)In order to understand the impact of these factors better we will conduct counterfactual experiments where we make one of these variables time-varying and set the other one constant. However, before moving to the counterfactual experiments, we present data on other features of this economy.
In Figure 6 we display the capital output ratio and the after tax return on capital for this economy. The model economy is able to generate movements in the capital output ratio and the interest rate that mimic the data.

Figure 5: Saving Rate

Figure 6: Additional Properties
4.2.1 Counterfactual Experiments

This framework allows us to conduct counterfactual experiments to assess the role of different factors that may have a role in the determination of saving rates in Japan. We continue with the OLG setup to conduct these experiments.

**Japanese Economy with U.S. TFP and High Initial K/Y** In the first set of counterfactual experiments, we examine the role of the TFP growth rate and the initial capital output ratio in determining the saving rate in Japan. While there are several ways we could carry out such a counterfactual experiment, we do it by asking the following question: If the Japanese TFP and the initial capital output ratio were the same as the U.S. TFP and the initial capital output ratio during this time period, what would the Japanese saving rate look like? In order to answer this question we need a measure of the U.S. TFP which we take from Jorgenson (2003).\(^\text{17}\) The noteworthy differences between the time series behavior of U.S. versus Japanese TFP’s are summarized in Table 2.

<table>
<thead>
<tr>
<th>Years</th>
<th>Japan</th>
<th>U.S.</th>
</tr>
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<tbody>
<tr>
<td>1962-1967</td>
<td>3.32</td>
<td>1.88</td>
</tr>
<tr>
<td>1968-1978</td>
<td>1.72</td>
<td>0.72</td>
</tr>
<tr>
<td>1979-1989</td>
<td>1.46</td>
<td>1.29</td>
</tr>
<tr>
<td>1990-1998</td>
<td>0.27</td>
<td>0.77</td>
</tr>
</tbody>
</table>

According to Table 2, the TFP growth rate in Japan is significantly higher than that of the U.S. in the 1962-1967 period. There is also a large decline in the U.S. TFP in period 1968-1978.

In Figure 7, we display the saving rate for Japan that would have occurred if both the initial conditions and the TFP growth rate for Japan were similar to conditions that existed

\(^{17}\)The data include TFP measures for Japan and the U.S. In the following tables we use the Hayashi and Prescott (2003) measure of TFP for Japan and the Jorgenson (2003) measure of TFP for U.S. Since Jorgenson’s data set ends in 1995, we have included our calculations for the 1995-1998 period for the U.S. In the sensitivity analysis we examine the Japanese saving rate based on the TFP measure provided in Jorgenson (2003) as well.

In addition, since there are large differences in the methodologies followed by Hayashi and Prescott (2003) and Jorgenson (2003) in calculating TFP growth rates, we also calculated a TFP series for the U.S. using the methodology followed by Hayashi and Prescott (2003). While there are differences between the TFP series generated by the two methods, the main results of this paper do not change dramatically. In both cases, it is clear that the Japanese saving rate would have looked very similar to the U.S. saving rate if the U.S. TFP and high initial conditions were present in Japan.
for the U.S.$^{18}$ In addition, we show the U.S. saving rate in the graph. With these two counterfactual features our model economy generates a saving rate for Japan that looks very similar to the actual U.S. saving rate. The average model-generated Japanese saving rate for the 1961-1998 period for this case is equal to 8.1%. The average empirical saving rate for the U.S. in the same time period was 8.4%.

![Figure 7: Japanese Saving with U.S. TFP and High Initial K/Y](image)

These results indicate that the time series behavior of the saving rate in Japan is mainly influenced by the TFP growth rate and low level of initial capital in this time period.

We can further analyze this result by separating the two features that are present in the above experiment. Table 3 summarizes the results where each feature is introduced one at a time. In the fourth column of Table 3 we summarize the results of the counterfactual experiment where only the U.S. TFP is used for the Japanese economy.$^{19}$

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Benchmark</th>
<th>U.S. TFP</th>
<th>Initial Cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962-1967</td>
<td>16.20</td>
<td>17.0</td>
<td>19.0</td>
<td>4.2</td>
</tr>
<tr>
<td>1968-1978</td>
<td>18.62</td>
<td>17.0</td>
<td>14.0</td>
<td>13.7</td>
</tr>
<tr>
<td>1979-1989</td>
<td>11.12</td>
<td>9.7</td>
<td>7.1</td>
<td>8.6</td>
</tr>
<tr>
<td>1990-1998</td>
<td>8.83</td>
<td>9.5</td>
<td>7.2</td>
<td>9.5</td>
</tr>
</tbody>
</table>

|         | 1962-1998 | 13.62 | 13.0 | 11.0 | 9.8 |

$^{18}$We change the initial level of capital from 32% of its detrended level in 1990 to 77% of its level in 1990.

$^{19}$As one can observe from these results, the relationship between the TFP growth rate and saving rate is a nonlinear one. The saving rate in a given period is affected by the growth rate of current and future TFP. When we use the U.S. TFP for Japan together with the low initial capital stock that was faced by the Japanese, we actually obtain a higher saving rate for the period 1962-1967. Notice also that the TFP growth rate declines more dramatically in the 1968-1978 period in U.S. than in Japan.
The same information is also displayed in Figure 8. That is, the counterfactual simulations are carried out for a case where the Japanese are assumed to be faced with the U.S. TFP. However, their initial K/Y ratio is still the value that was faced in Japan in 1960. The results indicate that the low initial conditions combined with the lower and declining TFP faced by the U.S. would have resulted in a higher saving rate in Japan than was observed for the first five years. In addition, notice that the two humps that are present in the data disappear when U.S. TFP is used.

Next we only change the initial level of capital from 32% of its detrended level in 1990 to 77% of its level in 1990. The last column of Table 3 shows that the change in the initial condition leads to a dramatic decrease in the saving rate between 1962-1967. In other words, the high saving rate that was observed in Japan during 1962-1967 may very well be due to the fact that Japan was catching up.

**Time Variation in Depreciation and Taxes** In the next set of experiments we examine the role of the depreciation rate, $\delta$, and the capital income tax rate, $\tau_\alpha$. Our benchmark is an economy where $\delta$ and $\tau_\alpha$ are set to their long-term averages, 10% and 35%, respectively. Now we change one feature at a time and examine its impact on the saving rate.

**Table 4: Accounting Exercise**

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Saving Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_\alpha$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>1968-1978</td>
<td>27.39</td>
<td>11.01</td>
</tr>
<tr>
<td>1979-1989</td>
<td>41.82</td>
<td>9.02</td>
</tr>
<tr>
<td>1990-1998</td>
<td>43.57</td>
<td>8.46</td>
</tr>
</tbody>
</table>
We can make several observations from Table 4. For example, if the capital income tax rate were to stay at its long run average of 35% instead of increasing to 41.82% between 1979-1989 the saving rate would have been higher by 2.5 percentage points. If the depreciation rate were equal to its long run average of 10% instead of the 16.14% in the 1962-1967 period, the saving rate in that period would have been higher by about 5 percentage points.

Overall, the above results indicate that the time series behavior of the saving rate in Japan is mainly influenced by the TFP growth rate and low level of initial capital in this time period. Given this finding it is unlikely that many of the factors that are discussed in the literature, such as the social security system, mortgage arrangements, family structure, monetary policy, equity or land price bubble, or the aging of the population in Japan would play a significant role in explaining the differences in saving rates between Japan and the U.S. Nevertheless our framework allows us to examine some of these factors.

In this paper we do not investigate the role of the changes in the population growth rate over time. However, we can examine the quantitative impact of such a change in the overall saving rate by changing the growth rate of the population that was used in our experiments. So far we had assumed the growth rate of population to be equal to its long run average of 1.2%. Instead, if we assume the population growth rate to be 0.9%, its average in the 1980-1998 period, the overall saving rate declines by 1 percentage point. In general, a decrease in the population growth rate has a similar effect to a decrease in the TFP growth rate, causing the saving rate to decline.

We can also analyze the role of social security by experimenting with different social security replacement rates at the steady state and along the transition. Our findings indicate that, an economy where the social security replacement rate is zero along the transition as well as at the steady state generates a saving rate that is 2 percentage points higher than that in an economy where the replacement rate is set to 50%. While these changes are not insignificant, they are not in the order of magnitude that would help resolve the difference between the U.S. and Japanese saving rates, especially since in the beginning of the 1960s there has been a unfunded social security system even if it was a modest one.

Overall, our results indicate that the major factors behind the behavior of the saving rate in Japan are the TFP growth rate and the initial low level of the capital output ratio.

4.2.2 Sensitivity Analysis

In this section we examine the impact of certain parameters and exogenous variables on the saving rate in the model.

**Jorgenson’s TFP** In our benchmark economy, we used the TFP measure provided by Hayashi and Prescott (2003). In the following experiment we use the data given in Jorgenson (2003) to examine the sensitivity of our results to the differences in TFP measures provided
by these two sources which are displayed in Table 5, where the initial capital output ratio in both cases is set to 1.06.\footnote{The TFP data for both countries are displayed in the Appendix.}

<table>
<thead>
<tr>
<th>Years</th>
<th>TFP Data</th>
<th>Saving Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Japan (HP)</td>
<td>Japan (Jorgenson)</td>
</tr>
<tr>
<td>1962-1967</td>
<td>3.3</td>
<td>5.9</td>
</tr>
<tr>
<td>1968-1978</td>
<td>1.7</td>
<td>2.4</td>
</tr>
<tr>
<td>1979-1989</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>1990-1995</td>
<td>0.3</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

The last two columns display the saving rates implied by these two measures of TFP. There are some visible differences between the two saving rates. However, both measures are able to capture some of the major fluctuations that took place in the Japanese saving rate between 1961-1998. Figure 9 repeats the experiment in Figure 4 with the TFP data for Japan taken from Jorgenson (2003).

**Figure 9: Sensitivity to TFP Measure**

**Taxes** In our initial calculations we had set the tax rate on capital income to be 35% which was taken from data provided in Mendoza, Razin, and Tesar (1994). However, Hayashi and Prescott (2003) estimate the tax rate on capital income to be 45% for a similar time period. Thus, we repeat our experiment for a constant tax rate of 0.45 for the entire period. Our results indicate that the average capital output ratio for the 1961-1998 period goes down by...
approximately 3.5% for all time periods with this higher tax rate. The average saving rate in the initial ten year period declines from 17.5% to 14.2%, but the major swings in the saving rate are still similar to those in the data.

Our benchmark economy also has taxes on labor income and consumption. When we examine the saving rate with these tax rates set to zero, we find an increase of about 1 percentage point in the average saving rate.

**Time Period** As we had mentioned before, we simulate our economies for the 1961-1998 time period because the TFP data given in Hayashi and Prescott (2003) start in 1960. However, the actual saving rate for Japan between 1956-1961 shows a dramatic increase. In order to see if our model economy can also generate this increase, we calculate the TFP series for Japan for this period using the data on capital that are provided by Hayashi and Prescott (2003) and assuming that the labor input, which is the missing series in that time period, is equal to its value in 1960. Figure 10 shows the results of this experiment where the only actual time series that is used is the TFP which demonstrates that except for the initial period, the simulated data capture the main changes in the saving rate in Japan very well.

![Figure 10: Sensitivity to Time Period](image)

### 5 Conclusions

In this paper, we use two models that are very different in many dimensions to explore the year-to-year fluctuations in the Japanese saving rate and capital-output ratio between 1961 and 1998. The first model is an environment with infinitely lived individuals who face perfect capital markets. The second model is an overlapping generations framework where individuals live for 80 periods and face mortality risk and borrowing constraints. Private annuity markets and credit markets are closed by assumption. Until the mandatory
retirement age, agents in this economy work an exogenously given number of hours. After retirement agents receive social security benefits that are financed by a payroll tax. In both models the return on asset holdings and the wage rate are determined endogenously by the profit maximizing behavior of a firm.

We calibrate both models to Japanese data for the 1961-1998 period. We use the average tax rates on capital income, labor income and consumption, observed social security replacement rate, average population growth rate and survival probabilities that prevailed in that time period. The simulations take the actual capital stock in 1960 as an initial condition and use the actual time path of TFP. We conduct deterministic simulations that allow us to identify the factors that may explain the differences in saving rates between Japan and U.S.

Our results demonstrate that both models do remarkably well in capturing the major movements in the saving rate in Japan in this time period. We show that two factors alone can account for most of the differences between the saving rates in Japan and U.S. These are the TFP growth rate and the low level of the initial capital stock in Japan. Our results indicate that if the Japanese were faced with the U.S. TFP during the 1961-1998 period as well as a relatively high initial capital stock like that of the U.S. in 1960, the time path of the saving rate in Japan would have looked very similar to that of the U.S.

Our finding contributes to the literature that identifies TFP as the main determinant of economic fluctuations. Analysis of the factors behind the TFP would further enhance our understanding of the Japanese saving behavior. This very important issue is left for future research.
6 Appendix A: Data

Japanese data are obtained from the following sources. Data on TFP, $A_t$, depreciation rate, $\delta_t$, government share in output, $G_t/Y_t$, and the capital output ratio, $K_t/Y_t$, are taken from Hayashi and Prescott (2003). Net national saving rate and the after-tax return on capital are obtained from Hayashi (1989). Tax rates on consumption, and capital and labor income are obtained from Mendoza, Razin, and Tesar (1994).\footnote{The data in Mendoza, Razin, and Tesar (1995) cover the period 1965-1996. We have assumed the 1961-1964 period tax rate on capital income to equal its value in 1965 and 1997-1998 tax rates to equal its level in 1996.}
Table 1: Time Series Data

<table>
<thead>
<tr>
<th>year</th>
<th>$A_t$</th>
<th>$\delta_t$</th>
<th>$C/Y_t$</th>
<th>$s/n$</th>
<th>$k/n$</th>
<th>Net national Saving</th>
<th>After-tax return on capital</th>
</tr>
</thead>
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<tr>
<td>1961</td>
<td>0.043</td>
<td>0.160</td>
<td>0.113</td>
<td>20.430</td>
<td>1.06</td>
<td>21.32</td>
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</tr>
<tr>
<td>1962</td>
<td>0.043</td>
<td>0.176</td>
<td>0.122</td>
<td>20.430</td>
<td>1.15</td>
<td>15.39</td>
<td>0.141</td>
</tr>
<tr>
<td>1963</td>
<td>0.045</td>
<td>0.160</td>
<td>0.124</td>
<td>20.430</td>
<td>1.13</td>
<td>15.98</td>
<td>0.152</td>
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<tr>
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<td>0.161</td>
<td>0.121</td>
<td>20.430</td>
<td>1.10</td>
<td>16.68</td>
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</tr>
<tr>
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<td>20.430</td>
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<td>1966</td>
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<td>1970</td>
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<td>1972</td>
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<td>0.067</td>
<td>0.102</td>
<td>0.136</td>
<td>30.230</td>
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<td>22.69</td>
<td>0.121</td>
</tr>
<tr>
<td>1974</td>
<td>0.065</td>
<td>0.108</td>
<td>0.140</td>
<td>34.940</td>
<td>1.48</td>
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<td>1975</td>
<td>0.064</td>
<td>0.094</td>
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<td>12.78</td>
<td>0.047</td>
</tr>
<tr>
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<td>1.61</td>
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<td>0.155</td>
<td>37.300</td>
<td>1.78</td>
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<td>0.053</td>
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<td>1982</td>
<td>0.071</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>0.085</td>
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<td>42.610</td>
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<td>0.147</td>
<td>42.610</td>
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<td>1998</td>
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<td>0.146</td>
<td>42.610</td>
<td>1.98</td>
<td>0.039</td>
<td></td>
</tr>
</tbody>
</table>

Figure A1 displays the TFP data used for Japan and the U.S. As mentioned in the text, Japanese TFP data are taken from Hayashi and Prescott (2003). U.S. TFP is calculated by using the differences in TFP levels between the two countries implied in Jorgenson (2003).
Appendix B: Computational Details:

In this paper, we follow Hayashi and Prescott (2003) in computing a transition path towards the final steady state, starting from a given set of initial conditions, taking as given the sequence of exogenous variables \( \{A_{s}, \delta_{s}, G_s/Y_s, N_s, \lambda_s, \tau_{a,s}, \tau_{h,s}\}^{92}_{s=s_1} \).

Our steps are as follows:
1. Compute the final steady state following the algorithm in İmrohoroglu et. al (1999). This step requires the detrending of aggregate variables by $A_s^{1/(1-\theta)}N_s$ so that we obtain a balanced growth path, after specializing the definition of recursive competitive equilibrium and numerically solving the two-dimensional fixed point problem. In particular, we iterate on an initial guess for the interest rate and the lump-sum transfer to the individuals $(r, \ell)$ until convergence. Note that the individuals can solve their optimization problems when we feed them the two factor prices and all policy parameters which is accomplished with our initial guesses and other calibrated parameters. Following Hayashi and Prescott (2002) we assume that the Japanese economy reaches the final steady state in 1999.

2. Use the actual 1960 and 1961 Japanese data as given initial conditions; in particular use the actual capital output ratio in Japan, and assume a uniform distribution of assets holdings (except for age 1 individuals who are born with zero assets) at the initial state.

3. Guess a time path for the vector $\{(r_s, \ell_s)\}_{s=1998}^{1998}$ of endogenous variables. Together with the sequence of exogenous variables $\{A_s, \delta_s, G_s, Y_s, N_s, \lambda_s, \tau_a,s, \tau_h,s\}_{s=1962}^{1998}$, all individuals can now solve their optimization problems as they have complete knowledge of the time paths of policy and prices.

4. Compute the transition path taking the initial conditions as given.

   (a) Starting from $S - 1 = 1999$ and working backward, obtain the decision rules of all cohorts through backward recursions.

   (b) Using the given initial asset distribution $\Phi_2$ over the initial cohorts in 1961, and the collection of decision rules just computed, calculate the new $\{(r_s, \ell_s)\}_{s=3}^{S-1}$ and $\{(r_s, \ell_s)\}_{s=3}^{S-1}$.

   (c) Compare the first sequence of $\{(r_s, \ell_s)\}$ to the latest and iterate on it until convergence.
References


