A Theory of Exchange Rates and The Term Structure of Interest Rates

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ABSTRACT

The purpose of this paper is to construct a model of exchange rate determination that is consistent with the stylized facts regarding the uncovered interest parity for short-term and long-term interest rates. This task is especially challenging because of the forward premium anomaly found for short-term interest rates and forward exchange rates. With an assumption that investors have a short investment horizon, the model is consistent with these stylized facts even when the degree of risk aversion is low. The model predicts a complicated relationship between exchange rates and the term structure of interest rates.

Keywords: forward premium anomaly, uncovered interest pairty for long-term bonds
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1 Introduction

The purpose of this paper is to construct a model of exchange rate determination that is consistent with the stylized facts for short-term and long-term interest rates. This task is challenging because the forward premium anomaly is found for short-term interest rates but not for long-term interest rates.

For short-term interest rates and forward exchange rates, uncovered interest parity is typically rejected (see, e.g., Hodrick (1987) and Engel (1996) for recent surveys). As Engel (1996) emphasizes, one form of the rejection found in many recent papers is that the regression of future depreciation on the current forward premium (which is equal to the short-term interest rate differential under the covered interest parity) yields negative estimates of the slope coefficient. This is called the forward premium anomaly (also see Backus, Foresi, and Telmer (2001) for a recent discussion).

For long-term interest rates, more favorable evidence for uncovered interest parity has been found. Direct evidence is given by recent papers, such as Meredith and Chinn (1998) and Alexius (2001). They find that regressions of future depreciation over a long-horizon on the current long-term interest rate differential typically yield significantly positive estimates of the slope coefficient.[1] Indirect evidence has been found in the standard exchange rate models, such as Meese and Rogoff (1988), Edison and Pauls (1993) and Baxter (1994). Under the uncovered interest parity and the long-run purchasing power parity assumptions, they show that the long-term interest rate differential is more consistent with these assumptions than the short-term rate

[1] Alexius (1999) finds similar results for returns on long-term bonds over short investment horizons

differential. Similarly, implications of standard exchange rate models hold better in long-horizon data than in short-horizon data (see, e.g., Mark (1995)).

To the best of our knowledge, this paper is the first to build a model that is consistent with these stylized facts for both short-term and long-term interest rates.[2] Many explanations have been provided to find an economic explanation for the forward premium anomaly for short-term interest rates. But neither the standard consumption-based asset pricing model with risk averse investors (see, e.g., Mark and Wu (1998)) nor the dynamic term structure model (see, e.g., Wu (2004)) can explain it. Alvarez, Atkeson, and Kehoe (2002) construct a model of segmented asset markets which can be consistent with the forward premium anomaly. McCallum (1994) and Meredith and Chinn (1998) provide an explanation for the forward premium anomaly based on policy reactions. However, in their models, an unspecified error term is necessary for the uncovered interest parity relationship. The model in the present paper gives an alternative explanation that is neither based on transactions costs nor on the assumption of an error term associated with the uncovered interest parity relationship.

Our model is a partial equilibrium model of exchange rate determination for a small open economy. The domestic investors have a constant absolute risk aversion utility function over their wealth in the next period and the asset returns are normally distributed conditional on the available information. We assume that there are three assets in the model: a risk free asset called domestic short-term bonds and two risky assets: domestic long-term bonds and foreign bonds. The investors are also assumed to have a short investment horizon in our model. Given that many professional traders

[2] Our model is consistent with these stylized facts in the sense that we observe these patterns with high probability in small samples.

who actively trade in foreign exchange markets are likely to be assessed based on their short-horizon performances by their employers, this assumption is justifiable.

Our intuition for constructing an economic model that is consistent with these stylized facts is based on effects of changes in risk premiums on foreign exchange rates. Given the conditional expectations and variances of all risky assets, we can decompose the effect of a change in the domestic short-term interest rate on the demand for foreign bonds into two components. The one is *the direct risk premium effect*. It is defined as the change in demand due to changes in the risk premium for foreign bonds when the risk premium for domestic long-term bonds is kept constant. The other is *the indirect risk premium effect*. It is the change in demand due to changes in the risk premium for domestic long-term bonds when the risk premium for foreign bonds is kept constant. The change in the demand for foreign bonds is the sum of these direct and indirect risk premium effects. In the special case of risk neutral investors, the indirect risk premium effect does not play any role. However, when investors are risk averse, it is necessary to evaluate both the direct and indirect risk premium effects in order to study how the foreign exchange rate changes when the domestic short-term interest rate changes.

The direct and indirect risk premium effects are properties of the demand for foreign bonds given the distributions of the asset returns and wealth conditional on the information available to the investors. In order to examine how these effects work in equilibrium, consider the term structure of interest rates where some shocks to short term interest rates are not transmitted to long term interest rates and the risk premium of long term bonds is only affected by them.

Suppose the domestic short term interest rates rise but the long term interest rates do not change. First, the risk premium for foreign bonds falls and the direct risk premium effect lowers the demand for foreign bonds without changing in the exchange rate. Since the supply for foreign bonds is essentially fixed in the short-run by the cumulative current balance in the model, the domestic currency appreciates now, creating expected future depreciation of the currency in order to restore an equilibrium. Second, the risk premium for domestic long-term bonds also falls as the domestic short-term interest rates rise. If the conditional covariance of the two risky assets returns is positive, the indirect risk premium effect increases the demand for foreign bonds. In order to restore an equilibrium, the domestic currency must depreciate this period, creating an expected appreciation in response to the indirect risk premium effect.

The sign and magnitude of the indirect risk premium effect depends on the conditional covariance. In the partial equilibrium model with given stochastic processes for interest rates, we endogenously derive the demand for foreign bonds by solving for the rational expectation equilibrium of the conditional expectation, variance, and covariance of the exchange rate. In equilibrium, the conditional covariance of the two risky assets returns is positive, and the direct and indirect risk premium effects have opposite signs. We show that under some reasonable parameter configurations, the indirect risk premium effect dominates the direct risk premium effect even when the degree of risk aversion is low. As a result, the domestic currency depreciates when the domestic short term interest rates rise and the long term interest rates do not rise. This feature of the model is the reason why the model is consistent with the forward premium anomaly for the short term interest rates.

On the other hand, when the domestic long term interest rates also rise with the domestic short term interest rates, then the risk premium for domestic long term bonds do not change. In this case, the indirect risk premium effect does not affect the equilibrium exchange rate. Therefore, the domestic currency appreciates when the short term and long term interest rates rise together. This feature of the model makes it consistent with the stylized facts of the exchange rate and the long term interest rates.[3]

In this paper, we show that the indirect risk premium effect is likely to be quantitatively important compared with the direct risk premium effect. In particular, we show that the indirect risk premium effect can even dominate the direct risk premium effect under reasonable parameter configurations. A Monte Carlo simulation for short-term regressions based on this parameter specification consistently shows the forward premium anomaly. The stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient.

The result in this paper is in sharp contrast to the conventional view that short-term capital is more internationally mobile than long-term capital. The 1960's Operation Twist, in which the Federal Reserve and the Treasury attempted to raise the short-term rate relative to the long-term interest rate, was evidently based on this view. However, empirical work by Fukao and Okubo (1984) suggests that international factors are more important in determining the domestic long-term interest rates than in short-term rates. Popper (1993) presents empirical evidence that long-term capital is as internationally mobile as short-term capital.

[3]The intuition behind these results for direct and indirect risk premium effects can be generalized with Ogaki's (1990) concepts of direct and indirect substitution effects. This generalization is explained in an earlier version of the present paper, Ogaki (1999).

The model in this paper has policy implications. It implies that the effectiveness of central bank attempts to affect exchange rates through the control of short-term interest rates depends on the responsiveness of long-term interest rates to changes in short-term interest rates.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 derives the rational expectation equilibrium. First, given the covariance and variance assumed by agents, the rational expectation of the mean of the exchange rate is used to solve for the exchange rate's law of motion. Then the condition for the rational expectation for the covariance is derived. Finally, the unique stable rational expectation equilibrium is found by equating the variance assumed by agents with the one implied by the demand function. Last section investigates the implications of the model on the relationship between the exchange rate and the term structure of interest rates. Conclusions are also given in the last section.

2 The Model

This paper adopts a simple partial equilibrium exchange rate model following Driskill and McCafferty (1980) and Fukao (1983). We endogenously derive the demand for foreign bonds by solving for the rational expectation of the covariance, so that the covariance assumed by agents is consistent with the one implied by the demand function.[4] It is technically difficult to solve for the rational expectation of the covariance in complicated asset pricing models. For this reason, we employ three asset models.

Consider a partial equilibrium model of exchange rate determination. For simplicity, the overall price level is assumed to be constant. Alternatively, all variables can be considered to be measured in real terms. Investors are assumed to live for two periods, and the same number of investors are born every period. There are 3 assets: domestic short-term bonds ($=B_{S,t}$), domestic long-term bonds ($=B_{L,t}$) and foreign bonds ($=B_{F,t}$). As the foreign interest rate will be assumed to be constant, the foreign short and long-term bonds are perfect substitutes and do not need to be distinguished. The domestic short and long-term bonds are discount bonds paying one unit of the domestic currency after one period and two periods, respectively. The foreign bonds behave in the same manner. At time t, a representative investor allocates his initial wealth ($=W_t$) among the three assets and he collects the payoffs paid by the assets he holds at the beginning of time $t+1$.

Let q_t be the price of domestic long-term bonds at time t and r_t be the domestic short-term interest rate. Then the rate of return on holding domestic long-term bonds

[4]The demand function depends on the covariance, conditional on the available information, between the exchange rates and the short-term interest rates. At the same time, the demand for foreign bonds affects the dynamics of the exchange rate and the covariance.

for one period, $r_{L,t}$, is

(2.1) $$r_{L,t} = \frac{1}{q_t}(\frac{1}{1+r_{t+1}} - q_t)$$

Since $q_t = 1/(1+R_t)^2$, where R_t is the domestic long-term interest rate

(2.2) $$r_{L,t} = (1+R_t)^2(\frac{1}{1+r_{t+1}} - \frac{1}{(1+R_t)^2}) \quad \approx \quad 2R_t - r_{t+1}$$

The risk premium for domestic long-term bonds, $\rho_{L,t}$, is defined to be the difference between the expected rate of return on holding long-term bonds for one period and that of short-term bonds;

(2.3) $$\rho_{L,t} = E_t(r_{L,t}) - r_t = 2[R_t - \frac{1}{2}\{r_t + E_t(r_{t+1})\}]$$

where E_t is the expectation operator conditional on the information set in period t, Ω_t. We assume that Ω_t includes the current and past values of r_t, R_t, r_t^*, R_t^*, and s_t, where r_t^*, and R_t^* are the foreign short and long-term interest rates, respectively, and s_t is the natural log of the exchange rate expressed in terms of the domestic currency.

The rate of return on holding foreign bonds for one period in terms of the domestic currency, $r_{F,t}$, is

(2.4) $$r_{F,t} = r_t^* + s_{t+1} - s_t$$

Let $\rho_{F,t}$, the risk premium for foreign bonds, denote the difference between the expected rate of return on holding foreign bonds for one period and that of short-term bonds;

(2.5) $$\rho_{F,t} = E_t(r_{F,t}) - r_t = r_t^* + E_t(s_{t+1}) - s_t - r_t$$

The model assumes that, at time t, a representative investor with a constant absolute risk aversion (CARA) utility function maximizes his expected utility of wealth at the beginning of the time $t+1$ ($=W_{t+1}$) subject to the budget constraint;

$$\text{(2.6)} \qquad max \qquad E_t\left(\frac{-e^{-kW_{t+1}}}{k}\right)$$

$$s.t. \qquad W_t = B^d_{S,t} + B^d_{L,t} + B^d_{F,t}$$

where k is the coefficient of absolute risk aversion, and the superscript d denotes demand, so domestic currency amounts invested in domestic short, long-term, and foreign bonds are $B^d_{S,t}$, $B^d_{L,t}$, and $B^d_{F,t}$, respectively. W_t is the initial wealth at time t, and the value of investor's assets at the beginning of time $t+1$, W_{t+1}, satisfies

$$\text{(2.7)} \qquad W_{t+1} = B^d_{S,t}(1+r_t) + B^d_{L,t}(1+r_{L,t}) + B^d_{F,t}(1+r_{F,t})$$

In the partial equilibrium model, the stochastic processes for the interest rates are exogenously given, and the utility function is parameterized. The equilibrium exchange rate satisfies the foreign bonds market clearing condition, $B^d_{F,t} = B^s_{F,t}$, where $B^s_{F,t}$ is the supply of foreign bonds to the domestic residents. It is assumed to be equal to the cumulative current account balance and to follow the dynamic equation;

$$\text{(2.8)} \qquad B^s_{F,t} = B^s_{F,t-1} + C_t$$

C_t is the current account balance in the period t satisfying[5]

$$\text{(2.9)} \qquad C_t = -a + bs_t + u_t,$$

where b is a positive number, and u_t is the trade shock which is assumed to be white noise with variance σ^2_u.

[5]Interest received by holders of foreign bonds is neglected

Suppose that W_{t+1} is normally distributed conditional on Ω_t and that the measure of the absolute risk aversion, k, is a positive constant. Under these assumptions, a representative investor's optimization problem is equivalent to maximizing

$$(2.10) \qquad max_{\{B^d_{F,t}, B^d_{L,t}\}} \quad E_t(W_{t+1}) - \frac{k}{2} var_t(W_{t+1})$$

where

$$(2.11) \qquad E_t(W_{t+1}) = W_t(1 + r_t) + B^d_{L,t}(\rho_{L,t}) + B^d_{F,t}(\rho_{F,t})$$

$$(2.12) \qquad \begin{aligned} var_t(W_{t+1}) &= (B^d_{L,t})^2 var_t(r_{t+1}) + (B^d_{F,t})^2 var_t(s_{t+1}) \\ &\quad -2(B^d_{L,t})(B^d_{F,t}) cov_t(r_{t+1}, s_{t+1}) \end{aligned}$$

First order conditions with respect to $B^d_{F,t}$ and $B^d_{L,t}$ are, respectively

$$(2.13) \qquad \rho_{F,t} - k(B^d_{F,t}) var_t(s_{t+1}) + k(B^d_{L,t}) cov_t(r_{t+1}, s_{t+1}) = 0$$

$$(2.14) \qquad \rho_{L,t} - k(B^d_{L,t}) var_t(r_{t+1}) + k(B^d_{F,t}) cov_t(r_{t+1}, s_{t+1}) = 0$$

Solving these FOCs for $B^d_{F,t}$ and $B^d_{L,t}$ gives demand functions for foreign bonds and domestic long-term bonds, respectively.

$$(2.15) \qquad B^d_{F,t}[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \rho_{F,t} - \psi \cdot \phi \cdot \rho_{L,t}$$

$$(2.16) \qquad B^d_{L,t}[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \frac{\sigma_s^2}{\sigma_r^2} \cdot \rho_{L,t} - \psi \cdot \phi \cdot \rho_{F,t}$$

where

$$(2.17) \qquad \psi = 1/k\sigma_s^2(1 - cor^2)$$

$$\phi = -cov/\sigma_r^2 \tag{2.18}$$

$$\sigma_s^2 = E_t[s_{t+1} - E_t(s_{t+1})]^2 \tag{2.19}$$

$$\sigma_r^2 = E_t[r_{t+1} - E_t(r_{t+1})]^2 \tag{2.20}$$

$$cov = E_t[\{s_{t+1} - E_t(s_{t+1})\}\{r_{t+1} - E_t(r_{t+1})\}] \tag{2.21}$$

$$cor = cov/(\sqrt{\sigma_s^2}\sqrt{\sigma_r^2}) \tag{2.22}$$

The demand function for foreign bonds, Equation (2.15), depends on *cov*, the covariance conditional on Ω_t between the exchange rate and the short-term interest rate, and σ_s^2, the conditional variance of the exchange rate. At the same time, the stochastic processes of the exchange rate and *cov* also rely on the demand function for foreign bonds. Therefore, it is required to solve for a rational expectation equilibrium in which the values of *cov* and σ_s^2 are consistent with the stochastic process of the exchange rate implied by the demand function for foreign bonds. In the next section, the rational expectation equilibrium will be derived.

When the short-term interest rate rises, there exist two opposite effects on the demand for foreign bonds given the second moments of the exchange rate and the short-term interest rate. The first effect, called *the direct risk premium effect*, is from the first term of Equation (2.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rates rise holding the risk premium for long-term bonds constant. This effect is equal to $-\psi$ and is negative. The second effect, called *the indirect risk premium effect*, is from the second term of Equation (2.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rate rises holding the risk premium for foreign bonds constant. This effect is equal to $\psi\phi$. In the rational expectations equilibrium derived in the next section, *cov* is negative, which implies that the indirect risk premium effect is positive.

An intuitive explanation of the indirect risk premium effect is as follows: If the short-term interest rate unexpectedly rises, the price of a long-term bond falls and this drop causes long-term bond holders to suffer an unexpected capital loss. When *cov* is negative, the exchange rate tends to appreciate and it causes investors an additional unexpected loss if they hold foreign bonds. Therefore, as long as an increase in the short-term interest rate is associated with an appreciation of the domestic currency, risk averse agents will want to avoid holding both long-term bonds and foreign bonds. The greater the appreciation of the domestic currency caused by an increase in the short-term interest rate, the stronger the substitutability of domestic long term bonds and foreign bonds. In particular, when an increase in short term interest rates reduces the risk premium for long term bonds, risk averse investors want to adjust a portfolio of risky assets toward holding more foreign bonds and less long-term bonds. This indirect risk premium effect allows the demand for foreign bonds to increase when the short-term interest rate rises.

The existence of two counter forces on the demand for foreign bonds implies that the effect of a rise in the short-term interest rate on the demand for foreign bonds depends on the relative strength of these two effects. The indirect risk premium effect dominates the direct risk premium effect if and only if $\phi > 1$. Therefore, ϕ may be referred to as the measure of the relative magnitude of the indirect risk premium effect. In the next section, it will be shown that ϕ is greater than 1 under reasonable parameter configurations.

3 The Rational Expectation Equilibrium

In this section, the model presented in the previous section will be used to derive the rational expectation equilibrium. The stochastic processes of interest rates are assumed to be as follows:

$$r_t = \mu + e_t + \varepsilon_t \tag{3.1}$$

$$R_t = \frac{1}{2}d + \mu + \frac{1}{2}(1 + c)e_t \tag{3.2}$$

$$r_t^* = \mu \tag{3.3}$$

$$R_t^* = \frac{1}{2}d + \mu \tag{3.4}$$

where e_t and ε_t are a persistent interest rate shock and a temporary interest rate shock, respectively. It is assumed that e_t follows an AR(1) process

$$e_t = ce_{t-1} + v_t, \qquad where \mid c \mid < 1 \tag{3.5}$$

and that it is independent of u_t. It is also assumed that ε_t, and v_t are white noise with variance σ_ε^2 and σ_v^2, respectively, and that they are independent of each other and of u_t. Finally, d and μ are positive numbers.

The conditional expectation is assumed to coincide with the best linear prediction. Since (3.2) is a fundamental representation in the sense of linear prediction theory (see, e.g., Rozanov (1967)), observing the current and past values of R_t is equivalent to observing the current and past values of e_t under an AR(1) process. It follows that

$$E_t(r_{t+1}) = \mu + ce_t \tag{3.6}$$

and from (2.3) and (3.6),

$$\rho_{L,t} = d - \varepsilon_t \tag{3.7}$$

For the purpose of this paper, we need to assume that the risk premium for long-term bonds, $\rho_{L,t}$, is nonzero. As is shown in (3.7), the assumption employed here is that only e_t is transmitted to the long-term interest rate, so that the risk premium is equal to the sum of the mean of long-term interest rate and a temporary interest rate shock.

Define $\eta = \sigma_\varepsilon^2/\sigma_e^2$, which may be called the measure of substitution between short-term bonds and long-term bonds. If $\eta = 0$, then the risk premium for long-term bonds will be the mean of the long-term interest rate, implying that the short-term bond and the long-term bond will become more substitutable. The greater the magnitude of η, the smaller the degree of the substitution.

Let L be the lag operator. Then the equilibrium condition in the period t is,

$$E_t[A_0(L)s_t] = a + D_0 \tag{3.8}$$

where

$$A_0(L) = -\psi L^{-1} + (b + \psi) \tag{3.9}$$

$$D_0 = -u_t - B^s_{F,t-1} - \psi\phi d - \psi e_t + \psi(\phi - 1)\varepsilon_t \tag{3.10}$$

The equilibrium condition for period $t+1$ is, if we take expectations conditional on Ω_t from both sides,

$$E_t[A(L)s_{t+1}] = a + D_1 \tag{3.11}$$

where

$$A(L) = -\psi L^{-1} + (b + 2\psi) - \psi L \tag{3.12}$$

$$D_1 = \psi(1-c)e_t - \psi(\phi-1)\varepsilon_t \tag{3.13}$$

The equilibrium condition for $t+\tau(\tau \geq 2)$ is, if we take expectations conditional on Ω_t from both sides,

$$E_t[A(L)s_{t+\tau}] = a + D_2 \tag{3.14}$$

where

$$D_2 = \psi(1-c)e_t c^{\tau-1} \tag{3.15}$$

Solving (3.8), (3.11), and (3.14) as a difference equation system of $E_t(s_{t+\tau})$ with respect to τ provides the unique saddle point solution,

$$s_t = \overline{s} - (\frac{1-\lambda}{b})u_t - (\frac{1-\lambda}{b})B^s_{F,t-1} - (\frac{\lambda}{1-\lambda c})e_t + \lambda(\phi-1)\varepsilon_t \tag{3.16}$$

where $\overline{s}$ $[= \frac{a}{b} - (\frac{1-\lambda}{b})\phi\psi d]$ is the long-run equilibrium exchange rate clearing the current account, and

$$\lambda = 1 + \frac{b}{2\psi} - \frac{b}{2\psi}\sqrt{1+\frac{4\psi}{b}} \tag{3.17}$$

It is shown that $0 < \lambda < 1$, $\partial\lambda/\partial\psi > 0$, $\lim_{\psi\to 0}\lambda = 0$, and $\lim_{\psi\to\infty}\lambda = 1$.

Equation (3.16) shows that the investor's expected values of cov and σ_s^2 affect the exchange rate dynamics through λ and ϕ. On the other hand, the exchange rate dynamics in (3.16) imply certain values of cov and σ_s^2, which need to be consistent with the investor's expected values in the rational expectation equilibrium. The equilibrium is analyzed in two steps. First, we solve for the rational expectation of cov. Second, we show the uniqueness and existence of the rational expectation equilibrium by solving for the rational expectation of σ_s^2.

Before solving for the equilibrium, note the nature of (3.16). The discrepancy between actual and long-run equilibrium exchange rates can be explained by several factors: the trade shock (the first bracket), the cumulative current account balance (the second bracket), the persistent interest rate shock (the third bracket), and the temporary interest rate shock (the fourth bracket). The trade shock, which tends to give rise to current account surplus, makes the domestic currency appreciate. As the cumulative current account balance becomes greater, the appreciation of the domestic currency increases; for an investor to have incentives to hold more foreign bonds, the domestic currency must appreciate at present, so that investors will anticipate it depreciating in the future. Prolonged increases in the short-term interest rate make the domestic currency appreciate. All of these effects are consistent with the expected directions. However, the temporary interest rate shock, ε_t, has a perverse effect if the relative magnitude of the indirect risk premium effect, ϕ, is greater than one.

The term ϕ may be obtained by solving for the rational expectation of covariance. Calculating $cov = E_t[\{s_{t+1} - E_t(s_{t+1})\}\{r_{t+1} - E_t(r_{t+1})\}]$ from (3.1) and (3.16) by taking the one period lead yields

$$cov = -(\frac{\lambda}{1-\lambda c})(1-c^2)\sigma_e^2 + \lambda(\phi - 1)\sigma_\varepsilon^2 \tag{3.18}$$

Substituting the definition of ϕ, (2.18), into (3.18), and solving for cov gives the rational expectation equilibrium;

$$cov = -[\frac{\lambda(1-c^2) + \lambda\eta(1-\lambda c)}{1-\lambda c}][\frac{1-c^2+\eta}{1-c^2+\eta(1+\lambda)}]\sigma_e^2 < 0 \tag{3.19}$$

Therefore, by (2.18),

$$\phi = [\frac{\lambda(1-c^2) + \lambda\eta(1-\lambda c)}{1-\lambda c}][\frac{1}{1-c^2+\eta(1+\lambda)}] > 0 \tag{3.20}$$

In the rational expectation equilibrium, the conditional covariance between the exchange rate and the short-term interest rate, cov, is negative and the measure of the relative magnitude of the indirect risk premium effect, ϕ, is positive. This implies that the indirect risk premium effect is positive as is shown in the previous section. The main issue for the purpose of this paper is whether ϕ is greater or less than one. In order to determine this, we will investigate the sign of

$$\phi - 1 = \frac{(1-c^2)\{\lambda(1+c)-1\} - \eta(1-\lambda c)}{(1-\lambda c)\{1-c^2+\eta(1+\lambda)\}} \tag{3.21}$$

In order to examine the sign of (3.21), we need to know how λ depends on the underlying parameters of the model. For this purpose, the existence and the uniqueness of the rational expectation equilibrium will be shown by solving for the rational expectation of the conditional variance of the exchange rate, $\sigma_s^2 = E_t[\{s_{t+1} - E_t(s_{t+1})\}^2]$. By taking one period lead of (3.16), we obtain

$$\sigma_s^2 = (\frac{1-\lambda}{b})^2\sigma_u^2 + [\frac{\lambda^2(1-c^2)+\lambda^2(\phi-1)^2\eta(1-\lambda c)^2}{(1-\lambda c)^2}]\sigma_e^2 \tag{3.22}$$

By (3.19) and the definition of cor, (2.22),

$$cor = -\sqrt{\frac{(1-c^2+\eta)\sigma_e^2}{\sigma_s^2}} \cdot [\frac{\lambda(1-c^2)+\lambda\eta(1-\lambda c)}{(1-\lambda c)(1-c^2+\eta(1+\lambda))}] \tag{3.23}$$

By using the definition of λ, (3.17), we obtain

$$\psi = \frac{b\lambda}{(1-\lambda)^2} \tag{3.24}$$

Substituting the definition of ψ, (2.17), into (3.24) gives

$$\frac{1}{k} = \sigma_s^2(1-cor^2)\frac{b\lambda}{(1-\lambda)^2} \tag{3.25}$$

The condition for the rational expectation equilibrium value for σ_s^2 is obtained by substituting (3.22) and (3.23) into (3.25);

$$\frac{1}{k} = g(\lambda), \tag{3.26}$$

where $g(\lambda) = \frac{\sigma_u^2 \lambda}{b} + \frac{b\sigma_e^2 \lambda^3 \{(1-c^2)+\eta(\phi-1)^2(1-\lambda c)^2\}}{(1-\lambda c)^2(1-\lambda)^2} - \frac{b\sigma_e^2(1-c^2+\eta)\lambda\{\lambda(1-c^2)+\lambda\eta(1-\lambda c)\}^2}{(1-\lambda c)^2(1-\lambda)^2(1-c^2+\eta(1+\lambda))^2}$

Let λ^* be the value of λ that satisfies (3.26). Any such λ^* corresponds to a rational expectation equilibrium. It can be checked that $\lim_{\lambda \to 0} g(\lambda) = 0$, and $\lim_{\lambda \to 1} g(\lambda) = \infty$. In particular, under the parameter configuration employed in the following Monte Carlo simulation, it can be shown that $g'(\lambda) > 0$. Hence, there exists a unique rational expectation equilibrium. Moreover, when k is smaller, λ^* is larger. It is shown that $\lim_{k \to 0} \lambda = 1$ and $\lim_{k \to \infty} \lambda = 0$. The value of ψ can be obtained by substituting λ^* for λ in (3.24). The value of ψ is decreased by a reduction in the variances σ_u^2 and σ_e^2 and by an increase in the measure of constant risk aversion, k, which in turn diminishes $g(\lambda)$.

Equation (3.21) shows that ϕ can be either greater or less than one, depending on the parameter values. One interesting case arises when the investor is close to being risk neutral. For a very small k, an approximate formula for (3.21) with $\lambda \cong 1$ is

$$\phi - 1 = \frac{(1+c)c - \eta}{1 - c^2 + 2\eta} \tag{3.27}$$

We investigate what condition is required to exhibit the forward premium anomaly under low degree of risk aversion. The forward premium regression for short-term interest rate differential is

$$s_{t+1} - s_t = \alpha + \beta(r_t - r_t^*) + error\ term \tag{3.28}$$

Let $\hat{\beta}$ be the estimate of β. The probability limit of estimator is

$$\begin{aligned} plim\ \hat{\beta} \quad &= \frac{cov(r_t - r_t^*, s_{t+1} - s_t)}{var(r_t - r_t^*)} \\ &\Rightarrow \frac{cov(r_t, s_{t+1} - s_t)}{var(r_t)} \end{aligned} \tag{3.29}$$

For this to be negative, we need

$$(3.30) \qquad \begin{aligned} &cov(r_t, s_{t+1} - s_t) < 0 \\ &\Rightarrow cov(r_t, s_{t+1}) < cov(r_t, s_t) \\ &\Rightarrow 1 < (\phi - 1)\eta \end{aligned}$$

However, substituting Equation (3.27) into Equation (3.30) does not produce a positive value of η satisfying Equation (3.30). Thus, the population limit of $\hat{\beta}$ is always positive in this model. The reason is that the persistent shock has the effect to make a positive slope coefficient because short term and long term interest rates move together in response to this shock in the model. This persistent shock dominates in the limit even though the temporary shock has the effect to make a negative coefficient because of the dominant indirect risk premium effect. However, given that the temporary shock has the effect to make a negative slope coefficient, the dominant direct risk premium effect is likely to cause downward small sample bias when the exchange rate is persistent. We conduct a Monte Carlo simulation[6] under these parameterizations to investigate this. Because the model is highly stylized so that we can analytically solve for the rational expectation of the covariance, we do not try to calibrate the data in this paper.

Suppose the AR(1) coefficient of the persistent interest rate shock, c, in Equation (3.5) is close to one (for example, $c = 0.9$), then ϕ in Equation (3.27) becomes greater than one as long as $\eta < 1.71$. When the investor is close to being risk neutral, the degree of substitution between short and long-term bonds must be high, and consequently, η should be very small. Under these parameter configurations, our model presented in the previous section predicts that when the measure of the

[6] We use Gauss for Windows NT/95 Version 3.2.38 to conduct the simulation

relative magnitude of the indirect risk premium effect, ϕ, is greater than one, the demand for foreign bonds increases as the short-term interest rate rises, resulting in the depreciation of domestic currency to cause an expected future appreciation of domestic currency. A Monte Carlo simulation based on these parameter configurations consistently generates a negative slope coefficient to show the forward premium anomaly. As Table (3.1) shows, the stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient.

Table 3.1: A Monte Carlo simulation for slope coefficient ($= \beta$) of short-term regression

$$(s_{t+1} - s_t) = \alpha + \beta(r_t - r_t^*) + \text{error term}$$
$$H_0 : \beta = 0$$

ϕ	1.6352 ($\eta = 0.7$)	2.7848 ($\eta = 0.3$)	3.5593 ($\eta = 0.2$)	5.1282 ($\eta = 0.1$)	9.0952 ($\eta = 0.01$)
mean of $\hat{\beta}$	0.2016	-0.3844	-0.7788	-1.5776	-3.5976
negative freq.[1)]	39.1	69.4	84.2	97.2	100.0
5 % level [2)]	1.9	8.5	18.0	49.6	93.2
(10 % level)	(3.7)	(13.8)	(28.1)	(62.5)	(96.8)

Note: 1) percentage of negative coefficients among total iteration(=1,000)
2) percentage of total iterations(=1,000) rejecting H_0 at five percent significance level. Numbers in parentheses are that of ten percent significance level.
3) sample size is 102 and $c = 0.9$

On the contrary, a Monte Carlo simulation for long-term interest differential still generates, as Table (3.2) shows, a positive slope coefficient under the same parameter configurations as standard exchange rate model predicts.

Table 3.2: A Monte Carlo simulation for slope coefficient ($=\beta$) of long-term regression

$$(s_{t+2} - s_t) = \alpha + \beta(R_t - R_t^*) + \text{error term}$$
$$H_0 : \beta = 0$$

ϕ	1.6352 ($\eta = 0.7$)	2.7848 ($\eta = 0.3$)	3.5593 ($\eta = 0.2$)	5.1282 ($\eta = 0.1$)	9.0952 ($\eta = 0.01$)
mean of $\hat{\beta}$	1.0267	1.0274	1.0278	1.0287	1.0309
positive freq.[1)]	92.2	91.9	92.1	91.3	87.0
5 % level [2)]	30.9	30.0	29.3	28.4	20.8
(10 % level)	(43.9)	(43.1)	(42.1)	(39.2)	(30.8)

Note: 1) percentage of positive coefficients among total iteration(=1,000)
2) percentage of total iteration(=1,000) rejecting H_0 at five percent significance level. Numbers in parentheses are that of ten percent significance level.
3) sample size is 102 and $c = 0.9$

4 Conclusion

In this paper, we derive the demand function for foreign bonds endogenously by solving for the rational expectation equilibrium and investigate how a rise in the short-term interest rate affects the demand for foreign bonds. It generates two opposite effects on the demand for foreign bonds. The direct risk-premium effect comes from the fact that risk averse agents with short investment time-horizons want to reduce the demand for foreign bonds to increase the amount invested in risk free assets. On the other hand, investors have another incentive, the indirect risk-premium effect, to increase the demand for foreign bonds to minimize potential capital losses resulting from holding both risky assets.

Monte Carlo simulation results show that under reasonable parameter configurations the indirect risk-premium effect is quantitatively important in finite sample. It can dominate the direct risk-premium effect causing demand for foreign bonds to increase. In this case, the forward premium anomaly about the short-term interest rate can be explained; the domestic currency depreciates now, creating expected future appreciation of the currency. For the long-term interest rate differential, this model still shows the same prediction on the exchange rate like standard exchange rate models. Byeon and Ogaki (1999) find such results for many of the G7 countries with cointegrating regressions of real exchange rates onto the short-term and long-term interest rate differentials. Ogaki and Santaella (2000) obtain similar results for Mexico.

If the indirect risk-premium effect is quantitatively important, then the effectiveness of central bank attempts to affect exchange rate by controlling the short-term interest rate depends on whether the long-term interest rate responds to changes in the short-term interest rate. Anecdotal evidence suggests that further empirical investigation is warranted. For example, from the middle of March 1982 to the end of November 1982, the Bank of Japan adopted a policy to increase the domestic short-term interest rate in order to cause an appreciation of the yen (see, e.g., Komiya and Suda [1983, pp. 347-354]). The short-term interest rate in Japan increased but the yen tended to depreciate, rather than appreciate, against the U.S. dollar during this period. One remarkable fact was that the long-term interest rate did not increase when the Bank of Japan began to increase the short-term interest rate (Komiya and Suda [1983, p.349]).

The model in this paper suggests that a much more complicated relationship might exist between the term structure of interest rates and the exchange rate than is implied by exchange rate models with risk neutral agents. In addition, the model can be applied to the relationship between the exchange rate and the term structure of various short-term rates if the investment horizon is very short (e.g., 1 month or shorter). In this sense, the model could help explain Clarida and Taylor's (1997) finding that the information given by the term structure of 1-month to 12-months forward premiums is useful in predicting the future exchange rate.

In this paper, we develop a highly stylized partial equilibrium model to obtain the rational expectation of the covariance between the exchange rate and the short term interest rate, which is a key parameter for the indirect risk premium effect. It is of interest to study if the qualitative implications of the model still holds in more realistic models. There has been little empirical work on the interaction between the exchange rate and the term structure of interest rates relative to the large volume of empirical work on the exchange rate. Further empirical investigation is warranted.

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