# Multi-sector Stochastic Dynamics and the Phillips Curve*

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Abstract

I provide a multi-sector stochastic model that illustrates the Phillips Curve relationship in the macro economy. While trade off between inflation and unemployment requires some assumption that distorts the neutrality of money, I do not attribute to money illusion the reason why change of real wage rates does not stimulate workers immediately, but to other frictions that both firms and workers face in the sectoral labor markets. According to Aoki and Yoshikawa (2003), I incorporate prices into a multi-sector model with stochastic quantity adjustment, and explain how the Phillips Curve arises from interaction in sectoral labor markets, and how the sectoral frictions affect the Phillips Curve.

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## 1 Introduction

The Phillips Curve, the macroeconomic phenomenon between unemployment and inflation, is a controversial issue since it seemed to break in the 1970s. Friedman (1968) suggests that it be a vertical in the long run when there is no gap between real inflation and expectations on it, that is, there is no money illusion. I argue that the decreasing slope of the Phillips Curve does not derive from money illusion, but from other market

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frictions, which never diminish in the long run. Namely, in this paper, I claim that the Phillips Curve will not be vertical even in the long run.

To begin with, trade off between inflation and unemployment requires some assumption that breaks the neutrality of money.[*1] For example, Lucas (1973) assumes that there are errors in expectation on prices, and this assumption is often described as money illusion. If workers are possessed by money illusion, they do not exactly perceive the increase or decrease in the real wage rate. Consequently, the change in the real wage rate does not immediately stimulate workers to move. Namely, the rise in the nominal wage rate induces workers to work more, just until they perceive the precise value of the real wage rate. Gordon (2004) calls the situation "inertia." Thus, the inflation and the unemployment rate correlate negatively when we observe this inertia.

However, the assumption of money illusion is weird. It is not clear why workers cannot perceive one part of prices while they can precisely know the other part of prices.[*2] What is more, workers with money illusion will increase labor supply when exogenous shock causes simultaneous macro inflation, even though the shock does not change the real wage rates at all. Therefore, I do not attribute to money illusion the reason why decrease in the real wage rate does not stimulate workers to separate their jobs immediately, but to other frictions that both firms and workers face in the sectoral labor markets.

There are several pieces of preceding literature focusing on the sectoral friction and its effect, like Lilien (1982). Among them, Aoki and Yoshikawa (2003) create a multi-sector quantity adjustment model that captures various frictions that are hard to describe directly, using a stochastic method.[*3] According to this model, I make a multi-sector stochastic model with prices,[*4] and examine the relationship between unemployment and inflation.

The main characteristic of the model is that we can focus on disequilibrium, which

[*1] The neutrality of money is not consistent with any relationship between inflation and unemployment. Generally speaking, the rise in the nominal wage rate of one sector brings about the macro inflation, but the inflation does not perfectly negate the rise in the real wage rate in the sector. Thus, labor supply in the sector will increase along with the rise in the real wage rate. However, the inflation also causes fall in the rest of the real wage rates in the other sectors, and then labor supply in those sectors will decrease and negate the rise in labor supply in the one sector above. As a result, the rise in the nominal wage rate has little effect on the labor supply. This is the neutrality of the money, and trade off never occurs as long as it holds.

[*2] See Yoshikawa (2000, pp.25).

[*3] Unlike a usual way of micro foundation, the model has a so-called "stochastic micro foundation" or "mesoscopic method," which summarizes micro agent's behavior into the parameters of stochastic process.

[*4] Aoki and Yoshikawa (2003) do not incorporate prices to their model.

haunts around equilibrium. In the model, the economy rarely reaches the equilibrium and keeps in disequilibrium.[*5] The Phillips Curve is often argued as a macroeconomic phenomenon that rises in disequilibrium. Iwai (1981) describes it as “swarm of mosquitoes,” where each mosquito moves towards the right and left but the whole swarm keeps its shape. This means that each micro agent does not directly represent the macroeconomic relationship, and the relationship can be captured only after aggregation of micro agent’s behavior. Tobin (1972) also points out this situation as follows:

> One rationalization might be termed a theory of stochastic macro-equilibrium: stochastic, because random intersectoral shocks keep individual labor markets in diverse states of disequilibrium; macro-equilibrium, because the perpetual flux of particular markets produces fairly definite aggregate outcomes of unemployment and wages.

In contrast to the assumption of money illusion,[*6] The model in this paper is so made that increase in the wage rate derives from sectoral labor inputs adjustment in disequilibrium, and not the other way around. This means that a shock in the macro price level does not produce any additional labor supply, as long as the shock has simultaneous impacts on all sectors, and then it does not change the real wage rates. In other words, the causality of the Phillips Curve in the model lies from quantity adjustment to change of the inflation rate, and not vice versa. This is also a distinguishable difference between money illusion and the frictions that I assume.

In this paper, I introduce the model structure including prices in section 2. The simulation results are exhibited in section 3, and then, I report the summary and interpretation as conclusion remarks in section 4.

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[*5] Aoki (2002) argues that these disequilibrium states permanently cause the business cycles.

[*6] Actually, money illusion is also behavior in disequilibrium, Iwai (1981, Ch.7) argues that the point of view based on disequilibrium includes money illusion in this sense. But I dare to divide money illusion from the other frictions in order to make the argument clear. In addition, even if we admit the existence of money illusion, disequilibrium will not disappear even in the long run when agents perfectly calibrate their expectations. This is because the frictions consist of other various components even after money illusion is removed.

# 2 The Model

## 2.1 Definitions

First of all, I set up a multi-sector stochastic model, according to Aoki and Yoshikawa (2003) except for a part of prices. We consider an economy that has $K$ distinct sectors denoted by $i$ ($i = 1, \cdots, K$), in which firms produce different goods with the only one factor inputs, labor. There are $N$ workers, who are either employed or unemployed. We do not incorporate the population growth, so that $N$ is constant.

Unlike a usual way of micro foundation, we do not directly describe firm's maximization problems. Instead, we interpret firm's behavior into that of a sector by regarding a sector as an economic agent. Namely, sectors act as economic agents to reflect firm's profit maximization, and then, we model the characteristics and behavior of sectors rather than firms.

Each sector has its own productivity coefficient $c_i$, output price $p_i$ and wage rate $w_i$. Note that productivity coefficients differ across all sectors, and describe heterogeneity of the sectors. The number of workers in sector $i$ is denoted by $n_i$, which represents the size of the sector.

For simplicity, I assume that the aggregated outputs of firms in sector $i$ is:

$$y_i = p_i c_i n_i, \tag{1}$$

that is, production function is linear for each sector. Note that it does not necessarily mean that production function is linear at the macro level or for each firm.

The nominal total outputs, $Y$, and the total labor inputs, $L$, are defined by:

$$Y = \sum_{i=1}^{K} y_i = \sum_{i=1}^{K} p_i c_i n_i. \tag{2}$$

$$L = \sum_{i=1}^{K} n_i \tag{3}$$

This $Y$ represents the nominal GDP, and is distributed to $K$ sectors according to the demand share of sector $i$, $s_i$, where $s_i > 0$, and $\sum_i s_i = 1$. The definition of the excess demand of sector $i$, denoted by $f_i$, is:

$$\begin{aligned} f_i &\equiv s_i Y - y_i, \\ &= s_i Y - p_i c_i n_i. \end{aligned} \tag{4}$$

Note that $\sum_i f_i = 0$ by definition, and all $f_i$ are not either positive or negative simultaneously.

## 2.2 Equilibrium

The economy settles in equilibrium if the excess demands are equal to zero in all sectors, that is:

$$\forall i,\ \ f_i = 0 \Leftrightarrow s_i Y = p_i c_i n_i. \tag{5}$$

This condition also means that all goods markets are in equilibrium.

From this condition, equilibrium values of $Y$, $n_i$, and $p_i$, denoted by $Y^e$, $n_i^e$, and $p_i^e$ for each, satisfy the following:

$$\forall i,\ \ s_i Y^e = p_i^e c_i n_i^e. \tag{6}$$

Then, the total employment in equilibrium, $L^e$, is:

$$\begin{aligned} L^e &= \sum_{i=1}^{K} n_i^e, \\ &= \left( \sum_i \frac{s_i}{p_i^e c_i} \right) Y^e. \end{aligned} \tag{7}$$

In fact, there is a continuum of equilibria because $Y^e$ and $L^e$ can take arbitrary values as long as they do not change the ratio of the two. In addition, an equilibrium exists for every $\{p_i^e\}_{i=1}^K$, that is, $\{p_i^e\}_{i=1}^K$ can also take arbitrary values. For every $\{p_i^e\}_{i=1}^K$, we can find an arbitrary set of $\{n_i^e\}_{i=1}^K$ and $L^e$ that satisfies the equilibrium condition (5). Combination of (6) and (7) leads to the sectoral labor ratio in equilibrium:

$$\frac{n_i^e}{L^e} = \frac{\left( \frac{s_i}{p_i^e c_i} \right)}{\left( \sum_i \frac{s_i}{p_i^e c_i} \right)}. \tag{8}$$

Since The right hand side of (8) is uniquely determined for every $\{p_i^e\}_{i=1}^K$, all we have to do is to set any $L^e$ and distribute it to $\{n_i^e\}_{i=1}^K$ according to the ratio.

## 2.3 Dynamics

We focus on disequilibrium in the model, rather than equilibrium, as I mentioned in the introduction. In disequilibrium, some sectors face positive excess demand, and others face negative one, that is, excess supply, though there may be a few sectors who happens to be in equilibrium. For the time being, we consider prices to be fixed and constant, for we discuss the dynamics of prices in section 2.5. Given fixed prices, we naturally assume that sectors will adjust their production level to reach the equilibrium. In other words, sectors tend to raise the production level when they have excess demand, and to shrink it when they face excess supply.

In adjustment of the production level, we assume that there are market frictions, and then the economy does not reach the equilibrium immediately. Thus, we have to set up some dynamics with frictions, which determine how sectors react to disequilibrium.

We employ a continuous-time jumping Markov chain in order to incorporate dynamics to the model. In such a continuous-time model, only one sector, called an active sector, have the right to adjust its factor inputs at any given time $t$. If the active sector does not adjust its inputs, no sector does at the moment. This assumption is not particular, but the same as the standard Poisson Process.

We have to set the transition rates among states in the Markov process in order to determine which sector becomes active at the moment. From the transition rates, we can find the "holding time" or "sojourn time" in terms of probability theory, that is, the time needed to move from one state to another. The holding time that it takes for sector $i$ to be active, $T_i$, is exponentially distributed, so we have $\Pr(T_i > t) = \exp(-b_i t)$. We assume this $b_i$ is equal to the size of the sector, $n_i$, without referring the transition rates themselves.[*7] In addition, we assume that the sector with the shortest holding time become active at the moment. This is only a technical assumption in order to translate a continuous-time structure into a discrete-time one for computer simulation. Namely, the probability that sector $i$ becomes active is equal to $\Pr(T_i = min_j\{T_j\}$ at time t). Lawler (1995) calculates this probability, and it is reduced into $n_i/(n_1 + \cdots + n_K) = \frac{n_i}{L}$. This implies that the larger the size of the sector is, the faster it becomes active.

When sector $i$ becomes active, it reacts to the disequilibrium that it faces. Here, we only make a reasonable weak assumption that an active sector facing excess demand raises its factor inputs, and vice versa. What is more, we assume that the sector always changes its inputs by only one unit; the size of the change in inputs is almost the same in any case. This assumption reflects the aggregation of various adjustments by firms in the sector. Briefly, if sector $i$ becomes active, it increases its factor inputs $n_i$ to $n_i + 1$

[*7] Usually, we first set the transition rates, and then calculate the holding time. So we take the other way around here.

when $f_i > 0$, and decreases $n_i$ to $n_i - 1$ when $f_i < 0$.[*8] When $f_i = 0$, it does not make any adjustment because it is in equilibrium for the time being.

The setting above enables us to avoid difficulty in capturing precise behavior of sectors. Since a sector is not really an economic agent, we need to summarize reactions of firms in a sector in order to characterize the sector's reaction. However, there are a huge number of firms[*9] that face their own maximization problems with heterogeneous constraints. Among so large numbers of firms, we reasonably assume that a few firms want to shrink their production, even though they belong to a sector with positive excess demand. Such firms are supposed to be in an idiosyncratic economic environment. For example, they may be recalling "lemons" that they have already sold, or unfortunately have a bad reputation in a market. Hence sector's reaction is highly influenced by heterogeneity of firms and idiosyncratic economic environment. We think that these frictions play important roles in macroeconomic phenomena, such as business cycles, Okun's law, or the Phillips Curve. But it is very hard to identify them into details for each. Consequently, we take them into account indirectly by using the stochastic method.

## 2.4 Unemployment pool

When an active sector increases its factor inputs, it means that one unit of labor will be employed from the unemployment pool. We assume that job creations and destructions are only caused by changes in labor demand. Or you can think that they occur only when firms and workers reach agreement, because the stochastic setting in the model can include frictions between firms and the workers.

A type of labor demand differs among sectors, so it is natural that the labor market be separated for each sector, that is, every sector has its own unemployment pool, $u_i$. The distinction of the unemployment pool reflects sectoral structures such as geographical diversity, difference in technology, and educational qualification.

We do not neglect intersectoral movement of labor. We assume that laid-off workers in sector $i$ directly enter the unemployment pool of sector $i$, while new workers hired in sector $i$ are not necessarily from the unemployment pool of sector $i$, and may be from that of a sector similar to sector $i$. We use "ultrametric distance"[*10] to introduce similarity across sectors.

[*8] In the model, unlike Aoki and Yoshikawa (2003), I do not incorporate a vacancy sign, $v_i$ for simplicity.

[*9] Yoshikawa (2003) points out that the number of firms is of the order of $10^6$.

[*10] See Appendix A., or Aoki (2003).

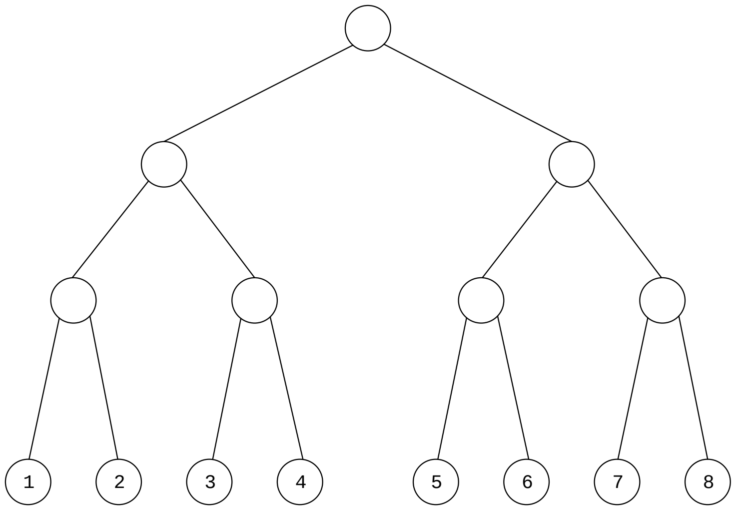


Figure 1 An example of ultrametric trees for $K = 8$. The numbers are identification of sectors.

In Figure 1, for example, sector 2 is very similar to sector 1, while sector 5 to 8 are different from sector 1 to a large extent. Actually, the ultrametric distances are: $d(1,2) = 1$, and $d(1,5) = d(1,6) = d(1,7) = d(1,8) = 3$. The ultrametric distance determines the probabilities that new workers are picked from the other sectors.

Before setting these probabilities, we have to define "job candidates" in sector $i$, denoted by $JC_i$ as follows:[*11]

$$JC_i = \sum_{j=1}^{K} \frac{u_j}{1 + d(i,j)} = u_i + \sum_{j \neq i} \frac{u_j}{1 + d(i,j)}. \tag{9}$$

The second equality derives from $\forall i,\ d(i,i) = 0$ by definition of ultrametric distance. This means that the job candidates consist of not only the unemployment pool of sector $i$, but also the other unemployment pools. Candidates from the other unemployment pools are weighted by $\frac{1}{1+d(i,j)}$. Namely, not all unemployed workers in the other sectors apply for jobs in sector $i$, because there are several frictions that discourage the unemployed in the other sectors, such as cost for acquiring new skills, difference in human capital, or geographical barrier. Note that the total number of job candidates exceeds that of the

[*11] Aoki and Yoshikawa (2003) use $U_i$ for $JC_i$, and $\tilde{u}_i$ for $\sum_{j \neq i} \frac{u_j}{1+d(i,j)}$.

unemployed workers in the economy. This is because each worker can apply for more than one job, as he or she does in the real world.

We assume that one unit of workers is picked from the job candidates with the same likelihood. Then, for all $j$, we have:

$$\begin{aligned}
&\Pr(\text{workers are picked from sector } j \mid \text{sector } i \text{ is active, and increases } n_i \text{ by one.}) \\
&= \frac{u_j/[1+d(i,j)]}{JC_i}, \\
&= \begin{cases} \frac{u_i}{JC_i}, & \text{if } j = i, \\ \frac{u_j/[1+d(i,j)]}{JC_i}, & \text{if } j \neq i. \end{cases}
\end{aligned} \tag{10}$$

For instance, setting $u_i = 10$ for all $i$ under the ultrametric tree in Figure 1, we calculate $JC_1 = 31.667$. When sector 1 employs one unit of new workers, the probability of picking from sector 1 is 31.58%, while the probability of picking from sector 2 and sector 8 are 15.79% and 7.89% for each.

## 2.5 Prices

While I set prices to be fixed and constant so far, I incorporate dynamics of prices from now on. For the dynamics of prices, I assume the following three conditions. (A1) Firms are price setters in the goods markets, and use the markup pricing strategy, that is, $\forall i,\ p_i = \alpha w_i.$, where $\alpha$ is the markup rate. (A2) Price adjustment in each sectoral labor market occurs as follows: if $n_i$ increase by one, $w_i$ will increase at $\eta_i^+$%, and if $n_i$ decrease by one, $w_i$ will decrease at $\eta_i^-$%. (A3) There is externality in each sectoral labor market, described as follows: the more job candidates exist, the lower $\eta_i^+$ becomes and the higher $\eta_i^-$ becomes, and vice versa.

I assume (A1) only for simplicity, and $\alpha$ is set to be one hereinafter. However, it is worthwhile to note that the markup pricing strategy does not distort behavior of firms in quantity adjustment argued in the preceding section.*[12] This is because the change of $p_i$ along with $w_i$ does not alter the sign of marginal profit.

The assumption (A2) means that labor supply and the wage rate are adjusted given labor demand in the labor market. This is based on the model structure that I mentioned

*[12] Another way, to achieve that price change does not distort the quantity adjustment, is to assume that firms are price takers rather than price setters. But if we assume that the market immediately set all prices to diminish excess demand partially at any rate, it is shown that the macro price level, calculated as is shown later, will hardly move from one. The proof mainly owes a property of $f_i$ as follows: $\sum_i f_i = 0$.

in the previous section, i.e. job creations and destructions are only caused by changes in labor demand. In order to match the labor supply to the labor demand, the wage rate necessarily goes up or down. Namely, (A2) implies that firms cannot hire a new employee without raising the wage rate, and lay off their workers without decreasing the wage rate. In other words, firms are supposed to be price setters in the labor markets, as well as in the goods markets. Note that (A2) actually gives rise to trade-off between unemployment and the price level, but not the inflation.

I also assume by (A2) that only the change in the real wage rate affects labor supply. The rise in the nominal wage rate of one sector, as is described above, brings about the macro inflation, but the inflation does not perfectly negate the rise in real wage in the sector. See Appendix B to know how the real wage rate changes in the model. Thus, labor supply in the sector will increase along with the rise in the real wage rate.

The assumption (A3) reflects tightness in each sectoral labor market. Since labor supply are fixed by the number of the unemployed workers, it is natural that there be a kind of externality concerning to the limit of human resource in the market. Such an externality obviously brings about the situation described as (A3). As argued later in section 3.2, (A3) is the most important assumption in order to produce the Phillips Curve. If there is no externality, the Phillips Curve does not appear in the model.

In the simulations conducted later, I modify a mathematical example of the externality (A3). I set $\eta_i^+$ and $\eta_i^-$ to be the functions of $JC_i$. For simplicity, I restrict the form of the functions to be linear.

$$\begin{aligned} \eta_i^+ &= a^+ - b^+(JC_i/D), \\ \eta_i^- &= a^- + b^-(JC_i/D), \end{aligned} \tag{11}$$

where $D$ is a constant discount factor introduced for technical reason.*13 I arbitrarily set the values of the parameters as follows: $a^+ = 0.024$, $a^- = 0$, and $b^+ = b^- = 0.06$.*14 Then, $\eta_i^+$ is decreasing with respect to $JC_i$, and $\eta_i^-$ is increasing with respect to $JC_i$.

Sectors are exposed to either excess labor demand or excess labor supply. Therefore, inflation occurs in some sectors and deflation occurs in the others. Consequently, in almost all cases, the inflation rate $\eta_i^+$ and the deflation rate $\eta_i^-$ in sectors are asymmetric, as discussed by Tobin (1972). Both rates, $\eta_i^+$ and $\eta_i^-$, happen to be equal with small

*13 The parameter $D$ adjusts $JC_i$ to be less than one, and is calculated as $D = \sum_{j=1}^{K} \frac{100}{1+d(i,j)}$, where $i$ is set to be arbitrarily one. Note that the value of $D$ does not depend on $i$. In other words, $D$ is the number of job candidates when all the population are unemployed, equally distributed for all sectors.

*14 I equate $b^+$ with $b^-$ in order to avoid the situation in which the inflation or the deflation always dominates the other.

likelihood. In the situation, if any, the unemployment rate would be the NAIRU. In (11), for example, the NAIRU[*15] corresponds to the set of $u_i$ that keeps $JC_i/D = \frac{a^+ - a^-}{b^+ + b^-} = 0.2$.

To examine the Phillips Curve, we also have to define the macro price level, as well as the dynamics of prices. In the model, I employ a weighted price index, $p$LI, according to "Laspeyres Index."[*16]

$$p\mathrm{LI}_t = \frac{\sum_i p_{i,t}\ c_i\ n_{i,T_{base}}}{\sum_i p_{i,T_{base}}\ c_i\ n_{i,T_{base}}}, \tag{12}$$

where $T_{base}$ is the base point in time. Note that there are additional subscripts of $p_{i,t}$ and $n_{i,t}$ to indicate the time period. Using this price level, I calculate the inflation rate per 100 time periods, $\pi$, as follows:

$$\pi_t = \frac{p_{i,t+100} - p_{i,t}}{p_{i,t}}. \tag{13}$$

The inflation rate between one time periods fluctuates very quickly, and almost looks like a random variable. This is why I take 100 lags in the calculation above.

# 3 Simulation

## 3.1 The Main Result

According to the dynamics introduced in the previous chapter, I conduct computer simulations because it is hard to take the analytical approach. The difficulty derives from a tremendous amount of states that the model has even around the equilibrium.[*17]

I simulate the model with 8 sectors with 100 members of the population in each sector, that is, $K = 8$ and $N = 800$. Without loss of generality, sectors are arranged in descending order of productivity, so that $c_i \geq c_j$ if $i \leq j$. Moreover, I set $c_i = (K - i + 1)/K$, and then we have $c_1 = 1$, $c_2 = 7/8$, and $c_8 = 1/8$. I use the demand shares, $[s_1, \cdots, s_8] = [6, 5, 4, 3, 2, 2, 2, 2]/26$, for all simulations here. The ultrametric distance across sectors is set as in Figure 1.

In the simulations, we set the initial states to be over-employed states, that is, almost

[*15] Note that the "natural unemployment rate" is not uniquely defined in the model, because it depends on GDP level. See Aoki and Yoshikawa (2003).

[*16] I also use the one like "Purshe Index," but omit it here because it makes only little difference.

[*17] The number of states is approximately calculated as $10^8$, under the assumption that prices are even all fixed and each $n_i$ takes about ten different values around the equilibrium.

all sectors, when they become active, start by firing employees.[*18] After certain periods of time, the number of employees in those sectors becomes small enough to be compatible with the demands for the sectors. This means that the Markov chain enter the closed set of states, from which the model does not escape. See Feller (1968, XV.8).

All assumptions considered, we firstly calculate how the sectoral inputs adjustment occurs, secondly determine wages, and finally set output prices equal to wages, for each time period. Prices are fixed at the initial periods,[*19] and then start to move at 2000 periods later when the model seems to enter the closed set after adequate quantity adjustment. While I compute the model up to 6000 time periods, I discard the first 3000 periods, for the model appears to be in transient states during the early stage. This also means that I set $T_{base} = 3000$ for calculating the price level index.

I take the average of 100 Monte Carlo runs,[*20] and illustrate the result in Figure 2 below. Note that I calculate the inflation rates after taking the average of prices.

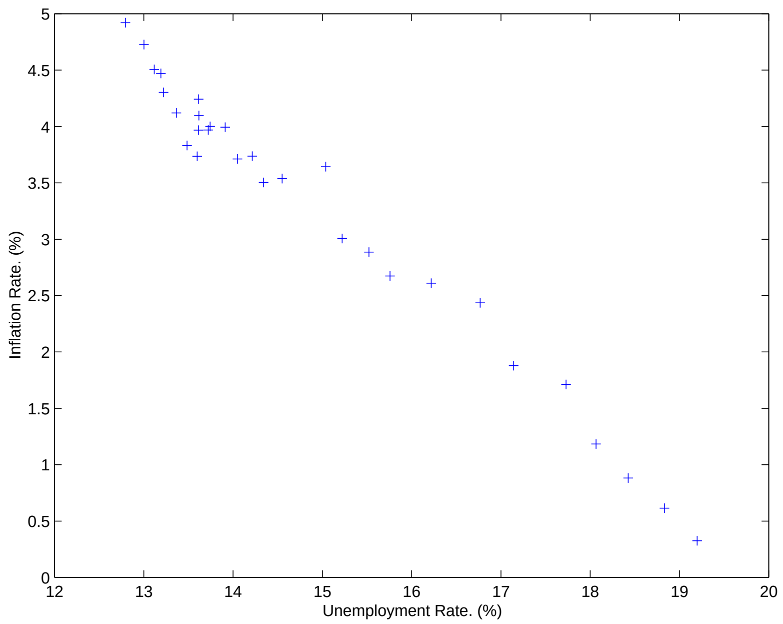


Figure 2 Phillips Curve.

[*18] With the settings above, the initial state that $\forall i,\ n_i(0) = 85$ (thus, $u_i(0) = 15$) satisfies this condition, so I choose it in the simulations.

[*19] This is because changes in prices tend to prevent the model from entering the closed set of states. The phenomenon that changes in prices make the economy unstable is consistent with preceding literature, e.g. Iwai (1981).

[*20] I use a free-licensed program, “Mersenne Twister,” to generate uniform random number tables on [0, 1). “Mersenne Twister” generates high-quality pseudorandom number sequences for Monte-Carlo simulations. See Matsumoto (1998), as to “Mersenne Twister” in details.

## 3.2 Additional cases

I conduct several cases of simulation with various parameters. Here, I show the two additional cases in order to make comparative analysis to the main result.

Firstly, I show that the assumption (A3), which is summarized as externality in each labor market, is essential for the Phillips Curve. In additional case 1, I only remove the setting that represents the assumption (A3) from the model. Without (A3), the rate of change of wage rates is supposed to be constant, and then I set $\eta_i^+ = \eta_i^- = 0.003$. As in Figure 3, the Phillips Curve does not seem to be decreasing. This means that (A3) is a sufficient condition for the descending slope of the Phillips Curve.

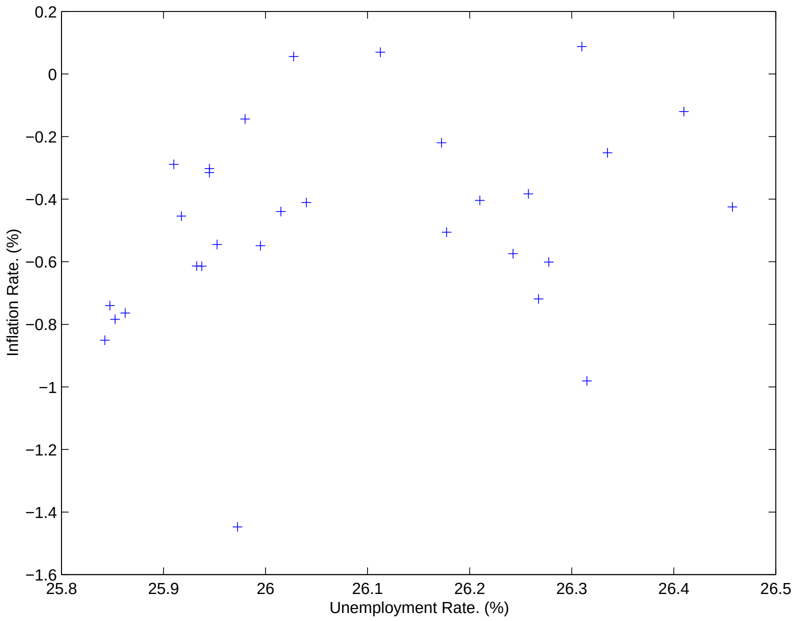


Figure 3 Additional case 1: without externality in each labor market.

Secondly, I argue how the frictions among labor markets affect the Phillips Curve. The sectoral market frictions are described by the ultrametric distance, though it represents similarity across sectors at the same time. I show the case with the longer ultrametric distance, in which the Phillips Curve almost disappears.

In additional case 2, I only change the ultrametric distance to be 1000 times as large as that in the previous cases. The ultrametric tree associated with it is stretched vertically, and have additional 999 nodes for each edge. This means that sectors are not similar to one another at all, and there is little intersectoral movement of labor. In fact, setting

$u_i = 10$ for all $i$ under this stretched ultrametric tree, we calculate $JC_1 = 10.0003$. When sector 1 employs one unit of new workers, the probability of picking from sector 1 is almost equal to one, while the probability of picking from both sector 2 and sector 8 are almost equal to zero.

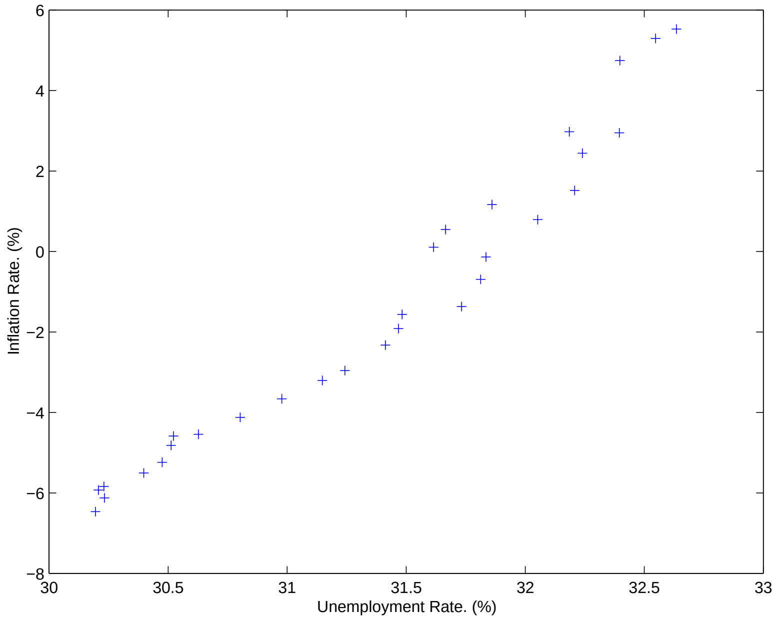


Figure 4 Additional case 2: with highly separated sectors.

In this case, each unemployment pool does not interact with one another, and is consequently independent of the macro labor market, that is, the total unemployment in the economy. In contrast, sectoral wages depend on their own job candidates, which seem to be composed of only the unemployment pool of the sector in this case. It is inferred from this argument that correlation between the total unemployment and inflation would be distorted, or even wiped out, like “cointegration” in terms of time-series analysis.

There seems to be even positive correlation between the total unemployment and inflation in Figure 4. This phenomenon resembles stagflation, although assumptions implemented to the model support the Phillips Curve relationship.

Conversely, the Phillips Curve comes back to life when the ultrametric distance across sectors goes down. In the relatively near distance, each sectoral labor market has an effect on one another, and then they are correlated with the total unemployment. Thus, sectoral wages also correlate with the total unemployment. It follows that the Phillips Curve becomes decreasing again. This means that the descending slope of the Phillips Curve remains after aggregation only if sectors are correlated with one another.

# 4 Concluding remarks

In this paper, I provide a multi-sector stochastic model with the Phillips Curve, as a result of interaction among sectors in disequilibrium. I use the fact that increase in labor supply associated with rise in the nominal wage rate in one sector is not negated immediately by decrease in labor supply in the rest of sectors, which is caused by the fall in real wage rates associated with the rise in the one sector. I attribute the fact to the frictions that both firms and workers face, which prevent the sectors from being active. This is because firm's decisions are totally opposite to worker's ones in the case, and the sectors are not supposed to be active when both of them face such frictions.[*21]

One of the frictions is described as the situation that cost for job switch of workers enables firms to employ at relatively low real wage rates, at which employees cannot make up their mind to quit the jobs. In this sense, neither firms nor workers are possessed by money illusion in the model. Namely, the above-mentioned inertia in quitting jobs can be caused by the frictions between firms and employees, and never disappears in the long run. In this way, we know how essential the stochastic property of the model is.

I also examine how the externality in the labor markets is essential for the descending slope of the Phillips Curve, by simulation analysis in additional case 1. Since I exogenously incorporate the externality, it fairly matters for comparative analysis how the condition is described in the model. The results depend on the functional form and its parameters that represent (A3) to some extent. Moreover, the value of the NAIRU is also determined exogenously. There would be more discussion if, at all possible, the value of the NAIRU depends on, for example, the ultrametric distance. In future work, some endogenous setting may improve robustness of the analysis.

Furthermore, additional case 2 illustrates how the sectoral structure has an effect on the Phillips Curve, in comparison with the longer ultrametric distance. The result is consistent with Hallett (2000), in that spill over effect among sectors is essential for the descending Phillips Curve. It is worthwhile to note that the long ultrametric distance leads to the relationship like stagflation. In other words, even if the Phillips Curve breaks its shape, we can attribute it to some structural changes in the economy, rather than correction of money illusion in the long run. It may somehow explain the break in the Phillips Curve in the 1970s, or so.

[*21] The model implicitly includes worker's decisions in this way.

While Aoki and Yoshikawa (2003) and Hallett (2000) also argue the importance of the demand distribution, I do not show the cases in which the demand distribution or productivity coefficient changes. This is because the arbitrariness in setting the externality (A3) prevents me from such a numerical comparison. The model has to be improved as to a couple of things for further analysis, but I hope that the model helps research further in the sectoral structure and its influence on the macro economy.

# Appendix A. Ultrametric distance

In Graph Theory, they define “tree distance” among terminal nodes of a hierarchical tree. Terminal nodes are defined by the nodes connected with only one edge, except for the root of the tree.

Definition A *tree distance* $d(i, j)$ associated with any two terminal nodes $i$ and $j$ of a hierarchical tree $T$ is defined by the number of the minimum nodes needed to ascent towards the root of the tree to move from $i$ to $j$.

Then, we have the definition of the ultrametric distance.

Definition A *tree distance* $d(i, j)$ with a tree $T$ is called *ultrametric distance*, (and $T$ is called *ultrametric tree*) if it satisfies the following:

(I) (symmetricity) $\forall i,\ j,\ d(i, j) = d(j, i)$,

(II) (transitivity) $\forall i,\ j,\ d(i, j) \leq \max_k\{d(i, k), d(k, j)\}$.

The second condition in the definition above is a kind of extension of the triangle inequality. See Aoki (2003), pp.81, to know why it is called “transitivity.” Note that even if all values of $d(i, j)$ are multiplied by a positive number at once, it keeps to satisfy the two properties above.

The ultrametric distance used in Figure 1. is described by the symmetric matrix $M$, whose $(i, j)$ element responds to $d(i, j)$.

$$M = \begin{bmatrix} 0 & 1 & 2 & 2 & 3 & 3 & 3 & 3 \\ 1 & 0 & 2 & 2 & 3 & 3 & 3 & 3 \\ 2 & 2 & 0 & 1 & 3 & 3 & 3 & 3 \\ 2 & 2 & 1 & 0 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 0 & 1 & 2 & 2 \\ 3 & 3 & 3 & 3 & 1 & 0 & 2 & 2 \\ 3 & 3 & 3 & 3 & 2 & 2 & 0 & 1 \\ 3 & 3 & 3 & 3 & 2 & 2 & 1 & 0 \end{bmatrix}. \tag{14}$$

# Appendix B. The Change in the Real Wage Rate

In the model, the change of the real wage rate only causes the change in the labor supply in the sector. Namely, the increase in the labor supply in sector $i$ is always followed by the rise in the real wage rate $w_i/p\mathrm{LI}$. In the other way around, the decline in the labor supply in sector $i$ is followed by the rise in $w_i/p\mathrm{LI}$. Here, I only show the former case because of the symmetricity of the argument.

When the labor supply increases, i.e. $u_i$ decreases by one, there will be $\eta_i^+$ percentage increase in the wage rate in sector $i$.

$$w_{i,t+1} = w_{i,t}(1+\eta_i^+). \tag{15}$$

From this equation, we have:

$$\begin{aligned}\eta_i^+ &= \frac{w_{i,t+1}-w_{i,t}}{w_{i,t}} \\ &= \frac{p_{i,t+1}-p_{i,t}}{p_{i,t}} \equiv \frac{\Delta p_{i,t}}{p_{i,t}},\end{aligned} \tag{16}$$

where $\Delta p_{i,t} \equiv p_{i,t+1} - p_{i,t}$. The second equality derives from the assumption of markup pricing, i.e. (A1).

In this case, the percentage increase of the price index, $pLI$, is calculated as follows:

$$\begin{aligned}\frac{\Delta p\mathrm{LI}_t}{p\mathrm{LI}_t} &= \frac{\frac{\Delta p_{i,t}\ c_i\ n_{i,T_{base}}}{\sum_i p_{i,T_{base}}\ c_i\ n_{i,T_{base}}}}{\frac{\sum_i p_{i,t}\ c_i\ n_{i,T_{base}}}{\sum_i p_{i,T_{base}}\ c_i\ n_{i,T_{base}}}} \\ &= \frac{\Delta p_{i,t}\ c_i\ n_{i,T_{base}}}{\sum_i p_{i,t}\ c_i\ n_{i,T_{base}}} \\ &= \eta_i^+ \ \frac{p_{i,t}\ c_i\ n_{i,T_{base}}}{\sum_i p_{i,t}\ c_i\ n_{i,T_{base}}} \\ &\simeq s_i\eta_i^+,\end{aligned} \tag{17}$$

where $\Delta p\mathrm{LI}_{i,t} \equiv p\mathrm{LI}_{i,t+1} - p\mathrm{LI}_{i,t}$. Since the economy fluctuates near the equilibrium, I use the equilibrium condition (5) for the approximation. This means that there will be $s_i\eta_i^+$ percentage increase in the price level.

Consequently, the real wage rate in sector $i$, $w_i/p\mathrm{LI}$, will be multiplied by $\frac{1+\eta^+}{1+s_i\eta^+}$, which is greater than 1. Namely, the rise in the nominal wage rate in one sector is not perfectly negated by that in the macro price level. This implies that additional labor supply occurs only if the real wage rate increases in the sector.

# Appendix C. Program flowchart

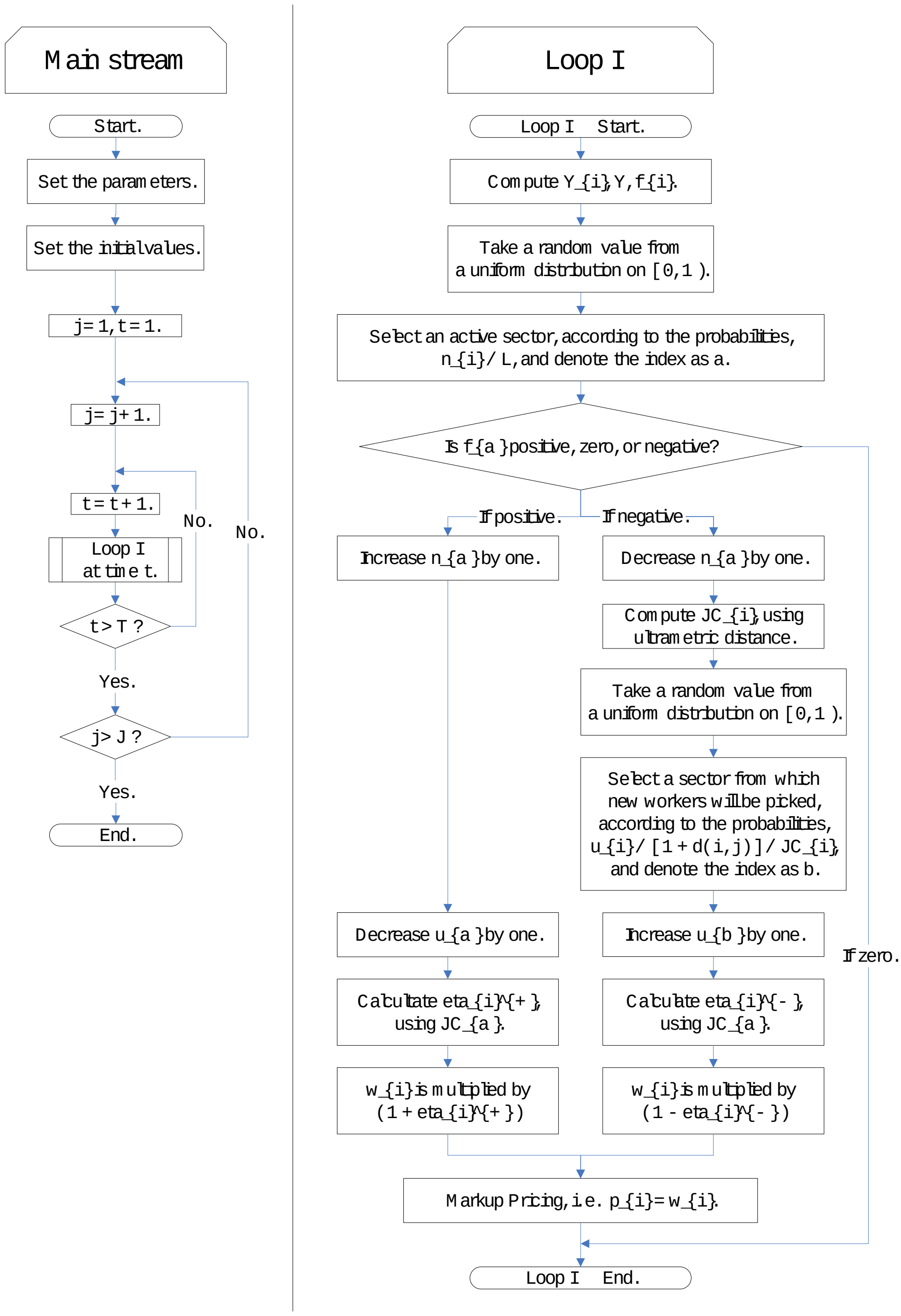


Figure 5 Program flowchart.

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