# Grants or Concessional Loans? <br> Aid to Low-Income Countries with a Participation Constraint 

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#### Abstract

This paper provides a theoretical framework to compare grants with concessional loans to low-income countries (LICs). Its main focus is on aid-dependency and debt sustainability. The issue of aid-dependency is important because, as the paper shows, standard growth regressions suggest that growth rates tend to stagnate around the threshold between low and lower-middle income countries. The model suggests that this stagnation may be due to the existence of an income level above which the country is ineligible for concessional loans or grants (which is referred to as the cutoff). The paper endogenizes debt sustainability by imposing a participation constraint-a constraint that ensures that the LIC recipient honors the aid contracts. The key result of the paper is that optimal concessional lending is better than its grant counterpart, provided that the perverse effects of concessional loans, namely over-borrowing and over-spending in the earlier periods due to the low concessional interest rate and the existence of the cutoff, are small enough. When the perverse effects are large enough, the country becomes permanently aid-dependent and remains at the cutoff, depending on the country's total factor productivity (TFP) level and initial conditions.


JEL Classification Codes: E21, F34, F35, F43, O16, O21
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[^0]
## 1 Introduction

In recent years, the effectiveness of grants and concessional loans has been widely discussed in financial and political circles ${ }^{2}$. A persuasive pro-grant argument is that grants do not add debt burdens to low-income countries (LICs). On the other hand, a strong pro-loan argument is that concessional loans can provide LICs with more funds than grants, given the same donor's budget across different aid schemes ${ }^{3}$. Unfortunately, there are not enough theoretical frameworks to think about these issues. This paper provides a theoretical framework to answer the question of whether low-income countries should be given concessional loans or grants, paying special attention to aid-dependency and debt sustainability problems.

We are concerned about aid-dependency because in standard growth regressions, we find that growth rates tend to stagnate around the threshold between low and lower-middle income countries (see section 2). Our model suggests that this stagnation may be due to the existence of a cutoff, the level of income above which countries lose their eligibility for aid. In particular, optimal concessional lending may motivate the recipient countries to permanently remain at the cutoff. This empirical finding is consistent with the facts that LICs receive significant amount of aid and that concessional loans comprise a large share of debt in these countries (Figure $1^{4}$ ).

Since the 1970s, LICs have suffered from costly debt crises followed by debt relief. Thus we would like to design an aid scheme that endogenizes debt sustainability. One approach is to impose a participation constraint-a constraint that motivates the LIC recipient to adhere to the aid contracts. In the absence of a participation constraint, the country can borrow more than it is willing to repay and therefore has an incentive to violate its contracts.

[^1]Figure 1
(a) Aid per capita (current US\$)


Source: World Development Indicators
(b) Concessional debt/Total debt (\%)


Source: Global Development Finance

In order to analytically compare grants with concessional loans, we introduce another scheme that generalizes them-transfers. Transfers are a sequence of lump-sump payments that can be positive or negative. I treat the optimal transfer scheme as a theoretical benchmark and compare it with two other schemes, which are arguably more realistic aid schemes. Grants, non-negative lump-sump payments, are a special case of transfers. Concessional loans in this paper are defined as subsidized loans at a low and fixed interest rate with certain eligibility criteria. Our concessional lending scheme is another special case of transfers (for more on the concessional interest rate rule, see section 3 ).

The most important assumptions of the paper are as follows. First, we focus on the role of benevolent donors, who are assumed to care purely about the welfare of LICs. Second, we assume that LIC governments maximize the welfare of households. This paper thus focuses on those LICs with good policy performance ${ }^{5}$. Some may think that we should instead consider "bad" governments who care about their own welfare, rather than that of households. An important point of the paper, however, is that even with good policies, the poverty trap may occur. Third, we mainly consider cases where a LIC is in financial autarchy, where the country has no access to private foreign financing, but has access to official aid. This is because official lenders make up the majority of aid in LICs and many of these

[^2]countries have little access to private foreign lending (figure 2).

Figure 2 Composition of Debt (\%)


## Source: Global Development Finance

The key result of the paper is that optimal concessional lending is better than its grant counterpart, provided that the perverse effects of concessional loans-over-borrowing and over-spending in the earlier periods due to the low concessional interest rate and the existence of a cutoff-are sufficiently small. When the perverse effects are large enough, the country becomes permanently aid-dependent and stays at the cutoff, depending on the country's total factor productivity (TFP) level and initial conditions. With regard to the optimal grant sequence (a one-time gift in period 1) a poverty trap does not result. Even though, in reality, it may be difficult for a donor to commit itself to such a grant sequence.

This paper is related to foreign aid, sovereign debt, and growth literature. First, our question, a comparison between different aid schemes in various environments, is a fundamental topic in aid literature. For example, there have been numerous debates over conditional versus unconditional aid ${ }^{6}$, and good versus poor policy environments ${ }^{7}$. However, there has been a lag in the development of theoretical literature to address the benefits of grants versus concessional loans (see for a recent example, Cordella and Ulku (2004)). This paper provides a very special dynamic contracting model where both donor and recipient maximize the welfare of households, but only the donor can commit itself to the contracts (for more description see section 3). Thus our model is different from those based on contract theory with asymmetric

[^3]information or with different objective functions among the agents. Second, this paper considers debt sustainability-an important topic in the sovereign debt literature (see for example Sachs and Cohen (1986) and a series of papers by Rogoff and Bulow). We endogenize debt sustainability by imposing a participation constraint. Furthermore, this paper gives a formal description of debt-overhang (see section 6.3.3). Our model suggests that countries with heavier initial debts tend to stagnate around the cutoff, because they are then able to manage their debts in the form of subsidized loans. This description of debt-overhang differs from those of the previous literature ${ }^{8}$. Third, we use a neoclassical growth model where we assume that the initial level of capital is below the steady state level. The combination of the three types of literature enables us to examine the role of each aid scheme in promoting growth along a sustainable debt path.

The structure of the paper is as follows. In the following section we document our empirical motivation and in section 3, we describe the features of the model. In section 4, 5, and 6, we analyze transfers, grants, and concessional loans, respectively, in the cases where the LIC is in financial autarchy but has access to foreign aid. We consider the environment where the country has access to foreign private financing in addition to foreign aid in section 7. And finally in section 8 we evaluate welfare implications.

## 2 Empirical Motivation

We start off by empirically documenting our motivation issues that growth rates tend to stagnate around the cutoff between low and lower-middle income countries. We consider the periods between 1980-2003 and run standard growth regressions for available countries. Our data from the World Bank's Global Development Finance and Summers-Heston consists of 92 countries in 1981 and 105 countries in $2001^{9}$. We do not include the 1970s when many LICs had accumulated significant amount of non-concessional

[^4]debts (thus the share of concessional loans had decreased in the 70s, see figure 1 (b)). Our dependent variable is the growth rate of Gross National Incomes (GNI) ${ }^{10}$ per capita (GNI, in 100 US dollars). The explanatory variables are those typically included in a standard growth regression (see Levine and Renelt (1992)): the average annual rate of population growth (GPO), the investment share of GDP (INV), the initial secondary-school enrollment rate (SEC80), and the initial level of real GDP per capita in 1980 from SH (RGDP80). In addition to these variables, we include the variable of interest, a measure of proximity to the cutoff (GNI per capita below which the country is categorized as a LIC) in the form of a Bartlett kernel:
\[

\operatorname{PROX}_{i t}=\left\{$$
\begin{array}{cc}
1-|x| & \text { for }|x| \leqq 1 \\
0 & \text { for }|x|>1
\end{array}
$$ \quad where x=\frac{\ln y_{i t}-\ln \bar{y}_{t}}{\ln (1+b)}\right.
\]

where $y_{i t}$ is the country $i$ 's GNI per capita in year $t, \bar{y}_{t}$ is the cutoff in year $t$ (in 100 US dollars), and $b$ is a scaling factor which controls the bandwidth of the kernel. We report cases where $b=1 / 2,1 / 3$, and $1 / 4$.

Table 1 (a) shows that the coefficient for PROX is positive and statistically significant at the level of significance 0.05 . When we consider country specific fixed effects ${ }^{11}$ the corresponding $t$-value goes down to -1.57 but we believe that this value is satisfactory for fixed effect estimation. We obtain similar results in the case where $b=1 / 3$ or $1 / 4$. This empirical evidence is consistent with our paper's theoretical result-the existence of a cutoff may result in economic stagnation around it. We obtain even higher $t$-values for the variable of interest when we drop countries which are considerably above the cutoff (see Appendix A).

[^5]Table 1 Dependent Variable: Growth Rate of Per Capita GNI, 1980-2003 (a) $\mathrm{b}=1 / 2$

|  | (a) OLS |  | (b) Fixed Effects |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Regressors | $\beta$ | $\mathrm{se}^{*}$ | $t$ | $\beta$ | se | $t$ |
| PROX | -0.0221 | 0.01042 | -2.13 | -0.0217 | 0.01379 | -1.57 |
| GPO | -0.0064 | 0.00176 | -3.66 | -0.0044 | 0.00208 | -2.10 |
| INV | 0.0011 | 0.00030 | 3.61 | $-6.07 \mathrm{e}-05$ | 0.00050 | -0.12 |
| SEC80 | 0.0003 | 0.00013 | 2.10 | - | - | - |
| RGDP80 | $3.17 \mathrm{e}-08$ | $6.57 \mathrm{e}-07$ | 0.05 | - | - | - |
| constant | 0.0023 | 0.00873 | 0.27 | 0.0359 | 0.01142 | 3.14 |

* standard errors
(b) $b=1 / 3$

|  | (a) OLS |  |  | (b) Fixed Effects |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Regressors | $\beta$ | se | $t$ | $\beta$ | se | $t$ |
| PROX | -0.0231 | 0.01216 | -1.90 | -0.0180 | 0.01505 | -1.20 |
| GPO | -0.0065 | 0.00176 | -3.67 | -0.0044 | 0.00208 | -2.10 |
| INV | 0.0011 | 0.00031 | 3.63 | $-4.28 \mathrm{e}-05$ | 0.00050 | -0.09 |
| SEC80 | 0.0003 | 0.00013 | 2.05 | - | - | - |
| RGDP80 | $1.17 \mathrm{e}-07$ | $6.52 \mathrm{e}-07$ | 0.18 | - | - | - |
| constant | 0.0014 | 0.00870 | 0.16 | 0.0348 | 0.01139 | 3.05 |

(c) $\mathrm{b}=1 / 4$

|  | (a) OLS |  | (b) Fixed Effects |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Regressors | $\beta$ | se | $t$ | $\beta$ | se | $t$ |
| PROX | -0.0274 | 0.01371 | -2.00 | -0.0208 | 0.01629 | -1.28 |
| GPO | -0.0065 | 0.00176 | -3.68 | -0.0044 | 0.00208 | -2.10 |
| INV | 0.0011 | 0.00031 | 3.68 | $-3.01 \mathrm{e}-05$ | 0.00050 | -0.06 |
| SEC80 | 0.0003 | 0.00013 | 2.01 | - | - | - |
| RGDP80 | $1.49 \mathrm{e}-07$ | $6.49 \mathrm{e}-07$ | 0.23 | - | - | - |
| constant | 0.0010 | 0.00868 | 0.11 | 0.0344 | 0.01138 | 3.02 |

## 3 The Features of the Model

In sections 4-6, we consider two types of agents, a benevolent donor and a sovereign LIC recipient. Our model is a very special dynamic contracting model because both agents maximize the welfare of the households but only the donor can commit to the contracts. The donor perfectly foresees the recipient's behavior and is thus able to determine capital and consumption paths in the recipient country by offering an aid scheme with a participation constraint. In section 7, we consider private foreign lenders in addition to these agents. The private foreign lenders loan funds to the LIC at the world interest rate, $r$. We assume that (i) there is only one good in the economy, (ii) the recipient's lifetime utility is given by:

$$
\begin{equation*}
\sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right) \tag{1}
\end{equation*}
$$

where $C$ denotes consumption and $\beta$ is the rate of time preference, and (iii) the LIC's initial capital level is lower than the steady state level.

Official creditors typically fix their concessional interest rates; for example, the rates for the World Bank's International Development Association (IDA) and for the International Monetary Fund are $0.75 \%$ and $0.5 \%$ respectively. In principle, only LICs are eligible for such concessional loans. We thus set our concessional lending rule as follows: the benevolent lender lends at a fixed concessional interest rate as long as the country's GNI per capita is below a certain level. The interest rates for concessional lending are set according to the following rule:

$$
\widetilde{r}_{t+1}= \begin{cases}\bar{r} & \text { if } Y_{t} \leq \bar{y}  \tag{2}\\ r & \text { otherwise }\end{cases}
$$

where $\bar{y}$ is the cutoff, $r$ is the world interest rate ${ }^{12}, \bar{r}$ is the concessional interest rate, $Y_{t}$ is the GNI per capita in the LIC. The production function

[^6]is given by $Y_{t}=f\left(K_{t}\right)$ where $K_{t}$ is capital per capita in period $t$.
The LIC's flow budget constraint is given by:
\[

$$
\begin{equation*}
\tau_{t}=C_{t}+I_{t}-f\left(K_{t}\right) \tag{3}
\end{equation*}
$$

\]

where $\tau_{t}$ is the transfer in period $t$ (positive or negative) and $I_{t}$ is the investment in period $t$. The transition equation for capital is given as $K_{t+1}=I_{t}+(1-\delta) K_{t}$, where $\delta$ is the rate of capital depreciation. Under the grant scheme, $\tau_{t} \geq 0$ for all $t^{13}$. Under the concessional lending scheme, the transfer in period $t$ is the amount of concessional debt the country accumulates minus the interest rate payments in period $t, \tau_{t}=\left(\widetilde{D}_{t+1}-\widetilde{D}_{t}\right)-\widetilde{r}_{t} \widetilde{D}_{t}$, where $\widetilde{D_{t}}$ is the debt in period $t$ that the LIC owes to the benevolent lender, and $\widetilde{r_{t}}$ is given by (2). This is a special case of transfers where the country has only two different levels of interest rates, $\bar{r}$ and $r$.

## 4 Transfers

In this and following two sections we consider the environment in which the country is in financial autarchy but has access to foreign aid. We are interested in this environment because official lenders provide the majority of loans in LICs and many of these countries have little access to private foreign lending (figure 2). In this section, we solve for the optimal transfer scheme with and without a participation constraint. Transfers always achieve the most efficient allocation in each environment because they are in a more generalized form than the other two schemes, grants and concessional loans.

We solve the optimal sequence of transfers given the donor's budget, $\alpha$. The value of $\alpha$ is determined by solving the optimal concessional lending scheme in section 6 . In this way, we can preserve the donor's budget across different schemes.

[^7]
### 4.1 No Participation Constraint: the first best allocation

In this subsection we assume that the LIC fully precommits itself to honoring the conditions of the aid scheme that is imposed by the benevolent donor so that there is no need to impose a participation constraint. This is an unrealistic assumption because it allows the LIC to have unlimited access to the donor's funds so the LIC reaches its steady state instantly. In reality, the donor's budget is limited and thus the results of this subsection are not realistic. Analyzing this no-participation constraint environment is still useful as a means of comparing different aid schemes.

The transfer problem is given by:

$$
\max _{C_{t}, K_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right)
$$

subject to the flow budget constraint and the donor's budget:

$$
\begin{aligned}
C_{t} & =f\left(K_{t}\right)+(1-\delta) K_{t}-K_{t+1}+\tau_{t} \\
\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} \tau_{t} & \leq \bar{T}
\end{aligned}
$$

where $\bar{T}$ is the net present value of transfers from the donor to the recipient. Note that if the country is initially bound to fulfill repayments to the donor ${ }^{14}$, then $\bar{T}$ can be negative, even if the country receives grants. Let $\alpha$ equal the donor's budget, found by solving for the optimal concessional lending scheme with no participation constraint (given by eq (27)). Thus $\bar{T}=\alpha-L$, where $L$ is the initial liability that the recipient owes to the donor. $K_{1}$, the initial capital stock, is given. The Lagrangian is as follows:

$$
\begin{aligned}
L= & \sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right) \\
& +\mu\left[\bar{T}-\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}\left(C_{t}+K_{t+1}-(1-\delta) K_{t}-f\left(K_{t}\right)\right)\right]
\end{aligned}
$$

[^8]The FOCs are given by:

$$
\begin{align*}
C_{t} & : u^{\prime}\left(C_{t}\right)=\mu  \tag{4}\\
K_{t+1} & : r=f^{\prime}\left(K_{t+1}\right)-\delta \tag{5}
\end{align*}
$$

Equation (4) implies that consumption is constant over time. (5) indicates that capital is constant from period 2 onward. In other words, the country receives a very large transfer and jumps to the steady state in period 1 . If $\alpha$ is not sufficiently large (which is a realistic assumption given a limited donor's budget), then negative transfers in subsequent periods must compensate for the transfer in period 1. The optimal path of transfers is thus a one-time transfer followed by constant negative transfers (i.e. $\left\{\tau_{1}^{*}, \tau_{s s}, \tau_{s s}, \ldots.\right\}$ where $\tau_{1}^{*}$ is positive and $\tau_{s s}$ is negative). This is the first best allocation. Note that this path is still optimal even if we impose an additional rule that transfers cannot be positive above a certain level of income (i.e. a cutoff, $\bar{y}$ ). We can pin down $\tau_{1}^{*}$ and $\tau_{s s}$ by combining the flow budget constraints in the first two periods (eq (6)) and the donor's budget equation (eq (7)):

$$
\begin{align*}
f\left(K_{s s}\right)-\delta K_{s s}+\tau_{s s} & =f\left(K_{1}\right)+(1-\delta) K_{1}-K_{s s}+\tau_{1}^{*}  \tag{6}\\
\tau_{1}^{*}+\frac{\tau_{s s}}{r} & =\bar{T}, \quad \tau_{1}^{*}>0, \tau_{s s}<0 \tag{7}
\end{align*}
$$

### 4.2 Participation Constraint: the constrained optimum

In the previous subsection, we did not impose any costs of defaulting and thus the LIC borrower had an incentive to default in period 2. In this section, we introduce a participation constraint to motivate the LIC to adhere to the contracts. Under the constraint, the value function under the aid scheme is always greater than or equal to the value function under the violation of the aid scheme.

Consider a LIC recipient which receives a positive or negative transfer in each period with the net present value of transfers from the donor to recipient defined as $\bar{T}$. Again, $\bar{T}=\alpha-L$ where $\alpha$ is the donor's budget, this time implied by solving for the optimal concessional lending with a participation constraint (given by eq (31)). $L$ is the initial liability that the recipient owes
to the donor. The LIC's flow budget constraint is given by (3). The benevolent donor has full access to world financial markets. In this environment, the benevolent donor chooses the optimal paths of consumption and capital. The donor's problem with a participation constraint (9) is formally given by:

$$
\begin{equation*}
\max _{\left\{C_{t}, K_{t+1}\right\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right) \tag{8}
\end{equation*}
$$

subject to:

$$
\begin{align*}
u\left(C_{t}\right)+\beta \sum_{j=1}^{\infty} \beta^{j-1} u\left(C_{t+j}\right) & \geq v^{A}\left(K_{t}\right), \forall t  \tag{9}\\
\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} \tau_{t} & \leq \bar{T}  \tag{10}\\
\tau_{t} & =C_{t}+I_{t}-f\left(K_{t}\right)  \tag{11}\\
K_{t+1} & =(1-\delta) K_{t}+I_{t} \tag{12}
\end{align*}
$$

where $K_{1}$ and $\bar{T}$ are given. The value function under default, $v^{A}($.$) , is$ the value function in autarchy with penalties for violating the participation constraint:

$$
v^{A}(K)=\max _{K^{\prime}}\left\{\begin{array}{c}
u\left((1-\lambda) f(K)-K^{\prime}+(1-\delta) K\right)  \tag{13}\\
+\beta v^{A}\left(K^{\prime}\right)
\end{array}\right\}
$$

where $\lambda$ is the fraction of output lost. We assume that such a violation incurs two types of costs: the exclusion of the violator from future aid and the loss of a fraction of the violator's output. We also assume that when the participation constraint is binding, the LIC adheres to the aid contract.

In order to solve this sequential problem, i.e. Eq. (8) subject to Eqs. (9) - (12), we write the problem recursively. Define $T_{t}$ as the NPV of transfers from the donor to the recipient, denoted in period $t$ value:

$$
\begin{equation*}
T_{t}=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \tau_{s} \tag{14}
\end{equation*}
$$

where the difference between $T_{t}$ and $\frac{T_{t+1}}{1+r}$ is equal to the transfer in period $t$, or equivalently the excess demand in period $t$ :

$$
\tau_{t}=T_{t}-\frac{T_{t+1}}{1+r}=C_{t}+I_{t}-f\left(K_{t}\right)
$$

The recursive formulation is given by:

$$
\begin{equation*}
v(K, T)=\max _{C, I} u(C)+\beta v\left(K^{\prime}, T^{\prime}\right) \tag{15}
\end{equation*}
$$

subject to:

$$
\begin{align*}
v\left(K^{\prime}, T^{\prime}\right) & \geq v^{A}\left(K^{\prime}\right)  \tag{16}\\
K^{\prime} & =(1-\delta) K+I  \tag{17}\\
T^{\prime} & =(1+r)[T+f(K)-C-I]  \tag{18}\\
T^{\prime} & \geq-B \quad B \text { is finite }  \tag{19}\\
T_{1} & \leq \bar{T} \tag{20}
\end{align*}
$$

where $K_{1}$ are $T_{1}$ are given. (16) is the participation constraint, and (17) and (18) are the transition equations for $K$ and $T$, respectively. $-B$ is the lower bound for $T^{\prime}$. If $B$ is large enough (so that $-B$ is a large negative number), then the optimal solution will never violate (19) and we can exclude Ponzi games.

### 4.3 Numerical Results

Since we cannot solve this problem analytically unless the participation constraint is absent, we solve it numerically using the value function iteration method. Here, regardless of the starting function, we obtain the same fixed point in the functional space. We specify the functional forms of the utility and production functions as $u(C)=\frac{C^{1-\frac{1}{\sigma}-1}}{1-\frac{1}{\sigma}}$ and $f(K)=A K^{\eta}$. We set $\eta=1 / 3$ and use Ostry and Reinhart's (1992) calibration results for "African" countries for the values of the elasticity of intertemporal substitution ( $\sigma=0.451$ ) and the discount factor $(\beta=0.945)$. We set the world interest rate $(r)$ at $1 / \beta-1 \simeq 5.82 \%$ in order for consumption in the steady state to be flat.

The benchmark numerical example As the benchmark example, consider the country with initial income $\left(Y_{1}\right)$ equal to $70 \%$ of steady state output $\left(Y_{s s}\right)$, initial liability-output ratio $\left(L / Y_{1}\right)$ of 1 , and level of TFP $(A)$ equal to 10 . Figure 3 shows the optimal path of transfers which is generated by the country's drive to achieve the first best allocation in the presence of a participation constraint. Since we assume that initial capital is lower than the steady state, the marginal product of capital is initially higher, so a large transfer in period 1 is preferable. When the donor's budget is not very large, the country has to compensate with negative transfers from period 2 onward. Again, note that this path is still optimal even if we impose an additional rule that transfers cannot be positive above a cutoff $(\bar{y})$.

Figure 3 Transfers (in fractions of $Y_{s s}$ )


## 5 Grants

In this section, we continue to consider the environment where the LIC is in financial autarchy but has access to foreign aid.

### 5.1 The optimal sequence of grants

We do not consider grant schemes with participation constraints because a country has no incentive to violate grant contracts since there are no repayment obligations. The optimal grant scheme is thus the same with or without a participation constraint.

A grant scheme is a special case of transfers where transfers are nonnegative $\left(\tau_{t} \geq 0\right)$. The donor' budget $(\alpha)$ is the same as in the transfer scheme case. We assume that the country makes interest payment of the initial liability $(L)$ to the donor in every period. Here, the optimal path of grants is a single grant of $\alpha$ in period 1 (i.e. $\left\{\tau_{t}\right\}_{t=0}^{\infty}=\{\alpha, 0,0,0 \ldots\}$ ). This path can be derived from the Euler equation and flow budget constraint:

$$
\begin{align*}
u^{\prime}\left(C_{t}\right) & =\beta u^{\prime}\left(C_{t+1}\right)\left[f^{\prime}\left(K_{t+1}\right)+1-\delta\right]  \tag{21}\\
C_{t} & =f\left(K_{t}\right)+(1-\delta) K_{t}-K_{t+1}-r L+\tau_{t}  \tag{22}\\
\tau_{t} & \geq 0
\end{align*}
$$

From (21), as long as $K_{t+1}<K_{s s}$, consumption is increasing ( $C_{t}<C_{t+1}$ ) and marginal utility is decreasing $\left(u^{\prime}\left(C_{t}\right)>u^{\prime}\left(C_{t+1}\right)\right)$ over time. The country stays below the steady state level in early periods if $\alpha$ is not very large. Below the steady state level, the country could increase its utility by reducing the future grant by one unit and increasing the present grant by the appropriate amount to preserve the total grant amount. This is because the marginal productivity of capital is greater in the present period. This alteration, however, is not possible because future grants are already at their minimum value, zero. Note that the optimal grant sequence is unchanged even if we impose an additional rule that grants must be zero above a certain level of income (i.e. cutoff, $\bar{y}$ ).

We numerically solve for the optimal grant scheme. Here we let the economy converge to the steady state within T periods and solve the path of consumption and capital, i.e. $\left\{C_{1}, \ldots, C_{T}\right\}$ and $\left\{K_{2}, \ldots, K_{T+1}\right\}$ using flow budget constraints (eq.(22)) and the Euler equations (eq.(21)), $\forall t=1,2, \ldots, T$, given the initial level of capital, $K_{1}$ and the path of grants. The numerical results are in section 6 .

### 5.2 A constant grant scheme

The optimal grant path, a one-time grant of $\alpha$ in period 1, may seem like an unrealistic assumption with respect to donor commitment. It may be more practical to consider a grant scheme that gives a fixed amount of grant $(\bar{g})$ every period as long as the GNI per capita is less than or equal to the cutoff.

$$
\tau_{t}=\left\{\begin{array}{c}
\bar{g} \quad \text { if } Y_{t} \leq \bar{y} \\
0 \quad \text { otherwise }
\end{array}\right.
$$

We use a numerical algorithm as in the previous subsection and solve this fixed-amount grant problem. We find that generally there exists a level of $\bar{g}$ above which the country is trapped at the cutoff. For example, in the benchmark example in section 4.3, the corresponding level of $\bar{g}$ is only $1.9 \%$ of the cutoff. In that example, this type of grant scheme can easily cause a poverty trap.

## 6 Concessional Loans

In this section, we again consider LICs that are financial autarchic but have access to concessional loans. We solve for the optimal lending scheme with and without a participation constraint. The flow budget constraint is given by eq (3) where the transfer in period $t$ has a specific form:

$$
\begin{equation*}
\tau_{t}=\left(\widetilde{D}_{t+1}-\widetilde{D}_{t}\right)-\widetilde{r}_{t} \widetilde{D}_{t} \tag{23}
\end{equation*}
$$

and the concessional interest rate rule is exogenously given by eq (2). Under this rule, there are only two levels of interest rates available ( $\bar{r}$ and $r$ ). Thus our concessional lending scheme is a special case of transfers. The donor's budget $(\alpha)$ is implied by solving the optimal concessional lending scheme. In order to preserve the LIC's initial liability to the donor across different schemes, we set $L=\left(1+\widetilde{r_{1}}\right) \widetilde{D}_{1}$, where $\widetilde{D}_{1}$ and $\widetilde{r_{1}}$ are the initial concessional debt and concessional interest rate, respectively.

### 6.1 No Participation Constraint

The concessional lending problem without a participation constraint is as follows:

$$
\max _{C_{t}, K_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right)
$$

subject to the intertemporal budget constraint:

$$
\begin{aligned}
& f\left(K_{1}\right)+(1-\delta) K_{1}+\sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+\widetilde{r_{s}}}\right)\left(f\left(K_{t}\right)+(1-\delta) K_{t}\right)\right) \\
= & \left(1+\widetilde{r_{1}}\right) \widetilde{D}_{1}+C_{1}+K_{2}+\sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+\widetilde{r_{s}}}\right)\left(C_{t}+K_{t+1}\right)\right)
\end{aligned}
$$

where $K_{1}, \widetilde{D}_{1}$, and $\widetilde{r_{1}}$ are given. The Lagrangian is given by:

$$
\begin{aligned}
L= & \sum_{t=1}^{\infty} \beta^{t-1} u\left(C_{t}\right) \\
& +\mu\left[\begin{array}{c}
f\left(K_{1}\right)+(1-\delta) K_{1}+\sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+r_{s}}\right)\left(f\left(K_{t}\right)+(1-\delta) K_{t}\right)\right) \\
-\left(1+\widetilde{r_{1}}\right) \widetilde{D}_{1}-C_{1}-K_{2}-\sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+r_{s}}\right)\left(C_{t}+K_{t+1}\right)\right)
\end{array}\right]
\end{aligned}
$$

FOCs are given by:

$$
\begin{align*}
u^{\prime}\left(C_{t}\right) & =\mu, \quad \text { for } t=1  \tag{24}\\
\beta^{t-1} u^{\prime}\left(C_{t}\right) & =\mu \prod_{s=2}^{t}\left(\frac{1}{1+\widetilde{r_{s}}}\right), \quad \text { for } t \geq 2  \tag{25}\\
\widetilde{r}_{t+1} & =f^{\prime}\left(K_{t+1}\right)-\delta, \quad \text { for } t \geq 1 \tag{26}
\end{align*}
$$

Initially, the country can borrow at the concessional interest rate (i.e. $\widetilde{r_{2}}=\bar{r}$ ) because we assume that initial output lies below the cutoff. At any level of capital above the cutoff, the country can borrow only at the world interest rate. The capital levels in period 2 and in the steady state, $K_{2}$ and $K_{s s}$, are pinned down by $\bar{r}=f^{\prime}\left(K_{2}\right)-\delta$ and $r=f^{\prime}\left(K_{s s}\right)-\delta$ (by (26)). These equations imply that $K_{2}$ is greater than $K_{s s}$ because the concessional interest rate is lower than the world interest rate $(\bar{r}<r)$. Thus capital overshoots steady state in period 1. However, from period 3 onwards, capital is at its steady state level (i.e. $K_{j}=K_{s s}$ for $j \geq 3$ ) because as of period 2 , the country no longer has access to concessional loans. Its capital level exceeds the cutoff $\left(K_{2}>\theta K_{s s}\right)$ and the capital level is $K_{s s}($ from $(26))$. Consumption, too, overshoots in period $1\left(C_{1}>C_{2}=C_{s s}\right)$. This is implied by the following Euler equations: $u^{\prime}\left(C_{1}\right)=\beta(1+\bar{r}) u^{\prime}\left(C_{2}\right)$ and $u^{\prime}\left(C_{2}\right)=$ $\beta(1+r) u^{\prime}\left(C_{3}\right)$ because we have $\beta(1+\bar{r})<1, \beta(1+r)=1$, and $u^{\prime}(C)$ is decreasing in $C$. Once we pin down $\left\{K_{2}, K_{s s}\right\}$ and $\left\{C_{1}, C_{s s}\right\}$, we can derive the path of debt, $\left\{\widetilde{D}_{2}, \widetilde{D}_{s s}\right\}$ via the budget constraint. The dynamics of concessional loans without a participation constraint are thus characterized by the overshootings of capital and consumption in period 1 due to the low concessional interest rate. The donor's budget, $\alpha$, is determined by:

$$
\begin{equation*}
\alpha \equiv \frac{\widetilde{D}_{2}(r-\bar{r})}{1+r} \tag{27}
\end{equation*}
$$

### 6.2 Participation Constraint

In this subsection, we solve for the optimal concessional lending in the presence of a participation constraint. We can solve this problem recur-
sively. In each period, the borrower country compares the value function under repayment, $v^{R}(K, \widetilde{D})$, with that under default, $v^{A}(K)^{15}$. When $v^{R}(K, \widetilde{D}) \geq v^{A}(K)$ the country repays, otherwise it defaults. The value function under repayment, $v^{R}(.,$.$) , is given by:$

$$
\begin{align*}
& v^{R}(K, \widetilde{D})=\max _{K^{\prime}, \widetilde{D}^{\prime}}\left\{\begin{array}{c}
u\left[\begin{array}{c}
f(K)-K^{\prime}+(1-\delta) K \\
+\widetilde{D}^{\prime}-\widetilde{D}(1+\widetilde{r})
\end{array}\right] \\
+\beta v^{R}\left(K^{\prime}, \widetilde{D}^{\prime}\right)
\end{array}\right\}( \\
& \text { subject to } v^{R}\left(K^{\prime}, \widetilde{D}^{\prime}\right) \geq v^{A}\left(K^{\prime}\right) \tag{29}
\end{align*}
$$

where $v^{R}(.,$.$) is increasing in K$ and is decreasing $\widetilde{D} . v^{A}($.$) , given by (13),$ is the value function if the recipient country violates the participation constraint.

We can replace the participation constraint with a debt ceiling function, $h(K)$, which is defined implicitly by $v^{R}(K, h)=v^{A}(K)$, where $\frac{\delta v^{R}(K, \widetilde{D})}{\delta \widetilde{D}}$ is strictly negative. In other words, given $K, h(K)$ is uniquely determined ${ }^{16}$. Thus the debt ceiling function is well-defined. The original value function under repayment can be rewritten as:

$$
v^{R}(K, \widetilde{D})=\max _{K^{\prime}, \widetilde{D}^{\prime}}\left\{\begin{array}{c}
u\left[\begin{array}{c}
f(K)-K^{\prime}+(1-\delta) K \\
+\min \left\{\widetilde{D}^{\prime}, h\left(K^{\prime}\right)\right\}-\widetilde{D}(1+\widetilde{r})
\end{array}\right]  \tag{30}\\
+\beta v^{R}\left(K^{\prime}, \widetilde{D}^{\prime}\right)
\end{array}\right\}
$$

This formulation with the debt ceiling (30) is the same as Sachs \& Cohen's $(1986)^{17}$ except that in this paper we numerically derive the value functions and the implied debt ceiling function using the value function iteration method. This paper also extends their model to analyze the dynamics of concessional loans to low-income countries. Note that we equate the initial

[^9]liabilities across schemes by setting $L=(1+\bar{r}) \widetilde{D}_{1}$ and the donor's budget is given by:
\[

$$
\begin{equation*}
\alpha \equiv \widetilde{D}_{2}+\sum_{t=2}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}\left(\widetilde{D}_{t+1}-\left(1+\widetilde{r}_{t}\right) \widetilde{D}_{t}\right) \tag{31}
\end{equation*}
$$

\]

### 6.3 Numerical Results

Again, we are unable to solve this constrained problem analytically, so we solve it numerically. We keep the same parameter values as in section 4.3. In addition, we set the concessional interest rate $(\bar{r})$ at $1 \%$ and fix the cutoff level $(\bar{y})$ at 0.7796 of steady state output $\left(Y_{s s}\right)$. This number ( 0.7796 ) is based on two additional assumptions. First, we introduce a TFP difference between the US and LICs. Appendix B reports the TFP ratios of forty LICs to the US between 1960 and $2000^{18}$. A cursory glance shows that about $3 / 8$ of LICs have TFP levels that are less than $1 / 3$ of the US level and are stable. We thus assume that the TFP ratio of LIC to the US $\left(A / A_{u s}\right)$ is $1 / 3$. Second, we believe it is reasonable to set $\bar{y}$ as a percentage of the steady state US output $\left(Y_{u s}\right)$. We set this percentage at $15 \%$ because in the data ${ }^{19}$ the PPP-adjusted real outputs per capita in most of lower-middle income countries ${ }^{20}$ are above this level. In this way, we can interpret our aid schemes as those that restrict eligibility for concessional loans only to LICs. Given these assumptions and our other parameter values ( $\sigma, \eta, \beta, r$ and $\delta$ ), it is easily shown that the steady state output ratio of the LIC to the US is 0.1924 . Therefore $\bar{y}$ is 0.7796 of $Y_{s s}$.

### 6.3.1 The benchmark numerical example

Recall the benchmark example in section 4.3 where the initial income is $70 \%$ (precisely $70.47 \%$ ) of the steady state output. This is equivalent to $90 \%$ of the cutoff. Figure 4 (a)-(c) show the paths of output, consumption-ouput

[^10]ratios, and investment-output ratios under the optimal concessional lending scheme (solid), under the optimal transfer scheme (dot), and under the optimal grant scheme (dash-dot). We find a striking result: with concessional loans the country becomes permanently aid-dependent and stays at the cutoff.

How can we intuitively interpret this poverty trap? It is important to understand the trade-offs between "high" and "low" steady states, where the low steady state is achieved under the concessional lending scheme and the high steady state is achieved under the other two schemes.

Figure 4 Benchmark example

loans(solid), grants(dash-dot), transfers(dot)
(c) investment/output

(d) debt (in fraction of $Y_{s s}$ )

loans(solid), grants(dash-dot), transfers(dot), non-concessional(dash)
The benefit of reaching the high steady state is that the country can achieve higher output in the long-run. Figure 4 (a) shows that output in the high steady state is roughly $23 \%$ more (in percentages of steady state output) than that in the low steady state. On the other hand, the realization of the low steady state enables the country to achieve higher investment and consumption in the short-run and also to sustain a higher debt level. Figure

4 (d) shows the debt paths with concessional (solid) and non-concessional (dash) loans. The only difference between these two loans is the interest rate level: for all $t$ under non-concessional loans $\widetilde{r}_{t}=r$. Note that the vertical dotted line is the cutoff. Figure 5 shows that if the country stays at the cutoff, concessional debt ceilings (solid) are much higher than the nonconcessional counterparts (dash). This means that a significant amount of concessional debt is sustainable below or at the cutoff.

Once the country is in this low steady state (point L), it is too painful to move to the high steady state (point H) because if the country were to surpass the cutoff, it would have to significantly reduce its debts in order to retain debt sustainability. Note that there is a sudden fall in the concessional debt ceiling when the country hits the cutoff. This is because when the capital level exceeds the cutoff, the country faces the world interest rate. Here, the only way to reduce its debts is to cut consumption (at least by $20 \%$ of $Y_{s s}$ ) in order not to default. Also, the country cannot reduce investment because in order to accumulate capital it must increase investment. (Note: the steady state investment is equal to capital depreciation.)

Figure 5 Debt Ceilings (the benchmark case) concessional (solid), non-concessional (dash)


### 6.3.2 TFP matters

In the benchmark example, the benefits of achieving the low steady state outweigh those of achieving the high steady state and consequently the coun-
try is stuck at the cutoff. This result is conditional on the country's TFP level. The higher the TFP, the higher the steady state output level relative to the cutoff. Therefore as TFP increases, the benefits of achieving the high steady state (i.e. long-run benefits) become greater. Now, we raise the TFP level by $20 \%$, ceteris paribus. Figure 6 shows the corresponding path of output, debt, consumption, and investment expressed as a fraction of $Y_{s s}$. Here, the country manages to converge to the high steady state, even though the distortionary effects of concessional loans are still apparent. The path of debt under concessional lending demonstrates that the country overborrows in early periods, enjoying the benefits from a low interest rate. The country then reduce its debt before it hits the cutoff to reach the high steady state. The path of output illustrates that the country stays right at the cutoff for a time, reducing its debt and making the transition from a low-income to a lower-middle income country. Economic growth slows until the country overcomes this challenging transition period.

As the TFP increases relative to the cutoff level, the country finds it more painful to be trapped in the low steady state because the gap between the high and low steady states increases. Thus if the TFP level is high enough, the benefit of achieving the high steady state can outweigh the benefits from its lower steady state counterpart.

Figure 6 With a higher TFP level

loans(solid), grants(dash-dot), transfers(dot)
(c) investment (in fraction of $Y_{s s}$ )

(d) debt (in fraction of $Y_{s s}$ )

loans(solid), grants(dash-dot), transfers(dot), non-concessional(dash)

### 6.3.3 Initial condition matters

Whether or not the country becomes permanently aid-dependent under concessional loans also depends on the country's initial condition. First, the lower the country's initial output, the larger the impact of short-run growth. Thus, the country tries to borrow a larger quantity of concessional loans in the short-run and is likely to be trapped in the low steady state. Second, the higher the initial debt level, the more likely the country is to converge to the low steady state, because this allows the country to manage heavier debt with a low interest rate. In short, the country chooses to converge to the high steady state only if initial income is high enough or initial debt is low enough, or both. Figure 7 shows the combinations of initial income and debt that can achieve the high and low steady states, the areas above and below the line respectively.

Figure 7 The initial income and debt (the benchmark case)


Debt overhang and cancellation In the benchmark example, the country's initial debt is too large causing it to be stuck at the cutoff in order to manage its heavy debt in the form of subsidized loans. The country does not grow; investment is equal to capital depreciation at the cutoff. Examining debt overhang in this way provides a unique description of the relationship between debt burden and stagnation.

Figure 7 captures the effectiveness of debt cancellation. Suppose we lower the country's initial debt level and the initial debt-output ratio is now 0.2 , ceteris paribus (Let call this case the "lower debt" case). In this case with lower debt, the country lies below the line so that the country converges to the high steady state.

The countries above the cutoff - stickiness from above Now, consider a situation where the country's output level is already above the cutoff. Suppose the initial output is $105 \%$ of the cutoff where the initial liability is high (say, $L / Y_{1}=1$ ). In this case, the country lies above the line in figure 7 so it is better off reducing output by one unit. The benefit of raising the debt ceiling by reducing capital is greater than the cost of lowering output. The capital level falls and eventually stays right below the cutoff. The country prefers to stay at the cutoff because significantly larger amounts of debt are sustainable due to higher concessional debt ceilings.

## 7 The Role of Private Foreign Lenders

In this section we discuss the role of non-concessional loans in the presence of foreign aid. Here, we consider foreign private loans, rather than official non-concessional loans because since the 70s it has been difficult for official lenders to commit themselves to lend on non-concessional terms. They have repeatedly granted LICs debt relief by replacing non-concessional loans with concessional loans. Thus in this paper, we use access to non-concessional loans and access to the world financial markets interchangeably.

### 7.1 Concessional Loans

Access to non-concessional loans does not change the optimal allocation under concessional loans. This is because the country maximizes the implicit transfers from the benevolent lenders by borrowing concessional loans when it is below or at the cutoff. In particular, with a participation constraint, the country borrows from benevolent lenders up to the debt ceiling, leaving no capacity to payback non-concessional lenders. Thus, there will not be any non-concessional lending as long as the country is eligible for concessional loans. When the country is above the cutoff, we can not distinguish between concessional and non-concessional loans because both lend at the world interest rate. As a result, there are multiple combinations of these loans that achieve the same allocation above the cutoff.

### 7.2 Transfers and Grants

The flow budget constraint with non-concessional loans is given by:

$$
\tau_{t}+D_{t+1}-(1+r) D_{t}=C_{t}+I_{t}+f\left(K_{t}\right)
$$

where $D$ is non-concessional loans. It is now impossible to distinguish between transfers and grants because we can always find a combination of grants and non-concessional loans that realizes the same allocation as any given path of transfers. Consequently, with access to financial markets, grants can achieve the same allocation as transfers.

It is easy to see that there are multiple solutions that realize the best allocation in each environment: without a participation constraint, there are two equations and four unknowns $\left(\tau_{1}, \tau_{s s}, D_{2}\right.$, and $\left.D_{s s}{ }^{21}\right)$; with a participation constraint, the sequential problem is given by:

$$
\begin{equation*}
\max _{\left\{C_{t}, K_{t+1}, \tau_{t}\right\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t} u\left(C_{t}\right) \tag{32}
\end{equation*}
$$

subject to:
${ }^{21} D_{s s}$ is the steady state debt level.

$$
\begin{gathered}
u\left(C_{t}\right)+\beta \sum_{j=1}^{\infty} \beta^{j-1} u\left(C_{t+j}\right) \geq v^{A}\left(K_{t}\right), \forall t \\
\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}\left[C_{t}+K_{t+1-}(1-\delta) K_{t}-f\left(K_{t}\right)\right]=\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} \tau_{t}-(1+r) D_{1} \\
\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} \tau_{t} \leq \bar{T}
\end{gathered}
$$

The solution to this problem is not unique. There are multiple paths of $\left\{\tau_{t}, D_{t}\right\}_{t=1}^{\infty}$ that realize the optimal consumption and capital paths, including a gift of $\alpha$ in period 1 combined with non-concessional loans.

## 8 Welfare Evaluation

We now evaluate the welfare cost of each scheme in various environments. We first evaluate welfare implications in environments without a participation constraint. Denote $\bar{c}_{\text {fb }}, \bar{c}_{\text {loan }}, \bar{c}_{\text {grant }}$, and $\bar{c}_{\text {private }}$ as the constant levels of consumption that realize the maximized lifetime utility under transfers (the first best allocation), grants in financial autarchy, concessional loans ${ }^{22}$, and no aid with access to financial markets (i.e. non-concessional financing only), respectively. Table 2 shows the consumption level under each scheme (i.e. $\bar{c}_{\text {loan }}, \bar{c}_{\text {grant }}$, and $\bar{c}_{\text {private }}$ ) as a percentage of $\bar{c}_{f b}$ where $\alpha$ is the donor's budget, and $Y_{s s}$ is the steady state output level. The optimal sequence of grants is less efficient than the concessional lending counterpart in financial autarchy. This is because grants cannot provide a large sum of funds in period 1. With access to financial markets, however, grants are better than concessional loans because they can achieve the first best allocation. Concessional loans cannot achieve this allocation due to the distortionary effect-the overshooting in period 1 (section 6.1).

[^11]Table 2 No Participation Constraint

| cases | $\alpha / Y_{s s}$ | financial autarchy |  |  | w/ access to financial markets |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  |  | loan | grant | loan | grant | private |  |
| benchmark | 16.7 | 82.6 | 78.2 | 81.7 | 100 | 81.7 |  |
| higher TFP | 17.4 | 99.9 | 92.7 | 99.9 | 100 | 98.7 |  |
| lower debt | 14.5 | 87.8 | 83.5 | 87.8 | 100 | 87.1 |  |
| above the cutoff | 14.7 | 84.0 | 82.8 | 84.0 | 100 | 83.3 |  |

benchmark (see section 6.3.1), higher TFP (6.3.2), lower debt and above the cutoff (6.3.3)

Table 3 With a Participation Constraint

| cases | $\alpha / Y_{s s}$ | financial autarchy |  | w/ access to financial markets |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | loan | grant | loan | grant | private |
| benchmark | 85.7 | $94.2^{*}$ | 99.7 | $94.2^{*}$ | 100 | 90.8 |
| higher TFP | 12.9 | 99.9 | 99.7 | 99.9 | 100 | 99.7 |
| lower debt | 3.9 | 100 | 99.4 | 100 | 100 | 99.9 |
| above the cutoff | 67.9 | $95.6^{*}$ | 99.9 | $95.6^{*}$ | 100 | 93.8 |

* cases where the LIC is in a "poverty trap"
benchmark (see section 6.3.1), higher TFP (6.3.2), lower debt and above the cutoff (6.3.3)
In environments with a participation constraint, transfers no longer achieve the first best allocation but do achieve the constrained optimum. Denote $\bar{c}_{\text {co }}, \bar{c}_{\text {cloan }}, \bar{c}_{\text {cgrant }}$ and $\bar{c}_{\text {cprivate }}$ as the constant levels of consumption that realize the maximized lifetime utility under transfers (constrained optimum), concessional loans, grants in financial autarchy, and no aid with access to financial markets (i.e. non-concessional financing only), respectively. Table 3 shows the consumption level under each scheme (i.e. $\bar{c}_{\text {cloan }}, \bar{c}_{\text {cgrant }}$, and $\left.\bar{c}_{\text {cprivate }}\right)$ as a percentage of $\bar{c}_{c o}$. When the country is in financial autarchy, the optimal sequence of grants is more efficient than the concessional lending counterpart when the perverse effects of concessional loans are large enough-these effects are especially large when the country is trapped in the low steady state. Whether or not this poverty trap occurs depends on the
country's TFP level and initial conditions (section 6.3). When the country has access to non-concessional loans, the optimal sequence of grants achieves the constrained optimum and is thus more efficient than concessional loans.

To summarize this section, whether or not optimal concessional lending is better than its grant counterpart depends on TFP levels, initial conditions, and the degree of access to financial markets.

## 9 Conclusion

Motivated by the political interest in the choice of aid schemes and our empirical finding that growth rates tend to stagnate around the cutoff, we have presented a theoretical framework to analyze the effectiveness of concessional loans versus grants.

Our model has shown that this stagnation is due to the existence of a cutoff. In particular, optimal concessional lending drives the recipient countries to permanently remain at the cutoff, depending on the country's TFP and initial conditions. The cost to the donor is large when the country is stuck at the cutoff. With regards to the optimal grant sequence (one-time gift in period 1), we find that even if we impose a cutoff rule (grants must be zero above the cutoff), a poverty trap does not result. In reality, it may be difficult for the donor to commit itself to such a grant sequence. Thus we have also analyzed a fixed-amount grant scheme (a more practical grant scheme); this non-optimal scheme may cause a poverty trap.

Since official lenders make up the majority of loans in LICs, we have mainly considered environments where the recipient countries are financial autarchic with access to foreign aid. In these environments, the optimal grant scheme is better than its concessional lending counterpart when the perverse effects of concessional loans are sufficiently large.

Finally, we present a few recommendations to the donors who make contributions to LICs which have little access to foreign private financing. If the LIC in question has good policy performance with relatively higher TFP or better initial conditions, concessional loans may be more effective than
grants simply because loans can provide more funds than grants. However if the LIC has relatively lower TFP or poorer initial conditions, grants may be more effective than loans. If the optimal grant scheme is not a practical option, it may be effective to tie grants to the projects that can directly raise TFP. This can improve output capacities in these countries so that eventually they find it too costly to remain at the cutoff and become independent from aid assistance.

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## APPENDIX A

When we drop countries considerably above the cutoff, we find even higher $t$-values for the variable of interest even though the significance levels for other explanatory variable decrease. In this appendix, we run regressions including only those countries where GNI per capita are less than two times the cutoff level. Our variable of interest is now defined as the distance between the actual GNI per capita and the cutoff in the absolute values:

$$
\operatorname{DIFF}_{i t}=\left|y_{i t}-\bar{y}_{t}\right|
$$

where $y_{i t}$ is the country $i$ 's GNI per capita in year $t$ and $\bar{y}_{t}$ is the cutoff in year $t$. We expect to obtain a positive coefficient for DIFF if there is stagnation around the cutoff. The OLS estimation in the following table shows that the growth rate of per capita GNI decreases by $1.68 \%$ when the gap between the GNI per capita and the cutoff decreases by $\$ 100$ for the period 1980-2003. The coefficient for DIFF is statistically significant at the level of significance 0.01 even when we consider country specific fixed effects.

Dependent Variable: Growth Rate of Per Capita GNI, 1980-2003

| Regressors | (a) OLS |  | (b) Fixed Effects |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\beta$ | se | t | $\beta$ | se | T |
| DIFF | 0.0168 | 0.00612 | 2.76 | 0.021 | 0.0081 | 2.59 |
| GPO | -0.0044 | 0.00697 | -0.63 | -0.0019 | 0.00763 | -0.25 |
| INV | 0.0013 | 0.00139 | 0.97 | 0.0001 | 0.00232 | 0.03 |
| SEC | 0.0007 | 0.0007 | 0.98 | --- | --- | --- |
| RDGP | $-3.18 \mathrm{E}-06$ | 0.00001 | -0.25 | -- | --- | --- |
| constant | -0.0547 | 0.04479 | -1.22 | -0.0371 | 0.0568 | -0.65 |

## APPENDIX B









[^0]:    ${ }^{1}$ Acknowledgement: I thank Atish Rex Ghosh for his introduction to the issue of debt sustainability in low-income countries while I was an intern at the International Monetary Fund in the summer of 2003. I am grateful to Carlos Vegh, Lee Ohanian, Harold Cole, Timothy Lane, Aaron Tornel, Yasuyuki Sawada, other UCLA faculty, and participants in macro and international proseminars at UCLA for helpful comments.

[^1]:    ${ }^{2}$ For a pro-grant argument, see the transcript of President Bush's speech at the World Bank (http://edition.cnn.com/2001/ALLPOLITICS/07/17/bush.speech.transcript/. For a pro-loan argument, see the EU's response to the US proposition, p. 14, the Courier ACP-EU, may-june 2002.
    ${ }^{3}$ For example, suppose the concessional interest rate and the world interest rate are 0 and $5 \%$, respectively. Then a concessional loan of 100 in period 1 is equivalet to a gift of 5 in period 2 or less than 5 in period 1 .
    ${ }^{4}$ HIPCs stand for Heavily Indebted Poor Countries.

[^2]:    ${ }^{5}$ Recently, the World Bank and other development banks consider good policy performance as a key factor to determine the eligibility for their assistance.

[^3]:    ${ }^{6}$ For e.g., see Svensson (2000), Summers and Pritchett (1993), and Krueger et al (1989).
    ${ }^{7}$ For e.g., see Burnside and Dollar (2000) and Easterly, Levine, and Roodman (2003).

[^4]:    ${ }^{8}$ The examples of the debt-overhang literature are: Krugman (1988); Sachs (1989); Cohen (1993). For a summary of the literature, see Patillo et al. (2002).
    ${ }^{9}$ The number of countries in a given year ranges from 92 in 1980 to 108 in 1998.

[^5]:    ${ }^{10}$ GNI is commonly denoted as GNP. GNI is the new terminology under the 1993 System of National Accounts (SNA), replacing the old terminology-GNP-under the 1968 SNA.
    ${ }^{11}$ We carry out fixed effect estimation because there may exist unobserved country specific effects which are not caputured by the initial conditions (SEC and RGDP) in the OLS estimation.

[^6]:    ${ }^{12}$ We assume $r \equiv 1 / \beta-1$ in order to have flat consumption in the steady state.

[^7]:    ${ }^{13}$ In section 5.1, we show that optimal grant sequence does not change even if we impose an additional rule that grants must be zero above a certain level of income.

[^8]:    ${ }^{14}$ External debts in many low income countries are still positive despite the fact that they have been receiving significant debt relief.

[^9]:    ${ }^{15}$ The superscript $A$ stands for autarchy with penalties for violating the contract imposed.
    ${ }^{16}$ Thus we can exclude the case where $h(K)$ is backward bending in $K$.
    ${ }^{17}$ An extension of Sachs \& Cohen (1986) can be seen in Borensztein and Ghosh (1989).

[^10]:    ${ }^{18}$ We use extended Penn World Tables by Adalmir Marquetti.
    (http://homepage.newschool.edu/~foleyd/epwt/)
    ${ }^{19}$ see the previous footnote.
    ${ }^{20}$ The World Bank's definition of LICs are those countries that have GNI per capita equal to $\$ 736-\$ 2,935$ in 2002, calculated using the World Bank Atlas method.

[^11]:    ${ }^{22}$ We have explained that the optimal allocation under concessional loans is the same with or without access to financial markets in section 7.1.

