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A New Model of Labor Dynamics: Ultrametrics, Okun's Law, and Transient Dynamics

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Abstract

This paper adds a labor sector to the model of outputs and fluctuations of Aoki (2002, Chapt. 8), and Aoki and Yoshikawa (2003) to produce a new model with labor sector.

The concept of ultrametrics is introduced to the labor dynamics of this paper to measure "distances" between clusters of unemployed workers in different geographical locations, with work experiences, and/or human capitals to reflect differences in probabilities of them being rehired from a pool of unemployed workers. We maintain the assumption of our previous model that marginal product of labor does not equalize instantaneously across sectors of our model

This model shares the same property with our earlier one that GDP responds to changes in demand patterns among the sectors. In addition, the model of this paper exhibits a relation between unemployment rates and GDP similar to that of the Okuns' law in its business cycle fluctuations. This Okun's coefficient increases as the average GDP increases. Our model also reaches stationary business cycles faster as more demands are put on more productive sectors of the model.

Introduction

There exists a large body of literature devoted to labor markets, some theoretical and some empirical, such as Blanchard and Diamond (1989, 1992), Mortensen (1989), Pissarides (2000) and Davis and Haltiwanger (1992). Importance of flows of job creations and job destructions as determinants of labor market dynamic behavior is illustrated by Davis, Haltiwanger and Schuh (1996).

Existing models of labor dynamics, however, do not successfully incorporate these empirical findings. For one thing, differences in geographical locations, human capitals, jobs experiences and the like are stressed in verbal discussions of labor markets, but are not incorporated into models convincingly, if at all. Unemployed workers are differentiated at most by their reservation wages in search models, or by the length of spell of unemployment.

In this paper, we treat clusters of different types of unemployed workers as forming a tree structure, and use ultrametrics to meassure similarities of wrokers in different clusters. When a sector hires a worker it does so randomly from a pool of workers composed of different clusters which are suitably weighted by the ultrametric distances as we show in a later section. See Aoki (1996, Sec.2.5) on ultrametrics.

In our earlier model, GDP responds to changes in demand patterns on the sectors of economy. This continues to hold for our new model.

This paper examines two new features. One is on the Okun's law. The Okun's law refers to a stable empirical relation between unemployment rates and rates of GDP changes: one percent incease (decrease) in GDP corresponds to x percent decrease (increase) in unemployment, where the value of x varies from country to country and from period to period, see Hamada and Kurosaka (1984) for example. It was about 4 in US when Okun announced this relation. With a Cobb-Douglas production function x is about 1. To obtain the value 4 we need some other effects such as increasing marginal product of labor or some other nonlinear effects. See Yoshikawa (2000). We report on the Okun's law based on a small scale simulation runs. In the simulation studies of our new model we obtain numbers larger than 4, even though our model use linear production functions for all sectors. See Yoshikawa (2000, 2003) on discussions of traditional literature on the Okun's law, and on the role of demands in macroeconomics.

The other feature is a very intriguing finding that the model exhibits faster transient dynamics of approaching stationary equilibrium distribution when larger shares of demand fall on more productive sectors than otherwise.

The Model

Consider an economy composed of K sectors, and sector i employs n_i workers, $i = 1, \ldots K$. The variable n_i needs not be the number of employees in literal sense. It should be a variable that represents 'size' of the sector in some sense. For example it may be the number of lines in assembly lines, and so on. In this paper we assume that K is fixed. In another model with growth, K can change as sectors enter and exit. See Aoki (2002, Sec. 8).

Sectors are in one of two status; either in normal time or in overtime. That is, each sector has two capacity utilization regimes. In normal time, which is indicated by variable $v_i = 0$, n_i workers produce output

$$Y_i = c_i n_i,$$

for $i = 1, 2, \ldots, K$, where c_i is the productivity coefficient, and n_i denotes the number of employees of sector i. In overtime, indicated by variable $v_i = 1$, n_i workers produce output equal to

$$Y_i = c_i(n_i + 1).$$

The total output (GDP) is given by the sum of all sectors

$$Y = \sum_{i=1}^{K} Y_i.$$

Note that we keep the constant coefficient structure of the production function used in our earlier model. The important difference is the notion of normal or overtime, and a pool of unemployed weighted by ultrametric distance which is discussed later.

Demand for good i is given by $s_i Y$, where s_i is a positive share of the total output Y which falls on sector i goods, with $\sum_i s_i = 1$.

Each sector has the excess demand defined by

$$f_i = s_i Y - Y_i, \tag{1}$$

for $i = 1, 2, \ldots, K$.

Changes in Y due to changes in any one of the sectors affect the excess demands of all sectors. That is, there exists an externality between aggregate output and demands for goods of sectors. Changes in the patterns of s's also affect these sets of excess demands.

The time evolution of the economy is modeled as a continous-time Markov chain, as described in Aoki (1996, 2002), for example.

At each point in time, the sectors of economy belong to one of two subgroups; one composed of sectors with positive excess demands for their products, and the other of sectors with negative excess demands.

We denote the sets of sectors with positive and negative excess demands by $I_+ = \{i : f_i \geq 0\}$, and $I_- = \{i : f_i < 0\}$, respectively. These two groups are used as proxies for groups of profitable and unprofitable sectors, respectively. All profitable sectors wish to expand their production. All unprofitable sectors wish to contract their production.

A novel feature of our model is that only one sector succeeds in adjusting its production up or down by one unit of labor at any given time. The sector that has the shortest holding or sojourn time is the sector that jumps first, that is, only the sector that jumps first succeeds in implementing the desired adjustment. See Lawler (1995) or Aoki (2002, p. 28) for the notion of holding or sojourn time of a continuous-time Markov chain. We call that sector that jumps first as the active sector. Variables of the active sector are denoted with subscript a.

Transition rates

Dynamics of this continous-time Markov chain are determined uniquely by the transition rates.

Sectors adjust their outputs by hiring or firing workers in response to the signs of excess demands which are used as proxies for profitability of the sectors. We assume that the economy has enough number of unemployed that sectors incur zero costs of firing or hiring, and do not hoard workers. To increase outputs the active sector calls back (one unit of) worker from the pool of workers who were earlier laid-off by various sectors. The actual rehired worker is determined by a probabilistic mechanism discussed in the section on ultrametrics.

To implement a simple model dynamics we assume the following. Other arrangements of the detail of the model behavior is of course possible. Each sector has three state vector components: the number of employed, n_i, the number of laid off workers, u_i, and a binary variable v_i, where $v_i = 1$ means that sector i is in overtime status producing $c_i(n_i + 1)$ output with n_i employees. Sectors in overtime status all post one vacancy sign during overtime status. When one of the sectors in overtime status becomes active with positive excess demand, then, it actually hires one additional unit of labor and cancells the overtime sign. When a sector in overtime becomes active with negative excess demand, then it cancels the overtime and returns to normal time and vacancy sign is removed. When $v_i = 0$, sector i is in normal time producing $c_i n_i$ output with n_i workers. When one of the sectors, sector i say, in normal time becomes active with positive excess demand, then it posts one vacancy sign and v_i changes into one. If this sector has negative excess demand when it becomes active, it fires one unit of labor. To summarize: When $f_a < 0$, n_a is reduced by one, and u_a is increased by one, that is one worker is immediately laid off. We also assume that v_a is reset to zero. When f_a is positive, we assume that it takes a while for the sector to hire one worker if it has not been in overtime status, i.e., v_a is not 1. If sector a had previously posted vacancy sign, then sector a now hires one worker, and cancels the vacancy sign, i.e., resets v_a to zero. If it has not previously posted a vacancy sign, then, it now posts a vacancy sign, i.e., sets v_a to 1, and increases its production with existing number n_a of workers by going into over-utilization state.

The transition path may be stated as $\mathbf{z}$ to $\mathbf{z}'$, where $(n_a, u_a, v_a = 0) \to (n_a, u_a, v_a = 1)$, and $(n_a, u_a, v_a = 1) \to (n_a + 1, u_a - 1, v_a = 0)$. In either case the output of the active sector changes into $Y_a' = Y_a + c_a$.

Continuum of equilibria

The equilibrium states of this model are such that all sectors are in normal time and have zero excess demands, that is,

$$s_i Y_e = c_i n_i^e, \; i = 1, 2, \ldots, K,$$

where subscript e of Y, and superscript e to n_i denote equilibrium values.

Denoting the total equilibrium employment by $L_e = \sum_i n_i^e$, we have

$$\left(\sum_i \frac{s_i}{c_i}\right) Y_e = L_e. \tag{2}$$

This equation is the relation between the equilibrium level of GDP and that of employment. We see that this model has a continuum of equilibria.

Transition to the closed set from initial states

In simulations described below we start the model from over-employed states, that is, there are more $n_i(0)$ to meet the demand $s_i Y(0)$ at least for some, and possibly for all i. Consequently such sectors, when they become active, start by firing employees. Eventually the number of employees become small enough to be compatible with the demands for the sectors. In other words, the Markov chain enter from transient states to the closed set of states, that is, those states from which the model does not escape. These states are the ones in the business cycle. See Feller (1968, XV.8).

We next describe the variations in the outputs and employments in business cycles near one of the equiibria.

Okun's Law

Okun's law in the economic literature usually refers to changes in gross domestic products (GDP) and unemployment rates measured at two different time instants, such as one year apart. There may therefore be growth or decline in the economies.

To avoid confusing the issues about the relations between GDP and unemployment rates during stationary business cycle fluctuations, that is, without growth of GDP, and those with growth, we run our simulations in stationary states assuming no change in the numbers of sectors, productivity coefficients, or the total numbers of labor force in the model.

The Okun's law refers to a stable empirical relation between unemployment rates and rate of changes in GDP: one percent increase (decrease) in GDP corresponds to x percent decrease (increase) in unemployment, where x is about 4 in the United States. It is well known that if labor market is a homogeneous single market operating under neoclassical setup, then the Okun's law does not hold. This numerical value of x is much larger than what one expects under the the standard neoclassical framework. Take, for example, the Cobb-Douglas production function with no technical progress factor. Then, GDP is given by $Y = K^{1-\alpha} L^{\alpha}$ with α of about 0.7, where the total population is $N = L + U$ of which U is the number of unemployed.

We have $\Delta U = -\Delta L$, where ΔK and ΔN are assumed to be negligible in the short run. The production function implies then that $\Delta Y/Y = \alpha \Delta L/L$ in the short run. That is, one percent decrease in Y corresponds to an increase of $\Delta U/N = -(1/\alpha)(\Delta Y/Y)(1 - U/N)$, i.e., an increase of a little over 1 percent of unemployment rate. To obtain the number 4, as in the Okun's law, we need some other effects, such as increasing marginal product of labor or some other nonlinear effects. See Yoshikawa (2000). In the simulation studies of our new model we can obtain numbers larger than 4, depending on the configurations of demand shares and productivity coefficients as in (2), even though the linear production functions for all sectors in our model may lead us to expect numbers closer to 1.

We assume that economies fluctuate about its equilibrium state, and refer to the relation

$$\frac{\Delta Y}{Y_e} = -x \frac{\Delta U}{N},$$

as Okun's law, where Y_e is the equilibrium level of GDP, approximated by the central value of the variations in Y in simulation. Similarly, ΔU is the amplitude of the business cycle oscillation in the unemployed labor force, and N is taken to be $L_e + U_e$, where U_e is approximated by the central value of the oscillations in U, and Y_e and L_e are related by the equilibrium relation (2).

The changes $\Delta Y/Y$ and $\Delta U/U$ are read off from the scatter diagrams in simulation after allowing for sufficient number of time to ensure that the model is in "stationary " state.

In simulations described below we note that after a sufficient number of time steps have elapsds, the model is in or near the equilibirum distribution. Then, Y and U are nearly linearly related with a negative slope i.e.,

$$\frac{\Delta Y}{Y} = -\beta \frac{\Delta U}{N} \frac{N}{Y},$$

where U is the total number of unemployed, so that U/N is the fraction of unemployed of the total number N of workers. The total output Y is related to N by $Y = \sum_i c_i n_i$. In the equilibrium we have $c_i n_i = s_i Y$ or $n_i = (s_i/c_i)Y$, hence $Y = \hat{c}N$, with $(\hat{c})^{-1} = \sum_i s_i/c_i$, so that we may express the Okun's law as $\Delta Y = -\beta \Delta U$, where $\beta = x\hat{c}$. That is, x is estimated by $\beta/\hat{c}$, where β is estimated from the scatter diagram in "$Y - U$" plane. This relation is not exactly true in business cycles, but may be used as an approximation. Alternatively we can use the average GDP value together with the average employed number to approximate c in business cycles.

This indicates that the dynamics are indeed nonlinear. By changing the initial conditions and observing how much time elapses before the model enters the closed set, we observe the relation that the larger the shares on more productive sectors the faster the model enters the stationary closed sets.

Ultrametric trees

We endogenize job destructions and creation differently from Pissarides (2000).

To present a simple model we ignore quits and on-the-job searches, and assume that only unemployed get jobs.

In this model, sectors are differentiated with different 'distances' between each other. These 'distances' reflect such factors as geographical differences, differences in technology, and educational qualifications. Workers in different sectors are different in job experiences and human capitals, and their differences affect probabilities of being hired of different sectors by using the concept of ultrametrics.

The stochastic process of filling vacancies of sector i by unemployed workers from the pool of sector j depends on the ultrametric distance $d(i, j)$ between the two sectors of the economy.

Transitions of the active sector depends on the sign of the excess demand, f_a as indicated above. When $f_a < 0$, then one unit of labor is fired immediately, and n_a become $n_a - 1$ as indicated at the end of the previous section.

Hiring a new unit occurs only with $f_a > 0$, and $v_a = 1$. Here we explain how the active sector employs one additional unit of labor. We need to distinguish u_a, which denotes the size of sector a's laid-off workers, from the total pool of unemployed from which sector a randomlly hires one unit of labor. This pool is composed of u_a and separate pools of laid-off workers from sector j $u_j, j \neq i$ suitably weighted by ultrametric distance. We denote the latter by $\tilde{u_a}$, that is $u_a + \tilde{u_a}$ is the total size of the pool of the unemployed units of labor for sector a.

These separate sub-pools are organized as a hierarchical tree with ultrametric distance.

The clusters or sub-pools of unemployed have different probabilities of being picked. The highest probability is for the pool of the workers who are laid-off from that sector. Its size is u_a. This reflects the empirical observation that often firms recall laid-off workers first as they become profitable again. Then pools of laid-off workers from other sectors are arranged in increasing order of the ultrametric distance from the pool of size$\tilde{u}_a$.

We illustrate this notion and its use in the case of $K = 3$ where sector 1 and 2 are at ultrametric distance 1, $d(1, 2) = d(2, 1) = 1$, and $d(1, 3) = d(2, 3) = 2$. Suppose that sector 1 is active. It draws from pools u_2 and u_3 after deflating them by $1 + d(1, 2)$ and $1 + d(1, 3)$ respectively. Thus a vacancy at sector 1 is filled from u_1 with probability

u_1/U_1, with $U_1 = u_1 + \tilde{u_1}$, where $\tilde{u_1} = u_2/[1 + d(1, 2)] + u_3/[1 + d(1, 3)]$. When sector 2 is the active sector, it hires one unit of labor from u_2 with probability u_2/U_2, with $U_2 = u_2 + \tilde{u_2}$, with $\tilde{u_2} = u_1/[1 + d(1, 2)] + u_3/[1 + d(2, 3)]$. Similarly when sector 3 is active.

The probability that a vacancy in sector 1, v_1, is filled from its own pool of unemployed, given that sector 1 jumps first is

$$\Pr(v_1 \text{is reduced by 1 to zero}|\text{sector 1 jumps}) = \frac{u_1}{U_1},$$

where

$$U_1 = u_1 + \tilde{u_1}.$$

Similarly v_1 is reduced by one from pool of unemployed of sector 2 with probability

$$\Pr(v_1 \text{is reduced by 1 from sector 2}|\text{sector 1 jumps}) = \frac{u_2/[1 + d(1,2)]}{U_1}.$$

In this example vacancy in sector 1 will be filled from own laid-off pool with probability $6/11$, from u_2 with probability $3/11$, and from u_3 with probability $2/11$. A vacancy in sector 3 will be filled from u_1 with probability $1/5$, from u_2 also with probability $1/5$, and from u_3 with probability $3/5$.

Simulation studies

Our model behaves randomly because the jumping sectors are random due to holding times being randomly distributed. This is different from the models in the literature which behave randomly by the technology shocks which are exoegnously imposed. Apparently, the model states have many basins of attractions each with near equal "potential energy" levels, much as spin glasses are.

Since the model is nonlinear and possibly possesses multiple equilibria, we use simulations to deduce some of the properties of the models. We pay attention to the phenomena of trade-offs between GDP and unemployment, and the scatter diagrams of GDP vs. unemployment to gather informtion on business cycle behaviors.

A large number of simulations have been run. See the descriptions of the case studies in the appendix. Here, we summarize the simulations of the three cases, Case 1, 3, and 5. In the first two cases the same demand share vector $\mathbf{s} = (5,4,3,2,1,1,1,1)/18$ is used. In Case 5, the share vector $\mathbf{s} = (3,3,4,4,1,1,2,2)/20$ is used. In all cases the productivity coefficients are $c_1 = 1$, and $c_8 = .225$ with equally spaced decrease in between. In Case 1 and 3, about 78 per cent of the demands fall on the top 4 productive sectors. In Case 5 the top 4 sectors account for 70 per cent of demand. The sum$\sum_i s_i/c_i = 1.3$ in Case 1. In Case 5 , the sum is 1.83. Case 1 and Case 3 uses different initial conditions. They appear to settle in different basins since Case 1 has a larger Y_e values than Case 3.

GDP values are in decreasing order from cases 1, 3, and 5. The number of unemployed is the largest in Case 3, then Case 5 and Case 1 in that order. The ratio of U_e/L_e are slightly higher in the Case 5. It is 2 per cent vs. 2.8 per cent.

The value of x is 3.5 in Case 1, 5.9 in Case 3 and 6.0 in Case 5. The values of x clearly depend on the basins of attractions, if the different starting points lead the model to different Y_e values. These figures are larger than Okun's. There are other empirical studies on the coefficients. For example, Hamada and Kurosaka (1984) examined the Japanese economy from 1953 to 1995, with the numbers ranging from 10.5 to 32, depending on the time spans and whether the economy was in high unemployment rate period or in low unemployment period.

In Case 1 and 3, the model reaches the closed set in about 600 time steps, while it takes about 750 steps in Case 5.

As a final remark we record that the simulations vary somewhat with the initial conditions.

We state the qualitative results of these simulation runs as follows:

1) larger shares of demands on more productive sectors result in larger average values of GDP.

2) Under the same circumstances, the systems with larger GDP reach 'near equilibrium' conditions faster.

3) Amplitudes of business cycles are larger, that is the amplitudes of GDP and unemployment rates are larger, the larger the average GDP.

4) Relations between unemployment and average GDP are described by a relation similar to the Okun's law.

Of these four qualitative conclusions, No.2 seems to be most interesting. In the existing literature this dynamic aspects of labor market characteristics has not been observed or commented on.

Conclusions

We have demonstrated by simulation that higher percentages of demands falling on more productive sectors produce three new results: Average GDPs are higher; the Okun's coefficients x is larger; and transient responses are faster and in near stationary states, excursions of GDP (hence of U) are larger as well. In addition the amplitudes of the business cycles are higher as well.

It is remarkable that we obtain a relation like Okun's law with linear constant coefficient production functions. The fact that we obtain larger numbers than Okun may indicate the importance of capacity utilization as well as nonlinear interactions among sectors. Capacity utilization of capital is crudely incorporated in our model by the fact that sectors go into overtime under positive excess demand conditions before they can fill the vacancy.

Unlike the Cobb-Douglas production or linear production fucntion which lead to x values of less than one, we observe the values well over 1 in our simulation. This indicates the importance of stochastic interactions among sectors introduced through the device of stochastic holding times.

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Appendix: Case studies

All the cases presented here have $K = 8$.

Case 1 Demand shares =[5,4,3,2,1,1,1,1]/18; The top four sectors have 78 per cent of total demand. We use three initial conditions, $n_i(0) = 90, n_i(0) = 85$, and $n_i(0)$=75.

The coefficients of the Okun's laws are 7.0, 6.6, and 5.4 with the three initial conditions. The average GDPs are Y_{av} =386.5, 364.7, and 321.9 respectively. The amplitudes of the business cycles are 0.26 per cent, .4 per cent, and .59 per cent respectively of the average GDP respectively.

In this case, consistent with demands, fewer number of workers are needed to meet the demands in productive sectors. The sizes of the sectors shrink before settling near the equilibrium levels of production after about 600 basic time steps. This is shown in Fig.1.1 which plots the average output over 8 sectors, Yav, and can be seen also from Fig.1.2, which is the plot of Yav vs $unav$.

The level of output average after 600 steps is about 386 units.

Case 2 The demand pattern of case 1 is reversed [1,1,1,1,2,3,4,5]. About 22 percent of demands fall on the top 4 sectors. Here we report results with the initial condition $n_i(0) = 85$. $x = 5.6$, $Y_{av} = 328.7$, and the amplitude of the business cycle is about .16 per cent of the average GDP.

All other settings are the same as case 1. Fig.2.1 shows that the model settles near stochastic equilibrium after about 1100 time steps. Fig. 2.2 is the scatter diagram of Yav vs $unav$. This case took a lot longer to settle near stochastic equilibrium than case 1.

The value of Yav after 1200 steps is about 178 units.

case 3 Effects of initial starting conditions are examined in case 3 and 4. In case 3, the initial condition, $n_i(0) = 75$, is used for all sectors. The model settles down to a stochastic equilibrium region after 500 steps or so, as in case 1. Fig.3.1 shows the trajectory of Y_{av} towards business cycles. Fig. 3.2 shows a scatter plot of Yav vs $unav$. The average level of output is about 321.1, and is less than that of case 1.

case4 This case is the same as case 2 with the only difference in $n_i(0) = 75$ for all sectors. It takes about 1000 steps to approach a stationary distribution. Yav is about 145, as shown in Fig.4. (The scale is too large to show the fluctuations in GDP in this plot.)

The next two cases, case 5 and 6, compare less concentrated demand pattern of [3,3,4,4,1,1,2,2] with [3,3,1,1,2,2,4,4]. The initial employees are set at $n_i(0) = 85$ for all sectors in both cases. The top 4 sectors occupy 70 per cent, and 40 per cent of total demand.

As expected, Yav of case 6 is less than that of case 5.

case 5 Fig.5.1 shows that the model settles down after 700 steps or so. Fig.5.2 is the scatter plot of Yav and $unan$.

case 6 Fig.6.1 shows that it takes about 1000 steps for the model to settle down. Fig.6.2 is the scatter plot of Yav vs $unav$.

case 7

This case is tried with the initial condition $n_i(0) = 85$. $x = 5.6$, $Y_{av} = 328.7$, and the amplitude of business cycle is about .16 per cent of Y_{av}.