Does Intra-Firm Bargaining Matter for Business Cycle Dynamics?*

Michael U. Krause
Deutsche Bundesbank†

Thomas A. Lubik
Federal Reserve Bank of Richmond‡

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Abstract

We analyse the implications of intra-firm bargaining for business cycle dynamics in models with large firms and search frictions. Intra-firm bargaining implies a feedback effect from the marginal revenue product to wage setting which leads firms to over-hire in order to reduce workers' bargaining position within the firm. The key to this effect are decreasing returns and/or downward-sloping demand. We show that equilibrium wages and employment are higher in steady state compared to a bargaining framework in which firms neglect this feedback. However, the effects of intra-firm bargaining on adjustment dynamics, volatility and comovement are negligible.

1 Introduction

We analyze the aggregate implications of intra-firm bargaining in a fully-fledged, yet simple, general equilibrium business cycle model with search frictions in the labor market. The issue of intra-firm wage bargaining arises whenever the scale of the firm changes non-linearly with its labor input. The two most prominent examples are concave production and downward-sloping demand. The first example has been studied by Smith (1999), Cahuc and Wasmer (2001), Cahuc, Marque, and Wasmer (2004) and Rotemberg (2006). The second example has been analyzed by Ebell and Haefke (2004) and also Rotemberg (2006).

We show how intra-firm bargaining implies a feedback effect in the bargaining process from a firm's marginal product to wage setting. Firms have an incentive to increase production in order to decrease the marginal product, and thus the wages of existing employees, in order to capture higher rents. In effect, firms reduce the bargaining position of the marginal worker by over-hiring. This partial equilibrium scenario, however, implies a general equilibrium feedback effect in that it leads to an expansion in production, and thus higher surplus to be shared among more workers. With a tighter labor market, the additional hiring of firms improves the outside options of workers, and thus raises their wage in general equilibrium.

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†Economic Research Center, Wilhelm-Epstein-Str. 14, D-60431 Frankfurt, Germany. Tel. +49 (0)69 9566-2382. Email: michael.u.krause@bundesbank.de

‡Research Department, 701 E. Byrd St., Richmond, VA 23219, USA. Tel: +1 804 697 8246. Email: Thomas.Lubik@rich.frb.org

The main contribution of this paper lies in the analysis of business cycle dynamics. When compared to a specification that neglects intra-firm bargaining, the dynamic response of the economy to a productivity shock is barely affected. The response of unemployment is slightly magnified, depending on the degree of returns to scale and elasticity of demand. Similarly, employment and vacancies rise slightly more. In this respect, intra-firm bargaining plays a role as the bargaining position of workers improves by less than is mandated by the rise in labor market tightness. However, intra-firm bargaining does not affect the qualitative response of the economy and an overall effect on output is virtually non-existent.

We interpret our findings to the effect that, in many circumstances, researchers may safely ignore intra-firm bargaining even when analyzing business cycle models with large firms that face decreasing returns or downward-sloping demand. This is not meant to imply that there may not be important and interesting effects on the steady state of a model. This has been explored, for example, by Ebell and Haefke (2004). However, if we – falsely – calibrate a model without this strategic feedback on wages to actual data where it is present, the mistake we make is likely to be small. Intra-firm bargaining is not the driving force of significant cyclical dynamics.

The gist of this argument can be illustrated by means of the following static example that abstracts from search and matching frictions. Consider a simple bargaining problem of a large firm that deals with each worker individually. Employed workers bargain over the wage w, with their outside option being unemployment which generates benefits b. The firm's bargaining position is given by the surplus that an additional worker generates, net of its outside option which is the value of leaving the job unfilled. This outside option is zero.

Let the firm's price be p and its output y. The firm pays wage w and employs n workers. Its value is given by its revenue minus cost, which consists of the wage bill and the hiring cost:

$$V = py - wn.$$

Consider first value maximization with respect to employment:[1]

$$\frac{\partial V}{\partial n} = \underbrace{\frac{\partial p}{\partial y}\frac{\partial y}{\partial n}y + p\frac{\partial y}{\partial n}p}_{mr} - \underbrace{\left[\frac{\partial w}{\partial n}n + w\right]}_{mc}.$$

The first term on the right-hand side would not be present if the firm were a price taker in the product market; the first term in square brackets would be absent if the firm were a price taker in the labor market. The latter would equal zero when firms can only hire one worker, or when the firm does not internalize the feedback from its employment choice to the wage schedule. The value of a marginal worker is therefore the difference between marginal revenue and marginal cost, $mr(n) - mc(n)$, which we indicate as depending on the level of employment.

The Nash bargaining solution maximizes the weighted product of the involved parties' surpluses. Given a worker's bargaining weight η, the solution is:

$$w(n) - b = \frac{\eta}{1-\eta}\left[mr(n) - mc(n)\right].$$

[1] For simplicity of exposition we abstract from fixed costs of hiring and intertemporal considerations.

Inserting the marginal cost term and taking account of the dependence of the wage on employment, yields:

$$w(n) = \eta \left[mr(n) - \frac{\partial w(n)}{\partial n} n \right] + (1-\eta)b.$$

The wage is a weighted average of the firm's marginal revenue and the worker's outside option. The second term in brackets captures the effect from intra-firm bargaining. Marginal revenue is adjusted for the feedback of the employment choice on the wage, which in turn affects the optimal number of employees. Stole and Zwiebel (1996) have shown that this prompts the firm to over-hire. This feedback effect crucially relies on the assumption that the firm's marginal revenue function is not independent of employment. Otherwise, as in the basic one-worker one-firm set-up of Pissarides (2000), the wage would not depend on n as $mr(n) = p$, for all n.

In the rest of the paper we proceed as follows. The next section outlines the model under the assumption of decreasing returns to labor and matching frictions in the labor market. This allows us to disentangle the relevant effect without excess complexity. We then add general equilibrium constraints, calibrate the model, and proceed to analyze the steady state and business cycle implications graphically and numerically. In section 4, we discuss the similarities of the results to the case of monopolistic competition, and show the robustness of our findings to its inclusion alongside decreasing returns. The final section concludes and highlights some further connections to the literature.

2 A Business Cycle Model with Search Frictions and Intra-Firm Bargaining

We illustrate the effects of (neglecting) intra-firm bargaining (IFB) by means of a simple model in which production is characterized by decreasing returns to labor and firms are large in the sense that they employ multiple workers. This contrasts with the standard search and matching framework in which production originates in one-worker one-firm pairs. We assume an economy with a continuum of firms that use labor as the only input in production. The production function of a typical firm is given by:

$$y_t = A_t n_t^{\alpha}, \tag{1}$$

where $0 < \alpha \leq 1$, and A_t is a stochastic productivity process common to all firms. n_t is the measure of workers employed by the firm. We assume that all firms behave symmetrically, and consequently suppress firm-specific indices. With the total labor force normalized to one, aggregate employment is identical to firm-level employment. Unemployment is defined as:

$$u_t = 1 - n_t. \tag{2}$$

The labor market is characterized by search and matching frictions encapsulated in the matching function $m(u_t, v_t) = m u_t^{\xi} v_t^{1-\xi}$. It describes the outcome of search behavior of firms and workers in that unemployed job seekers u_t are matched with vacancies v_t at rate

$m(u_t, v_t)$ to produce new employment relationships. $0 < \xi < 1$ is the match elasticity of the unemployed, and $m > 0$ describes the efficiency of the match process. Using the definition of labor market tightness $\theta_t = v_t/u_t$, the aggregate probability of filling a vacancy (taken parametrically by the firms) is $q(\theta_t) = m(v_t, u_t)/v_t$. The evolution of employment is then:

$$n_{t+1} = (1-\rho)[n_t + v_t q(\theta_t)]. \tag{3}$$

$0 < \rho < 1$ is the (constant) separation rate that measures inflows into unemployment.

Firms maximize profits by choosing employment next period and vacancies to be posted, subject to the firm-level employment constraint. This job creation comes at a flow cost $c > 0$. The Bellman equation is:

$$\mathcal{V}(n_t) = \max_{n_{t+1}, v_t} \left\{ A_t n_t^{\alpha} - w(n_t) n_t - c v_t + E_t \beta_t \mathcal{V}(n_{t+1}) \right\}. \tag{4}$$

$\mathcal{V}(\cdot)$ is the value of the firm, β_t is the time-varying discount factor, and $w(n_t)$ is the wage schedule, which will be determined below. The notation indicates that the wage of the marginal worker potentially depends on the existing number of workers in the firm. The first-order conditions are:

$$\begin{aligned} c &= \mu_t (1-\rho) q(\theta_t), \\ \mu_t &= E_t \beta_t \mathcal{V}'_{t+1}(n_{t+1}), \end{aligned}$$

where μ_t is the Lagrange-multiplier on the employment constraint (3). The corresponding envelope condition is:

$$\mathcal{V}'(n_t) = \alpha A_t n_t^{\alpha-1} - w(n_t) - \frac{\partial w(n_t)}{\partial n_t} n_t + E_t \beta_t \mathcal{V}'(n_{t+1}) \frac{\partial n_{t+1}}{\partial n_t}. \tag{5}$$

The presence of the derivative of the wage schedule reflects the impact of intra-firm wage bargaining. When choosing employment, firms take into account how an additional worker affects their bargaining position and thus wage setting.

We define the value of the marginal job $J(n_t) = \mathcal{V}'(n_t)$, and rewrite the envelope condition as an asset equation:

$$J(n_t) = \alpha A_t n_t^{\alpha-1} - w(n_t) - \frac{\partial w(n_t)}{\partial n_t} n_t + (1-\rho) E_t \beta_t J(n_{t+1}). \tag{6}$$

With constant returns to scale, $\alpha = 1$, the marginal product of labor is A_t (the 'one-worker one-firm' case), and the wage is independent of the firm's current employment level. The asset equation then reduces to the one in Pissarides (2000).

Combining this with the first-order conditions results in a vacancy-posting, or job creation, condition:

$$\frac{c}{q(\theta_t)} = (1-\rho) E_t \beta_t J(n_{t+1}), \tag{7}$$

which can alternatively be written as:

$$\frac{c}{q(\theta_t)} = (1-\rho) E_t \beta_t \left[\alpha A_{t+1} n_{t+1}^{\alpha-1} - w(n_{t+1}) - \frac{\partial w(n_{t+1})}{\partial n_{t+1}} n_{t+1} + \frac{c}{q(\theta_{t+1})} \right]. \tag{8}$$

To gain some intuition, suppose firms anticipate an increase in productivity A_{t+1}. This raises the present value of profits and thereby the marginal benefit of hiring more workers at given marginal cost $c/q(\theta_t)$. Other things being equal, more vacancies are posted, and n_{t+1} is expected to be higher, which, in turn, reduces the expected marginal product of labor until equality is restored.

This adjustment is affected by two additional channels. The first takes place within the firm, hence the label intra-firm bargaining. Adding a worker reduces the effective bargaining power of existing workers, and thus their wage. Assuming $E_t \partial w(n_{t+1})/\partial n_{t+1} < 0$, which we will show below to be true, this amplifies the incentive to post vacancies and employment increases further. In order to determine the quantitative significance of this effect, we need to solve for the equilibrium wage schedule $w(n_t)$, which is done below. The other channel is a feedback effect which arises in general equilibrium. As all firms post more vacancies, aggregate vacancies increase, the labor market tightens, and it becomes more costly to recruit additional workers with the rise in $c/q(\theta_t)$. Therefore, employment in each firm increases by less than it would if θ_t where constant.

2.1 Determining the Wage Schedule

Wages are determined based on the Nash bargaining solution: surpluses accruing to the matched parties are split according to a rule that maximizes the weighted average of the respective surpluses. Denoting the workers' weight in the bargaining process as $\eta \in [0, 1]$, this implies the sharing rule:

$$W_t - U_t = \frac{\eta}{1-\eta} J_t, \tag{9}$$

where W_t is the asset value of employment, U_t is the value of being unemployed, and J_t is, as before, the value of the marginal worker to the firm.[2]

The value of employment to a worker is described by the following Bellman equation:

$$W_t = w_t + E_t \beta_t [(1-\rho) W_{t+1} + \rho U_{t+1}]. \tag{10}$$

Workers receive the wage w_t, and transition into unemployment next period with probability ρ. The value of searching for a job, when currently unemployed, is:

$$U_t = b + E_t \beta_t [f_t (1-\rho) W_{t+1} + (1 - f_t(1-\rho)) U_{t+1}]. \tag{11}$$

An unemployed searcher receives benefits b and transitions into employment with probability $f_t(1-\rho)$. The job finding rate f_t is defined as $f(\theta_t) = m(v_t, u_t)/u_t$ which is decreasing in tightness θ_t. It is adjusted for the probability that a completed match gets dissolved before production begins next period.

We substitute the asset equations into the sharing rule (9) and, after some algebra, find the wage equation:

$$w(n_t) = \eta \left[\alpha A_t n_t^{\alpha-1} - \frac{\partial w(n_t)}{\partial n_t} n_t + c\theta_t \right] + (1-\eta) b. \tag{12}$$

[2] In models with one-worker firms, the net surplus of a firm is given by $J_t - V_t$, with V_t the value of a vacant job. By free entry, V_t is then assumed to be driven to zero.

Because of the the presence of the derivative of the wage schedule on account of intra-firm bargaining this a first-order differential equation, the solution of which is:

$$w(n_t) = \frac{\alpha\eta}{1-\eta(1-\alpha)} A_t n_t^{\alpha-1} + \eta c\theta_t + (1-\eta)b. \tag{13}$$

The derivative with respect to employment is given by:

$$\frac{\partial w(n_t)}{\partial n_t} = -\frac{(1-\alpha)\alpha\eta}{1-\eta(1-\alpha)} A_t n_t^{\alpha-2} < 0, \tag{14}$$

which, when inserted into (12), verifies the consistency with the solution.

For given employment, intra-firm bargaining increases the wage by virtue of the scale factor $1/\left[1-\eta(1-\alpha)\right] > 1$. The addition of a worker to the workforce implies a higher value to the firm as it lowers the marginal product of all incumbent workers. A new worker has therefore a higher value to the firm than just his marginal product because he contributes to lowering the firm's wage bill. By the logic of bargaining, the surplus is split, and workers get their share in terms of a higher wage. However, for the very reason that adding workers reduces the wage bill, firms post more vacancies to increase employment. This lowers the marginal impact of adding workers, which is declining in n_t. Thus, workers' marginal product decreases with employment and hence the wage. Equation (13) gives the overall effect of the falling marginal product on the wage, corrected for intra-firm bargaining.[3]

The wage schedule can be used in the job creation condition (8) to yield:

$$\frac{c}{q(\theta_t)} = (1-\rho)E_t\beta_t\left[\frac{(1-\eta)}{1-\eta(1-\alpha)}\alpha A_{t+1}n_{t+1}^{\alpha-1} - \eta c\theta_{t+1} - (1-\eta)b + \frac{c}{q(\theta_{t+1})}\right]. \tag{15}$$

Intra-firm bargaining leads to the term $1/\left[1-\eta(1-\alpha)\right]$ which reflects the firm's internalization of the effect of employment on the wage. It exerts a level effect in that the marginal benefit from adding workers is perceived to be higher. This induces more job creation. For the case of constant returns, $\alpha = 1$, the equation collapses to the usual form, and intra-firm bargaining is irrelevant. However, our argument has so far relied on partial equilibrium reasoning from the perspective of the firm. We will analyze below the general equilibrium feedbacks both on the steady state allocation and on the model's adjustment dynamics.

2.2 Wage Determination without Intra-Firm Bargaining

We assume from the outset that firms internalize the dependence of the wage schedule on employment (see Eqs. (4) and (5)). This allows them to act strategically and extract rents from the workers. As an alternative assume that firms behave myopically by taking the wage of its incumbent workforce as given when choosing employment. This amounts to $\partial w_t/\partial n_t = 0$ in the firms' problem. In this case, the value function of the firm is:

$$J(n_t) = \alpha A_t n_t^{\alpha-1} - w_t + (1-\rho)E_t\beta_t J(n_{t+1}). \tag{16}$$

[3] In a sense, this setup can be interpreted from the perspective of insider-outsider theory: firms are willing to expand employment and incur vacancy costs in order to reduce the bargaining power of insiders. The crucial assumption is that the incumbents' wages are not protected by long-term contracts, but are constantly renegotiated. The term 'bargaining power' is, of course, used loosely in the sense that the Nash bargaining parameter η is fixed.

Following the same steps as outlined above, we find the corresponding wage equation:

$$w_t = \eta\alpha A_t n_t^{\alpha-1} + \eta c\theta_t + (1-\eta)b, \tag{17}$$

and the job creation condition:

$$\frac{c}{q(\theta_t)} = (1-\rho)E_t\beta_t\left[(1-\eta)\alpha A_{t+1}n_{t+1}^{\alpha-1} - \eta c\theta_t - (1-\eta)b + \frac{c}{q(\theta_{t+1})}\right]. \tag{18}$$

When comparing the two job creation conditions, the only algebraic difference is the term multiplying the marginal product of labor, namely $(1-\eta) < (1-\eta)/[1-\eta(1-\alpha)]$. Intra-firm bargaining scales the marginal product of labor and thereby introduces an additional incentive for vacancy posting. The wage equations and job creation conditions under both scenarios will be the reference points from which we evaluate the general equilibrium effects of intra-firm bargaining.

2.3 Closing the Model

We assume that all workers belong to a representative household that insures its members perfectly against income risk implied by the two states of employment and unemployment. By means of a complete internal asset market, incomes are pooled in such a way that all households have the same level of income.[4] Assuming a CRRA-utility function for the household, we can thus construct an implied stochastic discount factor:

$$\beta_t = \beta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}, \tag{19}$$

which firms use to evaluate future revenue streams. $0 < \beta < 1$ is the household's discount factor, and $\sigma > 0$ is the intertemporal elasticity of substitution. c_t is the household's consumption, which draws from production as described by the social resource constraint:

$$c_t = y_t - cv_t. \tag{20}$$

Total hiring costs cv_t are subtracted form gross production as resources are lost in the search process.

3 The General Equilibrium Effects of Intra-Firm Bargaining

This simple search and matching model with concave production provides a laboratory for analyzing the qualitative and quantitative effects of intra-firm bargaining. We proceed in two steps. We first compute the model's steady state and compare allocations across the two wage-setting assumptions. This discussion parallels the results in Cahuc and Wasmer (2001). In the second step, we study the dynamic behavior of the model and the implications for business cycle statistics.

[4] This assumption is standard in the literature following Merz (1995) and Andolfatto (1996). Note that the unemployed enjoy a higher level of utility than the working since they do not suffer the disutility of employment.

In order to fix a baseline for the model's quantitative analysis, we calibrate the parameters to typical values found in the literature.[5] We set the discount factor $\beta = 0.98$, and choose $\sigma = 1$. The mean of the technology process A_t is normalized to unity. We assume that the input elasticity $\alpha = 2/3$, roughly the labor share in U.S. income. The separation rate is fixed at a value of $\rho = 0.1$, which is a mid-point of the range of values used in the literature. The match elasticity ξ is calibrated at 0.4 based on the empirical estimates in Blanchard and Diamond (1989), while the match efficiency parameter $m = 0.4$ is chosen to generate an unemployment rate of roughly 8-10%. To be consistent with this, we fix unemployment benefits b and vacancy creation costs c at 0.1. Finally, the Nash bargaining parameter $\eta = 0.5$, in absence of any supporting empirical evidence.[6]

3.1 Steady State Effects

The model's first-order conditions can be reduced to a two-equation system in unemployment u and vacancies v (see Pissarides, 2000). The first equation is the Beveridge curve, and is derived from the employment accumulation equation (3) in steady state, after substituting the expression for the firm-matching rate $q(\theta)$ and unemployment $n = 1 - u$. After rearranging, this results in a relationship between v and u:

$$v = \left[\frac{\rho(1-u)}{(1-\rho)mu} \right]^{\frac{1}{1-\xi}} u. \tag{21}$$

It is straightforward to show that this relationship is downward-sloping and concave in v-u space.

The second steady-state relationship is derived from the job creation condition (15). Substitution and rearrangement results in the following expression:

$$\frac{1-(1-\rho)\beta}{(1-\rho)\beta} \frac{c}{m} \left(\frac{v}{u}\right)^{\xi} = \frac{(1-\eta)}{1-\eta(1-\alpha)} \alpha A \left(1-u\right)^{\alpha-1} - \eta c \frac{v}{u} - (1-\eta)b, \tag{22}$$

for which no analytical solution in terms of v is available. Instead, we solve this equation numerically for our baseline calibration. It can be shown analytically, though, that the job creation curve is upward-sloping and mildly convex. Consequently, the two curves intersect once, so that the model delivers a unique steady state equilibrium. The two curves determining the steady state are depicted in Figure 1. The figure also contains the job creation curve that neglects the feedback from intra-firm bargaining (IFB), which is derived from (18).

Steady state equilibrium is at the intersection of both curves which yields an unemployment rate of 8.5%. Without IFB, the job creation schedule is flatter and tilts downwards, resulting in steady state unemployment of 10%. This confirms the result by Stole and Zwiebel (1996), subsequently refined by Cahuc and Wasmer (2001) and Ebell and Haefke (2004), that intra-firm bargaining leads to over-hiring. Firms have an incentive to add

[5] A more detailed discussion of the calibration of a closely related model can be found in Krause and Lubik (2006).

[6] Note that this violates the efficiency condition in Hosios (1990). We do not regard this as restrictive for our purposes since, as Cahuc and Wasmer (2001) have shown, the efficiency condition is modified under intra-firm wage bargaining, and second we are not explicitly concerned with welfare considerations.

more employees since the wage paid to all workers is falling in employment. This effect is mitigated by the feedback that hiring has on unemployment, as it raises labor market tightness and thus marginal hiring costs $c/q(\theta)$. Overall, the level of vacancies and employment are higher in the IFB-case since firms can generate higher surplus by diluting the effective bargaining power of their workers.[7]

The same reasoning can be illustrated with an alternative description of the steady state. We use the Beveridge curve to substitute out n in the wage equation (13), from which we derive a relationship between w and θ, labeled the 'wage curve'. The job creation condition can be rewritten in a similar way. Both schedules are depicted in Figure 2. We also plot the two schedules for the specification in which intra-firm bargaining is neglected. The figure shows that both wage and tightness are lower compared to the baseline with IFB.[8] Recall that, for given labor market tightness θ, higher employment allows a firm to reduce wages paid to workers, and to increase overall profits. However, when all firms act in this manner, labor market tightness rises both due to more vacancy postings and to a decline in unemployment. The overall effect on the wage is positive, so that intra-firm bargaining raises wages in general equilibrium, which Figure 2 illustrates.

3.2 Adjustment Dynamics and Business Cycle Statistics

We now turn to an analysis of the effects of intra-firm bargaining on the dynamic properties of the model. In order to do so, we linearize both the baseline specification and the model that neglects IFB around their respective steady states. Strictly speaking, this analysis conflates two effects: the differences in steady state, and the differences in the coefficients in the dynamic model. It is quite conceivable that models with identical steady states can have different dynamic properties. Similarly, differences in responses (which are themselves measured in percentage deviations from the steady state) have to be interpreted with care as they are relative to different steady states. This implied error in our framework is likely to be small since the differences in steady states are small.[9]

The resulting linear rational expectations models are solved using standard techniques. We first compare dynamic adjustment paths towards the steady state after a productivity disturbance. Secondly, we contrast their predictions for business cycle statistics based on simulated data. In order to describe the stochastic properties of the model we have to calibrate the technology process. We assume that productivity A_t follows an AR(1) process with autoregressive coefficient $\rho_A = 0.90$, and driven by a zero mean innovation ε_t with

[7] The underlying mechanism is not a labor supply effect in the traditional sense, which would require *increases* in the wage in order to attract additional workers. More searchers find employment since the increase in vacancy postings reduces labor market tightness, and thus increases the job-finding rate, which is enough to compensate the marginal unemployed worker for the lower wage rate.

[8] Since both schedules are affected under the different specifications, it may be conceivable that, say, the wage increased or decreased. Analytically, the schedules with and without IFB differ by a factor of $1/[1-\eta(1-\alpha)]$ that multiplies the marginal product of labor $\alpha A n^{\alpha-1}$. The schedules thus shift both in the same direction. Only for very small values of θ such a reversal can occur.

[9] The conceptual background we have in mind is that a researcher might ask how much of an error he commits when neglecting intra-firm bargaining. The reason for this neglect might be difficulty in solving differential equations of the type (12), and the possibly burdensome underlying first-order conditions. Alternatively, a researcher may be interested in exploring the implications of myopic behavior by firms that ignores the strategic incentives to expand employment.

variance $\sigma_\varepsilon^2 = 0.007^2$. This value is chosen to replicate the observed U.S. GDP standard deviation of 1.62%.

The impulse response function for both specifications are depicted in Figure 3. Two observations stand out immediately. First, the model exhibits an almost complete lack of internal propagation. The behavior of GDP follows virtually in its entirety the adjustment path of the productivity process. This observation has been emphasized by Krause and Lubik (2006), and is a corollary to the Shimer (2005) argument that the standard search and matching model is unable to replicate the volatility of unemployment and vacancies. Second, and more importantly for our discussion, the responses are remarkably similar in terms of shape, size, and direction. A persistent 1% increase in productivity raises current production and future marginal products of labor. This raises the value of jobs, and thus vacancies posted, per the job creation condition (15). This leads to increased employment in the following period (see equation (3)). Workers experience a rise in wages on account of higher productivity and labor market tightness. However, wages rise by less than productivity because of the strategic hiring decisions by firms. Thus, intra-firm bargaining does not change the basic dynamics of search and matching, but it (slightly) modifies its strength.

We also compare business cycles statistics computed from simulations of the two model specifications. The results are reported in Table 1. The baseline model is calibrated so as to replicate the standard deviation of U.S. GDP; the standard deviations of all other variables are then measured relative to this value. The overall impression is that the cyclical properties of the model with and without intra-firm bargaining are virtually identical. There is no difference in the behavior of output - which has already been apparent from the impulse response functions, and the real wage. However, when intra-firm bargaining is neglected, unemployment, vacancies and tightness are roughly 10% less volatile than in the baseline case. When compared to the corresponding business cycle facts for the U.S. economy, both models fall woefully short: the latter statistics are off by a factor of 10, the wage is 50% more volatile than in the data.

In terms of contemporaneous correlations, both specifications produce identical results. The models are reasonably successful in matching unemployment correlations. A benchmark statistic is the correlation between unemployment and vacancies. The model-implied value of -0.58 is not too far away from the value in U.S. data of -0.95. However, the models produce perfect correlation between the wage, θ, and output, which is inconsistent with the data. Overall, these result support the impression that a model with intra-firm bargaining is essentially observationally equivalent to one without. An empirical, likelihood-based test of both specifications would find it very difficult to distinguish between the two alternatives as they exhibit identical comovement and only minor differences between variable-specific volatilities. While intra-firm bargaining is a conceptually compelling idea, and quite conceivably relevant at the firm level, we conclude that it does not have a significant effect on aggregate dynamics.

4 Monopolistic Competition and Intra-Firm Bargaining

An alternative source of a declining marginal revenue product is downward-sloping demand in an environment with monopolistically competitive firms. Even with linear production, firms would be compelled, and are able to expand hiring since they can capture rents by moving down the demand curve. This assumption has been used, for instance, in New Keynesian models of output and inflation dynamics with search and matching in the labor market. Key examples are Trigari (2004) and Krause and Lubik (2006).

We assume that output of a representative monopolistically competitive firm is linear in labor: $y_t = A_t n_t$, and that each firm faces a downward-sloping demand function for the product variety it produces: $y_t = (p_t/\overline{p}_t)^{-\epsilon} Y_t$, where Y_t is aggregate demand, and $\overline{p}_t$, the aggregate price level, both taken as given by the firm; $\epsilon > 1$ is the substitution elasticity between competing varieties, and p_t is the individual firm's price. The firm's real revenue is then given by:

$$\left(\frac{p_t}{\overline{p}_t}\right) y_t = A_t^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} n_t^{\frac{\epsilon-1}{\epsilon}}. \tag{23}$$

The asset equation for the value of a marginal job can be derived following the same steps as before:

$$J(n_t) = \frac{\epsilon-1}{\epsilon} A_t^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} n_t^{\frac{\epsilon-1}{\epsilon}-1} - w(n_t) - \frac{\partial w(n_t)}{\partial n_t} n_t + (1-\rho) E_t \beta_t J(n_{t+1}). \tag{24}$$

Note that despite linear production, marginal revenue is responds elastically to changes in employment, which opens the possibility of intra-firm bargaining.

The asset equation for workers remain unchanged, and so does the sharing rule. We can consequently derive a wage equation as before:

$$w(n_t) = \eta \left[\frac{\epsilon-1}{\epsilon} A_t^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} n_t^{-\frac{1}{\epsilon}} - \frac{\partial w(n_t)}{\partial n_t} n_t + c\theta_t\right] + (1-\eta) b. \tag{25}$$

The solution to this differential equation is:

$$w(n_t) = \frac{\frac{\epsilon-1}{\epsilon}\eta}{1-\eta(1-\frac{\epsilon-1}{\epsilon})} A_t^{\frac{\epsilon-1}{\epsilon}} Y_t^{\frac{1}{\epsilon}} n_t^{-\frac{1}{\epsilon}} + \eta c\theta_t + (1-\eta) b. \tag{26}$$

It is straightforward to verify that this expression corresponds to the wage equation (13), derived under concave production, if $\alpha = \frac{\epsilon-1}{\epsilon}$. However, this neglects the general equilibrium feedback effect from aggregate demand condition, captured by Y_t, which both parties in the bargaining process take as given. Substituting $Y_t = y_t = A_t n_t$, i.e. assuming a symmetric equilibrium, results in:

$$w_t = \frac{\frac{\epsilon-1}{\epsilon}\eta}{1-\eta/\epsilon} A_t + \eta c\theta_t + (1-\eta) b. \tag{27}$$

The aggregate wage equation is now independent of employment (on account of constant returns in production), but the feedback effect from intra-firm bargaining modifies the productivity coefficient. If IFB is neglected, this coefficient is $\frac{\epsilon-1}{\epsilon}\eta < \frac{\frac{\epsilon-1}{\epsilon}\eta}{1-\eta/\epsilon}$.

This wage equation can be used to derive the job creation condition, which closely parallels (15):

$$\frac{c}{q(\theta_t)} = (1-\rho) E_t \beta_t \left[\frac{(1-\eta)\frac{\epsilon-1}{\epsilon}}{1-\eta/\epsilon} A_{t+1} - \eta c\theta_{t+1} - (1-\eta) b + \frac{c}{q(\theta_{t+1})}\right]. \tag{28}$$

Since the employment equation (3) is unaffected in the monopolistic competition framework, we can describe the steady state solution by reference to Figures 1 and 2. In the former graph, the shape of the curves is unaffected, there is a unique equilibrium, and intra-firm bargaining results in over-hiring, as the job creation curve tilts downward when IFB is neglected. Similarly, the steady state relationships depicted in Figure 2 remain the same qualitatively. In the literature, the substitution elasticity ϵ is often calibrated with a value of 11, which implies a steady state mark-up of 10%. Given our baseline specification with $\eta = 0.5$, the IFB feedback coefficient is $1/(1 - \eta/\epsilon) \approx 1.05$, which is negligible with respect to steady state values and dynamics.

5 A Final Generalization

Concave production and downward-sloping demand do not produce substantial effects of intra-firm bargaining on their own for plausible calibrations. We therefore combine both elements from before in the simple search and matching framework. Following the steps outlined before, the wage equation that takes into account the feedback from IFB, is:

$$w_t = \frac{\alpha\eta\frac{\epsilon-1}{\epsilon}}{1-\eta\left(1-\alpha\frac{\epsilon-1}{\epsilon}\right)}A_t n_t^{\alpha-1} + \eta c\theta_t + (1-\eta)b. \tag{29}$$

The job creation condition is:

$$\frac{c}{q(\theta_t)} = (1-\rho)E_t\beta_t\left[\frac{(1-\eta)\alpha\frac{\epsilon-1}{\epsilon}}{1-\eta\left(1-\alpha\frac{\epsilon-1}{\epsilon}\right)}A_{t+1}n_{t+1}^{\alpha-1} - \eta c\theta_{t+1} - (1-\eta)b + \frac{c}{q(\theta_{t+1})}\right]. \tag{30}$$

The specification without IFB results in the same equations, the difference being the denominator of the term pre-multiplying the marginal product of labor. The scale factor which measures the feedback from IFB is now $1/\left[1-\eta\left(1-\alpha\frac{\epsilon-1}{\epsilon}\right)\right]$. This factor is increasing in η, decreasing in α, and decreasing in ε. In other words, intra-firm bargaining affects steady state allocations and business cycle dynamics more in economies in which workers enjoy higher bargaining power (large η), the labor share of income is small (low α), and markets are not very competitive (low ε).[10]

We illustrate the role of IFB in the extended model by a few numerical examples, which are reported in Table 2. We compute various model statistics for variations of the parameters affecting the scale factor. In particular, we contrast our baseline calibration with a high worker bargaining parameter ($\eta = 0.9$), a lower labor share ($\alpha = 0.5$), and inelastic demand ($\varepsilon = 2$). We first note that for an extreme parameterization, shown in the tight-most column, the scale factor goes up to 3, compared to a baseline of 1.25. That this implies stronger effects of IFB is confirmed by the percentage increase of steady state employment and wage over the case when IFB is neglected, as the percentage change is monotonically related to the scale factor. For baseline bargaining power, the change in employment is, however, fairly small, but more substantial for wages. With higher worker bargaining power, these numbers increase dramatically. What the percentages hide, however, are the actual

[10] This reasoning underlies Ebell and Haefke's (2004) finding that product market deregulation can have substantial employment and welfare effects. In fact, their implied values for the substitution elasticity is $\varepsilon = 3$.

steady state levels. The second row in the table shows that employment actually falls with increases in the scale factor.

An increase in the scale factor also has a monotonic effect on the percentage change in the standard deviation of labor market tightness. For a given parameterization, the inclusion of intra-firm bargaining improves the predictive power of the model as far as the volatility of key labor market variables is concerned. However, this scale factor effect again masks the fact that with high η and low ε the standard deviation of θ is implausibly low. We conclude that the combination of concave production and downward-sloping demand can increase the strength of the feedback effect of intra-firm bargaining. From a pure calibration perspective, there is, however, a trade-off between 'maximizing' the IFB effect and the plausibility of key model predictions. For empirically relevant parameter values, the IFB effect still remains negligible as far as business cycle dynamics are concerned.

6 Conclusions

Intra-firm bargaining yields a strategic incentive for firms to expand employment in order to weaken their workers' bargaining position. This expands employment and raises wages in general equilibrium because lower unemployment and higher vacancies posted raise workers' outside options, offsetting the partial equilibrium effect. While this is a conceptually compelling story of hiring behavior at a microeconomic level, we have shown in this paper that the aggregate effects of intra-firm bargaining are negligible in standard search and matching framework with concave production and downward-sloping product demand.

The results in this paper should not be taken to imply that we regard intra-firm bargaining as irrelevant *per se*. The specification that combines both sources of declining marginal revenue product shows that somewhat extreme, but still plausible calibrations can imply large effects. This raises a few questions for further research. Given aggregate data, do the restrictions implied by an IFB specification help with parameter specification? Specifically, the bargaining parameter η is difficult to pin down. Furthermore, it is often difficult to fit the the behavior of the marginal product of labor, which might be ameliorated by the inclusion of the scale factor. A related question is to what extent it is possible to distinguish between the two specification in aggregate data. A second line of research delves deeper into the production side. Cahuc, Marque, and Wasmer (2004) have shown that intra-firm bargaining has different effects in models with capital and heterogenous labor. Depending on the bargaining power of workers, it may actually lead to underemployment. Their analysis, however, is restricted to the steady state only.

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Table 1: Business Cycle Statistics

Standard Deviations

	u	v	θ	w	y
Intra-Firm Bargaining	0.78	0.95	1.55	0.98	1.62
Neglecting IFB	0.68	0.84	1.36	1.02	1.62
U.S. Data	6.90	8.27	14.96	0.69	1.62

Correlations

	u	v	θ	w	y
u	1	-0.58	-0.85	-0.84	-0.86
v		1	0.91	0.92	0.91
θ			1	0.99	0.99
w				1	0.99
y					1

Table 2: Intra-Firm Bargaining: Robustness

	$\eta = 0.5$		$\eta = 0.9$	
	$\alpha = 2/3$ $\varepsilon = 11$	$\alpha = 2/3$ $\varepsilon = 2$	$\alpha = 2/3$ $\varepsilon = 2$	$\alpha = 1/2$ $\varepsilon = 2$
Scale Factor	1.25	1.50	2.50	3.08
Employment w/ IFB	0.92	0.88	0.73	0.72
% Increase due to IFB				
Employment	3.7	6.0	35.2	41.2
Wage	30.6	48.4	137	167
Std. Deviation of θ				
Rel. to Output	1.60	1.90	0.55	0.53
% Increase due to IFB	16.8	35.7	52.8	60.6

Figure 1: The Steady State Effects of Intra-Firm Bargaining

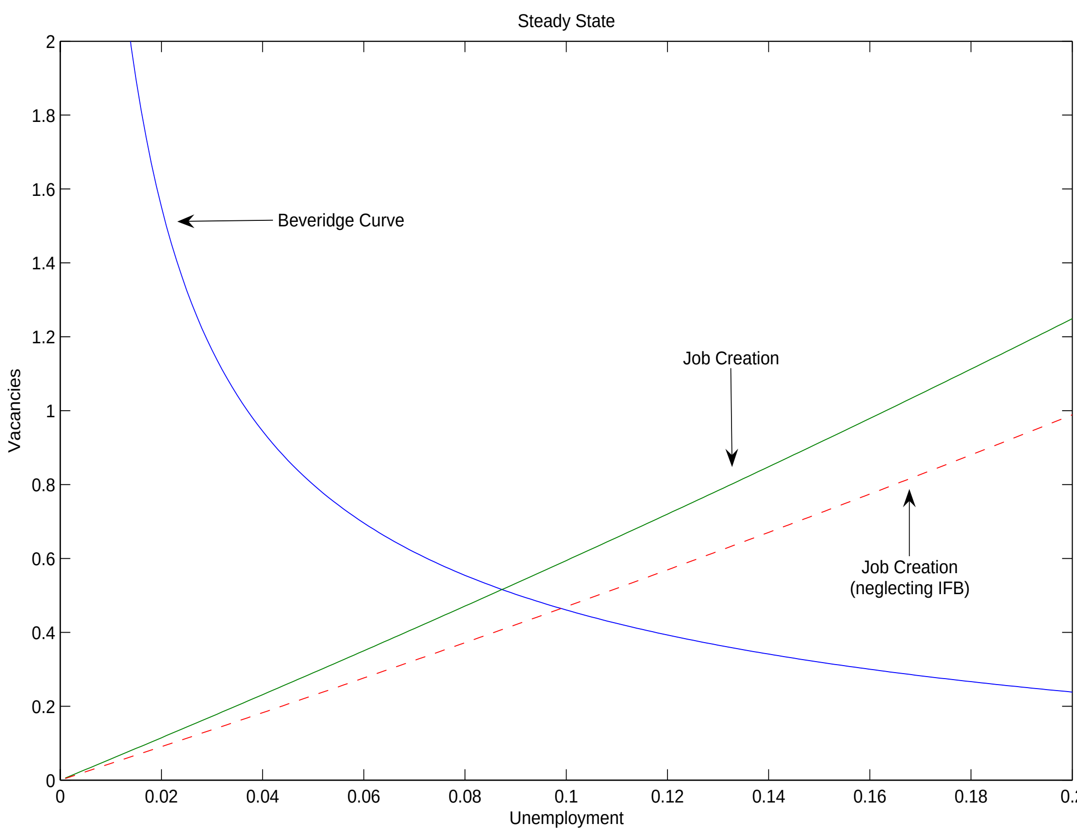


Figure 2: Wage Determination in Steady State

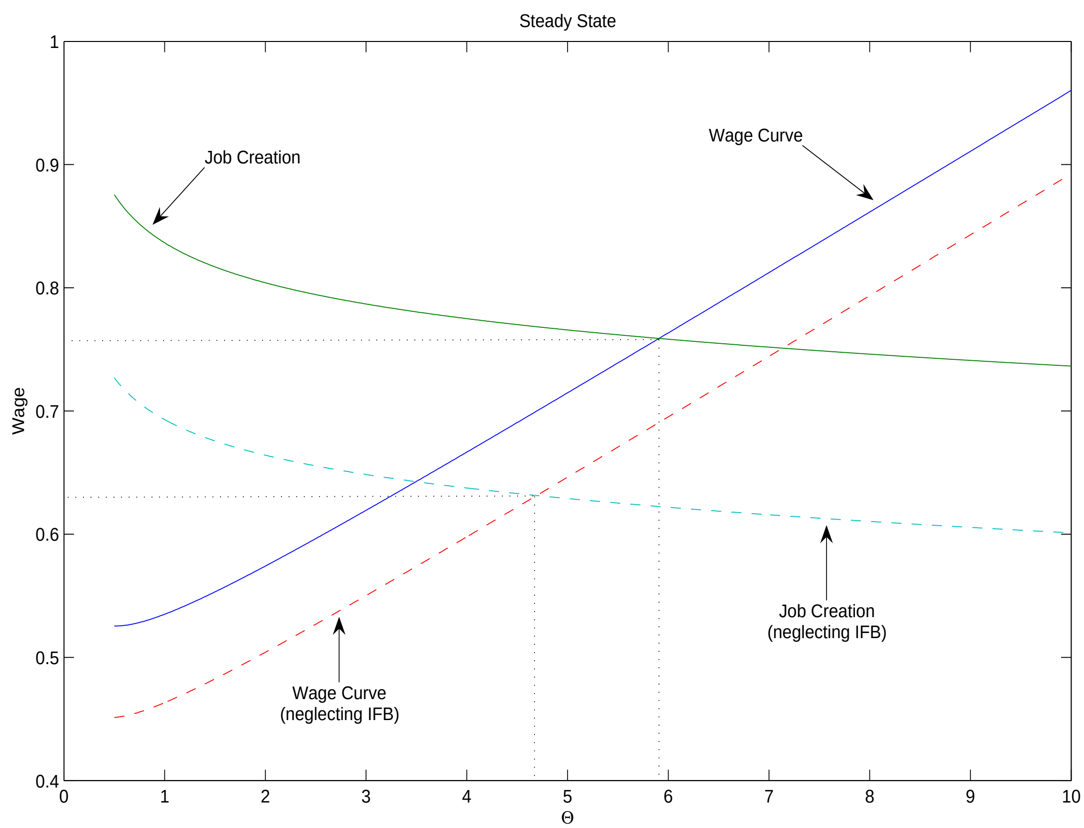
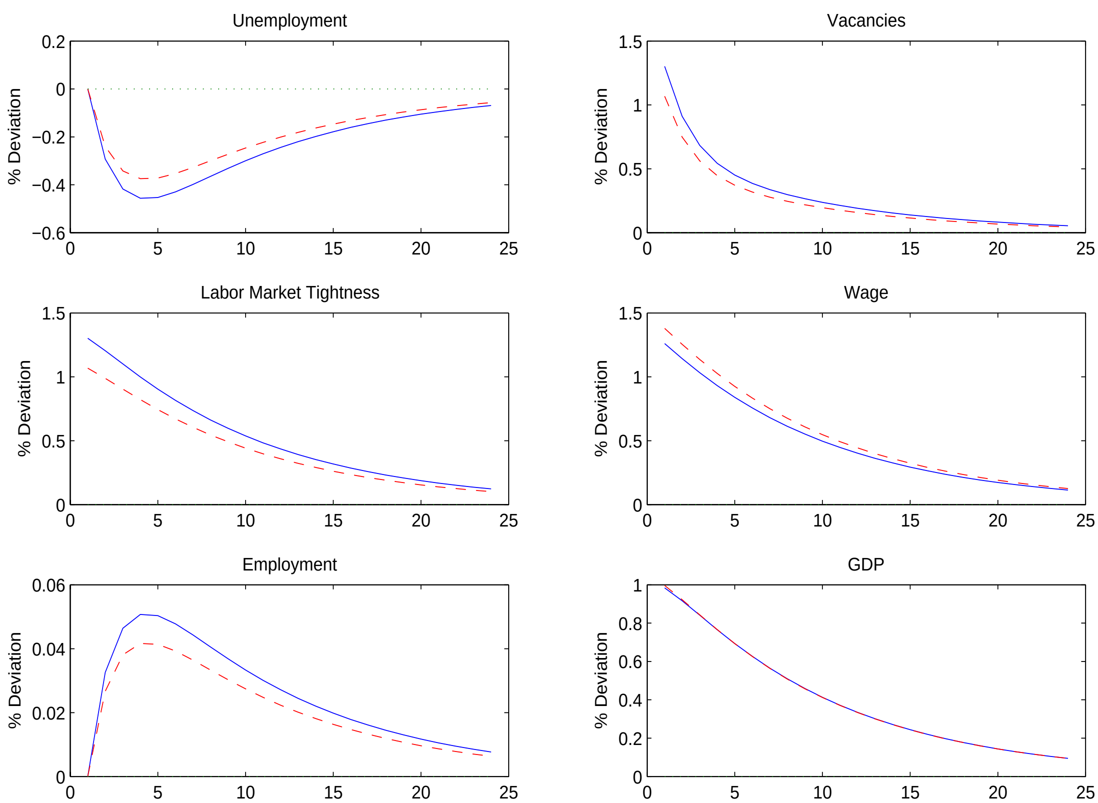


Figure 3: The Dynamic Effects of Intra-Firm Bargaining



Notes: Impulse responses to a 1% productivity shock. Solid lines refer to the baseline, dashed lines to the model without IFB