Flat Tax Reforms in the U.S.: A Boon for the Income Poor

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Summary: We use a version of the neoclassical growth model economy to evaluate two revenue neutral flat-tax reforms. In the less progressive flat-tax reform the households face a 22 percent integrated flat tax and a labor income tax exemption of $16,000 per household. In the more progressive flat-tax reform the flat-tax rate is 29 percent and the labor income tax exemption is $32,000 per household. The households in our economy have identical preferences, they are altruistic towards their descendants and they go through the life cycle stages of working-age and retirement. The benchmark model economy replicates the main features of the current U.S. tax and transfer systems, and it accounts for the main aggregate and distributional features of the U.S. economy in very much detail. We find that both reforms result in a significant increase in wealth inequality. We also find that while the less progressive reform is expansionary (output increases by 2.4 percent), the less progressive reform is contractionary (output decreases by 2.6 percent). On the other hand, while the less progressive tax reform results in a more unequal distribution of income after taxes (its Gini index increases from 0.510 in the benchmark model economy to 0.524), the more progressive tax reform is significantly more egalitarian (the Gini index of its after-tax income distribution is only 0.497). Finally, we compute the steady-state welfare costs of the reforms and we find that equality wins the trade-off: in the less progressive reform aggregate welfare falls by −0.17 percent of consumption, and in the more progressive reform it increases by +0.45 percent of consumption. Both flat-tax reforms result in significant boons for the income poor who pay less income taxes and obtain sizeable welfare gains.

Keywords: Flat-tax reforms; Efficiency; Inequality; Earnings distribution; Income distribution; Wealth distribution.

JEL Classification: D31; E62; H23

*Díaz-Giménez, Universidad Carlos III de Madrid and CAERP <kueli@eco.uc3m.es> and Pijoan-Mas, CEMFI and CEPR <pijoan@cemfi.es>. This paper follows up from the research conducted by Ana Castañeda for her PhD Dissertation and from later work with José-Victor Ríos-Rull. Ana is no longer in academics and has gracefully declined to sign this paper, which is hers as much as ours. We have benefitted enormously both from her input and from her code. José-Victor moved on to other projects when we were well into this research. To both of them we are most grateful. Díaz-Giménez gratefully acknowledges the financial support of the Fundación de Estudios de Economía Aplicada (FEDEA) and of the Spanish Ministerio de Ciencia y Tecnología (Grant SEC2002-004318).

1 Introduction

The debate on fundamental tax reforms is heating up as some countries, mostly in Eastern Europe, are starting to adopt flat-tax systems. In recent years academic work has simulated the consequences of such reforms for the U.S. economy. Starting with the seminal book of Hall and Rabushka (1995), academics have been pushing for a simplification of the tax code, a broadening of the tax base and a reduction of marginal taxes. Ventura (1999) was among the first to carry out a quantitative general equilibrium simulation of a flat-tax reform for the U.S. economy. He concluded that flat-tax reforms generate large gains in output and productivity at the expense of significant increases in inequality. Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), in a complementary exercise, also found that a flat-tax reform of the current U.S. income tax system would increase aggregate output. In their analysis, they showed that the very income poor as well as the very rich would benefit from the reform at the expense of the middle classes. This paper follows closely in their tracks, but it simulates the flat-tax reforms in a model economy that replicates the main features of the current U.S. tax and transfer system and that accounts for the main aggregate and distributional features of the U.S. economy in much greater detail.

Specifically, we quantify the aggregate, distributional and welfare consequences of two revenue neutral fundamental flat-tax reforms in a version of the neoclassical growth model that we calibrate to the U.S. economy. Our model economy is an extension of the model economy described in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Its main features are the following: The households have identical preferences and they are altruistic towards their descendants. They live exponential life-times which they start out as workers and they end up as retirees. When a retired household leaves the economy it is replaced by a working-age descendant that inherits its estate and part of its earning ability. Working-age households also face a non-insurable idiosyncratic shock to their endowments of efficiency labor units. Every household makes optimal consumption, labor and saving decisions. Every firm behaves competitively and every price is flexible.

We also model the U.S. tax system and the lump-sum part of U.S. transfers in very much detail. The model economy firms pay a payroll tax and a capital income tax. The model economy households pay a payroll tax, an income tax, a consumption tax and an estate tax. Every tax instrument in the model economy is designed to replicate the main features and to collect the same revenues as the corresponding tax instrument in the U.S. economy.

To simulate the flat-tax reform, we replace our versions of the corporate income tax and the personal income tax with an integrated flat-tax on all incomes. To make its average rates progressive, the labor income part of the flat-tax has a large exemption. We study two revenue neutral reforms that differ in their flat-tax rate and in the amount of the labor income tax exemption. In the first flat-tax reform the households face a 22 percent integrated flat-tax and a labor income tax exemption of $16,000 per household. In the second flat-tax reform the flat-tax rate is 29 percent and the labor income tax exemption is $32,000 per household. For obvious reasons, we call the first reform the less progressive flat-tax reform and we call the second reform the more progressive flat-tax reform.

We find that the two flat-tax reforms have very different steady-state aggregate, distributional and welfare consequences. The less progressive flat-tax reform is more efficient than the current progressive tax system. Under this reform, aggregate output increases by 2.4 percent and labor productivity increases by 3.2 percent, when compared with the corresponding values of the benchmark model economy. In contrast, the more progressive flat-tax reform is less efficient than the current progressive income tax system. Under this reform, aggregate output and labor productivity are is 2.6 percent smaller 1.4 percent smaller than the corresponding values of the benchmark model economy.

We also find that both reforms result in a significant increase in wealth inequality. Under the current income tax system the Gini index of wealth in our benchmark model economy is 0.818 (it is 0.803 in the U.S. economy according to the 1998 Survey of Consumer Finances). Under the less progressive flat-tax system the Gini index of wealth increases to 0.839, and under the more progressive flat tax system it increases further to 0.845.

Other distributional implications of the reforms are very different. The less progressive flat-tax reform results in more unequal distributions of earnings and, more importantly, of after-tax income (their Gini indexes are 0.613 and 0.524), while the distributions of earnings and after-tax income under the more progressive flat-tax reform are more egalitarian (their Gini indexes are 0.610 and 0.497). Therefore, a policymaker who tried to decide between these reforms would face the classical trade-off between efficiency and equality. Which economy should she choose? The more efficient but less egalitarian model economy E_1, or the less efficient but more egalitarian model economy E_2?

To quantify this trade-off and to answer this question, we compare the steady-state welfare of our three model economies using a Benthamite social welfare function. It turns out that the less progressive tax-reform results in a steady-state welfare loss equivalent to -0.17 percent of consumption, and that the more progressive tax-reform results in a steady-state welfare gain equivalent to $+0.45$ percent of consumption. Finally, we compute the individual welfare changes for each household-type and we find that both reforms are a significant boon for the income poor. Specifically, the households in the bottom 40 percent of the after-tax income distribution of the benchmark model economy would be happier under the less progressive flat-tax reform, and this percentage of happier households increases to an impressive 70 percent under the more progressive flat-tax reform.

A detailed description of the model economy and its calibration, and an intuitive analysis of our findings follows in the ensuing pages.

2 The model economy

The model economy analyzed in this article is a modified version of the stochastic neoclassical growth model with uninsured idiosyncratic risk and no aggregate uncertainty. The key features of our model economy are the following: *(i)* it includes a large number of households with identical preferences; *(ii)* the households face an uninsured, household-specific shock to their endowments of efficiency labor units; *(iii)* the households go through the life cycle stages of working-age and retirement; *(iv)* retired households face a positive probability of dying, and when they do so they are replaced by a working-age descendant; and *(v)* the households are altruistic towards their descendants.

2.1 Population dynamics and information

We assume that our model economy is inhabited by a continuum of households. The households can either be of working-age or they can be retired. Working-age households face an uninsured idiosyncratic stochastic process that determines the value of their endowment of efficiency labor units. They also face an exogenous and positive probability of retiring. Retired households are endowed with zero efficiency labor units. They also face an exogenous and positive probability of dying. When a retired household dies, it is replaced by a working-age descendant who inherits the deceased household estate, if any, and, possibly, some of its earning abilities. We use the one-dimensional shock, s, to denote the household's random age and random endowment of efficiency labor units jointly . We assume that this process is independent and identically distributed across households, and that it follows a finite state Markov chain with conditional transition probabilities given by $\Gamma_{SS} = \Gamma(s' \mid s) = Pr\{s_{t+1} = s' \mid s_t = s\}$, where s and $s' \in S = \{1, 2, \ldots, n_s\}$.

We assume that every household is endowed with ℓ units of disposable time, and that the joint age and endowment shock s takes values in one of two possible J–dimensional sets, $s \in S = \mathcal{E} \cup \mathcal{R} = \{1, 2, \ldots, J\} \cup \{J{+}1, J{+}2, \ldots, 2J\}$. When a household draws shock $s \in \mathcal{E}$, we say that it is of working-age, and we assume that it is endowed with $e(s) > 0$ efficiency labor units. When a household draws shock $s \in \mathcal{R}$, we say that it is retired, and we assume that is is endowed with zero efficiency labor units. We use the $s \in \mathcal{R}$ to keep track of the realization of s that the household faced during the last period of its working-life. This knowledge is essential to analyze the role played by the intergenerational transmission of earnings ability in this class of economies.

The notation described above allows us to represent every demographic change in our model economy as a transition between the sets $\mathcal{E}$ and $\mathcal{R}$. When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$, we say that it has retired. When it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$, we say that it has died and has been replaced by a working-age descendant. Moreover, this specification of the joint age and endowment process implies that the transition probability matrix Γ_{SS} controls: *(i)* the demographics of the model economy, by determining the expected durations of the households' working-lives and retirements; *(ii)* the life-time persistence of earnings, by determining the mobility of households between the states in $\mathcal{E}$; *(iii)* the life cycle pattern of earnings, by determining how the endowments of efficiency labor units of new entrants differ from those of senior working-age households; and *(iv)* the intergenerational persistence of earnings, by determining the correlation between the states in $\mathcal{E}$ for consecutive members of the same dynasty. In Section 3.1.2 we discuss these issues in detail.

We assume that every household inherits the estate of the previous member of its dynasty at the beginning of the first period of its working-life. Specifically, we assume that when a retired household dies, it does so after that period's consumption and savings have taken place. At the beginning of the following period, the deceased household's estate is liquidated, and the household's descendant inherits a fraction $1 - \tau_e(z_t)$ of this estate. The rest of the estate is instantaneously and costlessly transformed into the current period consumption good, and it is taxed away by the government. Note that variable z_t denotes the value of the households' stock of wealth at the end of period t.

2.2 Preferences

We assume that households value their consumption and leisure, and that they care about the utility of their descendents as much as they care about their own utility. Consequently, the households' preferences can be described by the following standard expected utility function:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \; u(c_t, \ell - h_t) \mid s_0 \right\}, \tag{1}$$

where function u is continuous and strictly concave in both arguments; $0 < \beta < 1$ is the time-discount factor; $c_t \geq 0$ is consumption; ℓ is the endowment of productive time; and $0 \leq h_t \leq \ell$ is labor. Consequently, $\ell - h_t$ is the amount of time that the households allocate to non-market activities.

2.3 Production possibilities

We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t)$. Aggregate capital is obtained aggregating the wealth of every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, δ, and we use r and w to denote the prices gross of all taxes of, respectively, capital and labor.

2.4 The government sector

We assume that the government in our model economies taxes households' capital income, labor income, consumption and estates, and that it uses the proceeds of taxation to make real transfers to retired households and to finance an exogenously given level of government consumption.

Capital income taxes are described by function $\tau_k(y_k)$, where y_k denotes capital income; labor income taxes are described by function $\tau_l(y_a)$, where y_a denotes the labor income tax base; social security contributions paid by firms are described by function $\tau_{sf}(y_l)$ where y_l denotes labor income; social security contributions paid by households are described by function $\tau_{sh}(y_l)$; household income taxes are described by function $\tau_y(y_b)$, where y_b denotes the household income tax base; consumption taxes are described by function $\tau_c(c)$; and estate taxes are described by function $\tau_e(z_t)$; and public transfers are described by function $\omega(s_t)$. Therefore, in our model economies, a government policy rule is a specification of $\{\tau_k(y_k),\ \tau_l(y_a),\ \tau_{sf}(y_l),\ \tau_{sh}(y_l),\ \tau_y(y_b),\ \tau_c(c),\ \tau_e(z_t),\ \omega(s_t)\}$ and of a process on government consumption, $\{G_t\}$. Since we also assume that the government must balance its budget every period, these policies must satisfy the following restriction:

$$G_t + Z_t = T_t, \tag{2}$$

where Z_t and T_t denote aggregate transfers and aggregate tax revenues, respectively.[1]

[1]Note that social security in our model economy takes the form of transfers to retired households, and that these transfers do not depend on past contributions made by the households. We make this assumption

2.5 Market arrangements

We assume that there are no insurance markets for the household-specific shock.[2] Moreover, we also assume that the households in our model economy cannot borrow.[3] Partly to buffer their streams of consumption against the shocks, the households can accumulate wealth in the form of real capital, a_t. We assume that these wealth holdings belong to a compact set $\mathcal{A}$. The lower bound of this set can be interpreted as a form of liquidity constraints or, alternatively, as the solvency requirement mentioned above. The existence of an upper bound for the asset holdings is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference. This condition is necessarily satisfied in equilibrium.[4] Finally, we assume that firms rent factors of production from households in competitive spot markets. This assumption implies that factor prices are given by the corresponding marginal productivities.

2.6 The households' decision problem

The individual state variables are the shock realization s and the stock of assets a.[5] The problem that the household solves is:

$$v(a,s) \;=\; \max_{\substack{c \geq 0 \\ z \in \mathcal{A} \\ 0 \leq h \leq \ell}} \; u(c, \ell - h) \;+\; \beta \sum_{s' \in S} \Gamma_{ss'} \; v[a', s'(z)], \tag{3}$$

$$\text{s.t.} \qquad c + z = y - \tau + a, \tag{4}$$

$$y = a\,r + e(s)\,h\,w + \omega(s), \tag{5}$$

$$\tau = \tau_k(y_k) + \tau_l(y_a) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c), \tag{6}$$

in part for technical reasons. Discriminating between the households according to their past contributions to a social security system requires the inclusion of a second asset-type state variable in the household decision problem, and this increases the computational costs significantly.

[2]This is a key feature of this class of model worlds. When insurance markets are allowed to operate, our model economies collapse to a standard representative household model, as long as the right initial conditions hold. In a recent article, Cole and Kocherlakota (1997) have studied economies of this type with the additional characteristic that private storage is unobservable. They conclude that the best achievable allocation is the equilibrium allocation that obtains when households have access to the market structure assumed in this article. We interpret this finding to imply that the market structure that we use here could arise endogenously from certain unobservability features of the environment —specifically, from both the realization of the shock and the amount of wealth being unobservable.

[3]Given that leisure is an argument in the households' utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world.

[4]Huggett (1993) and Marcet, Obiols-Homs, and Weil (2003) prove this proposition.

[5]Since there is no aggregate uncertainty and since we only look at steady states, there are no aggregate state variables.

$$a'(z) = \begin{cases} z - \tau_e(z) \text{ if } s \in \mathcal{R} \text{ and } s' \in \mathcal{E}, \\ z \text{ otherwise.} \end{cases} \tag{7}$$

where v denotes the households' value function, r denotes the interest rate on assets, and w denotes the wage rate. Note that the definition of income, y, includes three terms: capital income, $y_k = a\,r$, that can be earned by every household; labor income, $y_l = e(s)\,h\,w$, that can be earned only by working-age households —recall that $e(s) = 0$ when $s \in \mathcal{R}$; and social security income, $\omega(s)$, that can be earned only by retired households —recall that $\omega(s) = 0$ when $s \in \mathcal{E}$. The household policy that solves this problem is a set of functions that map the individual state into choices for consumption, gross savings, and hours worked. We denote this policy by $\{c(a, s),\, z(a, s),\, h(a, s)\}$.

2.7 Equilibrium

Each period the economy-wide state is a probability measure, x_t, defined over $\mathcal{B}$, an appropriate family of subsets of $S \times \mathcal{A}$ that counts how many household are of each type. The steady-state has the property that the measure of households remains invariant, even though both the state variables and the actions of the individual households change from one period to the next.[6]

Definition 1 *A steady state equilibrium for this economy is a household value function, $v(a, s)$; a household policy, $\{c(a, s),\, z(a, s),\, h(a, s)\}$; a government policy, $\{\tau_k(y_k)$,$\tau_l(y_a)$, $\tau_{sf}(y_l)$,$\tau_{sh}(y_l)$,$\tau_l(y_b)$,$\tau_c(c)$, $\tau_e(z)$,$\omega(s)$,$G\}$; a stationary probability measure of households, x; factor prices, (r, w); and macroeconomic aggregates, $\{K, L, T, Z\}$, such that:*

(i) *When households take factor prices and the government policy as given, the household value function, and the household policy solve the households' decision problem described in expression (3).*

(ii) *Firms also behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities:*

$$r = f_1\,(K, L) - \delta \quad \textit{and} \quad w = f_2\,(K, L)\,. \tag{8}$$

where K and L denote the aggregate capital and labor inputs, respectively.

[6] See Hopenhayn and Prescott (1992) and Huggett (1993)

(iii) *Factor inputs, tax revenues, and transfers are obtained aggregating over households:*

$$
\begin{aligned}
K &= \int a \; dx && (9)\\
L &= \int h(a,s)\; e(s)\; dx && (10)\\
T &= \int [\tau_k(y_k) + \tau_l(y_a) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c)] dx + && (11)\\
& \int \mathbf{I}_{s\in\mathcal{R}} \gamma_{s\mathcal{E}} \tau_e(z)\, z(a,s) dx && (12)\\
Z &= \int \omega(s)\; dx. && (13)
\end{aligned}
$$

where household income, $y(a,s)$, is defined in equation 5; $\mathbf{I}$ denotes the indicator function; $\gamma_{s\mathcal{E}} \equiv \sum_{s'\in\mathcal{E}} \Gamma_{s,s'}$; and, consequently, $(\mathbf{I}_{s\in\mathcal{R}}\, \gamma_{s\mathcal{E}})$ is the probability that a household of type s dies —recall that this probability is 0 when $s \in \mathcal{E}$, since we have assumed that working-age households do not die. All integrals are defined over the state space $S \times \mathcal{A}$.

(iv) *The goods market clears:*

$$
\int \; [c(a,s) + z(a,s)] \;\; dx + G = f\,(K, L) + (1-\delta)\; K. \tag{14}
$$

(v) *The government budget constraint is satisfied:*

$$
G + Z = T \tag{15}
$$

(vi) *The measure of households is stationary:*

$$
x(B) = \int_B \left\{ \int_{S,\mathcal{A}} \left[\mathbf{I}_{z(a,s)} \;\; \mathbf{I}_{s\notin\mathcal{R}\vee s'\notin\mathcal{E}} + \mathbf{I}_{[1-\tau_e(z)]z(a,s)} \;\; \mathbf{I}_{s\in\mathcal{R}\wedge s'\in\mathcal{E}} \right] \;\; \Gamma_{s,s'} \;\; dx \right\} dz \; ds' \tag{16}
$$

for all $B \in \mathcal{B}$, where $\vee$ and $\wedge$ are the logical operators "or" and "and". Equation (16) counts the households, and the cumbersome indicator functions and logical operators are used to account for estate taxation. We describe the procedure that we use to compute this equilibrium in Section B of the Appendix.

3 Calibration

To calibrate our model economy we must choose the functional forms and parameters that describe its preferences, technology, government policy and its age and endowment of efficiency labor units process. When all is told, this amounts to choosing the values of a total of 42 parameters. In order to make these choices we impose 6 normalization conditions and we target 36 statistics that describe the relevant features of the U.S. economy. Ten of these statistics describe the U.S. macroeconomy, 11 describe the current U.S. fiscal policy and the remaining 15 describe the Lorenz curves of the U.S. earnings and wealth distributions. A detailed discussion of this last, non-standard feature of our calibration procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

3.1 Functional forms and parameters

3.1.1 Preferences

Our choice for the households' common utility function is

$$u(c, l) = \frac{c^{1-\sigma_1}}{1-\sigma_1} + \chi \frac{(\ell - l)^{1-\sigma_2}}{1-\sigma_2} \tag{17}$$

We make this choice because the households in our model economies face very large changes in productivity which would result in extremely large variations in hours worked, had we chosen the standard non-separable preferences. Therefore, to characterize the households' preferences we must choose the values of five parameters: the four utility function parameters and of the time discount factor, β.[7]

3.1.2 The joint age and endowment of efficiency labor units process

In Section 2.1, we have assumed that the joint age and endowment of efficiency labor units process, $\{s\}$, takes values in set $S = \{\mathcal{E} \cup \mathcal{R}\}$, where $\mathcal{E}$ and $\mathcal{R}$ are two J-dimensional sets. Since the number of realizations of this process is $2J$, to specify it completely we must choose $(2J)^2 + J$ parameters. Of these parameters, $(2J)^2$ correspond to the transition probability matrix on s, and the remaining J correspond to the endowments of efficiency labor units, $e(s)$.[8] However, we impose some additional restrictions on the transition probability matrix of process $\{s\}$, Γ_{SS}, that reduce the number of parameters to only J^2+J+4.

To understand these restrictions better, it helps to consider the following partition of this matrix:

$$\Gamma_{SS} = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix} \tag{18}$$

[7] Note that we have assumed that retired households do not work and, consequently, the second term in expression (17) becomes an irrelevant constant for these households.

[8] Recall that we have assumed that $e(s) = 0$ for all $s \in \mathcal{R}$.

Submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ contains the transition probabilities of working-age households that are still of working-age one period later. Since we impose no restrictions on these transitions, to identify submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ we must choose the values of J^2 parameters.

Submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states. The value of this matrix is $\Gamma_{\mathcal{E}\mathcal{R}} = p_{e\varrho}I$, where $p_{e\varrho}$ is the probability of retiring and I is the identity matrix. This is because we assume that that every working-age household faces the same probability of retiring and because, to keep track of the earnings ability of retired households, we use only the realization of their last working-age period. Therefore, to identify submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ we must choose the value of only one parameter.

Submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retired household dies and it is replaced by its working-age descendant. The rows of this matrix contain a two parameter transformation of the stationary distribution of $s \in \mathcal{E}$, which we denote $\gamma^*_{\mathcal{E}}$. We have designed this transformation to approximate both the life-cycle profile and the intergenerational correlation of earnings. Intuitively, our procedure amounts to shifting the probability mass from $\gamma^*_{\mathcal{E}}$ both towards the first row of $\Gamma_{\mathcal{R}\mathcal{E}}$ and towards its diagonal. The definitions of the two shift parameters can be found in Appendix A and a detailed justification of our procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Consequently, to identify submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ we must choose the value of two parameters.

Finally, submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ contains the transition probabilities of retired households that are still retired one period later. The value of this submatrix is $\Gamma_{\mathcal{R}\mathcal{R}} = p_{\varrho\varrho}I$, where $(1-p_{\varrho\varrho})$ is the probability of dying. This is because we assume that every retired household faces the same probability of dying and because the type of retired households never changes. Again, to identify this submatrix we must choose the value of only one parameter.

To keep the dimension of process $\{s\}$ as small as possible while still being able to achieve our calibration targets, we choose $J=4$. Therefore, to specify process $\{s\}$, we must choose the values of $J^2+J+4=20$ parameters.[9]

3.1.3 Technology

In the U.S. after World War II, the real wage has increased at an approximately constant rate —at least until 1973— and factor income shares have displayed no trend. To account for these two properties, we choose a standard Cobb-Douglas aggregate production function in capital and labor. Therefore, to specify the aggregate technology, we must choose the values

[9]Note that in counting these parameters we have not yet required that that Γ_{SS} must be a Markov matrix.

of two parameters: the capital share of income, θ, and the depreciation rate of capital, δ.

3.1.4 Government Policy

To describe the government policy in our benchmark model economy, we must choose the capital income tax function, the payroll tax function, the household income tax function, the estate tax function, the consumption tax function, and the values of government consumption, G, and of the transfers to retired households, $\omega(s)$.

Capital income taxes: Our choice for the model economy's capital income tax function is

$$\tau_k(y_k) = a_1 y_k \tag{19}$$

Payroll taxes: Our choice for the model economy's payroll tax function is

$$\tau_{sf}(y_l) = \begin{cases} a_2 y_l & \text{for } 0 \leq y_l \leq a_3 \\ a_2 a_3 & \text{otherwise} \end{cases} \tag{20}$$

where τ_{sf} are the payroll taxes paid by firms. We chose this function because it approximates the shape of the U.S. payroll tax function where the marginal payroll tax rate is a positive constant up to a certain level of labor income and it is zero from that level of income onwards. Note that, as is the case in the U.S. Social Security tax code, we assume that the payroll taxes paid by the model economy households τ_{sh} are identical to those paid by the model economy firms, that is, $\tau_{sh}(y_l) = \tau_{sf}(y_l)$.

Household income taxes: Our choice for the model economy's income tax function is

$$\tau_y(y_b) = a_4(y_b - (y_b^{-a_5} + a_6)^{-1/a_5}) \tag{21}$$

where $y_b = y_k + y_l - \tau_k - \tau_{sf}$. We choose this function because it is the function chosen by Gouveia and Strauss (1994) to model the 1989 U.S. effective federal personal income taxes. Note that, following the U.S. personal income tax code, both capital income taxes and social security contributions made by firms are excluded from the household income tax base in the model economy.

Estate taxes: Our choice for the model economy's estate tax function is

$$\tau_e(z) = \begin{cases} 0 & \text{for } \; z < a_7 \\ a_8(z - a_7) & \text{otherwise} \end{cases} \tag{22}$$

We chose this function because it replicates the main features of the current U.S. effective estate taxes.[10]

Consumption taxes: Our choice for the model economy's consumption tax function is

$$\tau_c(c) = a_9 c \tag{23}$$

These choices imply that to specify the government policy in the model economy we must choose the values of 11 parameters.

3.1.5 Adding Up

Our modeling choices and our calibration strategy imply that we must choose the values of a total of 42 parameters to specify our model economy fully. Of these 42 parameters, 5 describe household preferences; 2 describe the aggregate technology; 24 parameters describe the joint age and endowment process; and the remaining 11 parameters describe the government policy.

3.2 Targets

To determine the values of the 42 model economy parameters described above, we impose 6 normalization conditions and we target 36 statistics that describe relevant features of the U.S. economy

3.2.1 Model period

The U.S. tax code defines tax bases in annual terms. Since the income tax, the payroll tax and the estate tax are non-proportional taxes, the obvious choice for our model period is one year. Moreover, the Survey of Consumer Finances which is our main inequality data source is also yearly, and a shorter model period would have increased our computational costs significantly.

[10]See, for example, Aaron and Munnell (1992).

3.2.2 Normalization conditions

The household endowment of disposable time is an arbitrary constant and we choose it to be $\ell = 3.2$. This choice makes the aggregate labor input approximately equal to one. Next, we normalize the endowment of efficiency labor units of the least productive households to be $e(1) = 1.0$. Finally, since matrix Γ_{SS} is a Markov matrix, its rows must add up to one. This property imposes four additional normalization conditions on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$.[11] Therefore, normalization allows us to determine the values of 6 of the model economy parameters.

3.2.3 Macroeconomic and demographic targets

Ratios: We target a capital to output ratio, K/Y, of 3.58; a capital income share of 0.376; and an investment to output ratio, I/Y, of 22.5 percent. We obtain our target value for the capital output share dividing $288,000, which was average household wealth in the U.S. in 1997 according the 1998 Survey of Consumer Finances, by $80,376, which was per household Gross Domestic Product according to the Economic Report of the President (2000), U.S. 1997.[12] Our target for the capital income share is the value that obtains when we use the methods described in Cooley and Prescott (1995) excluding the public sector from the computations.[13] To calculate the value of our target for I/Y, we define investment as the sum of gross private fixed domestic investment, change in business inventories, and 75 percent of the private consumption expenditures in consumer durables using data for 1997 from the Economic Report of the President (2000).[14]

Allocation of time and consumption: We target a value of $H/\ell = 33$ percent for the average share of disposable time allocated to working in the market.[15] For the curvature of consumption we choose a value of $\sigma_1 = 1.5$. This value falls within the range (1–3) that is standard in the literature.[16] Finally, we want our model economy to replicate the relative cross-sectional variability of U.S. consumption and hours. To this purpose, we target a value

[11]Note that our assumptions about the structure of matrix Γ_{SS} imply that once submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ has been appropriately normalized, every row of Γ_{SS} adds up to one without imposing any further restrictions.

[12]This number was obtained using the U.S. population quoted for 1997 in Table B-34 of the Economic Report of the President (2000) and an average 1998 SCF household size of 2.59 as reported in Budría, Díaz-Giménez, Quadrini, and Ríos-Rull (2002).

[13]See Castañeda, Díaz-Giménez, and Ríos-Rull (1998) for details about this number.

[14]This definition of investment is approximately consistent with the 1998 Survey of Consumer Finances definition of household wealth, which includes the value of vehicles, but does not include the values of other consumer durables.

[15]See Juster and Stafford (1991) for details about this number.

[16]Recent calibration exercises find very similar values for σ_1. For example, Heathcote, Storesletten, and Violante (2004) find 1.44 and Pijoan-Mas (2003) finds 1.46.

of $cv(c)/cv(h) = 3.5$ for the ratio of the cross-sectional coefficients of variation of these two variables.

The age structure of the population: To replicate the U.S. economy counterparts, we target the expected durations of the working-lives and retirements of the model economy households to be 45 and 18 years, respectively.

The life-cycle profile of earnings: We want our model economy to replicate the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21 in the U.S. economy. According to the Panel Study of Income Dynamics the average value of this statistic was 1.303 in the 1972–1991 period and this is the value that we target.

The intergenerational transmission of earnings ability: We want our model economy to replicate the intergenerational transmission of earnings ability in the U.S. economy. To measure this feature we use the cross-sectional correlation between the average life-time earnings of one generation of households and the average life-time earnings of their immediate descendants. Solon (1992) and Zimmerman (1992) have measured this statistic for fathers and sons in the U.S. economy, and they have found it to be approximately 0.4.

Total: These choices give us a total of 10 macroeconomic and demographic targets.

3.2.4 Government Policy

In Table 1 we report the revenues obtained by the combined U.S. Federal, State, and Local Governments for the 1997 fiscal year. Our calibration task is to allocate the different U.S. economy tax revenue items to the tax instruments of the benchmark model economy. To this purpose, we calibrate the benchmark model economy household income tax, capital income tax, estate tax, and payroll taxes so that they collect the revenues levied by the U.S. personal income taxes, corporate profit taxes, estate and gift taxes, and payroll taxes, respectively.

The remaining sources of government revenues in the U.S. are sales and gross receipts taxes, property taxes, excise taxes, custom duties and fees, and other taxes. Added together, in 1997 these tax instruments collected 7.09 percent of GDP which we allocate to the consumption tax in the model economy.[17]

[17] Note that, since we also target government transfers and government expenditures (see below), the model economy's consumption tax rate is determined residually to balance the government budget.

Fiscal Year	1997	
	$Billion	%GDP
Gross Domestic Product (GDP)	8185.20	100.00
Total Federal, State and Local Gvt Receipts	2252.75	27.52
Individual Income Taxes	896.54	10.95
Social Insurance and Retirement	539.37	6.60
Sales and Gross Receipts Taxes	261.73	3.20
Property Taxes	218.83	2.67
Corporate Profit Taxes	216.11	2.64
Excise Taxes	56.92	0.70
Estate and Gift Taxes	19.85	0.24
Custom Duties and Fees	17.93	0.22
Other Taxes	25.47	0.30

Table 1: Federal, State, and Local Government Receipts Source: Tables B78, B81, and B86 of the Economic Report of the President 2000.

In 1997, in the U.S., total tax revenues collected from these sources amounted to 27.52 percent of GDP. Since in our model economy we require the government budget to be balanced, we must allocate these revenues to either government consumption or transfers. We target a value for the model economy's aggregate transfers to output ratio of $Z/Y = 5.21$ percent. This value corresponds to the share of U.S. GDP accounted for by Medicare and two thirds of Social Security transfers in 1997. We make this choice because transfers in our model economies are lump-sum, and Social Security transfers in the U.S. economy are mildly progressive. This gives us a residual share for government expenditures to GDP of 22.30 which is our target for the G/Y ratio in our model economy.[18] We discuss the details of the various tax functions in the paragraphs below.

Capital income taxes: We choose a_1 in the capital income tax function described in expression (19) so that the revenues obtained from this tax instrument in the benchmark model economy match the revenues collected by the corporate profit tax in the U.S. economy.

Payroll taxes: In 1997 in the U.S. the payroll tax rate paid by both households and firms was 7.65 percent each and it was levied only on the first $62,700 of gross labor earnings. This number corresponded to approximately 78 percent of the U.S. per household GDP. To replicate these features, in our model economy we make $a_2 = 0.0765$ and $a_3 = 0.78\bar{y}$, where

[18] Note that our target for G/Y is 4.48 percentage points larger than the 17.89 obtained for the Government Expenditures and Gross Investment entry in the NIPA tables. The difference is essentially accounted for by the sum of net interest payments and the deficit (3.58 percent of GDP).

$\bar{y}$ denotes per household output.

Household income taxes: To identify the function that replicates the progressivity of U.S. effective personal income taxes described in expression (21), we must choose the values of parameters a_4, a_5 and a_6. Since a_4 and a_5 are unit-independent, we use the values reported by Gouveia and Strauss (1994) for these parameters, namely, $a_4 = 0.258$ and $a_5 = 0.768$. To determine the value of a_6 we require that the tax rate levied on average household output in our benchmark model economy is the same as the effective tax rate on per household GDP in the U.S. economy. These choices imply that the household income tax collections in our model economy are endogenous and they can be interpreted as an overidentification restriction.

Estate taxes: We want our model economy to mimic the tax exempt level specified in the U.S. estate tax code, which was \$600,000 during the 1987–1997 period. Since U.S. average per household GDP was approximately \$60,000 during that period, our target for the value of estates that are tax exempt in our model economy is $a_7 = 10\bar{y}$. Finally we choose parameter a_8 so that our model economy's estate tax collections replicate the U.S. economy estate tax collections.[19]

Consumption taxes: We choose parameter a_9 in the consumption tax function described in expression (23) so that the government in the model economy balances its budget. Therefore, the consumption tax collections in our model economy are also endogenous, and they can be interpreted as an overidentification restriction.

Total: These choices give us a total of 11 government policy targets.

3.2.5 The distributions of earnings and wealth

The conditions that we have described so far specify a total of 27 targets. Since to solve our model economy we have to determine the values of 42 parameters, we need 15 additional targets. These 15 targets are the Gini indexes and 13 additional points form the Lorenz curves of U.S. earnings and wealth reported in Table 8. In practice, instead of targeting 13 specific points, we searched for a set of parameter values such that, overall, the Lorenz curves of the model economies are as similar as possible as their U.S. counterparts.

[19] See, for example, Aaron and Munnell (1992).

3.3 Choices

The values of some of the model economy parameters are obtained directly either because they are normalization conditions or because they are uniquely determined by one of our targets. In this fashion, we make $\ell = 3.2$ and $e(1) = 1.0$ and we choose four of the transition probabilities of $\Gamma_{\mathcal{E}\mathcal{E}}$ so that the rows add up to one, and we choose $\sigma_1 = 1.5$, $\theta = 0.376$, $p_{e\varrho} = 0.022$, and $1 - p_{\varrho\varrho} = 0.056$. Similarly, the values for the payroll tax rate $a_3 = 0.0765$ is taken directly from the U.S. payroll tax code, and the values of two of the parameters of the income tax function, $a_4 = 0.258$ and $a_5 = 0.768$ are taken directly from the values estimated by Gouveia and Strauss (1994) for the U.S. economy. These choices pin down a total of 13 out of the 42 model economy parameters

The values of the remaining 29 parameters are determined solving the system of 29 non-linear equations that results from imposing that the relevant statistics of the model economy should be equal to their corresponding targets. The details of the procedure that we used to solve this system can be found in Appendix C.

4 Findings

4.1 The calibration exercise

In this subsection we discuss our calibration results very briefly. A detailed discussion of the reasons that allow our model economy to account for the main aggregate and distributional statistics of the U.S. economy can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

Calibrating our model economy amounts to finding a stochastic process for the endowment of efficiency labor units that aligns our benchmark model economy to the U.S. economy. This process, however, is not to be taken literally, since it represents everything that we do not know about the workings of our economy. An ambitious continuation of this research would be to endogenize its main features which we now describe.

In Table 3 we report the relative endowments of efficiency labor units and the invariant measures of each type of working-age households. We have normalized the endowment of efficiency labor units of workers of type $s = 1$ to be $e(s) = 1$. The endowments of workers of $s = 2$, $s = 3$, and $s = 4$ are, approximately, 3, 10, and 635. This means that, in our model economy, the luckiest workers are 635 times as lucky as the unluckiest ones. The stationary distribution shows that each period 85 percent of the workers are unlucky and draw shocks $s = 1$ or $s = 2$, while one out of every 1,567 workers is extremely lucky and draws shock

Table 2: Parameter values for the benchmark model economy

Preferences		
Time discount factor	β	0.930
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.119
Relative share of consumption and leisure	χ	1.050
Endowment of discretionary time	ℓ	3.200
Technology		
Capital income share	θ	0.376
Capital depreciation rate	δ	0.050
Age and endowment process		
Probability of retiring	$p_{e\varrho}$	0.022
Probability of dying	$1 - p_{\varrho\varrho}$	0.056
Life cycle earnings profile	ϕ_1	1.000
Intergenerational persistence of earnings	ϕ_2	0.733
Fiscal policy		
Government consumption	G	0.369
Retirement pensions	ω	0.800
Capital income tax function	a_1	0.146
Payroll tax function	a_2	1.262
	a_3	0.076
Household income tax function	a_4	0.258
	a_5	0.768
	a_6	0.456
Estate tax function	a_7	16.179
	a_8	0.246
Consumption tax function	a_9	0.099

Table 3: The relative endowments of efficiency labor units, $e(s)$, and the stationary distribution of working-age households, $\gamma^*_{\mathcal{E}}$ (%)

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.17	9.91	634.98
$\gamma^*_{\mathcal{E}}$ (%)	47.78	37.24	14.91	0.0638

$s = 4$.

Table 4: The transition probabilities of the process on the endowment of efficiency labor units for working-age households that remain of working-age one period later, $\Gamma_{\mathcal{E}\mathcal{E}}$ (%)

	To s'			
From s	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.15	1.39	0.23	0.009
$s = 2$	1.60	96.00	0.18	0.000
$s = 3$	1.19	0.00	96.56	0.028
$s = 4$	6.63	0.45	6.52	84.18

In Table 4 we report the transition probabilities between the working-age states. Every row sums up to 97.78 percent plus or minus rounding errors. This is because the probability that a worker retires is 2.22 percent. Table 4 shows that the first three shocks are very persistent. Their expected durations are 25.7, 25.3 and 29.4 years. On the other hand shock $s = 4$ is relatively transitory and its expected duration is only 7.6 years. As far as the transitions are concerned, we find that a worker whose current shock is $s = 1$ is most likely to make a transition to shock $s = 2$ than to any of the other shocks. Likewise, a worker whose current shock is either $s = 2$ or $s = 3$ is most likely to move back to shock $s = 1$. Only very rarely workers whose current shock is either $s = 1$ or $s = 2$ will make a transition to either shock $s = 3$ or shock $s = 4$. Finally, when a worker draws shock $s = 4$, it is most likely that it will draw either shock $s = 3$ or shock $s = 1$ shortly afterwards.

In the first two rows of Tables 6 and 7 and the first two rows of each of the panels of Table 8 we report the statistics that describe the main aggregate and distributional features of the U.S. and the benchmark model economies. A glance at the numbers confirms that, overall, our model economy succeeds in replicating the main relevant features of the U.S. economy in very much detail. Naturally, there are some exceptions. For instance, the last two columns of Table 6 show that our parsimonious modelling of the life cycle does not allow us to match life-cycle profile of earnings and the intergenerational correlation of earnings simultaneously. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) give a detailed discussion of this issue, and they show that this class of model economies can account for these two statistics one at a time. We are particularly encouraged by the ability of our model economy to replicate the fiscal policy ratios (see Table 7) and the earnings, income and wealth distributions (see Table 8) that constitute the main focus of this article.[20]

4.2 The flat tax reform

We study two fundamental, revenue neutral, tax reforms similar to the one proposed by Hall and Rabushka (1995). These reforms replace the current personal income with a flat tax on labor income with a large deduction, and the current corporate income taxes with an integrated flat tax on business income. The labor income tax, $\tau_l(y_a)$, is defined as follows

$$\tau_l(y_a) = \begin{cases} 0 & \text{for } y_a < a_{10} \\ a_{11}(y_a - a_{10}) & \text{otherwise} \end{cases} \tag{24}$$

where the tax base, $y_a = y_l - \tau_{sf}(y_l)$, is labor income net of social security taxes paid by firms, a_{10} is the fixed deduction, and a_{11} is the flat tax rate. The business income tax is identical to the capital income tax defined in expression (19) above. Since capital and labor income are taxed at the same marginal tax rate, we make $a_1 = a_{10}$. In this tax reform, the progressivity of direct taxes arises from the fixed deduction on labor income. On the other hand, capital income taxes are not progressive. Another defining feature of this reform is that it eliminates the double taxation of capital income.

To find the values of the tax parameters of our reformed model economies we do the following: first we choose the values for the labor income exemption, a_{10}. In model economy E_1 this exemption is $a_{10} = 0.3236$ which corresponds to 20 percent of per household GDP of the benchmark model economy, or approximately \$16,000. In model economy E_2 it is $a_{10} = 0.6472$ which corresponds to 40 percent of per household GDP in the benchmark economy, or approximately \$32,000.[21] Then we search for the flat tax rates that make the reforms revenue neutral. These tax rates turn out to be $a_1 = a_{10} = 21.5$ percent in model economy E_1 and $a_1 = a_{10} = 29.2$ percent in model economy E_2. Henceforth we refer to model economy E_1 as the less progressive reform and to model economy E_2 as the more progressive reform.

[20] Recall that the income distribution is not one of our calibration targets.

[21] Ventura (1999) carries out two policy experiments that are very similar to ours.

4.2.1 Taxes, taxes, taxes

Taxes in the U.S. interact in interesting and somewhat surprising ways. The personal income tax, τ_y, is progressive in the classical sense since both marginal and average tax rates are increasing in income. But, as we have already mentioned, the payroll tax is not progressive. The marginal payroll tax on labor incomes below a_3 (\$62,700 in the U.S. in 1997) is constant and equal to $2a_2 = 15.3\%$, and the marginal payroll tax rate on labor incomes above this threshold is zero. Since in our model economies households exchange leisure for consumption and since pensions are independent from contributions, the households are concerned with total effective income taxes put together, and not in those collected with the various tax instruments individually. Additionally, this lack of progressivity has some important mathematical implications because it makes the returns to hours worked increasing in hours at certain points, and therefore introduces non-convexities in the household problem. See appendix B for details

To illustrate the interactions between these two tax instruments, in Figure 1 we represent the sum of the payroll and personal income taxes paid by the households in our three model economies. Since the personal income tax rates depend on both capital and labor income, in each graph we plot the tax rates paid by households whose net worths are \$0, \$9,370, and \$454,120 which puts them in the first, the fifth, and the ninth deciles of the wealth distribution.[22]

A careful inspection of Figure 1 reveals various features of the current U.S. tax system that we have found both surprising and interesting. First, in the benchmark model economy the marginal tax rates paid by the households in the bottom half of the wealth distribution are almost identical. Naturally, in the flat-tax economies this is the case by design. Therefore, flat income taxes are not too different from the current taxes, as far as their wealth progressivity is concerned.

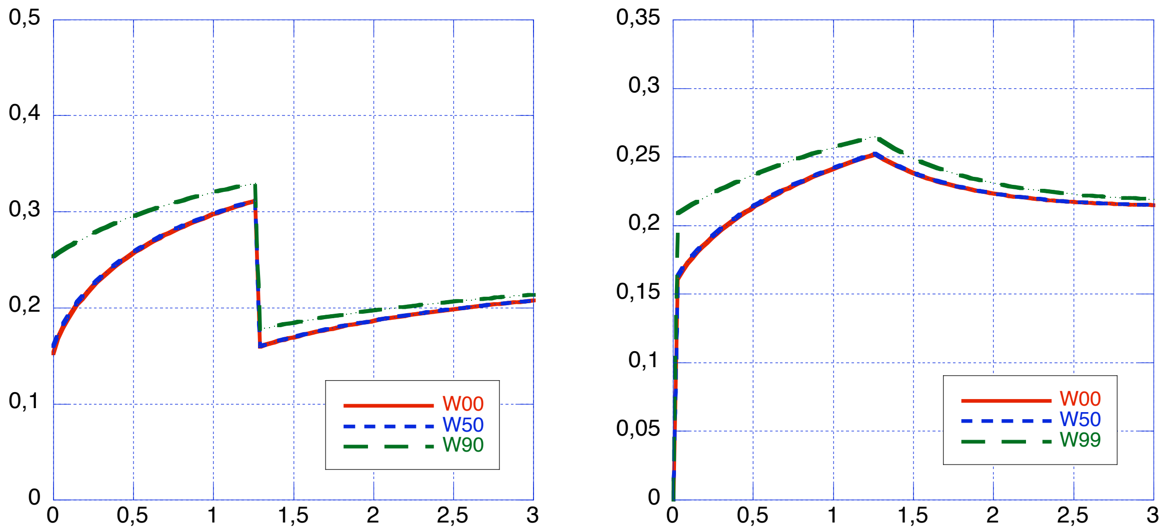
Second, if we add the payroll and the personal income taxes, it turns out that the current labor income taxation, far from being progressive, is actually regressive.[23] Specifically, the first panel of Figure 1 shows that the marginal tax rate on labor income faced by wealth-poor

[22]We have transformed the model economy units into U.S. dollars to give the reader a better sense of the magnitudes involved.

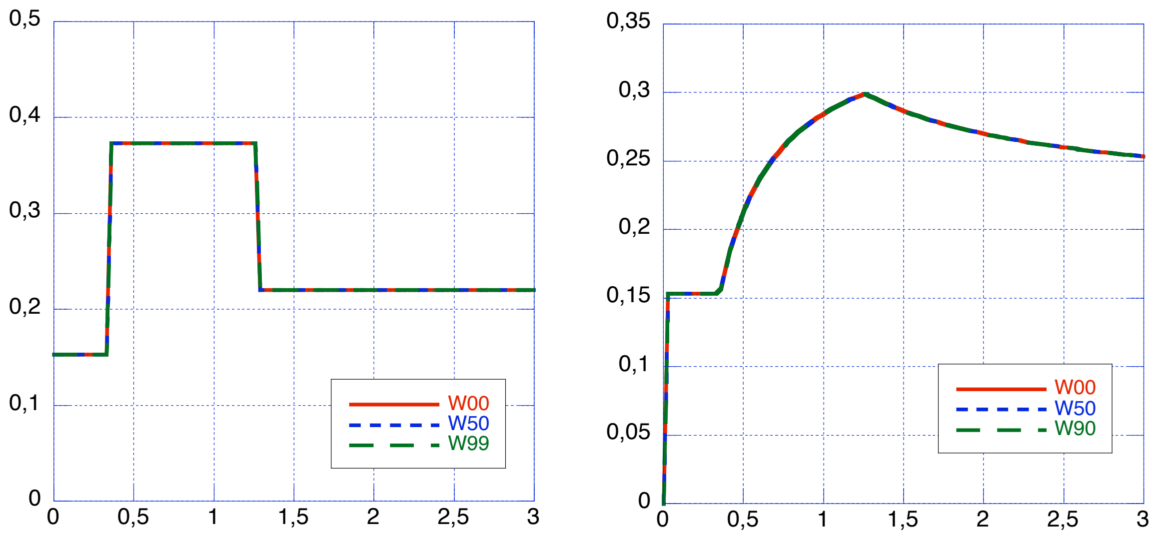
[23]In this article we use the adjectives "progressive", "proportional", and "regressive" because they give us an intuitive description of tax instruments, but we do not use them to imply any normative judgement about fairness. We do this for two reasons. First, because to evaluate the fairness of a tax instrument, we should evaluate its consequences for the distribution of welfare or, at least, for the distribution of after-tax income; and second because to measure the fairness of a tax instrument, we should use the lifetime tax burden and the lifetime taxable income, and not the tax burden and the taxable income of any single period.

Figure 1: Marginal and Average Tax Rates on Labor Income in the Model Economies

(a) Benchmark Model Economy (E0)



(b) Reformed Model Economy (E1)



(c) Reformed Model Economy (E2)

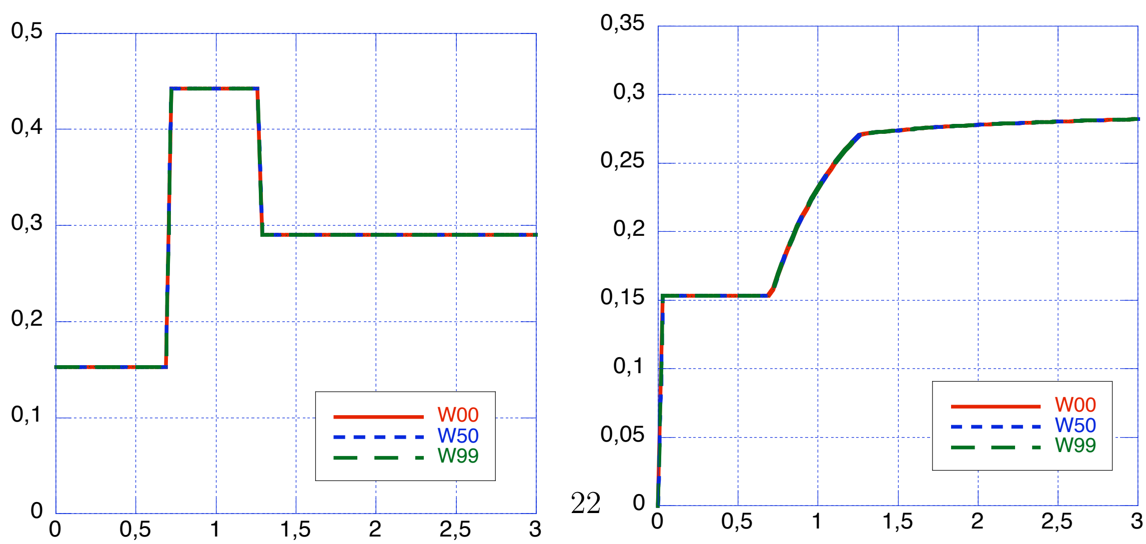
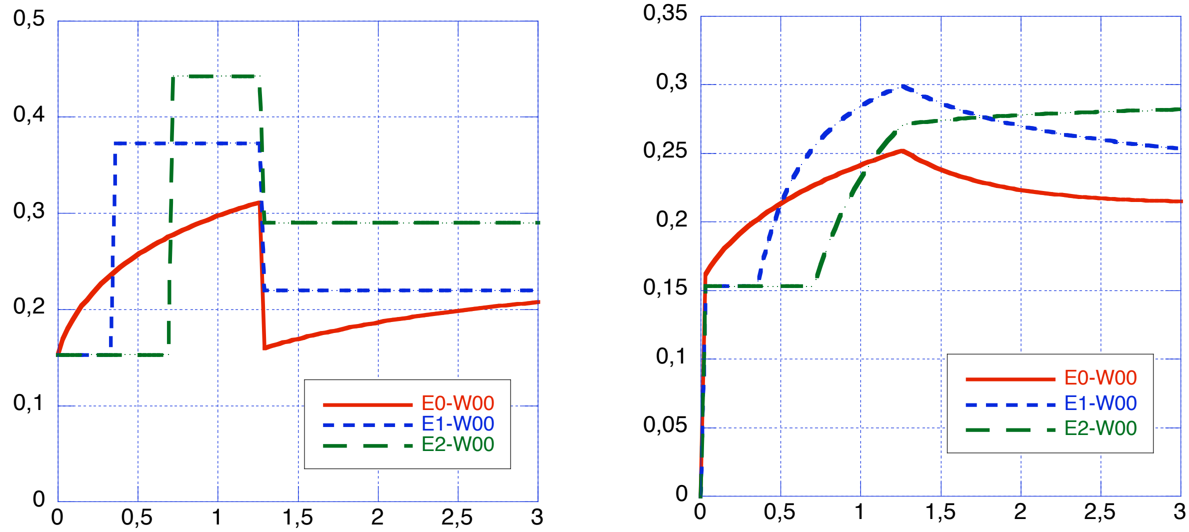
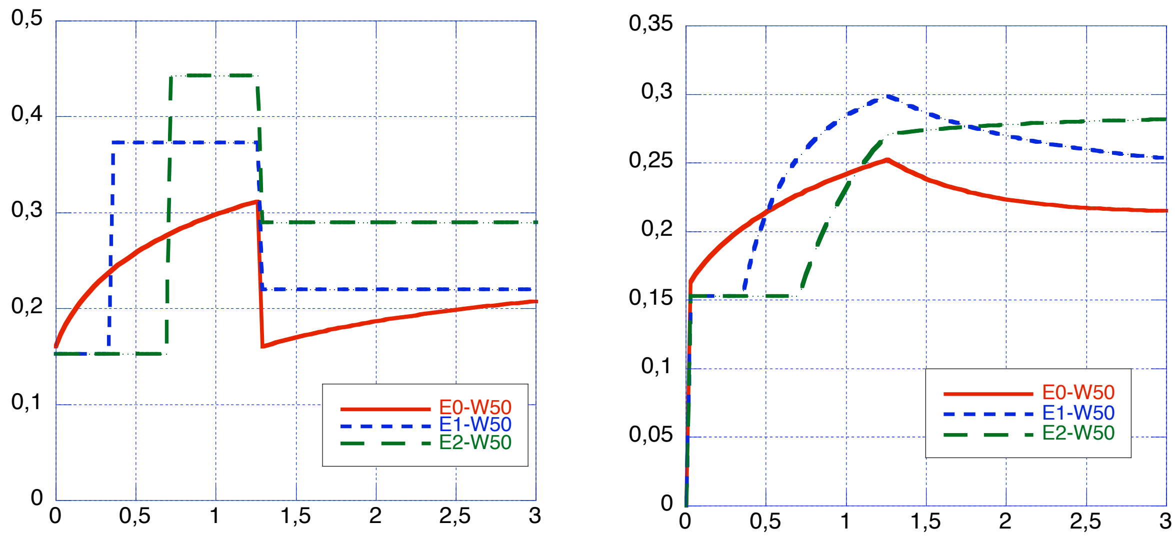


Figure 2: Marginal and Average Tax Rates on Labor Income in the Model Economies Compared

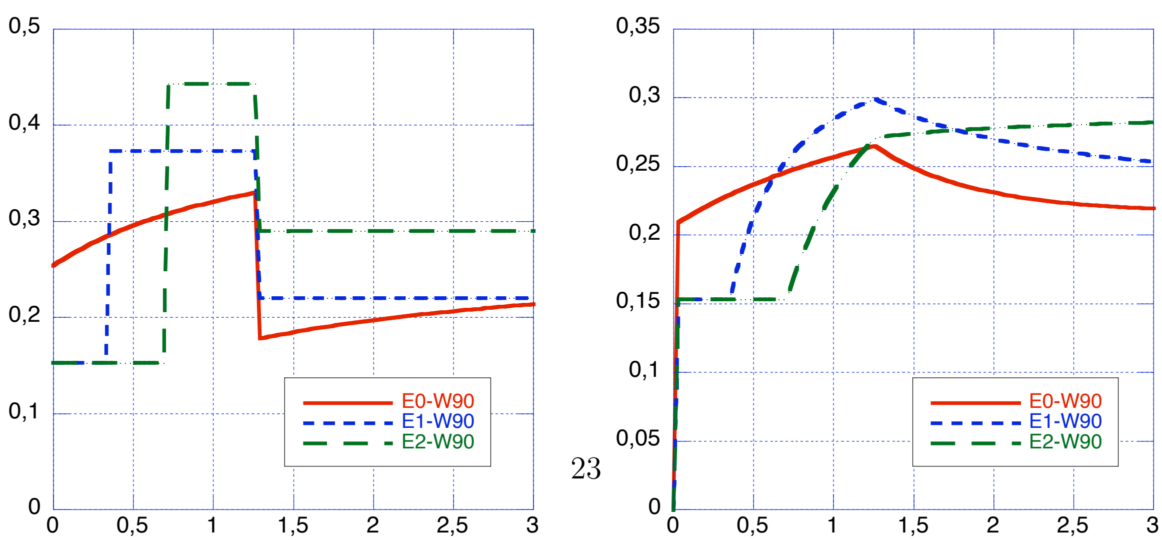
(a) The Wealth Poor (W10)



(b) Median Wealth (W50)



(c) The Wealth Rich (W90)



households starts at about 15% and it reaches its maximum value of approximately 30% when labor income is $62,700. At this income level, the marginal payroll tax rate drops to zero and the total marginal tax on labor income drops back to approximately 15% which is the marginal tax rate paid by households with zero labor income. In the case of very wealthy households, the regressivity of marginal labor income tax rates is even more remarkable: the marginal tax on labor income paid by households who earn $62,700 is 17%, which is only two-thirds of the 25% paid by equally wealthy households who earn zero labor income.

This interaction between payroll taxes and labor income taxes is also present in the flat tax economies, albeit in a smaller degree (see Panels (b) and (c) of Figure 1). In both cases, the marginal income tax rates are step functions, and in both cases the middle labor incomes pay the highest marginal taxes. But, unlike the current system, in both flat tax reforms, the labor income rich pay higher marginal labor income taxes than the labor income poor.

As far as average labor income taxes are concerned, we find that they are only progressive in model economy E_2 (see the right-hand-side panels of Figure 1). In the benchmark model economy and in model economy E_1 average taxes peak at the payroll tax income cap and they decrease for higher levels on income. In contrast, in model economy E_2, once the payroll tax income cap is reached, the average tax rate increases asymptotically to that economy's flat tax rate (29.0%).[24]

To compare the labor income taxes before and after the reform, in Figure 2 we plot the marginal and average tax rates of the three model economies in the same graphs for the three values of wealth mentioned above. We find that in all cases the flat tax reforms favor the labor income poorest at the expense of all other labor income earners.

As far as capital income taxes are concerned, under both flat tax reforms, the marginal tax rate on capital income is higher than the capital income tax in the benchmark model economy, where it was only 14.6%. However, as we have already mentioned, in the benchmark model economy capital income is taxed twice: first by the capital income tax, and then by the household income tax. Since the marginal income tax rates in the current system are increasing with income, it turns out that the capital income poor end up facing higher marginal capital income tax rates under the flat tax reforms, while the capital income rich end up facing lower marginal capital income taxes. Consequently, the flat tax reforms are regressive as far as marginal capital income taxes are concerned.

[24]This happens because in model economy E_2 the flat tax rate is higher than the average tax that obtains at the payroll tax cap.

Table 5: Production, inputs and input ratios in the model economies

	Y	K	L^a	H^b	K/L	L/H	Y/H
E_0	1.62	5.76	0.75	33.70	7.64	2.24	4.80
E_1	1.66	6.16	0.75	33.46	8.19	2.25	4.95
E_2	1.58	5.43	0.75	33.26	7.26	2.25	4.74
$E_1/E_0(\%)$	2.44	6.93	–0.16	–0.69	7.10	0.53	3.15
$E_2/E_0(\%)$	–2.64	–5.72	–0.73	–1.30	–5.03	0.58	–1.35

[a]Variable L denotes the aggregate labor input.
[b]Variable H denotes the average percentage of the endowment of time allocated to the market.

4.2.2 Macroeconomic aggregates and ratios

The aggregate consequences of the two flat tax reforms turn out to be very different. The less progressive reform is expansionary (output increases by 2.4 percent and labor productivity by 3.15 percent). But the more progressive reform is contractionary (output decreases by 2.6 percent and labor productivity by 1.4 percent).

In Tables 5 and 6 we report the main macroeconomic aggregates and ratios of our model economies. We find that the expansion in model economy E_1 output is brought about by an increase in aggregate capital of 6.9 percent, a reduction in the aggregate labor input of slightly less than -0.2 percent and an increase in the capital to labor ratio of 7.3 percent. In contrast, the contraction in model economy E_2 is brought about by a reduction in aggregate capital (5.7 percent), in the aggregate labor input (0.7 percent) and in the capital labor ratio (5.0 percent). Consequently, in model economy E_1 the steady-state interest rate is lower and the wage rate is higher than in the benchmark economy, and that the opposite is true for economy E_2. If we look at the last two columns of Table 5 we find out that the changes in labor productivity are the result of large changes in the capital to labor ratio, K/L, which dwarf the changes in the average efficiency of labor, L/H.

The reasons that justify these results are the following: first, flat tax reforms reduce the marginal capital income tax rates faced by the wealthy and increase the capital income tax rates faced by the wealth-poor. Consequently, their incentives to accumulate capital change in different directions. Second, the flat tax reforms increase the marginal taxes on labor income paid by every household except the very labor income poor. Since the reforms affect different households in different ways, their overall effects can vary significantly from one reform to another. Overall, we find that the flat tax reforms that we consider are expansionary as

Table 6: The values of the targeted ratios and aggregates in the U.S. and in the benchmark model economies

	$C/Y(\%)$	$I/Y(\%)$	$G/Y(\%)$	K/Y	$H^a/\ell(\%)$	$(cv_c/cv_l)^b$	$e_{40/20}$	$\rho(f,s)$
U.S.	54.2	22.5	23.3	3.58	33.3	3.5	1.30	0.40
E_0	59.2	18.0	22.8	3.56	33.7	3.3	1.23	0.14
E_1	59.0	18.8	22.3	3.71	33.5	3.6	1.24	0.15
E_2	59.2	17.4	23.4	3.45	33.3	3.5	1.23	0.16
E_0	59.2	18.0	22.8	3.56	–	–	–	–
E_1/Y_0	60.4	19.2	22.8	3.81	–	–	–	–
E_2/Y_0	57.6	17.0	22.8	3.35	–	–	–	–

[a]This ratio denotes the average share of disposable time allocated to the market.
[b]This statistic is the ratio of the coefficients of variation of consumption and of hours worked.

long as the integrated flat rate is small enough, and that they become contractionary as we increase the tax-exempt level of labor income. Consequently, it turns out that flat tax reformers indeed face the classical trade-off between efficiency and equality, since they must choose between the large exemptions that make the tax system more progressive and the economy less efficient, and the opposite effects brought about by low tax exemptions.

In Table 6 we report additional aggregate statistics. Overall, we find that the changes are rather small. Not surprisingly, the most noteworthy changes are those in the investment to output ratio which is 18.0 in the benchmark model economy, 18.8 in model economy E_1 and 17.4 in economy in model economy E_2.

In Table 7 we report the main fiscal policy ratios of the model economies. In model economy E_1 the tax revenue to output ratio is smaller than in the benchmark economy, and both its government expenditures to output ratio and its transfers to output ratio are reduced accordingly. The opposite is the case in model economy E_2. These results arise because both reforms are revenue neutral, and while aggregate output in model economy E_1 is larger than in the benchmark economy, in model economy E_2 it is smaller.

In accordance with Hall and Rabushka (1995) predictions, the labor income tax of the reformed economies collects less revenues than the personal income tax of the benchmark model economy, and these revenue losses are compensated by the higher revenues collected by the capital income tax. Moreover, in the three model economies considered, the revenues collected by the payroll and consumption taxes, and the combined revenues of the labor and capital income taxation are very similar.

Table 7: Fiscal policy ratios in U.S. and in the model economies (%)

	G/Y	Z/Y	T/Y	T_y/Y	T_l/Y	T_k/Y	T_s/Y	T_c/Y	T_e/Y
U.S.	23.3	5.2	27.5	11.0	–	2.6	6.6	7.1	0.24
E_0	22.8	4.5	27.2	11.6	–	2.9	5.9	6.5	0.37
E_1	22.3	4.3	26.6	–	9.9	4.1	5.8	6.5	0.46
E_2	23.4	4.6	28.0	–	9.3	5.9	5.9	6.5	0.43
E_0	22.8	4.5	27.2	11.6	–	2.9	5.9	6.5	0.37
E_1/Y_0	22.8	4.5	27.3	–	10.1	4.2	5.9	6.6	0.47
E_2/Y_0	22.8	4.5	27.3	–	9.0	5.7	5.8	6.3	0.41

4.2.3 Earnings, income and wealth inequality

In Table 8 we describe the earnings, income and wealth inequality in the U.S. and in the model economies. Since the three economies have identical processes on the endowments of efficiency labor units, it is not surprising that the changes in the distribution of earnings are very small. Before-tax income and, especially, wealth become more unequally distributed under both reforms. Most interestingly, the distributional consequences of the reforms on after-tax income differ significantly: while after-tax income inequality increases in the less progressive tax reform, it decreases significantly in the more progressive tax reform. It turns out that along this dimension policymakers truly face the classical trade-off: the gains in efficiency of the less progressive flat tax reforms are obtained at the expense of greater after-tax income inequality.

In Table 8 we report the Gini indexes and some points of the Lorenz curves of the earnings, income and wealth distributions to describe these results quantitatively. The Gini index of wealth increases significantly. It is 0.813 in the benchmark economy and 0.839 and 0.845 in the two reformed economies. Both reforms bring about a large increase in the share of wealth owned by the top quintile (2.3 and 3.4 percentage points, respectively) at the expense of the shares owned by other quintiles. The increases in the shares of wealth owned by the households in the top 1 percent of the wealth distributions are even larger (3.2 and 4.9 percentage points, respectively).

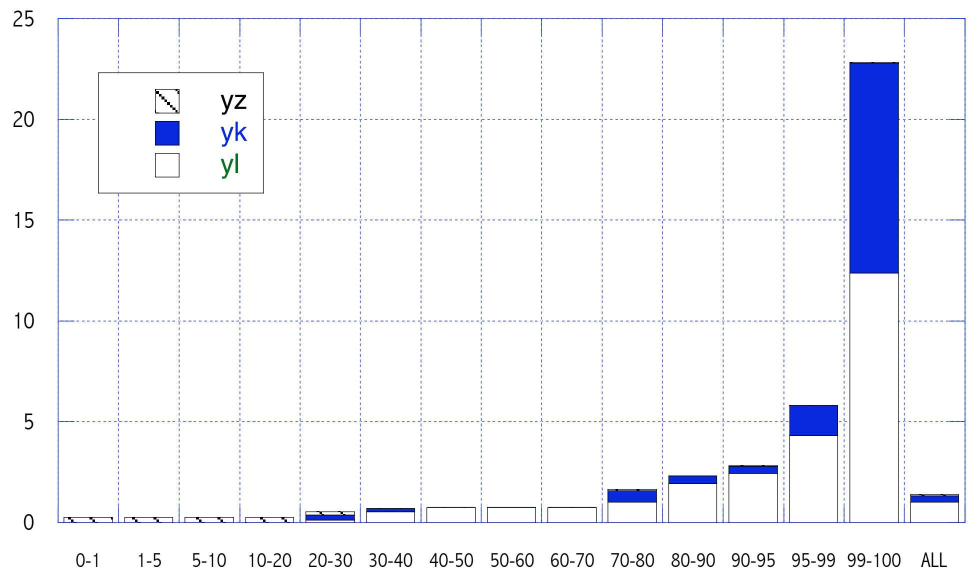
We report the Gini indexes and the Lorenz curves of the income distribution in the last two panels of Table 8. The changes in before tax income are rather small. The gini indexes increase from 0.533 to 0.541 under both reforms, and the changes between the shares earned by the quintiles are minor (less than 0.5 percentage points in every case).

Most of the distributional action shows up in the after tax income distribution. Under

Table 8: The distributions of earnings, wealth and income in the U.S. and in the model economies

The earnings distributions									
	Gini	*Quintiles (%)*					*Top groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
U.S.	0.611	–0.4	3.2	12.5	23.3	61.4	12.4	16.4	14.8
E_0	0.613	0.0	4.2	14.4	18.8	62.5	11.8	16.7	15.2
E_1	0.615	0.0	4.2	14.2	18.7	62.9	12.0	16.8	15.0
E_2	0.610	0.0	5.2	13.8	18.2	62.8	11.7	16.9	15.4
The wealth distributions									
	Gini	*Quintiles (%)*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
U.S.	0.803	–0.4	1.7	5.7	13.4	79.5	12.6	24.0	29.6
E_0	0.818	0.0	0.3	1.5	15.9	82.2	12.6	19.8	34.7
E_1	0.839	0.0	0.1	1.0	14.4	84.5	12.5	20.4	37.9
E_2	0.845	0.0	0.1	1.2	13.0	85.6	12.3	20.7	39.6
The income distributions (before all taxes and after transfers)									
	Gini	*Quintiles (%)*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
U.S.	0.550	2.4	7.2	12.5	20.0	58.0	10.3	15.3	17.5
E_0	0.533	3.7	8.9	10.9	17.2	59.3	10.1	16.6	16.0
E_1	0.541	3.6	8.5	10.8	17.1	60.0	10.3	16.8	16.2
E_2	0.541	3.7	8.9	10.3	16.9	60.2	10.1	17.0	17.0
The income distributions (after all income taxes and transfers)									
	Gini	*Quintiles (%)*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
E_0	0.510	4.6	9.3	11.0	17.2	57.9	10.1	16.3	14.9
E_1	0.524	4.7	8.9	10.4	16.7	59.3	10.1	16.7	16.5
E_2	0.497	4.9	9.8	11.4	17.1	56.7	9.5	15.9	15.9

Figure 3: Labor income, capital income, and transfer income in the benchmark model economy



the less progressive reform the Gini index of after-tax income increases form 0.510 to 0.524 and the shares of after-tax income earned by the households in the top quintile and the top 1 percent increase by 1.4 and 1.6 percentage points. In contrast, under the more progressive tax reform the Gini index of after-tax income decreases to 0.497 and the share earned by the bottom 60 percent of the distribution increase in 1.2 percentage points. But in spite of the progressivity of this reform, the share of after-tax income earned by the income richest still increases by one percentage point.

4.2.4 The tax burden

To discuss the distribution of the tax burden, in Tables 9 and 10 we report the average tax rates and the shares of taxes paid by the quantiles of the before-tax income distribution. To make the three model economies comparable, we also report the decomposition of household income taxes into labor income taxes, which we label τ_{yl}, and capital income taxes, which we label τ_{yk}.[25]

Average tax rates in the benchmark economy. We find that, under the current tax system, total average tax rates are almost proportional in spite of the fact that the personal

[25] This decomposition of household income taxes is similar to the one used by Mendoza, Razin, and Tesar (1994).

income tax code is designed to make the tax system progressive. It is true that average income tax rates are clearly progressive (see the first panel of Table 9). But, with the exception of the average taxes paid by the households in the first quintile, this progressivity all but disappears when we add up all income taxes together and when we consider the entire tax system (see the first rows of the last two panels of Table 9). The average tax rates of the remaining tax instruments display an interesting see-saw pattern. The peculiarities of labor income taxation discussed above are some of the reasons that justify this pattern. Other reasons can be found in Figure 3 where we represent the sources of income of the households in the deciles of the income distribution. As Figure 3 illustrates, the shares of capital income also display the see-saw pattern, since the households in the second quintile, many of them rich retirees, own a disproportionally large share of total capital.

Average tax rates in the reformed economies. If we compare the distribution of average tax rates before and after the reforms we find the following: First, the income poorest households pay significantly less income taxes in the reformed economies than in the benchmark model economy (see the sixth panel of Table 9). Second, in model economy E_1, the income rich pay less taxes than the households in the third and fourth income quintiles who are left to foot the bill (see the last panel of Table 9). Third, in model economy E_2 the tax burden is distributed more progressively than in the benchmark economy: while the households in the bottom four quintiles pay lower average taxes, the households in the top quintile pay higher average taxes. Fourth, we also find that in both flat tax economies capital income taxes replicate the see-saw pattern of the benchmark economy and that in every quantile, average capital income taxes are higher in economy E_2 than in economy E_0, while in economy E_1 they are lower.[26] Finally, we find that the large labor income tax exemption of model economy E_2 generates a very unequal distribution of average labor income tax rates. If we exclude payroll taxes, the bottom 60 percent of the income distribution of model economy E_2 pays no labor income taxes, whereas the average tax rates paid by the households in the top quintile are 4.4 percentage points higher than in the benchmark economy, and 1.5 percentage points higher than in model economy E_1 (see the third panel of Table 9).

The distribution of the tax burden. Table 10 shows that the total income tax burden, the total tax burden and, especially, the estate tax burden are very progressively distributed in the three model economies. Moreover, the distributions of the total income tax burden and

[26]This finding is consistent with the fact that in model economy E_1 there is more aggregate capital than in model economy E_0, while in model economy E_2 there is less aggregate capital.

Table 9: Average taxes paid by the quantiles of the distribution of income before taxes and after transfers (%)

	All	*Quintiles*					*Top Quantiles*		
	0–100	1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Panel 1: Personal income taxes (τ_y)									
E_0	8.5	4.5	6.7	7.7	9.1	14.6	14.1	17.6	19.5
Panel 2: Capital income taxes ($\tau_k + \tau_{yk}$)									
E_0	3.6	0.4	7.0	0.2	4.7	5.6	3.1	8.5	13.9
E_1	2.9	0.3	6.5	0.1	3.8	3.8	2.0	5.6	9.7
E_2	3.8	0.3	7.2	0.3	6.1	5.3	3.0	7.8	13.9
Panel 3: Labor income taxes ($\tau_l + \tau_{yl}$)									
E_0	6.0	0.0	3.5	7.6	7.0	11.7	12.6	13.0	11.6
E_1	7.7	0.0	3.4	10.7	9.8	14.6	16.2	14.4	11.1
E_2	3.8	0.0	0.0	0.0	3.1	16.1	17.9	17.7	13.8
Panel 4: Consumption taxes (τ_c)									
E_0	9.1	10.1	12.5	7.7	8.5	6.5	5.8	6.7	6.5
E_1	9.1	10.4	12.9	7.4	8.4	6.5	5.6	6.8	7.1
E_2	9.2	10.3	11.8	8.4	9.3	6.2	5.5	6.2	6.6
Panel 5: All labor income taxes ($\tau_l + \tau_{yl} + \tau_s$)									
E_0	14.1	0.0	10.9	22.7	18.4	18.4	19.7	16.3	13.1
E_1	15.9	0.0	11.1	25.9	21.2	21.2	23.1	17.6	12.5
E_2	12.1	0.0	8.4	15.2	14.1	23.0	25.2	20.9	15.1
Panel 6: All income taxes ($\tau_k + \tau_l + \tau_y + \tau_s$)									
E_0	18.8	4.8	19.0	22.9	23.2	24.0	22.8	24.8	27.0
E_1	18.8	0.3	17.7	26.0	25.0	25.0	25.2	23.2	22.2
E_2	16.0	0.3	15.6	15.4	20.3	28.3	28.2	28.8	28.9
Panel 7: All Taxes ($\tau_k + \tau_l + \tau_y + \tau_s + \tau_c + \tau_e$)									
E_0	27.9	14.9	31.5	30.6	31.9	30.7	28.8	31.6	35.5
E_1	28.0	10.6	30.5	33.4	33.8	31.6	30.9	30.1	32.0
E_2	25.3	10.6	27.4	23.9	29.8	34.7	33.9	35.1	38.1

Table 10: Shares of tax revenues paid by the quantiles of the distribution of income before taxes and after transfers (%)

	Quintiles					*Top Quantiles*		
	1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Panel 1: Capital income taxes ($\tau_k + \tau_{yk}$)								
E_0	0.2	8.5	0.3	15.1	75.8	5.9	20.1	38.0
E_1	0.2	10.9	0.2	17.1	71.5	5.6	19.2	35.2
E_2	0.2	8.5	0.4	18.4	72.6	5.8	19.0	37.1
Panel 2: Labor income taxes ($\tau_l + \tau_{yl}$)								
E_0	0.0	3.5	8.5	13.2	74.8	12.8	22.5	21.0
E_1	0.0	2.9	10.0	15.1	71.9	14.1	21.4	15.7
E_2	0.0	0.0	0.0	9.2	90.8	16.5	28.9	22.6
Panel 3: Consumption taxes (τ_c)								
E_0	4.9	14.2	11.0	19.3	50.6	7.9	14.6	13.0
E_1	4.9	13.8	10.4	19.0	51.9	7.8	14.9	14.5
E_2	5.1	13.4	11.5	20.0	50.0	7.6	14.0	14.3
Panel 4: Estate taxes (τ_e)								
E_0	0.0	0.0	0.0	13.5	86.5	5.8	5.0	75.7
E_1	0.0	0.0	0.0	15.4	84.6	3.6	5.3	75.7
E_2	0.0	0.0	0.0	10.9	89.1	4.6	3.5	81.1
Panel 5: All income taxes ($\tau_k + \tau_l + \tau_y + \tau_s$)								
E_0	0.8	7.4	10.6	17.1	64.2	9.8	17.4	20.1
E_1	0.0	6.9	12.2	18.4	62.4	11.1	17.0	15.6
E_2	0.0	6.0	6.5	15.9	71.5	11.7	20.3	20.7
Panel 6: All Taxes ($\tau_k + \tau_l + \tau_y + \tau_s + \tau_c + \tau_e$)								
E_0	1.7	8.9	10.5	17.6	61.2	9.3	16.6	19.1
E_1	1.2	8.5	11.5	18.5	60.2	10.2	16.3	16.3
E_2	1.2	7.6	7.6	16.8	66.8	10.6	18.6	20.1

of the total tax burden in model economy E_2 are significantly more progressive than in model economy E_1 (see the last two panels of Table 10). This findings justify in part the different distributional consequences of the two flat tax reforms that we have already discussed.

4.2.5 Welfare

In model economy E_1, the less progressive flat-tax economy, aggregate output, consumption, productivity and leisure are all higher than in model economy E_0. In contrast, in model economy E_2, the more progressive flat-tax economy, aggregate output, consumption and productivity are lower, and only aggregate leisure is higher than in model economy E_0.

These results are consistent with the idea that high tax rates and small tax bases are more distortionary than low tax rates and big tax bases. At least on aggregate.

However, it is also true that the after-tax income distribution in model economy E_1 is significantly more unequal than in model economy E_2. Therefore, a policymaker who tried to decide between these reforms would face the classical trade-off between efficiency and equality. Which economy should she choose? The more efficient but less egalitarian model economy E_1, or the less efficient but more egalitarian model economy E_2? In this section we use a Benthamite social social welfare function to quantify the trade-off and answer this question.[27]

To carry out the welfare comparisons, we define $v_0(a, s, \Delta)$ as the equilibrium value function of a household of type (a, s) in model economy E_0 whose equilibrium consumption allocation at all points in time is increased by a fraction Δ and whose leisure remains unchanged. Formally,

$$v_0(a, s, \Delta) = u\left(c_0(a, s)(1 + \Delta), \ell - h_0(a, s)\right) + \beta \sum_{s' \in S} \Gamma_{ss'}\ v\left(z_0(a, s), s', \Delta\right) \tag{25}$$

where $c_0(a, s)$, $h_0(a, s)$ and $z_0(a, s)$ are the optimal decision rules that solve the household decision problem defined in Section 2.6. Next, we define the welfare gain of living in flat-tax economy E_i (where $i = 1, 2$), as the fraction of additional consumption, Δ_i, that is needed to attain the steady-state social welfare of the flat-tax model economy E_i in the benchmark model economy. Formally, Δ_i is the solution to the equation

$$\int v_0(a, s, \Delta_i)\, dx_0 = \int v_i(a, s)\, dx_i \tag{26}$$

where v_i and x_i are the equilibrium value function and the equilibrium stationary distribution of households in the flat-tax model economy E_i.

We find that the equivalent variation in consumption for the less progressive flat-tax reform is $\Delta_1 = -0.17$, and that the equivalent variation in consumption for the more progressive flat-tax reform is $\Delta_2 = 0.45$. This means that, using a Benthamite welfare criterion, the flat-tax reform with a low tax rate and a low labor income exemption results in an aggregate welfare loss and that the flat-tax reform with a high tax rate and a high exemption results in an aggregate welfare gain. Therefore, in Benthamite welfare terms, equality wins the trade-off and the social planner should choose the more progressive flat-tax reform.

[27] Benthamite social welfare functions give identical weights to every household in the economy. Consequently, with concave utility functions, equal sharing is the welfare maximizing allocation. Also notice that in this section we compare the welfare of steady-state allocations and we remain conspicuously silent about the transitions between these steady-states.

Flat-tax reforms are fundamental tax reforms that change every margin of the households' and the firms' decision problems. These changes in the individual behavior of households and firms translate into changes in aggregate allocations and prices which result in further changes in the individual decisions. Finally, the solutions to these fundamentally different decision problems generate equilibrium distributions of households that are also fundamentally different. To improve our intuitive understanding of our welfare findings, it helps to decompose the equivalent variation in consumption discussed above into these three components.

To this purpose, we define two auxiliary measures of the equivalent variations in consumption associated with each one of the flat-tax reforms. First, we compute the equivalent variation in consumption between model economy E_0 and the reformed economy E_i ignoring the distributional changes brought about by the flat-tax reform. We denote this variation by Δ_i^a, and we define it as follows:

$$\int v_0\left(a, s, \Delta_i^a; r_0, w_0\right) dx_0 = \int v_i\left(a, s; r_i, w_i\right) dx_0 \tag{27}$$

Notice that, in this welfare measure, we calculate the aggregate welfare of the flat-tax economy using its equilibrium price vector, (r_i, w_i), and the equilibrium stationary distribution of the benchmark model economy, x_0.

Second, we compute the equivalent variation in consumption between model economy E_0 and the reformed economy E_i ignoring both the distributional and the general equilibrium changes brought about by the flat-tax reform. We denote this variation by Δ_i^b, and we define it as follows:

$$\int v_0\left(a, s, \Delta_i^b; r_0, w_0\right) dx_0 = \int v_i\left(a, s; r_0, w_0\right) dx_0 \tag{28}$$

Notice that now we calculate the aggregate welfare of the flat-tax economy using both the equilibrium stationary distribution and the equilibrium price vector of the benchmark model economy.

These two equivalent variations allow us to decompose the total equivalent variation that we have defined above as follows:

$$\Delta_i = \Delta_i^b + \left(\Delta_i^a - \Delta_i^b\right) + \left(\Delta_i - \Delta_i^a\right) \tag{29}$$

The first term of expression (29) measures the welfare gains ignoring the general equilibrium and the distributional changes, the second term measures the welfare gains that are due to the general equilibrium effects only, and the third term measures the welfare gains that are due to the distributional changes. In Table 11 we report this decomposition for the two reforms that we study in this article.

Table 11: Decomposing the aggregate welfare changes

	Equivalent variation of consumption (%)			
Economy	Δ^b	$\left(\Delta_i^a - \Delta_i^b\right)$	$(\Delta_i - \Delta_i^a)$	Δ_i
E_1	-0.27	0.75	-0.65	-0.17
E_2	3.64	-0.58	-2.62	0.45

We find that, if we abstract from the distributional and general equilibrium changes, the less progressive flat-tax reform results in a welfare loss that is equivalent to -0.27 percent of consumption. In contrast, the more progressive flat-tax reform results in a welfare gain that is equivalent to 3.64 percent of consumption. These welfare changes are the direct consequence of the redistribution of the tax burden, and of the new individual allocations of consumption and leisure that the new tax system generates.

When we consider the general equilibrium effects brought about by the change in prices, the less progressive flat-tax reform results in a welfare gain that is equivalent to 0.75 percent of consumption, and the more progressive tax reform results in a welfare loss that is equivalent to -0.58 percent of consumption. These welfare changes are the consequence of the efficiency gains or losses that result form the new aggregate values of consumption and leisure in the flat-tax economies.

Finally, the new equilibrium distributions of the flat-tax model economies result in welfare losses that are equivalent to -0.65 percent of consumption in the less progressive flat-tax reform, and to -2.62 percent of consumption in the more progressive flat-tax reform. These welfare losses arise because the reforms move the households to points in the state space which have a lower utility.

Table 12: Welfare inequality

	Equivalent variation of consumption (%)									
Economy	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
E_1	0.3	3.0	2.7	0.5	-0.2	-0.5	-0.5	-0.9	-1.1	0.0
E_2	2.4	4.7	4.5	2.2	4.1	5.1	5.1	-0.4	-2.9	-3.3

Finally, we compute the individual welfare changes brought about by the reforms for the various types of households. Formally, for each household type $(a, s) \in \mathcal{A} \times S$, we compute the equivalent variation $\Delta_i(a,s)$ as follows:

$$v_0(a,s,\Delta_i(a,s);r_0,w_0) = v_i(a,s;r_i,w_i) \tag{30}$$

The interpretation of these welfare measures is the following: they measure the change in consumption that would make each household in the steady-state of the benchmark economy indifferent to being dropped in the steady-state of a given flat-tax economy keeping its assets and its household-specific shock (and, therefore, its age and its endowment of efficiency labor units). By construction these household-specific welfare measures do not account for any of the distributional changes but they do account for the general equilibrium effects.

Once we have computed these individual welfare changes, we aggregate them over the deciles of the before-tax income distribution of model economy E_0. We report these statistics in Table 12. That table unambiguously shows that both flat tax reforms are a significant boon for the income poor. Specifically, we find that the households in the bottom 40 percent of the income distribution of the benchmark model economy would be happier in the less progressive flat-tax model economy (the households in the top decile would also be marginally happier), and that this percentage of happier poor households increases to an impressive 70 percent in the more progressive flat-tax re model economy. On the other hand, the remaining households, who happen to be the income rich (and therefore who are friends of President Bush) would be happier under the current tax system.

5 Concluding comments

Hall and Rabushka (1995) claimed that revenue neutral flat-tax reforms would be expansionary. We find that they can be, if we choose the appropriate flat-tax rate and the associated labor income tax exemption. Many suspected that flat-tax reforms would increase wealth inequality. We find that indeed they do, but that in steady-state Benthamite welfare terms it matters little. Policy-makers fret that flat-tax reforms increase the tax-rates on capital paid by the wealthy. We find that they don't. Economists have puzzled about the trade-off between efficiency and equality. This research uncovers a large Robin Hood effect in flat-tax reforms, and it suggests that, as far as the model economy households are concerned, equality wins the trade-off. Finally, economic folklore has defended progressive taxation on the grounds that it is good for the poor. We find that flat-taxes are better. The next step in this research is to compute the transitions. We know that this task will not be easy, and that it will probably require a large cluster of parallel computers, but we are surely going to give it a try.

Appendix

A The definition of parameters ϕ_1 and ϕ_2

Let p_{ij} denote the transition probability from $i \in \mathcal{R}$ to $j \in \mathcal{E}$, let γ_i^* be the invariant measure of households that receive shock $i \in \mathcal{E}$, and let ϕ_1 and ϕ_2 be the two parameters whose roles are described in Section 4.1.2, then the recursive procedure that we use to compute the p_{ij} is the following:

• *Step 1:* First, we use parameter ϕ_1 to displace the probability mass from a matrix with vector $\gamma_{\mathcal{E}}^* = (\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*)$ in every row towards its diagonal, as follows:

$$
\begin{aligned}
p_{51} &= \gamma_1^* + \phi_1\gamma_2^* + \phi_1^2\gamma_3^* + \phi_1^3\gamma_4^* \\
p_{52} &= (1-\phi_1)[\gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^*] \\
p_{53} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{54} &= (1-\phi_1)\gamma_4^* \\
p_{61} &= (1-\phi_1)\gamma_1^* \\
p_{62} &= \phi_1\gamma_1^* + \gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^* \\
p_{63} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{64} &= (1-\phi_1)\gamma_4^* \\
p_{71} &= (1-\phi_1)\gamma_1^* \\
p_{72} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{73} &= \phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^* + \phi_1\gamma_4^* \\
p_{74} &= (1-\phi_1)\gamma_4^* \\
p_{81} &= (1-\phi_1)\gamma_1^* \\
p_{82} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{83} &= (1-\phi_1)[\phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^*] \\
p_{84} &= \phi_1^3\gamma_1^* + \phi_1^2\gamma_2^* + \phi_1\gamma_3^* + \gamma_4^*
\end{aligned}
$$

• *Step 2:* Then for $i = 5, 6, 7, 8$ we use parameter ϕ_2 to displace the resulting probability mass towards the first column as follows:

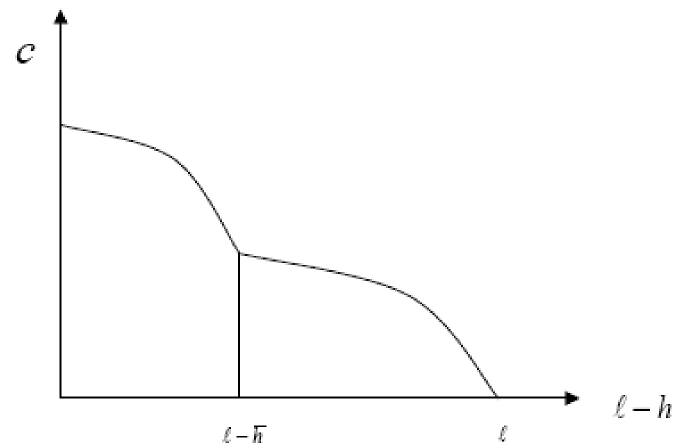
$$
\begin{aligned}
p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p_{i4} \\
p_{i2} &= (1-\phi_2)[p_{i2} + \phi_2 p_{i1} + \phi_2^2 p_{i4}] \\
p_{i3} &= (1-\phi_2)[p_{i3} + \phi_2 p_{i4}] \\
p_{i4} &= (1-\phi_2) p_{i4}
\end{aligned}
$$

B Non-convexities

In Section 4.2.1 we made clear that adding up household income taxes and social security contributions introduces a fall in the marginal tax to labor income. Of course, this fall in the marginal tax to labor income implies a fall in the marginal tax to work effort. This poses a little nagging problem when solving for the household policy. In particular, for a given choice of next period assets z, the budget set of the intra-temporal labor decision is not convex. In Figure 4 we illustrate this point. Take a given pair of individual state variables $\{a, s\}$ and a choice z of assets next period. Then, equations (4), (5) and (6) plus the boundary constraints in c and h define the frontier of possibilities between c and h. In Figure 4 we plot this frontier for a case with $a = 0$. When households supply zero hours and enjoy ℓ units of leisure consumption is zero. As the household starts to supply work effort its consumption increases albeit at a falling rate. The reason is that due to the progressive income tax $\tau_y(y_b)$ every extra hour of work effort generates less after tax income. Let's define $\bar{h}$ as the work effort such that $e(s)\bar{h}w = a_3$. From this point on the social security marginal tax is zero. Therefore the slope of the frontier increases discretely at $h = \bar{h}$ and then, as we increase h, it decreases monotonically due to the progressivity embedded in $\tau_y(y_b)$.

This lack of convexity is twice unfortunate. First, the first order necessary condition is not sufficient for the optimum and therefore it does not identify uniquely the optimal solution. Indeed, there are potentially two points that satisfy the first order condition, one above and one below the threshold a_3 and only one corresponds to the optimal solution. Second, and more troublesome, as we change the choice of assets for next period z the optimal choice of hours will have a point of discontinuity exactly when moving from a solution at one side of a_3 to a solution at the other side.

Figure 4: Non-convex constraints



C Computation

As we have mentioned in Section 3, to calibrate our model economy we must solve a system of 29 non-linear equations in 29 unknowns. Actually, we solve a smaller system of 25 non-linear equations in 25 unknowns because the value of government expenditures, G, is determined residually from the government budget, and because three of the tax parameters are functions of our guess for aggregate output. This non-linear system is only the outer loop of our computational procedure because we must also find the stationary equilibrium values of the capital labor ratio, K/L, and of aggregate output, Y, for each vector of unknowns. The details of our computational procedure are the following:

• *Step 1:* We choose a vector of weights, one for each of the 25 non-linear equations. These weights measure the relative importance that we attach to each one of our targets.

• *Step 2:* We guess a value for the 25 unknowns

• *Step 3:* We guess an initial value for aggregate output, Y_0 (which determines the values of the three tax parameters mentioned above).

• *Step 4:* We guess an initial value for the capital labor ratio $(K/L)_0$

• *Step 5:* We compute the decision rules, the stationary distribution of households and the new value of the capital labor ratio, $(K/L)_1$

• *Step 6:* We iterate on K/L until convergence

• *Step 7:* We compute the new value of aggregate output, Y_1, that results from the converged value of K/L

• *Step 8:* We iterate on Y until convergence

• *Step 9:* We iterate on the 25-dimensional vector of unknowns until we find an acceptable solution to the system of 25 non-linear equations.

To find the solution of the system of 25 non-linear equations in 25 unknowns, we use a standard non-linear equation solver (specifically a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package).

To calculate the decision rules, we discretize the state space and we use a refinement of the discrete value function iteration method. Our refinement uses upper bounds and monotonicity to reduce the size of the control space and Howard's policy improvement algorithm to reduce the number of the searches. The size of our state space is $n_k \times n_s = 681 \times 8 = 5,448$ points. The size of our control space is $n_k \times n_n = 681 \times 201 = 136,881$ points for workers and $n_k = 681$ for retirees. Since the numbers of working-age and retirement states are $n_w = n_r = 4$, the total number of search points is $[(n_k \times n_w) \times (n_k \times n_n)] + [(n_k \times n_r) \times n_k] = 374,718,888$ points.

We approximate the stationary distribution, x^*, with a piecewise linearization of its associated distribution function. The grid for this approximation has 80,000 unequally spaced points which are very close to each other near the origin (see Aiyagari (1994), Huggett (1995) or Ríos-Rull (1998) for details).

To compute the model economy's distributional and aggregate statistics we compute the integrals with respect to the stationary distribution, x^*. We evaluate these integrals directly using our approximation to the distribution function for every statistic except for those that measure mobility, the earnings life cycle, and the intergenerational correlation of earnings. To compute these three statistics, we use a representative sample of 20,000 households drawn from x^* (see Ríos-Rull (1998) for details).

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