# The Welfare Enhancing Effects of a Selfish Government in the Presence of Uninsurable, Idiosyncratic Risk*


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**Abstract**

This paper poses the following question: Is it possible to improve welfare by increasing taxes and throwing away the revenues? This paper demonstrates that the answer to this question is "yes." We show that there may be welfare gains from taxing capital income even when the additional capital income tax revenues are wasted or consumed by a selfish government. Previous literature has assumed that government expenditures are exogenous or productive, or allowed for redistribution of tax revenue either via lump-sum transfers, unemployment compensation or other redistributive schemes. In our model a selfish government taxes capital above a given threshold and then consumes the proceeds. This raises the before-tax real return on capital and and thereby enhances the ability of agents to self-insure when they are long-term unemployed and have low savings. Since all agents have positive probability of finding themselves in that state there are cases where all agents prefer a selfish government to no government at all.

# 1 Introduction

Is it possible to improve welfare by increasing taxes and throwing away the revenues? This paper demonstrates that the answer to this question is "yes."

This question arises naturally in the recent literature on the role of self-interested governments in providing insurance, see in particular Acemoglu-Golosov-Tsyvinski (2005). These authors show in a model of idiosynchratic productivity risk, that a direct mechanism implemented by a selfish government can improve welfare over a laissez-faire market mechanism with no government. In their analysis, agents receive insurance from the government at the price of paying for its selfish consumption. Our analysis shows that there are simple competitive mechanisms with a selfish government that Pareto dominate the laissez-faire competitive equilibrium. We provide intuition and insight into the key mechanisms at work by analyzing two simple benchmark cases and then document the quantitative relevance of these effects in a version of the Bewley-Aiyagari-model with nonlinear capital taxation,

To keep matters simple, we analyze the government from a positive, not a normative perspective: we directly assume that tax revenues are wasted or - alternatively - consumed by a selfish government, without directly modeling its preferences. Another interpretation of our assumption is that are very large frictions in collecting taxes and spending them. While we find any of these interpretations appealing, our focus here is instead entirely on the implied welfare effects to private agents.

Put differently, the point of this paper is to illustrate that ruling out insurance markets for individuals experiencing idiosyncratic shocks opens up some particularly blatant opportunities to improve welfare. In particular, our paper provides a rationale for government and taxation that is distinct from the traditional rationale which relies on the presence of public goods that can only be financed with tax revenues.[1]

We illustrate our point in several steps. In Section 2 we describe some situations where levying a distortionary tax and throwing away the proceeds does *not* enhance welfare. This analysis serves to highlight the roles of complete markets, linear tax rates and homothetic preferences. We then show that if these assumptions are relaxed, there is a positive role for a wasteful government. We illustrate using two simple analytical examples in Section 3.

To ascertain the quantitative relevance of this possibility we conduct some computational experiments using a Bewley-Aiyagari model in Section 4. More specifically, we analyze the welfare effects of an increase in the capital income tax in a model with borrowing constraints and idiosynchratic, uninsurable income shocks.

Previous work by Chamley(1986), Judd (1985) and Lucas (1990) has shown that

[1] see e.g. Glomm and Ravikumar (1994) for an example where the public good aspect of government purchases is modeled.

all agents will prefer a zero tax rate on capital in the long-run when tax rates are linear, markets are complete, government purchases are exogenous and the government is benevolent. Aiyagari (1995) shows that if instead individuals are subject to idiosyncratic uninsured risk and borrowing constraints, the optimal tax rate on capital is positive. This result has found its restatement in the recent work by Golosov and Tsyvinski (2006) within the context of the new dynamic public finance literature, see Kocherlakota (2005) and Golosov, Tsyvinski and Werning (2006).

Our quantitative model departs from the previous literature in three respects. First, we explicitly compare welfare in an equilibrium where capital taxes are used to finance wasteful government purchases with a laissez-faire equilibrium with no government. The previous Ramsey tax literature (examples include Chamley (1986), Judd (1985) or Aiyagari (1995)), in contrast assumes that there is an exogenously fixed and positive amount of government purchases that must be financed. In that environment under some relatively weak assumptions described in Section 2, agents are always better off if the level of government purchases is reduced towards zero. Second, we assume markets are incomplete. This assumption introduces the possibility that a government could improve over a laissez faire competitive equilibrium by enhancing the set of insurance possibilities. Third, we assume that the capital income taxation is nonlinear. More specifically, we assume that a linear tax is levied only on capital income above some exempt level.

We do not provide a direct-mechanism foundation for this assumption, but it is likely that such a foundation can be obtained in at least two ways. First, assume that savings of agents are not observable. Assume furthermore that agents have access to an unobservable "backyard" technology, employing capital up to some threshold $\bar{k}$, while renting out their additional capital holdings to firms via the market place. In that situation, the government can only tax the market payments of firms to households for renting capital at its source, and thus can only tax capital income of households exceeding the threshold. If agents are free to distribute their additional savings across these firms, the government might as well choose the tax schedule to be linear, as in Golosov and Tsyvinski (2006). Second, one might imagine a tax authority requiring reports of agents not only regarding their productivity level, but also their savings, and assume, that the tax authority could verify the savings level at a cost. We conjecture that this environment too could give rise to the type of tax scheme considered here, but leave this for future research.

The provision of insurance emphasized in the recent dynamic public finance literature is key to our results. Even though tax revenues are waste in our computational examples all agents, including the persistently unemployed, the transitorily unemployed, and the employed, prefer a selfish government that imposes a positive income tax on capital over no government. The reason is that when the capital tax is increased, it lowers the stock of capital and thereby raises the before-tax return. As a result, agents are better able to self-insure when they find themselves unemployed with low levels of savings. This positive effect of higher taxes is balanced against two negative effects. Higher taxes on capital lower after-tax income of individuals who save above the threshold. In addition, a higher tax rate on capital is associated with a lower wage rate for the employed. Still, if agents are sufficiently likely to eventually find themselves persistently unemployed with low savings, the insurance benefits exceed the other two costs and all agents in the economy prefer a positive tax rate on capital.

Our quantitative experiments indicate that this result occurs when the insurance motive is large. All of our examples have the property that the real return on assets is negative when there is no government. Moreover, unemployment risk is also large. If these assumptions are not satisfied some agents continue to prefer a positive tax rate on capital. Other agents, however, prefer no taxes and measures of average welfare may either rise or fall with the size of the tax rate.

The remainder of the paper is organized as follows. Section 2 establishes conditions under which there is no role for a selfish government enhance welfare over autarky by taxing and consuming the proceeds. Section 3 uses two examples to show that if these conditions are relaxed that there is a role for a selfish government to tax and consume. Section 4 describes our quantitative model, Section 5 reports results from a set of computational experiments and Section 6 contains our concluding remarks.

## 2 No effect in some benchmark cases

Before analyzing economies in which welfare can be improved by introducing a government that taxes and consumes the proceeds it is useful to first describe conditions under which this *cannot* happen. First we show that if the equilibrium is Pareto-optimal, the result will not obtain. Then we demonstrate the importance of the price channel: if prices are fixed, this channel is shutdown and again, the effect can not take place. Finally, we show in that starting from a second-best case with complete markets, linear taxes on capital income, heterogenous endowments of capital and preferences that satisfy a regularity condition, a tax increase will not increase welfare if the proceeds are thrown away.

The first two claims above are truly trivial. We state them nonetheless for the sake of clarity and completeness. We assume the usual Arrow-Debreu-McKenzie framework, which needs no restating here.

**Proposition 1** *In a Pareto optimal equilibrium, any tax increase cannot result in a Pareto-improvement. In particular, in an equilibrium of an economy, in which the* *first welfare theorem holds, any tax increase which improves the welfare of some agent must decrease the welfare of another agent*

**Proof:** *This is a tautology: there is nothing to prove.* •

This proposition says, that to generate the effect one must either consider an environment where markets are incomplete or start from an allocation that is not Pareto optimal. For instance, if the initial situation is one where there are distortionary taxes than the initial allocation will typically not be Pareto optimal and there is a possibility that increasing a tax and thowing away the proceeds can improve welfare.

If all agents are identical, and attention is restricted to symmetric allocations and supporting price systems that satisfy the first welfare theorem then a tax increase can make no agent better off.

**Proposition 2** *Consider the decision problem of an agent under fixed, nonnegative prices. Assume that the agent has completely ordered preferences, that there is free disposal, and that there are no consumption externalities, i.e. that his (or her) utility does not depend on the consumption of some goods by some other agent.*[2] *Then a tax increase cannot make the agent better off.*

**Proof:** *Using revealed preferences, this is a trivial consequence of the fact that a tax increase reduces the budget set.* •

This proposition indicates that the endogenous response of prices to a tax change is crucial if one is to find situations where welfare can be improved by increasing a tax and throwing the proceeds. If the price channel is shut down then the result will not occur.

To show the third claim about the impossibility of an improvement in expected utility due to a wasted tax increase if taxes are linear and preferences are homothetic, we provide an example rather than a general result. Consider the following model. Suppose that households are identical and solve the following optimization problem:

$$\max U(c_1, c_2, 1-n) \tag{2.1}$$

*s.t.*

[2]Admittedly, these are superfluous assumptions as we have already stated that we are in the usual Arrow-Debreu-McKenzie framework. These assumptions are added here for emphasis, since the proposition ceases to hold, if they are violated.

$$(1 + \tau_c)pc_1 + pc_2 \leq (1 - \tau_n)wn \tag{2.2}$$

where households choose the combination of the two consumption goods and work effort that maximizes their utility given prices and tax rates. We assume that utility is strictly increasing in each of its three arguments. A typical competitive firm chooses its labor input to solve:

$$\max pzn - wn. \tag{2.3}$$

The feasibility constraint for this economy is:

$$c_1 + c_2 \leq zn \tag{2.4}$$

where $z$ is an index of the productivity of labor input. Finally, note that given the structure of this problem taxation has no productive role.

Given taxes $\tau_c$ and $\tau_n$, a competitive equilibrium for this economy is an allocation $(c_1^*, c_2^*, n^*)$ and a set of prices $(p^*, w^*)$ that

1. is feasible
2. solves the household's problem given values of $\tau_c$ and $\tau_n$
3. solves the firm's problem.

In this economy, it is straightforward to show that an increase in either of the tax rates cannot improve welfare. To do this, consider the following (myopic) planner's problem:

$$\max U(c_1, c_2, 1 - n) \tag{2.5}$$

*s.t.*

$$(1 + \tau_c)c_1 + c_2 \leq (1 - \tau_n)zn. \tag{2.6}$$

Notice first, that given taxes $\tau_c$ and $\tau_n$, the allocations from this planners problem can be supported as a competitive equilibrium for the economy described above with the prices $p = 1$ and $w = z$, and vice versa.

Notice next that any increase in $\tau_c$ or $\tau_n$ reduces the resources available to the planner and thus, reduces consumption much as in the proof of Proposition 2. From this we see that it is not feasible to improve the welfare of identical households by increasing a tax on either consumption or labor and throwing away the proceeds.

We will see in the next section that if we allow households to be different and producer prices to adjust to changes in tax rates as they do in general equilibrium that there are nontrivial interactions between heterogeneity and price effects and it becomes possible to improve welfare for at least some agents by levying taxes and throwing away or consuming the proceeds.

# 3 Two simple examples

We now provide two examples in which a tax increase improves utility for some of the agents even though the proceeds are thrown out. These examples are deliberately simple so that it is straightforward to see the economic mechanisms that produce our result.

The first example shows how relaxing homotheticity of preferences can give rise to welfare-increases due to wasted tax increases. The second example shows how a nonlinear tax schedule can improve welfare, even if the tax proceeds are thrown away.

## 3.1 Example 1: Bread or Steak

This section considers a very simple example that demonstrates the theoretical possibility that increasing a tax and throwing away the proceeds can improve welfare of some of the agents. The key here is non-homotheticity of preferences.

There are $n+1$ agents. Only one of these $n$ agents is rich. Poor agents receive an endowment $W_p$ in some numeraire, whereas the rich agent receives the endowment $W_r$. Agents have a choice between eating bread (B) or steak (S). They care greatly about reaching a certain satisfactory level (L) of calory intake, but otherwise prefer to eat steak rather than bread. We formalize this by assuming that agents maximize

$$u(B,S) = v(\alpha \min\{B+S;L\} + (1-\alpha)S) \tag{3.1}$$

*s.t.*

$$p_B B + (1+\tau)p_S S = W,$$

over choices $B \geq 0$, $S \geq 0$ for some increasing and concave function $v(\cdot)$. Here, $\alpha$ measures the relative importance of reaching the level of calory intake $L$, $p_B$ and $p_S$ are the prices of bread and steak in terms of the numeraire, $\tau$ is the consumption tax on steak and $W = W_r$ or $W = W_p$ is the endowment of the agent. We restrict attention to equilibria in which bread is sufficiently cheap, $p_B < \alpha(1+\tau)p_S$, the poor are sufficiently poor, $W_p < p_B L$, and the rich sufficiently rich, $W_r > (1+\tau)p_S L$, so that the poor agents only consume bread and the rich agent only consumes steak.

To produce steak or bread, grain is needed of which there is a total quantity $G$. The production function is assumed to be linear: one unit of grain can be turned into one unit of bread or into $\gamma$ units of steak. The technology is operated by a perfectly competitive sector of price-taking and profit-maximizing firms. These firms solve:

$$\max_{B\geq 0, S\geq 0} p_B nB + p_S S \tag{3.2}$$

*s.t.*

$$nB + S/\gamma = G.$$

Restricting attention to equilibria with interior solutions, it follows that $\gamma p_S = p_B$. We assume that $\gamma < \alpha$, justifying our prior restriction above to equilibria with $p_B < \alpha(1+\tau)p_S$ for positive tax rates, $\tau \geq 0$. Market clearing requires that the demand for bread and steak equals supply. The government receives the tax revenues and is assumed to simply throw them away.

What happens as taxes $\tau$ are increased from zero to some positive amount? To analyze the effect, rewrite the agent's budget constraint as:

$$S = \frac{W_r}{p_S(1+\tau)} = \frac{W_r}{W_p}\frac{\gamma}{1+\tau}B. \tag{3.3}$$

Substituting (3.3) into the resource constraint yields:

$$B\left(n + \frac{W_r}{W_p}\frac{1}{1+\tau}\right) = G, \tag{3.4}$$

demonstrating that the bread consumption of the poor increases with increasing taxes.

On the other hand, welfare for the rich agent declines. With enough curvature on $v$, and a large enough gap between the endowments of the poor and the rich, overall welfare would increase too for a large range of welfare weights.

It is also interesting to note that if agents are allowed to agree on the tax rate $\tau$ before they observe their type, all agents will choose a positive tax rate rather than zero. The reason is clear intuitively: such a tax results in decreased demand for grain, since the rich agent can no longer afford to sustain the zero-tax steak consumption. This decrease in the demand for grain lowers the price of bread in terms of the numeraire, which benefits the poor.

The example above is partial-equilibrium in nature: for example, the ownership of grain is not stated. The mechanism of the example has a general equilibrium flavour, however: the poor are made better off via price changes in highly interdependent markets. The argument depends critically on stating endowments in terms of a numeraire, and assuming these endowments are constant under tax changes. If the model were restated instead, so that the endowment was constant in units of grain, the tax increase would reduce welfare: the bread consumption of the poor would not change, and the steak consumption of the rich would fall. Thus, in evaluating policies of this type in general, the income source matters greatly.

The preference structure is also important. If, for instance, preferences were homothetic in bread and steak then the expenditure share of each of these goods would be constant as income was varied and the result would disappear.

Finally, it should be noted that the fraction of poor and rich is exogenous in this example as well as the other two examples in this section. Thus, only a fraction of agents benefits from a selfish government. When we consider the Aiyagari-Bewley model in Sections 4 and 5 the fraction of each type is endogenous and a given individual will find himself in different situations at various points of time.

## 3.2 Example 2: Capital and Labor

As pointed out in the "bread versus steak" example in Section 3.1 above, the source of income matters. We therefore reinvestigate the issue of capital income taxation. The key in this example will be a nonlinear tax schedule. To keep things simple, we assume that rich agents are subject to a different capital income tax rate than poor agents.

We will assume that poor as well as rich agents are each endowed with one unit of time per period, which they supply inelastically as labor. Thus, their income in the second period is wage income, which in turn is tied to the level of the capital stock: if the capital stock is low, wages will be too, which, ceteris paribus, will hurt the poor agent. With these two opposing forces - the rise in the interest rate versus the fall in wages - at work, is it still possible to construct an example in which the rise in $\tau$ improves welfare for the poor? The purpose of this section is to show that this can indeed occur, and offer a prelude to the computational example of Section 4.

We make some more detailed functional form assumptions. Each agent is endowed with one unit of time per period, which he or she supplies inelastically as labor. Production of output $Y$ is Cobb-Douglas with

$$Y = \frac{1}{\rho} K^{\rho} L^{1-\rho}, \tag{3.5}$$

where $K$ is capital ("machines") and $L$ is labor. The initial capital stock *per agent* is assumed to be $K_1 = \bar{K}$ and is owned entirely by the rich agent. Capital does not depreciate. Output and old capital can be used for consumption and new capital,

$$\frac{1}{n+1} \left( nC_{1,p} + C_{1,r} \right) + K_2 = Y_1 + K_1, \tag{3.6}$$

where $Y_t = K_t^{\rho}/\rho$ is the per capita output in period $t$. Capital per capita in period 2 can be written as

$$K_2 = \frac{1}{n+1} \left( nS_p + S_r \right), \tag{3.7}$$

where $S_p$ is the saving of a poor agent and $S_r$ is the saving of the rich agent. The budget constraint of the poor agent is

$$C_{1,p} + \frac{1}{d} C_{2,p} = W_1 + \frac{1}{d} W_2 \tag{3.8}$$

with $S_p = W_1 - C_{1,p}$, whereas the budget constraint of the rich agent is

$$C_{1,r} + \frac{1}{(1-\tau)d} C_{2,r} = W_1 + \frac{1}{(1-\tau)d} W_2 + (n+1)\left(\rho Y_1 + \bar{K}\right). \tag{3.9}$$

Wages $W_t$ and the dividend on capital $d$ are given, as usual, by

$$W_t = (1-\rho) Y_t \tag{3.10}$$

$$d = \rho Y_2 / K_2 \tag{3.11}$$

We restrict attention to equilibria which permit an interior solution to the savings problem of the rich agent, so that $(1-\tau)d = 1$. This implies that the second period capital stock is

$$K_2 = (1-\tau)^{(1/(1-\rho))} \tag{3.12}$$

and hence second period wages are

$$W_2 = \frac{1-\rho}{\rho} (1-\tau)^{(\rho/(1-\rho))} \tag{3.13}$$

The poor agent will only consume in period 2, i.e. $C_{1,p} = 0$ and

$$\begin{aligned} C_{2,p}(\tau) &= dW_1 + W_2 \\ &= \frac{1-\rho}{\rho}\left(\bar{K}^{\rho} \frac{1}{1-\tau} + (1-\tau)^{(\rho/(1-\rho))}\right) \end{aligned} \tag{3.14}$$

To evaluate, whether a poor agent (and thus every agent ex ante) would favour an increase in $\tau$, starting from $\tau = 0$, evaluate the derivate of the right hand side of (3.14) with respect to $\tau$ at $\tau = 0$ to find

$$\frac{d}{d\tau} C_{2,p}(0) = \frac{1-\rho}{\rho}(\bar{K}^{\rho} - \frac{\rho}{1-\rho}). \tag{3.15}$$

Thus, the tax increase is favored, iff

$$\bar{K}^{\rho} > \frac{\rho}{1-\rho}. \tag{3.16}$$

In that case, the positive interest rate effect ($dW_1$) outweighs the negative second-period wage effect ($W_2$).

To close the model, we need to verify that consumption is positive in both periods for the rich agent, as we have assumed. To that end, we make the following additional assumptions. We assume that $\rho = 1/2 - \epsilon$ for some small but nonzero $\epsilon > 0$, and that $\bar{K} = 1$, so that the inequality above is satisfied, i.e. so that the poor agents are in favour of a tax increase. Their savings $S_p = W_1$ satisfy $S_p < 1 + 5\epsilon$, if $\epsilon$ is sufficiently small, so that the savings of the rich agent at $\tau = 0$ is

$$S_r = (n+1)K_2 - nS_p > 1 - 5n\epsilon. \tag{3.17}$$

If $\epsilon$ is sufficently small, $S_r$ is positive. Since $S_r$ is smaller than $(n+1)$, the initial capital endowment of the rich agent, it follows that the consumption of the rich is indeed positive in both periods.

Thus, here too a tax increase whose proceeds are thrown away improves welfare for the poor. One can use other values for $\rho$ as well, as long as the initial capital stock is chosen "just right": otherwise, the consumption of the rich according to the calculations above becomes negative, which is inadmissable.

# 4 A Bewley-Aiyagari-model with nonlinear capital income taxation.

We next turn to provide a more elaborate computational example that illustrates that increasing the capital income tax and throwing away the proceeds can also enhance welfare in a quantitative model. More specifically, we examine the welfare effects of a nonlinear capital income taxation scheme in the context of an infinite horizon economy with idiosyncratic risk. We have two specific rationales for considering a quantitative model. First, it allows us to relax the assumption that agent types are exogenous and thereby opens the door to the possibility that all agents might prefer a selfish government over no government. Second, we are interested in understanding our theoretical result is quantitatively relevant.

The rationale for investigating a nonlinear tax scheme is based on the results of Section 2. As shown in Section 2 in a simple example, if the tax schedule on capital is linear increasing the tax rate reduces resources everywhere in a quasi-social planners problem, and therefore also reduces welfare. This coincides with numerical results obtainable here: without an exemption in the capital income tax rate, none of our quantitative examples provide higher welfare over a laissez-faire equillibrium with no government.

The welfare analysis reported here is performed by investigating steady state results, rather than considering transitions to the steady state. We do this to faciliate comparisions with the previous literature which has focused on the combined effects of taxation and distribution.

Time is discrete, $t = 1, 2, \ldots$. There is a continuum of agents $i \in [0; 1]$ , who live forever and care about

$$v(n_{i,1}, k_{i,0}) = E\left[\sum_{t=1}^{\infty} \beta^t \frac{c_{i,t}^{1-\eta} - 1}{1 - \eta}\right]. \tag{4.1}$$

Agents may be in one of three states. They can either be employed ($s = 3, n_{i,t} = 1$), short-term unemployed ($s = 2, n_{i,t} = 0$) or long-term unemployed ($s = 1, n_{i,t} = 0$), where we denote the status of employment by $n_{i,t} \in \{0; 1\}$. The state evolves according to a three-state stationary Markov process, which is assumed to be identical, but independent across agents. The transition from state $s \in \{1; 2; 3\}$ to $s' \in \{1; 2; 3\}$ is described by probabilities $\pi_{s,s'}$.

If agents are employed, they work and receive an aftertax wage $w_t$ in terms of period-$t$ consumption. Agents hold capital $k_{i,t}$, where $k_{i,0}$ is the initial endowment of agent i. They rent capital out to firms, for which they receive a rental rate of $d_t$ per unit of capital before taxes and depreciation. Capital depreciates at an exogeneously given rate $\delta$. The consumption good in period $t$ can also be used for investing in the capital good: negative investments ("resales") are permitted. Thus, the before-tax return ("one plus the interest rate") on capital is

$$R_t = d_t + 1 - \delta \tag{4.2}$$

per unit of capital. Capital income is subject to taxes, labor income is assumed to be assessed on an aftertax basis. Since our primary focus is on the effects of capital taxation we hold the labor tax fixed throughout our analysis.

The tax function for capital is given by $\tau(\cdot, \cdot)$. The after-tax return $\rho_{i,t}$ per unit of capital realized by agent $i$ at time $t$ depends on the amount of capital held by the agent and on the market return via

$$\rho_{i,t} k_{i,t-1} = R_t k_{i,t-1} - \tau(R_t, k_{i,t-1}). \tag{4.3}$$

There are no asset markets other than capital. In particular, there is no insurance against the idiosynchratic shocks arising from the Markov process for the employment status. Thus, agent $i$ maximizes the utility function (4.1), subject to the initial conditions $n_{i,1}$ and $k_{i,0}$ and subject to the period-by-period budget constraints

$$c_{i,t} + k_{i,t} = w_t n_{i,t} + R_t k_{i,t-1} - \tau(R_t, k_{i,t-1}). \tag{4.4}$$

with $k_{i,t} \geq 0$.[3]

There is a competitive sector of firms, which hires labor and rents capital to produce output with a linearly homogenous production function. Firms solve

$$\max_{k,n} y(k, n) - d_t k - w_t n \tag{4.5}$$

[3]We are imposing an ad hoc borrowing constraint here that rules out un-collateralized borrowing.

where output is produced using a Cobb-Douglas production function $y = Ak^{\alpha}n^{1-\alpha}$. As usual, profits will be zero in equilibrium.

There is a government, which chooses the tax function $\tau$. The government is assumed to waste these tax revenues or, equivalently, to consume them: agents neither receive transfers from the government nor does government consumption enter their utility function.

Let $G_t$ denote total government consumption or, equivalently, the total wasted tax revenue.

Take a particular tax function $\tau(\cdot,\cdot)$ as given. An equilibrium consists of stochastic sequences $((c_{i,t}, k_{i,t}, n_{i,t}, \rho_{i,t})_{i,t}, w_t, d_t, R_t)$, that solve the maximization problem of the agents and the firm, satisfy the Markov law of motion, and satisfy the market clearing conditions[4],

$$K_t = \int_0^1 k_{i,t} di \tag{4.6}$$

$$N_t = \int_0^1 n_{i,t} di \tag{4.7}$$

$$G_t = \int_0^1 \tau(R_t, k_{i,t}) di \tag{4.8}$$

$$K_t + C_t + G_t = Y_t + (1-\delta)K_{t-1}, \tag{4.9}$$

where $Y_t = AK_{t-1}^{\alpha}N_t^{1-\alpha}$ is aggregate production.

To do the welfare comparison, it is desirable to make all agents identical initially, i.e. to give them the same initial capital and employment status, and to then consider the dynamic adjustment path of the economy. This turns out to be hard numerically and makes comparisons with the previous literature more difficult. We have thus chosen the easier route of only comparing steady states and calculating average welfare: the usual caveats apply. To do the calculations, we alternate between two phases of computation. In phase 1, we iterate on Bellman's equation to calculate household decision rules, given dividends and the wage. In phase 2, we iterate on the steady state capital-employment distribution, exploiting ergodicity, given the decision rules. Alternating between these two phases of calculations usually resulted in convergence to an equilibrium. As the equilibrium must be characterized by $\beta R < 1$, as Huggett (1995) has shown, it follows that the steady state distribution for capital has compact (i.e. bounded) support. The numerical method is in many ways similar to the approach used in Uhlig (1990) or Aiyagari (1994, 1995).

[4]The integrals here are Pettis integrals, see Uhlig (1996), i.e. we employ the law of large numbers.

# 5 Results

The model parameters are set so that the capital share, $\alpha = 0.30$, the discount factor $\beta = 0.96$, the depreciation rate $\delta = \{0.15, 0.7\}$ and the coefficient of relative risk aversion of $\eta = \{1, 2, 5\}$. The Markov transition matrix for the three states of long-term unemployment (state 1), short-term unemployment (state 2) and employment (state 3) is

$$\pi = \begin{bmatrix} 0.97 & 0.03 & 0 \\ 0.03 & 0.2 & 0.77 \\ 0 & 0.1 & 0.9 \end{bmatrix} \tag{5.1}$$

implying a steady state distribution of

$$\mu = \begin{bmatrix} 0.10 \\ 0.1 \\ 0.79 \end{bmatrix} \tag{5.2}$$

We discretize the state-space when solving Bellman's equation. The minimum capital holding is set to 0.0001 and a minimum consumption level of 0.001 is assumed.

We provide three sets of simulation results that vary the risk aversion parameter $\eta$. We will see that all of our examples require a negative real return on assets when there is no government. This is accomplished by varying the depreciation rate on capital. The choice of the Markov process implies that there is a state with persistent unemployment. We view this as a simple way to proxy for e.g. retirement or disability in an infinite horizon model.

We first considered the case where capital income is taxed linearly, i.e. where

$$\tau(R, k) = (1 - \bar{\tau})Rk. \tag{5.3}$$

In this case, no agent benefits from the tax increase, because the after-tax return is lower for everybody.

Next we considered a more general specification of the capital tax schedule by allowing for an exemption:

$$\tau(R, k) = \bar{\tau}R \max\{k - \bar{k}; 0\} \tag{5.4}$$

where $\bar{\tau}$ is the marginal tax rate on capital above the exempt amount $\bar{k}$. The German tax code turns out to have roughly this structure, with the exemption amount to 12200 DM (approx 9000 \$) for a married couple. Also, in the US, there are indirect provisions of that sort, since capital income due to the appreciation of a home is tax free once in a tax payers life. This tax schedule can, more generally, be thought of as a parsimonious way of capturing progressivity in the tax code.

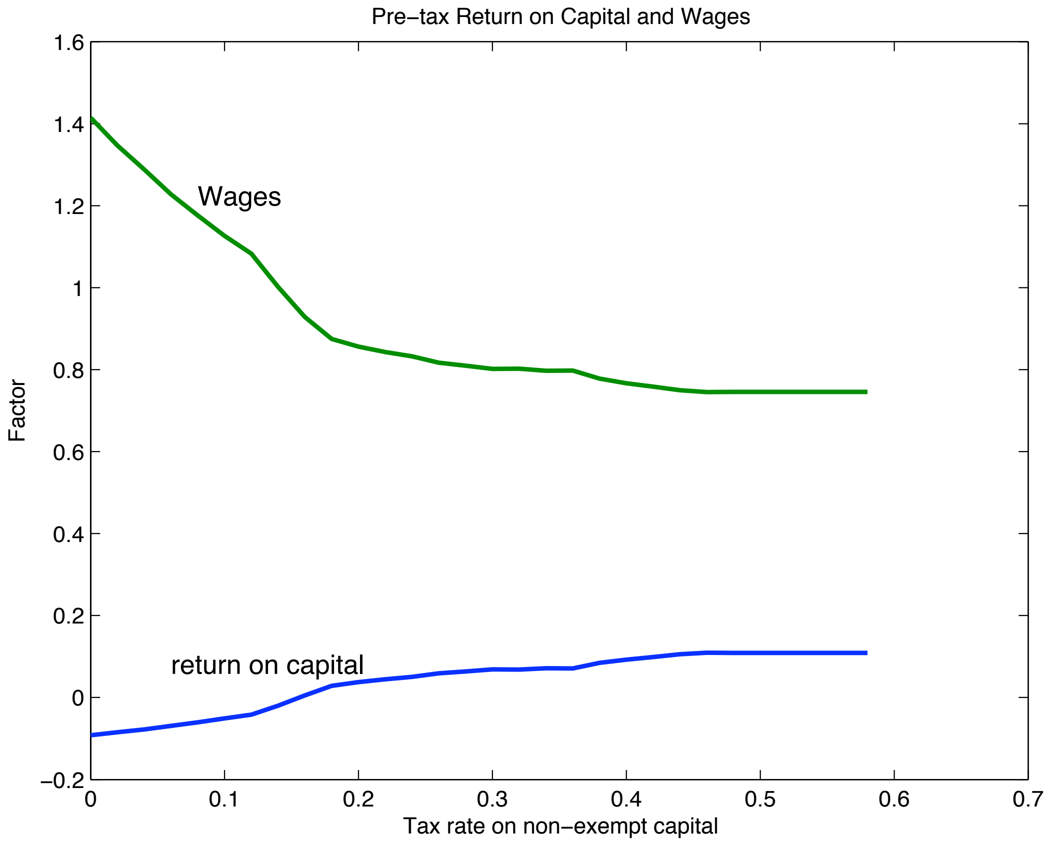


Figure 1: This figure shows how pre-tax dividends and wages vary with the tax rate $\bar{\tau}$.

Note, however, that our tax rates are applied to the entire return $R = d + 1 - \delta$ on capital and not just to the capital gain $d - \delta$. The reason for this choice is that we are interested in a region of the parameter space where $d - \delta < 0$. Consider first the results for the case where $\eta = 2$. This simulation also assumes a value of $\delta = 0.15$ and that the first unit of capital is exempt from taxation.

Figure 1 reports values of the before tax real return on capital and the wage for alternative values of the tax rate on capital. From this figure we can see that the before tax return on capital rises monotonically with the tax rate while the wage rate falls monotonically. Note also, that at low levels of the tax rate the before tax return on capital is negative. In the various simulations we have run, we have found that this feature is important in producing a welfare enhancing role for a selfish government. Households not only face idiosyncratic risk but their ability to self-insure is limited due to the fact that the real value of their assets declines over time. Figure 2 shows how after-tax capital income varies with holdings of the capital stock at three

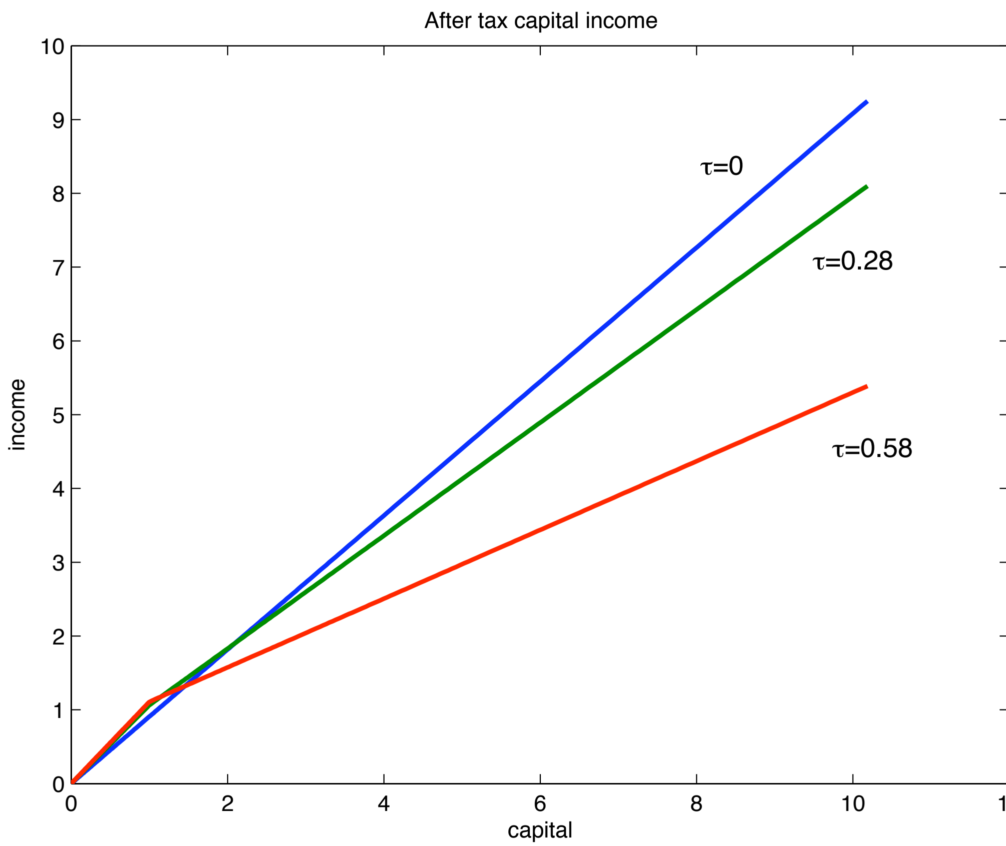


Figure 2: This figure shows how the pattern of after-tax capital income changes as the tax rate is increased when there is an exemption on taxation below a threshold.

alternative tax rates. When the tax rate on capital income is zero capital income rises linearly with holdings of capital. As the tax rate is increased capital income increases at a faster rate for holdings of capital that are below the exemption threshold. Then the slope becomes much flatter at levels of capital income above the threshold. This effect becomes more pronounced as the tax rate is increased from 0.28 to 0.58. From this we see that the general equilibrium effect of the higher tax rate on the before tax return on capital acts to increase capital income for households with low holdings of capital. This mechanism acts as insurance for households when they find themselves persistently unemployed and thus have no labor income and their savings are run down. However, it is not clear at all that this policy is welfare enhancing. When households are employed and have high holdings of capital this policy lowers both their wages and their capital income. Indeed, if households are employed most of the time one might expect this latter effect to dominate the former effect.

Figure 3 indicates that the value of the insurance is substantial in this computational example.

Figure 3 reports the overall welfare effect in terms of expected welfare and welfare for each of the three employment states. It should be emphasized that the plots of each state are averages across all holdings of capital. This figure makes very clear that the biggest beneficiaries of a higher tax rate on capital are the long-term unemployed. The largest benefits occur when the tax rate ranges between 0 and 0.2. It can also be seen from this figure that average welfare is also increasing for short-term unemployed and employed households too. Even though average welfare increases for both employed and unemployed there may be some individuals who prefer no government. This is, in fact, the case in the baseline simulation. For values of the tax rate that are less than 4 percent some agents prefer no taxes. We summarize this property of the simulation in Table 1. When the tax rate reaches 4 percent, as reported in the upper panel of the table, welfare is higher for individuals in all states that receive positive probability in the ergodic distribution. The before tax real return on capital is -0.08 percent and the fraction of output this is consumed by the government is 13 percent. At tax rates above this level welfare is increasing for all agents. The upper limit on the gains to a selfish government occur when government revenue is maximized. This is reported in the lower panel of Table 1, government revenue is maximized when the tax rate is 10 percent. This produces a share of output of 19 percent. Consumption's share of output is 0.33.

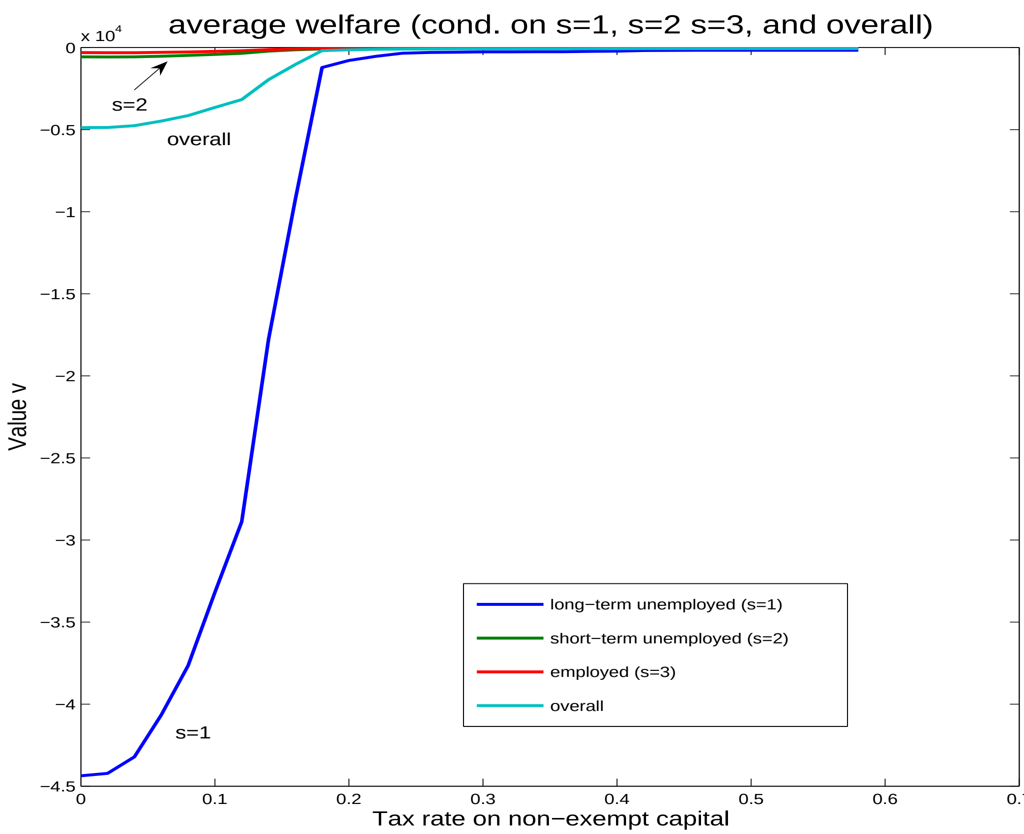


Figure 3: This figure shows the change in average welfare when capital income taxes are increased. The overall effect is divided into the three components: the welfare gain for long-term unemployed agents, short-term unemployed agents and the welfare gain for employed agents.

Table 1

Three parameterizations where the presence of a selfish government provides higher welfare for all agents than laissez-faire (no government).

| | | | |
|---|---|---|---|
| *Parameters* | | | |
| Risk aversion coefficient (η) | 1 | 2 | 5 |
| depreciation rate (δ) | 0.7 | 0.15 | 0.15 |
| amount of capital subject to tax exemption | 0.1 | 1 | 1 |
| *Lowest capital tax rate that produces higher welfare than laissez-faire* | 0.22 | 0.04 | 0.1 |
| Fraction of output that is wasted | 0.048 | 0.13 | 0.33 |
| before tax rental rate on capital | 0.775 | 0.072 | 0.071 |
| level of the capital stock | 0.205 | 6.04 | 6.25 |
| consumption share of output | 0.68 | 0.25 | 0.033 |
| output | 0.53 | 1.459 | 1.474 |
| *Revenue maximizing capital tax rate* | 0.28 | 0.1 | 0.48 |
| Fraction of output that is wasted | 0.055 | 0.19 | 0.63 |
| before tax rental rate on capital | 0.82 | 0.0989 | 0.136 |
| level of the capital stock | 0.19 | 3.88 | 2.437 |
| consumption share of output | 0.69 | 0.3263 | 0.037 |
| output | 0.52 | 1.2778 | 1.11 |

Table 1 also reports results for two other parameterizations of the model. The right most column contains results for a scenario with higher risk aversion $\eta = 5$. For tax rates that range between 0 and 10 percent some but not all agents are better off. However, for tax rates in excess of 10 percent, all agents are better off than laissez-faire. Note also that the fraction of output consumed by the government is larger here it ranges from 0.33 to 0.63 and consumptions share of output is very small.

The left-most column of Figure 1 reports results for log preferences. If the depreciation rate is left at 0.15 then there are always a positive measure of agents who prefer laissez-faire over a positive tax rate on capital. However, if the real return on capital is driven negative by increasing the depreciation rate on capital we find that there are scenarios where all agents prefer a positive tax rate on capital. The key is once again a negative return on capital. Here the range of tax rates that improves welfare and also lies in the increasing portion of the Laffer curve is narrow. The income tax rate must lie between 22 and 28 percent. The fraction of output that is wasted though is less than 6 percent and well within the range of estimates of corruption in emerging economies. In addition, consumption's share of output is a bit below 0.7 which is also not wildly at odds with data from emerging economies. The capital output ratio is low, only about 0.37 and the before tax real return on capital is positive and ranges between 7.5 percent and 12 percent. Overall, this simulation is not is not all that far from the experiences of some emerging economies.

# 6 Conclusion

This paper studied the welfare implications of raising the capital income tax on savings and throwing away the proceeds in an economy with idiosynchratic, uninsurable income shocks and borrowing constraints. This paper has shown, that introducing a selfish government that taxes and consumes the revenues can raise welfare of each type of agent when there is an exemption on the tax for households with low levels of saving.This is because the wasted tax increase raises the before return on capital and thereby increases income of poor unemployed households. The computational examples provided in this paper indicate that this result is most likely to be relevant in very poor economies where households face negative real returns on their savings and occasional highly persistent shocks to their earnings due to e.g. disability. The results also suggest that in economies where property rights are protected and private insurance markets are well developed individuals will be less tolerant of governments that tax capital and consume the proceeds.

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