Monetary Policy in a Life-Cycle Economy: Distributional Consequences of Monetary Policy Rule

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Abstract

In this paper, we answer to practically important questions concerning monetary policy implementation: whether the monetary policy scheme needs to be changed as societal aging deepens; and how monetary policy affects heterogeneous agents, namely workers and retirees, unevenly. According to simulation results from the dynamic stochastic general equilibrium model with nominal rigidity that incorporates lifecycle behavior *a la* Gertler (1999), monetary policy does not have to be altered significantly as societal ageing deepens. On the distributional aspects of monetary policy, however, we find that the optimal instrument rule for workers is quite different from the one for retirees. In an economy where even workers save for their retirement as is the case in Japan, workers prefer more inflation-fighting monetary policy than retirees do and, therefore, the central bank faces policy trade-off between maximizing worker's and retiree's welfare even in a cash-less economy.

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1. Introduction

Societal aging is one of the biggest economic issues in many developed countries. In Japan in particular, society is aging so rapidly that not only is the working population (those older than 15 but younger than 65) already shrinking, but the total population is also expected to start decreasing by 2007. In such a situation, the central bank has an even greater interest in identifying optimal operational monetary policy rule within a framework in which there are heterogenous agents, namely workers and retirees. Seminal research by Woodford (2003) summarizes the various forms of optimal monetary policy corresponding with different economic conditions and has had a significant influence on central banks' views of how monetary policy should be conducted. However, to date very little research has paid attention to the optimal monetary policy rule under heterogenous agents,[1] particularly within a lifecycle setting.[2] One reason for the paucity of research in this area is the difficulty of obtaining the social loss function to be minimized by the central bank within a heterogenous agent setting. This is far from a trivial problem, and makes the analytical derivation of the optimal monetary policy rule extremely difficult. However, use of a second-order approximation of the social loss function, introduced by Rotemberg and Woodford (1998), has recently become popular as way of solving the model and computing the social welfare (utility) based on mean and variances. Recently, Sutherland (2002), Kim, Kim, Schaumburg and Sims (2003), Juillard, Karam, Laxton and Pesenti (2004), and Schmitt-Grohe and Uribe (2004) show how to compute the optimal operational rule, which maximizes unconditional utility.

[1] There exist several researches on the optimality of the Friedman rule with heterogeneous agents, for example, Albanesi (2005), Bhattacharya, Haslag and Martin (2005), and Williamson (2005). As a related research, Doepke and Schneider (2005) show that young borrowers benefits more from inflation than retirees, and inflation can be welfare-enhancing since it acts like a tax on foreign share holders. They, however, pay no attention to the possible distributional effects from choosing different instrument rules even in cash-less economy as examined in this paper.

[2] As a large-scale dynamic general equilibrium model used for central bank projections and policy simulations, the Bank of Finland constructed a model with lifecycle behavior as examined in this paper (see Kilponen, Ripatti and Vilmunen, 2004, and Kilponen and Ripatti, 2005).

With this method, we can obtain the welfare-based instrument rule in a lifecycle economy by maximizing the weighted average utility of different agents. At the same time, we can analyze the nature of optimal operation rules for workers and retirees if they are different.

In this paper, we first set up a dynamic stochastic general equilibrium model with nominal rigidity and capital adjustment costs that incorporates lifecycle behavior *a la* Gertler (1999). We examine the nature of the optimal welfare-based instrument rule within a lifecycle economy. We look at how optimal monetary policy rule differs as we vary the parameter sets defining lifecycle behavior, such as the survival rate of retirees. We then deduce the impact of societal aging on the conduct of monetary policy by looking at the performance of the welfare-based rule for different settings of lifecycle behavior. Of course, as mentioned in Bean (2004), it is true that "the glacial nature of demographic change appears to suggest that the implications for monetary policy should be modest." The role of monetary policy may therefore be limited to reducing short-term distortions. At the same time, however, the optimal monetary policy may change in the face of societal aging. This impact of societal aging on monetary policy has two separate aspects, and it is important to maintain the distinction between the two. First, the "transition" toward the aging society can be most naturally considered in terms of a macro shock, which affects monetary policy decisions. Second, the optimal monetary policy in a "steady state" may be quite different for an elderly society. For the purposes of the current paper, we focus on the second of these aspects.[3]

Bean (2004) summarizes the previous research in this field and points out their implications to the central bank such that: (1) demographic developments represent a macroeconomic shock, which may lead to abrupt movements in asset prices and

[3] We may be able to obtain the Ramsey optimal responses of monetary policy against transitional societal ageing under deterministic world. Yet, the computation of the Ramsey policy under the assumption of exotic preference is not very trivial.

sharp movements in saving behavior; (2) the natural rate of interest becomes lower both along the transition path and in the steady state; (3) the natural rate of unemployment may also be affected through the matching mechanism;[4] (4) the wealth channel is likely to become a more important transmission channel of monetary policy than the intertemporal substitution; (5) the Phillips curve can be flatter due to immigration and the increased participation of retired workers whose supply of labor is considered to be elastic; (6) the constituency for keeping inflation low will be larger thanks to higher average wealth accumulation; and (7) societal aging may induce diversification and risk-shifting with a securitized market rather than bank-intermediated finance, and has some implications for financial stability. Although not all the topics raised by Bean (2004) can be covered in this paper, we formally verify these points using the dynamic general equilibrium model with sticky price and lifecycle behavior. In addition, we compare the shape of the welfare-based instrument rule for a steady-state with the equivalent for an aged society by changing the deep parameters governing the demographics. This leads us to recognize another important point of whether the welfare target – namely the weighted average of utility between workers and retirees – changes as the population of retirees becomes larger. Although the steady state assumed in the lifecycle economy may be very different from the current state of the economy, the simulation results in this paper have some useful implications for how monetary policy should be conducted in the face of an aging society. Retired people rely more on interest income than on wages from their labor supply. It is natural to suppose that the relative importance of income effects over substitution effects becomes larger in an aging society. Hence, monetary policy must have some distributional effects on the welfare of both workers and retirees. For example, more inflation-fighting policy may be welfare-enhancing for the former but not for the latter. Many anecdotal opinions have been

[4] We consider incorporating matching mechanism similar to Merz (1995) and Andolfatto (1996).

heard so far on these points, but there have been very few studies that have tackled this problem in a theoretically consistent dynamic general equilibrium framework,[5] which is a work horse model for modern monetary policy analysis.

We therefore use a theoretical model with micro-foundations to answer such important questions, which have never been analyzed seriously: whether the monetary policy scheme needs to be changed as societal aging deepens; and how monetary policy affects heterogeneous agents, namely workers and retirees, unevenly. From simulations in this paper, we have found several intriguing findings. First, as long as we compute aggregate welfare by assuming one-for-one voting on public policy making under the existence of only two political parties: worker's and retiree's party, monetary policy does not have to be altered significantly as societal aging deepens. Under the reasonable setting of a demographic change, the degree of increase in population of retirees is not enough to alter the optimal instrument rule for aggregate welfare. Second, on the distributional aspects of monetary policy, however, we find that the optimal instrument rule for workers is quite different from the one for retirees. In an economy where even workers save for their retirement as is the case in Japan, workers prefer more inflation-fighting monetary policy than retirees do. Therefore, the central bank faces policy trade-off between maximizing worker's and retiree's welfare due to heterogeneity in agents. Interesting finding here is that distributional effects of monetary policy can be found even in a cash-less economy. Finally, as societal aging deepens, the policy trade-off which the central bank faces becomes less severe. This may also sound counterintuitive. Retirees, however, need to work more as society becomes greyer so that they can maintain the optimized level of consumption in the model examined in this paper. Therefore, the degree of heterogeneity between workers and retirees becomes lessened.

This paper is put together as follows. In section two, we explain the model

[5] Miles (2002) is one exception, but has more interest in the transmission mechanism, and therefore no interest in the forms of policy rules.

employed in this analysis. Section three first discusses how monetary policy can be useful as a stabilization tool in a lifecycle economy. Then, we evaluate how societal aging affects the steady states of the economy as well as impulse responses against technology, cost push, and monetary policy shocks. In section four, we first derive welfare-based operational rules for different steady states of the lifecycle economy and discuss its nature. We, then, inquire into the trade-off which the central bank faces when trying to maximize the welfare of both workers and retirees. Finally, section five concludes and shows future extensions.

2. Model

The model examined here is based on Gertler (1999). We add a sticky price mechanism with endogenous capital, referred to as the "canonical model" by Edge (2003).

2.1. Firms

Firms are assumed to face a cost minimization problem via price setting subject to a Rotemberg (1982) - type adjustment cost.[6]

2.1.1. Marginal Cost

Marginal cost where there exist two inputs, namely labor L and capital K, is computed as examined in Christiano Eichenbaum and Evans (2005). By denoting real wages by $\frac{W}{P}$ as nominal wage over price level and real cost of capital by r^K, each firm j minimizes costs:

$$\frac{W_t}{P_t} L_{j,t} + r_t^K K_{j,t}$$

[6] For tractability in level Phillips curve relation, we here choose Rotemberg adjustment cost for nominal rigidity. This, however, means a departure from standard analytical evaluation of monetary policy using theoretically consistent loss function based on Calvo pricing. In first order, it does not make any difference whether the Phillips curve is based on Calvo or Rotemberg, but it does in second order. We leave the extension to employ Calvo pricing in this life cycle model as examined in Schmitt-Grohe and Uribe (2004) for our future research.

subject to standard Cobb-Douglas production technology with capital share being α,

$$Y_{j,t} = [Z_t \exp(z_t) L_{j,t}]^{1-\alpha} K_{j,t}^{\alpha},$$

where Y is output, Z is deterministic technology growth, and technology shock z is assumed to follow an AR(1) process:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{1}$$

$$\varepsilon_{z,t} \sim N(0, \sigma_z).$$

The Lagrangian multiplier of this optimization problem is the real marginal cost φ. This is assumed to be symmetric across monopolistically competitive firms:

$$\varphi_t = \left[\frac{W_t}{(1-\alpha) Z_t \exp(z_t) P_t}\right]^{1-\alpha} \left(\frac{r_t^K}{\alpha}\right)^{\alpha}.$$

Similarly, real wages and cost of capital are also defined as follows:

$$\frac{W_t}{P_t} = (1-\alpha) \varphi_t [Z_t \exp(z_t)]^{1-\alpha} L_{j,t}^{-\alpha} K_{j,t}^{\alpha},$$

$$r_{t+1}^K = \alpha \mathrm{E}_t \varphi_{t+1} [Z_{t+1} \exp(z_{t+1})]^{1-\alpha} L_{j,t+1}^{1-\alpha} K_{j,t+1}^{\alpha-1}.$$

2.1.2. Price Setting

Under monopolistic competition and a Rotemberg (1982) type adjustment cost ϕ, each firm sets prices in order to maximize its real dividend D:

$$D_{j,t} = \frac{P_{j,t,}}{P_t} Y_{j,t} - \varphi_t Y_{j,t} - \frac{\phi}{2}\left(\frac{P_{j,t}}{\pi P_{j,t-1}} - 1\right)^2 Y_t,$$

subject to a downward sloping demand curve with elasticity of substitution κ:

$$Y_{j,t} = \left(\frac{P_{j,t,}}{P_t}\right)^{-\kappa} Y_t.$$

In symmetric equilibrium where $P_{j,t} = P_t$ and the target level of inflation is zero, from the first order necessary condition, we can derive the New Keynesian Phillips curve:

$$(1-\kappa) + \kappa\varphi_t - \phi\left(\pi_t - 1\right)\pi_t + \mathrm{E}_t \frac{1}{R_{t+1}} \phi\left(\pi_{t+1} - 1\right)\pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} + u_t = 0, \quad (2)$$

where gross inflation rate π is defined by

$$\pi_t = \frac{P_t}{P_{t-1}},$$

and R is gross nominal interest rate set by the central bank and cost push shock u is assumed to follow AR process:[7]

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}, \quad (3)$$

$$\varepsilon_{u,t} \sim N\left(0, \sigma_u\right)$$

and the macro-level real dividend is defined as follows:

$$D_t = \left[1 - \varphi_t - \frac{\phi}{2}\left(\pi_t - 1\right)^2\right] Y_t. \quad (4)$$

[7] Cost push shock is included in this model as a stochastic markup as suggested by Clarida, Gali and Gertler (2001, 2002).

2.1.3. Capital Producers

Competitive capital producers[8] make new capital goods, which is sold at the competitive price Q, using financial assets of households A and investment I. Each optimizes following profit:

$$Q_t K_{t+1} - I_t - \frac{R_t}{\pi_t}\frac{A_t}{P_t} + r_t^K K_t,$$

subject to production technology of capital used by firms:

$$K_{t+1} = (1-\delta) K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

financial market equilibrium:

$$\frac{A_t}{P_t} = Q_{t-1} K_t,$$

and adjustment cost function $S(\cdot)$ used in Christiano, Eichenbaum and Evans (2005):[9]

$$S\left(\frac{I_t}{I_{t-1}}\right) = \left[(1+z)(1+n)\right]^2 S'' \left\{ \frac{\left(\frac{I_t}{I_{t-1}}\right)^2}{2\left[(1+z)(1+n)\right]^2} - \frac{I_t}{I_{t-1}(1+z)(1+n)} + \frac{1}{2} \right\}.$$

From first order necessary conditions, we can obtain the equation for rental cost of capital:

$$Q_{t+1}(1-\delta) - \frac{R_{t+1}}{\mathrm{E}_t \pi_{t+1}} Q_t + r_{t+1}^K = 0, \tag{5}$$

[8] This is the behavior of type II firm in Ljungqvist and Sargent (2004).

[9] As shown in such researches as Dupor (2001), Carlstrom and Fuerst (2005) and Woodford (2003), realistic model property, especially in responses to policy shock, is not obtained without capital adjustment cost.

and the price of capital:

$$Q_t \left\{ 1 - [(1+z)(1+n)]^2 S'' \left[\frac{\left(\frac{I_t}{I_{t-1}}\right)^2}{2(1+z)^2(1+n)^2} - \frac{\frac{I_t}{I_{t-1}}}{(1+z)(1+n)} + \frac{1}{2} \right] \right\} = 1 \quad (6)$$

$$+Q_t [(1+z)(1+n)]^2 S'' \left[\frac{\frac{I_t}{I_{t-1}}}{(1+z)^2(1+n)^2} - \frac{1}{(1+z)(1+n)} \right] \frac{I_t}{I_{t-1}}$$

$$-\mathrm{E}_t \frac{Q_{t+1}\pi_{t+1}}{R_{t+1}} [(1+z)(1+n)]^2 S'' \left[\frac{\frac{I_{t+1}}{I_t}}{(1+z)^2(1+n)^2} - \frac{1}{(1+z)(1+n)} \right] \left(\frac{I_{t+1}}{I_t} \right)^2 .$$

Furthermore, the law of motion for capital is now expressed as:

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t \qquad (7) \\ &+ \left\{ 1 - [(1+z)(1+n)]^2 S'' \left[\frac{\left(\frac{I_t}{I_{t-1}}\right)^2}{2(1+z)^2(1+n)^2} - \frac{\frac{I_t}{I_{t-1}}}{(1+z)(1+n)} + \frac{1}{2} \right] \right\} I_t . \end{aligned}$$

2.2. Households

In a lifecycle economy assumed in this model, there are two types of households: retirees and workers.[10]

2.2.1. Retiree

Retirees, denoted by superscript r, who were born at j and become retired at k, are assumed to maximize their recursive utility V from consumption C and leisure $1-L$:[11]

$$V_t^{rjk} = \left\{ \left[\left(C_t^{rjk} \right)^v \left(1 - L_t^{rjk} \right)^{1-v} \right]^\rho + \beta\gamma \mathrm{E}_t \left(V_{t+1}^{rjk} \right)^\rho \right\}^{\frac{1}{\rho}} ,$$

where ρ determines the intertemporal elasticity of substitution and ν defines the marginal rate of transformation between consumption and leisure. Since retirees rate of survival is γ, future welfare is discounted by common subjective discount

[10] It is quite possible to extend the model to incorporate, for example, three generations, such as young worker's, old worker's and retirees.

[11] A Cobb-Douglas utility function satisfies the balanced growth restriction.

factor β multiplied with γ. This optimization problem is subject to their intertemporal budget constraint:

$$\frac{A_{t+1}^{rjk}}{P_t} = \left(\frac{R_t}{\gamma}\right)\frac{A_t^{rjk}}{P_t} + \frac{W_t}{P_t}\xi L_t^{rjk} + D_t^{rjk} - C_t^{rjk},$$

where $\xi \in [0,1]$ is the relative marginal product of labor of retirees to workers.[12] It is natural to assume that retirees receive less compensation than workers. From the first order necessary conditions, we can derive the relationship between consumption and labor supply:

$$L_t^{rjk} = 1 - \frac{1-v}{v}\frac{P_t}{\xi W_t}C_t^{rjk},$$

and the consumption Euler equation:

$$\mathrm{E}_t C_{t+1}^{rjk} = \left[\beta \mathrm{E}_t R_{t+1}\left(\frac{P_t}{P_{t+1}}\right)^{1-\rho+v\rho}\left(\frac{W_t}{W_{t+1}}\right)^{(1-v)\rho}\right]^{\frac{1}{1-\rho}} C_t^{rjk}. \tag{8}$$

Next, we derive a solved-out consumption function, in the form of total wealth multiplied by the marginal propensity to consume out of wealth $\epsilon\theta$:

$$C_t^{rjk} = \epsilon_t\theta_t\left[\left(\frac{R_t}{\gamma}\right)\frac{A_t^{rjk}}{P_t} + H_t^{rjk} + F_t^{rjk}\right]. \tag{9}$$

By iterating the budget constraint forward, human wealth H and financial wealth F can be expressed in a recursive manner:

$$F_t^{rjk} = D_t^{rjk} + \mathrm{E}_t\frac{P_{t+1}\gamma}{P_t R_{t+1}}F_{t+1}^{rjk},$$

$$H_t^{rjk} = \frac{W_t}{P_t}\xi L_t^{rjk} + \mathrm{E}_t\frac{P_{t+1}\gamma}{P_t R_{t+1}}H_{t+1}^{rjk}.$$

[12] There are other interpretations for ξ. It can be considered to reflect the incentive and legal structure pension system or the relative labour income tax rate.

We can then derive the dynamic equation for the marginal propensity to consume $\varepsilon\theta$ for retirees:

$$\epsilon_t\theta_t = 1 - \mathrm{E}_t \frac{\epsilon_t\theta_t}{\epsilon_{t+1}\theta_{t+1}} \gamma R_{t+1}^{\frac{\rho}{1-\rho}} \beta^{\frac{1}{1-\rho}} \left(\frac{P_t}{P_{t+1}}\right)^{\frac{v\rho}{1-\rho}} \left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}}. \tag{10}$$

Furthermore, we can find a value function that satisfies the above conditions:

$$V_t^{rjk} = (\epsilon_t\theta_t)^{-\frac{1}{\rho}} C_t^{rjk} \left(\frac{1-v}{v}\frac{P_t}{\xi W_t}\right)^{1-v}. \tag{11}$$

2.3. Workers

Workers, denoted by superscript w, who were born at j, also maximize their recursive utility:

$$V_t^{wj} = \left\{ \left[\left(C_t^{wj}\right)^v \left(1 - L_t^{wj}\right)^{1-v} \right]^\rho + \beta \mathrm{E}_t \left[\omega V_{t+1}^{wj} + (1-\omega) \left(V_{t+1}^{rj}\right) \right]^\rho \right\}^{\frac{1}{\rho}}$$

subject to

$$\frac{A_{t+1}^{wj}}{P_t} = R_t \frac{A_t^{wj}}{P_t} + \frac{W_t}{P_t} L_t^{wj} + D_t^{wj} - C_t^{wj},$$

where ω is the probability that the current worker will remain a worker in the next period. From the first order necessary conditions, we can derive the relationship between consumption and labor supply:

$$L_t^{wj} = 1 - \frac{1-v}{v}\frac{P_t}{W_t} C_t^{wj}.$$

and the consumption Euler equation:

$$\begin{aligned}
& \left[\left(C_t^{wj}\right)^v \left(1-L_t^{wj}\right)^{1-v}\right]^{\rho-1} \left(1-L_t^{wj}\right)^{1-v} v\left(C_t^{wj}\right)^{v-1} \\
= \quad & \beta \mathrm{E}_t \left[\omega V_{t+1}^{wj} + (1-\omega)\left(V_{t+1}^{rj}\right)\right]^{\rho-1} \\
& v\frac{R_{t+1}}{P_{t+1}} \left(\begin{array}{c} \omega P_t \left(V_{t+1}^{wj}\right)^{1-\rho} \left(C_{t+1}^{wj}\right)^{v\rho-1} \left(1-L_{t+1}^{wj}\right)^{(1-v)\rho} \\ +(1-\omega) P_t \left(V_{t+1}^{rj}\right)^{1-\rho} \left(C_{t+1}^{rj}\right)^{v\rho-1} \left(1-L_{t+1}^{rj}\right)^{(1-v)\rho} \end{array}\right).
\end{aligned}$$

Then, we assume that the value function takes the form:

$$V_t^{wj} = (\theta_t)^{-\frac{1}{\rho}} C_t^{wj} \left(\frac{1-v}{v}\frac{P_t}{W_t}\right)^{1-v}, \tag{12}$$

where θ is the marginal propensity to consume for workers. Then, the Euler condition becomes:

$$\begin{aligned}
& \omega \mathrm{E}_t C_{t+1}^{wj} + (1-\omega)(\epsilon_{t+1})^{-\frac{1}{\rho}} \mathrm{E}_t C_{t+1}^{rj(t+1)} \left(\frac{1}{\xi}\right)^{1-v} \\
= \quad & \left\{\beta \mathrm{E}_t \frac{P_t R_{t+1}}{P_{t+1}} \left[\omega + (1-\omega)(\epsilon_{t+1})^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}\right] \left(\frac{P_t}{P_{t+1}}\frac{W_{t+1}}{W_t}\right)^{(v-1)\rho}\right\}^{\frac{1}{1-\rho}} C_t^{wj}.
\end{aligned} \tag{13}$$

As with the case for retirees, we are looking to identify the marginal propensity of consumption out of wealth in the solved-out consumption equation:

$$C_t^{wj} = \theta_t \left(R_t \frac{A_t^{wj}}{P_t} + H_t^{wj} + F_t^{wj}\right). \tag{14}$$

Then, by using the consumption Euler equation and the solved out consumption function, we can derive a dynamic equation as follows

$$
\begin{aligned}
&\left[1-\beta^{\frac{1}{1-\rho}}\mathrm{E}_t\left(\frac{P_tR_{t+1}}{P_{t+1}}\Psi_{t+1}\right)^{\frac{\rho}{1-\rho}}\left(\frac{P_t}{P_{t+1}}\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}}\frac{\theta_t}{\theta_{t+1}}-\theta_t\right]R_t\frac{A_t^{wj}}{P_t} \\
= \quad &\left[-1+\theta_t+\beta^{\frac{1}{1-\rho}}\mathrm{E}_t\left(\frac{P_tR_{t+1}}{P_{t+1}}\Psi_{t+1}\right)^{\frac{\rho}{1-\rho}}\left(\frac{P_t}{P_{t+1}}\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}}\frac{\theta_t}{\theta_{t+1}}\right]\left(H_t^{wj}+F_t^{wj}\right) \\
&\left(H_t^{wj}+F_t^{wj}\right)-\left(\frac{W_t}{P_t}L_t^{wj}+D_t^{wj}\right)-\mathrm{E}_t\frac{P_{t+1}}{P_tR_{t+1}}\frac{\omega}{\Psi_{t+1}}\left(H_{t+1}^{wj}+F_{t+1}^{wj}\right) \\
&-\mathrm{E}_t\frac{P_{t+1}}{P_tR_{t+1}}\frac{1}{\Psi_{t+1}}\left(1-\omega\right)\left(\epsilon_{t+1}\right)^{-\frac{1}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}\epsilon_{t+1}\left(H_{t+1}^{rj}+F_{t+1}^{rj}\right),
\end{aligned}
$$

where we define

$$
\Psi_t=\omega+\left(1-\omega\right)\left(\epsilon_t\right)^{-\frac{1-\rho}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}. \tag{15}
$$

This equation holds, if

$$
\theta_t=1-\beta^{\frac{1}{1-\rho}}\mathrm{E}_t\left(\frac{P_t}{P_{t+1}}\right)^{\frac{\rho v}{1-\rho}}\left(R_{t+1}\right)^{\frac{\rho}{1-\rho}}\Psi_{t+1}^{\frac{\rho}{1-\rho}}\left(\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}}\frac{\theta_t}{\theta_{t+1}}, \tag{16}
$$

$$
H_t^{wj}=\frac{W_t}{P_t}L_t^{wj}+\omega\mathrm{E}_t\frac{P_{t+1}H_{t+1}^{wj}}{P_tR_{t+1}\Psi_{t+1}}+\left(1-\omega\right)\mathrm{E}_t\left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}\frac{P_{t+1}H_{t+1}^{rj}}{P_tR_{t+1}\Psi_{t+1}},
$$

and

$$
F_t^{wj}=D_t^{wj}+\omega\mathrm{E}_t\frac{P_{t+1}F_{t+1}^{wj}}{P_tR_{t+1}\Psi_{t+1}}+\left(1-\omega\right)\mathrm{E}_t\left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}\frac{P_{t+1}F_{t+1}^{rj}}{P_tR_{t+1}\Psi_{t+1}}.
$$

With these three equations satisfied, the surmised value function has a solution.

2.4. Aggregation

First, we summarize the population growth in this model. Then, the equations that define individual behavior are transformed into aggregate form.

2.4.1. Population Growth

The dynamics of the population of workers N is expressed as follows:

$$\begin{aligned} N_{t+1} &= (1-\omega+n)\,N_t + \omega N_t \\ &= (1+n)\,N_t, \end{aligned}$$

where n is the growth rate of workers, while that of retirees is:

$$N^r_{t+1} = (1-\omega)\,N_t + \gamma N^r_t.$$

Hence, around the steady state, the ratio of the number of retirees to that of workers becomes constant:

$$\frac{N^r}{N} = \frac{1-\omega}{1+n-\gamma} = \Gamma,$$

which means that both the working and retired populations grow at the same rate n.[13]

2.4.2. Aggregation

If we assume the existence of a non-profit life insurance company that distributes wealth among retirees, the marginal propensity to consume becomes the same among them. Therefore, subscripts j and k in the equations above can just be removed. The law of motion of assets held by retirees in aggregate is defined by:

$$\frac{A^r_{t+1}}{P_t} = R_t\frac{A^r_t}{P_t} + \frac{W_t}{P_t}\xi L^r_t + D^r_t - C^r_t + (1-\omega)\left(R_t\frac{A^w_t}{P_t} + \frac{W_t}{P_t}L^w_t + D^w_t - C^w_t\right), \quad (17)$$

[13] Analysis around demographic steady state is rather unrealistic. However, we believe that our research around steady state is useful to understand the nature of how monetary policy should be conducted when the relative importance of retirees increases.

while the equivalent expression for assets held by workers is:

$$\frac{A_{t+1}^w}{P_t} = \omega R_t \frac{A_t^w}{P_t} + \frac{W_t}{P_t} L_t^w + D_t^w - C_t^w,$$

with financial market equilibrium:

$$\frac{A_{t+1}^r + A_{t+1}^w}{P_{t+1}} = Q_t K_{t+1} \tag{18}$$

We assume that dividends are distributed in line with the amount of financial assets held:

$$D_t^w = \frac{A_t^w}{A_t^w + A_t^r} D_t, \tag{19}$$

and

$$D_t^r = \frac{A_t^r}{A_t^w + A_t^r} D_t. \tag{20}$$

Next, we aggregate individual labor supply. This simply involves multiplying by the population of each category:

$$L_t^r = \Gamma N_t - \frac{1-v}{v} \frac{P_t}{\xi W_t} C_t^r, \tag{21}$$

and

$$L_t^w = N_t - \frac{1-v}{v} \frac{P_t}{W_t} C_t^w. \tag{22}$$

According to the labor supply decision above, the production function also changes as follows:[14]

$$Y_t = \left[Z_t \exp(z_t) \left(L_t^w + \xi L_t^r \right) \right]^{1-\alpha} K_t^\alpha. \tag{23}$$

[14] This holds as a result of approximation, as shown in Yun (1996).

Therefore, real wages and cost of capital are also defined as:

$$\frac{W_t}{P_t} = (1-\alpha)\,\varphi_t \frac{Y_t}{L_t^w + \xi L_t^r} \tag{24}$$

$$r_{t+1}^K = \alpha \mathrm{E}_t \varphi_{t+1} \frac{Y_{t+1}}{K_{t+1}}. \tag{25}$$

The resource constraint is expressed as:

$$Y_t = C_t^r + C_t^w + I_t \tag{26}$$

Furthermore, because the population growth rate in each category is $(1+n)$, the discount rate when computing financial and human wealth also changes. Therefore, :

$$F_t^r = D_t^r + \mathrm{E}_t \frac{P_{t+1}}{P_t} \frac{\gamma}{(1+n)\,R_{t+1}} F_{t+1}^r, \tag{27}$$

$$H_t^r = \frac{W_t}{P_t} \xi L_t^r + \mathrm{E}_t \frac{P_{t+1}}{P_t} \frac{\gamma}{(1+n)\,R_{t+1}} H_{t+1}^r, \tag{28}$$

$$\begin{aligned} H_t^w \quad = \quad & \frac{W_t}{P_t} L_t^w + \omega \mathrm{E}_t \frac{H_{t+1}^w}{(1+n)\,R_{t+1}\Psi_{t+1}} \frac{P_{t+1}}{P_t} \\ & + (1-\omega)\,\mathrm{E}_t \left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{H_{t+1}^r}{(1+n)\,R_{t+1}\Psi_{t+1}} \frac{P_{t+1}}{P_t}, \end{aligned} \tag{29}$$

and

$$\begin{aligned} F_t^w \quad = \quad & D_t^w + \omega \mathrm{E}_t \frac{F_{t+1}^w}{(1+n)\,R_{t+1}\Psi_{t+1}} \frac{P_{t+1}}{P_t} \\ & + (1-\omega)\,\mathrm{E}_t \left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{F_{t+1}^r}{(1+n)\,R_{t+1}\Psi_{t+1}} \frac{P_{t+1}}{P_t}. \end{aligned} \tag{30}$$

2.5. Monetary Policy

As examined in Juillard, Karam, Laxton and Pesenti (2004), and Schmitt-Grohe and Uribe (2004), we restrict our attention to the class of instrument rule that maximizes the aggregate welfare defined below:

$$V = \frac{v^w}{v^w_{opt}} + \Gamma \frac{v^r}{v^r_{opt}}, \tag{31}$$

where v^w and v^r are de-trended unconditional mean of simulated welfare of workers and retirees respectively while v^w_{opt} and v^r_{opt} are their maximized level within the class of rules examined in this paper. We assume that aggregate welfare is defined by the weighted average of percentage deviations of unconditional welfare from its optimal level by population.[15]

Concerning monetary policy rule, there are several candidates for the instrument rule. We follow Juillard, Karam, Laxton and Pesenti (2004) and use very standard policy rule:

$$R_{t+1} = \vartheta R_t + (1-\vartheta) R_{SS} + \eta_1 (\pi_t - 1) + \eta_2 \left(\frac{Y_t}{Y_{t-1}} - 1 \right) + \tau_t, \tag{32}$$

where the monetary policy shock, which is used only for deriving impulse responses, is assumed to follow n.i.d. process:

$$\tau_t = \varepsilon_{\tau,t}$$

$$\varepsilon_{z,t} \sim N(0, \sigma_z).$$

[15]In this model, we assume one vote for each individual and existence of only two political parties, namely party for workers and that for retirees. Therefore, even the worker very close to the average retirement age would vote for worker's party under this voting scheme, since each agent faces constant probability of transition. Workers cannot know exactly when they will turn to be retirees. Therefore, dominance of worker's party supported by large population of workers is stronger in this paper than in the actual economy. This problem should be partially alleviated by dividing workers into two generations such as young and old workers.

Furthermore, we exclude any strategic interaction between workers and retirees.

2.6. De-trended System of Equations

The system of equations consists of 27 equations: (1), (2), (3), (4), (5), (6), (7), (9), (10), (14), (15), (16), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26), (27), (28), (29), (30), and (32). Since we assume both deterministic technology and population growth, endogenous variables are de-trended: Y, C, I, K, D, H, F are de-trended by ZN; A is de-trended by ZNP; W is de-trended by ZP; L is de-trended by N; and V is by $Z^v N$. Now the de-trended system of equations becomes:

$$r^K : \mathrm{E}_t Q_{t+1}(1-\delta) - \frac{R_{t+1}}{\mathrm{E}_t \pi_{t+1}} Q_t + r^K_{t+1} = 0$$

$$Q : Q_t \left\{ 1 - \left[(1+z)(1+n)\right]^2 S'' \left[\frac{\left(\frac{i_t}{i_{t-1}}\right)^2}{2} - \frac{i_t}{i_{t-1}} + \frac{1}{2} \right] \right\}$$
$$-Q_t \left[(1+z)(1+n)\right]^2 S'' \left(\frac{i_t}{i_{t-1}} - 1\right) \frac{i_t}{i_{t-1}}$$
$$+E_t \frac{Q_{t+1}\pi_{t+1}}{R_{t+1}} \left[(1+z)(1+n)\right]^3 S'' \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2 - 1 = 0$$

$$k : k_{t+1} = (1-\delta) \frac{k_t}{(1+z)(1+n)}$$
$$+ \left\{ 1 - \left[(1+z)(1+n)\right]^2 S'' \left[\frac{\left(\frac{i_t}{i_{t-1}}\right)^2}{2} - \frac{i_t}{i_{t-1}} + \frac{1}{2} \right] \right\} i_t$$

$$\varphi : r^K_{t+1} = \alpha \mathrm{E}_t \varphi_{t+1} \frac{Y_{t+1}}{K_{t+1}} (1+z)(1+n)$$

$$\pi : (1-\kappa) + \kappa \varphi_t - \phi(\pi_t - 1)\pi_t$$
$$+\mathrm{E}_t \frac{1}{R_{t+1}} \phi(\pi_{t+1} - 1)\pi^2_{t+1} \frac{y_{t+1}}{y_t} (1+z)(1+n) + u_t = 0$$

$$d : d_t = \left(1 - \varphi_t - \frac{\phi}{2}\pi_t^2\right) y_t$$

$$l^r : l^r_t = \Gamma - \frac{1-v}{v} \frac{1}{\xi w_t} c^r_t$$

$$\epsilon : \epsilon_t \theta_t = 1 - \mathrm{E}_t \frac{\epsilon_t \theta_t}{\mathrm{E}_t \epsilon_{t+1} \theta_{t+1}} \gamma R_{t+1}^{\frac{\rho}{1-\rho}} \beta^{\frac{1}{1-\rho}} \left(\frac{1}{\pi_{t+1}}\right)^{\frac{v\rho}{1-\rho}} \left(\frac{w_t}{w_{t+1}} \frac{1}{\pi_{t+1}(1+z)}\right)^{\frac{(1-v)\rho}{1-\rho}}$$

$$l^w : l^w_t = 1 - \frac{1-v}{v} \frac{1}{w_t} c^w_t$$

$$c^w : c^w_t = \kappa_t \left[\frac{R_t}{\pi_t} \frac{a^w_t}{(1+z)(1+n)} + h^w_t + f^w_t \right]$$

$$\Psi : \Psi_t = \omega + (1-\omega)(\epsilon_t)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}$$

$$\theta : \theta_t = 1$$
$$-\beta^{\frac{1}{1-\rho}} \mathrm{E}_t \left(\frac{1}{1+\pi_{t+1}}\right)^{\frac{\rho v}{1-\rho}} (R_{t+1}\Psi_{t+1})^{\frac{\rho}{1-\rho}} \left[\frac{w_{t+1}}{w_t}(1+\pi_{t+1})(1+z)\right]^{\frac{(v-1)\rho}{1-\rho}} \frac{\theta_t}{\theta_{t+1}}$$

$$c^r : c^r_t = \epsilon_t \theta_t \left(\frac{R_t}{\pi_t} a^r_t + h^r_t + f^r_t \right)$$

$$f^r : f^r_t = d^r_t + \mathrm{E}_t \frac{\gamma \pi_{t+1}(1+z)}{R_{t+1}} f^r_{t+1}$$

$h^r : h_t^r = \xi w_t l_t^r + \mathrm{E}_t \frac{\gamma \pi_{t+1}(1+z)}{R_{t+1}} h_{t+1}^r$

$h^w : h_t^w = w_t l_t^w + \mathrm{E}_t \frac{\omega \pi_{t+1}(1+z)\mathrm{E}_t h_{t+1}^w}{R_{t+1}\Psi_{t+1}}$
$\quad + \mathrm{E}_t \left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{(1-\omega)\pi_{t+1}(1+z)h_{t+1}^r}{R_{t+1}\Psi_{t+1}}$

$f^w : f_t^w = d_t^w + \mathrm{E}_t \frac{\omega \pi_{t+1}(1+z) f_{t+1}^w}{R_{t+1}\Psi_{t+1}} + \mathrm{E}_t \left(\epsilon_{t+1}\right)^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{(1-\omega)\pi_{t+1}(1+z) f_{t+1}^r}{R_{t+1}\Psi_{t+1}}$

$a^r : a_{t+1}^r = \frac{R_t a_t^r}{(1+z)(1+n)\pi_t} + w_t \xi l_t^r + d_t^r - c_t^r$
$\quad + (1-\omega)\left[\frac{R_t a_t^w}{(1+z)(1+n)\pi_t} + w_t l_t^w + d_t^w - c_t^w\right]$

$d^w : d_t^w = \frac{a_t^w}{a_t^w + a_t^r} d_t$

$d^r : d_t^r = \frac{a_t^r}{a_t^w + a_t^r} d_t$

$y : y_t = \left[\exp\left(z_t\right)\left(l_t^w + \xi l_t^r\right)\right]^{1-\alpha} k_t^\alpha$

$a^w : a_{t+1}^r + a_{t+1}^w = Q_t k_{t+1}$

$w : w_t = (1-\alpha)\varphi_t \frac{y_t}{(l_t^w + \xi l_t^r)}$

$i : y_t = c_t^r + c_t^w + i_t$

$R : R_{t+1} = \vartheta R_t + (1-\vartheta)\left(R_{SS}\right) + \eta_1\left(\pi_t - 1\right) + \eta_2\left(\frac{y_t}{y_{t-1}} - 1\right) + \tau_t$

$u : u_t = \rho_u u_{t-1} + \varepsilon_{u,t}$

$z : z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$

$\tau : \tau_t = \varepsilon_{\tau,t}$

Definition 1 *(**Competitive Equilibrium**) A competitive equilibrium is a sequence of endogenous predetermined variables $\{a^r,\ a_t^w,\ k,\ i,\ R\}$ and a sequence of endogenous variables $\{r^K,\ Q,\ \varphi,\ \pi,\ d,\ l^r,\ \epsilon,\ l^w,\ c^w,\ \Psi,\ \theta,\ c^r,\ f^r,\ h^r,\ h^w,\ f^w,\ d^w,\ d^r,\ y,\ w\}$ given the sequence of exogenous predetermined variables $\{u,\ z,\ \tau\}$*

Values in equations (11) and (12) are also expressed as de-trended variables:

$v^r : v_t^r = \left(\epsilon_t \kappa_t\right)^{-\frac{1}{\rho}} c_t^r \left(\frac{1-v}{v}\frac{1}{\xi w_t}\right)^{1-v}$

$v^w : v_t^w = \left(\kappa_t\right)^{-\frac{1}{\rho}} c_t^w \left(\frac{1-v}{v}\frac{1}{w_t}\right)^{1-v}$

3. Model Properties

In this section, we inquire model properties. First, we show that monetary policy is useful as a stabilization tool in this lifecycle economy. Second, effects of societal aging on the steady states are discussed. Third, we theoretically show the possibility that the central bank faces policy trade-off between maximizing worker's and retiree's welfare due to heterogeneity in agents. Finally, we compare impulse responses against technology and cost push shock in different lifecycle economies.

3.1. Necessity for Stabilization Policy

Since nominal rigidity is embedded in this model, monetary policy is indispensable so as to avoid explosive solutions in the model. Something similar to the Taylor principle must be satisfied so as to obtain unique rational expectation equilibrium.

The tricky question is, however, whether monetary policy is useful as a stabilization tool in this economy. Since the periodic utility is a pure Cobb-Douglas function, indirect utility is linear in expenditure. Risk neutrality seems to be assumed in agents' preferences. Here, we show that monetary policy does indeed reduce the loss stemming from agents' risk aversion from the standpoints of pure Cobb-Douglas function and non expected utility function employed in this paper.

3.1.1. Cobb-Douglas Utility

Here, we first show that agents become risk averse against technology shock even with pure Cobb-Douglas utility. Assume that agent maximizes Cobb-Douglas utility

$$\bar{U}_t = \bar{C}_t^{\bar{\alpha}} \left(1 - \bar{L}_t\right)^{1-\bar{\alpha}},$$

subject to resource constraint with production technology:

$$\bar{C}_t = \bar{A}_t \bar{L}_t.$$

First order necessary conditions are

$$\bar{\alpha} \bar{A}_t \left(\bar{A}_t \bar{L}_t\right)^{\bar{\alpha}-1} \left(1 - \bar{L}_t\right)^{1-\alpha} - (1-\alpha) \left(\bar{A}_t \bar{L}_t\right)^{\alpha} \left(1 - \bar{L}_t\right)^{-\alpha} = 0,$$

$$\bar{L}_t = \bar{\alpha}.$$

Therefore

$$\bar{C}_t = \bar{\alpha} \bar{A}_t.$$

Hence, utility becomes:

$$\begin{aligned} \bar{U}_t &= \left(\bar{\alpha} \bar{A}_t\right)^{\bar{\alpha}} (1-\bar{\alpha})^{1-\bar{\alpha}} \\ &= \bar{A}_t^{\bar{\alpha}} (\bar{\alpha})^{\bar{\alpha}} (1-\bar{\alpha})^{1-\bar{\alpha}} \\ &= \bar{A}_t^{\bar{\alpha}} * CONSTANT \end{aligned}$$

Since

$$0 \leq \bar{\alpha} \leq 1,$$

this agent is risk averse in aggregate (technology) shock and dislikes volatile consumption due to technology shock. Therefore, monetary policy can be used to facilitate motivation for consumption smoothing by agents with pure Cobb-Douglas utility.

3.1.2. Non Expected Utility

Retirees Recursive utility of retirees can be expressed as a function of consumption and marginal propensity to consume out of wealth. By combining equation

(11) and the relationship between consumption and leisure, we can express recursive utility with periodic utility defined above:

$$\begin{aligned} V_t^{rjk} &= (\epsilon_t \theta_t)^{-\frac{1}{\rho}} \left(C_t^{rjk}\right)^v \left(1 - L_t^{rjk}\right)^{1-v} \\ &= (\epsilon_t \theta_t)^{-\frac{1}{\rho}} \bar{U}_t^r. \end{aligned}$$

Since we already know that the periodic utility is a concave function against technology shock, this retirees' recursive utility is also a concave function against aggregate shocks. Furthermore, as ρ is set to be negative and larger than unity, retirees dislike volatile marginal propensity to consume out of wealth, namely uncertainty.[16]

Workers Similar to the case of retirees above, recursive utility can be expressed with periodic utility:

$$\begin{aligned} V_t^{wj} &= (\theta_t)^{-\frac{1}{\rho}} \left(C_t^{wj}\right)^v \left(1 - L_t^{wj}\right)^{1-v} \\ &= (\theta_t)^{-\frac{1}{\rho}} \bar{U}_t^w. \end{aligned}$$

Again, we can see that recursive utility is a concave function against aggregate uncertainty and marginal propensity to consume.

3.2. Heterogeneity in Agents

In this subsection, we inquire into how agents' heterogeneity assumed in this model may request different optimal operational rules for each agent.

3.2.1. Consumption Euler Equations

First, we inquire into the worker's and retiree's Euler equations. Differences in Euler equations can result in a different optimal operational rule for workers from

[16]By substituting equation (11) into recursive utility, we can find that recursive utility is a concave function with respect to periodic utility as of now and the next period.

that for retirees. Equation (8) shows retiree's consumption Euler equation:

$$\mathrm{E}_t C_{t+1}^r = \left[\beta \mathrm{E}_t R_{t+1} \left(\frac{P_t}{P_{t+1}}\right)^{1-\rho+v\rho} \left(\frac{W_t}{W_{t+1}}\right)^{(1-v)\rho}\right]^{\frac{1}{1-\rho}} C_t^r,$$

while worker's consumption Euler equation is expressed in equation (13):

$$\begin{aligned} & \omega \mathrm{E}_t C_{t+1}^w + (1-\omega)\left(\epsilon_{t+1}\right)^{-\frac{1}{\rho}} \mathrm{E}_t C_{t+1}^r \left(\frac{1}{\xi}\right)^{1-v} \\ = & \left[\beta \mathrm{E}_t \frac{P_t R_{t+1}}{P_{t+1}} \left(\omega + (1-\omega)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}\right) \left(\frac{P_t}{P_{t+1}} \frac{W_{t+1}}{W_t}\right)^{(v-1)\rho}\right]^{\frac{1}{1-\rho}} C_t^w. \end{aligned}$$

The latter can be modified by denoting the consumption of agents who are workers at t by C^W:

$$\mathrm{E}_t C_{t+1}^W = \left[\beta \mathrm{E}_t R_{t+1} \left(\frac{P_t}{P_{t+1}}\right)^{1-\rho+v\rho} \left(\frac{W_t}{W_{t+1}}\right)^{(1-v)\rho}\right]^{\frac{1}{1-\rho}} \mathrm{E}_t \Upsilon_{t+1} C_t^W,$$

where

$$\begin{aligned} \mathrm{E}_t \Upsilon_{t+1} &= \mathrm{E}_t \frac{\left(\omega + (1-\omega)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}\right)^{\frac{1}{1-\rho}}}{\omega + (1-\omega)\left(\epsilon_{t+1}\right)^{-\frac{1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}} \\ &= \mathrm{E}_t \frac{\Psi_{t+1}^{\frac{1}{1-\rho}}}{\omega + (1-\omega)\frac{\partial V_{t+1}^t / \partial C_{t+1}^t}{\partial V_{t+1}^w / \partial C_{t+1}^w}}. \end{aligned}$$

Clearly, the difference between worker's Euler equation and retiree's Euler equation is the existence of Υ, whose denominator is utility weight of consumption. However, as long as Υ does not fluctuate significantly different from unity even though shocks hit the economy, the central bank does not have to really care about difference between workers and retirees. Since the steady state level of ϵ is close to unity, the welfare-based optimal rule obtained here might maximize the welfare of both workers and retirees.

3.2.2. Leisure-Consumption Relation

After de-trending deterministic technology and population growth, the labor supply of workers and retirees is expressed as follows:

$$l_t^w = 1 - \frac{1-v}{v}\frac{1}{w_t}c_t^w,$$

$$l_t^r = \Gamma - \frac{1-v}{v}\frac{1}{\xi w_t}c_t^r.$$

Differences between them are consumption and ξ. Therefore, there may be minuscule difference in leisure-consumption relation between workers and retirees.

3.2.3. Values

Difference can be found in the marginal propensity to consume included in recursive welfare between in worker's welfare in equation (11) and that of retirees in equation (12) as below:

$$\begin{aligned} V_t^r &= (\epsilon_t\theta_t)^{-\frac{1}{\rho}} C_t^r \left(\frac{1-v}{v}\frac{P_t}{\xi W_t}\right)^{1-v} \\ &= (\epsilon_t\theta_t)^{-\frac{1}{\rho}} (C_t^r)^v (1-L_t^r)^{1-v}, \end{aligned} \tag{33}$$

$$\begin{aligned} V_t^w &= (\theta_t)^{-\frac{1}{\rho}} C_t^w \left(\frac{1-v}{v}\frac{P_t}{W_t}\right)^{1-v} \\ &= (\theta_t)^{-\frac{1}{\rho}} (C_t^w)^v (1-L_t^w)^{1-v}. \end{aligned} \tag{34}$$

Therefore, we need to inquire into marginal propensity to consume out of wealth in equations (10) and (16):

$$\epsilon_t\theta_t = 1 - \beta^{\frac{1}{1-\rho}} \mathrm{E}_t \frac{\epsilon_t\theta_t}{\epsilon_{t+1}\theta_{t+1}} R_{t+1}^{\frac{\rho}{1-\rho}} \left(\frac{P_t}{P_{t+1}}\right)^{\frac{v\rho}{1-\rho}} \left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}} \gamma,$$

$$\theta_t = 1 - \beta^{\frac{1}{1-\rho}} \mathrm{E}_t \frac{\theta_t}{\theta_{t+1}} \left(R_{t+1}\right)^{\frac{\rho}{1-\rho}} \left(\frac{P_t}{P_{t+1}}\right)^{\frac{\rho v}{1-\rho}} \left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}} \Psi_{t+1}^{\frac{\rho}{1-\rho}},$$

where

$$\mathrm{E}_t \Psi_{t+1} = \omega + (1-\omega)\, \mathrm{E}_t \left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}.$$

As is obvious from the above two equations, dynamics in γ and $\Psi_{t+1}^{\frac{\rho}{1-\rho}}$ must be different qualitatively. At a minimum, movements in worker's marginal propensity to consume is more complicated than that of retirees. Furthermore, the central bank can affect these movements through interest rate changes.

3.3. Long-Run Effects of Societal Aging

We study steady state effects of societal aging. Three steady states are computed: (1) baseline lifecycle economy, (2) aged lifecycle economy, and (3) no lifecycle economy.[17] First, we show parameter values of (1) baseline lifecycle economy calibrated using previous studies. The model is solved with quarterly frequency. Therefore, parameters are on quarterly bases.

[17] See appendix.

Table 1

Parameter values

Parameter	Value	Description and Definitions
θ	5	$\theta/(\theta-1)$ is markup
σ	.25	Intertemporal elasticity of substitution
ϕ	100	Price adjustment cost
β	$1.04^{-.25}$	Subjective discount factor
ω	$1.023^{-.25}$	Probability of remaining as worker
α	0.667	Labor share
δ	0.015	Capital depreciation rate
ρ	-3	$(\sigma-1)/\sigma$
ν	.4	Utility weight on consumption
ξ	.6	Labor productivity of retirees
γ	$1.1^{-.25}$	Probability of remaining as retirees
S''	2.48	Second derivative of adjustment cost
Z	$1.01^{0.25}-1$	Technology growth rate
n	$1.01^{0.25}-1$	Population growth rate
ϑ	$0.0 \sim 0.9$	Policy parameter 1
η_1	$0.1 \sim 4.0$	Policy parameter 2
η_2	$0.1 \sim 4.0$	Policy parameter 3
Γ	0.214	Steady state population ratio of retirees over workers
ρ_u	.4	Cost push shock persistence
ρ_z	.9	Technology shock persistence
σ_u	.01	Standard deviation of cost push shock
σ_z	.01	Standard deviation of technology shock

In (1) baseline lifecycle economy, in yearly bases, such a lifecycle economy is assumed that workers are from age 21 to 65, which is defined by ω, and retirees are from age 66 to 75, defined by γ, based on Auerbach and Kotlikoff (1987). Other parameters which defines lifecycle behavior are mostly based on Gertler (1999) and size of capital adjustment cost is taken from Christiano Eichenbaum and Evans (2002). Sizes of two shocks are calibrated so that simulated second moments of the model are similar to those of actual data.[18]

The steady state of this economy[19] is as follows.

Table 2

Steady state of (1) baseline lifecycle economy

	c/y	k/y	R	a^r/k	Ψ	l^w	l^r	$\varepsilon\theta$	θ
Value	.78	11.09	.0090	.16	1.012	.34	.02	.03	.02

Individuals' life expectancy becomes longer in (2) aged lifecycle economy.[20] In this economy, individuals retire at the age of 65 but are supposed to live until 85. The parameter that determines the probability of remaining as retirees, γ, is altered from $1.1^{-.25}$ to $1.05^{-.25}$. By this alteration, the percentage of retiree to worker population, namely Γ, becomes from 21% to 39%. The steady state of this economy is expressed in Table 3.

Table 3

Steady state of (2) aged lifecycle economy

	c/y	k/y	R	a^r/k	Ψ	l^w	l^r	$\varepsilon\theta$	θ
Value	.78	11.17	.0088	.22	1.008	.34	.08	.02	.01

[18] Simulation using a model with more empirical plausibility via Bayesian estimaion initiated by Schorfeide (2000) is our future challenge.

[19] Unlike such researches as Christiano, Eichenbaum and Evans (2005), there exists positive profit from monopolistic competition even in the steady state. We follow Juillard, Karam, Laxton and Pesenti (2004).

[20] We will later show the case where the number of retirees becomes larger but there is no change in life expectancy.

Since workers need to prepare for a longer retirement period, their incentive for saving becomes more significant in this economy, which is reflected in decrease in marginal propensity to consume. Therefore, interest rates as well as worker's discount factor become lower.[21] Hence, inefficiency of too much saving deepens. These movements result in higher capital output ratio. On the labor supply side, although workers save more to prepare for longer retirement, retirees need to work more in this economy so that it can maintain an optimized level of consumption.

Table 4 shows the steady state of no lifecycle economy. No lifecycle economy is simulated so as to provide a benchmark for an economy with homogenous agents.

Table 4

Steady state of (3) no lifecycle economy

	c/y	k/y	R	l
Value	.83	8.35	.017	.30

As indeed mentioned above, since households do not have to prepare for retiring, they save less. Therefore, nominal interest rates are highest in this economy. An interesting question is indeed how monetary policy should be conducted in these different settings of lifecycle economy.

3.4. Impulse Responses[22]

We show impulse responses of major variables against technology shock, cost push shock and monetary policy shock under (1) baseline lifecycle economy, (2) aged lifecycle economy and (3) no lifecycle economy.[23] Since this model is very

[21] However, we think that this does not hold generally. Under some parameter set, it is possible that interest rate may be higher due to more labor supply from retirees in greyer society.

[22] In all simulations examined in this paper, we ignore the existence of a zero lower bound of nominal interest rate.

[23] Only impulse responses for nominal interest rates, output, inflation, and real markup are shown for (3) no lifecycle economy.

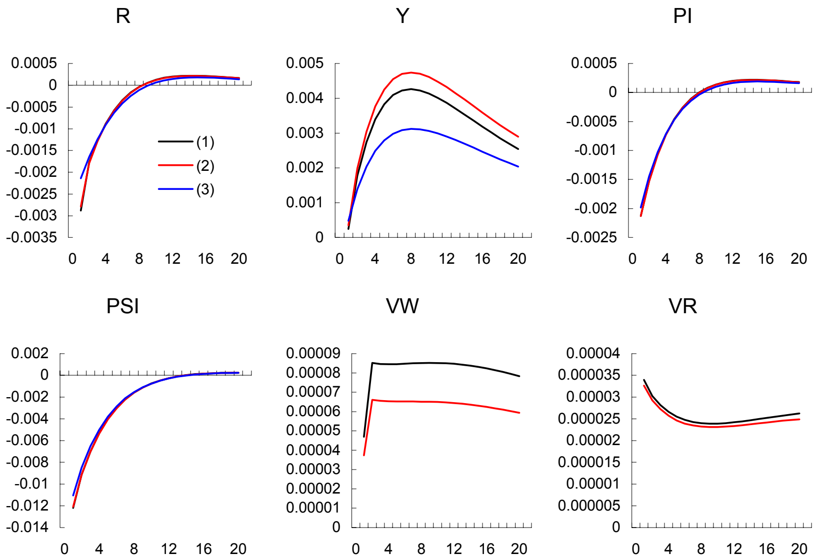

Figure 1: Impulse responses against technology shock

theoretical and not calibrated so as to mimic the actual tendencies found in the data, the focus here is on how worker's choice variables differ from those of retirees.

As a baseline operational monetary policy rule, we employ an instrument rules as below:

$$R_t = 0.5R_t + 0.5R_{SS} + 1.5\pi_t + 1.5\left(\frac{y_t}{y_{t-1}} - 1\right).$$

3.4.1. Technology Shock

Figure 1 shows the impulse responses against level technology shock. The black lines show responses under (1) baseline lifecycle economy while the red line is for (2) aged lifecycle economy and the blue line is for (3) no lifecycle economy.

A shock to the level of technology naturally increases the level of output. However, deflation occurs, and therefore nominal interest rate is lowered. Unlike the technology shock usually employed in the standard dynamic new Keynesian model as well-explained in Walsh (2003) and Woodford (2003), the shock assumed in this model is a level shock. Since level shock here considered decays gradually, it is recognized as a negative growth shock in the Euler equations of a log linearized system, where the percentage deviation of current consumption from its steady state value receives positive effects from growth shock of technology.[24] Therefore, even though GDP increases, the inflation rate and nominal interest rates are decreased.

Intriguing responses are found in those of value of workers and retirees. Their movements are different. Furthermore, the degree of changes in responses when the lifecycle assumption is altered also differs for workers and for retirees. This may hint that an operational rule that is optimal for workers may be different from what is optimal for retirees.

Moreover, while responses of inflation and real interest rate do not significantly differ between different lifecycle economies, that of output is quite different. This suggests that intertemporal elasticity of substitution has changed in aggregate. Hence, the form of optimal operational monetary policy can be different between in (1) baseline lifecycle economy and (2) aged lifecycle economy.

3.4.2. Cost Push Shock

Figure 2 shows responses against a cost push shock. After a shock raises the inflation rate temporarily, nominal interest rates are raised by the central bank via policy reaction function. Reflecting this monetary contraction, output is reduced. Again, the shape of impulse responses of values is quite different between different lifecycle economies as well as between workers and retirees, although the directions of responses are the same.

[24] Although, to be exact, impulse responses are computed via second order approximation, such responses are almost the same as ones obtained from a log linear approximation.

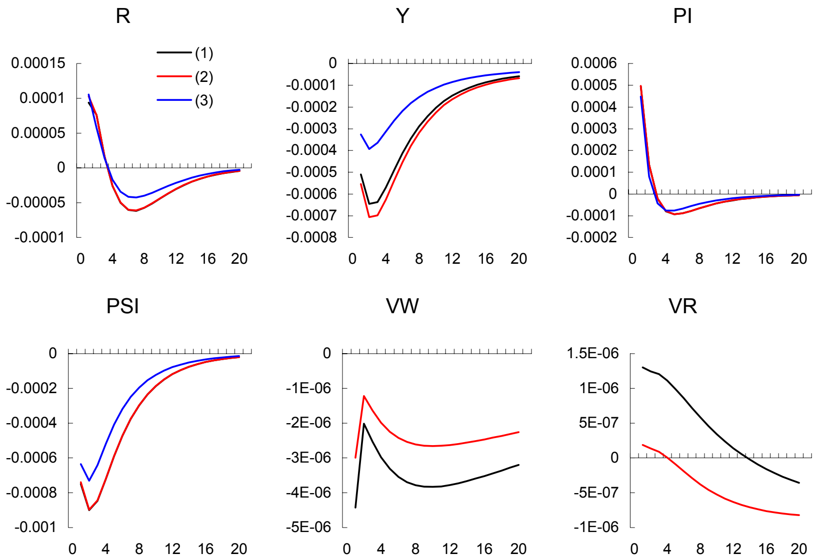

Figure 2: Impulse responses against cost push shock

3.4.3. Monetary Policy Shock

Impulse responses against monetary policy shock are shown in Figure 3. Here, we assume that monetary policy shocks are not serially correlated. Without capital adjustment cost, even though a positive interest rate shock is added to the model, nominal interest rates are immediately decreased. Unlike the model without capital under usual setting,[25] according to the first order condition which relates marginal productivity of capital to cost of capital, a positive monetary policy shock decreases output and therefore decreases marginal productivity of capital and real interest rate eventually. These result in immediate decrease in nominal interest rate after a positive policy shock. With capital adjustment cost as examined here, however, a jump in the theoretical stock price can bring increase in nominal interest rate.

Again, the most intriguing point is that responses of values of workers are quite

[25] Even a dynamic new Keynesian model without capital can result in decrease in nominal interest rates after positive monetary shock under some setting.

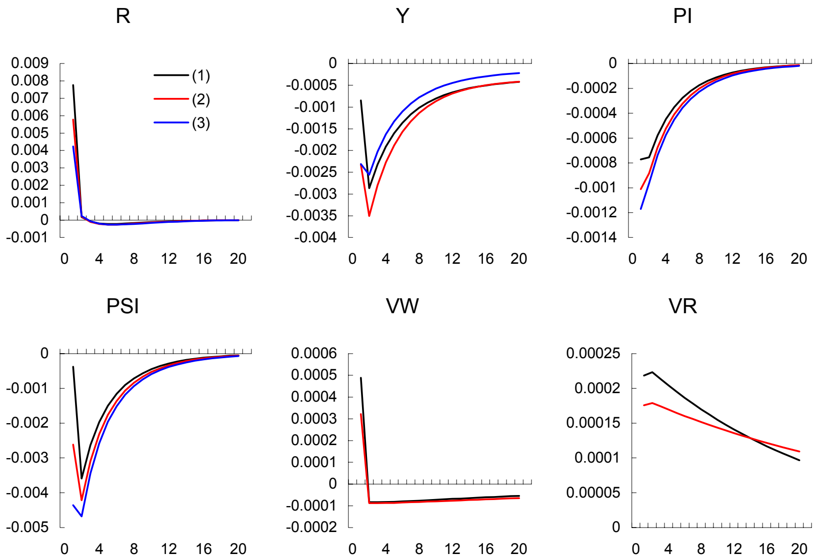

Figure 3: Impulse responses against monetary policy shock

different from those of retiree, which suggests that optimal operational rules takes different forms between for workers and retirees. After a positive monetary tightening shock, welfare of workers becomes soon worse while that of retirees is improved. Since retirees rely more on interest rate income which they save as workers, income effects from increase in nominal interest rates dominates substitution effects for retirees. This implies severe policy trade-off. If the central bank cares more about retirees, or if monetary policy is determined mainly through opinions of older people because of their bargaining power in politics over younger people, there may be a bias towards higher nominal interest rates albeit temporarily.

4. Results

We first seek an optimal instrument rule in equation (32) that maximizes the weighted average of the recursive utility (value) of two agents in equation (31). Hence, we solve the model with the second order approximation method embedded in Dynare.

4.1. Welfare-Based Optimal Operational Rule

The operational rule examined in this paper is already shown in equation (32). We alter ϑ from 0 to .9 by .1, η_1 and η_2 from .1 to 4.0 by .1.[26] Therefore, in each case, we have 16000 (40*40*10) simulation results.[27]

Figure 4 demonstrates a standard policy frontier which depicts the relationship between inflation variability and output variability[28] for both (1) baseline lifecycle economy (black plot), (2) aged lifecycle economy (red plot) and (3) no lifecycle economy (blue plot). We can monitor the standard trade-off between inflation and output stabilization in this model.

Then, what are the optimal welfare based operational rules for each economy? Table 5 shows optimal operational rules for aggregate, workers and retirees, in (1) baseline lifecycle economy, (2) aged lifecycle economy, and (3) no lifecycle economy.[29]

[26] As can be seen from results below, optimal operational rules are on the boundary of η_1. Since the form of operational rule should not be the Ramsey optimal policy rule as examined in Levin and Lopez-Salido (2004) and Levin, Onatski, Williams and Williams (2005), η_1 in the optimal operational rule can be very large. We indeed have found that there is the upper bound of η_1, above which welfare tends to decrease, but it is extremely large. Therefore, we concentrate on the range from 0.1 to 4.0, within which parameters of major previous researches on estimated Taylor rule, as summarized in Woodford (2003), are included well adequately. We believe that coefficients of a monetary policy rule larger than this range is very unrealistic.

Yet, even if we extend the parameter range from 0.1 to 10.0 with less fine grids, qualitative results below do not change at all.

[27] It is true that we do not have to obtain the optimal operational rule from stochastic simulation if we can compute the Ramsey optimal rule derived from first order conditions. Then, all we need to do is to inquire into the nature of the optimal Ramsey policy rule. It is, however, not very trivial to analytically derive the optimal Ramsey policy rule under exotic preferences as employed in this paper.

[28] We use normalized standard deviation, which is standard deviation / mean.

[29] We derive optimal operational rules around distorted steady state, which is analyzed in Benigno and Woodford (2005). There is, however, no significant differences in results from analyses around both distorted and non-distored steady states, as far as we compute optimal operational rules with less fine grids around non-distorted steady state.

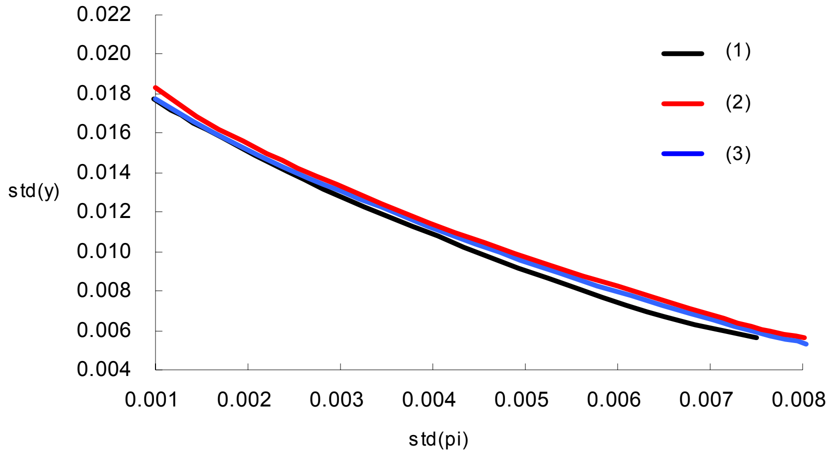

Figure 4: Policy frontiers

Table 5

Optimal operational rules

Economy	ϑ	η_1	η_2	V[30]	v^w	v^r
(1)	0.6	4.0	0.1	1.214 (1)	0.1819 (1)	0.05267 (13362)
	0.6	4.0	0.1	1.214 (1)	0.1819 (1)	0.05267 (13362)
	0.9	0.2	0.3	1.213 (11098)	0.1817 (11520)	0.05273 (1)
(2)	0.5	4.0	0.1	1.214 (1)	0.1686 (2)	0.07286 (11362)
	0.9	4.0	0.1	1.214 (2)	0.1686 (1)	0.07286 (11405)
	0.9	0.2	0.1	1.213 (8673)	0.1683 (9160)	0.07292 (1)
(3)	0.6	4.0	0.1	0.17	-	-

The number in parentheses shows the ranking in each category. Welfare-based optimal operational rules for workers and that in (3) no lifecycle economy are almost

[30] For (3) no lifecycle economy, unconditional mean of v is shown.

the same and have a smaller coefficient on output growth and large one for inflation rate.[31]

There are several intriguing findings in Table 5. First, the optimal operational rules for aggregate are almost the same in both (1) baseline lifecycle economy and (2) aged lifecycle economy. Furthermore, they are also almost the same as the optimal operational rules for workers and in (3) no lifecycle economy. These suggest that under the heterogeneity of agents in this model setting, the monetary policy scheme does not have to be altered as long as societal aging deepens in a reasonable manner.

Second, and most importantly, the optimal operational rule for workers welfare is quite different from that for retirees in lifecycle economy as indeed expected in the previous section. Therefore, the central bank faces a trade-off between maximizing workers' and retirees' welfare. For example, the optimal operational rule is also the best rule for workers in (1) baseline lifecycle economy, but it is just the 11362nd rule out of 13798 rules[32] for retirees, not an optimal rule at all for retirees!

4.2. Welfare Trade-Off

Can we find a clear trade-off between the worker-retiree welfare maximization problem? Figure 5 shows the south-west frontier of scattered plots for ranking in the welfare of workers and retirees in (1) baseline lifecycle economy and Figure 6 shows that in (2) aged lifecycle economy.[33] If an operational rule can increase the welfare of workers and retirees in equal proportions, there should not be any frontier but just scattered plots must look like a 45-degree line. Yet, both Figures 5 and 6 demonstrate that increasing worker welfare through monetary policy does not

[31] There exists an upper bound for coefficient on inflation rate, after which utility becomes smaller.

[32] Although 16000 simulations are conducted, some cannot satisfy Blanchard and Kahn (1980) condition.

[33] In the conference version paper, figures 5 to 12 are actual scattered plots. Since we conduct numerous number of simulations, they are very large in byte. Hence, we only show frontiers in this paper.

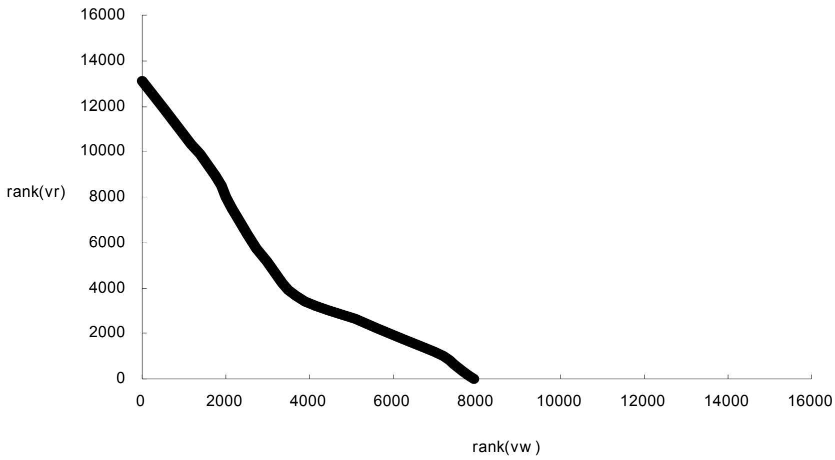

Figure 5: Frontier of ranking plots in baseline lifecycle economy

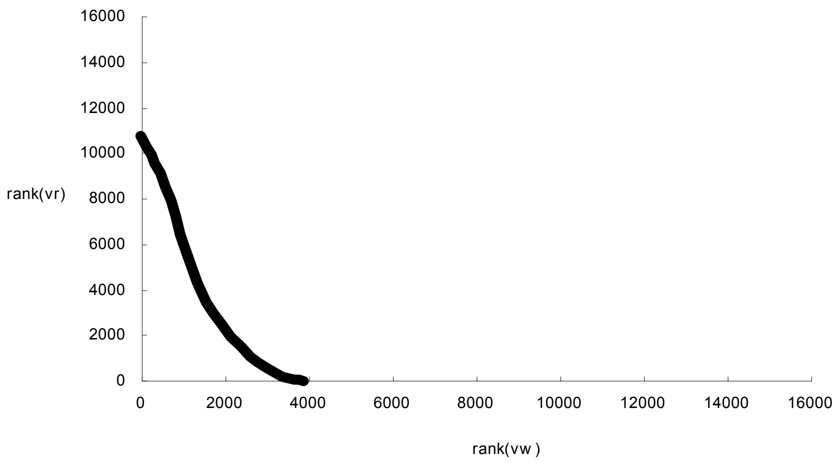

Figure 6: Frontier of ranking plots in aged lifecycle economy

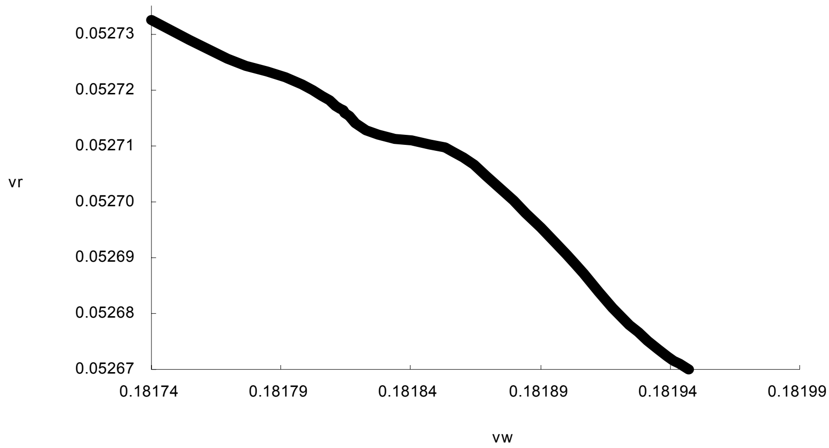

Figure 7: Worker's and retiree's welfare in baseline lifecycle economy

necessarily result in the higher welfare of retirees. Both charts show unambiguous policy trade-off between workers' and retirees' welfare. The frontiers on the scattered plots demonstrate the region where the central bank should choose the Pareto optimal operational rules given monopolistic competition.[34] Another intriguing point is that such a trade-off becomes less severe in an aged lifecycle economy. This seems somehow counterintuitive, but the reason becomes very clear if we compare Tables 2 and 3 in subsection 3.3 again. In an aged lifecycle economy, retirees need to work more to have an optimized level of consumption for their longer retirement period. Due to this increased labor supply by retirees, they becomes more similar to workers. We can find a similar trade-off in north-east frontiers of v^w-v^r plots as shown in Figures 7 and 8. We assume that aggregate welfare can be summarized by the weighted average of worker's and retiree's welfare, namely one for one voting by population. Moreover, the population of workers is much larger than that

[34] Here, we consider the Pareto optimality only between workers and retirees. Therefore, contrary to the standard analysis using conventional overlapping generations model as in McCandless and Wallace (1991), welfare differences among heterogenous agents with different age are not considered in this paper.

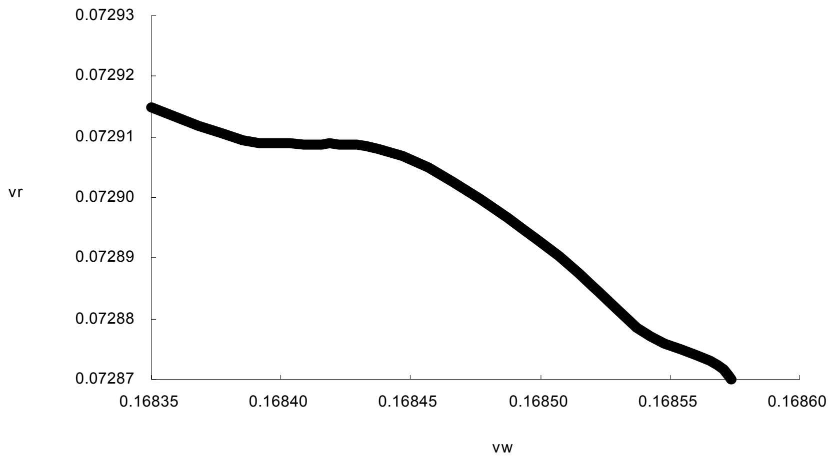

Figure 8: Worker's and retiree's welfare in aged lifecycle economy

of retirees Therefore, right-end plots demonstrate the optimal operational rules in both (1) baseline lifecycle economy and (2) aged lifecycle economy. Figures from 5 through 9 demonstrate that the degree of convexity in ranking plots, or concavity in welfare plots, is more significant in (1) baseline economy than (2) aged lifecycle economy. This is again due to the fact that longer life expectancy requests more work from retirees as well as more saving from workers. While the population ratio merely determines the marginal rate of transformation between the welfare of workers and retirees, steady state level of labor supply, especially by retirees, defines the curvature in the frontiers in Figures 5 to 10.

How does the simple rule as in equation (32) affect the welfare of both workers and retirees? We inquire into this point by looking at north-west frontiers of v^w-v^r plots by different combinations of policy parameters. Figures 9 and 10 below show those of v^w-v^r plots in Figures 7 and 8 for $0 \leq \vartheta \leq 0.1$, $0.2 \leq \vartheta \leq 0.3$, $0.4 \leq \vartheta \leq 0.5$, $0.6 \leq \vartheta \leq 0.7$ and $0.8 \leq \vartheta \leq 0.9$. We have found that the frontier of plots with $0.8 \leq \vartheta \leq 0.9$ are almost always on the most north-east. This

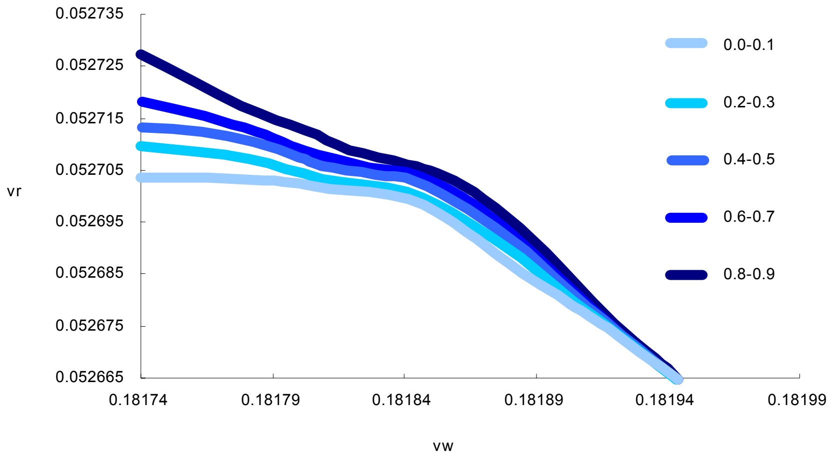

Figure 9: Worker's and retiree's welfare by history dependency in baseline lifecycle economy

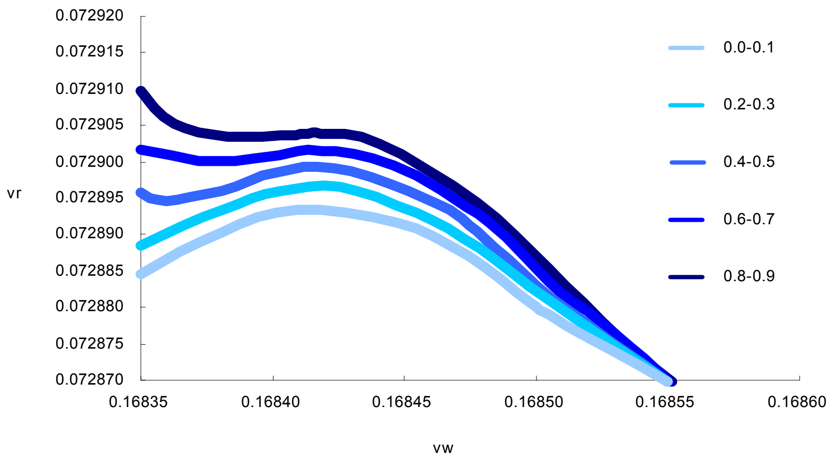

Figure 10: Worker's and retiree's welfare by history dependency in aged lifecycle economy

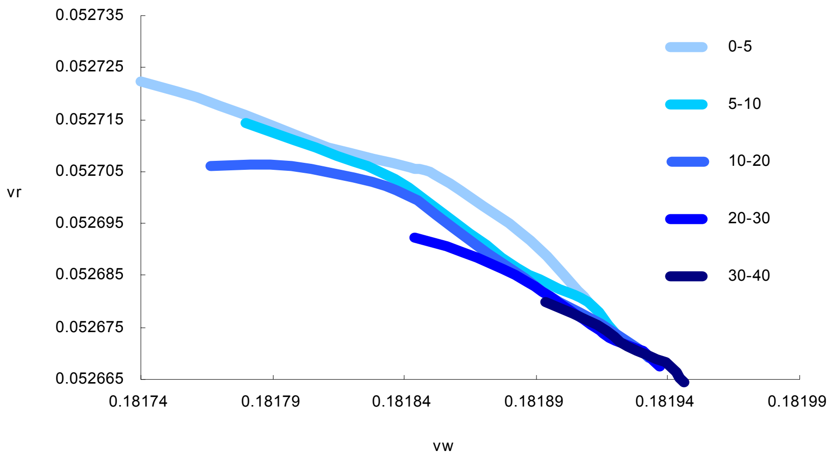

Figure 11: Worker's and retiree's welfare by inflation-fighting in baseline lifecycle economy

result imply the effectiveness of history dependent monetary policy in a forward-looking economy as models simulated in this paper. Then, we inquire into the role of η_1 (policy parameter on inflation) and η_2 (policy parameter on output growth) on the effectiveness of monetary policy. Parameter sets are divided by relative importance of inflation over output growth in monetary policy decision, which is defined by η_1/η_2. In Figures 11 and 12, we again show north-east frontiers of v^w-v^r plots for $0 < \eta_1/\eta_2 \leq 5$, $5 < \eta_1/\eta_2 \leq 10$, $10 < \eta_1/\eta_2 \leq 20$, $20 < \eta_1/\eta_2 \leq 30$, and $30 < \eta_1/\eta_2 \leq 40$. We can clearly see that more weight on inflation rate is desirable for workers, while the opposite is true for retirees under highly history dependent monetary policy.[35] This is somewhat counterintuitive, and contrary to the conclusion obtained in Doepke and Schneider (2005), which shows that the younger generation benefits more from inflation than does the older generation.

[35] At the Central Bank Workshop on Macroeconomic Modelling at the Reserve Bank of South Africa, our discussant, Laura Pitschelli, showed that optimal operational rule becomes less inflation-fighting as societal aging with the Bank of England's large-scale dynamic general equilibrium model, BEQM. This is consistent with our findings.

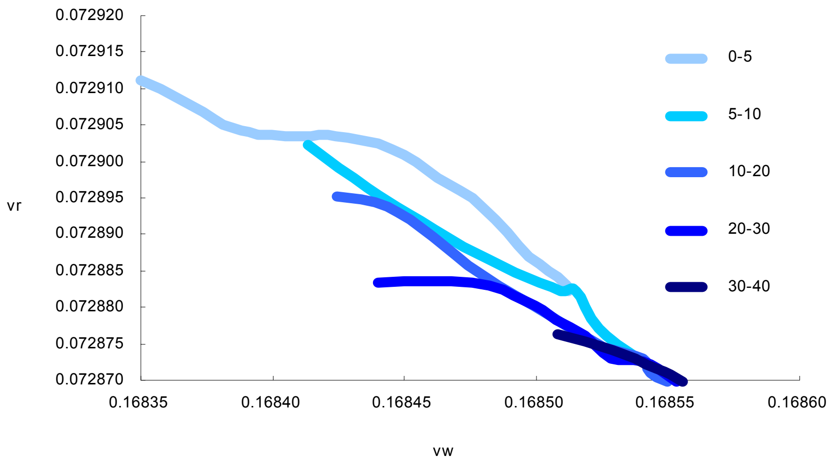

Figure 12: Worker's and retiree's welfare by inflation-fighting in aged lifecycle economy

The main reason for this difference is that younger people borrow from older people in Doepke and Schneider (2005) and other conventional overlapping generations models.[36] On the other hand, as is consistent with Japan's current situation, both workers and retirees are net shareholders of firms and not borrowing. To understand why workers prefer inflation-fighting monetary policy in a non-borrowing economy in detail, arguments in section 3.2.3 are useful. There, we have found that the largest difference between worker's and retiree's welfare can be found in the marginal propensity to consume. Retiree's marginal propensity to consume is as in equation (16):

$$\epsilon_t\theta_t = 1 - \beta^{\frac{1}{1-\rho}} \mathrm{E}_t \frac{\epsilon_t\theta_t}{\epsilon_{t+1}\theta_{t+1}} R_{t+1}^{\frac{\rho}{1-\rho}} \left(\frac{1}{1+\pi_{t+1}}\right)^{\frac{v\rho}{1-\rho}} \left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}} \gamma,$$

[36] Moreover, Doepke and Shneider analyze the case of unexpected inflation. We rather inquire into the systematic response of monetary policy against inflation.

while worker's marginal propensity to consume is as in equation (10):

$$\theta_t = 1 - \beta^{\frac{1}{1-\rho}} \mathrm{E}_t \frac{\theta_t}{\theta_{t+1}} \left(R_{t+1}\right)^{\frac{\rho}{1-\rho}} \left(\frac{1}{1+\pi_{t+1}}\right)^{\frac{\rho v}{1-\rho}} \left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}} \Psi_{t+1}^{\frac{\rho}{1-\rho}},$$

where

$$\mathrm{E}_t \Psi_{t+1} = \omega + (1-\omega) \, \mathrm{E}_t \left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} \left(\frac{1}{\xi}\right)^{1-v}.$$

Both include terms concerning inflation rate as $\left(\frac{1}{1+\pi_{t+1}}\right)^{\frac{v\rho}{1-\rho}}$. The only difference between above two equations is γ and $\Psi_{t+1}^{\frac{\rho}{1-\rho}}$ and such difference should represent different responsiveness of nominal asset held by workers and retirees respectively to inflation. While γ is naturally constant, Ψ_{t+1} is the discount factor for workers, which includes the possibility they become retirees in the future. Therefore, the worker's discount factor incorporates some component of retirees' marginal propensity to consume out of wealth, which partially depends on again inflation rate. Therefore, according to the fact that each agent is risk averse in marginal propensity to consume, workers dislike fluctuating inflation rates more than retirees do, and this results in higher welfare for workers with higher η_1/η_2.[37] This is because workers care not only how they dislike inflation but also how retirees, that they may become next period, do, under longer life expectancy. According to Gertler (1999), "Intuitively, everything else equal, the marginal utility gain from a unit of wealth for a retired person is less than for a worker, since the former consumes out of wealth at a faster pace than does the latter (i.e., consumption out of a dollar of wealth received early in life can be smoothed over more periods than consumption out of a dollar received (unexpectedly) late in life)." Hence, worker's consumption smoothing motivation is higher than that of retirees. This is an argument which

[37] We feel the necessity to inquire into the determination of Ψ to understand why workers dislike inflation more than retirees do.

can also be explained by two Euler equations (8) and (13). An intriguing conclusion from this result is that monetary policy should become less inflation-fighting as societal aging deepens. However, since this model does not produce relative price distortions as in cases with Calvo (1983) or Taylor (1979) type pricing, the results here do not necessarily mean that weight on the price dispersion becomes larger in the theoretically consistent loss function. They just simply mean that within the class of optimal rule examined here, it is beneficial for the central bank to have more weight on inflation for worker's welfare to suppress economic fluctuations around steady states.

The above are only a tentative interpretation of our results. It is possible that we obtain different results with different parameter sets. We need to compile results from various sensitivity analysis for the difference between the optimal operational rule for workers and that for retirees to know whether our results are robust or not. So far, albeit with less fine grids, we compute the optimal operational rules for $\sigma \in \{0.25, 0.5, 0.75\}$, $v \in \{0.2, 0.4, 0.6, 0.8\}$, and $\xi \in \{0.4, 0.6, 0.8\}$. The obtained optimal rules for workers always has very high inflation-fighting stance, namely η_1/η_2. Overall, as explained above, we believe that the point below is important to determine the optimal operational rule: how much each agent dislikes fluctuations in real assets from re-evaluation of nominal assets via volatile inflation, which eventually lead to more variation in consumption.

Another interesting point to inquire is how large the trade-off becomes in terms of consumption, or how much consumption each retiree may lose on average if such an optimal operational rule is employed by the central bank. Figure 13 shows the plots of worker's and retiree's welfare against ranking in (1) baseline lifecycle economy while those in (2) aged lifecycle economy are shown in Figure 14. If the rule is ranked in the bottom 2000 in ranking, welfare significantly worsens. As shown in Table 5, the optimal rule for aggregate and workers is the 13362nd rule for retirees in (1)

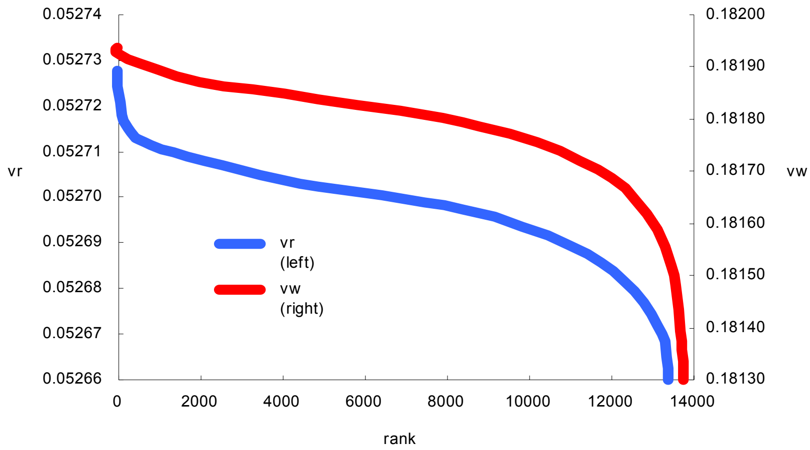

Figure 13: Ranking and welfare in baseline lifecycle economy

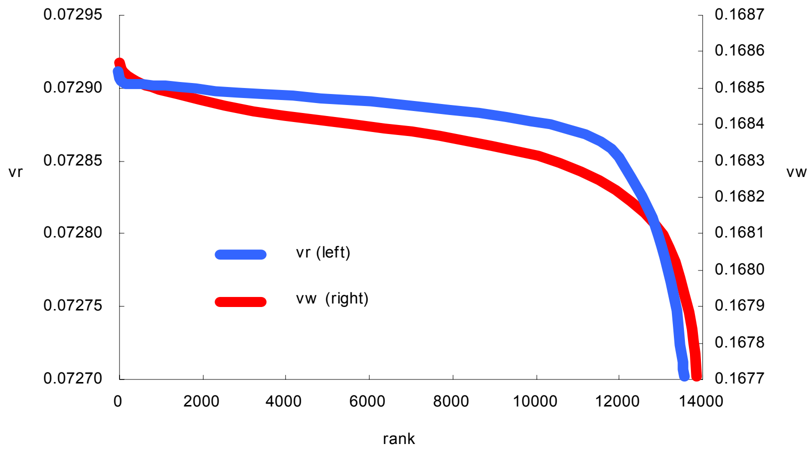

Figure 14: Ranking and welfare in aged lifecycle economy

baseline lifecycle economy. Therefore, retiree's welfare and, eventually, consumption level, may be considerably reduced. Table 6 shows the optimal operational rules for workers and retirees respectively in (1) baseline lifecycle economy and (2) aged lifecycle economy. Welfare loss from taking the other's optimal operational rules is also computed: $\log\left(v^w/v^w_{opt}\right)$ demonstrates the percentage welfare loss for workers when the optimal operational rule for retirees is employed while $\log\left(v^r/v^r_{opt}\right)$ shows that for retirees.

Table 6

Welfare Loss

Economy	ϑ	η_1	η_2	v^w	v^r	$\log\left(v^w/v^w_{opt}\right)$	$\log\left(v^r/v^r_{opt}\right)$
(1)	0.6	4.0	0.1	0.1819 (1)	0.05267 (13362)	0.00%	-0.12%
	0.9	0.2	0.3	0.1817 (11520)	0.05273 (1)	-0.13%	0.00%
(2)	0.9	4.0	0.1	0.1686 (1)	0.07286 (11405)	0.00%	-0.07%
	0.9	0.2	0.1	0.1683 (9160)	0.07292 (1)	-0.14%	0.00%

According to equations (33) and (34) with $v = 0.4$, one percent welfare loss implies $\left(1.01^{2.5} - 1\right) * 100\%$ permanent loss of consumption given that labor supply and marginal propensity to consume are unchanged. Therefore, for example, in (1) baseline lifecycle economy, if the central bank uses the optimal operational rule for workers as guidance for monetary policy, which is the optimal operational rule in aggregate, retirees need to endure 0.3% permanent loss of consumption. On the other hand, if the optimal operational rule for retirees is employed in this economy, workers will also lose 0.3% consumption permanently.

5. Conclusion

In this paper, we try to answer to practically important questions concerning monetary policy implementation: whether the monetary policy scheme needs to be changed as societal aging deepens; and how monetary policy affects heterogeneous agents, namely workers and retirees, unevenly. Using the dynamic stochastic general equilibrium model with nominal rigidity that incorporates lifecycle behavior, we have found several intriguing findings. First, as long as the main sources of asymmetry between workers and retirees are their longevity and marginal product of labor, and we compute aggregate welfare by assuming one-for-one voting on public policy making under the existence of only two political parties: worker's and retiree's party, monetary policy does not have to be altered significantly as societal aging deepens in a reasonable manner. This is because the degree of increase in population of retirees is not enough to alter the optimal instrument rule for aggregate population under one-for-one voting on public policy making. Therefore, a monetary policy scheme which aims at maximizing worker welfare is enough for optimizing aggregate welfare as well. Second, on the distributional aspects of monetary policy, however, we find that the optimal instrument rule for workers is quite different from the one for retirees, even with slight heterogeneity between two agents in our model. Therefore, the central bank faces policy trade-off between maximizing worker's and retiree's welfare due to heterogeneity in agents. For example, impulse responses show that after a positive monetary tightening shock, welfare of workers becomes soon worse while that of retirees is improved. Since retirees rely more on interest rate income which they save as workers, income effects from increase in nominal interest rates dominates substitution effects for retirees. Such a trade-off is validated by the stochastic simulation. Intriguing finding here is that unlike the recent growing literature on the optimality of the Friedman rule with heterogeneous agents, distributional effects of monetary policy can be derived even under the cash-less economy. Furthermore, by inquiring into how monetary policy instrument rules can affect the welfare of both workers and retirees unevely, we found that in an economy where even workers save for their retirement, as is the case in Japan, workers prefer more inflation-fighting monetary policy than retirees do. Since workers own more financial assets than retirees do, they suffer more against nominal asset revaluation caused by changes in the price level. Finally, as societal aging deepens, the difference between workers and retirees becomes smaller and, therefore, the policy trade-off which the central bank faces becomes less severe. Retirees need to work more as society becomes greyer so that they can maintain the optimized level of consumption in the model examined in this paper. Thus, the degree of heterogeneity between workers and retirees becomes lessened.

In an aging economy like Japan's, it is becoming more and more important to recognize agents' heterogeneity and their political power when conducting public policy, including monetary policy. We have seen that under a reasonable setting of societal aging, the central bank should aim at maximizing worker welfare according to one-for-one voting decision-making in public policy. This, however, in turn, implies that if retirees have more bargaining or political power than workers do, a risk exists that optimal policy for aggregate may not be chosen by the central bank. Our future research includes: (1) how we should design a social security system in order that monetary policy always has even effects on workers and retirees, where the central bank should not worry about agents' heterogeneity and is enough to care about aggregate welfare; (2) how the results in this paper can be altered when the model is extended to open economy setting, where the steady state real interest rate is set abroad and demographics would determine the net foreign asset position at the same time.

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Appendix

No Lifecycle Economy

No lifecycle economy comparable to models in this paper is derived by assuming $\omega = 1$. Then, households stay workers for infinite horizon. Since firms' behavior does not have to be altered with this modification, we show just households' choice problem briefly and new system of equations.

Household

Maximize:

$$V_t = \left\{ \left[(C_t)^v (1 - L_t)^{1-v} \right]^\rho + \beta \mathrm{E}_t (V_{t+1})^\rho \right\}^{\frac{1}{\rho}}$$

subject to

$$\frac{A_{t+1}}{P_t} = R_t \frac{A_t}{P_t} + \frac{W_t}{P_t} L_t + D_t - C_t.$$

From first order conditions, we can derive equations which defines household's behavior:

$$L_t = 1 - \frac{1-v}{v} \frac{P_t}{W_t} C_t,$$

$$C_t = \epsilon_t \left[R_t \frac{A_t}{P_t} + H_t + F_t \right], \tag{35}$$

$$F_t = D_t + \frac{P_{t+1}}{P_t} \frac{F_{t+1}}{R_{t+1}},$$

$$H_t = \frac{W_t}{P_t} L_t + \frac{P_{t+1}}{P_t} \frac{H_{t+1}}{R_{t+1}},$$

$$\epsilon_t = 1 - \frac{\epsilon_t}{\mathrm{E}_t \epsilon_{t+1}} R_{t+1}^{\frac{\rho}{1-\rho}} \beta^{\frac{1}{1-\rho}} \left(\frac{P_t}{P_{t+1}} \right)^{\frac{v\rho}{1-\rho}} \left(\frac{W_t}{W_{t+1}} \right)^{\frac{(1-v)\rho}{1-\rho}}, \tag{36}$$

and

$$V_t = (\epsilon_t)^{-\frac{1}{\rho}} C_t \left(\frac{1-v}{v} \frac{P_t}{W_t} \right)^{1-v}. \tag{37}$$

Aggregation

As in the lifecycle models, we assume that total population is N and its growth rate is $1+n$.

$$L_t = N_t - \frac{1-v}{v}\frac{P_t}{W_t}C_t \tag{38}$$

$$H_t = \frac{W_t}{P_t}L_t + \frac{P_{t+1}}{P_t}\frac{1}{(1+n)\,R_{t+1}}\mathrm{E}_t H_{t+1}. \tag{39}$$

$$F_t = D_t + \frac{P_{t+1}}{P_t}\frac{1}{(1+n)\,R_{t+1}}F_{t+1}. \tag{40}$$

$$Y_t = \left[Z_t \exp(z_t)\,L_t\right]^{1-\alpha} K_t^{\alpha} \tag{41}$$

$$\frac{W_t}{P_t} = (1-\alpha)\,\varphi_t \frac{Y_t}{L_t} \tag{42}$$

$$\frac{A_t}{P_t} = K_t \tag{43}$$

$$Y_t = C_t + I_t \tag{44}$$

System of Equations

The system of equations consists of 18 equations: (1), (2), (3), (4), (5), (6), (7), (32), (35), (36), (38), (39), (40), (41), (42), (25), (43) and (44). De-trended system of equations are expressed as follows:

r^K: $\mathrm{E}_t Q_{t+1}(1-\delta) - \frac{R_{t+1}}{1+\mathrm{E}_t\pi_{t+1}}Q_t + \mathrm{E}_t r^K_{t+1} = 0$

Q: $Q_t\left\{1-\left[(1+z)(1+n)\right]^2 S''\left[\frac{\left(\frac{i_t}{i_{t-1}}\right)^2}{2} - \frac{i_t}{i_{t-1}} + \frac{1}{2}\right]\right\}$

$-Q_t\left[(1+z)(1+n)\right]^2 S''\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}$

$$+\mathrm{E}_t Q_{t+1}\left[(1+z)(1+n)\right]^3 S''\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 - 1 = 0$$

$$k: k_{t+1}(1+z)(1+n) = (1-\delta)k_t$$
$$+\left\{1-\left[(1+z)(1+n)\right]^2 S''\left[\frac{\left(\frac{i_t}{i_{t-1}}\right)^2}{2} - \frac{i_t}{i_{t-1}} + \frac{1}{2}\right]\right\} i_t$$

$$\varphi : \mathrm{E}_t r^K_{t+1} = \alpha \mathrm{E}_t \varphi_{t+1}\frac{Y_{t+1}}{K_{t+1}}$$

$$\pi : (1-\kappa) + \kappa\varphi_t \exp(u_t) - \phi\pi_t(\pi_t+1)$$
$$+\mathrm{E}_t \frac{1}{R_{t+1}}\phi(\mathrm{E}_t\pi_{t+1})(\mathrm{E}_t\pi_{t+1}+1)\frac{y_{t+1}}{y_t}(1+z)(1+n) = 0$$

$$d : d_t = \left(1-\varphi_t - \frac{\phi}{2}\pi_t^2\right) y_t$$

$$l : l_t = 1 - \frac{1-v}{v}\frac{c_t}{w_t}$$

$$\epsilon : \epsilon_t = 1 - \frac{\epsilon_t}{\mathrm{E}_t\epsilon_{t+1}} R_{t+1}^{\frac{\rho}{1-\rho}} \beta^{\frac{1}{1-\rho}}\left(\frac{P_t}{P_{t+1}}\right)^{\frac{v\rho}{1-\rho}}\left(\frac{W_t}{W_{t+1}}\right)^{\frac{(1-v)\rho}{1-\rho}}$$

$$c : c_t = \epsilon_t(R_t a_t + h_t + f_t)$$

$$f : f_t = d_t + \mathrm{E}_t\frac{(1+\pi_{t+1})(1+z)}{R_{t+1}} f_{t+1}$$

$$h : h_t = w_t l_t + \mathrm{E}_t\frac{(1+\pi_{t+1})(1+z)}{R_{t+1}} h_{t+1}$$

$$y : y_t = \left[\exp(z_t)\, l_t\right]^{1-\alpha} k_t^\alpha$$

$$a : a_t = Q_t k_t$$

$$w : w_t = (1-\alpha)\varphi_t \frac{y_t}{l_t}$$

$$i : y_t = c_t + i_t$$

$$R : R_t = \vartheta R_t + (1-\vartheta)(R_{SS}) + \eta_1\pi_t + \eta_2\left(\frac{y_t}{y_{t-1}}-1\right) + \tau_t$$

$$u : u_t = \rho_u u_{t-1} + \varepsilon_{ut}$$

$$z : z_t = \rho_z z_{t-1} + \varepsilon_{zt}$$

Definition 2 ***(Competitive Equilibrium)*** *A competitive equilibrium is a sequence of endogenous predetermined variables* $\{a,\ k,\ i,\ R\}$ *and a sequence of endogenous variables* $\{r^K,\ Q,\ \varphi,\ \pi,\ d,\ l,\ \epsilon,\ c,\ f,\ h,\ y,\ w\}$ *given the sequence of exogenous predetermined variables* $\{u,\ z,\ \tau\}$

Value in equation (37) is also expressed as de-trended variables:

$$v : v_t = (\epsilon_t)^{-\frac{1}{\rho}} c_t \left(\frac{1-v}{v}\frac{1}{w_t}\right)^{1-v}$$