Credit Scoring and Competitive Pricing of Default Risk[1]

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Abstract

When people cannot commit to pay back their loans and there is limited information about their characteristics, lending institutions must draw inferences about their likelihood of default. In this paper, we examine how this inference problem impacts consumption smoothing. In particular, we study an environment populated by two types of people who differ with respect to their rates of time preference and receive idiosyncratic earnings shocks. Impatient types are more likely to borrow and default than patient types. Lenders cannot directly observe a person's type but make probabilistic assessments of it based on the person's credit history. The model delivers an integrated theory of terms of credit and credit scoring that seems broadly consistent with the data. We also examine the impact of legal restrictions on the length of time adverse events can remain on one's credit record for consumption smoothing and welfare.

1 Introduction

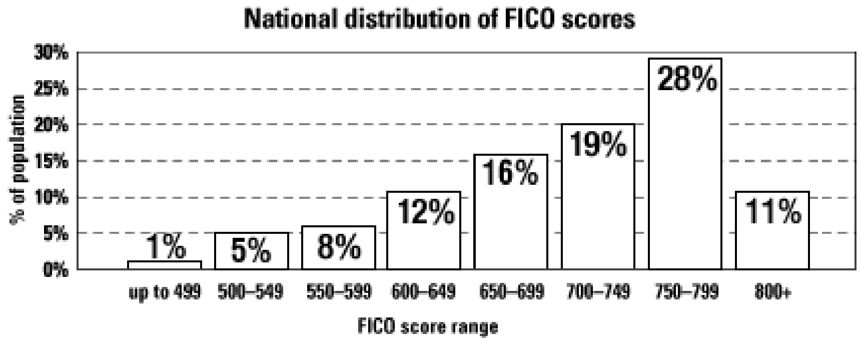
The legal environment surrounding the U.S. unsecured consumer credit market is characterized by the following features. Broadly speaking, the U.S. Constitution gives private individuals an inalienable right to declare bankruptcy under Chapter 7 or Chapter 13. A Chapter 7 bankruptcy permanently discharges net debt (liabilities minus assets above statewide exemption levels). A Chapter 7 filer is ineligible for a subsequent Chapter 7 discharge for 6 years. During that period, the individual is forced into Chapter 13 which is typically a 3-5 year repayment schedule followed by discharge. Over two-thirds of household bankruptcies in the U.S. are Chapter 7. Finally, the Fair Credit Reporting Act requires credit bureaus to exclude the filing from credit reports after 10 years (and all other adverse items after 7 years).

This lack of commitment does not prevent private unsecured borrowing. Currently, the level of unsecured consumer credit in the U.S. is between 10 to 15 percent of annual aggregate consumption *and* there is 1 bankruptcy filing per year for every 75 U.S. households. Evidently, the current market arrangement works. The question is *how* does it work and could it work *better* without legal restrictions?

Given the inability of people to commit, it's important for a lender to assess the probability that a borrower will fail to pay back – that is, assess the risk of default. In the U.S., lenders use *credit scores* as an index of the risk of default. The credit scores most commonly used are produced by a single company, the Fair Isaac and Company, and are known as FICO scores.[1] These scores range between 300 and 850, where a higher score signals a lower probability of default. The national distribution of FICO scores are given in Figure 1.

[1]Over 75% of mortgage lenders and 80% of the largest financial institutions use FICO scores in their evaluation and approvals process for credit applications.

Figure 1



0010.gif

Source: http://www.myfico.com/myfico/Credit Central/ScoringWorks.asp

A FICO score aggregates information from an individual's credit record like his payment history (most particularly the presence of adverse public records such as bankruptcy and delinquency) and current amounts owed.[2] These scores appear to affect the extension of consumer credit in four primary ways.

1. Credit terms appear to improve with a person's credit score.
2. The presence of a bankruptcy flag constrains individual's access to credit.
3. The removal of adverse public records can raise scores substantially and boosts access to credit.
4. Taking on more debt (paying off debt) tends to lower (raise) credit scores.

[2]The score also takes into account the length of a person's credit history, the kinds of credit accounts (retail credit, installment credit etc.) and the borrowing capacity (or line of credit) on each account. It's also worth noting the kinds of information that are not used in credit scores. By law, credit scores cannot use information on race, color, national origin, sex, and marital status. Further, FICO scores do not use age, assets, salary, occupation, and employment history.

Table 1

FICO Score	Auto Loan	Mortgage
720-850	4.94%	5.55%
700-719	5.67%	5.68%
675-699	7.91%	6.21%
620-674	10.84%	7.36%
560-619	15.14%	8.53%
500-559	18.60%	9.29%

Source: http://www.myfico.com/myfico/Credit Central/LoanRates.asp

Table 1 provides information on the relationship between FICO scores and the average interest rate on a new 60-month auto loan or a new 30-year fixed mortgage consistent with item 1. Item 2 is consistent with evidence in Fisher, Filer, and Lyons [5]. Using data from the PSID and SCF, they document that a higher percentage of post-bankruptcy households were denied access to credit. Item 3 is consistent with evidence provided in Musto [9]. Musto studied the impact of striking an individual's bankruptcy record from his or her credit history after 10 years (as required by the Fair Credit and Reporting Act). He found (p.735) "there is a strong tenth year effect for the best initial credits...these consumers move ahead of 19% of the nonfiler population in apparent creditworthiness when their flags are removed." Furthermore, he states (p.740) "...the boost translates to significant new credit access for these filers over the ensuing year". In conjunction with Table 1, items 2 and 3 suggest that an individual who fails to pay back an unsecured loan will experience an adverse change in the terms of (unsecured) credit. Thus, a failure to pay back a loan adversely impacts the terms of credit and may result in outright denial of credit. Item 4 is consistent with the advice given by FICO for improving one's credit score.[3] Additionally, item 4 in conjunction with Table 1 indicates that even absent default, the terms of credit on unsecured credit worsen as an individual gets further into debt – people face a rising marginal cost of funds.

These facts suggest the following characterization of the workings of the unsecured con-

[3]To improve a score, FICO advises to "Keep balances low on credit card and 'other revolving credit"' and "[p]ay off debt rather than moving it around". Source:www.myfico.com/CreditEducation/ImproveYour Score

sumer credit market. Given the inability of borrowers to commit to pay back, lenders condition the terms of credit (including whether they lend at all) on an individual's credit history. This history is somehow encapsulated by a credit score. Individuals with higher scores are viewed by lenders as less likely to default and receive credit on more attractive terms. The failure to pay back a loan (default) leads to a drop in the individual's credit score. Consequently, post-default access to credit is available on worse terms and may not be available at all. Even absent default, greater indebtedness leads to a lower credit score and worse terms of credit. Finally, there is considerable amount of unsecured credit extended under these circumstances and a non-trivial fraction of borrowers default.

There is now a fairly substantial literature on how (and to what extent) borrowing can occur when agents cannot commit to pay back (some key papers will be noted below). The challenge, as we see it, is to use the insights of this literature to specify a structure that can make quantitative sense of the characterization of the unsecured consumer credit market offered in the previous paragraph.

This paper takes some tentative steps toward meeting this challenge. We consider an environment with a continuum of infinitely-lived people who at any point in time may be one of two types who differ in their discount factor. An agent's type is drawn independently from others and follows a two-state Markov process which is identical for everyone. Types differ in their discount factors. Importantly, a person's type is private information.

These people interact with a competitive financial intermediation sector that can borrow in the international credit market at some fixed risk-free rate and make one-period loans to individuals at an interest rate that reflects that person's risk of default. Because differences in preference bear on the willingness of each type of person to pay back a loan, intermediaries must form some assessment of a person's type. We model this as a Bayesian inference problem: intermediaries use the recorded history of a person's actions in the credit market to update their prior probability of the person being of a given type and then charge an interest rate that is appropriate for the posterior probability.

We model the pricing of unsecured consumer loans in the same fashion as in our predecessor paper Chatterjee, *et.al.* [2]. As in that paper, all one-period loans are viewed as discount bonds and the price of these bonds depend on the size of the bond. This is necessary because the probability of default (for any type) will depend on the size of the bond (i.e., on the person's liability). In addition, and this is a feature that is new to this paper, the price of the bond also depends on the posterior probability of a person being of a given type, *conditional* on selling that particular sized bond. This is necessary because the two types will not have the same probability of default for any given sized bond and a person's asset choice is potentially informative about the person's type. With this asset market structure, competition implies that the expected rate of return on each type of bond is equal to the (exogenous) risk-free rate.

This is, possibly, the simplest environment one could imagine that could make sense of the observed connection between credit history and the terms of credit. Suppose it turns out that, in equilibrium, one type of person, say type g, always has a lower probability of default. Then, under competition, the price of a discount bond (of any size) could be expected to be positively related to the probability of a person being of type g. Further, default will lower the *posterior* probability of being of type g because type g people default less frequently. If we interpret a person's credit score as (some positive transform of) the probability of a person being of type g, we would explain Table 1 and item 1. We caution the reader, however, that although this sounds intuitive the statement that a person of type g is *always* less likely to default is a very strong restriction on equilibrium behavior and may require correspondingly strong assumptions on fundamentals.

There are two strands of existing literature to which our paper is closely related. One strand relates to the banking literature where Diamond's [4] well-known paper on acquisition of reputation in debt markets is a key reference.[4] Diamond considers a situation where there are two types of infinitely-lived risk-neutral entrepreneurs who interact with a competitive financial intermediation sector. Financial intermediaries make one-period loans without directly observing the entrepreneur's type. One type of entrepreneur always chooses the safe project but the other type chooses between a safe project and risky project and an entrepreneur defaults if the project fails. Since an entrepreneur's loss is bounded be-

[4]Phelan [10] studies reputation aquisition by a government in a related framework.

low, the second type has an incentive to choose the risky project. In this environment, an entrepreneur's payment history (did the project ever fail?) reveals something about an entrepreneur's type. Consequently, the terms of credit offered to an entrepreneur will depend on the entrepreneur's payment history. Diamond's set-up clearly has parallels to our own. The main difference is that for us the decision to default is the key decision (in Diamond this happens only when the project fails) and we don't permit any choice with regard to the riskiness of the income stream.

The second strand of literature to which our paper is related is the literature on sovereign debt. This literature shares with us the concern about the inability of the borrower to commit to pay back. The inability of the sovereign to commit stems from the fact that the sovereign does not (by definition) answer to a higher authority. In this strand, the paper that is most closely related to ours is Cole, Dow and English [3]. They focus on an interesting aspect of sovereign defaults, namely, that a sovereign who defaults is shut out of international credit market until such time as the sovereign makes a payment on the defaulted debt. In our case, the inability to commit stems from a right to bankruptcy granted to an individual by the legal system. Consequently, a bankruptcy results in a *discharge* of existing debt and individuals do not have the option of making payment on discharged debt in the future.[5]

Our framework has the ability to address an interesting question that arises from Musto's empirical work. What are the effects on consumption smoothing and welfare of imposing legal restrictions (like the Fair Credit Reporting Act), which requires adverse credit information (like a bankruptcy) to be stricken from one's record after a certain number of years (10 in the U.S.)? Specifically, Musto p. 726 states that his empirical "results bear on the informational efficiency of the consumer credit market, the efficacy of regulating this market with reporting limits, and the quality of postbankruptcy credit access, which in turn bears on the incentive to file in the first place." He finds p. 747 "the removal of the flag leads to excessive credit, increasing the eventual probability of default. This is concrete evidence that the flag regulation has real economic effects. This is market efficiency in reverse." We

[5]Given the choice between Chapter 7 and 13, individuals would choose to file Chapter 13 only if they wished to keep assets they would lose under a Chapter 7 filing. Since borrowers in our model have negative net worth (there is only one asset), Chapter 7 is always the preferred means to file for bankruptcy.

use our model to assess this efficiency concern. In a world of incomplete markets and private information, flag removal may provide insurance to impatient agents in our framework that competitive intermediaries may not be able to provide. Hence extending the length of time that bankruptcy flags remain on credit records may not necessarily raise ex-ante welfare. This issue echoes Hart's [8] examples where the opening of a market in a world of incomplete markets may make agents worse off.

2 Model Economy

We begin by describing the market arrangement in our model economy. This is followed by a recursive formulation of the individual's decision problem and a description of profit maximizing behavior of firms serving the unsecured credit industry.

2.1 Default Option and Market Arrangement

We model the default option to resemble, in procedure, a Chapter 7 bankruptcy filing. If an individual files for bankruptcy, the individual's beginning of period liabilities are set to zero (i.e., the individual's debt is discharged) but during the filing period (when the individual's books are open) he is not permitted to enter new contracts.[6]

There is a competitive credit industry that accepts deposits and makes loans to individuals. An individual can borrow at an interest rate that depends on the size of the loan and on the market's belief about the individual's type. We will assume that there are only two types of people denoted type g and b. As noted earlier, belief about an individual's type is important because an individual cannot commit to repay and the probability of repayment can vary across types. An individual can also save via deposits and all deposits fetch a constant risk-free rate.

Time is discrete and indexed by t. Let $\ell_t \in L \subset R$ be an agent's beginning of period t asset holding (chosen in period $t-1$), where $\ell_t < 0$ denotes debt and $\ell_t \geq 0$ denotes

[6] Unlike Chatterjee, et. al. [2], the agent is not bound from borrowing until the bankruptcy filing leaves his record.

deposits. The set L is finite. Let d_t be an indicator variable that takes on the value of 1 if the individual defaults in period t on loan ℓ_t and zero otherwise (in the event of default at t, ℓ_{t+1} is constrained to be 0). An individual's history of observed actions (asset choices and default decisions) at the beginning of period t is given by $(\ell_t, h^t(T))$ where $h^t(T) = (d_{t-1}, \ell_{t-1}, d_{t-2}, ..., \ell_{t+1-T}, d_{t-T})$ where $T \geq 1$.[7] We assume an asset market structure where in period t the price of a loan of size $\ell_t + 1 \in L$ made to an individual with history (ℓ_t, h^t) is given by $q(\ell_t + 1, \ell_t, h^t) \geq 0$.[8]

2.2 People

There is a unit measure of people. At any time t, people can be one of two types, indexed by $i_t \in \{g, b\}$.

Within a period, the timing of events is as follows. At the start of a period, each person learns his type and this type is drawn in an i.i.d. fashion from a two-state Markov process with transition matrix $\delta(i'|i) = \Pr(i_{t+1} = i'|i_t = i)$. In particular, if an agent was of type i in period t, he will remain type i in the current period with probability $1 - \delta_i$ and change type with probability δ_i. Next, each individual receives a random endowment of goods and this endowment is an i.i.d. draw with measure η on a compact support $E \subset R_{++}$. After observing his type and endowment, an individual chooses whether to default on his borrowings if $\ell_t < 0$. Finally, the individual chooses his asset position ℓ_{t+1} and consumes c_t. While an individual's type, endowment, and consumption are private information, his default decisions and asset position are observable.

Given our asset market structure, it is natural to adopt a recursive formulation of an individual's decision problem with state variables given by $(i, e, \ell, h(T))$. The value function of an agent of type i, denoted by $v_i(e, \ell, h)$, solves the following functional equation:

[7] We do not include prices of these one-period contracts in the history $(\ell_t, h^t(T))$ since they are considered proprietary and excluded from standard credit reports.

[8] Since current filers cannot enter into contracts, it should be understood that these prices are extended to individuals with $d_t = 0$ allowing us to neglect burdensome notation.

Case 1: When $\ell < 0$

$$v_i(e, \ell, h; q) = \max_{d \in \{0,1\}} v_i^d(e, \ell, h; q) \tag{1}$$

where the value function when the agent decides not to default ($d = 0$) is given by

$$\begin{aligned} v_i^0(e, \ell, h; q) = & \max_{(c, \ell') \in B(e, \ell, h; q) \neq \varnothing} u_i(c) + \\ & \beta_i \int_E [(1 - \delta_i) v_i(e', \ell', h') + \delta_i v_{-i}(e', \ell', h')] \ \eta(de') \end{aligned} \tag{2}$$

where

$$B(e, \ell, h; q) = \{c \geq 0, \ell' \in L \mid c + q(\ell', \ell, h) \cdot \ell' \leq e + \ell\}$$

and the value function when the agent chooses to default ($d = 1$) or $B(e, \ell, h; q) = \varnothing$ (in which case default is the only option) is given by

$$v_i^1(e, \ell, h; q) = u_i(e) + \beta_i \int_E [(1 - \delta_i) v_i(e', 0, h'; q) + \delta_i v_{-i}(e', 0, h'; q)] \ \eta(de').$$

Here, $u_i(c)$ is the utility that an individual of type i receives from consuming c units of the good and β_i is his discount factor. The continuation of $h = (d_{-1}, \ell_{-1}, ..., \ell_{+2-T}, d_{+1-T}, \ell_{+1-T}, d_{-T})$ following action (ℓ', d) is given by

$$\begin{aligned} h'(T) &= \lambda(\ell, d, h; T) \\ &= (d, \ell, ..., \ell_{+2-T}, d_{+1-T}). \end{aligned}$$

The value function under default, $v_i^1(e, \ell, h)$, assumes that default $d = 1$ wipes out all debt and that a defaulting individual cannot accumulate any asset in the period of default (i.e. $\ell' = 0$).

Case 2: When $\ell \geq 0$,

$$v_i(e, \ell, h; q) = v_i^0(e, \ell, h; q). \tag{3}$$

In what follows we denote the set of earnings for which an individual of type i and history (ℓ, h) defaults on a loan of size ℓ by $D_i(\ell, h; q) = \{e \mid d_i(e, \ell, h; q) = 1\} \subseteq E$. We will also denote by $E_i(\ell', \ell, h; q) = \{e \mid \ell_i'(e, \ell, h; q) = \ell'\} \subseteq E$ the set of earnings for which an individual of type i in history (ℓ, h) chooses ℓ'.

2.3 The Credit Industry

Financial intermediaries have access to an international credit market where they can borrow or lend at the risk-free interest rate $r \geq 0$. The profit on a loan of size $\ell' < 0$ made to an individual with history (ℓ, h) is the present discounted value of inflows less the current value of outflows and the profit on deposit of size $\ell' > 0$ made to an individual with history (ℓ, h) is the current value of the inflows less the present discounted value of outflows. Then the profit on a contract of type (ℓ', ℓ, h), denoted $\pi(\ell', \ell, h; q, p)$, is:

$$\pi(\ell', \ell, h; q, p) = \begin{cases} (1+r)^{-1}[1 - p(\ell', \ell, h)](-\ell') - q(\ell', \ell, h)(-\ell') & \text{if } \ell' < 0 \\ q(\ell', \ell, h)\ell' - (1+r)^{-1}\ell' & \text{if } \ell' \geq 0 \end{cases} \tag{4}$$

where $p(\ell', \ell, h)$ is the fraction of individuals with history (ℓ, h) expected to default on a loan of size ℓ' tomorrow. If $\alpha(\ell', \ell, h)$ is the measure of type (ℓ', ℓ, h) contracts sold, the decision problem of an intermediary is to maximize $\sum_{\ell',(\ell,h)} \pi(\ell', \ell, h; q, p) \cdot \alpha(\ell', \ell, h)$ subject to the constraint that $\alpha(\ell', \ell, h) \geq 0$.

3 Equilibrium

Let μ be a distribution of individuals over $\{g, b\} \times L \times H$ where H is the set of all possible h. An equilibrium is a set of decision rules, a price menu, default probabilities, beliefs, and a distribution (that is, $((\ell_i'(\ell, h; q^*), d_i(\ell, h; q^*)), q^*, p^*, \Psi^*, \mu^*)$ which satisfies the following five sets of conditions.

The first set are the optimization conditions of individuals. That is, given q^*, $(\ell_i'(\ell, h; q^*), d_i(\ell, h; q^*))$ must be consistent with (1) and (3).

The second set are zero profit conditions for loans and deposits

$$q^*(\ell', \ell, h; p^*) = \begin{cases} (1+r)^{-1}[1 - p^*(\ell', \ell, h; \Psi^*)] & \ell' < 0 \\ (1+r)^{-1} & \ell' \geq 0 \end{cases} . \tag{5}$$

The third set of conditions involve the default probabilities which must be consistent with decision rules and beliefs

$$p^*(\ell', \ell, h; \Psi^*) = \left\{ \begin{array}{c} [1 - \eta(D_g(\ell', h'; q^*))] \cdot \Psi^*(\ell', 0, \ell, h) \\ +[1 - \eta(D_b(\ell', h'; q^*))] \cdot (1 - \Psi^*(\ell', 0, \ell, h)) \end{array} \right\}. \tag{6}$$

where the belief function $\Psi^*(\ell', d, \ell, h)$ denotes the probability that an individual who chose (ℓ', d) in history (ℓ, h) is of type g at the beginning of the following period after the realization of the type change shock.

The fourth and most important set of conditions concern the equilibrium belief (updating) function $\Psi^*(\ell', d, \ell, h)$. We require that beliefs must be consistent with Bayes' rule whenever applicable.[9] Recall that according to Bayes' rule, the probability that event A is true given that event B is true is given by $\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$ provided $\Pr(B) > 0$.[10] Translating to our problem, the financial intermediary evaluates the probability that an individual is type g conditional on observing their credit history (ℓ, h) and current actions (ℓ', d). Thus, we let event A be defined as "the agent's type is g" and event B be defined as "observing

[9]This notion of assigning beliefs "whenever possible" as to individual types on the basis of Bayes Rule is similar to what is assumed as part of a definition of Perfect Bayesian Equilibrium (see Fudenberg and Tirole [6], p. 331-333).

[10]In Bayesian terminology, $\Pr(A)$ is the prior probability that A is true and $\Pr(A \mid B)$ is the posterior probability that A is true given that B is observed.

(ℓ', d, ℓ, h)". Applying Bayes law gives[11]

$$\begin{aligned} \Pr(g|\ell', d, \ell, h) &= \frac{\Pr(\ell', d, \ell, h|g)\Pr(g)}{\Pr(\ell', d, \ell, h)} \\ &= \frac{\Pr(\ell', d|g, \ell, h)\Pr(g|\ell, h)}{\Pr(\ell', d|g, \ell, h)\Pr(g|\ell, h) + \Pr(\ell', d|b, \ell, h)\Pr(b|\ell, h)}. \end{aligned} \tag{7}$$

There are two mutually exclusive events associated with possible (ℓ', d) choices that are partitioned on the basis of the default decision. First, a type i individual with history (ℓ, h) defaults on loan ℓ so that $\ell' = 0, d = 1$. In that case $\Pr(0, 1|i, \ell, h) = \eta(D_i(\ell, h; q^*))$. In the second case, a type i individual with history (ℓ, h) does not default on loan ℓ (which is obviously the case when $\ell \geq 0$) and chooses $\ell' \in L$. In this case $\Pr(\ell', 0|i, \ell, h) = \eta(E_i(\ell', \ell, h; q^*))$. The other terms in (7) are given by

$$\Pr(i|\ell, h) = \frac{\mu^*(i, \ell, h)}{\sum_{j \in \{g,b\}} \mu^*(j, \ell, h)}. \tag{8}$$

Therefore, prior to the type shock realization, the posterior probability that an individual in history (ℓ, h) who defaults on his loan is of type g is given by:

$$\Pr(g|0, 1, \ell, h) = \frac{\eta(D_g(\ell, h; q^*))\mu^*(g, \ell, h)}{\eta(D_g(\ell, h; q^*))\mu^*(g, \ell, h) + \eta(D_b(\ell, h; q^*))\mu^*(b, \ell, h)} \tag{9}$$

[11] This expression follows from: (i)

$$\Pr(\ell', d, \ell, h|g) = \Pr(\ell', d|g, \ell, h)\Pr(\ell, h|g),$$

where another application of Bayes' law to the last expression yields

$$\Pr(\ell, h|g) = \frac{\Pr(g|\ell, h)\Pr(\ell, h)}{\Pr(g)}$$

so that the numerator $\Pr(\ell', d, \ell, h|g)\Pr(g)$ can be written

$$\left[\Pr(\ell', d|g, \ell, h)\frac{\Pr(g|\ell, h)\Pr(\ell, h)}{\Pr(g)}\right]\Pr(g);$$

and (ii) the fact that the denominator $\Pr(\ell', d, \ell, h)$ can be written

$$\Pr(\ell', d|\ell, h)P(\ell, h)$$

where

$$\Pr(\ell', d|\ell, h) = \Pr(\ell', d|g, \ell, h)\Pr(g|\ell, h) + \Pr(\ell', d|b, \ell, h)\Pr(b|\ell, h).$$

and the posterior that an individual in history (ℓ, h) who chooses not to default and to enter the asset market is of type g is given by

$$\Pr(g|\ell', 0, \ell, h) = \frac{\eta(E_g(\ell', \ell, h; q^*))\mu^*(g, \ell, h)}{\eta(E_g(\ell', \ell, h; q^*))\mu^*(g, \ell, h) + \eta(E_b(\ell'\ell, h; q^*))\mu^*(b, \ell, h)} \tag{10}$$

Then, given that type can change at the beginning of the next period,

$$\Psi^*(\ell', d, \ell, h) = (1 - \delta_g) \Pr(g|\ell', d, \ell, h) + \delta_b \left[1 - \Pr(g|\ell', d, \ell, h)\right]. \tag{11}$$

Since Bayes' rule is applicable only if the conditioning event has positive probability, we may also need to assign values to $\Psi(\ell', d, \ell, h)$ in some fashion when the conditioning set is empty. Theory does not restrict the assignment but there may be existence and computational issues involved in the choice.

The fifth condition requires that for a steady state equilibrium μ^* is a fixed point of the mapping

$$\mu'(i', \ell', h') = \left[\sum_{i,\ell,h} \int_e \delta(i'|i)\mathbf{1}\{\ell'_i(e, \ell, h; q^*) = \ell', \lambda(\ell, d_i(e, \ell, h; q^*), h; T) = h'\} d\eta(e)\mu(i, \ell, h)\right]. \tag{12}$$

With a finite number of histories (i.e. when $T < \infty$), the computation of an equilibrium is straightforward and a mapping to type scores, $\Pr(g|\ell, h)$, follows using (8). For the $T = \infty$ case, computation of an equilibrium requires that we encode the infinite history in a manageable way. This can be done provided we make an assumption about the updating formula (11). To state the assumption, consider any two distinct histories (ℓ, h) and $(\ell, \widehat{h})$ with the property that $\Pr(g|\ell, h) = \Pr(g|\ell, \widehat{h})$. Then, we assume that $\Psi(\ell', d, \ell, h) = \Psi(\ell', d, \ell, \widehat{h})$ for all ℓ', d. In other words, if two individuals with the same beginning-of-period asset positions have exactly the same *prior* probability of being of type g, the *posterior* probability of their being of type g is the same if they take the same actions. That is, the updated probability depends not on how they arrived at ℓ provided $\Pr(g|\ell, h) = \Pr(g|\ell, \widehat{h})$. Under this assumption, we can replace the infinite-dimensional state variable h by the scalar s, where s is the prior (or, equivalently, the beginning-of-period) probability of being of type g (i.e. $s = \Pr(g|\ell, h(\infty))$. The law of motion of this new state variable s is given by $s' = \Psi(\ell', d, \ell, s)$ in equation (11). Intuitively, if the updating rule does not distinguish between these two people then there is no reason for these two people to behave differently conditional on type. Hence the prior probability s and beginning-of-period asset position ℓ are the only two state variables needed in the household decision problem.

4 When is a Type Score a Credit Score?

We start with the case where the entire history of asset market actions are kept in the individual's record (i.e. $T = \infty$ so that no information is discarded). We will set aside the question of whether an equilibrium exists and simply provide an example of an equilibrium below. We are simply interested in knowing whether a type score has the four properties of a credit score noted in the introduction. For instance, under what conditions on $u_i(.)$ and β_i is $D_g(\ell, s; q^*) \subseteq D_b(\ell, s; q^*)$ for any s and ℓ? Such a ranking would give content to the statement that, from the perspective of lenders, type g is the good type and type b is the bad type and, therefore, give some basis for identifying the *type score* (the probability that a person is of type g) with a *credit score*. However, the potentially complex dependence of a person's decision rule on the q and Ψ functions makes it challenging to provide such a ranking – unless very strong assumptions are made on preferences and choice sets.

To make progress, we will specialize the model to a case that is simple enough so that with a combination of reasoning and numerical simulation we can develop some intuition on the basic economics of the situation. With this mind, we will make the following assumptions:

A1. $\beta_b = 0$ and $0 < \beta_g$.

A2. $\delta_i \in (0, 1)$ and $1 - \delta_g > \delta_b$.

A3. $L = \{-x, 0, x\}$.

A4. If $\eta(D_i(\ell, s; q)) = 0$ for all i, $\Psi(0, 1, \ell, s) = (1-\delta_g)s + \delta_b(1-s)$ and if $\eta(E_i(\ell', \ell, s; q)) = 0$ for all i, $\Psi(\ell', 0, \ell, s) = (1 - \delta_g)s + \delta_b(1 - s)$.

The strong assumption is that type b agents are myopic. This strong assumption will enable us to characterize their decision rule independent of their type score s. The second assumption on type transition probabilities will yield a nondegenerate interval for type scores.

The third assumption, while making asset choices simple to characterize, limits how much separation can go on. The fourth assumption concerns off-the-equilibrium-path beliefs; if no one with type score s takes a certain action, then we update using their current score.

Assumption A1 provides a very simple characterization of type b behavior.

Proposition 1. For any s, type b agents borrow if $\ell \in \{0, x\}$ and default if $\ell = -x$ and prices are strictly positive. Specifically: (i) $E_b(x, \ell, s; q^*) = \varnothing$; (ii) $D_g(-x, s; q) \subseteq D_b(-x, s; q) = E$; and (iii) for $\ell \in \{0, x\}$, $E_g(-x, \ell, s; q^*) \subseteq E_b(-x, \ell, s; q^*) \in \{\varnothing, E\}$.

These results follow because a type b person cares about an action *only* to the extent it affects current consumption – what any action might entail about the person's future type-score is not relevant because the person does not care about the future at all. This is true even though the type b agent may switch to being type g at the start of the next period simply because switches happen in the *future* and a type b person does not give *any* weight to the future. Therefore, if choosing $\ell' = x$ is feasible it is strictly dominated by choosing $\ell' = 0$ (– the latter is a feasible choice if the former is feasible) and part (i) follows. To see part (ii), observe that paying the debt back and not borrowing (i.e., choosing $(d, \ell') = (0, 0)$) results in a reduction of current consumption and is strictly dominated by choosing $(d, \ell') = (1, 0)$ for a type b. Paying the debt back and borrowing also results in a drop in current consumption since current consumption under this action is $-(1 - q^*(-x, \Psi^*(-x, 0, -x, s)) < 0$ by virtue of the fact that in equilibrium the $q^*(-x, \sigma) \leq 1/(1+r) < 1$ for any σ.[12] Therefore, for a type b person with debt, the optimal decision is to default independent of his earnings. To see part (iii), consider the following two cases. First, if $q(-x, \Psi^*(-x, 0, \ell, s)) > 0$, the optimal decision for a type b agent is to borrow since this maximizes current period consumption and that's all the person cares about. Therefore, $E_b(-x, \ell, s; q^*) = E$. Second, if $q(-x, \Psi^*(-x, 0, \ell, s)) = 0$, then agents are borrowing constrained and neither type can choose $\ell'_i = -x$. Hence $E_g(-x, \ell, s; q^*) = E_b(-x, \ell, s; q^*) = \varnothing$.

Given that type b people behave in this way, we can now partially characterize the equi-

[12] In order to economize on notation, rather than expressing q as a function of ℓ', ℓ, and s, we express q as a function of the updated score $s' = \Psi(\ell', d, \ell, s)$. We can do this, because the only reason that ℓ, s matters for prices is for the inference about an individual's type at the time of repayment.

librium updating function Ψ^*. We have:

Proposition 2. If an individual does not choose $-x$, his updated score will generally rise. Specifically: (i) if $\ell \in \{0, x\}$ and $\eta(E_g(x, \ell, s; q^*)) > 0$, Bayesian updating implies $\Psi^*(x, 0, \ell, s) = 1 - \delta_g$; and (ii) If $\ell \in \{0, x\}$, $\eta(E_g(0, \ell, s; q^*)) > 0$, and $E_b(-x, \ell, s; q^*) = E$, Bayesian updating implies $\Psi^*(0, 0, \ell, s) = 1 - \delta_g$.

For (i), if the person saves, then by Proposition 1(i), he is not of type b. Provided in equilibrium there is some e for which type g agent with ℓ, s chooses to save (i.e. $\eta(E_g(x, \ell, s; q^*)) > 0$, a requirement that is necessary to apply Bayes' formula), then by (10) $\Pr(g|\ell', 0, \ell, s) = 1$.[13] The Proposition follows from (11). For (ii), we know by Proposition 1(iii) that in equilibrium $E_b(-x, \ell, s; q^*) \in \{\varnothing, E\}$. If all type b borrow, then lenders can correctly infer, provided $\eta(E_g(0, \ell, s; q^*)) > 0$, that an agent who chooses $\ell' = 0$ is of type g.

The next two propositions address issues that are at the heart of this project. These propositions establish that in equilibrium the type score s has properties that resemble the properties of credit scores, namely, that credit scores decline with default and decline (improve) with increasing (decreasing) indebtedness (consistent with item 4).

Proposition 3. A default lowers an agent's pre-type-shock score. Specifically, Bayesian updating implies $\Pr(g|0, 1, -x, s) \leq s$.

To see why the proposition is true, observe first that given the assumptions on δ_i and the definition in (11), it follows that $\Psi^*(\ell', d, \ell, s) \in [\delta_b, 1 - \delta_g]$ so we need only consider $s \in (0, 1)$. Next, by Proposition 1(ii), $\eta(D_b(-x, s; q^*)) = 1$. Therefore, by (9) we have:

$$\Pr(g|0, 1, -x, s) - s = \frac{(1-s)[\eta(D_g(\ell, s; q^*)) \cdot s - s]}{[\eta(D_g(\ell, s; q^*))] \cdot s + (1-s)}.$$

If $\eta(D_g(\ell, s; q^*)) < 1$, then $\Pr(g|0, 1, -x, s) - s < 0$; that is, if some type g persons do not default, default increases the probability that a person is of type b. If $\eta(D_g(\ell, s; q^*)) = 1$, default does not provide any information about type and $\Pr(g|0, 1, -x, s) = s$.

Proposition 4. Running down assets lowers an agent's pre-type-shock score and running them up raises that score. Specifically, suppose $E_b(-x, \ell, s; q^*) = E$. (i) If $\ell \in \{0, x\}$,

[13] If this requirement is not met, then $\Pr(g|\ell', 0, \ell, s) = \frac{0}{0}$.

Bayesian updating implies $\Pr(g|-x,0,\ell,s) \leq s$. (ii) If $\ell' \in \{0,x\}$ and $\eta(E_g(\ell',-x,s;q^*)) > 0$, Bayesian updating implies $\Pr(g|\ell',0,-x,s) = 1$.

Part (i) of the proof is analogous to that of Proposition 3. Intuitively, if type b people can borrow and some type g do not borrow then taking on debt strictly increases the likelihood that the person is of type b (otherwise the score does not change). Since there is only a single level of debt in this model, this property is the model analog of taking on debt in item 4 mentioned in the introduction. Again, since all type b are borrowing, part (ii) follows since paying down debt signals the agent is of type g.

It is worth pointing out that Proposition 4 does not hold for people who have debt and choose to continue to be in debt. The next result follows from Proposition 1(ii) and (10).

Proposition 5. Maintaining debt helps raise an agent's pre-type-shock score. Specifically, if $\eta(E_g(-x,-x,s;q^*)) > 0$, Bayesian updating implies $\Pr(g|-x,0,-x,s) = 1$.

It is important to recognize that Propositions 4 and 5 refer to the function $\Pr(g|\ell',d,\ell,s)$.The impact of a person's action on s' will depend not only on how $\Pr(g|\ell',d,\ell,s)$ is affected, but also on the possibility that the person may change type by the following period. This induces "mean-reversion" in the Ψ function.[14] This is consistent with Musto's finding that (p.735) "FICO scores are mean-reverting."

$$\begin{aligned} \Pr(g|\ell',d,\ell,s) < \frac{\delta_b}{\delta_g+\delta_b} &\Longrightarrow \Psi(\ell',d,\ell,s) > \Pr(g|\ell',d,\ell,s) \\ \text{and } \Pr(g|\ell',d,\ell,s) > \frac{\delta_b}{\delta_g+\delta_b} &\Longrightarrow \Psi(\ell',d,\ell,s) < \Pr(g|\ell',d,\ell,s). \end{aligned}$$

This feature makes it important to distinguish between $\Pr(g|\ell',d,\ell,s)$ and $\Psi(\ell',d,\ell,s)$ in discussing the impact of current actions on a person's type score. In particular, if the person's

[14] To see this, let $\varphi = \Pr(g|\ell',d,\ell,s)$ and simply manipulate (11):

$$\begin{aligned} \Psi &= (1-\delta_g)\varphi + \delta_b\,[1-\varphi] \\ &\Longleftrightarrow \Psi - \varphi = -(\delta_g+\delta_b)\varphi + \delta_b. \end{aligned}$$

Hence

$$\Psi - \varphi > 0 \Longleftrightarrow \frac{\delta_b}{(\delta_g+\delta_b)} > \varphi.$$

current period score is low it is possible for his next period score to *rise* after default. For example, consider a person with $s = \delta_b$. If this person defaults his $\Pr(g|0, 1, \ell, \delta_b)$ will be less than δ_b but positive (provided $\eta(D_g(\ell, \delta_b; q^*)) > 0$). Since $\Pr(g|\ell', d, \ell, s)$ is positive, it follows from the definition of the Ψ function that $\Psi^*(0, 1, -x, \delta_b) > \delta_b = s$! Basically, when a person's score is low the mean reverting force can end up being the dominant one and can raise a person's score in the period following default.

Propositions 1-5 exhaust what we can say analytically about the nature of the equilibrium. Notably, these propositions do not say anything about item 1 in the introduction. By Proposition 1(ii), the probability of default on a loan made to a person with (updated) score σ is

$$p(-x, \sigma; q^*) = \eta(D_g(-x, \sigma; q^*)) \cdot \sigma + (1 - \sigma).$$

If $\eta(D_g) < 1$ then, holding fixed $\eta(D_g)$, it is clear that a higher σ is associated with a lower probability of default. However, $\eta(D_g)$ is not in general independent of σ. Therefore, unless we can characterize the behavior of type g people we cannot prove that item 1 is true in this model. But the behavior of type g people is hard to characterize because unlike type b, their decisions are affected by (q^*, Ψ^*) which itself is determined by their actions. Thus, we turn to exploring the behavior of type g people numerically.

At this stage, we have not calibrated the model. Here we simply take $\beta_g = 0.75$, $r = (0.8/0.75) - 1$, $\delta_g = 0.1$, $\delta_b = 0.5$, $x = 0.5$, and consider a uniform distribution over a 61 element grid of earnings given by $\{10^{-10}, 0.35, 0.7, 1.05, \ldots, 21.0\}$.

We will start by describing the equilibrium Ψ function. It is useful to start here because what people reveal about themselves by their actions will be key to understanding how type g individuals behave. Given our assumptions about δ_g and δ_b in A2 it follows from (11) that $\Psi^* \in [0.5, 0.9]$ so that in a stationary equilibrium a person's beginning of period score $s \in [0.5, 0.9]$.

Equilibrium Ψ Function $\Psi^*(\ell', d, \ell, s)$

- $\ell = -x$.

  1. $\Psi^*(0, 1, -x, s) < s$ for $s > 0.69$

2. $\Psi^*(-x, 0, -x, s) = (1 - \delta_g)s + \delta_b(1 - s)$

3. $\Psi^*(0, 0, -x, s) = (1 - \delta_g)s + \delta_b(1 - s)$

4. $\Psi^*(x, 0, -x, s) = 1 - \delta_g$

- $\ell = 0$.

1. $\Psi^*(-x, 0, 0, s) < s$ for $s > 0.51$

2. $\Psi^*(0, 0, 0, s) = 1 - \delta_g$

3. $\Psi^*(x, 0, 0, s) = 1 - \delta_g$

- $\ell = x$.

1. $\Psi^*(-x, 0, x, s) = \delta_b$

2. $\Psi^*(0, 0, x, s) = 1 - \delta_g$

3. $\Psi^*(x, 0, x, s) = 1 - \delta_g$

Consider first the case where the individual is in debt ($\ell = -x$). While we know from Proposition 1(ii) that type b always default, in this particular equilibrium, for every s some type g default (those with low earnings) and the remaining type g choose $d = 0$ and $\ell' = x$ (those with high earnings). In fact, in this equilibrium the default decision of a type g agent depends only on earnings and is independent of s. Figure 2 plots $\eta(D_g(-x, s; q^*)) = 0.41$ for all $s \in [0.5, 0.9]$. The fact that for every s some type g choose to pay back their loan makes equilibrium loan prices positive, i.e., $q^*(-x, \sigma) > 0$ for $\sigma \in [0.5, 0.9]$. If an individual defaults then $\Pr(g|0, 1, \ell, s) < s$ in accordance with the discussion following Proposition 5 since default strictly reduces the probability of an individual being of type g. Note however that because of the mean reversion in Ψ the person's score at the start of next period, $\Psi^*(0, 1, -x, s)$, is lower than s only if $s > 0.69$. See Figure 3 which plots $\Psi^*(0, 1, -x, s)$ against a 45° line. Next, if an individual chooses $\ell' = x$ his $\Pr(g|\ell', 0, -x, s)$ rises to 1 because only type g people take this action. Finally, in this equilibrium a type g person with debt never chooses $\ell' \in \{-x, 0\}$ so that $\eta(E_g(\ell', -x, s; q^*)) = 0$ for $\ell' \in \{-x, 0\}$. For these actions, off-the-equilibrium-path beliefs are assigned according to the rule specified in assumption A4.

Consider next the case where the individual does not have any assets ($\ell = 0$). In this particular equilibrium, for every s all type b borrow and for every s most type g choose $\ell' = x$, some choose $\ell' = 0$, and some with very low earnings choose $\ell' = -x$. Then, in accordance with the discussion following Proposition 4, we know that $\Pr(g| -x, 0, 0, s) < s$. Choosing any other action ($\ell' \in \{0, x\}$) reveals the person to be of type g and raises his pre-type shock score to 1.

Finally, consider the case where the individual has assets ($\ell = x$). In this particular equilibrium, for every s all type b borrow and for every s most type g choose $\ell' = x$ and some choose $\ell' = 0$. If the individual chooses $\ell' \in \{0, x\}$, then his φ will rise to 1 since type b people never choose $\ell' \in \{0, x\}$.

Next we plot the equilibrium q function in Figure 4. Since $q^*(-x, \sigma) = (1 + r)^{-1} \times [1 - \eta(D_g(-x, \sigma; q^*)] \cdot \sigma$ and $\eta(D_g(-x, \sigma; q^*))$ is independent of σ in this equilibrium, the price rises linearly throughout $[0.5, 0.9]$. It's worth emphasizing that the loan price depends on the person's updated score, and therefore depends on his ℓ and s, a point that is not evident in Figure 4. This is true because the price paid for a loan is given by $q^*(-x, \Psi^*(-x, 0, \ell, s))$. This is graphed in Figure 5 and for completeness we plot $\Psi^*(-x, 0, \ell, s)$ in Figure 6. This feature of the environment can have surprising implications. For instance, the price offered on a loan is strictly *decreasing* in the person's initial asset position. As discussed earlier, type g people with $\ell = x$ never borrow regardless of s and so the price of a loan offered to someone *with* assets is lowest because only type b borrow. But some type g people with $\ell = 0$ borrow regardless of s so the price of a loan offered to people *without* assets is higher. Finally, the price on a loan offered to someone with debt is higher than the other two because in this off-the-equilibrium-path case $\Psi^*(-x, 0, -x, s) = (1 - \delta_a)s + \delta_b(1 - s)$. Basically, the market views running down one's assets as a signal that a person is more likely to be of type b – hence the interest rate offered to people who take such actions is correspondingly high.

It may be possible to find regions where the price could decline. This could happen if $D_g(-x, \sigma; q^*)$ is not independent of σ and $D_g(-x, \sigma_1; q^*) \subset D_g(-x, \sigma_2; q^*)$ for $\sigma_1 < \sigma_2$.

The reason for this is that a type g person's opportunity cost of default is *inversely* related to his score. To understand this point, it is important to remember that the *only* reason anyone cares about his score is the effect the score has on the terms of credit. We can determine part of his opportunity cost by noting what happens to his score if he defaults versus if he repays. To see this, consider Figure 7, which plots the difference in the loan price he faces tomorrow if he chooses $\ell' = x$ today relative to the price he faces if he defaults today: $q(-x, \Psi^*(x, 0, -x, s)) - q(-x, \Psi^*(0, 1, -x, s))$. As is evident, the difference is generally declining in s.

Put somewhat differently, a person's benefit from investing in reputation is high when the person's current reputation is low. As the person's reputation improves, the value of enhancing it further declines and the person repays less frequently. Returning to Figure 3, the enhancement to one's reputation from not defaulting is given by the difference between one's posterior if he doesn't default, which in this equilibrium is $\Psi^*(x, 0, -x, s) = 1 - \delta_g$, and one's posterior if he does default $\Psi^*(0, 1, -x, s)$.

Finally, in Figure 8 we plot the invariant distribution of agents across type scores and asset holdings. As can be seen, a little under 12% of the population are borrowers and have low to medium type scores. Most people with high scores either hold positive assets (66%) or no assets (14%). Finally, there are some agents with no assets but medium type scores (a little over 8%).

At this point it is useful to summarize the extent to which a *type score* has properties similar to a *credit score* in the equilibrium studied so far. A type score is like a credit score in the following regard:

- The relationship between a type score and loan price is positive.
- Default reduces type-score.
- For people without assets, taking on debt reduces type-score.

In closing, one final comment is order. So far we have studied cases where one type is entirely myopic. This is, of course, a rather stringent assumption but we may view it as an approximation to the case where type b people aren't totally myopic but are greatly more impatient than type g. Specifically, we have verified that we can get an equilibrium with β_b positive but small ($\beta_b = 0.10$) in which type b people continue to behave as they did under complete myopia – they default if they have debt and borrow otherwise.

5 Legal Restrictions on the Length of Credit History

As mentioned in the introduction, the Fair Credit Reporting Act requires credit bureaus to exclude a bankruptcy filing from credit reports after 10 years (and all other adverse items after 7 years). In terms of our model, this is a restriction on T. In this section, we consider the implications of setting $T = 1$ in our environment for equilibrium behavior and welfare. When $T = 1$, there are 4 possible $(\ell_t, h^t(1)) = (\ell_t, d_{t-1})$ histories at any date t. This set is smaller than $\#(\{0,1\})^{\#(L)} = 2^3$ since default decisions imply certain asset choices (i.e. $d_{t-1} = 1$ implies $\ell_t = 0$).

While all the results about behavior of $i = b$ types holds in this environment, it is still the case that it is hard to prove things about the behavior of type g people since their decisions are once again affected by (q^*, Ψ^*) which itself is determined by their actions. Thus, we again turn to exploring the behavior of type g people numerically. We use the identical parameter values as the previous section.

The surprising result is that even with this high degree of restriction of credit recording, agents behave as if in the $T = \infty$ case. This is somewhat less surprising when one realizes that there is a large degree of discreteness in the action space. All equilibrium objects are in fact identical (prices, the invariant distribution). Specifically, the equilibrium posteriors are given by:

Equilibrium Ψ Function $\Psi^*(\ell', d, \ell, h(1))$

- $\ell = -x, d_{-1} = 0 \Longrightarrow \Pr(g|-x, 0) = 0.51$

    1. $\Psi^*(0, 1, -x, 0) = 0.62 > \Pr(g|-x, 0)$
    2. $\Psi^*(-x, 0, -x, 0) = (1 - \delta_g)\Pr(g|-x, 0) + \delta_b(1 - \Pr(g|-x, 0))$
    3. $\Psi^*(0, 0, -x, 0) = (1 - \delta_g)\Pr(g|-x, 0) + \delta_b(1 - \Pr(g|-x, 0))$

4. $\Psi^*(x, 0, -x, 0) = 1 - \delta_g$

- $\ell = 0$, $d_{-1} = 1 \Longrightarrow \Pr(g|0, 1) = 0.62$.

1. $\Psi^*(-x, 0, 0, 1) = 0.51$
2. $\Psi^*(0, 0, 0, 1) = 1 - \delta_g$
3. $\Psi^*(x, 0, 0, 1) = 1 - \delta_g$

- $\ell = 0$, $d_{-1} = 0 \Longrightarrow \Pr(g|0, 0) = 1 - \delta_g$.

1. $\Psi^*(-x, 0, 0, 0) = 0.55$
2. $\Psi^*(0, 0, 0, 0) = 1 - \delta_g$
3. $\Psi^*(x, 0, 0, 0) = 1 - \delta_g$

- $\ell = x$, $d_{-1} = 0 \Longrightarrow \Pr(g|x, 0) = 1 - \delta_g$.

1. $\Psi^*(-x, 0, x, 0) = \delta_b$
2. $\Psi^*(0, 0, x, 0) = 1 - \delta_g$
3. $\Psi^*(x, 0, x, 0) = 1 - \delta_g$

Using these figures, we can compute the changes in average score following a removal of the bankruptcy flag from one's record to compare it to Figure 1 in Musto. The credit score with default is given by $\Pr(g|0, 1) = 0.62$. The average score after the default leaves the credit record (next period for $T = 1$) is given by $\sum_{i,(\ell',0)\supset(0,1)} \eta(E_i(\ell', 0, 0, 1))\Psi^*(\ell', 0, 0, 1) = 0.75$. In this case, the jump in score is 21%. Musto found that for individuals in the highest pre-default quintile of credit scores, they jumped ahead of 19% of households after the score left their record. This provides an (admittedly highly stylistic) example where ex-ante welfare is identical for the $T = 1$ and $T = \infty$ equilibria despite large movements in credit scores associated with the legally imposed removal of adverse events from individual credit histories.

6 Conclusion

It is well known that lenders use credit scores to regulate the extension of consumer credit. People with high scores are offered credit on more favorable terms. People who default on their loans experience a decline in their scores and, therefore, lose access to credit on favorable terms. People who run up debt also experience a decline in their credit scores and have to pay higher interest rates on new loans. While credit scores play an important role in the allocation of consumer credit they have not been adequately studied in the consumption smoothing literature. This paper attempts to remedy this gap.

We described an economic environment in which a credit score – i.e., a person's index of creditworthiness – could be given a precise meaning. Specifically, the two types of people in our environment differed with respect to their rates of time preference. Lenders could not directly observe a person's type but made probabilistic assessments of it based on the person's financial history. We referred to the probability that a person is of a given type as a person's *type score*. We showed, via an example, that if one of the types discounted the future heavily then the probability that a person is of the *patient* type (i.e., the type that does *not* discount the future heavily) behaves like a *credit score*. That is (i) the terms of credit depend favorably on the probability of a person being of the patient type, (ii) this probability declines if a person defaults on a loan and (iii) this probability declines also when a person takes on new debt.The paper also provided an (admittedly highly stylistic) example where ex-ante welfare is identical for equilibria where records face legal restrictions on the time that adverse events can remain on individual credit histories vis-a-vis the unrestricted case, despite what might seem like inefficiently large movements in credit scores after the flag is removed.

Many questions remain. First, how robust is our theory of credit scores to a richer asset space or endowment process? Richness of the asset space has important implications for signalling. Second, will type scores behave like credit scores for other type differences - such as attitude to risk or occupational risk - between people? Third, do high credit scores encourage the "good risks" (the patient types in this model) to default more frequently in the data? Because maintaining a good reputation is costly, our theory implied that the "good risks" have a greater incentive to default when their scores are high. This implication seems integral to the reputation-based theory of credit scores developed in this paper. Finally, when do credit scores fail to be a "sufficient statistic" in the updating function?

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Figure 2: Default Probability of Type g

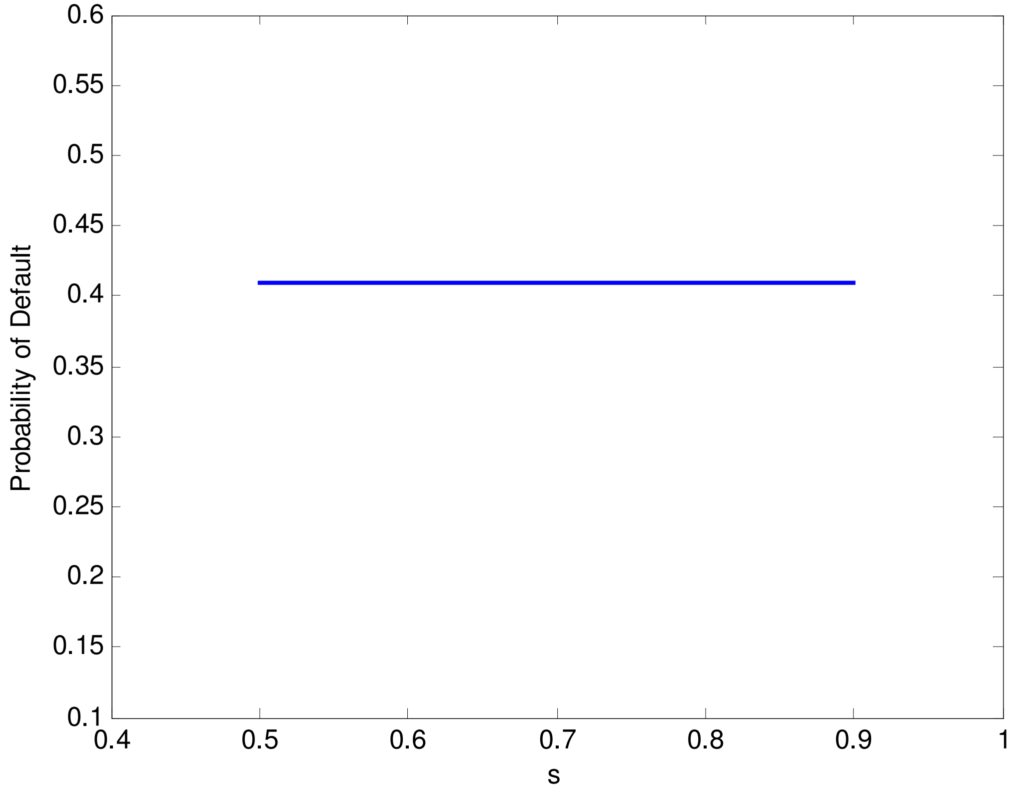


Figure 3: Posterior for Defaulters

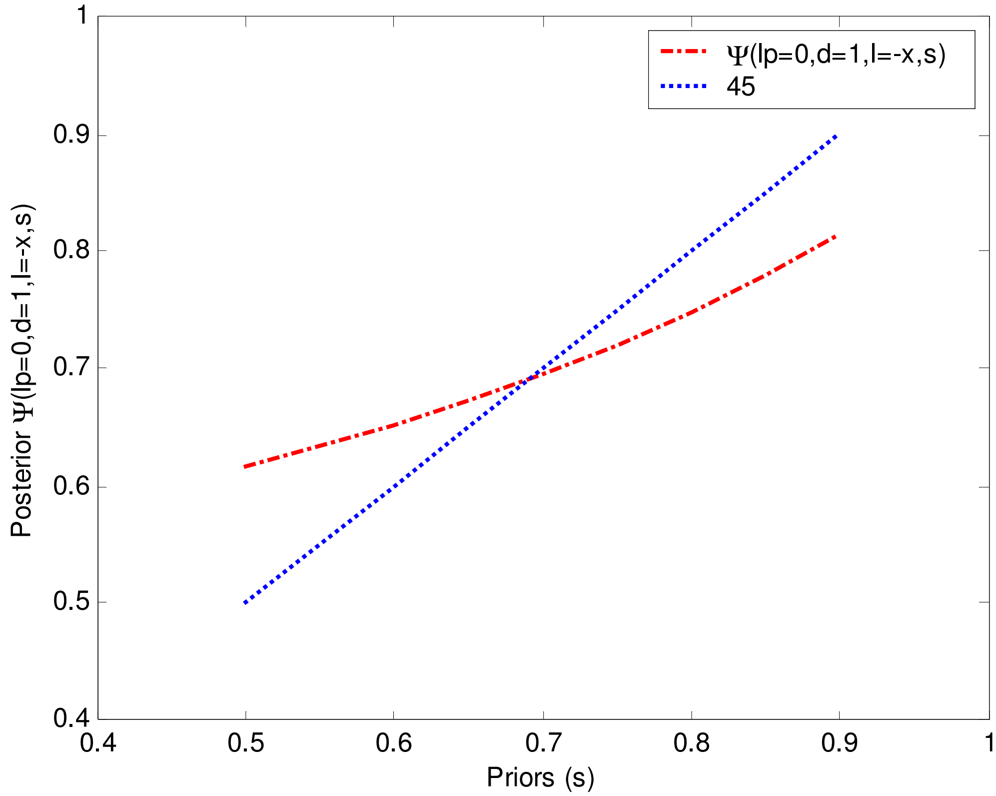
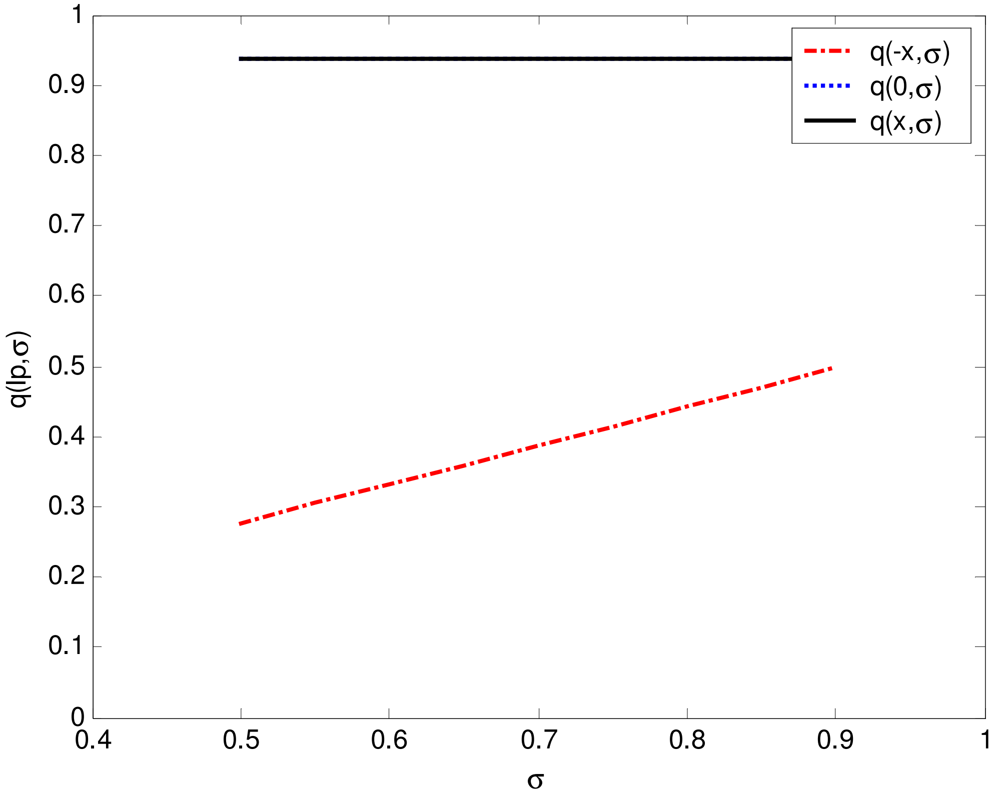


Figure 4: Equilibrium q Function



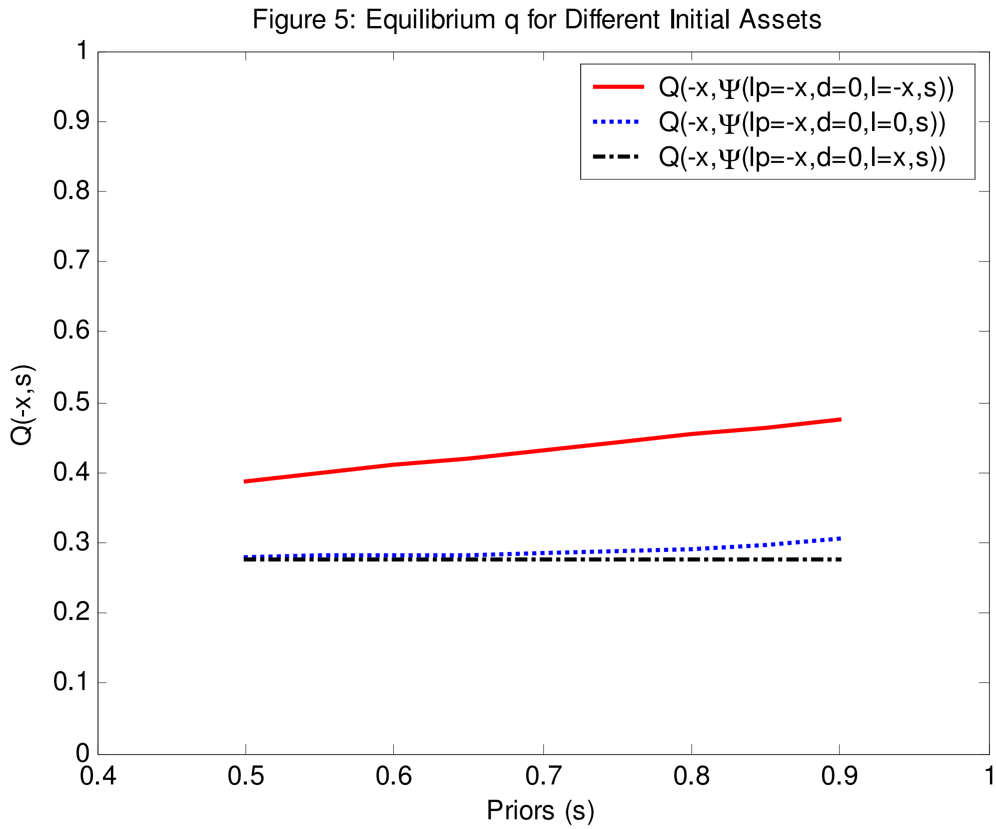


Figure 5: Equilibrium q for Different Initial Assets
Q(-x,Ψ(lp=-x,d=0,l=-x,s))
Q(-x,Ψ(lp=-x,d=0,l=0,s))
Q(-x,Ψ(lp=-x,d=0,l=x,s))
Q(-x,s)
Priors (s)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 6: Posterior for Borrowers for Different Initial Assets

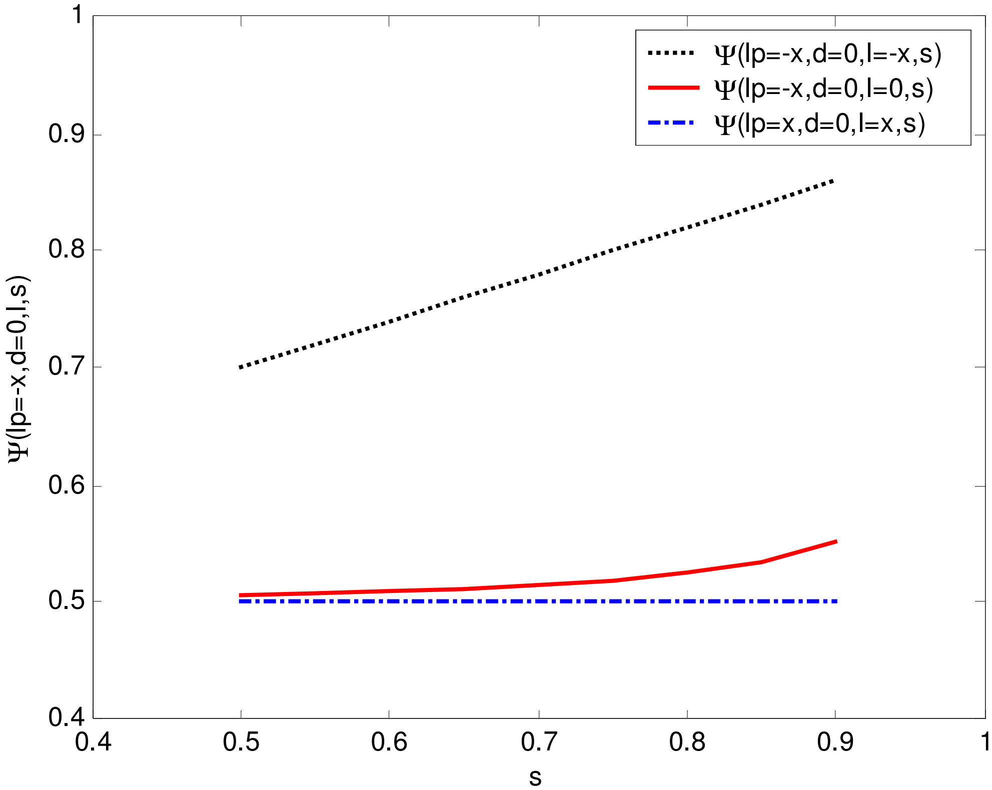


Figure 7: Partial Value of a Reputation

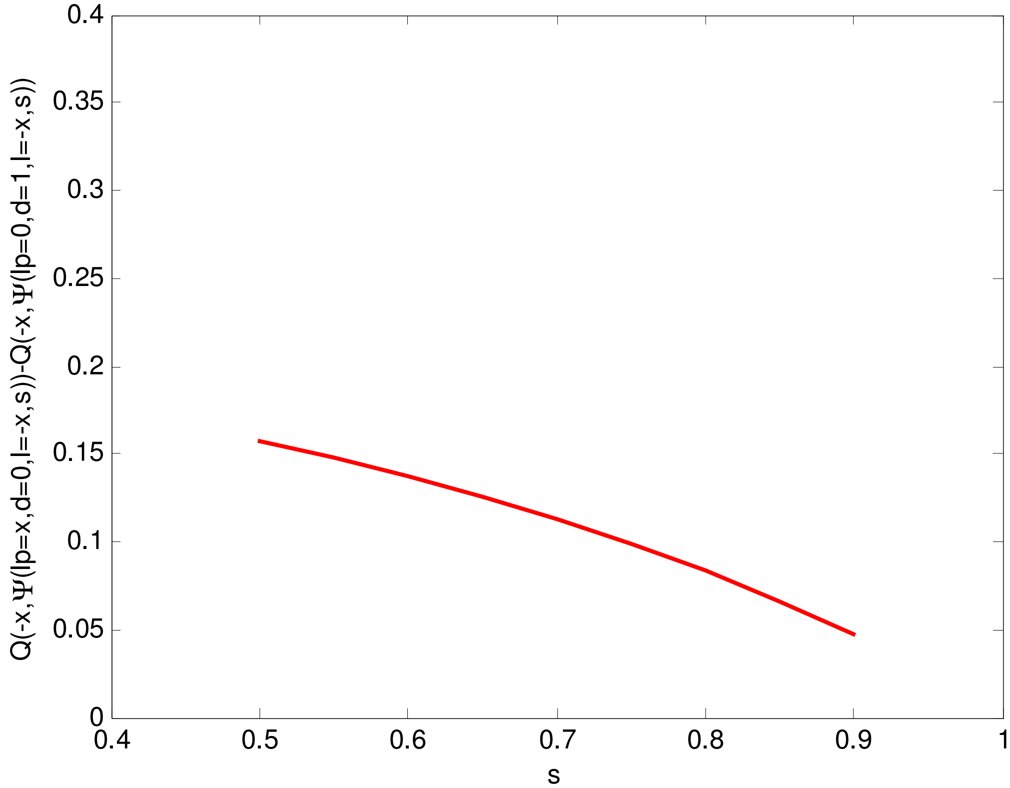


Figure 8: Fraction of Agents Across type Scores at the Stationary Distribution