Measuring the Welfare Costs of Inflation in a Life-cycle Model*

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Abstract

The welfare costs of inflation are analyzed in a life-cycle model. In the benchmark model, money is held to satisfy a cash-in-advance constraint. An inflation rate over 200% per annum maximizes lifetime utility because it leads to better smoothing of utility over the life-cycle, in essence because inflation taxes rich, old agents and makes net transfers to poor, young ones. This version of the model suggests that high inflation in developing countries may be part of an optimum policy. Introducing other taxes into the model gives the government alternative sources of revenue and reduces the optimal inflation rate to something close to the Friedman rule. Allowing some goods to be purchased with costly credit also reduces the optimal (lifetime utility-maximizing) inflation rate. However, if seigniorage revenue can be used to lower these other taxes, high inflation is again optimal. Finally, the transitional dynamics following a disinflation are traced out for the costly credit version of the model with U.S. taxes. This policy leads to a Pareto superior allocation.

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1 Introduction

Measured costs of inflation seem small compared to the costs of reducing inflation. Using Bailey (1956) welfare triangle analysis, Fischer (1981) found that a 10 percent inflation leads to a loss of between 0.5 percent and 0.8 percent of output. Measures from general equilibrium models are similar; by way of example, Cooley and Hansen (1989) found that a 10 percent inflation yields a cost of 0.4 percent of output. While these costs are 'forever,' the short-term costs of reducing inflation can be considerable. In the early 1980s, the U.S. Federal Reserve reduced inflation from 14 percent to 4 percent, but at the cost of two major recessions during which the unemployment rate rose from 6 percent to nearly 11 percent.

If the benefits of reduced inflation are small relative to the costs of achieving lower inflation, why do economists and policymakers worry so much about inflation? One answer is that there is something missing from the economic models used to assess the costs of inflation, and that the costs of inflation are in fact much larger than these models suggest. This paper explores whether adding a particular form of heterogeneity, age, is important in measuring the costs of inflation. The model benchmark presented below is essentially a life-cycle version of Cooley and Hansen (1989). Individuals hold money to satisfy a cash-in-advance constraint on their purchases of consumption goods. As in Cooley and Hansen, new money balances are injected via a lump sum transfer.

The key life-cycle features of the model are as follows. First, individuals live exactly T periods; there is no random death as in Ríos-Rull (1996). Second, individuals face a hump-shaped human capital. This feature is included so as to match up with evidence on real wages over the life-cycle. Third, individuals start life with no capital (real assets), and must end their lives with non-negative capital holdings. Since there is no bequest motive, individuals will, in fact, end life with no capital. Between birth and death, individual are unconstrained with respect to their capital holdings, and so *may* go into debt if they wish. Finally, individual start life with some real money balances. This feature is included so that there is not a 'trivial' reason for inflation to be welfare-improving: If individuals have no initial real balances, then if there is no lump-sum transfer of money balances, the cash-in-advance constraint implies that individuals would be unable to purchase consumption in the first period of their lives. So that money is not simply created "out of thin air," it is assumed that agents end life with the same level of real balances with which they started.

The principal findings are as follows. In the benchmark model in which seigniorage is the only source of government revenue, annual inflation rates around 210 percent maximize steady state lifetime utility. A number of other welfare metrics are considered; for the most part, they confirm that finding that high inflation is welfare-maximizing; see Section 4 for details. Why this surprising result? Under the benchmark calibration, the real interest rate is positive, and the Euler equation governing asset accumulation then implies that individual consumption profiles rise with age. Owing to the cash-in-advance constraint, real money balances also increase with age. Since new money enters the economy via lump-sum transfers that are independent of age, the young receive more in transfer than they pay out in inflation taxes. The reverse is true for the old. In effect, inflation leads to a transfer of resources from the old to the young – a sort of reverse social security system. These transfers tend to flatten the utility-age profile in a way that agents find desirable, at least from a lifetime utility point of view. The results for the benchmark model may be most applicable to developing countries that do not have access to sources of government revenue apart from seigniorage.

In an alternative calibration, taxes on labor and capital income, as well as on consumption purchases, are set at levels seen in the United States. In the model, the proceeds of these "fiscal" taxes are lump-sum rebated to households. The first result of interest is that, holding the other taxes fixed, the lifetime utility-maximizing inflation rate is essentially the Friedman rule (that is, the negative of the real rate of interest). Alternatively, suppose that as the inflation tax is varied, *one* of the other taxes is adjusted so that total government revenue is unchanged. Once more, fairly high inflation rates maximize lifetime utility. Again, other welfare metrics give similar results.

Perhaps the cash-in-advance constraint is too rigid a payment technology. To investigate this possibility, costly credit is introduced allowing for an endogenous determination of "cash" and "credit" goods; see Prescott (1987); Schreft (1992); Ireland (1994). Specifically, households consume a continuum of goods and must choose which goods to purchase with cash, and which with credit. The cost of using credit increases with "distance" from the household's "home" market. As in Dotsey and Ireland (1996), households will use credit in those markets close to its home, and cash in all other markets. When there are no other taxes, the optimal money growth (inflation) rate is around 5% per annum. The only case in which high inflation maximizes lifetime utility is when U.S. tax rates are in place and seigniorage revenue is used to reduce the labor income tax; in this case, the optimizing inflation rate is in the neighborhood of 70% per year.

A final experiment considers the transition from one money growth rate to another. In the costly credit version of the model with U.S. tax rates, reducing the money growth rate to zero maximizes lifetime utility in steady state. As shown by XXX, a policy that generates a welfare gain across steady states may not generate a welfare gain after accounting for the transition between these steady states. As shown in section 7, all generations are made better off by switching to zero money growth.

There are two other notable papers that suggest the importance of heterogeneity in assessing the costs of inflation: İmrohoroğlu (1992) and Erosa and Ventura (2002). The environment considered by İmrohoroğlu is one in which individuals hold money balances as a buffer against uninsurable income shocks (spells of unemployment). She finds that Bailey welfare triangles understate the costs of inflation by as much as a factor of 3. Erosa and Ventura's model has two types of agents, rich and poor. There is also a technology to allow for credit transactions, and individuals choose which range of goods to purchase with credit, and which with cash. They calibrate their model to match observations for the United States which implies that the poor purchase a greater proportion of their goods with cash, and so experience a greater burden of the inflation tax.

The remainder of the paper is organized as follows. The model is presented in Section 2, and calibrated in Section 3. Welfare results can be found in Section 4, and transition dynamics are presented in Section 7. Section 8 concludes.

2 The Economic Environment

The model setup is more general than is necessary for the benchmark (cash-in-advance) model in order to accommodate later extensions. To later allow for an endogenous cash-credit good distinction, it is assumed that at each date t, a continuum of markets operate on the circumference of a circle; the length of the circumference is 2. Each location along the circumference is occupied by a continuum of goods producing firms, financial intermediaries, and households of each cohort. Enough symmetry is assumed that the analysis can focus on a representative firm, a representative financial intermediary, and a representative household of each cohort.

2.1 Households

At each date t is 'born' a unit mass of identical individuals. Each individual will experience exactly T periods of 'economic life.' The term *economic life* is used to refer to individuals who have entered the labor force and so participate in economic activity. Early childhood development and education are not considered here. Altruism between parents and their offspring is also suppressed. In order to analyze fairly realistic life-cycle dynamics, the lifespan T will be long. In the calibration section, a period will be specified as one quarter, and T will be set to 220, corresponding to 55 years of economic life.

Since individuals differ only as to their date of birth, individual-specific variables need to specify an individual's date of birth, and their current period of life. By way of example, n_t^i denotes the hours of work of an individual born at date t who is in their i^{th} period of life. In *calendar time*, these hours are supplied occurs at date $t+i$.

In each period of life, an individual has a taste for variety with respect to consumption goods. In particular, a household at location j' cares about the range of goods, $[j', \text{mod}(j'+1,2)]$. In the presentation below, attention will be focused on the household at location 0 which consumes goods on the interval $j \in [0,1]$, denoted $\{c_t^i(j)\}_{j=0}^1$. These consumption goods are aggregated according to a Leontief technology,

$$c_t^i = \inf_{j\in[0,1]} \left\{ c_t^i(j) \right\}. \quad (1)$$

Use of this aggregator is common in the costly credit literature; see, for example, Prescott (1987). An implication of equation (1) is that the household will choose to consume the *same* quantity of all goods.

Preferences for a member of generation t (that is, someone born at t) are given by:

$$E_t \sum_{i=0}^{T-1} \beta^i U(c_t^i, \ell_t^i), \quad \beta > 0. \quad (2)$$

The period utility function, U, is defined over consumption, c_t^i, and leisure, ℓ_t^i, and is assumed to possess standard properties. Future utility is discounted at the rate β.

Households face a number of constraints. To start, the nominal budget constraint is

$$\begin{aligned} P_{t+i}(1+\tau_c)\int_0^1 c_t^i(j)\,dj + P_{t+i}[k_t^{i+1}-(1-\delta)k_t^i] + \int_0^1 I_t^i(j)Q_{t+i}(j)\,dj + M_t^{i+1} = \\ (1-\tau_n)W_{t+i}h^i n_t^i + (1-\tau_k)R_{t+i}k_t^i + M_t^i + X_{t+i}^M + X_{t+i}^R, \quad i=0,\ldots,T-1. \end{aligned} \tag{3}$$

The right-hand side gives sources of funds. The first term is after-tax labor income; the tax rate on labor income is τ_n. The variable h^i, denoting the 'human capital' of an individual aged i, is included in the model so that the life-cycle profile of labor earnings resembles that observed in the U.S. data. The human capital profile is exogenous and known to an individual from birth. At age i, an individual combines human capital with time supplied to the market, n_t^i, earning a pre-tax wage W_{t+i} on human capital-augmented hours. The *observed* pre-tax wage for an individual aged i will be $W_{t+i}h^i$.

The second term on the right-hand side of equation (3) is after-tax capital income. The household starts period $t+i$ with real assets (or capital) k_t^i. It rents this capital for a nominal rental payment of R_{t+i} which is taxed at the rate τ_k.

The household also starts period $t+i$ with money balances, M_t^i. It receives two lump-sum transfers from the government: a purely monetary transfer, X_{t+i}^M, and a 'real' transfer which when expressed in nominal terms is X_{t+i}^R.

The left-hand side of equation (3) represents uses of funds. The price level at $t+i$ is P_{t+i}. The household purchases the range of consumption goods $\{c_t^i(j)\}_{j=0}^1$; these purchases are taxed at the rate τ_c. The household also expends funds on investing in capital, given by the second term on the left-hand side. Here, δ is the depreciation rate of capital. Negative investment is permitted and corresponds to a change in ownership in capital goods.

The household can use either cash or credit to purchase its consumption goods. If the household uses credit in market j, it incurs a lump-sum cost of $Q_{t+i}(j)$. The indicator function $I_t^i(j)$ equals 1 if the household chooses to purchase good j with credit, and equals 0 if it buys good j with cash. Consequently, the integral on the left-hand side of (3) represents to total outlay on credit services.

Finally, the household departs period $t+i$ with nominal money balances M_t^{i+1}.

The household faces the following cash-in-advance constraint:

$$(1+\tau_c)P_{t+i}\int_0^1 [1-I_t^i(j)]c_t^i(j)\,dj \le M_t^i + X_{t+i}^M, \quad i=0,\ldots,T-1. \tag{4}$$

Recalling that $I_t^i(j)=0$ for goods purchased with cash, the term on the left-hand side of the cash-in-advance constraint is the value of consumption purchased with money. These purchases are constrained by the sum of beginning-of-period money balances and the monetary lump-sum payment from the government, X_{t+i}^M.

The time endowment of an individual is normalized to unity; thus, labor and leisure must satisfy

$$\ell_t^i + n_t^i \le 1, \quad i=0,\ldots,T-1. \tag{5}$$

The only constraints that will be placed on capital holdings are that individuals start life with no capital, and they must end life with non-negative capital:

$$k_t^0 = 0, \quad k_t^T \geq 0. \tag{6}$$

$k_t^{i+1} < 0$ would mean that at age i a member of generation t went into debt.

The final two constrains are on money holdings. It is assumed that individuals start life with real balances, $\overline{m} > 0$, and must end life with the same level of money balances:

$$\frac{M_t^0}{P_{t-1}} = \overline{m}, \quad \frac{M_t^T}{P_{t+T-1}} \geq \overline{m}. \tag{7}$$

If $\overline{m} = 0$, then the cash-in-advance constraint, (4), would imply that positive first period of life consumption is feasible only if the transfer, X_t^M, is strictly positive. This transfer can be strictly positive only if money growth, and so inflation, is strictly positive. Absent positive initial money balances there would be a trivial reason for positive inflation to dominate the Friedman rule (deflate at the real interest rate) since this would be the only way for individuals to enjoy positive first period consumption.

The initial real balances could be thought of as a transfer made from a parent to an offspring, or as coming from earnings of a child prior to entering the labor force. The constraint on end-of-life real balances is imposed to conserve on aggregate private money balances (money balances are not being magically introduced through the endowment of the just-born).

Most of the constraints faced by an individual will be satisfied owing to nonsatiation. The cash-in-advance constraint will bind if inflation is sufficiently high to ensure that the return on capital exceeds that on money (so that no one would hold money as a store of value). It is assumed that this condition is, in fact, satisfied.

2.2 Financial Intermediaries

For the household to use credit in market j, it must purchase the right to use credit in that market at the price $Q_t(j)$. This cost might be thought of as that associated with verifying the identity of the household in market j. An intermediary located in market j requires $\gamma(j)$ units of labor to identify the household. This labor input increases with distance: $\gamma'(j) > 0$. The nominal cost to the intermediary is $W_t\gamma(j)$. Owing to competition among the financial intermediaries in market j, in a competitive equilibrium each earns zero profits; thus,

$$Q_t(j) = W_t\gamma(j). \tag{8}$$

2.3 Goods Producing Firms

Firms face a sequence of static problems. Each period, the typical firm rents capital, K_t, and hires effective units of labor (that is, human capital-augmented labor), N_t^g, to maximize real profits,

$$F(K_t, N_t^g; z_t) - r_t K_t - w_t N_t^g, \tag{9}$$

where F is a standard constant-returns-to-scale production function and z_t is a shock to technology. Since F is constant-returns-to-scale, in equilibrium firms will earn zero profits. Consequently, there was no need to tackle the tricky issue of firm ownership when specifying the households' problems.

2.4 Government

Each period, the government levies a set of taxes and creates (or destroys) money balances subject to its budget constraint. The monetary transfer is

$$X_t^M = \frac{(\mu_t - 1)M_t}{T} \tag{10}$$

where μ_t is the gross growth rate of money, M_t is aggregate money balances, and T is the number of generations alive at t. Consequently, each generation receives its 'share' of new money balances.

The transfer from the fiscal authority is

$$X_t^R = \frac{\tau_c P_t C_t + \tau_n W_t N_t^e + \tau_k R_t K_t}{T} \tag{11}$$

where C_t denotes aggregate consumption, N_t^e is the total supply of labor (measured in efficiency units), and K_t is the aggregate capital stock; these variables are defined below in subsection 2.6. Notice that the government runs a balanced budget each period; it does not issue debt.

2.5 Analysis: Cash or Credit?

In choosing whether to use cash or credit to purchase a particular good, a household balances two different costs. In general, for the household to use cash, it must have acquired this money in the previous period which entails an opportunity cost: the household could, instead, have acquired more of the real asset which presumably pays a higher rate of return than money. While using credit does not require 'advanced planning,' it does involve the direct cost $Q_t(j)$. Clearly, the household will choose to use cash when it is relatively cheap to do so, else it will use credit.

Recall that the price of credit is given by equation (8), or in real terms,

$$q_t(j) = w_t \gamma(j) \tag{12}$$

where $q_t(j) \equiv Q_t(j)/P_t$ and $w_t \equiv W_t/P_t$. A straight cash-in-advance version of the model is a special case in which $\gamma(j)$ is so high that using credit is prohibitively expensive. Suppose that $\gamma(0) = 0$ and $\lim_{j \to 1} \gamma(j) = \infty$. That is to say, the labor input required to identify the household in its 'home market' is zero while it requires an infinite input at the farthest market that the household shops in. Then both cash and credit will be used. Furthermore, since $\gamma'(j) > 0$, it follows that there is a cutoff, s_t^i, such that credit is used for goods $j \in [0, s_t^i]$ while cash is used for goods $j \in (s_t^i, 1]$; see Dotsey and Ireland (1996). Consequently, the choice of $\{I_t^i(j)\}_{j=0}^1$ is simplified greatly. The simpler problem is presented in the Appendix along with first-order conditions and a conversion to real magnitudes.

A feature of the fixed cost nature of credit services is that rich agents are more willing to incur the cost of using credit; see Erosa and Ventura (2002). In the current environment, it is the older agents who are rich, and it is they who should use credit more frequently.

2.6 Competitive Equilibrium

A competitive equilibrium for this economy is defined in the usual way:

(1) Each member of cohort t chooses contingency plans for consumption, hours of work, capital and money holdings, so as to maximize lifetime utility taking as given the process generating prices and the evolution of the aggregate state.

(2) Firms maximize period-by-period profits taking as given prices.

(3) The government satisfies its budget constraint.

(4) Markets clear:

$$K_t = \sum_{i=0}^{T-1} k_{t-i}^i, \tag{13}$$

$$N_t^e = \sum_{i=0}^{T-1} h^i n_{t-i}^i, \tag{14}$$

$$M_{t+1} = \sum_{i=0}^{T-1} M_{t-i}^{i+1}, \tag{15}$$

$$\underbrace{\sum_{i=0}^{T-1} c_{t-i}^i}_{C_t} + \underbrace{\sum_{i=0}^{T-1} \left[k_{t-i}^{i+1} - (1-\delta)k_{t-i}^i\right]}_{I_t} = F(K_t, N_t^e; z_t) \tag{16}$$

In the market clearing conditions that the summations are across individuals alive at date t. By way of example, c_{t-i}^i is the consumption at date t of a typical member of cohort $t-i$; at time t, this individual is aged i.

3 Calibration

The length of a period is set to one quarter, and individuals live exactly 55 years; thus, $T = 220$.

The period utility function is parameterized as

$$U(c,\ell) = \frac{[c\ell^{\omega}]^{1-\sigma} - 1}{1-\sigma}.$$

In the benchmark model, the coefficient of relative risk aversion, σ, is set to unity and so $U(c,\ell) = \ln c + \omega \ln \ell$.

The goods production function is

$$F(K, N^g; z) = zK^{\alpha}(N^g)^{1-\alpha}.$$

The parameters governing production are taken from Gomme and Rupert (2003). The capital share parameter, α, is set to 0.3 and corresponds to capital's share of income from the U.S. National Income and Product Accounts. The technology shock, z_t, follows a first-order autoregressive process,

$$\ln z_t = \rho \ln z_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2).$$

Over the sample 1954–2001, Gomme and Rupert estimate $\rho = 0.9499$ and $\sigma_\varepsilon^2 = 0.008463$. The depreciation rate for capital, δ, is set to 0.02, implying an annual depreciation rate of 8%, a value that corresponds closely to the average depreciation rate implicit in the capital stock and depreciation data reported by the Bureau of Economic Analysis.

Money growth also follows a first-order autoregressive process,

$$\mu_t = \psi \mu_{t-1} + (1-\psi)\bar{\mu} + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2)$$

where $\bar{\mu}$ is the long run money growth rate. The parameters governing the behavior of money growth are estimated from U.S. data on per capita currency and M1 growth. These parameter estimates are summarized in Table 2. By either measure – currency or M1 – average money growth has been fairly low. The stochastic processes for the technology shock and money growth are assumed to be uncorrelated.

The credit technology is

$$\gamma(j) = \gamma\left(\frac{j}{1-j}\right)^{\theta}. \tag{17}$$

The benchmark model is a straight cash-in-advance model without credit as in Cooley and Hansen (1989). Setting $\gamma = \infty$ ensure that no credit is used (except, perhaps, for the home market which is of measure zero). The implications of more general formulations with credit use are explored in Section 6.

The human capital profiles are smoothed parameters based on the *Panel Study on Income Dynamics* and is taken from Gomme et al. (2004); see Figure 1.

There are two preference parameters that have yet to be assigned values: the discount factor, β, and the leisure weight, ω. These parameters are set such that in steady state: (1) the real interest rate is 7.77% per annum which is the average return on the S&P 500 over the period 1800 and 1990 as reported in Siegel (1992); and, (2) households work, on average $\frac{1}{3}$ of the time, a value consistent with time-use surveys.

The benchmark calibration is designed to correspond as closely as possible to Cooley and Hansen (1989). As a consequence, all the taxes are set to zero.

Finally, $\overline{m}$, initial and final money balances, are set to 0.1, which constrains first period consumption (see the discussion of steady state below).

The values of the parameters for the benchmark calibration are summarized in Table 1.

3.1 Steady State

The age-profiles of consumption, human capital, real money balances, hours of work, capital (real assets) and utility are graphed in Figure 1, along with the profiles corresponding to a non-monetary version of the model (in which case all goods are effectively credit goods). The non-monetary steady state is presented to verify that the introduction of money into the life-cycle model does not severely alter the nature of the model's steady state.

The human capital profiles, taken from Gomme et al. (2004), indicate that real wages rise fairly quickly, peak around age 55 ($i = 140$), then gradually decline. Hours of work peak around age 35 ($i = 60$).

The consumption profile rises monotonically with age. It is, perhaps, easiest to understand the shape of this profile in the non-monetary version of the model. In this case, one of the Euler equations is

$$U_c(c_t^i, 1 - n_t^i) = \beta E_{t+i} \left\{ U_c(c_t^{i+1}, 1 - n_t^{i+1})[1 + r_{t+i+1} - \delta] \right\}.$$

Given logarithmic preferences, in steady equation this equation reads

$$\frac{c^{i+1}}{c^i} = \beta[1 + r - \delta].$$

The term in square brackets is the gross real interest rate which is fixed in the calibration process (for the monetary steady state). In fact, The value of β is calibrated in order to match that real interest rate target. It turns out that the product of the discount factor and the gross real interest rate is larger than unity implying that individual will chose a path for consumption that grows over their lifetimes.

Real money balances also rise with age owing to the cash-in-advance constraint.

Early in the life-cycle, households run up debt: their capital holdings are negative. Between ages 25 years ($i = 20$) and 55 years ($i = 160$), they save, followed by a prolonged period of dissaving. Since there is no bequest motive, individuals choose to end their lived with no real assets.

The age-profile of utility initially falls, then rises throughout the remainder of life.

The fact that the monetary and non-monetary steady states are so close to each other suggests that money is not distorting individual behavior too much. The observation is not too surprising in light of the modest money growth (and consequently inflation) rates. Given that in the benchmark model money growth is calibrated to the growth rate of U.S. currency per capita, net money growth is 5% per annum.

3.2 Business Cycle Moments

Another litmus test for the model is whether its predictions for business cycle moments are similar to those reported in the literature. Table 3 reports business cycle moments for the U.S. economy, the benchmark model, and the non-monetary model.

There are two important points. First, the model's performance (whether benchmark or non-monetary) is on par with that of standard real business cycle models (with a representative, infinitely lived agent). This finding should not be too surprising since Ríos-Rull (1996) found that an annual version of the life-cycle model generated business cycle moments similar to that of the standard real business cycle model.

Second, adding money and money growth fluctuations has a fairly minor impact on the model's predictions for business cycle fluctuations. Cooley and Hansen (1989) made a similar observation for a representative, infinitely lived agent model.

In summary, nothing in this section suggests that there is anything odd about the benchmark model.

4 Welfare Costs of Inflation

4.1 Lifetime Utility in Steady State

One obvious criterion for evaluating money growth (or inflation) rates is steady state lifetime utility. Since steady state decisions differ across money growth rates, index these decision rules by μ. Steady state lifetime utility, condition on money growth μ, can be expressed as:

$$V(\mu) \equiv \sum_{i=0}^{T-1} \beta^i U[c^i(\mu), \ell^i(\mu)]. \tag{18}$$

Figure 2 plots $V(\mu)$ against a range of money growth rates. Remarkably, steady state lifetime utility is maximized at a money growth (inflation) rate of 210% per annum. By way of contrast, in models with an infinitely-lived representative agent, like Cooley and Hansen (1989), steady state utility is maximized by setting $\mu = \beta$ which implies a negative (net) money growth rate. Such a money growth rate results in a zero nominal interest rate, a result known as the 'Friedman rule.'

That the utility-maximizing money growth rate is so high is even more surprising given the similarity in the life-cycle profiles of consumption and leisure (hours of work) across the benchmark and non-monetary models' steady states presented in Figure 1. That is to say, money growth is not introducing a substantial distortion into the steady state of the model.

Some insight into why the lifetime utility-maximizing inflation rate is so high can be garnered from Figure 3 which presents life-cycle profiles for the benchmark money growth rate (4%) and the optimal money growth rate (210%). Notice that the higher money growth (inflation) rate twists the utility profile, making it flatter.

Why should higher inflation lead to improved utility-smoothing over the life-cycle? Recall that consumption (and, via the cash-in-advance constraint, real money balances) grows over the life-cycle. Consequently, older agents pay a higher inflation tax than young agents – but the proceeds of the inflation tax are rebated independent of age. Figure 3(d) shows that for the optimal inflation rate, net taxes paid – that is, the inflation tax paid less the lump-sum transfer – are big and positive for old households while young households receive transfers on net. In other words, inflation is a means of transferring resources from old, rich households to young, poor ones.

Of course, there is a cost to inflation. As is standard in cash-in-advance models, inflation introduces a distortion into the labor supply decision since cash earned in the current period cannot be spent until the subsequent period when inflation has eroded its purchasing power. Presumably, tax-transfer schemes that avoid this deleterious effect of inflation would deliver even higher lifetime utility.

One might think that households should be able to achieve, on their own, any utility-smoothing that they desire since they are free to go into debt. No doubt, introducing period-by-period non-negativity constraints on capital holdings would worsen the ability of individuals to smooth their utility over their lifetimes, thus perhaps increasing the potential benefits of inflation in this environment. However, when individuals go into debt, they eventually must repay this debt. The government, on the other hand, can in effect 'borrow' on behalf of the young at essentially a zero real interest rate. That is to say, the government faces a different feasibility constraint than that implied by the sequence of budget constraints confronting households.

4.2 Welfare Metrics

The next task is to obtain a 'unit free' measure of how agents care about alternative inflation (money growth) rates. A common approach in the literature is to find an 'equivalent variation payment' – that is, how much consumption must be given to agents to make them indifferent between two alternative money growth rates. When there is a representative agent, this calculation is relatively straightforward; see, for example, Cooley and Hansen (1989). This calculation is more complicated in the current environment owing to heterogeneity over the life-cycle. Consequently, a number of alternative measures of the welfare costs of inflation are explored.

Welfare costs will be expressed relative to a zero inflation rate. Let $V(\mu_0)$ denote the lifetime utility associated with a zero money growth rate. For the first two welfare metrics, find the age-independent addition to consumption, $\Delta c(\mu)$, that makes households indifferent (in a lifetime utility sense) between μ_0 and some alternative money growth rate. That is, find the value of $\Delta c(\mu)$ that satisfies

$$\sum_{i=0}^{T-1} \beta^i U[c^i(\mu) + \Delta c(\mu), \ell^i(\mu)] = V(\mu_0). \tag{19}$$

To render this measure of the welfare cost unit-free, express the total transfer relative to either total consumption or total output:

$$W^1 = \frac{T\Delta c(\mu)}{C(\mu)} \times 100\%, \tag{20}$$

$$W^2 = \frac{T\Delta c(\mu)}{Y(\mu)} \times 100\%. \tag{21}$$

A closely related way to measure the costs of inflation is to find the (again, age-independent) fraction of consumption, $\lambda^c(\mu)$, that must be given to agents to make them as well off as under money growth μ_0:

$$\sum_{i=0}^{T-1} \beta^i U[(1 + \lambda^c(\mu))c^i(\mu), \ell^i(\mu)] = V(\mu_0), \tag{22}$$

$$W^3 = \lambda^c(\mu) \times 100\%. \tag{23}$$

Alternatively, the welfare cost can be expressed as the constant fraction of *income* needed to give lifetime utility $V(\mu_0)$:

$$\sum_{i=0}^{T-1} \beta^i U[c^i(\mu) + \lambda^y(\mu)y^i(\mu), \ell^i(\mu)] = V(\mu_0), \tag{24}$$

$$W^4 = \lambda^y(\mu) \times 100\% \tag{25}$$

where

$$y^i(\mu) = (1 - \tau_n)w(\mu)h^i n^i(\mu) + [1 - \delta + (1 - \tau_k)r(\mu)]k^i(\mu) + x^M(\mu) + x^R(\mu). \tag{26}$$

The remaining measures of the costs of inflation make the equivalent variation payments age-specific. In this case, for each age i, find $\Delta c^i(\mu)$ such that

$$U[c^i(\mu) + \Delta c^i(\mu), \ell^i(\mu)] = U[c^i(\mu_0), \ell^i(\mu_0)]. \tag{27}$$

One pair of welfare metrics is obtained by simply adding up all of the individual equivalent variation payments and dividing by either aggregate consumption or aggregate output:

$$W^5 = \frac{\sum_{i=0}^{T-1} \Delta c^i(\mu)}{C(\mu)} \times 100\%, \tag{28}$$

$$W^6 = \frac{\sum_{i=0}^{T-1} \Delta c^i(\mu)}{Y(\mu)} \times 100\%. \tag{29}$$

Suppose that some money growth rate generates a welfare *benefit*. Thus far, all of the welfare measures presented have the property that, at a point in time, a benevolent government could, in principle, implement a set of lump-sum taxes and transfers (corresponding to the equivalent variation payments) that would lead to a Pareto superior allocation.

Two final welfare metrics discount the equivalent variation payments in equation (27):

$$W^7 = \frac{\sum_{i=0}^{T-1} \beta^i \Delta c^i(\mu)}{C(\mu)} \times 100\%, \tag{30}$$

$$W^8 = \frac{\sum_{i=0}^{T-1} \beta^i \Delta c^i(\mu)}{Y(\mu)} \times 100\%, \tag{31}$$

Measure W^8 is essentially the same as that of Summers (1981) who used the percentage change in lifetime income to measure the welfare costs of income taxation.

Table 8 and Figure 4 summarize the welfare calculations. The welfare-maximizing money growth rate associated with welfare metrics, W^1–W^4, conform quite closely with the money growth rate that maximizes life-time utility. The largest welfare benefit (i.e., negative welfare cost) occurs around 220% and 230% annual money growth rates. The welfare benefits are quite sizeable: 3.45% of income according to W^2 and 3.8% according to W^4. However, the maximum welfare benefit associated with welfare metric W^6 is quite small (less than 0.1% of income) and occurs at -3% annual money growth. This result seems odd in the sense that W^6 computes age-specific lump-sum payments while W^2 computes an age-independent lump-sum payment. Finally, W^8 – which discounts the age-specific lump-sum payments computed for W^6 – is maximized at 140% money growth, and yields a welfare benefit just under 0.5% of income.

5 Introducing Other Taxes

The analysis in subsection 4.1 suggests that the utility smoothing afforded by the combination of a high inflation tax and large lump-sum transfers is at the heart of the high life-time utility-maximizing money growth rates. If that is the case, then arming the government with alternative sources of revenue – consumption, labor income and capital income taxes – should lessen its reliance on the inflation tax, and so reduce the optimal money growth rate. This is the exercise considered in this section.

The tax rates are taken from Mendoza et al. (1994), and correspond to average effective tax rates for the U.S. The specific values used are: $\tau_c = 5.8\%$, $\tau_n = 24.8\%$ and $\tau_k = 42.9\%$, corresponding to the consumption tax, labor income tax and capital income tax, respectively.

Life-time utility is plotted against money growth in Figure 5(a) while the welfare metrics W^2, W^4, W^6 and W^8 are plotted in 5(b). Now, very moderate deflation maximizes life-time utility; all of the welfare metrics lead to the same conclusion. These results confirm the conjecture that it is the lump-sum transfers that are driving the high optimal money growth rates found in section 4.

As shown in Table 8, the welfare benefit of -3% inflation is between 0.1% and 0.2% of income, depending on the welfare metric. The costs of moderate inflations are similar to those found by Cooley and Hansen (1989). For example, relative to a zero inflation rate, a 10% inflation rate generates a welfare cost between 0.3% and 0.5% of income, depending on the welfare metric.[1]

5.1 Revenue Neutral Experiments

Now, suppose that the government uses seigniorage revenue to lower *one* of the other taxes, subject to raising the same revenue as under zero inflation. Specifically, as money growth, μ, is varied, adjust one of τ_c, τ_n and τ_k to satisfy

$$(\mu-1)M(\mu)+\tau_c C(\mu)+\tau_n W(\mu)N^e(\mu)+\tau_k R(\mu)K(\mu)=\tau_c(0)C(0)+\tau_n(0)W(0)N^e(0)+\tau_k(0)R(0)K(0). \tag{32}$$

The results of these experiments are summarized in Table 6. Replacing the U.S. consumption tax of 5.8% with a consumption *subsidy* of 0.4%, financed by 30% annual money growth, results in either a modest welfare gain (around 0.2% of income according to welfare metrics W^2 and W^4) or a welfare loss (0.1% of output according to metrics W^6 and W^8. As with the benchmark cash-in-advance model with no other taxes, the optimal policy depends on the welfare criterion applied. In this case, welfare metrics W^6 and W^8 suggest that the optimal inflation rate is slightly positive ($1-2\%$), yielding a negligible welfare benefit.

Next, lowering the tax rate on labor income from 24.8% to a subsidy rate of 9.3% (associated with a 200% inflation rate) is associated with a substantial welfare gain: between 2.3% and 3.1% of income. Welfare metrics W^2 and W^4 are also maximized at 200% money growth while metrics W^6 and W^8 are maximized at an annual money growth rate of 160%.

Finally, life-time utility is maximized by replacing the capital income tax of 42.9% with a subsidy of 4.5% and a money growth rate of 40% per annum. As with the labor income replacement experiment, the welfare gains of this policy are sizeable: between 5.5% and 7.4% of income. For the most part, the welfare welfare metrics indicate that the welfare benefit is maximized at an inflation rate near 40%.

To summarize the results from this section, holding the tax rates on consumption, labor income and capital income at their U.S. values, the optimal inflation rate is approximately the Friedman rule (deflate at the negative of the real interest rate). However, allowing the proceeds of the inflation tax to replace *one* of these other taxes again generates high optimal inflation rates – as high as 200% in the case in which seigniorage revenue is used to replace the labor income tax rate.

[1]Cooley and Hansen (1989) report welfare costs relative to an optimal inflation rate (the Friedman rule) whereas the results above are expressed relative to a zero inflation rate.

6 Credit

Perhaps the high optimal inflation rates obtained above are due to the very rigid payments technology associated with the cash-in-advance constraint. When credit is not available (as in the benchmark economy), the cash-in-advance constraint implies that the burden of the inflation tax is borne by those who consume the most. In the benchmark model, that burden falls on older agents; see subsection 3.1. Borrowing on an idea in Erosa and Ventura (2002) who use a similar credit technology, rich agents (in the benchmark economy, older agents), are better able to afford to use the credit technology. Consequently, old, rich agents should be able to afford to make greater use of the credit technology, reducing the role of the inflation tax in financing transfer payments.

The model is now calibrated to allow some goods to be purchased with credit. Specifically, the parameters in the credit technology equation (17) are now calibrated as in Dotsey and Ireland (1996). They used two pieces of information to pin down the two parameters, γ and θ: evidence on the use of money for transactions in the U.S. from Avery et al. (1987), and the long-run interest semi-elasticity of money demand. Suppose that 'money' in the model corresponds to currency in the U.S. economy. Then 30% of transactions in the U.S. use money, and the interest semi-elasticity is 2.73. The first observation requires that, in steady state, the average value of s must be 0.7 (70% of transactions are made with credit):

$$\frac{1}{T}\sum_{i=0}^{T-1} s_{t-i}^{i} = 0.7.$$

Using the second observation requires solving the model for two different money growth rates (implying two different inflation rates, and so nominal interest rates). As in Dotsey and Ireland (1996), inflation rates of 0% and 10% are used, and so

$$\frac{\ln v_{10} - \ln v_0}{R_{10} - R_0} = 2.73$$

where v_{10} denotes the annual velocity of money under a 10% inflation, R_{10} is the corresponding nominal interest rate, and variables with 0 subscripts correspond to an inflation rate of 0%. The nominal interest rate is computed from a Fisher equation,

$$R = (1+\pi)(1+r-\delta)$$

where r is the real rental price of capital, and so $r-\delta$ is the real interest rate.

6.1 Inflation Tax Only

To start, consider the environment in which there are no other taxes: $\tau_c = 0$, $\tau_n = 0$ and $\tau_k = 0$. This version of the model corresponds to the benchmark (cash-in-advance) economy, except that individuals are now able to use credit for some of their purchases. Recall that in the benchmark model, the optimal inflation rate was very high: 210% maximized lifetime utility; see Table 8. Results for the costly credit version of the model are summarized in Table 7. As with the benchmark model, the optimal inflation rate depends on the welfare criterion used. Allowing for the use of credit, an inflation rate of 5% maximizes lifetime utility – much smaller than in the benchmark economy.

Figure 6(a) illustrates that net transfers are greatly diminished when credit use is permitted. For comparison with the benchmark model, net transfers are computed for a money growth rate of 210% – the rate that maximizes lifetime utility in the straight cash-in-advance model. Under the straight cash-in-advance, such a money growth rate allowed the government to raise considerable revenue from old agents, and make large transfers to the young. By way of comparison, in the costly credit version of the model, these taxes and transfers are extremely modest. This result lends further credibility to the notion that in the benchmark model of section 4, it is the large taxes and transfers that leads to the very high optimal inflation rates.

Introducing other taxes to the costly credit version of the model leads to uniformly lower optimal inflation rates. In this case, it is difficult to solve for steady state when money growth is negative, and the optimal inflation rate (by any metric) is zero. Since welfare costs (or benefits) are measured relative to a zero inflation rate, the welfare costs of the (constrained) optimal inflation rate are also zero.

Next, the revenue neutral experiments of subsection 5.1 are repeated for the costly credit version of the model. As above, as the money growth rate is varied, so too does the amount of seigniorage revenue raised. This seigniorage revenue is used to reduce one other tax. The results of these experiments are summarized in Table 8.[2] When seigniorage revenue is used to reduce the consumption tax, the optimal inflation rate is quite modest: No more than 2% per annum. This policy reduces the consumption tax rate by 0.2 percentage points, and generates a very modest welfare gain (less than 0.01% of income). However, when the proceeds of the inflation tax are used to lower the labor income tax rate, rather more substantial inflation rates, between 60% and 70%, maximize welfare. The labor income tax rate falls by over 4 percentage points, and this policy results in a welfare gain of between 0.3% and 0.4% of income, depending on the welfare metric used. While these welfare gains are smaller than seen in the straight cash-in-advance model, they are sizeable when compared to welfare gains typically seen in the literature.

7 Transition Dynamics

As shown by XXX, comparing welfare across steady states can be misleading. In that paper, adding a constraint (which cannot possibly make agents better off) raises steady state welfare. However, along the transition path, welfare is unambiguously lowered. Consequently, it is not obvious that the welfare gains identified above (computed across

[2]Results for reducing the capital income tax are not included in Table 8 because the economy appears to be on the "wrong" side of the Laffer curve. Consequently, as inflation rises, the capital income tax actually has to rise as well to maintain a fixed level of government revenue.

steady states) would hold up if the transition path from one steady state to another is considered.

The particular example analyzed in this section is the costly credit version of the model with U.S. taxes. The policy switch is from five percent money growth (the U.S. historical average for currency per capita) to zero percent. In steady state, the welfare benefit of this change in policy is around 0.1% of income (using welfare metric W^6 which computes age-specific lump-sum payments). The policy change is unanticipated and implemented at time zero. The transition path is computed by using decision rules that have been linearized around the zero money growth steady state, but with initial conditions given by the five percent money growth steady state.

The welfare cost of this policy change is computed in a similar fashion as to how the welfare metric W^6 is computed. The difference is that along the transition path, the age-specific lump-sum payments are computed for each date of the transition. If there is, in fact, a welfare benefit at each date, then it would be possible, in principle, to actually implement the set of taxes and transfers that would lead to a Pareto superior allocation.

The transition path for key macroeconomic variables are summarized in Figures 7(a) and 7(c). Relative to the initial steady state, output, consumption and hours all rise on impact, and stay above their previous steady state values. This result is common in models of inflation: A permanent reduction in inflation lowers the inflation tax which, in a model with a cash-in-advance constraint, operates like a tax on wage income. In the fact of a higher effective real wage rate, individuals are willing to work more. The transition to the new steady state is rapid: Most of the action occurs within 10 quarters.

Figure 7(b) gives the welfare *benefit* of following this disinflation policy. The welfare benefit rises sharply on impact, falls somewhat, then asymptotically approaches its long run value – with a longer transition than for the macroaggregates. Notice, in particular, that the welfare benefit is uniformly *positive*.

Finally, Figure 7(d) plots the lifetime utility of those generations living through the transition. Unlike the other figures in Figure 7, the horizontal axis in Figure 7(d) gives the cohort, not time. For example, the observation at -200 is the lifetime utility of the cohort born at date -200. It lives from $t = -200$ through $t = 19$. This cohort experiences 200 quarters of life under the five percent money growth regime, and 20 quarters of zero percent money growth. Relative to lifetime utility in the five percent money growth steady state, *every* generation living through the transition is made better off. It is, perhaps, interesting to note that those who are very young when the policy is implemented have higher lifetime utility than those born a few quarters after policy implementation. Those who are young at date 0 have received the benefit of the net transfer payments associated with positive money growth, but then: (a) do not have to make the net payments when they are old, and (b) do not have their labor supply decisions distorted as much by inflation.[3] If monetary policy were subject to a popular vote, the move to zero inflation would be accepted unanimously.

[3]The intuition underlying the Friedman rule suggests that, a negative inflation rate would distort individual decisions even less than zero inflation.

8 Conclusion

Life-cycle heterogeneity has been shown to lead to some interesting results. In the benchmark cash-in-advance model with no other taxes, high inflation is optimal (maximizes lifetime utility) because it allows the government to tax old, rich households and make (net) transfers to young, poor households. In the benchmark model, these taxes and transfers lead to better utility smoothing over the life-cycle. Adding consumption, labor income and capital income taxes gives the government alternative sources of revenue; in this case, the optimal inflation rate is essentially the Friedman rule. The benchmark model may, then, be suitable for understanding why developing countries often have very high inflation rates: Since markets in developing countries are not very well developed, it is difficult for governments in such countries to levy the full range of taxes found in developed countries. The benchmark model suggests that high inflation may be optimal in developing countries. Koreshkova (2001) provides an alternative explanation of why high inflation may be optimal in developing countries based on the size of their underground economies.

Adding costly credit to the model, thus allowing for an endogenous determination of cash-vs-credit goods, lowers the optimal inflation rate. In this version of the model, two effects are at work. First, as in Erosa and Ventura (2002), rich agents can better afford to use the credit technology and so can avoid the burden of the inflation tax. Second, as the inflation rises, *all* agents make greater use of credit. The combination of these two effects causes the inflation tax base (real money balances) to fall rapidly with inflation. Relative to the straight cash-in-advance model, at a given inflation rate the government can no longer raise as much revenue. Net taxes and transfers are much smaller in this case.

A set of revenue neutral experiments were run in which seigniorage revenue was used to lower one tax at a time. In the straight cash-in-advance model, an inflation rate as high as 200% maximizes lifetime utility if the proceeds are used to lower the labor income tax rate from 24.8% to -9.3%; this policy results in a welfare gain of between 2 and 3 percent of income, depending on the welfare metric used. The largest measured welfare gains are associated with lowering the capital income tax rate. In this case, an inflation rate of 40% maximizes lifetime utility, and results in a welfare gain between 5.5% and 7.5% of income. In the costly credit version of the model, the biggest gain is associated with lowering the labor income tax rate from 24.8% to just over 20%; the lifetime utility-maximizing money growth rate is around 70%. While the welfare gains are more modest in this case – 0.3–0.4% of income – they are still large when compared to typical welfare benefits of alternative government policies.

Elsewhere, it has been shown that comparing steady state utility may be a misleading measure of the desirability of alternative policies. It is not obvious that along the transition path that agents are made better off by a change in policy, and there may be distributional effects to consider. To evaluate this possibility, the transition path was computed for the costly credit version of the model with U.S. tax rates in place. The policy change is to lower money growth once-and-for-all from 5% per annum to 0%. In steady state, this policy generates a welfare benefit of 0.1% of income. Two results stand out. First, along the transition path, the welfare benefit measured at each date is strictly positive. Second, the lifetime utility of all generations experiencing the change in policy is higher than it would have been if money growth had remained at 5%. This change in policy leads to a Pareto superior outcome.

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Technical Appendix

A.1 Benchmark Model

A.1.1 Household's Problem

The household's Bellman equation is:

$$
\begin{aligned}
V(k_t^i, M_t^i; i) \equiv \max \Bigg\{ & U(c_t^i, 1-n_t^i) + \beta E_{t+i} V(k_t^{i+1}, M_t^{i+1}; i+1) \\
& + \Lambda_{1t}^i \Bigg[(1-\tau_n) W_{t+i} h^i n_t^i + [(1-\delta)P_{t+i} + (1-\tau_k) R_{t+i}]\, k_t^i + M_t^i + X_{t+i}^M + X_{t+i}^R \\
& \quad - P_{t+i}(1+\tau_c) c_t^i - P_{t+i} k_t^{i+1} - \int_0^{s_t^i} Q_{t+i}(j)\, dj - M_t^{i+1} \Bigg] \\
& + \Lambda_{2t}^i \left[M_t^i + X_{t+i}^M - P_{t+i}(1+\tau_c)(1-s_t^i) c_t^i \right] \Bigg\}
\end{aligned}
\tag{A.1}
$$

The choice variables are: c_t^i, n_t^i, s_t^i, k_t^{i+1} and M_t^{i+1}. Recall that credit is used for consumption purchases on markets $j \in [0, s_t^i]$ while cash is used for the remainder, $j \in (s_t^i, 1]$. Keep in mind the are boundary conditions, (6) and (7). The relevant first-order conditions are:

$$
U_c(c_t^i, 1-n_t^i) = P_{t+i}(1+\tau_c) \left[\Lambda_{1t}^i + \Lambda_{2t}^i (1-s_t^i) \right], \quad i = 0, \ldots, T-1 \tag{A.2}
$$

$$
U_\ell(c_t^i, 1-n_t^i) = \Lambda_{1t}^i (1-\tau_n) W_{t+i} h^i, \quad i = 0, \ldots, T-1 \tag{A.3}
$$

$$
\Lambda_{1t}^i Q_{t+i}(s_t^i) = \Lambda_{2t}^i P_{t+i}(1+\tau_c) c_t^i, \quad i = 0, \ldots, T-1 \tag{A.4}
$$

$$\beta E_{t+i}\Lambda_t^{i+1}\left[(1-\delta)P_{t+i+1}+(1-\tau_k)R_{t+i+1}\right]=\Lambda_{1t}^{i}P_{t+i}, \quad i=0,\ldots,T-2 \tag{A.5}$$

$$\beta E_{t+i}\left(\Lambda_{1t}^{i+1}+\Lambda_{2t}^{i+1}\right)=\Lambda_{1t}^{i}, \quad i=0,\ldots,T-2 \tag{A.6}$$

These equations, along with the budget constraint, (3), and cash-in-advance constraint, (4), and the boundary conditions characterize the solution to the household's problem, including the multipliers, Λ_{1t}^{i} and Λ_{2t}^{i}.

A.1.2 Goods Producing Firms

The problem faced by a typical goods producer is given in the text in equation (9). The associated first-order conditions are:

$$P_tF_1(K_t,N_t^e;z_t)=R_t \tag{A.7}$$

$$P_tF_2(K_t,N_t^e;z_t)=W_t \tag{A.8}$$

A.1.3 Financial Intermediaries

The equations of interest here are the price of using credit, equation (8), and the total labor used in this sector,

$$N_t^c=\sum_{i=0}^{T-1}\int_0^{s_{t-i}^{i}}\gamma(j)dj \tag{A.9}$$

A.1.4 Government

In addition to the expressions for lump-sum transfers, (10) and (11), the stock of money evolves according to

$$M_{t+1}=\mu_tM_t. \tag{A.10}$$

A.1.5 Aggregates

Total capital, effective labor, money and consumption are given, respectively, by:

$$K_t=\sum_{i=0}^{T-1}k_{t-i}^{i} \tag{A.11}$$

$$N_t^e=\sum_{i=0}^{T-1}h^in_{t-i}^{i} \tag{A.12}$$

$$M_{t+1}=\sum_{i=0}^{T-1}M_{t-i}^{i+1} \tag{A.13}$$

$$C_t=\sum_{i=0}^{T-1}c_{t-i}^{i}. \tag{A.14}$$

A.1.6 Conversion to Real Magnitudes

Normalize by the aggregate price level:

$$w_{t+i} \equiv \frac{W_{t+i}}{P_{t+i}}, \quad r_{t+i} \equiv \frac{R_{t+i}}{P_{t+i}}, \quad x^M_{t+i} \equiv \frac{X^M_{t+i}}{P_{t+i}}, \quad x^R_{t+i} \equiv \frac{X^R_{t+i}}{P_{t+i}}, \quad q_{t+i}(j) \equiv \frac{Q_{t+i}(j)}{P_{t+i}},$$
$$m^{i+1}_t \equiv \frac{M^{i+1}_t}{P_{t+i}}, \quad \lambda^i_{1t} \equiv \frac{\Lambda^i_{1t}}{P_{t+i}}, \quad \lambda^i_{2t} \equiv \frac{\Lambda^i_{2t}}{P_{t+i}}, \quad \pi_{t+i} \equiv \frac{P_{t+i}}{P_{t+i-1}}, \quad m_{t+1} \equiv \frac{M_{t+1}}{P_t},$$

Notice that money balances are normalized by the 'previous period' price level; this is done so that the household's budget constraint does not involve next period's price level which is not known at the time household decisions are made.

The equations governing the solution of this economy are:

$$\begin{aligned}(1+\tau_c)c^i_t + k^{i+1}_t + \int_0^{s^i_t} q_{t+i}(j)dj + m^{i+1}_t \\ = (1-\tau_n)w_{t+i}h^i n^i_t + [1+\delta+(1-\tau_k)r_{t+i}]\,k^i_t + \frac{m^i_t}{\pi_{t+i}} + x^M_{t+i} + x^R_{t+i}, \quad i=0,\dots,T-1\end{aligned} \tag{A.15}$$

$$(1+\tau_c)(1-s^i_t)c^i_t = \frac{m^i_t}{\pi_{t+i}} + x^M_{t+i}, \quad i=0,\dots,T-1 \tag{A.16}$$

$$U_c(c^i_t, 1-n^i_t) = (1+\tau_c)\left[\lambda^i_{1t} + \lambda^i_{2t}(1-s^i_t)\right], \quad i=0,\dots,T-1 \tag{A.17}$$

$$U_\ell(c^i_t, 1-n^i_t) = \lambda^i_{1t}(1-\tau_n)w_{t+i}h^i, \quad i=0,\dots,T-1 \tag{A.18}$$

$$\lambda^i_{1t} q_{t+i}(s^i_t) = \lambda^i_{2t}(1+\tau_c)c^i_t, \quad i=0,\dots,T-1 \tag{A.19}$$

$$\lambda^i_{1t} = \beta E_{t+i}\left\{\lambda^{i+1}_{1t}\left[1-\delta+(1-\tau_k)r_{t+i+1}\right]\right\}, \quad i=0,\dots,T-2 \tag{A.20}$$

$$\lambda^i_{1t} = \beta E_{t+i}\left\{\frac{\lambda^{i+1}_{1t}+\lambda^{i+1}_{2t}}{\pi_{t+i+1}}\right\}, \quad i=0,\dots,T-2 \tag{A.21}$$

$$k^0_t = 0, \quad k^t_t = 0 \tag{A.22}$$

$$m^0_t = \overline{m}, \quad m^t_t = \overline{m} \tag{A.23}$$

$$r_t = F_K(K_t, N^e_t; z_t) \tag{A.24}$$

$$w_t = F_N(K_t, N^e_t; z_t) \tag{A.25}$$

$$K_t = \sum_{i=0}^{T-1} k^i_{t-i} \tag{A.26}$$

$$N^e_t = \sum_{i=0}^{T-1} h^i n^i_{t-i} \tag{A.27}$$

$$m_{t+1} = \sum_{i=0}^{T-1} m^{i+1}_{t-i} \tag{A.28}$$

$$m_{t+1} = \mu_t \frac{m_t}{\pi_t} \tag{A.29}$$

$$C_t = \sum_{i=0}^{T-1} c_{t-i}^i \tag{A.30}$$

$$x_t^M = \frac{(\mu_t - 1)m_t/\pi_t}{T} \tag{A.31}$$

$$x_t^R = \frac{\tau_c C_t + \tau_n w_t N_t^e + \tau_k r_t K_t}{T} \tag{A.32}$$

$$q_t(j) = w_t \gamma(j) \tag{A.33}$$

$$N_t^c = \sum_{i=0}^{T-1} \int_0^{s_{t-i}^i} \gamma(j) dj \tag{A.34}$$

$$N_t^e = N_t^g + N_t^c \tag{A.35}$$

A.2 Non-monetary Model

A.2.1 Household's Problem

The household's Bellman equation is:

$$\begin{aligned} V(k_t^i; i) \equiv \max \Big\{ & U(c_t^i, 1 - n_t^i) + \beta E_{t+i} V(k_t^{i+1}; i+1) \\ & + \lambda_t^i \big[(1-\tau_n) w_{t+i} h^i n_t^i + [(1-\tau_k) r_{t+i} + 1 - \delta] k_t^i - (c_t^i + k_t^{i+1})\big] \Big\} \end{aligned} \tag{A.36}$$

Euler equations and budget constraint:

$$c_t^i + k_t^{i+1} = (1-\tau_n) w_{t+i} h^i n_t^i + [(1-\tau_k) r_{t+i} + 1 - \delta] k_t^i + x_t^R, \quad i = 0, \ldots, T-1 \tag{A.37}$$

$$U_2(c_t^i, 1 - n_t^i) = \lambda_t^i (1-\tau_n) w_{t+i} h^i, \quad i = 0, \ldots, T-1 \tag{A.38}$$

$$U_1(c_t^i, 1 - n_t^i) = \lambda_t^i, \quad i = 0, \ldots, T-1 \tag{A.39}$$

$$\lambda_t^i = \beta E_{t+i} \lambda_t^{i+1} [(1-\tau_k) r_{t+i+1} + 1 - \delta], \quad i = 0, \ldots, T-2 \tag{A.40}$$

$$k_t^0 = 0, \quad k_t^t = 0 \tag{A.41}$$

A.3 Computational Issues

Notice that the aggregate state vector includes the capital and money holdings of all cohorts alive at a particular date. Individual decision rules depend on the entire state vector (not merely a few selected moments) since the state vector is needed to form expectations of future prices (which, in turn, depend on the future state vector). Fortunately, as pointed out by Ríos-Rull (1996), matters are greatly simplified if decision rules are *linear*. When solving for decision rules when the stochastic elements of the model are in play, it is then opportune to use a log linearization technique. See Klein (2000) for details on the particular technique employed in this paper.

Table 1: Benchmark Model Parameter Values

Preferences		
β	0.9841	discount factor
ω	1.6353	labor-leisure weight
σ	1.0	coefficient of relative risk aversion
Technology		
α	0.3	capital's share of income
δ	0.02	depreciation rate of capital
ρ	0.9499	technology shock, autoregressive parameter
σ_ε	0.0085	standard deviation of innovation to technology shock
$\{h^i\}_{i=0}^{T-1}$		human capital profiles
Money Growth		
ψ	0.8327	autoregressive parameter
$\bar{\mu}$	1.0416	long run annual money growth rate
σ_u	0.0045	standard deviation of innovation to money growth
Other		
T	220	number of periods of life
Calibration Targets		
$\bar{h}$	0.33	average hours worked
$\bar{r}$	0.0192	real interest rate (quarterly)

Table 2: Estimates of the Money Growth Process

	Currency	M1
$\bar{\mu}$	1.012362	1.008852
ψ	0.8327293	0.6448415
	(0.0382036)	(0.0581623)
σ_ξ^2	0.00446666	0.0088325
sample	1954Q1–2003Q2	1959Q2–2003Q2

Table 3: Selected Moments

	Standard	Cross Correlation of Real Output With								
	Deviation	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S.										
G.D.P.	1.66	0.13	0.37	0.62	0.85	1.00	0.85	0.62	0.37	0.13
Consumption	0.95	0.34	0.53	0.68	0.79	0.80	0.67	0.48	0.28	0.07
Investment	4.95	0.29	0.47	0.68	0.84	0.90	0.76	0.53	0.26	−0.00
Aggregate Hours	1.79	−0.11	0.11	0.38	0.66	0.88	0.91	0.80	0.63	0.41
Productivity	0.86	0.49	0.48	0.41	0.26	0.10	−0.26	−0.48	−0.59	−0.59
Benchmark										
Output	1.43	0.07	0.23	0.44	0.69	1.00	0.69	0.44	0.23	0.07
Consumption	0.62	−0.08	0.02	0.16	0.36	0.60	0.49	0.39	0.29	0.22
Investment	5.09	0.12	0.27	0.45	0.68	0.95	0.63	0.36	0.16	0.00
Hours	0.71	0.15	0.30	0.49	0.72	0.98	0.64	0.35	0.13	−0.03
Effective Hours	0.72	0.15	0.30	0.49	0.72	0.99	0.64	0.35	0.13	−0.03
Productivity	0.74	0.00	0.16	0.38	0.65	0.99	0.73	0.51	0.32	0.17
Capital	0.34	−0.42	−0.33	−0.19	0.03	0.32	0.50	0.60	0.63	0.62
Non-monetary										
Output	1.42	0.07	0.23	0.44	0.69	1.00	0.69	0.44	0.23	0.07
Consumption	0.41	−0.14	0.02	0.25	0.54	0.91	0.76	0.61	0.46	0.34
Investment	4.90	0.13	0.28	0.47	0.71	0.99	0.65	0.38	0.16	0.00
Hours	0.70	0.15	0.30	0.49	0.72	0.99	0.63	0.35	0.13	−0.03
Effective Hours	0.71	0.15	0.30	0.49	0.72	0.99	0.63	0.35	0.13	−0.03
Productivity	0.74	0.00	0.16	0.38	0.65	0.99	0.73	0.51	0.32	0.17
Capital	0.33	−0.44	−0.35	−0.20	0.02	0.33	0.52	0.62	0.65	0.64

Table 4: Welfare Costs of Inflation, Cash-in-advance Model, No Other Taxes

Inflation	V	W^1	W^2	W^3	W^4	W^5	W^6	W^7	W^8
−3.0	−25.6006	0.2618	0.2218	0.3963	0.3271	−0.0860	−0.0729	0.0339	0.0287
−2.0	−25.5167	0.1720	0.1458	0.2581	0.2138	−0.0575	−0.0487	0.0224	0.0189
−1.0	−25.4365	0.0848	0.0719	0.1262	0.1049	−0.0288	−0.0244	0.0111	0.0094
0.0	−25.3597	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	−25.2861	−0.0825	−0.0699	−0.1209	−0.1010	0.0289	0.0245	−0.0108	−0.0092
2.0	−25.2153	−0.1628	−0.1380	−0.2370	−0.1984	0.0579	0.0491	−0.0214	−0.0181
3.0	−25.1471	−0.2410	−0.2042	−0.3486	−0.2924	0.0871	0.0738	−0.0317	−0.0269
4.0	−25.0814	−0.3172	−0.2687	−0.4561	−0.3831	0.1163	0.0985	−0.0418	−0.0354
5.0	−25.0180	−0.3915	−0.3316	−0.5597	−0.4708	0.1457	0.1234	−0.0517	−0.0438
10.0	−24.7305	−0.7362	−0.6236	−1.0281	−0.8685	0.2936	0.2487	−0.0979	−0.0830
20.0	−24.2671	−1.3156	−1.1142	−1.7784	−1.5039	0.5939	0.5030	−0.1767	−0.1496
50.0	−23.3804	−2.4860	−2.1044	−3.1982	−2.6640	1.5041	1.2732	−0.3331	−0.2820
100.0	−22.6744	−3.4707	−2.9364	−4.3141	−3.4891	2.9741	2.5162	−0.4419	−0.3738
120.0	−22.5261	−3.6878	−3.1194	−4.5469	−3.6410	3.5349	2.9901	−0.4555	−0.3853
130.0	−22.4699	−3.7720	−3.1904	−4.6349	−3.6945	3.8090	3.2217	−0.4581	−0.3874
140.0	−22.4237	−3.8428	−3.2500	−4.7073	−3.7360	4.0790	3.4498	−0.4584	−0.3877
150.0	−22.3861	−3.9018	−3.2997	−4.7661	−3.7672	4.3449	3.6744	−0.4566	−0.3861
160.0	−22.3560	−3.9505	−3.3406	−4.8131	−3.7896	4.6067	3.8956	−0.4531	−0.3831
200.0	−22.2953	−4.0629	−3.4348	−4.9080	−3.8120	5.6155	4.7475	−0.4248	−0.3591
210.0	−22.2918	−4.0746	−3.4446	−4.9134	−3.8047	5.8585	4.9526	−0.4149	−0.3508
220.0	−22.2921	−4.0811	−3.4498	−4.9129	−3.7934	6.0980	5.1548	−0.4042	−0.3417
230.0	−22.2958	−4.0828	−3.4511	−4.9072	−3.7787	6.3341	5.3541	−0.3927	−0.3319
240.0	−22.3024	−4.0802	−3.4487	−4.8968	−3.7608	6.5669	5.5507	−0.3805	−0.3216
250.0	−22.3118	−4.0737	−3.4431	−4.8822	−3.7402	6.7965	5.7445	−0.3676	−0.3107
260.0	−22.3237	−4.0637	−3.4345	−4.8637	−3.7172	7.0230	5.9356	−0.3542	−0.2994
270.0	−22.3378	−4.0505	−3.4232	−4.8417	−3.6919	7.2465	6.1242	−0.3403	−0.2876
300.0	−22.3914	−3.9941	−3.3751	−4.7578	−3.6051	7.8995	6.6753	−0.2960	−0.2501
350.0	−22.5097	−3.8574	−3.2590	−4.5726	−3.4336	8.9347	7.5486	−0.2159	−0.1824

Notes: V is life-time utility. W^1 through W^8 are metrics of the welfare costs of inflation and are defined in subsection 4.2. Other taxes are zero: $\tau_c = 0$, $\tau_n = 0$ and $\tau_k = 0$.

Table 5: Welfare Costs of Inflation, Cash-in-advance Model, U.S. Taxes

Inflation	V	W^1	W^2	W^3	W^4	W^5	W^6	W^7	W^8
−3.0	17.5958	−0.0931	−0.0789	−0.1039	−0.0796	−0.1945	−0.1648	−0.1874	−0.1588
−2.0	17.5241	−0.0636	−0.0539	−0.0706	−0.0541	−0.1294	−0.1096	−0.1247	−0.1056
−1.0	17.4496	−0.0325	−0.0275	−0.0359	−0.0275	−0.0646	−0.0547	−0.0622	−0.0527
0.0	17.3725	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	17.2933	0.0337	0.0285	0.0369	0.0283	0.0643	0.0544	0.0619	0.0525
2.0	17.2121	0.0684	0.0579	0.0747	0.0573	0.1282	0.1086	0.1236	0.1047
3.0	17.1293	0.1041	0.0882	0.1133	0.0869	0.1919	0.1626	0.1850	0.1567
4.0	17.0449	0.1406	0.1191	0.1526	0.1170	0.2554	0.2163	0.2461	0.2085
5.0	16.9593	0.1778	0.1506	0.1926	0.1475	0.3185	0.2698	0.3070	0.2600
6.0	16.8725	0.2157	0.1828	0.2331	0.1784	0.3814	0.3231	0.3676	0.3114
7.0	16.7846	0.2543	0.2154	0.2741	0.2097	0.4439	0.3761	0.4279	0.3625
8.0	16.6959	0.2933	0.2485	0.3156	0.2412	0.5063	0.4288	0.4880	0.4134
9.0	16.6063	0.3329	0.2820	0.3574	0.2730	0.5683	0.4814	0.5478	0.4640
10.0	16.5160	0.3728	0.3158	0.3996	0.3050	0.6301	0.5337	0.6074	0.5145
20.0	15.5881	0.7886	0.6678	0.8344	0.6304	1.2338	1.0448	1.1897	1.0075
30.0	14.6434	1.2182	1.0314	1.2789	0.9558	1.8134	1.5353	1.7491	1.4809
40.0	13.7036	1.6500	1.3967	1.7231	1.2739	2.3709	2.0069	2.2874	1.9362
50.0	12.7783	2.0785	1.7591	2.1623	1.5823	2.9081	2.4612	2.8063	2.3750
100.0	8.4652	4.1119	3.4774	4.2349	2.9702	5.3403	4.5163	5.1576	4.3618
150.0	4.6704	5.9434	5.0236	6.0932	4.1433	7.4455	6.2933	7.1950	6.0815
200.0	1.3042	7.5992	6.4205	7.7693	5.1595	9.3120	7.8676	9.0025	7.6061
250.0	−1.7187	9.1106	7.6948	9.2970	6.0585	10.9956	9.2869	10.6337	8.9812
300.0	−4.4635	10.5030	8.8683	10.7029	6.8672	12.5341	10.5833	12.1248	10.2376
350.0	−6.9793	11.7960	9.9576	12.0074	7.6038	13.9543	11.7795	13.5016	11.3973

Notes: V is life-time utility. W^1 through W^8 are metrics of the welfare costs of inflation and are defined in subsection 4.2. The taxes are: $\tau_c = 5.8\%$, $\tau_n = 24.8\%$ and $\tau_k = 42.9\%$.

Table 6: Revenue Neutral Experiments, Cash-in-advance, U.S. Taxes

	$\tau_c^* = -0.4\%$	$\tau_n^* = -9.3$	$\tau_k^* = -4.5\%$
μ^*	30%	200%	40%
$-W^2$	0.1884	3.1495	5.8430
$-W^4$	0.1842	2.5410	5.4890
$-W^6$	-0.1058	2.3220	7.4326
$-W^8$	-0.1009	2.2961	7.1210

Notes: μ^* is the money growth (inflation) rate that maximizes life-time utility. W^2 through W^8 are metrics of the welfare costs of inflation and are defined in subsection 4.2. The initial U.S. tax rates are: $\tau_c = 5.8\%$, $\tau_n = 24.8\%$ and $\tau_k = 42.9\%$.

Table 7: Welfare Results Summary, Costly Credit Model, Inflation Tax Only

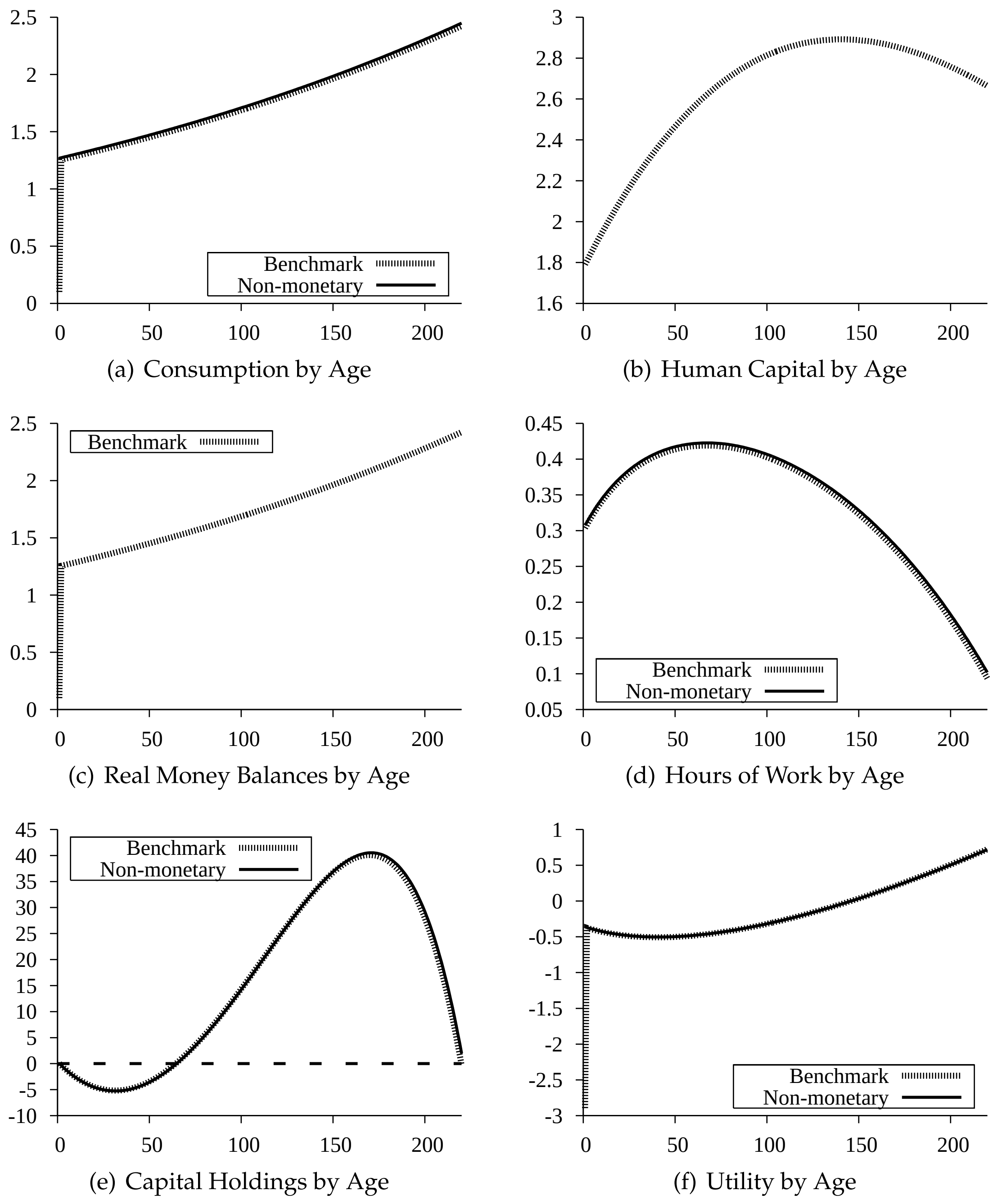
Criterion	Maximizing Inflation Rate (%)	Welfare Gain (%)
Lifetime utility	5	
W^2	5	0.0276
W^4	5	0.0371
W^6	-3	0.0968
W^8	3	0.0032

Notes: See subsection 4.2 for an explanation of the calculation of the welfare metrics W^2 through W^8. Welfare costs are computed relative to a zero inflation rate.

Table 8: Welfare Results Summary, Costly Credit Model, Revenue Neutral

Criterion	Maximizing Inflation Rate (%)	Welfare Gain (%)	Tax Rate (%)
Replace consumption tax			τ_c
Lifetime utility	2		5.62
W^2	2	0.0095	5.62
W^4	2	0.0092	5.62
W^6	0	0.0000	5.80
W^8	0	0.0000	5.80
Replace labor income tax			τ_n
Lifetime utility	70		20.32
W^2	70	0.3964	20.32
W^4	70	0.3733	20.32
W^6	60	0.2980	20.70
W^8	60	0.2949	20.70

Notes: At zero inflation, taxes are set to their values for the U.S. economy: $\tau_c = 5.8\%$, $\tau_n = 24.8\%$ and $\tau_k = 42.9\%$. Seigniorage revenue is used to replace one other tax as explained in subsection 5.1. See subsection 4.2 for an explanation of the calculation of the welfare metrics W^2 through W^8.



(a) Consumption by Age

(b) Human Capital by Age

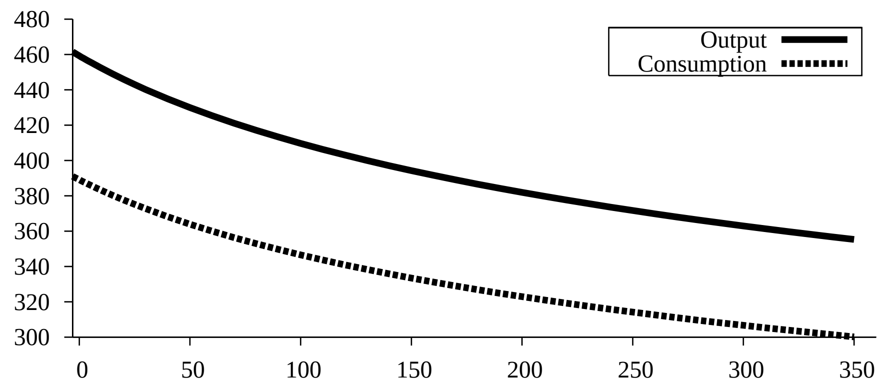
(c) Real Money Balances by Age

(d) Hours of Work by Age

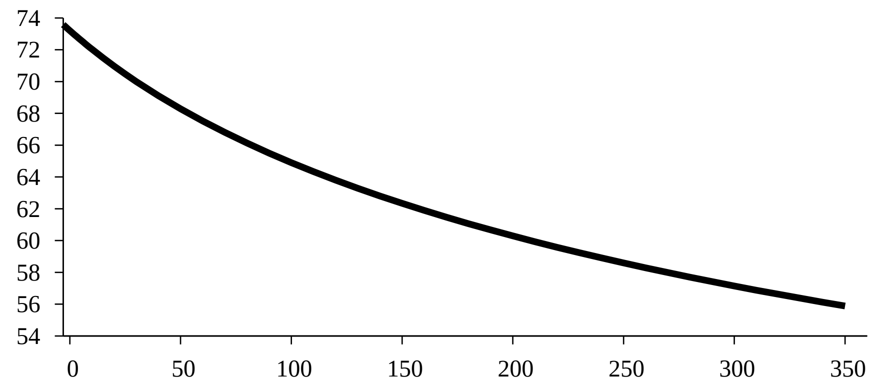
(e) Capital Holdings by Age

(f) Utility by Age

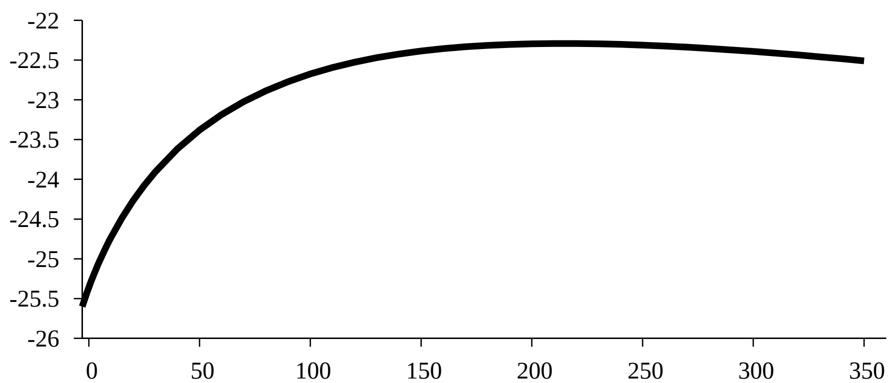
Figure 1: Steady State



(a) Aggregate Output and Consumption

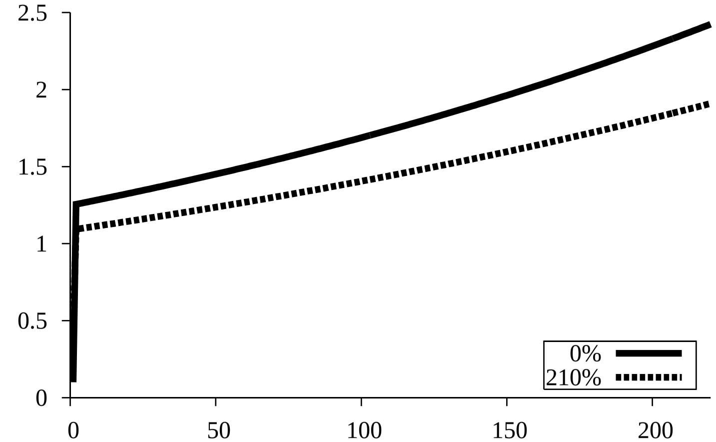


(b) Aggregate Hours

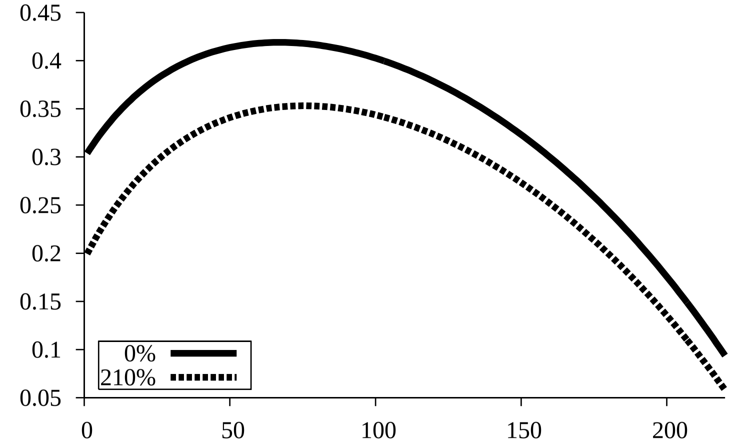


(c) Lifetime Utility

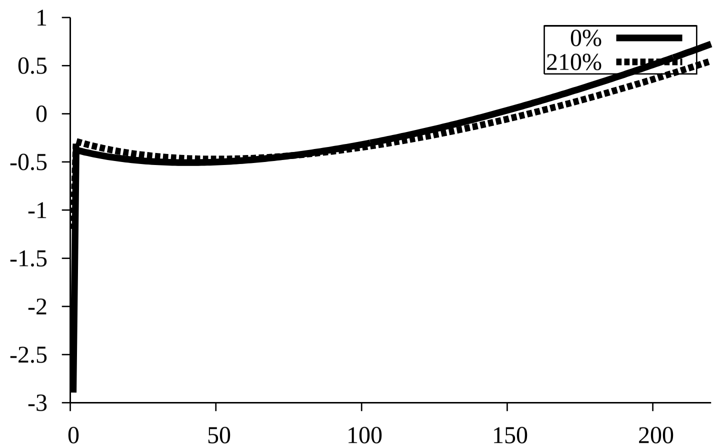
Figure 2: Steady State Values Plotted Against Money Growth



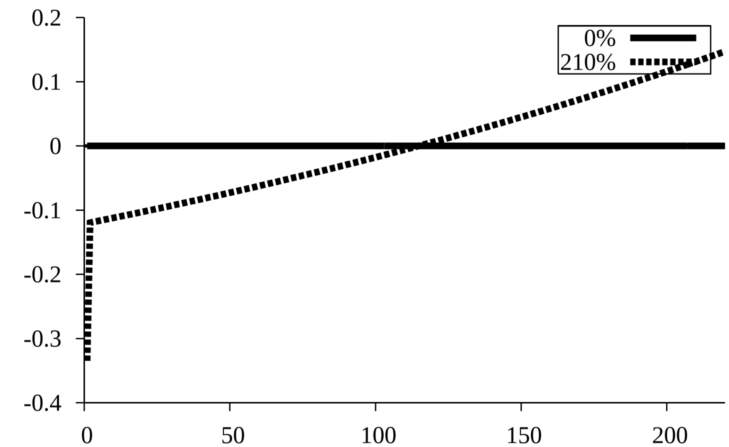
(a) Consumption by Age



(b) Hours of Work by Age

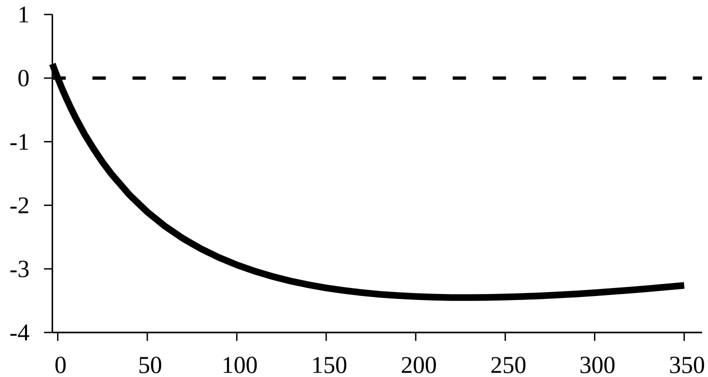


(c) Utility by Age

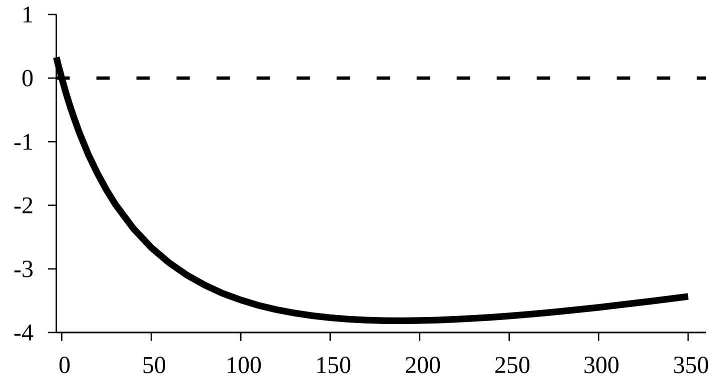


(d) Net Taxes by Age

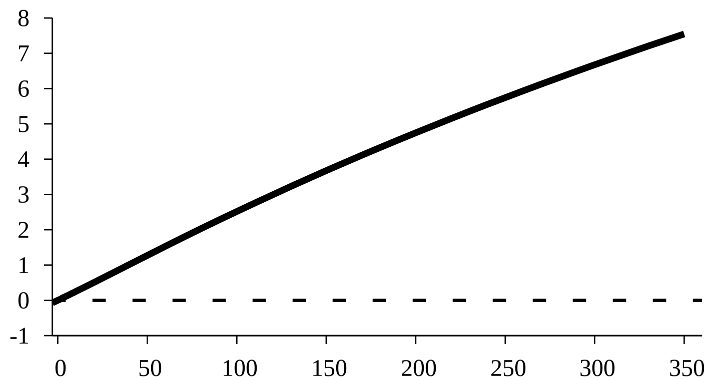
Figure 3: Utility-Maximizing Money Growth (210%) vs. 0%



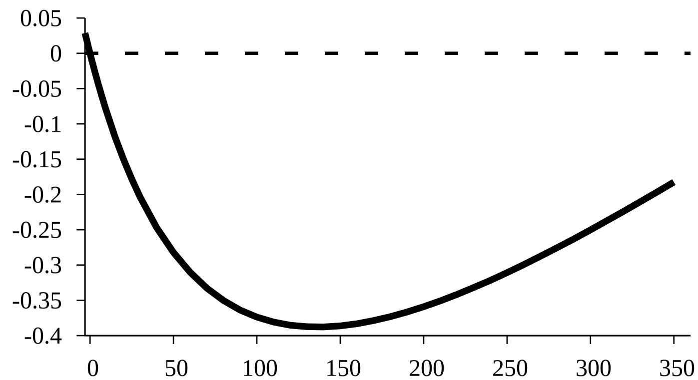
(a) Welfare Metric: W^2



(b) Welfare Metric: W^4

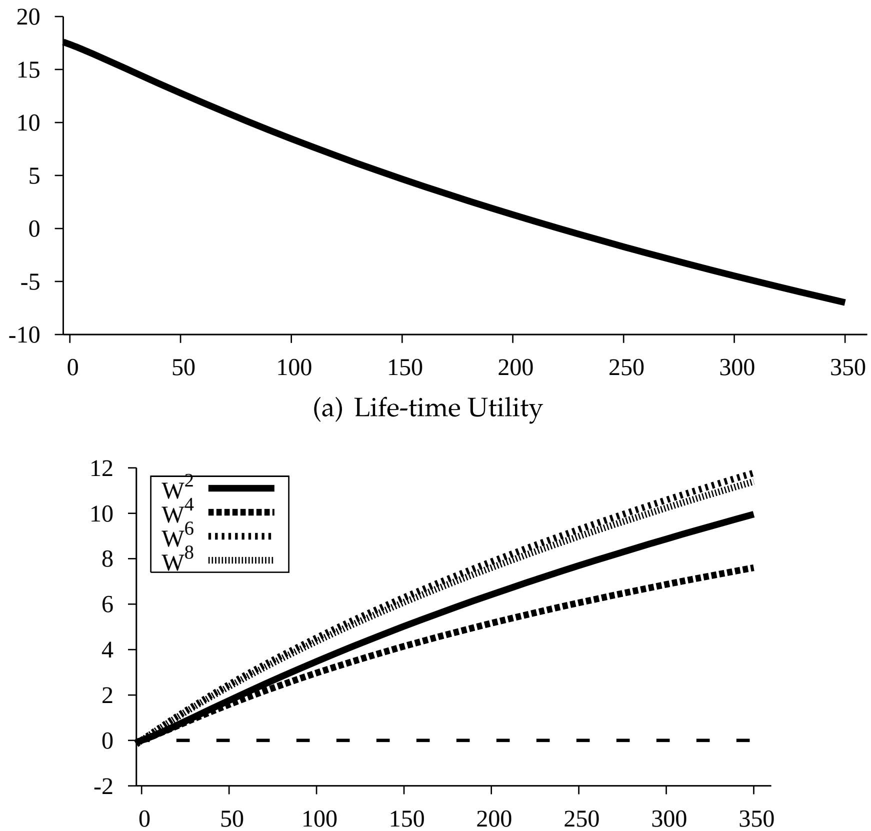


(c) Welfare Metric: W^6



(d) Welfare Metric: W^8

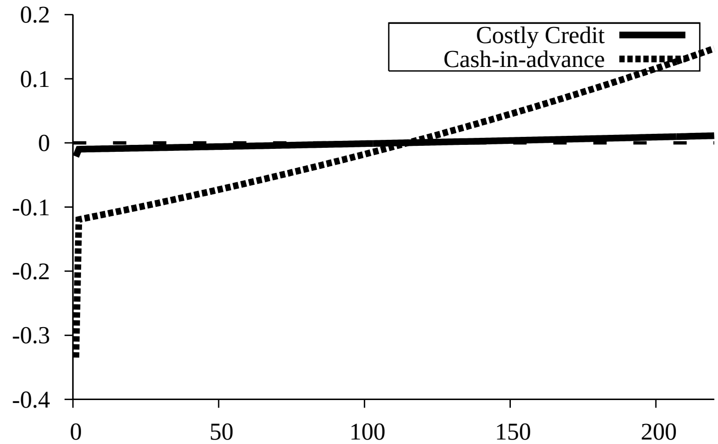
Figure 4: Welfare Metrics, Cash-in-advance, No Other Taxes



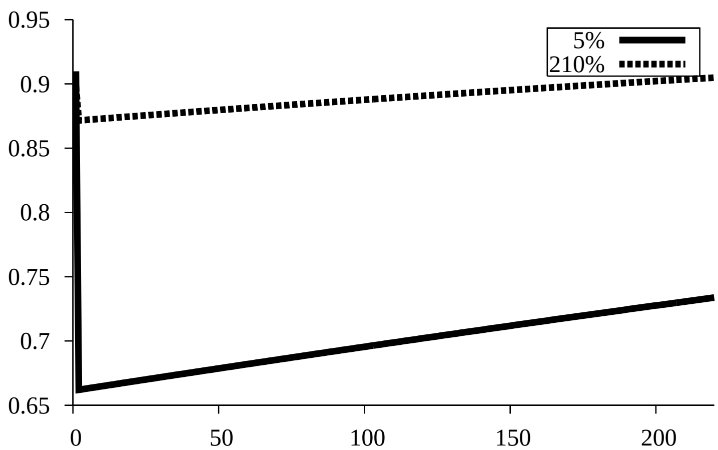
(a) Life-time Utility

(b) Welfare Metrics

Figure 5: Optimal Money Growth Rate With U.S. Taxes ($\tau_c = 5.8\%$, $\tau_n = 24.8\%$ and $\tau_k = 42.9\%$)

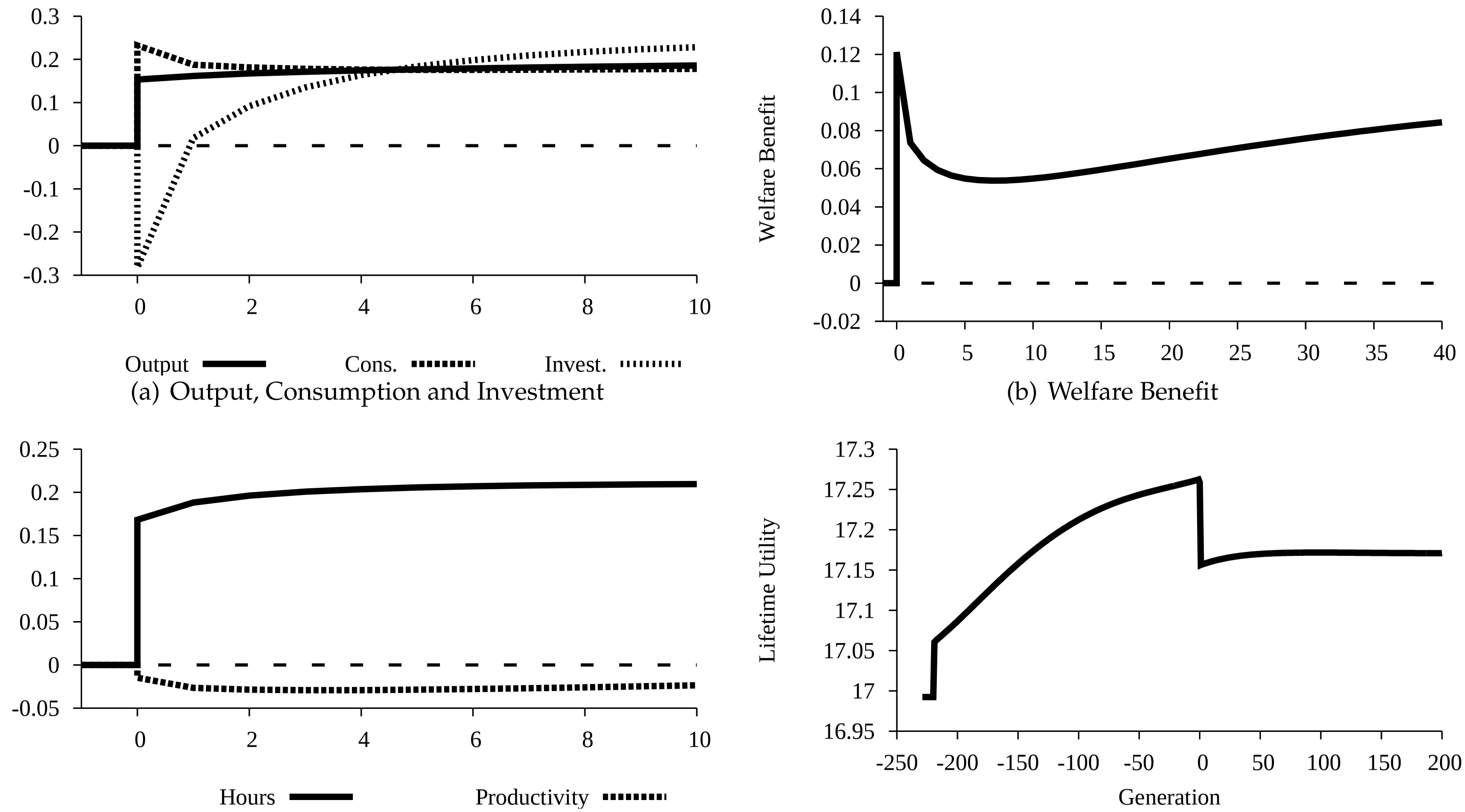


(a) Net Transfers by Age



(b) Fraction of Goods Purchased with Credit, by Age

Figure 6: Costly Credit Model, Inflation Tax Only



(a) Output, Consumption and Investment

(b) Welfare Benefit

(c) Hours and Productivity

(d) Lifetime Utility

Figure 7: Transition Dynamics Following a Permanent, Unanticipated Change in Money Growth From 4% to 0% Per Annum, Occurring at Date 0