Consumption over the Life Cycle: The Role of Annuities*

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1 Introduction

Among the many sources of uncertainty that an individual faces when planning for consumption in old age, one of the more significant is uncertainty about how long the individual will live. This source of uncertainty could be easily insured against if the individual were to purchase an annuity that provides a constant flow of income until death. But, annuity markets in the U.S. are quite thin. A standard explanation for the lack of annuity markets is adverse selection—those with long expected lifetimes will be attracted to annuities, which might cause them to be unattractively priced for most people.[1]

In this paper we explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a calibrated general equilibrium overlapping generations model where markets are otherwise complete. A large literature has documented that individual household consumption increases early in life, with a peak sometime around age 45-55 and a decline after that.[2] This is generally regarded as posing a puzzle for a standard life cycle model because, with complete markets, the model implies that consumption should be smooth over a lifetime. Depending on the relative magnitudes of the household's time discount rate and the market interest rate, consumption can be constant, or monotonically decreasing or increasing as an individual ages.

If an annuity market (or its equivalent) is unavailable, this intuition no longer applies. If survival probabilities decrease as an individual ages, individuals will more heavily discount the future as they grow older. This allows for the possibility, depending on the value of the interest rate, that consumption might increase early in life when survival probabilities are high and the effective rate of discount is low. As survival probabilities fall, the slope of the consumption profile may become negative.

In addition, because social security provides some insurance against uncertain lifetimes and may provide an adequate substitute for missing annuity markets, we also study the shape of the consumption profile in a model with missing annuity markets and a pay-as-you-go social security system as in

[1] See, for example, Friedman and Warshawsky (1990) and Mitchell, Poterba, Warshawsky, and Brown (1999).

[2] Thurow (1969) is an early example from this empirical literature. More recent contributions include Attanasio and Browning (1995), Attanasio, Banks, Meghir and Weber (1999), Gourinchas and Parker (2002), and Fernández-Villaverde and Krueger (2002). In addition, a recent paper by Aguiar and Hurst (2004) argues that, while consumption expenditures may be hump shaped, home production is used to smooth actual consumption relative to expenditures.

the U.S. Social security turns out to matter significantly, but life cycle consumption continues to be hump shaped.

We are not the first to note the impact of annuity markets on consumption over the life cycle. Yaari (1965) is perhaps the first to study the impact of uncertain lifetime on the shape of the life cycle consumption profile in an overlapping generations model. Levhari and Mirman (1977) extend Yaari's work by providing results on how risk averse consumers respond to a change in the distribution of lifetime uncertainty. That is, they obtain results showing how uncertain lifetimes affect the level of consumption at a particular age, as opposed to how consumption changes over the course of the life cycle.

Davies (1981) is perhaps the first to use a life cycle model with uncertain lifetimes to interpret actual consumption and savings behavior, in particular the savings behavior of retired individuals. More recently, İmrohoroğlu, İmrohoroğlu, and Joines (1995) develop an applied general equilibrium model with long but randomly-lived households to study the welfare effects of social security reform. They were able to generate age-consumption profiles with a hump by closing annuity markets, though they also had individual income uncertainty and borrowing constraints. Bütler (2001) provides a continuous-time overlapping generations model and gives an example of how a lack of annuity markets can yield a hump-shaped consumption profile. In this paper, our goal is to assess, using a calibrated general equilibrium model with social security, the extent to which a lack of annuity markets by itself can account for the observed hump shaped consumption profile.

Most of the consumption literature, however, has explored other factors that potentially play an important role in determining consumption over the life cycle. One possibility is that the hump shape may be due to demographic factors–Attanasio and Browning (1995) and Attanasio, Banks, Meghir, and Weber (1999) argue that the change in the size of a household over time is a significant determinant of the hump in consumption. However, more recent research has generally found that demographic factors alone cannot account for the pattern of lifetime consumption.[3]

Thurow (1969), for example, suggested that the age-consumption profile may be hump shaped due to borrowing constraints. That is, individuals are prevented from shifting as much wealth as they would like from later in life to finance consumption earlier. Another possibility is that individuals face

[3] For example, compare the findings of Attanasio and Browning (1995) with those of Attanasio, Banks, Meghir and Weber (1999). The first paper concludes that the hump can be entirely explained by demographic factors while the second finds an important role for income uncertainty.

income uncertainty and must die with non-negative assets. This creates a motive for precautionary savings that could lead to consumption rising with income early in life. Both Attanasio, Banks, Meghir and Weber (1999) and Gourinchas and Parker (2002) emphasize this point. Fernández-Villaverde and Krueger (2001) argue that households accumulate durables early in life as a way of insuring against income uncertainty. In their model, the stock of durables provides insurance by acting as collateral for consumption loans.

In addition, Heckman (1974) and Bullard and Feigenbaum (2003) explore the possibility that substitutability between consumption and leisure, rather than market incompleteness, may play an important role. That is, if preferences are such that consumption and leisure are substitutes, individuals may choose to consume more during the periods of their life when they spend the largest fraction of their time engaged in market work.

The main finding of our paper is that lack of annuity markets by itself might provide a quantitatively plausible explanation for actual life cycle consumption profiles when we abstract from social security. Once social security is introduced, the model displays consumption profiles that, while still hump shaped, are too steep early in life and consumption peaks too late in life.

The remainder of the paper is organized as follows. The next section surveys some of the empirical literature estimating life cycle consumption profiles. From this we obtain some basic statistics from data that we can also compute for our model economies. In the third section, we present a simple partial equilibrium model to provide intuition on how a lack of annuity markets can deliver a hump-shaped consumption profile. A general equilibrium model, one that incorporates social security, and its calibration are described in Section 4. Numerical findings are reported in Section 5, and Section 6 provides some concluding remarks.

2 Empirical Consumption Profile

Both Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2002) have estimated life cycle consumption profiles using data from the Consumer Expenditure Survey. Both find that consumption rises early in life, peaks sometime between age 45 and 55, and declines after that. This work is useful for our purposes because their estimation procedure controls for family composition and cohort (growth) effects. We abstract from the first in our theoretical analysis, and, although we incorporate technological progress in our model, we also correct for growth in computing our theoretical consumption profiles.

Fernández-Villaverde and Krueger (2002) obtain the profile for nondurable consumption expenditures of an adult equivalent for ages 22 through 87 shown in Figure 1.

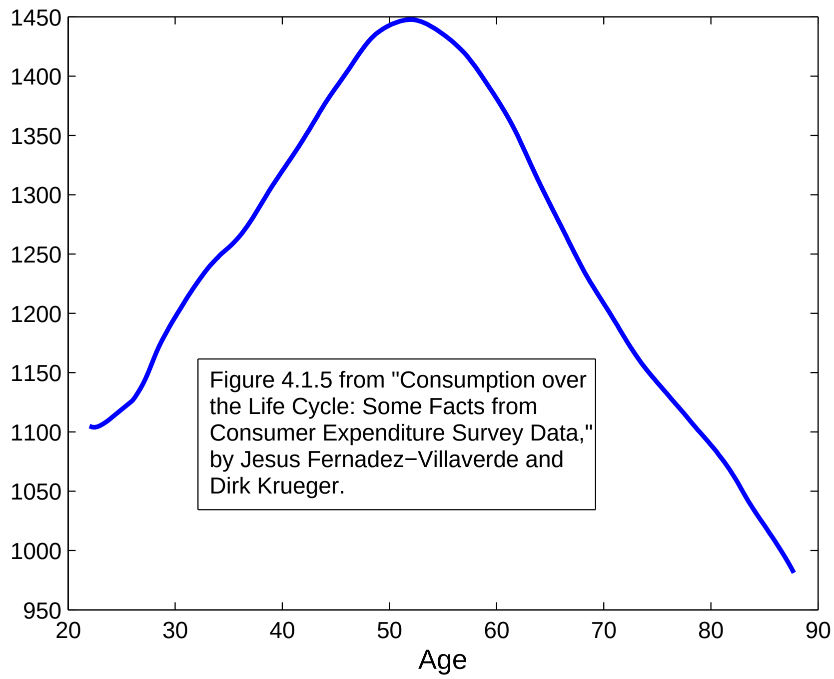

Figure 1. Nondurable Consumption over the Life Cycle

According to their estimates, consumption peaks at age 52 and the ratio of consumption at this maximum to consumption at age 25 is equal to 1.29. We use these numbers as data benchmarks with which to compare our model results. Gourinchas and Parker (2002), using a broader definition of consumption, compute a consumption profile for ages 26 through 65. They find that consumption peaks near age 45 and the ratio of peak consumption to age 25 consumption is close to 1.12.

3 A Simple Partial Equilibrium Model

To clarify why a lack of annuity markets can lead to a hump shaped lifetime consumption profile we first study a very simple endowment economy. Each period one agent is born that lives a maximum of I periods. Assume that the lifetime endowment pattern is given by

$$y_i = \begin{cases} 1 & \text{for } i < I_M \\ 0 & \text{for } i \geq I_M \end{cases},$$

where I_M is the mandatory retirement age. That is, individuals receive one unit of income in each period until they retire, at which point they must finance consumption with accumulated savings. A new born in this economy solves the following problem:

$$\max \sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=0}^{i-1} s_j \right) \frac{c_i^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{aligned} & c_i + \Lambda_i a_{i+1} = R(a_i + b) + y_i, \qquad i = 1, 2, \ldots, I, \\ & a_1 = 0. \end{aligned}$$

Here c_i is consumption of an age-i individual, a_i is asset holdings, y_i is endowment income, and s_j is the conditional probability of surviving from age j to age $j+1$. Non-annuitized assets of individuals who die in a given period are distributed to all living individuals as a lump sum transfer b. The interest factor, R, is taken parametrically in this partial equilibrium model.

We allow for zero, partial or complete annuitization of wealth by assuming a value for $\lambda \in [0, 1]$, which is the fraction of assets that are annuitized. For a given value of λ, the savings required of an individual who would like a_{i+1} assets available at the beginning of the following period is $\Lambda_i a_{i+1}$, where $\Lambda_i = 1 - \lambda(1 - s_i)$. This implies that

$$\Lambda_i a_{i+1} = \lambda s_i a_{i+1} + (1 - \lambda) a_{i+1}.$$

The first term on the right hand side of this expression is savings in the form of annuitized assets and the second term is savings in the form of assets that are not annuitized. Note that s_i is the actuarially fair price for a one period annuity sold to an individual of age i. If $\lambda = 1$, then there are complete annuity markets. As long as $\lambda < 1$, there will be unintended bequests b. These are computed as

$$b = \frac{\sum_{i=1}^{I-1} \left[\prod_{j=1}^{i} s_{j-1} \right] (1-\lambda)(1-s_i) a_{i+1}}{\sum_{i=1}^{I} \prod_{j=1}^{i} s_{j-1}}.$$

Given values for the model parameters and the interest rate R, it is straight forward to solve for the lifetime consumption path that would be chosen by individuals in this economy. To calibrate reasonable survival probabilities, we assume that individuals start their economic life at age 21 (corresponds to $i = 1$) and live to a maximum age of 100 ($I = 80$). They retire at age 65 ($I_M = 45$). We use survival probabilities published by the Social Security Administration for the cohort born in 1950. In addition, we assume log utility, which corresponds to $\gamma = 1$. In this case, the life cycle consumption-saving decision is determined by a sequence of Euler equations that can be written as follows:

$$\frac{c_{j+1}}{c_j} = \frac{\beta s_j R}{\Lambda_j},$$

where the right hand side reduces to $\beta s_j R$ in the absence of annuity markets and to βR when all assets are annutitized.

We are interested in determining how the consumption profile depends on the value of λ and the value of the interest factor R relative to the subjective discount factor β. Assuming $\beta = 0.96$, we first consider a value of R such that $\beta R = 1$. Figure 2 shows the consumption profile in this case for three values of λ: 0, 0.3, and 1. One can see that, if there are perfect annuity markets, consumption will be constant over the individual's lifetime, which is also clear from the Euler equation, as the right hand side equals one in this case. If there are no annuities available, individual consumption is declining over time. The intuition for this is that individuals, because they face a probability of not surviving to enjoy the fruits of their savings, discount the future more heavily than if actuarially fair annuities are available. This finding is robust to allowing individuals to hold a substantial amount of their saving in the form of annuities ($\lambda = 0.3$).

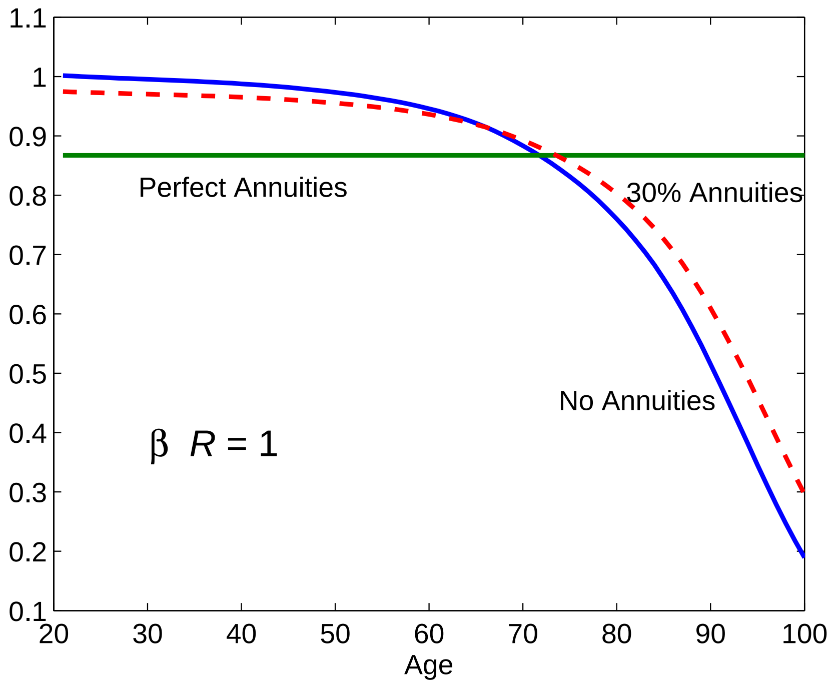

Figure 2. Consumption Profiles

If the consumption profile is to be hump shaped, consumption must increase early in life. To illustrate this, we choose R so that $\beta R = 1.02$. In this case, consumption would rise throughout life if individuals have access to perfect annuity markets. This is because the price of the annuity, which is falling as an individual ages, compensates for the increasing effective rate of discount due to survival probabilities falling as an individual ages.

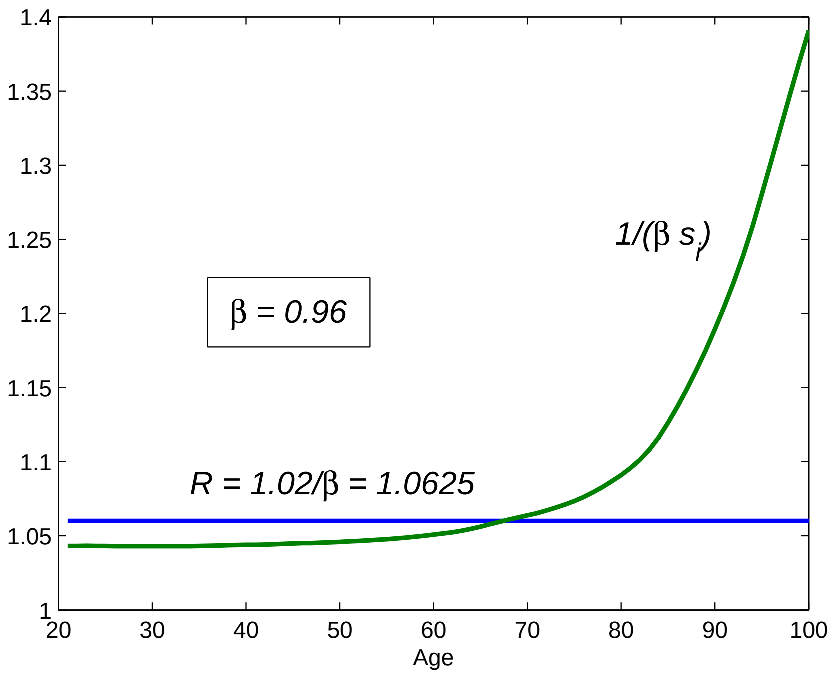

Figure 3. Subjective and Market Discount Factors

A lack of annuity markets, however, means that individuals are not compensated for their increasing effective rate of discount and consumption may decline in the later stages of life. This happens when an individual's effective rate of discount is larger than the interest rate. Figure 3 shows the period-by-period trade-off that the individual faces in his consumption-saving choice for $\beta = 0.96$ and $R = 1.02/\beta$. As long as the market discount factor given by the gross real interest rate exceeds the subjective discount factor adjusted for the conditional survival probability of that age, consumption grows. Once the reverse is true, consumption has reached its peak and starts to decline.

Figure 4 illustrates consumption profiles when there are perfect annuity markets, when there are no annuities, and when 30% of assets is annuitized.

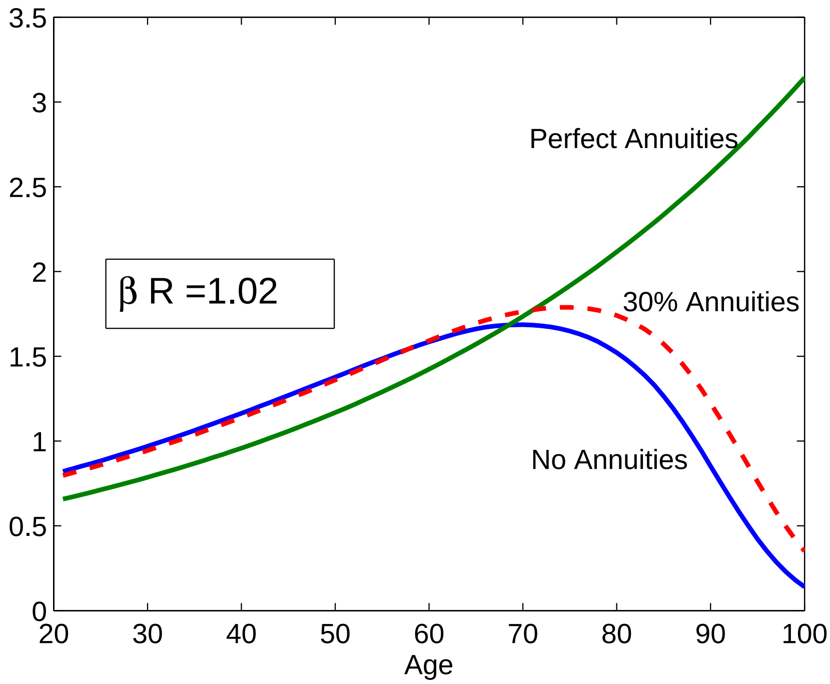

Figure 4. Consumption Profiles

4 A General Equilibrium Model

In this section, we will describe a fully calibrated general equilibrium life cycle model of the sort studied by Auerbach and Kotlikoff (1987), İmrohoroğlu, İmrohoroğlu, and Joines (1995), Ríos-Rull (1996), and Fernández-Villaverde and Krueger (2001), among others.

4.1 The Environment and Demographics

We use a stationary overlapping generations setup. At each date t, a new generation of individuals is born and the population growth rate is η. Individuals face long but random lives with a maximum possible age I. Lifespan uncertainty is described by s_i, the conditional probability of surviving from age i to $i+1$. We assume a stationary population by making the survival probabilities and the population growth rate time-invariant.[4] Total population is given by

$$N_t \sum_{i=1}^{I} \frac{\Pi_{j=1}^{i} s_{j-1}}{(1+\eta)^{i-1}},$$

where N_t denotes the number of individuals born in period t. Given that we assume stationary demographics, the fraction of the total population that is of age i is constant over time. These cohort shares, $\{\mu_i\}_{i=1}^{I}$, are given by

$$\mu_i = \frac{s_{i-1}}{(1+\eta)} \mu_{i-1}, \text{ for } i = 2, ..., I, \tag{1}$$

and

$$\sum_{i=1}^{I} \mu_i = 1.$$

4.2 Technology

There is a representative firm with access to a constant returns to scale Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} (A_t H_t)^{1-\alpha}, \tag{2}$$

where K_t and H_t are aggregate capital and labor inputs, respectively, and α is capital's output share. There is exogenous labor-augmenting technological growth at the rate $g > 0$:

$$A_{t+1} = (1+g) A_t. \tag{3}$$

The capital stock depreciates at the rate δ and follows the law of motion

$$K_{t+1} = (1-\delta) K_t + X_t, \tag{4}$$

where X_t is aggregate investment in period t.

4.3 Households

Individuals differ by their date of retirement. There are M possible retirement dates (I_m for $m = 1, \ldots, M$) and individuals know the date of their

[4] For studies that examine the quantitative impact of time-variation in either demographic variable on social security reform, see Kotlikoff, Smetters and Walliser (1999) and De Nardi, İmrohoroğlu and Sargent (1999), among others.

retirement at birth. The fraction of individuals with retirement date I_m is denoted by π_m.

An individual of type m born at time t solves the following problem:

$$\max \sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=1}^{i-1} s_j \right) \frac{\left[c_{i,m,t+i-1}^{\phi} (1 - h_{i,m,t+i-1})^{1-\phi} \right]^{1-\gamma}}{1-\gamma}, \tag{5}$$

subject to

$$\begin{aligned} c_{i,m,t+i-1} + \Lambda_i a_{i+1,m,t+i} \quad &= \quad R_{t+i-1}(a_{i,m,t+i-1} + b_{t+i-1}) \\ &\quad +(1-\tau_s) w_{t+i-1} \varepsilon_i h_{i,m,t+i-1} + S_{i,m,t+i-1}, \end{aligned} \tag{6}$$

where β is the subjective discount factor, R_{t+i-1} is the interest rate factor, τ_s is the social security payroll tax, $S_{i,m,t+i-1}$ is the social security benefit paid to an individual of age i and type m, $a_{i+1,m,t+i}$ is the amount of assets to be available at age $i+1$, ε_i is the efficiency weight of an individual at age i, and $h_{i,m,t+i-1}$ is hours supplied by an age-i individual of type m at time $t+i-1$. As in Section 3, we use $\lambda \in [0,1]$ to indicate the degree of completeness of private annuity markets, and $\Lambda_i = 1 - \lambda(1 - s_i)$. We assume that accidental bequests, if they exist, are returned to all surviving individuals, regardless of age, in a lump sum denoted by b_{t+i-1}.

Finally, we assume that all individuals are born with zero wealth and will exhaust all accumulated wealth at the maximum achievable age I, so that $a_{1,m,t} = a_{I+1,m,t+I} = 0$ for all m and t.

4.4 Social Security

There is an unfunded social security system in our economy. Benefits are linked to average lifetime earnings in a manner consistent with the Social Security Administration's (SSA) computation. An individual born at date t receives total labor income over the life cycle equal to

$$\sum_{j=1}^{I_m-1} w_{t+j-1} \varepsilon_j h_{j,m,t+j-1},$$

where I_m is the retirement age for this individual and $t + I_m - 1$ is the date of retirement. To obtain the indexed annual income (similar to the notion of Average Indexed Monthly Earnings calculated by the SSA), we need to multiply past earnings up to the time of retirement by a 'productivity factor', with earnings that are in the more distant past getting a higher factor. For an individual who retires at age I_m at date $t+I_m-1$, past earnings are scaled up so that the most recent income before retirement (at date $t+I_m-2$) is multiplied by $(1+g)^0$, income from the period preceding that one is multiplied by $(1+g)^1$, and so on, until the first working age income for this individual, $w_t\varepsilon_1 h_{1,m,t}$, is multiplied by $(1+g)^{I_m-2}$. Therefore, for an individual who retires at time $t+I_m$, total indexed labor income over the life cycle is given by

$$\sum_{j=1}^{I_m-1} w_{t+j-1}(1+g)^{I_m-1-j}\varepsilon_j h_{j,m,t+j-1}.$$

Retirement benefits for an age i individual who retires at age I_m in date $t+i-1$ is a fraction θ_m of average lifetime indexed income (the replacement rate depends on the age of retirement).

$$S_{i,m,t+i-1} = \begin{cases} \frac{\theta_m}{I_m-1}\sum_{j=1}^{I_m-1} w_{t+j-1}(1+g)^{I_m-1-j}\varepsilon_j h_{j,m,t+j-1} & \text{for } i \geq I_m \\ 0 & \text{for } i < I_m \end{cases}.$$

We calibrate the replacement rate to data and use the pay-as-you-go requirement

$$\sum_{m=1}^{M}\pi_m \sum_{i=I_m}^{I}\mu_i S_{i,m,t} = \tau_s \sum_{m=1}^{M}\pi_m \sum_{i=1}^{I_m-1}\mu_i w_t \varepsilon_i h_{i,m,t}$$

to endogenously calculate the social security tax rate. In this formula, $\{\pi_m\}_{m=1}^{M}$ is the fraction of individuals who retire at age I_m.

5 Competitive Equilibrium

A competitive equilibrium with stationary demographics consists of a social security tax rate τ_s, and sequences indexed by t for unintended bequests b_t, household allocations $\{\{c_{i,m,t}, a_{i+1,m,t+1}, h_{i,m,t}\}_{i=1}^{I}\}_{m=1}^{M}$, factor demands K_t and H_t, and factor prices w_t and R_t such that

1. The household allocation solves the individuals' maximization problem.

2. Factor demands solve the stand-in firm's profit maximization problem, which implies that

$$\begin{aligned} w_t &= (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} A_t^{1-\alpha}, \\ R_t &= \alpha\left(\frac{A_t H_t}{K_t}\right)^{1-\alpha} + 1 - \delta. \end{aligned}$$

3. Aggregate quantities are obtained as weighted averages of optimal cohort decision rules where the weights are the constant population shares.

$$
\begin{aligned}
K_t &= \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I} (a_{i,m,t} + b_t)\mu_i, \\
H_t &= \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} \mu_i \varepsilon_i h_{i,m,t}, \\
b_t &= \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I-1} \frac{\mu_i (1 - s_i)(1 - \lambda) a_{i+1,m,t}}{1 + \eta}.
\end{aligned}
$$

4. The social security system is unfunded:

$$
\tau_s = \frac{\sum_{m=1}^{M} \pi_m \sum_{i=I_m}^{I} \mu_i S_{i,m,t}}{\sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} \mu_i w_t \varepsilon_i h_{i,m,t}}.
$$

6 Solving for the Steady State Equilibrium

The competitive equilibrium defined above has the property that $c_{i,m,t}$, $a_{i,m,t}$, $S_{i,m,t}$, K_t, and w_t, for all i and m, grow at the constant rate of technological progress, g. For each variable Z_t define

$$
\widehat{Z} \equiv \frac{Z_t}{A_t}.
$$

Also, let $\widehat{\beta} = \beta(1+g)^{\phi(1-\gamma)}$. Then, we can find the time-invariant quantities $\{\{\widehat{c}_{i,m},\, h_{i,m},\, \widehat{a}_{i+1,m},\, \widehat{S}_{i,m}\}_{i=1}^{I}\}_{m=1}^{M}$, $\widehat{K}$, H, $\widehat{b}$, and prices $\widehat{w}$ and R by solving the following set of equations:

1. A set of first order conditions for work effort for $i = 1, 2, \ldots, I_m - 1$ and $m = 1, 2, \ldots, M$:

$$
\begin{aligned}
& \phi \widehat{c}_{i,m}^{\,\phi(1-\gamma)-1} (1 - h_{i,m})^{(1-\phi)(1-\gamma)} (1 - \tau_s) \widehat{w} \varepsilon_i \\
& + \sum_{j=I_m}^{I} \widehat{\beta}^{\,j-i} \left(\prod_{k=i}^{j-1} s_k \right) \frac{\theta_m \widehat{w} \varepsilon_i}{I_m - 1} \phi \widehat{c}_{j,m}^{\,\phi(1-\gamma)-1} \\
= \;& (1 - \phi) \widehat{c}_{j,m}^{\,\phi(1-\gamma)} (1 - h_{i,m})^{(1-\phi)(1-\gamma)-1},
\end{aligned}
$$

where the second term on the left hand side accounts for the effect of current work effort decision on future retirement benefits.

2. A set of intertemporal first order conditions for $i = 1, 2, \ldots, I-1$ and $m = 1, 2, \ldots, M$:

$$\widehat{c}_{i,m}^{\phi(1-\gamma)-1}(1+g)\Lambda_i(1-h_{i,m})^{(1-\phi)(1-\gamma)} = R\widehat{\beta} s_i \widehat{c}_{i+1,m}^{\phi(1-\gamma)-1}(1-h_{i+1,m})^{(1-\phi)(1-\gamma)},$$

[note that $h_{i,m} = 0$ for $i \geq I_m$].

3. A set of budget constraints,

   (a) for $i = 1, 2, \ldots, I_m - 1$ and $m = 1, 2, \ldots, M$:

$$\widehat{c}_{i,m} + (1+g)\Lambda_i \widehat{a}_{i+1,m} = R(\widehat{a}_{i,m} + \widehat{b}) + (1-\tau_s)\widehat{w}\varepsilon_i h_{i,m},$$

   (b) for $i = I_m, I_m + 1, \ldots, I$ and $m = 1, 2, \ldots, M$:

$$\widehat{c}_{i,m} + (1+g)\Lambda_i \widehat{a}_{i+1,m} = R(\widehat{a}_{i,m} + \widehat{b}) + \frac{\theta_m}{I_m - 1}(1+g)^{I_m - i - 1} \sum_{j=1}^{I_m - 1} \widehat{w}\varepsilon_j h_{j,m}.$$

4.
$$\widehat{a}_{I+1,m} = 0.$$

5.
$$\widehat{a}_{1,m} = 0.$$

6.
$$\widehat{K} = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I} \mu_i (\widehat{a}_{i,m} + \widehat{b}).$$

7.
$$H = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} \mu_i \varepsilon_i h_{i,m}.$$

8.
$$\widehat{b} = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I-1} \frac{\mu_i (1-s_i)(1-\lambda)\widehat{a}_{i+1,m}}{1+\eta}.$$

9.
$$\widehat{w} = (1-\alpha)\left(\frac{\widehat{K}}{H}\right)^{\alpha}.$$

10.
$$R = \alpha \left(\frac{H}{\widehat{K}}\right)^{1-\alpha} + 1 - \delta.$$

6.1 Calibration

Individuals in our economy are assumed to begin their economic life at age 21 (that is, $i = 1$ corresponds to age 21) and live until a maximum age of 100 ($i = 80$). The conditional survival probabilities from age i to age $i + 1$, $\{s_i\}_{i=1}^{I}$, are taken from estimates provided by the Social Security Administration (SSA) for a cohort born in 1950 [see Bell and Miller (2002)]. The population growth rate, η, is assumed to be 1.2 percent per year. The age specific efficiency weights for labor hours, $\{\varepsilon_i\}_{i=1}^{I}$, are based on estimates from Hansen (1993).

Retirement can occur at $M = 9$ possible ages, which correspond to ages 62-70 ($I_m \in \{42, 43, ..., 50\}$)[5]. The fraction of individuals that retire and begin collecting social security at age I_m, π_m, was obtained from the SSA.[6] In addition, we calculated the age adjusted social security replacement rate, θ_m, from SSA data on benefits as a percentage of the Primary Insurance Amount (PIA) by the age at which benefits begin.[7] In particular, we chose $\{\theta_m\}_{m=1}^{M}$ to be consistent with these benefit percentages, the fraction of individuals that retire at these ages, and the U.S. average social security replacement rate of 0.45.

Since the primary focus of this paper is on the role of annuities, as opposed to the role of nonseparable utility studied in Bullard and Feigenbaum (2003), we choose the risk aversion parameter, γ, equal to one in our benchmark calibration. That is, the period utility function is separable, $U(c_i, h_i) = \phi \log c_i + (1 - \phi) \log(1 - h_i)$.

The remaining parameters of preferences and technology are chosen so that our model is consistent with various facts characterizing the U.S. macroeconomy. The growth rate of labor augmenting technological progress, g, is chosen so that our model is consistent with the measured growth rate of real

[5] In our model, the retirement age is the exogenously determined age at which individuals begin collecting social security and also stop working. In the U.S. economy, these two events do not need to coincide. French (2004) studies a life cycle model where labor participation and the date at which social security payments begin are separate decisions made by utility maximizing individuals.

[6] Age 62 is the first age at which individuals are eligible to collect social security. By delaying, an individual will receive higher payments (θ_m increases with m), but there is no incentive to delay beyond age 70. Hence, the fraction of individuals that begin collecting social security after age 70 is trivial. The values we used for $\{\pi_m\}_{m=1}^{M}$ were obtained directly from Social Security Administration and are similar to numbers found in Table 6B5 of the 2004 Annual Statistical Supplement of the Social Security Administration.

[7] These data can be found at the following web site:
`www.ssa.gov/OACT/ProgData/ar_drc.html`.

per capita income. Given this and the average capital output ($K/Y = 3.32$) and investment output ($X/Y = 0.25$) ratios measured from U.S. data, we obtain the depreciation rate as follows:

$$\delta = \frac{X/Y}{K/Y} - g - \eta - g\eta.$$

The capital share parameter, α, is set equal to 0.36, which is consistent with measures of capital's share from NIPA data. Finally, the preference parameters β and ϕ are chosen to target the capital output ratio and the fraction of time spent on market activities (taken to be 0.31).

Table 1 summarizes our calibration.

Table 1. Benchmark Calibration

Demographics		
first age	$i = 1$	21
maximum age	$I = 80$	100
population growth rate	η	0.012
conditional survival probabilities	$\{s_i\}_{i=1}^{I}$	SSA, cohort born in 1950
efficiency weights	$\{\varepsilon_i\}_{i=1}^{I}$	Hansen(1993)
Technology		
capital share parameter	α	0.36
depreciation rate	δ	0.047
productivity growth rate	g	0.0165
Preferences		
subjective discount factor	β	0.9726
coefficient of relative risk aversion	γ	1.0
share of consumption	ϕ	0.368

Social Security Parameters ($\Sigma_{m=1}^{M}\theta_m/M = 0.45$)

I_m	62	63	64	65	66	67	68	69	70
π_m	0.46	0.14	0.06	0.19	0.06	0.02	0.02	0.01	0.05
θ_m	0.39	0.43	0.46	0.49	0.52	0.56	0.59	0.62	0.65

In alternative calibrations, we explore the relative impact of nonseparable utility in addition to lack of annuity markets on the shape of the consumption profile. Hence, we also consider cases where $\gamma = 4$ and $\gamma = 7$. This requires re-calibrating the parameters β and ϕ in order to hit our targets. Table 2 summarizes the values used in these alternative calibrations.

Table 2. Alternative Calibrations

	β	ϕ
$\gamma = 1$	0.9726	0.368
$\gamma = 4$	1.0127	0.380
$\gamma = 7$	1.0545	0.386

7 Results

Here we describe consumption profiles for three model economies. The first is one with complete annuity markets and no social security. The second has no annuity markets and still no social security. The third case, the one used for calibration in section 6.1, adds social security, which serves as a partial substitute for the missing annuity markets.

In each case, we consider three values for the risk aversion parameter, $\gamma = 1$, 4, and 7. The $\gamma = 1$ case is one in which the utility function is separable between consumption and leisure, so leisure (retirement, in particular) has no effect on the marginal utility of consumption. The other two cases involve non-separable utility, so changes over the life cycle in the amount of time individuals spend working will also affect the shape of the consumption profile.[8] A summary of our quantitative findings is contained in Table 3.

[8] The effect of this non-separability on consumption profiles is studied in Heckman (1974) and Bullard and Feigenbaum (2003).

Table 3. Quantitative Findings								
	θ	τ_s	X/Y	K/Y	R	H	S_1	S_2
DATA	0.45	0.10	0.25	3.32		0.310	52	1.29
Case 1	No Social Security and Perfect Annuities							
$\gamma = 1$	0.0	0.0	0.280	3.718	1.050	0.310	NA	1.34
$\gamma = 4$	0.0	0.0	0.300	3.979	1.044	0.318	59	1.34
$\gamma = 7$	0.0	0.0	0.309	4.103	1.041	0.323	59	1.39
Case 2	No Social Security and No Annuities							
$\gamma = 1$	0.0	0.0	0.269	3.573	1.054	0.297	60	1.21
$\gamma = 4$	0.0	0.0	0.311	4.131	1.041	0.311	57	1.28
$\gamma = 7$	0.0	0.0	0.355	4.719	1.030	0.327	57	1.30
Case 3	Social Security and No Annuities							
$\gamma = 1$	0.45	0.107	0.25	3.32	1.062	0.310	67	1.61
$\gamma = 4$	0.45	0.107	0.25	3.32	1.062	0.310	61	1.60
$\gamma = 7$	0.45	0.107	0.25	3.32	1.062	0.310	61	1.59

Note: NA : the ratio is still rising at maximum attainable age.
S_1 is the age at which life cycle consumption attains its maximum.
S_2 is the ratio of maximum consumption to consumption at age 25

Table 3 reports various statistics including the three that were used to calibrate the model in section 6.1: the investment-output ratio, the capital-output ratio, and the average fraction of time spent working. Since we calibrated the model under the assumptions of Case 3 (social security and no annuities), the results hit our targets exactly in this case.[9] In addition, we report the social security tax rate, the steady state interest rate, the age at which consumption reaches its maximum, and the ratio of maximum consumption to age 25 consumption.[10]

[9] Note that, as shown in Table 2, we have different calibrated parameters depending on the value of γ.

[10] Note that the social security tax rate that maintains the pay as you go system in Case 3 is close to the actual social security tax rate of about 10%.

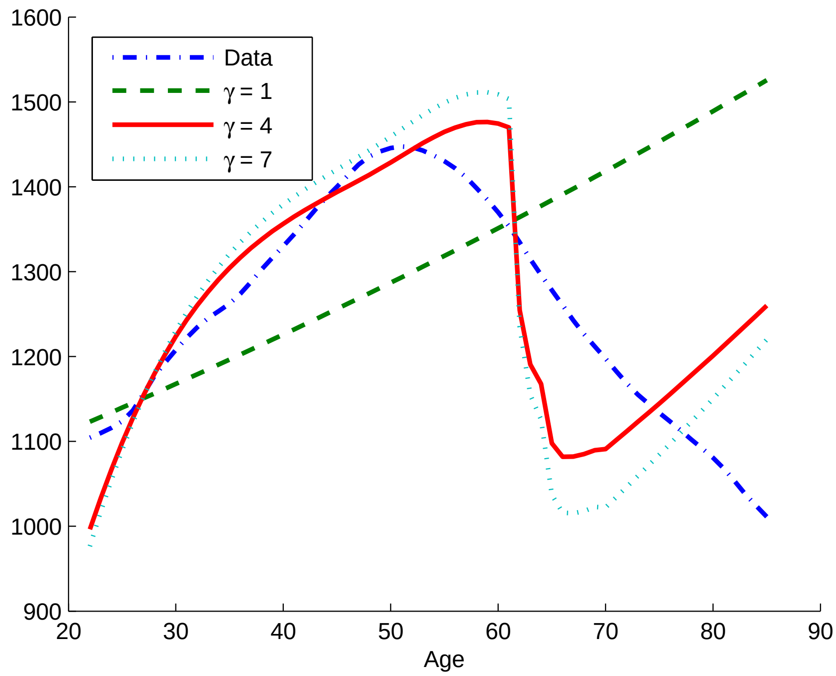

Figure 5. No Social Security and Perfect Annuities

Figure 5 shows the consumption profiles for our economy with complete annuity markets and no social security, along with the consumption profile estimated from the Consumer Expenditure Survey shown in Figure 1. Consumption from the model economies has been normalized so that the life cycle profile has the same mean as the one estimated from actual data. Given perfect annuity markets, consumption profiles are hump shaped only when utility is non-separable. Consumption monotonically increases throughout life in the $\gamma = 1$ case. In the other two profiles shown, consumption peaks at about age 59 and then rises again after retirement (consumption peaks at age 52 in the data). This is very much inconsistent with what is observed in actual data.

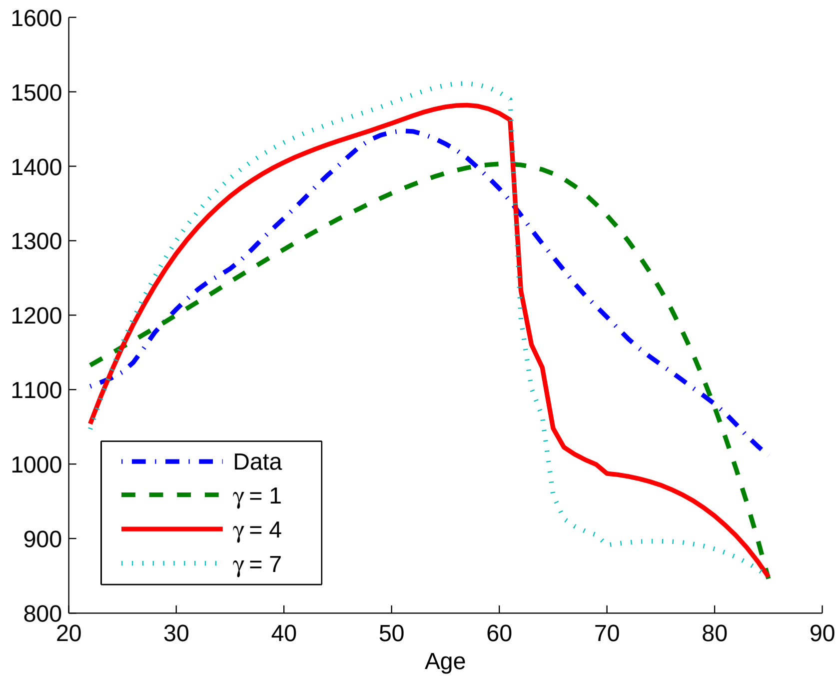

Figure 6. No Social Security and No Annuity Markets

When annuity markets are shut down, all consumption profiles have a hump shape (see Figure 6). The intuition for why a hump is observed in the $\gamma = 1$ case is the same as was discussed in section 3; the effective rate of discount that combines the subjective rate of discount and the unconditional probability of survival eventually exceeds the market rate of discount measured by the interest rate. Although consumption peaks a bit later than in the data, we view these results as indicating that lack of annuity markets is a quantitatively plausible explanation for the shape of the consumption profiles estimated from actual data. This conclusion is reinforced by our robustness experiments that we describe in section 7.1.

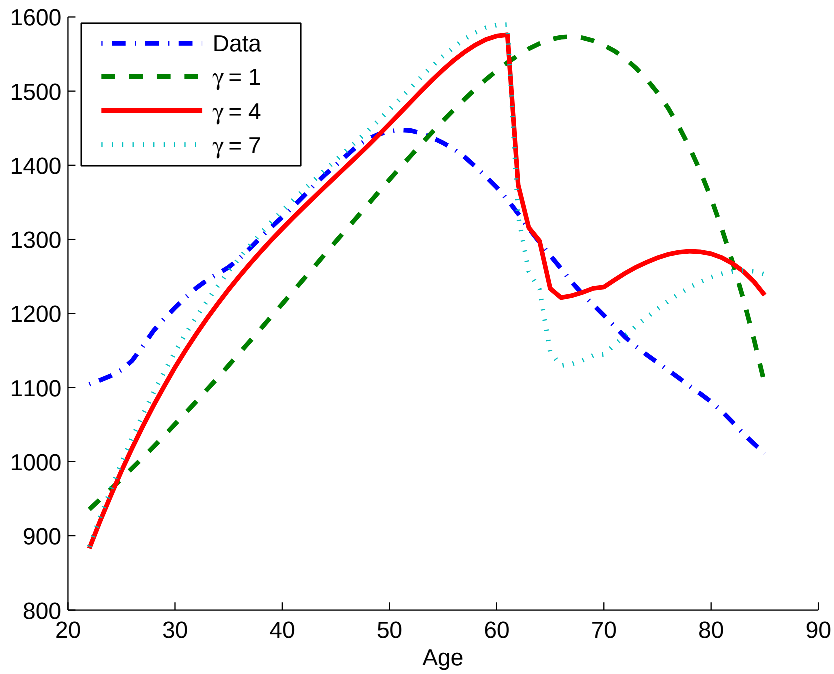

Figure 7. Social Security and No Annuity Markets

Once social security is introduced, consumption peaks too late relative to the data and the ratio of maximum consumption to age 25 consumption is too large. This can be seen both in Table 3 and in Figure 7. In the figure, we also see an upward sloping profile after retirement (except for the $\gamma = 1$ case) that we do not see in actual data.

It is also the case, however, that the interest rate for Case 3 is quite a bit higher than the interest rates obtained in our experiments without social security. To see if this higher interest rate, rather than the partial annuity provided by social security, is responsible for our findings, we re-calibrated the parameters in Case 2 to hit the targets defined in section 6.1. This implies an interest rate that is the same as in Case 3. We find consumption profiles that are essentially identical to those reported on for Case 2 in Table 3 and Figure 6. Hence, it is not the higher interest rate that is driving the differences in the results obtained for Cases 2 and 3 in Table 3.

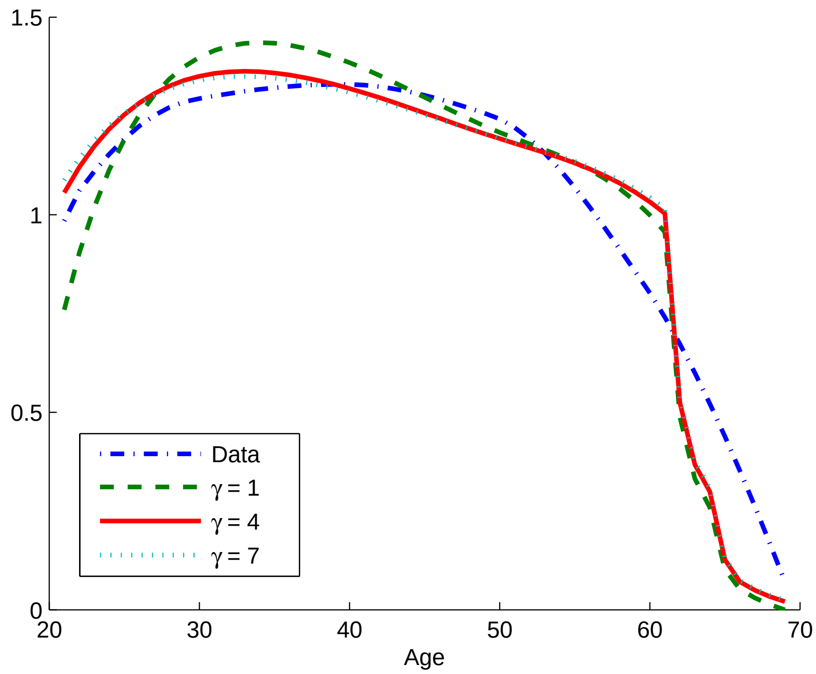

Figure 8. Hours Profile, Social Security and No Annuity Markets

Our model also has implications for the life cycle profile of hours worked. It turns out that the hours profiles are very similar in all the cases considered, so we only show the profiles for the economy with no annuity markets and social security in Figure 8. These profiles are almost completely determined by the efficiency weights and by our retirement assumptions. Annuity markets, or lack thereof, have essentially no impact on the shape the hours profile.

7.1 Robustness

In order to assess the robustness of our results on the lack of annuity markets, we have solved the model for a wide range of plausible parameter values, both with and without social security. In particular, we have considered all possible combinations of the parameter values shown in Table 4.

Table 4. Parameter Space for Robustness Check

Parameter	Min Value	Max Value	Increment
γ	1	4	3
β	0.98	1.02	0.01
δ	0.03	0.09	0.02
α	0.30	0.40	0.02
ϕ	0.30	0.40	0.02

In Table 5 we summarize our findings regarding the shape of the steady state life cycle consumption profile. We report the minimum and maximum values for the two key statistics across all parameterizations considered. The two statistics are the age at which consumption is at its maximum (S_1) and the ratio of maximum consumption to consumption at age 25 (S_2).

Table 5. Results of Robustness Check

Case	min S_1	max S_1	min S_2	max S_2
Social Security, $\gamma = 1$	60	74	1.21	2.74
Social Security, $\gamma = 4$	57	61	1.33	1.78
No Social Security, $\gamma = 1$	22	70	1	1.99
No Social Security, $\gamma = 4$	37	60	1.09	1.49
Data	$S_1 = 52$		$S_2 = 1.29$	

S_1 is the age at which life cycle consumption attains its maximum.
S_2 is the ratio of maximum consumption to consumption at age 25.

This exercise reinforces the findings from our calibrated model. If one ignores the existence of social security, a life cycle model with no annuity markets appears to account for the key properties of consumption over the life cycle. However, once social security is introduced, consumption is predicted to peak too late in life and the size of the hump in consumption is too large.

8 Concluding Remarks

The empirical life cycle consumption profile in the U.S. has a hump around age 50. This is typically considered a puzzle since the complete markets life cycle model would produce a smooth consumption profile over the life cycle. In this paper we explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a calibrated general equilibrium life cycle model where markets are otherwise complete.

If an annuity market (or a partial substitute) is unavailable, then the decline in the survival probabilities over the life cycle as an individual ages leads to a heavier discounting of the future as they grow older. This allows for the possibility, depending on the value of the interest rate, that consumption might increase early in life when survival probabilities are high and the effective rate of discount is low. As survival probabilities fall, the slope of the consumption profile may become negative.

In addition, because social security provides some insurance against uncertain lifetimes and therefore may provide an adequate substitute for missing annuity markets, we also study the shape of the consumption profile in a model with missing annuity markets and a pay-as-you-go social security system as in the U.S. Social security turns out to matter significantly.

The main finding of our paper is that lack of annuity markets by itself may provide a quantitatively plausible explanation for actual life cycle consumption profiles when we abstract from social security. Once social security is introduced, the model displays consumption profiles that are too steep and where consumption peaks too late in life. This finding would likely be reinforced if we were to introduce to the model assets held in other defined benefit pension plans. On the other hand, we have abstracted from many potentially relevant factors such as altruism, uninsurable income risk and borrowing constraints, and uninsurable risk after retirement such as a possibility of out-of-pocket expenses during illness. One or more of these factors may help generate a consumption profile that more closely matches the empirical profile.

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