# The Adjusted Solow Residual and Asset Returns

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**Abstract**

The purpose of this study is to examine effects of a measured aggregate productivity shock on asset returns. To achieve this, a simple equilibrium business cycle model is presented to show that an aggregate productivity shock can be identified as a factor affecting asset returns. Then, Solow's productivity residual is chosen as a proxy for the measured aggregate productivity shock. One valuable contribution of the study is its incorporation of recent macroeconomic developments on variable factor utilizations. In particular, this paper deviates from the conventional growth accounting framework to adjust for cyclical variations of the Solow residual. This study first shows that asset returns tie with capital returns based on the standard business cycle model without adjustment costs, and then empirically evaluates the relationship between the measured aggregate productivity shock and asset returns. Investigations based on post-World War II U.S. data uncover significant differences between the conventional Solow residual and the adjusted Solow residual in their dynamic effects on asset returns. The results from the VARs show that once variable capital utilization is controlled for, the measured aggregate productivity shock generates dynamics similar to what Basu, Fernald, and Kimball (2004) documented. More importantly, technology improvements have delayed effects on asset returns, which is somewhat difficult to be rationalized based on the frictionless model studied in this paper.

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# 1 Introduction

This paper's primary focus is on a macroeconomic factor and its implications on asset returns. More specifically, the purpose of this study is to identify an aggregate productivity shock as a macroeconomic factor, to measure it appropriately, and then to empirically evaluate its effects on asset returns using post World War II U.S. data.

It is a well-known fact that asset returns have been affected not only by firm-specific risks but also by macroeconomic risks. Although the firm-specific risks, (diversifiable risks) can be avoided by building a good portfolio, the macroeconomic risks (undiversifiable risks) cannot be avoided due to their nature. Since the macroeconomic risks are unavoidable yet have significant effects on the asset returns, many researchers have attempted to identify one or more variables as the macroeconomic risks and analyzed their impacts.

In the empirical finance literature, macroeconomic risks are considered *factors*. The first significant study in this area was conducted by Chen, Roll, and Ross (1986), who explored a set of economic variables systematically affecting asset returns.[1] Although a wide range of variables have been chosen as factors in this line of research, there has been little consensus on why such variables have to be the factors. In other words, the existing literature often fails to provide a theoretical justification for the factors that are chosen, especially when they are selected by fitting returns, rather than by being derived from explicit theoretical frameworks.

This study takes a different approach by establishing a link between theory and empirics. In particular, it identifies one of the factors based on a simple equilibrium model, and then empirically assesses its effects on asset returns. Based an equilibrium business cycle model, this study shows that an aggregate productivity shock can be identified as one of the factors affecting asset returns. In other words, this study provides a theoretical justification for an identifiable factor before it turns to the empirics.

Over the years, consumption-based asset pricing models have provided theoretical foundations for analyzing asset returns. Under the models, the key relationship between asset returns and a stochastic discount factor is closely related to the first-order condition of an investor's consumption and portfolio choice problem.[2] Another similar approach has looked at the production side. Cochrane (1991, 1996), Lamont (2000), Hall (2001), Jermann (1998), and Rouwenhorst (1995) studied asset pricing implications from a perspective of the production side of the economy. They derived the asset pricing relationship from the first order condition of the producer's problem. Indeed, this paper takes an approach similar to existing production-side asset pricing models, where asset returns are equal to investment returns.

[1] The variables include (1) the spread between long and short-term interest rates, (2) expected inflation, (3) unexpected inflation, (4) industrial production, and (5) the spread between high- and low-grade bonds.

[2] For a good survey on the equity premium puzzle, see Kocherlakota (1996).

This line of study is particularly useful to show how equilibrium business cycle models can be used to study various issues in finance. In the standard one-sector business cycle model, an aggregate productivity shock is important because it is considered one of the major sources of fluctuations in most macroeconomic variables in the absence of other shocks, such as preference shocks and monetary shocks. Clearly, a single source of uncertainty, the aggregate productivity shock in the model can be the natural candidate for a macroeconomic factor. The investment returns are exposed to the aggregate productivity shock in the equilibrium business cycle model, and the investment returns equal the asset returns in the production-side asset pricing model. Thus, if the two models are combined, the link between the aggregate productivity shock and the asset returns can be established.

To evaluate quantitative aspects of the relationship between aggregate productivity shock and asset returns, this paper attempts to estimate the fundamental equations using the U.S. data, rather than to calibrate the model and to run simulations to match the observed data. In that sense, this study shares a spirit with Lettau and Ludvigson (2001), who first identified the consumption-wealth ratio as one of factors based on an equilibrium model, and then empirically evaluated its effects.

For empirical investigations, Solow's productivity residual is chosen as a proxy for the measured aggregate productivity shock. This study, however, deviates from the conventional approach by incorporating recent macroeconomic developments on variable factor utilizations to adjust for the cyclical variation of the Solow residual. In particular, the *conventional* Solow residual is constructed based on the standard growth accounting framework.[3] The *adjusted* Solow residual, on the other hands, adjusts for variable capital utilization. The major differences between the two are their cyclical variations.

The cyclicality of the conventional Solow residual has been well-documented in the business cycle literature. A number of previous studies have already argued that variable factor utilization is one way to explain the observed cyclicality of the conventional Solow residual. Starting from Greenwood, Hercowitz, and Huffman (1988), a growing number of authors have studied variable factor utilization and its implications on equilibrium business cycle models. In particular, Shapiro (1996) presented a framework to adjust for the cyclicality of the conventional Solow residual using capacity utilization. Basu, Fernald, and Shapiro (2001) also considered variable factor utilization along with adjustment costs when they measured the Solow residual. Finally, Paquet and Robidoux (2001) examined the exogeneity of the adjusted Solow residual using Canadian data. To accommodate these recent developments, this study follows a methodology primarily built on the work by Paquet and Robidoux (2001) and Shapiro (1996).

The adjustment is not trivial because it substantially changes some characteristics of the conventional Solow residual. First, the variability of the Solow residual gets much smaller after the adjustment. Second, the Solow residual be-

[3]The *conventional* Solow residual indicates the usual Solow residual throughout the paper.

comes far less procyclical.[4] Indeed, Basu, Fernald, and Kimball (2004) argued that the technology improvements were contractionary after the adjustments.

This study contributes to the literature by exploring asset pricing implications of the variable capital utilization adjustment. While the existing literature examines business cycle implications of the adjustment, few studies investigate asset pricing implications. This study examines effects of the aggregate productivity shock on the asset returns based on two alternative measures: the conventional Solow residual and the adjusted Solow residual.

Empirical investigations of the study consists of two parts. First, linear approximation of the fundamental equations, derived from the equilibrium model, are estimated to evaluate the size of the effects of the adjusted Solow residual (and the conventional Solow residual) on asset returns. The results show that the adjustment of variable capital utilization does make big differences on the outcomes of estimations. However, the Granger causality test, which could empirically verifies the direction of the causality implied by the model, suggests that data do not support the implied casuality; the test results are at best ambiguous. Since the first step of the analysis uncovers interesting dynamic effects coming from the adjusted Solow residual, Vector Autoregressions (VARs) are employed as a second step to better understand dynamic effects of the measured productivity shock on the asset returns. The results from the VARs show that once variable capital utilization is controlled for, the effects of the measured aggregate productivity shock on the asset returns dramatically change. In particular, while the conventional Solow residual has a contemporaneous effect on the asset returns, the adjusted Solow residual has a delayed effect on the asset returns.

What is the intuition behind the results? It appears that the depreciation rate, which depends on the utilization rate, plays a non-negligible role in explaining observed dynamics in the model with variable capital utilization. In a simple equilibrium business cycle model, the depreciation rate is constant and a favorable shock measured by conventional Solow residual would increase asset returns on impact unambiguously. On the hands, when variable utilization is allowed, the asset returns become a function of the non-constant depreciation rate, which is determined by the variable utilization rate. Because the optimal utilization rate is a non-linear function of capital, labor, and the aggregate productivity shock, the degree of substitutability and complementarity among them appear to matter. Thus, if favorable aggregate productivity shock induces to decrease in labor, overall effects from the aggregate productivity shock on the asset returns on impact could be smaller than those in the convectional case.

The remainder of the paper is organized as follows. Section 2 and 3 describes the model economy and empirical specifications, respectively. Section 4 illustrates how the aggregate productivity shocks are measured. Section 5 describes data and time series characteristics of variables, accompanied by estimation

[4] The procyclicality, a comovement between the conventional Solow residual and output growth is one of the key features in RBC models. This study finds that the correlation coefficient between the conventional Solow residual and output growth is 0.9 while the correlation coefficient between the adjusted Solow residual and output growth is 0.2.

results. Section 6 concludes.

# 2 Model

A simple version of the equilibrium business cycle model is presented to derive key equations for empirical investigations. The economy is composed of a large number of homogeneous households whose utilities are determined by consumption of goods and labor. On the production side of the economy, identical firms produce homogenous goods. The firms own capital stock of the economy. There is only one source of uncertainty, an aggregate productivity shock. Markets are competitive and complete.

## 2.1 Households

A representative agent maximizes her expected lifetime utility subject to her budget constraint. The agent's utility function is assumed concave, strictly increasing, and twice continuously differentiable, while the type of the utility function is not assumed.[5]

$$\max_{C,L} E_o \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1}$$

$$0 < \beta < 1$$

where $\beta$ is the discount factor, $C_t$ is consumption in period $t$, and $L_t$ is labor in period $t$. The sequential budget constraint is:

$$C_t + P_t Z_t = w_t L_t + Z_{t-1}(P_t + D_{t-1}), \tag{2}$$

where $P_t$ is the asset price measured in consumption goods at time $t$ and $Z_t$ denotes the number of shares owned by the consumer at the beginning of $t$ and $D_t$ is dividend. Then, the consumption Euler equation associated with this problem becomes:[6]

$$u_{c_t} = \beta E_t u_{c_{t+1}} R_t, \tag{3}$$

where $R_t = \frac{P_{t+1}+D_t}{P_t}$ represents the rate of return on the asset between time $t$ and $t+1$.

[5]The purpose of this section is not to solve the model in details by specifying all functional forms but to derive the key relationship between an aggregate productivity shock and asset returns.

[6]See Appendix A.1 for the details of the derivations.

## 2.2 Firms

A representative firm has a constant-return-to-scale production function with output augmenting (Hicks-neutral) technical progress. $A_t$ is the aggregate productivity level (level of technology) in period $t$. The firm chooses labor, $L_t$, and utilized capital stock, $u_t K_t$, to maximize the expected discounted present value of the firm. A general production function, $G$ allows for variable capital utilization,

$$Y_t = G(A_t, u_t K_t, L_t), \tag{4}$$

where $Y_t$ is the output in period $t$, $K_t$ the capital stock at the beginning of the period, $u_t$ the rate of capital utilization, and $L_t$ labor in period $t$. When the technical progress is output augmenting, the production function can be rewritten as:

$$Y_t = A_t F(u_t K_t, L_t). \tag{5}$$

The capital stock is accumulated with the variable depreciation rate, $\delta_t$. There are no adjustment costs for capital. As in Greenwood et al. (1988), the depreciation rate is the increasing function of the utilization rate. This study assumes that the time $t$ depreciation rate of capital, $\delta_t$, is given by

$$\delta_t = \delta_0 + \frac{1}{\phi} u_t^{\phi}, \tag{6}$$

where $0 < \delta_0 < 1$ and $\phi > 1$. Since $\phi > 1$, depreciation increases with capital utilization in a convex manner. $\delta_0$ indicates the amount of depreciated capital stock due to rust, which does not depends on the level of utilization. On the other hands, $\frac{1}{\phi} u_t^{\phi}$ represents the amount of depreciated capital stock due to wear. And $\phi$ can be interpreted as the elasticity of marginal rate of depreciation. As $\phi$ increases, the shape of the depreciation rate curve become more convex, indicating that it is more costly to change the utilization rate.

The stock of capital is given by,

$$K_{t+1} = (1 - \delta_t) K_t + I_t. \tag{7}$$

When output is produced, the payments to labor, $w_t$, and investment, $I_t$, for the next period are made. The remaining portion becomes dividend, $D_t$, which is paid out at the end of the period,

$$D_t = Y_t + -w_t L_t - I_t. \tag{8}$$

The firm maximizes its net present discounted value:[7]

$$\max_{K,L,u} E_0 \sum_{t=0}^{\infty} \beta^t \nu_t (Y_t - w_t L_t - I_t), \tag{9}$$

subject to (5) & (7),

[7] In the firm's problem, choice variables are capital stock, $K$, labor, $L$ and the utilization rate, $u$.

where $\nu_t$ represents the price of capital or the marginal rate of substitution between time 0 and $t$ of the firm owners. Then, the first order conditions for capital, labor, and the utilization rate are:

$$\beta E_t \nu_t (A_t F_{1,t} u_t + 1 - \delta_t) = \nu_{t-1}, \tag{10}$$

$$A_t F_{2,t} = w_t, \tag{11}$$

$$A_t F_{1,t} K_t = u_t^{\phi-1} K_t, \tag{12}$$

where $F_{1,t}$ is the derivative of $F$ with respect to the first argument,$u_t K_t$, and $F_{2,t}$ the derivative of $F$ with respect to the second argument, $L_t$.

According to equation (10) and (11), the price of capital, $\nu_t$ is equal to the next period's expected marginal value product, and the wage rate is equal to the marginal product of labor. The left side of equation (12) shows the marginal benefit, which is the value of additional output from a higher utilization rate. The right side of equation (12) is the marginal cost, measured in terms of the replacement cost. This is the cost in terms of additional unit of capital being worn out due to a higher utilization rate. Thus, for an optimal choice of the utilization rate, the firm needs to equate the marginal benefit and the marginal cost.

$$u_t^* = (A_t F_{1,t})^{\frac{1}{\phi-1}} \tag{13}$$

Further more, once the optimal utilization rate is chosen, its associated level of depreciation rate is determined.[8]

$$\delta_t^* = \delta_0 + \phi (A_t F_{1,t})^{\frac{\phi}{\phi-1}} \tag{14}$$

## 2.3 Equilibrium

An equilibrium is defined as a set where firms maximize their present discounted values given their production technology and households maximize their utilities subject to their budget constraints. The equilibrium is efficient since it satisfies all the efficient allocation conditions. In addition, all goods that are produced are either invested or consumed.

$$Y_t = C_t + I_t. \tag{15}$$

Labor markets and financial markets also clear so that no excess demand or supply exists.

## 2.4 Key Relationships

Under the assumption of constant-returns-to-scale production and competitive markets, the key relationship between the asset returns and the aggregate productivity shock can be derived. The capital payment is the remainder of the

[8]One thing to notice here is that $F_{1,t}$, the derivative of $F$ with respect to the first argument is a function of $u_t, L_t$, and $K_t$.

value of the output after the payment to labor because of the homogeneity of degree one. Then, the dividend equation can be written as:

$$D_t = Y_t - w_t L_t - I_t = Y_t - (Y_t - A_t F_{1,t} u_t K_t) - (K_{t+1} - K_t - \delta_t K_t). \quad (16)$$

Rearranging terms, equation(16) can be written as:

$$\frac{D_t + K_{t+1}}{K_t} = A_t F_{1,t} u_t + 1 - \delta_t. \quad (17)$$

Using (10), the first order condition for the capital from the firm's problem and (3), the household Euler equation from the households problem, it is shown that $K_t = P_t$.[9] Then, the equation above becomes:

$$\frac{D_t + K_{t+1}}{K_t} = \frac{D_t + P_{t+1}}{P_t} = R_t. \quad (18)$$

Equation(18) describes asset returns, composed of the dividend at time $t$, the asset price at time $t$, and the asset price at time $t+1$. Combined with equation (17), equation (18) becomes,

$$R_t = A_t F_{1,t} u_t + 1 - \delta_t. \quad (19)$$

Using the optimal utilization rate in Equation (14), the above can be written as:

$$R_t = A_t F_{1,t} u_t + 1 - [\delta_0 + \phi (A_t F_{1,t})^{\frac{\phi}{\phi-1}}] \quad (20)$$

Equation (20) represents the relationship between asset returns and net marginal product of capital.

## 2.5 Constant capacity utilization

In this section, a case of constant utilization rate is considered as a special case of variable capital utilization. An extreme case would be where $u_t = u = 1.00$, then $\delta_t = \delta_0 + \frac{1}{\phi} = \delta$. Thus, the depreciation rate becomes constant. As in previous example, $A_t$ is the aggregate productivity level in period $t$. Then, the modified production function becomes:

$$Y_t = G(A_t, u_t K_t, L_t) \underbrace{\Rightarrow}_{when\ u_t=1} G(A_t, K_t, L_t). \quad (21)$$

As technical progress is output augmenting, the production function can be rewritten as:

$$Y_t = A_t F(K_t, L_t). \quad (22)$$

[9]For the detailed derivation, see Appendix A.2. Without adjustment costs, the Tobin's $q$ is equal to 1. Consequently, the price of the equity in the model is equal to the value of capital stock. When the adjustment costs are introduced, the price of equity in the model could be different from the value of capital stock. In fact, Hall (2003) found relatively strong evidence against substantial adjustment costs using annual data from two-digit industries. The result supported his earlier work (2001), where he measured intangible capital based on the value of the capital stock market, assuming a low rate of adjustment.

The stock of capital is accumulated according to equation (23),

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{23}$$

The objective function for the firm has not been changed; it maximizes its net present discounted value.[10] However, there are only two first order conditions:

$$\beta E_t \nu_t (A_t F_{1,t} + 1 - \delta) = \nu_{t-1}, \tag{24}$$

$$A_t F_{2,t} = w_t, \tag{25}$$

where $F_{1,t}$ is the derivative of $F$ with respect to the first argument,$K_t$, and $F_{2,t}$ the derivative of $F$ with respect to the second argument, $L_t$.

Finally, the key relationship is derived in the same way as in the previous section, using the dividend relationship and the linear homogeneity assumption of the production function. In particular,

$$D_t = Y_t - w_t L_t - I_t = Y_t - (Y_t - A_t F_{1,t} K_t) - (K_{t+1} - K_t - \delta K_t). \tag{26}$$

Rearranging terms, equation(26) can be written as:

$$\frac{D_t + K_{t+1}}{K_t} = A_t F_{1,t} + 1 - \delta. \tag{27}$$

Equation(27) describes asset returns:

$$R_t = A_t F_{1,t} + 1 - \delta. \tag{28}$$

When $u_t = 1$, for all $t$, equation (19) becomes equation (28).

# 3 Empirical Strategies

According to equation (20), the asset returns in period $t$ depend on the level of the aggregate productivity $A_t$, the capital stock $K_t$, the labor input $L_t$, the utilization rate $u_t$. At this stage, one of difficult tasks is to choose a particular functional form for the aggregate production function and to derive the exact relationship. This study starts with a general function, $H$, which simply assumes that the asset returns are non-linearly related to the level of aggregate productivity, $A_t$, and other variable, $K_t$, $L_t$, and $u_t$. Thus, equation (20) can be written as:

$$R_t = H(A_t, K_t, L_t, u_t). \tag{29}$$

Then, the first-order Taylor approximation around the stationary equilibrium is applied to derive a linear estimation regression equation. Equation (29) now becomes:

$$R_t = H^* + H_1^*(A_t - A^*) + H_2^*(K_t - K^*) + H_3^*(L_t - L^*) + H_4^*(u_t - u^*) + \text{Approximation Error}. \tag{30}$$

[10] In the model of the constant utilization rate, choice variables are capital stock and labor only. The utilization rate is no longer a choice variable.

where $H^*$ is evaluated at the stationary equilibrium levels of all its arguments and $H_1^*$, $H_2^*$, $H_3^*$, and $H_4^*$ are the derivatives of the function $H$ with respect to $A_t$, $K_t$, $L_t$, and $u_t$, respectively, evaluated at the long-run equilibrium levels of all its arguments.

By rearranging the terms, the linear approximation of the asset return equation is obtained:

$$R_t = \beta_0 + \beta_1 A_t + \beta_2 K_t + \beta_3 L_t + \beta_4 u_t + \epsilon_t, \tag{31}$$

where $\epsilon_t$ includes an approximation error and factors other than $A_t, K_t, L_t$, and $u_t$ affecting asset returns at time $t$. $\beta_0 = H^* - H_1^* A^* - H_2^* K^* - H_3^* L^* - H_4^* u^*$, $\beta_1 = H_1^*$, $\beta_2 = H_2^*$, $\beta_3 = H_3^*$, $\beta_4 = H_4^*$.

In this specification, there are five parameters to be estimated, $\beta_0$, $\beta_1$, $\beta_2$, $\beta_3$, and $\beta_4$. Before the estimation, this study examines time series properties of all variables. To avoid possible spurious regression results in the presence of unit roots, a set of unit root tests is conducted. In particular, a recently developed test, Ng and Perron's unit root test, and a conventional unit root test, the Augmented Dickey Fuller test, are performed to determine whether those variables are indeed I(0).[11]

The unit root test results show that all the variables except the utilization rate are I(1), indicating that all of them have unit roots. As a result, estimating the specification (31) as it is could be problematic. Thus, the first-differenced specification of equation (31) is considered:

$$\Delta R_t = \beta_1 \Delta A_t + \beta_2 \Delta K_t + \beta_3 \Delta L_t + \beta_4 \Delta u_t + \eta_t, \tag{32}$$

where $\{\eta_t\} \sim i.i.d.(0, \sigma_\eta^2)$.

Equation (32) is the final specification for empirical investigations under the variable utilization. In this specification, the parameters in equation (32) have elasticity interpretations because this study takes the natural log of all the level variables in equation (31).

Lastly, the final specification under the constant utilization can be obtained as a special case of equation (32). Once the utilization rate is constant, $\Delta u_t = 0$, and Equation (32) becomes,

$$\Delta R_t = \beta_1 \Delta A_t + \beta_2 \Delta K_t + \beta_3 \Delta L_t + \nu_t. \tag{33}$$

where $\{\nu_t\} \sim i.i.d.(0, \sigma_\nu^2)$.

# 4 Measuring Aggregate Productivity Shock

The aggregate productivity shock is one of the key variables in the empirical investigations. To measure it, this study uses the growth accounting framework.

[11]Ng and Perron (2001) argued that many unit root tests suffered from size distortion and that, in some cases, unit root tests tend to produce an over-rejection of the unit root hypothesis. They developed a unit root test based on GLS de-trended series, and provided modified information criteria to determine the number of lagged variables to be used in the unit root test.

Under the assumptions of competitive markets, and constant returns to scale, the growth accounting framework decomposes growth in output into two parts: portions attributed to growth in inputs and changes in aggregate productivity.[12]

From the production function introduced in the previous section, the *conventional* Solow residual can be constructed:

$$\Delta A_{conv} = \Delta Y - \alpha \Delta K - (1 - \alpha)\Delta L, \tag{34}$$

where $\Delta Y$ is the growth rate of output, $\alpha$ is the factor share distributed to capital, $\Delta L$ is the rate of growth of labor hours, $\Delta K$ is the rate of growth of physical capital, and $\Delta A_{conv}$ is the conventional Solow residual.[13]

One of the well-known properties of the conventional Solow residual is its procyclicality. A growing number of studies point out that one source of the procyclicality might come from unaccounted variations of inputs. Because a rise in factor utilization leads to an increases in output, factor utilization should be considered when the Solow residual is constructed - otherwise, the Solow residual will be spuriously procyclical. In his study, Shapiro (1996) argued that capital stock needed to be adjusted for variable capital-utilization rates to properly measure the Solow residual and showed that the procyclicality of the Solow residual almost disappeared after the adjustment.[14] Prior to his work, Burnside, Eichenbaum, and Rebelo (1995) took electricity use as a proxy for the flow of capital service and argued that the Solow residual was not very procyclical when cyclical variation of capital was taken into account. In addition, Paquet and Robidoux (2001, 1999) used the adjusted Solow residual when they tested its exogeneity based on U.S. and Canadian data. To accommodate these recent developments, this study follows Paquet and Robidoux (2001) and Shapiro (1996)'s approach when it constructs the adjusted Solow residual.

More specifically, *utilized* capital stock as an input in the production function is analogous to the standard measure of labor input. For labor input, total labor hours (average hours worked times the number of workers) are often used. This is the *utilized* labor because it measures the amount of hours that workers spend in production. And it is used to calculate the proportion attributed to labor contribution in the growth accounting, rather than the number of workers. Analogously, the *utilized* capital should be used as a capital input to calculate the proportion attributed to capital contribution instead of physical capital

[12] Hall(1990) said that the following theorem would hold under Solow's assumptions: *the productivity residual is uncorrelated with any variable that is uncorrelated with the rate of growth of true productivity.* This is the restatement of Solow's basic results in which the residual measures the shift of the production function.

[13] The conventional Solow residual is not sensitive to the capital share parameter, $\alpha$. In fact, this study constructs the conventional Solow residual and the adjusted Solow residual using both the actual shares and the average shares. The results of the study are robust regardless of the share data.

[14] He showed that correlation between the adjusted Solow residual and output growth was close to zero. In this study the correlation coefficient is about 0.2.

stock. Under this consideration, the *adjusted* Solow residual can be constructed,

$$\begin{aligned}\Delta A_{adj} &= \Delta Y - \alpha\Delta(uK) - (1-\alpha)\Delta L \\ &= \Delta Y - \alpha\Delta K - (1-\alpha)\Delta L - \alpha\Delta u \\ &= \Delta A_{conv} - \alpha\Delta u \end{aligned} \tag{35}$$

where $\Delta A_{adj}$ is the adjusted Solow residual, $\Delta Y$ the growth rate of output, $\alpha$ the factor share distributed to capital, $\Delta uK$ the rate of growth of utilized capital, and $\Delta L$ the rate of growth of labor hours, $\Delta K$ the rate of growth of capital, and $\Delta u$ the rate of growth of utilization rate.

Equation (35) shows that as long as the utilization rate is not constant $\Delta u \neq 0$, the adjusted Solow residual and the conventional Solow residual are not equivalent, $\Delta A_{adj} \neq \Delta A_{conv}$.

# 5 Data and Estimation

## 5.1 Empirical Investigation - Benchmark Cases

The sample period of the study runs from 1949 to 2001. Asset returns are calculated based on the Standard & Poors 500 composite index. An one-year time horizon begins on January 1st and ends on December 31st. Thus, an investor purchases one unit of asset at the beginning of the period (January 1st) and then sells it at the end of the period (December 31st). In the mean time, dividend payment for her share is made at the end of the period before the asset is sold so that the dividend is included for one period's returns. Real asset returns are computed after adjusting for inflation from nominal asset returns. To measure aggregate productivity shocks, (both the conventional Solow residual and the adjusted one one), real Real Gross Domestic Product (GDP), the number of employees, average hours worked, non-residential real capital stock, and the capacity utilization rate, the average labor share are used. The real GDP and the labor share are from the Bureau of Economic Analysis (BEA). The average labor share is computed from annual series. The real capital stock is from the BEA. The capital stock includes private and public capital stock, excluding residential capital stock. Consumer Price Index (CPI) from the Bureau of Labor Statistics (BLS) is used as a deflator. The capacity utilization rate is from the Federal Reserve Board.[15] Finally, all labor data (hours worked and the number

[15]Shapiro (1989) criticized the official utilization series produced by the Federal Reserve Board as an economically meaningful measure of utilization. It is true that the official series is less than a satisfactory measure of the capacity utilization rate. It is, however, the changes in the capital utilization rate, not the levels that this study uses in the empirical analysis. In addition, the characteristics of the adjusted Solow residual are qualitatively very similar to the ones found in the previous studies based on other measures of the capacity utilization rate: Burnside, Eichenbaum, and Rebelo (1995), Shapiro (1996), Basu, Fernald and Kimball (2004), and Galí (1999). A similar justification was made by Paquet and Ribidoux (1997) when they used the official capacity utilization rate from the Federal Reserve Board to adjust the Solow residual. This study compares the official capacity utilization rate series from the FRB with the one used in Burnside, Eichenbaum, and Rebelo. Those two series move close together and the correlation coefficient is over 0.9

of employees) are from the BLS.

As a first step, the Ng-Perron unit root test and the Augmented Dickey-Fuller unit root test are performed. This step justifies the final specifications derived in the previous section. Table 1 and Table 2 show the results of the unit root tests for the variables used in the empirical investigations.[16] $A_{conv}, A_{adj}, L, K$, and $R$ appear to have unit roots.[17] Table 3 summarizes descriptive statistics. There is no significant difference between the Solow residual and the adjusted Solow residual in terms of the average annual growth rates. Average annual aggregate productivity growth is 1.58% based on the conventional Solow residual and 1.48% based on the adjusted Solow residual. The volatility, however, is quite different between the two. The standard deviation of the adjusted Solow residual is 0.01, while that of the conventional Solow residual is 0.02.[18] The contemporaneous correlations in Table 4 are consistent with what others have documented in the existing literature. After the adjustment, the correlation between the Solow residual and output growth reduces dramatically from 0.9 to 0.2. In addition, the correlation between the adjusted Solow residual and labor-hour growth becomes negative. It could imply that a positive aggregate technological shock might be contractionary.[19]

According to the model in this study, the aggregate productivity shock that hits the economy, affects the asset returns. Thus, the causality runs from the aggregate productivity shock to the asset returns. However, Kiyotaki and Moore (1997) argued for a reverse causality.[20] Their model also generates the relationship between the aggregate productivity and the asset returns, but the direction of the causality is exactly the opposite of what this study argues. To empirically verify the relationship, the Granger-causality tests are performed. Table 5 shows that the measured aggregate productivity changes, proxied by the adjusted Solow residual, Granger-cause changes in the asset returns, and changes in the asset returns also Granger-cause the adjusted Solow residual. However, the test results based on the conventional Solow residual are different. The conventional Solow residual do not Granger-cause changes in the asset returns, and changes in the asset returns do not Granger-cause the conventional Solow residual either. Thus, the overall test results based on the Granger-causality are at best inconclusive. In other words, the data indicate that the casuality could run in both directions. To better understand dynamic effects of the model, Vector Autoregression (VAR) analysis is considered in the later section.

[16] The critical values for the Ng-Perron test are from Ng and Perron (2001).

[17] The utilization rate, $u$, does not follow a unit root process according to the results from the Ng-Perron and the Augmented Dickey-Fuller tests.

[18] The difference is quite substantial in terms of the coefficient of variation. While the coefficient of variation for the conventional Solow residual is 129.6%, it is 84.6% for the adjusted Solow residual.

[19] In fact, Galí (1999), and Basu, Fernald, and Kimball (2004) documented a negative contemporary correlation between the adjusted Solow residual and input growth. They argued that a technology shock could reduce input usages to accommodate an increase in productivity, especially in the short run with some price rigidity.

[20] A consumer can only borrow against her collateral, and the value of the collateral is linked to the asset price. When the asset market booms, investors can invest, which leads to productivity improvements.

Figure 1 shows the plots of changes in the asset returns and the measured aggregate productivity shock proxied by the two Solow residuals.[21] Table 6 presents the estimation results for the regression specifications, (32) and (33). The estimated coefficients on the conventional Solow residual are significant at a 5% level of significance level for both Case I and Case III. The sign of the estimated coefficient is consistent with what the model predicts. According to the model, the productivity shock is a driving force of the macro variables including the asset returns. Thus, the measured productivity shock, proxied by the conventional Solow residual should have a positive effect on asset returns.

The story becomes somewhat different when variable capital-utilization rate is introduced. Although the sign is consistent with the model's prediction, the coefficient becomes statistically insignificant. Instead, the utilization rate appears quite important to explain the asset returns.

Another thing to notice in Table 6 is that the adjusted Solow residual appears to generate a dynamic effect on the asset returns. Perhaps, the delayed effect might come from the changes in the depreciation rate. Once variable capital utilization is taken into account, the contemporaneous effect of the aggregate productivity shock on the asset returns become weaker and the utilization rate becomes important. As a consequence, the shock generates interesting dynamic effects in the model. The implication of the dynamic effects in the presence of the aggregate productivity shock is not trivial but difficult to analyze in this section. To better understand the dynamic effects of the measured aggregate productivity shock on the asset returns, this study considers a VAR analysis.

Before this study turns to the VAR analysis, the Ramsey's RESET test is conducted to test possible specification errors in the regression specifications.[22] The testing results are informative because this study derives the final specifications by the first-order linear approximation, where the higher-order approximation terms are dropped. Thus, the final specifications may not describe the correct relationship between the asset returns and the measured aggregate productivity. The test results in Table 7, however, suggest no specification errors in this study. The study fails to reject the null hypotheses of no specification errors at a 5% significance level for both cases.[23]

## 5.2 Vector Autoregressions: VARs

The purpose of running the VAR is to better understand dynamic effects of the measured aggregate productivity shock on the asset returns. Based on the

[21] For the first two plots in Figure 1, the effects of other variables on changes in the asset returns are already partialled out in the Solow residuals. Thus, these plots show the relationship between the conventional Solow residuals and the asset returns (the first figure), and the relationship between the adjusted Solow residual and the asset returns (the second figure) after controlling for other variables.

[22] The Ramsey RESET test is for any or all of the specification errors, which produce a non-zero mean for an error term. Thus, $H_o : u \sim N(0, \sigma^2), H_a : u \sim N(\mu, \sigma^2), \mu \neq 0$.

[23] In general, rejection of the null hypothesis indicates one or all of the possible specification errors, including omitted variables, an incorrect functional form, measurement errors in regressors, and serially correlated disturbance.

final specifications, [(32) and (33)], the first-differenced form of asset returns ($\Delta R$), measured aggregate productivity shock proxied by both the conventional Solow residual ($\Delta A_{conv}$) and the adjusted Solow residual ($\Delta A_{adj}$), capital stock growth ($\Delta K$), labor input growth ($\Delta L$), and changes in the utilization rate ($\Delta u$) are considered in the VARs.[24]

### 5.2.1 Specifications

First, this study considers the following VAR(1) specification for the measured aggregate productivity shock proxied by the adjusted Solow residual.

$$\begin{bmatrix} \Delta A_{adj,t} \\ \Delta L_t \\ \Delta K_t \\ \Delta u_t \\ \Delta R_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} \Delta A_{adj,t-1} \\ \Delta L_{t-1} \\ \Delta K_{t-1} \\ \Delta u_{t-1} \\ \Delta R_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4t} \\ \epsilon_{5t} \end{bmatrix}. \tag{36}$$

For the conventional Solow residual, VAR(1) is specified as follows:

$$\begin{bmatrix} \Delta A_{conv,t} \\ \Delta L_t \\ \Delta K_t \\ \Delta R_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \Delta A_{conv,t-1} \\ \Delta L_{t-1} \\ \Delta K_{t-1} \\ \Delta R_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \end{bmatrix}. \tag{37}$$

### 5.2.2 Identifications

Since the VAR is a reduced form model, the reduced-form errors are linear combinations of primitive shocks to the system.[25] This study follows Sim's (1980) method of orthogonalizing the innovations. For identification purposes, this study introduces a lower triangular matrix with 1 on the main diagonal.

While the triangular identification scheme provides a set of residuals that are uncorrelated with residuals associated with the equations ordered before them, it is a well-known fact that the order of the variables is quite important in this identification scheme.[26] Thus, in the impulse responses and the variance decompositions, this paper chooses the orders consistent with the theoretical parts of the study. In addition, a sensitivity analysis is conducted to shows that different orders do not change the results.

[24] The first-differenced form of variables are used because of stationarity considerations. The impulse responses from level specifications show that impacts never die out even after 20 years, which do not provide meaningful interpretations of the dynamic effects of the shock.

[25] Shocks to unorthogonalized innovations (the reduced form errors) generally do not provide useful interpretations, especially when the shocks are correlated with each other.

[26] One of the problems of this identification scheme is that the decomposition of the variance of the reduced form error is not unique. Also, the lower triangularity of the transition matrix imposes a recursive structure on the system.

For benchmark cases, this study assumes that an aggregate productivity shock hits the economy at the beginning of period, and then, firms optimally choose capital and labor, then finally asset returns are determined.[27] In addition, alternative specifications consistent with the model studied by Kiyotaki and Moore (1997) are considered. In their model, a shock hits the asset market at the beginning of period. Thus, the asset returns change first. Then, firms hire labor and capital. As a result, aggregate productivity changes.

#### 5.2.3 Impulse Response Functions and Variance Decompositions

Figure 2 shows impulse response functions of one standard deviation perturbation to the aggregate productivity based on equation (36). Two interesting results emerge from the analysis. First, the shock generates the delayed effects on asset returns: it does not affect the asset returns in the first period and generates dynamic effects on the asset returns. Second, the shock reduces labor usages and lowers the utilization rate. Indeed, the results are robust to different orderings as long as the aggregate shock comes first and the asset returns come last. In other words, changes in orders among inputs(capital, labor, and utilization) do not affect results.

The impulse response functions of one standard deviation perturbation to aggregate productivity based on equation (37) are shown in Figure 3. In this case, there are immediate effects on two inputs and the asset returns. After 7 years, the effects of the shock on the asset returns completely disappear.

As Figures 4 and 5 show, this study reports additional impulse response functions with one standard deviation shock to the asset returns. These alternative specifications are consistent with the work by Kiyotaki and Moore (1997). Figure 4, where the adjusted Solow residual is used, the dynamic effects of the shock seem hard to be explained by their model. With a positive shock, there is no contemporaneous effect on the aggregate productivity. Furthermore, in the next period, the aggregate productivity declines. However, Figure 5, where the conventional Solow residual is used as a proxy for measured aggregate productivity, shows that the shock to asset returns has an contemporaneous effect on two inputs and the aggregate productivity as their model predicts.

Finally, forecast error variance decompositions are considered to summarize impacts of the shock.[28] Table 8 shows the results for the benchmark cases and Kiyotaki and Moore specifications. For the benchmark cases, in the first period, about 0.3% of the variance of changes in the asset returns are affected by the adjusted Solow residual. In the second period, however, the proportion jumps to 13% and it stays around that level after the third period. When the conventional Solow residual is used as a proxy for the measured aggregate productivity shock, about 27% of the variance are influenced in the first period. The proportion

[27]For the specification (36), where the adjusted Solow residual is used, firms optimally choose capital, labor, and, the utilization rate.

[28]The idea behind constructing the forecast error variance decomposition is to determine the proportion of the variability of the errors in forecasting the variable of interest at time $t+s$ based on the information available at time $t$ and $t+s$.

goes up a little bit in the second period to 33%. Over the long forecast horizon, about 15% to 30% of the variance of changes in asset returns are attributed to the measured aggregate productivity. The variance decompositions reveal two aspects about the dynamic effects of the shock. First, the conventional Solow residual has larger impacts on the asset returns than the adjusted Solow residual. Second, while most of the effects of the conventional Solow residual take place in the first period, the effects of the adjusted Solow residual do not happen until the second period; the first period effects are substantially different between the two.

In alternative specifications consistent with Kiyotaki and Moore (1997), where a shock hits the asset returns first, the sizes of the effects on the asset returns are smaller than those from the benchmark cases because of the ordering. In fact, the measured aggregate productivity plays a minimal role in these specifications. In particular, when the adjusted Solow residual is used, about 4% of the variance are affected in the second period and the proportion goes up a bit to 7% in the third period.[29] When the conventional Solow residual is used, the variance of the changes in asset returns is not affected even in the second period. In the third period, about 2% of the variance are attributed to the conventional Solow residual. Overall, the adjusted Solow residual has larger impacts than the conventional Solow residual, which is the opposite of what is seen in the benchmark cases.

In summary, the results from the impulse responses and the variance decompositions suggest the followings. First, the measured aggregate productivity shock is important in understanding in asset returns. And the causality could run from the technology to asset returns for both the conventional Solow residual and the adjusted Solow residual. Thus, the model studied in this paper provides a framework to understand the direction of the relationship between the aggregate productivity shock and asset returns. Second, the dynamic effects of the aggregate productivity shock measured by the adjusted Solow residual are remarkably different from the ones based on the conventional Solow residual. The results imply the variable utilization could play a critical role in explaining the observed dynamic effects of the aggregate productivity shock on the asset returns. Once variable utilization is controlled for, technology improvements appear input saving. Thus, firms use less labor and decrease capital utilization. Those changes could offset positive impact effects of a favorable technology on asset returns. As a consequence, asset returns might not respond to technology improvements on impact. While the explanation is consistent with what Basu, Fernald, and Kimball (2004) argued, the observed results are somewhat difficult to be rationalized based on the frictionless model studied in this paper.

Under the equilibrium business cycle model with constant utilization, asset returns increases on impact with a favorable aggregate productivity shock. When utilization is allowed to vary, it is believed that variable utilization would create an amplification effect. The idea is that when technology improves, it is

[29]The variance of the changes in asset returns in the first period is not affected by the aggregate productivity in the Kiyotaki and Moore specifications.

relatively easier for a firm to change capital services by adjusting the level of utilization of capital without changing the level of capital. Thus, capital service supply becomes upward sloping rather than vertical in the short run. As a consequence, improvements in technology could increase utilized capital and would create an amplification effect.

According to the prediction of the model, technology improvements have a positive impact effect on utilization. However, the results from the impulse responses show the opposite. A favorable technology improvement actually reduces utilization, which is somewhat difficult to explain based on the model studied in this paper. Indeed, in order to rationalize the observed phenomenon, the model needs to have some kinds of mechanism, which generates slow adjustments. As Basu, Fernald, and Kimball (2004) suggested, the prediction from a stick-price model might be consistent with the findings of the study. Given output level, improvements in technology could be input saving.

# 6 Conclusion

This paper examines and documents the effects of a measured aggregate productivity shock on the asset returns. Using a simple equilibrium business cycle model, this study derives the relationship between the aggregate productivity shock and the asset returns. Then, it uses Solow's productivity residual as a proxy for the aggregate productivity shock to empirically evaluate the size of the effects of the shock on the asset returns. For empirical investigations, the variable capital utilization is considered to accommodate the relatively recent macroeconomic developments on the measurement of the aggregate productivity shock.

By comparing the two aggregate productivity measures, the conventional residual and the adjusted Solow residuals, this study documents the empirical implications of the variable capital utilization on the asset returns. This study re-confirms that the variable capital utilization substantially reduces cyclical fluctuations of the measured aggregate productivity. The first step estimation reveals that while the conventional Solow residual has strong contemporaneous effects on the asset returns, the adjusted Solow residual loses contemporaneous effects on the the asset returns. Instead, the adjust Solow residual appears to generate dynamic effects on the asset returns.

Further analyses based on the VARs uncover the delayed effects of the aggregate productivity on asset returns. In addition, the variance decompositions suggest that the size of the effects is substantially different in the first period when the variable capital utilization is introduced.

While the equilibrium business cycle model is successfully identify the aggregate productivity shock as a macroeconomic factor affecting asset returns and help us to understand the direction of the causality, the model does not appear to rationalize empirical findings presented in the study. In particular, once variable utilization is controlled for, improvement technology becomes input saving and generates delayed effects on asset returns.

The results of the study re-iterate the importance of the variable capital utilization when the Solow residual is constructed as a proxy for the measured productivity shock. Given these results, Solow residual should be used with caution for its relevance in the analysis of asset returns.

# A Appendix

In this appendix the details of derivation for the model are presented.

## A.1 Details of the Derivation for Section 2.1

This section is to derive the Euler equation for a consumer problem. From the household's sequential budget constraint, equation (2):

$$C_t + P_t Z_t = w_t L_t + Z_{t-1}(P_t + D_{t-1})$$

Define the consumer's wealth at time $t$,$W_t = Z_{t-1}(P_t + D_{t-1}) + w_t L_t$ Using the definition of asset returns, $R_t = \frac{P_{t+1}+D_t}{P_t}$, the budget constraint can be re-written in terms of $w_t$,

$$W_{t+1} = R_t(W_t - C_t) \tag{38}$$

In this problem, $(W_t)$ can be defined as a state variable, and $(W_t - C_t)$ as a control variable, $h_t$ along with $L_t$. The transition equation of $W_t$ is given by $W_{t+1} = R_t(W_t - C_t) = R_t h_t$. The Bellmans functional equation of the problem becomes:

$$V(W_t) = \max_{h_t}\{u(W_t - h_t) + \beta E_t V(h_t R_t)\} \tag{39}$$

The first-order condition is:

$$-u_{c_t} + \beta E_t V_1(w_{t+1}) R_t = 0 \tag{40}$$

By the Envelope theorem, it is known that:

$$V_1(W_{t+1}) = u_{c_{t+1}} \tag{41}$$

Using equations (40) and (41) can be written as:

$$u_{c_t} = \beta E_t u_{c_{t+1}} R_t, \tag{42}$$

which is the consumption Euler equation shown in equation (3).

## A.2 Details of the Derivation for Section 2.4

The first order condition for capital, equation (10) is:

$$\beta E_t \nu_t (A_t F_{1,t} u_t + 1 - \delta_t) = \nu_{t-1}$$

As it is defined, $v_t$ is the price of capital or the marginal rate of substitution between time 0 and time $t$. In other words, $v_t = \frac{u_{c_t}}{u_{c_0}}$. From equation (17),

$$\frac{D_t + K_{t+1}}{K_t} = A_t F_{1,t} u_t + 1 - \delta_t \tag{43}$$

Using equations (10), (43) can be written as:

$$\beta E_t v_t (\frac{D_t + K_{t+1}}{K_t}) = v_{t-1} \tag{44}$$

Since $v_t = \frac{u_{c_t}}{u_{c_0}}$, equation (44) becomes:

$$\beta E_t \frac{u_{c_t}}{u_{c_0}} (\frac{D_t + K_{t+1}}{K_t}) = \frac{u_{c_{t-1}}}{u_{c_0}} \tag{45}$$

By multiplying $u_{c_0}$ , equation (45) becomes:

$$\beta E_t u_{c_t} (\frac{D_t + K_{t+1}}{K_t}) = u_{c_{t-1}} \tag{46}$$

Equation (46) is the consumption Euler equation, (42), when $\frac{D_t + K_{t+1}}{K_t}$ is replaced by $R_t$. Therefore, it is shown that $P_t = K_t$ in equation (A.13).

$$R_t = \frac{D_t + K_{t+1}}{K_t} = \frac{D_t + P_{t+1}}{P_t} \tag{47}$$

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Table 1: Ng-Perron Unit Root Tests

This table reports the results of the Ng-Perron unit root tests. Significant coefficients at a five percent level are indicated with an asterisk, *. The sample period is from 1949 to 2001. All variables are in natural logarithm. REALR stands for gross real asset returns, K for capital stock, L for labor hours, and $u$ for utilization rate, SRL for aggregate productivity level $A_{conv}$, and ADJSRL for adjusted aggregate productivity level $A_{adj}$.

| Ng-Perron test statistics | | MZa | MZt | MSB | MPT |
|---|---|---|---|---|---|
| REALR | | -1.357 | -0.682 | 0.503 | 14.549 |
| K | | -1.925 | -0.700 | 0.363 | 9.854 |
| L | | -7.364 | -1.709 | 0.232 | 4.056 |
| u | | -20.754* | -3.131* | 0.151* | 1.494* |
| SRL | | -1.425 | -0.549 | 0.385 | 11.23 |
| ADJSRL | | 1.186 | 0.965 | 0.814 | 50.15 |
| | 1% | -13.8 | -2.58 | 0.174 | 1.78 |
| Asymptotic Critical Values | 5% | -8.1 | -1.98 | 0.233 | 3.17 |
| | 10% | -5.7 | -1.62 | 0.275 | 4.45 |

Table 2: Augmented Dickey-Fuller Unit Root Tests

This table reports the results of the Augmented Dickey-Fuller unit root tests. Significant coefficients at a five percent level are indicated with an asterisk, *. The sample period is from 1949 to 2001. All variables are in natural logarithm. REALR stands for gross real asset returns, K for capital stock, L for labor hours, and $u$ for utilization rate, SRL for aggregate productivity level $A_{conv}$, and ADJSRL for adjusted aggregate productivity level $A_{adj}$.

| | | Augmented Dickey-Fuller test statistics |
|---|---|---|
| REALR | | -1.45 |
| K | | -2.39 |
| L | | 0.28 |
| u | | -4.188* |
| SRL | | -1.52 |
| ADJSRL | | -2.11 |
| MacKinnon's critical values | 1% | -3.56 |
| | 5% | -2.92 |
| | 10% | -2.60 |

Table 3: Descriptive Statistics

GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GY for real GDP growth, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The sample period is from 1949 to 2001.

| | GREALR | SR | ADJSR | GY | GL | GK | GCU |
|---|---|---|---|---|---|---|---|
| Mean | 0.0271 | 0.0158 | 0.0148 | 0.0358 | 0.0139 | 0.0300 | 0.0026 |
| Median | -0.0434 | 0.0174 | 0.0147 | 0.0375 | 0.0153 | 0.0308 | 0.0023 |
| Max | 0.9318 | 0.0879 | 0.0499 | 0.0875 | 0.0474 | 0.0473 | 0.1000 |
| Min | -0.3317 | -0.0220 | -0.0106 | -0.0203 | -0.0227 | 0.0138 | -0.1151 |
| SD | 0.2454 | 0.0205 | 0.0126 | 0.0243 | 0.0149 | 0.0071 | 0.0452 |

Table 4: Contemporary Correlations

GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GY for real GDP growth, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The sample period is from 1949 to 2001.

| | GREALR | SR | ADJSR | GY | GL | GK | GCU |
|---|---|---|---|---|---|---|---|
| GREALR | 1.00 | | | | | | |
| SR | 0.48 | 1.00 | | | | | |
| ADJSR | 0.19 | 0.53 | 1.00 | | | | |
| GY | 0.45 | 0.91 | 0.23 | 1.00 | | | |
| GL | 0.14 | 0.17 | -0.57 | 0.55 | 1.00 | | |
| GK | -0.09 | 0.02 | 0.02 | 0.22 | 0.24 | 1.00 | |
| GCU | 0.42 | 0.79 | -0.09 | 0.90 | 0.61 | 0.01 | 1.00 |

Table 5: Pairwise Granger Causality Tests

This table reports the results of the Granger-Causality tests. GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual. The sample period is from 1949 to 2001. The number of lagged variable is 1. Significant coefficients at a five percent level are indicated with an asterisk, *.

| Null Hypothesis: | F-Statistic | Probability |
|---|---|---|
| SR does not Granger Cause GREALR | 1.699 | 0.199 |
| GREALR does not Granger Cause SR | 1.801 | 0.186 |
| | | |
| ADJSR does not Granger Cause GREALR | 11.328 | 0.002* |
| GREALR does not Granger Cause ADJSR | 18.409 | 0.000* |

Table 6: Regression Results

The results are based on the regressions of gross real asset return growth on conventional Solow residual, adjusted Solow residual, capital stock growth, labor-hour growth, utilization growth, lagged Solow residual, and lagged adjusted Solow residual. The sample period is from 1950 to 2001 (endpoints adjustment for lagged specifications). Case III is based on GMM estimations where lagged variables are used as instruments. Significant coefficients at a five percent level are indicated with an asterisk, **, and at a ten percent level are indicated with an asterisk, *. The numbers in the parentheses are standard errors. GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, GCU for utilization growth, and OBS for the number of observations.

| *Adjusted Solow Residual* | | | |
|---|---|---|---|
| | Case I | Case II | Case III |
| ADJSR | 5.87 | | 4.41 |
| | (3.24) | | (9.20) |
| GK | -3.04 | | -2.49 |
| | (3.07) | | (4.24) |
| GL | 1.71 | | -18.36* |
| | (3.69) | | (9.24) |
| GCU | 2.12** | | 4.78* |
| | (0.96) | | (2.72) |
| ADJSR(-1) | | 3.44** | |
| | | (1.70) | |
| OBS | 51 | 50 | 52 |

| *Conventional Solow Residual* | | | |
|---|---|---|---|
| | Case I | Case II | Case III |
| SR | 5.67** | | 10.20** |
| | (1.50) | | (4.32) |
| GK | -2.85 | | -4.07 |
| | (1.52) | | (3.62) |
| GL | 1.50 | | -15.31** |
| | (2.16) | | (4.21) |
| SR(-1) | | -1.99 | |
| | | (1.31) | |
| OBS | 51 | 51 | 52 |

Table 7: Ramsey RESET Tests

The table reports results of the Ramsey RESET tests. The tests are based on the results from the regressions (Case I) in Table 6. Significant coefficients at a five percent level are indicated with an asterisk, *. The sample period is from 1950 to 2001.

| *Adjusted Solow Residual* | | | |
|---|---|---|---|
| F-statistic | 1.996 | Probability | 0.164 |
| Log likelihood ratio | 2.214 | Probability | 0.136 |

| *Conventional Solow Residual* | | | |
|---|---|---|---|
| F-statistic | 1.348 | Probability | 0.252 |
| Log likelihood ratio | 1.442 | Probability | 0.230 |

Table 8: Variance Decompositions for Changes in Real Asset Returns due to Adjusted Solow Residual & Conventional Solow Residual

The table reports the results of the variance decompositions. GREALR stands for gross real asset return growth, SR for conventional Solow residual and ADJSR for adjusted Solow residual. Forecast error (FE) for each period is computed by Monte Carlo simulations (500 repetitions). GK stands for for capital stock growth, GL for labor-hour growth, GCU for utilization growth. The orders for each case: Case 1 (Benchmark with adjusted Solow residual): ADJSR, GL, GCU, GK, GREALR; Case 2 (Benchmark with conventional Solow residual): SR, GL, GK, GREALR; Case 3 (Kiyotaki & Moore with adjusted Solow residual): GREALR, GL, GCU, GK, ADJSR; Case 4 (Kiyotaki & Moore with conventional Solow residual): GREALR, GL, GK, SR.

| | Case 1 | | Case 2 | | Case 3 | | Case 4 | |
|---|---|---|---|---|---|---|---|---|
| Period | FE | (%) | FE | (%) | FE | (%) | FE | (%) |
| 1 | 0.175 | 0.33 | 0.192 | 26.84 | 0.175 | 0.00 | 0.187 | 0.00 |
| 2 | 0.247 | 13.47 | 0.253 | 32.46 | 0.246 | 3.35 | 0.252 | 0.01 |
| 3 | 0.255 | 15.76 | 0.255 | 32.20 | 0.255 | 6.49 | 0.257 | 2.22 |
| 4 | 0.259 | 15.45 | 0.255 | 32.45 | 0.259 | 6.28 | 0.269 | 2.04 |
| 5 | 0.260 | 15.89 | 0.256 | 32.50 | 0.260 | 6.50 | 0.271 | 2.54 |
| 6 | 0.260 | 15.89 | 0.256 | 32.51 | 0.260 | 6.52 | 0.272 | 2.54 |
| 7 | 0.261 | 15.87 | 0.256 | 32.52 | 0.261 | 6.51 | 0.273 | 2.61 |
| 8 | 0.261 | 15.89 | 0.256 | 32.52 | 0.261 | 6.53 | 0.273 | 2.61 |
| 9 | 0.261 | 15.89 | 0.256 | 32.52 | 0.261 | 6.53 | 0.273 | 2.61 |
| 10 | 0.262 | 15.89 | 0.256 | 32.52 | 0.261 | 6.53 | 0.273 | 2.62 |

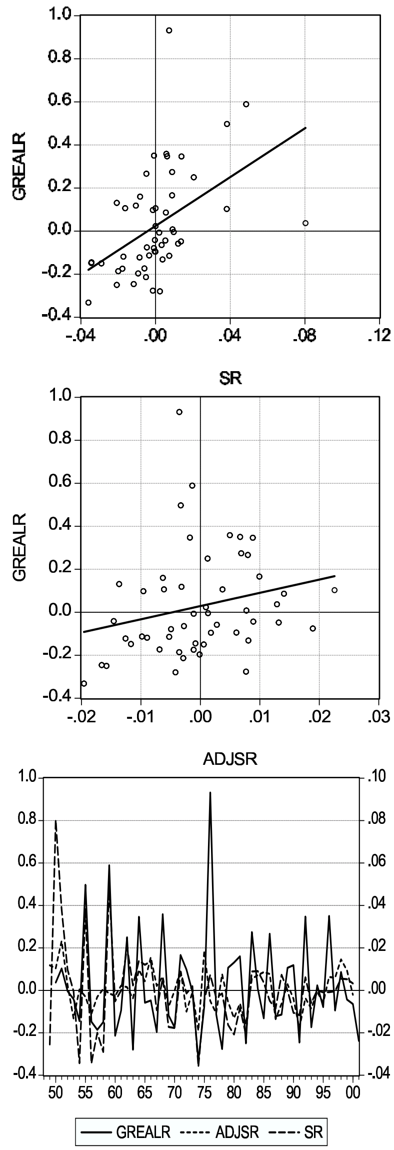


Figure 1: Changes in Asset Returns vs. the Adjusted Solow Residual & Changes in Asset Returns vs. the Conventional Solow Residual : In the first plot, the effects of other variables (GK, GL) are partialled out in SR. In the second plot, the effects of other variables (GK, GL, GCU) affecting changes in asset returns are partialled out in ADJSR. GREALR stands for gross real asset return growth, SR for conventional Solow residual, and ADJSR for adjusted Solow residual. GK stands for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The sample period is from 1949 to 2001.

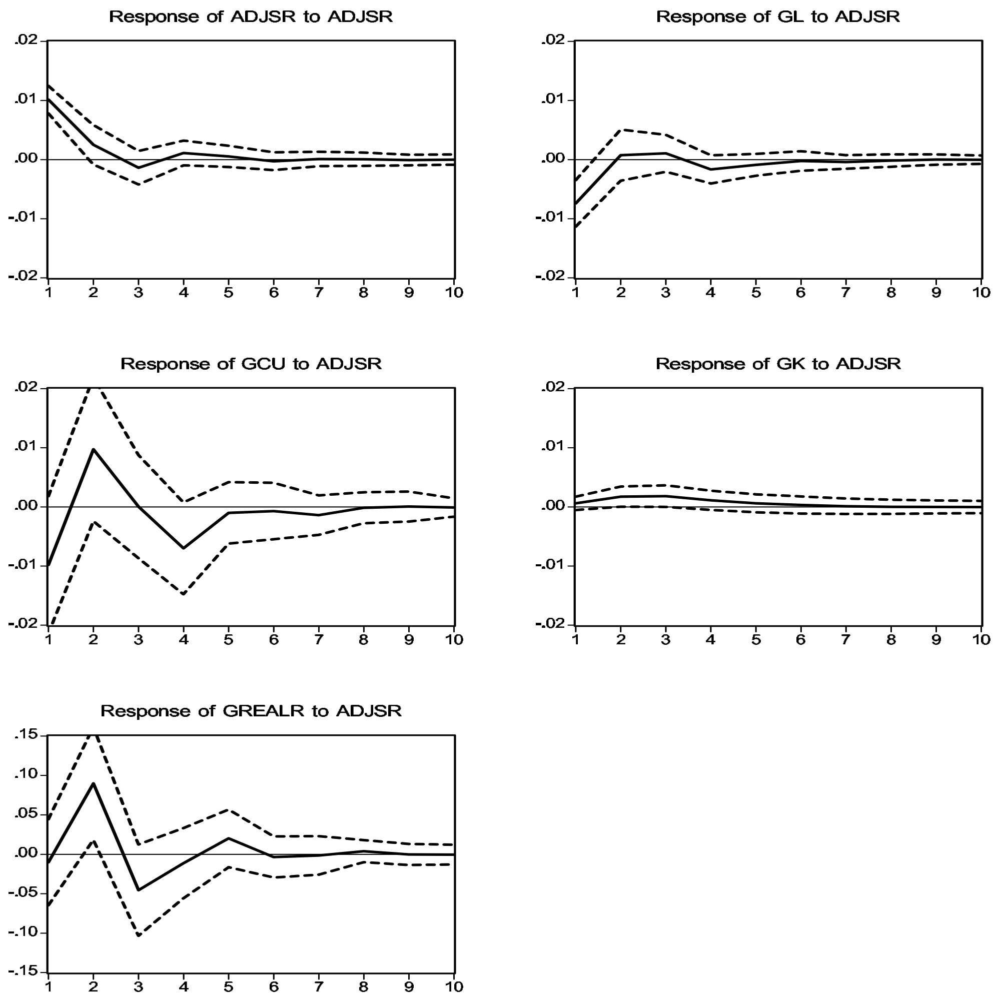


Figure 2: Impulse Response Functions: one standard deviation perturbation to adjusted Solow residual. GREALR stands for gross real asset return growth, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The order: ADJSR, GL, GCU, GK, GREALR.

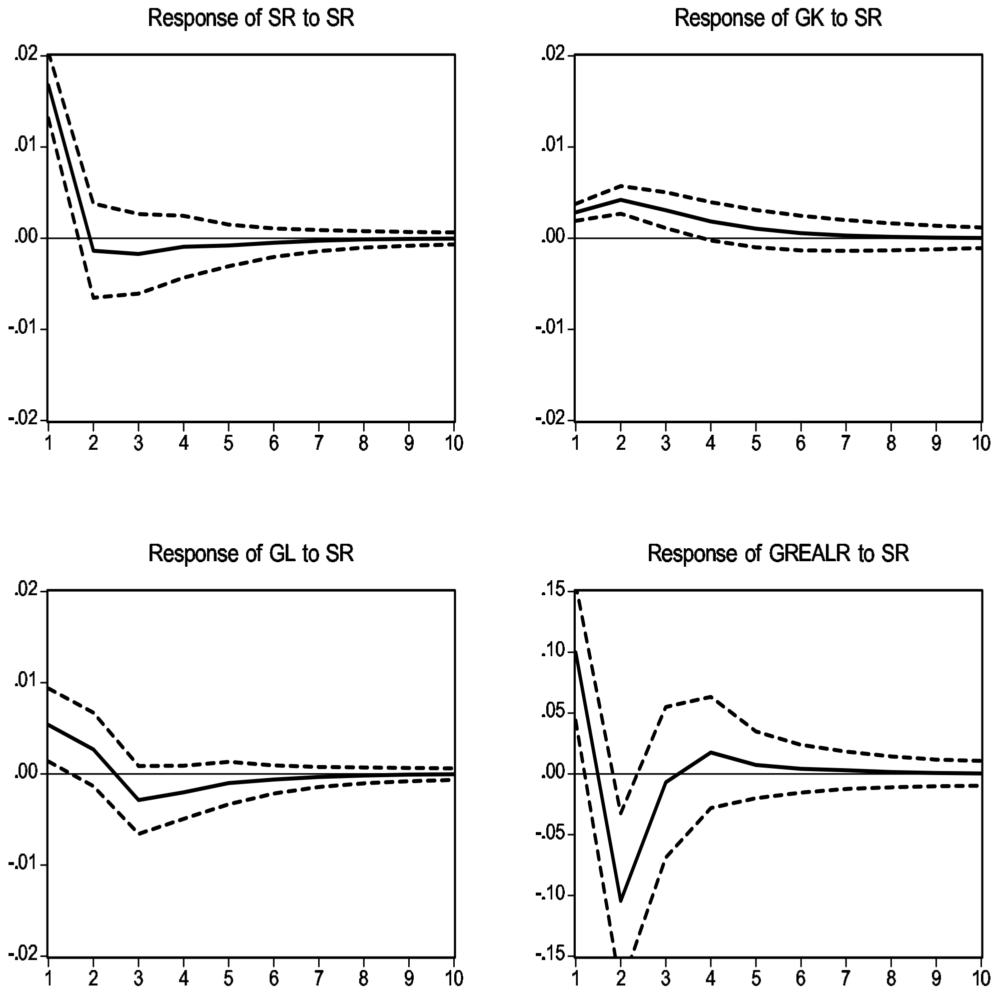


Figure 3: Impulse Response Functions: one standard deviation perturbation to conventional Solow residual. GREALR stands for gross real asset return growth, SR conventional Solow residual, GK for capital stock growth, and GL for labor-hour growth. The order: SR, GL, GK, GREALR.

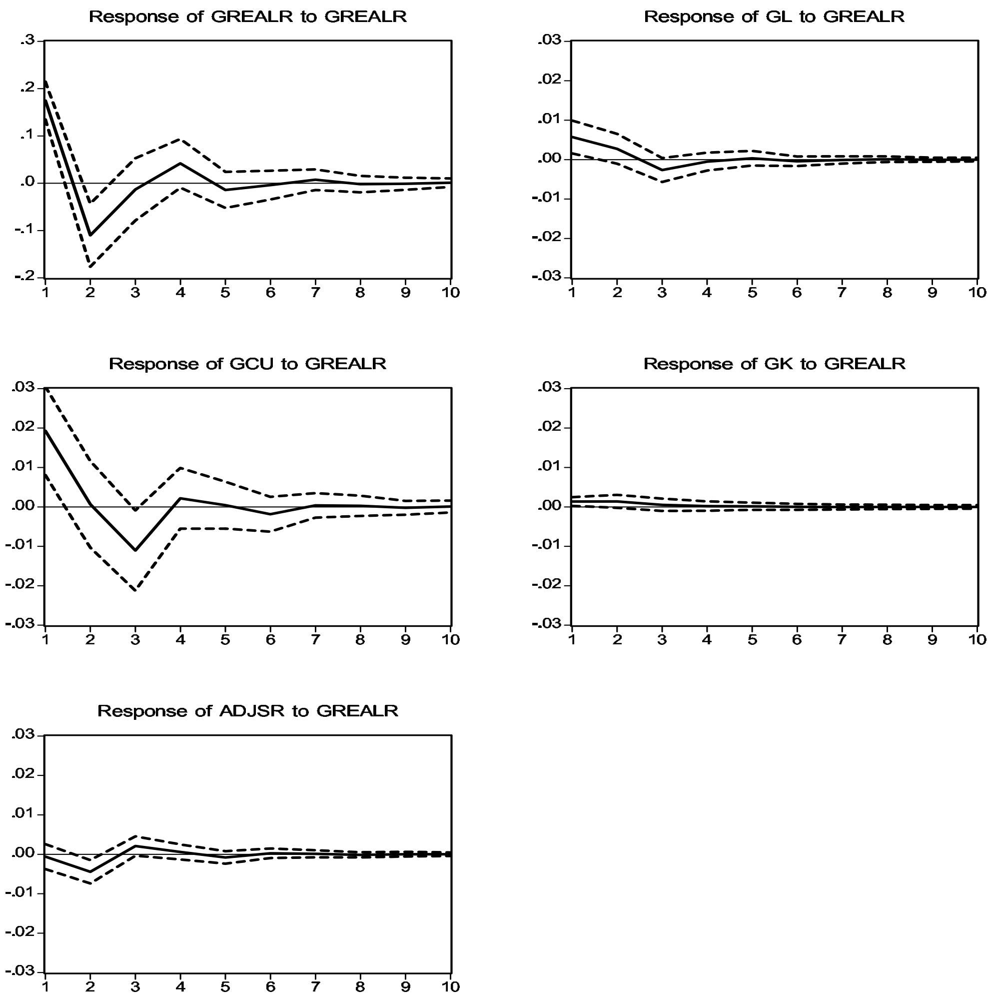


Figure 4: Impulse Response Functions: one standard deviation perturbation to changes in gross real asset returns. GREALR stands for gross real asset return growth, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The order: GREALR, GL, GCU, GK, GLADEP, ADJSR.

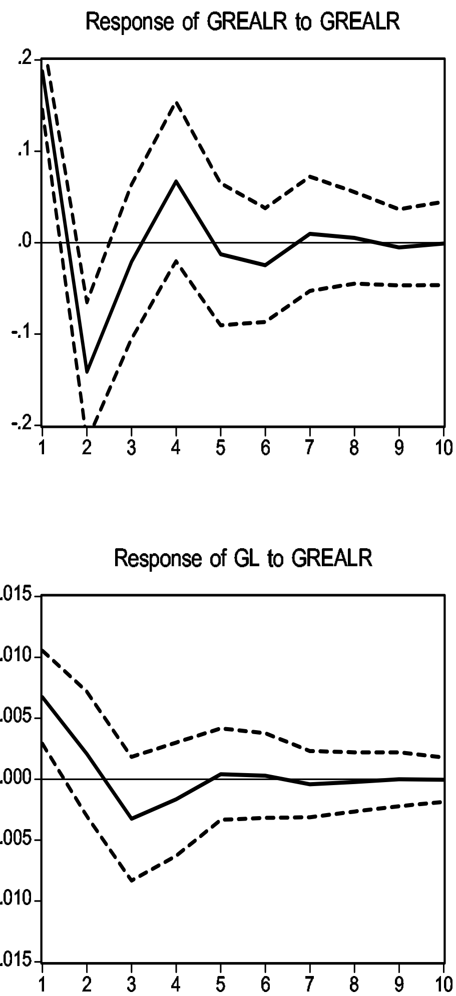


Figure 5: Impulse Response Functions: one standard deviation perturbation to changes in gross real asset returns. GREALR stands for gross real asset return growth, SR for conventional Solow residual, GK for capital stock growth, GL for labor-hour growth. The order: GREALR, GL, GK, SR.