Advertising and Consumers’ Communications

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Until recently, brand identities were built by firms via brand image advertising. However, the flourishing consumer communication weakened the firms’ grip on their brands. The interaction between advertising and consumer communications and their joint impact on brand identity is the focal point of this paper.

We present a model in which consumer preference for functional attributes may correlate with the identity they desire to project of themselves. This correlation is known to the firm but not to the consumers. Both the firm and the consumers can communicate their desired brand identity, although the actual brand identity is determined endogenously by the composition of consumers who purchase it (i.e., what types of people consume the brand).

We find that sometimes the firm can strengthen the identity of its brand by refraining from advertising. This result is based on the following intermediate finding: advertising can diminish the endogeneous informativeness of consumer communications by making it one-sided. Furthermore, it turns out that refraining from brand image advertising may be optimal for the firm when the product is especially well positioned to create a strong identity—i.e., when consumer preferences for functional and self-expressive attributes are highly correlated.

Key words: advertising; communication; word of mouth; branding; brand image

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1. Introduction

Many brands are valued by consumers not only for their functional attributes but also for the identity that consumers can project of themselves while consuming the brand (Aaker 1997 and 1999). For example, certain brands communicate that their user is gay (Williams 2007). Given this, firms may be interested in creating self-expressive (identity) attributes of their products. Advertising that firms use for this purpose is termed, here, brand image advertising.

However, these self-expressive attributes are not under the direct control of the firm. One reason for this is that the identity the brand will project depends on who is using it (Berger and Heath 2007). Another reason is that the product users can also communicate with each other to help establish the identity they would prefer the product to have. For example, when the urban consumers embraced Timberland’s boots and shoes, the identity of the brand that used to be associated with rural culture experienced a dramatic change. Although Timberland proclaimed that it would like to promote the functional value only and stay away from association with any image, its brand became an identity statement in certain demographics because of its use and because of the communications of the users (Walker 2008). The optimal advertising strategy of the firm in the presence of consumer communication and their joint impact on the brand identity is the focal point of this paper.

Consumer desire to affect the brand identity is not a recent phenomenon. In addition to the Timberland case mentioned above, the New Coke fiasco in 1985 provides another well-known example. In 1985, the Coca-Cola Company replaced the original formula of its flagship soft drink. Although a majority of consumers preferred the new taste in blind tests, the new product sparked consumer outrage. According to Oliver (1986), many of the angry consumers were southerners who considered the drink a fundamental part of their regional identity. Although Coke promoted the new formula as “the same thing, only better,” they viewed the company’s decision to change the formula through the prism of the Civil War, as another surrender to the “Yankees.” According to Oliver (1986), it is widely believed that the failure of New Coke was due to outraged customers,
such as retiree Gay Mullins, who used upward of $50,000 of his own money to promote his opinion that New Coke does not represent the same values as the original.

More recently, consumer communication is becoming even more widespread through channels enabled by the Internet, such as Facebook and YouTube. To give an example, the “Starbucks Ad” posted on YouTube (http://www.youtube.com/watch?v=VnbT7qt6RF4; accessed November 11, 2012) starts with a young blonde model holding a Starbucks cup and saying, “I don’t know anybody who doesn’t love a Frappuccino on a hot summer day” and ends with her summarizing what she thinks about the coffee: “…[T]hey’re freaking delicious!” Sounds like a hip commercial for Starbucks. Right? Think again. This is actually a user-generated video with a quite ironic subtext about the social image of the company.

Such consumer-generated ads are just one way in which consumers communicate to affect brand identity. Whereas in the past, brand identity was mostly created by the firms, today, consumers are deeply involved in this process.

The observation that both firms and consumers influence brand identity raises many interesting questions. In this study, we focus only on one of them: How should the firm adjust its advertising to take into account consumer communication. This is related to some more specific questions: How do ads affect consumer communication and its informativeness? How does the existence of consumer communication affect the incentives to advertise? If firm’s advertising affects the informativeness of consumer communication, how should the firm adapt its advertising strategy? How do the firm’s incentives depend on the strength of interrelation between consumer preferences for functional and self-expressive attributes?

Because of the impact of consumer communication on brand identity, firms and marketers are sailing in new territories, and to the best of our knowledge, there is no theoretical framework to assist them in this new and challenging journey. We present a model that captures the major ingredient of this new environment and identifies the interesting relationship between consumer communication and image advertising.

In the model, a monopolist sells a product over two periods. The product has two attributes: the functional one and the identity (self-expressive) one. The latter is endogenously determined by the type of people who will be using the product. There are two different types of consumers with this respect, and the identity attribute directly depends on the proportion of consumers of each type who bought by the end of the second period.

Although consumers have rational expectations about the brand identity, they do not know it for sure because they do not know the correlation of preferences between the functional and identity attributes in the general population. On the other hand, given that firms have more resources and conduct marketing research, we assume the firm knows this correlation.

The timing is as follows. First, the firm decides whether to engage in brand-image advertising and, if so, decides on the advertising content, which is the firm’s claim (or prediction) about the proportion of consumers of each type that will buy its brand by the end of the second period. Then, consumers, having observed the functional attribute and the ad, decide whether to buy the brand (period 1). Those who bought the brand can communicate their type to others. Then, in period 2, the rest of the consumers decide whether to purchase the brand. Finally, all consumers obtain their utilities given the realized image of the brand.

We find that in the absence of consumer communication, the firm always prefers to advertise to strengthen the brand identity. Furthermore, it turns out that the content of the ad is informative about the type of consumers for which the brand is more appropriate, i.e., of the direction of the brand identity, but not about the strength of the brand identity.

When consumers communicate, their communication turns out to be more informative than the ads, but only if consumers of both identity types communicate. Therefore, to make sure that the second-period consumers can predict the brand identity precisely, the firm must encourage consumers of both identity types to buy. Sending an ad in the first period decreases the heterogeneity of the consumers who buy in the first period, sometimes to such a degree that the consumer communication becomes one-sided (i.e., only done by consumers of one of the identity types) and therefore uninformative. In contrast, the firm can facilitate the informativeness of consumer communication by abstaining from advertising. We show that this incentive to refrain from advertising may be higher when the correlation between consumer preferences for functional and self-expressive attributes is higher. Furthermore, we show that it may be that, in equilibrium, the firm advertises when the correlation is low and does not when it is high. Moreover, we show that, in equilibrium, only the firm facing the higher correlation may refrain from advertising because of its effect on consumer communication.

This equilibrium has an intuitive appeal. It means that when consumers’ preferences on the functional attributes are a good predictor of their identity, the firm can rely on its consumers to spread the word on the brand because their message is going to be clear (i.e., quite homogeneous) and credible. Furthermore,
advertising in such a case can only do harm by homogenizing the early consumers so much so that the consumer communication is one-sided and thus uninformative. Interestingly, in the last couple of decades, we have seen the emergence or reemergence of brands that satisfy two conditions: (a) they either refrained from using ads or used ads in a very limited form, and (b) they have a very clear image.

Starbucks and Red Bull provide some examples illustrating the above idea in the beverage market. Both firms have spent very little on ads during their diffusion phase (Ries and Ries 2004). Furthermore, both have a very clear image. Starbucks’s image (which has also been frequently criticized) is of a social and environmental responsible company, whereas Red Bull has a wild image (wild parties, extreme sports, etc.). Moreover, it seems that two additional characteristics of the above equilibrium apply to these companies: (1) their diffusion benefited from interactions among consumers that was facilitated by the firm (e.g., theme coffeehouses in the Starbucks case, delivering free samples to student parties in the Red Bull case), and (2) there seems to be a high correlation between the functional and self-expressive preferences (where this is interpreted as a high consistency between the identity and the functional characteristics of the brand). For example, the functional attribute of Red Bull—an energy drink—is also highly consistent with the wild identity that it aims to project. In other words, people who prefer to behave wildly are more likely to need an energy drink. Although these examples can be interpreted in various ways and do not constitute a proof of the validity of the results of the model, it is encouraging to find that the idea that a clear image brand can be built without advertising is at least consistent with some business practices.

To the best of our knowledge, this is the first work on the interaction between marketing communications (i.e., brand-image ads) and consumers’ communications. Furthermore, surprisingly, there is very little theoretical work that lays the foundation for brand-image advertising. A notable exception is Wernerfelt (1990), who considers the firms’ decision to invest in brand-image advertising and assumes that the image is determined by the advertising spending. Alternatively, in this paper, we consider how the image is endogenously affected by the correlation between user preferences for the functional and self-expressive attributes and how the firm’s advertising decision is affected by the possibility of consumers’ communication. Endogenizing the identity is not unique to our model. Becker (1990) and Karni and Levin (1994) have allowed the image of the brand to depend on consumption decisions. In their case it was the popularity of the restaurant that comprised its image but the composition of the demand (i.e., which people are visiting the restaurant) did not matter. This assumption is modified in the later models of conspicuous goods to account for consumer caring about the composition of the demand (Bagwell and Bernheim 1996, Amaldoss and Jain 2005, Kuksov 2007, Balachander and Stock 2009, Yoganarasimhan 2012). In Amaldoss and Jain (2005), for example, snobs wish to differentiate themselves from the “conformists.” In these models, price enabled the differentiation between the different segments of consumers in the two types of products. In our model, this is done via communication of the firm and/or consumers.

Although there is extensive analytical marketing literature on advertising (e.g., Mahajan and Muller 1986, Desai 1997, Fruchter and Kalish 1997, Rajiv et al. 2002, Dukes and Gal-Or 2003, Shaffer and Zettelmeyer 2004, Villas-Boas 2004, Iyer et al. 2005, He and Chen 2006), this literature normally either models advertising as informative about the product existence or directly assumes how advertising affects the consumer utility or demand. Although we do not assume that advertising has a direct effect on the utility, in the equilibrium of our model, advertising content is informative. Furthermore, in equilibrium, the advertising decision of the firm sometimes serves as a signal (about the correlation between consumers’ functional and self-expressive preferences). The idea that advertising can serve as a signal is also not new (see, e.g., Nelson 1974, Kihlstrom and Riordan 1984, Milgrom and Roberts 1986, Simester 1995 for theoretical development or Horstmann and MacDonald 1994 and Ackерberg 2001 for empirical tests). More recently, Anand and Shachar (2009) have suggested that the targeting of the ads can also serve as a signal on the brand’s horizontal attributes, and Mayzlin and Shin (2011) present a case in which the absence of information in the ad can serve as a quality signal. Our paper adds to this literature by considering how advertising interacts with consumer communication.

It is also important to realize what we do not do in this study: we do not formulate the process of self-regulation that occurs for consumers over time as they construct their self-perceptions of who they are and get feedback from the environment about how well they are able to “validate” that sense of self. Elements of such process are addressed by others (e.g., Benabou and Tirole 2011). Here, we take consumers’ self-concept as given and consider how their desire to express it affects the advertising strategy of the firm.

2. Model Setup
A monopoly firm produces one product at zero marginal cost and sells it in two periods to consumers of total mass two. Consumers derive utility from the
product itself, which we will call the functional value, \( u_f \), and from the self-expression the product enables, which we will call the image value, \( u_i \).

Consumers are heterogeneous in their tastes. With regard to functional value, there are two segments, each of unit mass. Within one segment, which we call \( L \)-functional segment, the consumer functional value \( u_f \) is positive and distributed uniformly on \([0, V_f]\). Within the other segment, which we call \( R \)-functional segment, consumers’ functional value is negative and distributed uniformly on \([-V_f, 0]\). This means that the functional attribute of the brand is more appealing to the \( L \)-functional segment.

With regard to the value of self-expression, there are again two segments, also each of unit mass. One segment, which we call \( L \)-identity segment, consists of consumers who derive a positive utility from projecting the \( L \) identity and negative utility from projecting \( R \) identity. The other \((R\text{-identity})\) segment has the reverse preferences. As discussed below, there is no one-to-one mapping from the functional segments to the identity segments. For example, belonging to the \( L \)-functional segment does not imply that this consumer belongs to the \( L \)-identity segment.

Following prior evidence (Berger and Heath 2007) that the identity of a product depends on the composition of its consumers, we will consider the self-expressive attribute as endogenously dependent on the set of consumers who purchased the brand. Specifically, if we denote by \( D_L \) the mass of consumers from the \( R \)-identity segment who bought the product and by \( D_R \) the mass of consumers from the \( L \)-identity segment who purchased it, then the image of the product is defined as \( I = 2D_L/(D_L + D_R) - 1 \). This expression linearly maps the proportion of the demand coming from the \( L \)-identity segment to the interval \([-1, 1]\), so that \( I = 1 \) indicates that everyone who buys the product is from \( L \), whereas \( I = -1 \) indicates that everyone who bought the product is from \( R \)-image segment.

The self-expressive utility \( u_i \) is formulated as \( u_i = I \cdot v_i \) for \( L \)-identity consumers and \( u_i = -I \cdot v_i \) for \( R \)-identity consumers, where \( v_i \)—the consumer value of self-expression—is distributed uniformly on \([0, V_i]\). In other words, projecting the right identity increases the utility and projecting the wrong identity decreases the utility and \( v_i \) represents the magnitude of this effect, which is also the value of projecting exactly the right identity. Although this model captures the social utility of brand image through an exogenous variable \( v_i \), this could be consistent with a more primitive formulation of the consumer utility function. For example, as suggested by Kuksov (2007), \( v_i \) can capture the consumer value of social matching with other consumers, where the value is higher if the consumer identity types match. As argued in Kuksov (2007), one can then justify that brands with images provide more credible information transmission than pure conversations and may be valuable to consumers. Although this formulation may be appealing, we do not want to restrict the value of self-expression to this particular mechanism and thus model it as an exogenous variable.

It is important to note that the terms “functional” and “self-expressive” can be considered interpretations. Removing these interpretations, all we have is a product with two attributes: one that is determined by the firm and one that is a function of consumers’ composition. Although it is perhaps possible to interpret the second attribute in other ways, we find the term “self-expressive” appealing because it reflects recent findings on identity (Berger and Heath 2007). Furthermore, it is also possible to extend the interpretation of the first attribute beyond “functional.” For example, some might consider that humor is an attribute of some brands and that it is determined by the firm. If the latter is true, such an attribute would be classified as functional in our framework. To focus the discussion, we only consider brand characteristics that are either clearly controlled by the firm—the functional ones—or dependent on the consumers’ composition—the self-expressive ones.

Although there is not a one-to-one mapping between the functional and identity segments, these segments are not independent. It is reasonable to assume that the functional and self-expressive attributes are not independent for at least two reasons. First, it is likely that people with different personalities/identities have different functional needs. For example, is the action-packed environment of Club Med suitable for a low-key person? Second, some functional attributes disable the possibility of signaling of some identities. For example, can owning a car with the fuel efficiency of the original Hummer possibly signal an identity that suggests concern for the environment? Accordingly, we denote the mass of the \( LL \)-subsegment consisting of consumers who are both in \( L \)-functional and \( L \)-identity segments by \( q \), and to use a symmetric setup, we assume that the mass of the \( RR \)-subsegment is also \( q \). Accordingly, the mass of cross-matched subsegments \( RL \) and \( LR \) is \( 1 - q \) each. Note that in this subsegment notation, we use the first letter to indicate the functional segment and the second to indicate the identity segment.

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1 Note that this is equivalent to one segment of consumers with values of the functional attribute distributed uniformly on \([-V_f, V_f]\). Dividing this distribution into the two segments is helpful for the conceptual discussions.

2 Note that in such a framework, because the value of the brand image comes from the expected change in the type of the partner found, expected consumer utility depends on the brand image even if the brand image is never observed by the consumer.
Assume that $q$ may take one of the following four values: $q_t$, $1 - q_t$, $q_h$, or $1 - q_h$, for some $\frac{1}{2} < q_t < q_h$. In other words, (a) the correlation between consumer preferences for the functional and self-expression attributes can be either high ($q_h$) or low ($q_t$), and (b) LL and RR can be either more likely than $LR$ and $RL$ (i.e., $q \in [q_t, q_h]$) or less likely (i.e., $q \in [1 - q_t, 1 - q_h]$).

Because marketers conduct research on their consumers and their tastes, it is reasonable to assume that the firm knows the distribution of consumer preferences. On the other hand, it is unreasonable to assume that the consumers have as much information. Therefore, we assume that the distribution of consumer preferences is private information of the firm. Notice also that if this distribution was perfectly known to consumers, then in equilibrium, there would be no uncertainty about the brand identity, and there would be no value of communication either of the firm or of consumers. We assume that before receiving any communication (advertising from the firm or communication from other consumers), the consumer belief about $q$ is that it is equally likely to be any one of the four possible values ($q_t, 1 - q_t, q_h, \text{ or } 1 - q_h$). Thus, prior to receiving any communication, the consumer expectation of $q$ is exactly equal to $\frac{1}{2}$. It could also be reasonable to assume that consumers have some expectation of the direction of the correlation, i.e., whether $q$ is above or below $\frac{1}{2}$, but introducing such an expectation would complicate the analytical tractability. It is worth noting that other than consumer uncertainty about $q$, everything else is known to both the consumers and the firms. Specifically, they know the above description of the market as well as the exact value of the parameters $V_I$, $V_l$, $q_t$, and $q_h$.

Consumer uncertainty might be resolved, at least partially, by advertising. Technically, a firm would normally advertise about the image of the product, but because in our case the uncertainty about image stems from uncertainty about $q$, what rational consumers would be trying to infer from any advertising is the value of $q$. Thus, we directly assume that the firm can send an ad with a declaration about the value of $q$ (e.g., the ad can say “$q$ is equal to $q_h$”). Once again, this formulation captures brand image advertising well because it represents the firm’s effort to associate an identity with its product, which is exactly what such ads are trying to do (e.g., in the case of Virginia Slims, projecting a liberated image). Furthermore, the firm is allowed to send any declaration—i.e., the declaration is not restricted to be truthful. Note that although truth-in-advertising laws may ensure that some types of advertising, such as about prices and functional attributes, are truthful, they may have little bite in restricting the identity that a firm may associate with its product in its ads, because such advertising reflects statements about image expectations and not about physical attributes. However, because consumers do not forget the content of the ad, the firm is not allowed to change the content of the ad after sending it. This restriction immediately implies that if the firm advertises in the first period, advertising in the second period serves no purpose. In other words, the firm advertises in the first period, in the second period, or in neither.

The firm’s decision to send an ad or not, and the content of the ad, can be used by the consumers in updating their prior beliefs on $q$. Furthermore, whereas some consumers like to be the first to buy a new product (Rogers 2003), others tend to wait and get some additional information from these early consumers. To account for this, we assume that there are two time periods in which consumers may buy. A consumer considering to purchase the brand in the first period can only have the firm’s message (or absence thereof) to use in her belief updating, whereas a consumer considering to purchase the brand in the second period can use both the firm’s message (or its absence) and the first-period consumer messages (if any).

We assume that a consumer who bought in the first period communicates her preferred identity (i.e., which identity segment she is in) if and only if her value of projecting image $v_i$ is high enough, i.e., $v_i \geq v_i^*$ for some $v_i^* > 0$. Because this generates either infinitely many or no consumer messages, we assume that the second-period consumers do not observe each of the messages but only the proportion of $R$ to $L$ messages (this is a common assumption; see, e.g., Kuksov and Xie 2010, Joshi and Musalem 2012).

To abstract from the pricing decision and concentrate on the firm’s incentives to advertise, we consider the price as exogenously fixed at some value $p > 0$. Furthermore, we assume that the price is the same in both periods. There is no doubt that endogenizing the price can enrich the model, but given that this is the first model to incorporate communication of both the firm and the consumers, it makes sense to focus on the communication aspect of this interaction.\(^3\) The previous theoretical analysis of image advertising has adopted a similar modeling strategy. For example, in his “variety model,” Wernerfelt (1990) also abstracts from price considerations.

Following previous studies that demonstrate the social rewards from early adoption behavior (Arnould 1989, McCracken 1988, Fisher and Price 1992), we assume that the total value of the product to a consumer in the second period is lower than its value in the first period. Specifically, if a consumer bought in the first period, her utility is $u_i = u_j + u_i - p$, where

\(^3\) In §4, we discuss some issues with endogenizing price in the current model.
We do not have a subscript indexing consumers.

When \( V_F \) is small enough, this model could result in multiple equilibria, some of which involve consumer coordination on the image, especially in the absence of the firm’s advertising. To avoid multiple equilibria, we assume that in the absence of the firm’s advertising, consumer expectation of the image is 0 (i.e., consistent with expectation of \( q \) being \( \frac{1}{2} \)) and look for an equilibrium where the firm’s advertising \( L(R) \) identity in period 1 increases demand from the \( L(R) \)-identity segment. Note that if advertising is not costly, an equilibrium where consumer demand is uncorrelated with advertising and advertising uncorrelated with the image always exists (see, for example, the discussion in Kuksov 2007).

We analyze the model under two scenarios: (1) \( V_F^- > V_l \) and (2) \( V_F^- < V_l \). The first scenario represents the state of communication when consumers’ communication was very costly and thus marketing communication was done only by the firms. We would refer to this case as no consumer communication. The second scenario represents the state of communication when the cost of consumers’ communication is low enough so that some consumers communicate. We would refer to this case as with consumer communication.

### 3. Model Analysis

#### 3.1. Advertising Strategy When Consumers Do Not Communicate

In this subsection we analyze the firm’s incentives to send an image ad and characterize its impact and informativeness. To achieve this goal, we start by describing the demand when the firm does not advertise and then proceed to the case in which it does. The comparison between these two cases enables us to identify the conditions under which advertising is optimal.

##### 3.1.1. The Case of No Ad.
Because the firm does not send an ad in either period and there is no consumer communication, consumers do not learn anything between the periods. Thus, the expected value of \( u_f + u_i \) is the same in both periods. However, because consumers who purchase the product in the second period do not enjoy the full value of \( u_f + u_i \), all the demand will be concentrated in the first period.

Based on the symmetric nature of the prior beliefs (which implies that the expectation of the prior belief about \( q \) is exactly \( \frac{1}{2} \)), the identity expected by the consumers (denoted by \( \hat{I} \)) is 0 for all consumers. Therefore, consumers make decisions based on their functional value only, and as a result, consumer demand is

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D_{\text{noAd}} = \frac{V_F - p}{V_F}. \tag{2}
\]
3.1.2. The Case of Using an Ad. Recall that if the firm advertises, it does it either in the first or in the second period. We start by considering the case of an ad in the first period. In such a case, like in the one above, consumers do not learn anything between periods, and thus all the demand is concentrated in the first period.

Next we want to show that, in such a case, the firm has an incentive to project a homogeneous identity. Specifically, we wish to show that as the expected identity moves farther from 0 (i.e., more homogeneous), the demand becomes higher. Without loss of generality, consider consumer expectation to be that the brand is more appealing to the \( I \)-identity consumers. In other words, consumers expect \( \hat{I} \) to be positive.

Whereas the functional attribute of the brand is appealing to the \( LL \) and \( LR \) subsegments, the self-expressive attribute is attractive to the \( LL \) and \( RL \) subsegments. Thus, the demand is a sum of demands from these three subsegments (\( LL, RL, \) and \( LR \)). Accordingly, the stronger the \( L \)-identity of the brand, the higher the demand from \( LL \) and \( RL \) subsegments and the lower the demand from \( LR \) subsegment.

An interesting case to examine is where the expected identity is in the same direction as the actual correlation; i.e., the consumers expect the brand to be more appealing to the \( L \)-identity consumers, and these consumers have indeed a higher than average utility from the functional attribute of this brand. More formally, in such a case, \( \hat{I} > 0 \) and \( q > \frac{1}{2} \). Below, we refer to this case and to its symmetric analog (i.e., \( \hat{I} \leq 0 \) and \( q \leq \frac{1}{2} \)) as directionally consistent beliefs. The main reason that this case is interesting is that, as we will show later (see Proposition 1), beliefs are likely to be directionally consistent. It turns out that in such a case, the more homogeneous the expected identity is, the higher the demand. This is stated in the following lemma.

**Lemma 1.** When consumers do not receive any new information after the first period and beliefs are directionally consistent, the total demand increases in the absolute value of \( \hat{I} \). Furthermore, the demand is higher when beliefs are directionally consistent than when they are exactly the opposite (e.g., when \( q > \frac{1}{2} \), the demand is higher when consumers expect the identity to be \( \hat{I} > 0 \) than when they expect it to be \( -\hat{I} \)).

The proofs of all lemmas and propositions are in the appendix. The logic of this result is the following. Consider the case of \( q > \frac{1}{2} \). When \( \hat{I} \) increases, the \( L \)-identity consumers find the brand more appealing and the \( R \)-identity ones find it less so. Thus, the demand from the subsegments \( LL \) and \( RL \) increases, whereas the demand from the \( LR \) consumers decreases. It turns out that the increase in the demand due to the \( LL \) consumers is larger than the decrease due to the \( LR \) ones. The main reason for this is that the size of the \( LL \) subsegment is larger than the size of the \( LR \) one.\(^5\) This is related to the assumption that there is a correlation of preferences for the functional and self-expressive attributes. Specifically, this assumption implies that the size of the \( LL \) subsegment is larger than the size of the \( LR \) one (i.e., \( q > \frac{1}{2} > (1-q) \)).

Note that the condition that consumers do not receive any new information after the first period is satisfied if the firm advertises in the first period and when there is no consumer communication. Therefore, the lemma implies that as long as the firm engages in image advertising at the time of new product introduction, it is likely to make strong statements such as “this product is for type \( X \)” as opposed to “this product will be 75% consistent with type \( X \).”

Next, consider the case in which the firm does not advertise in the first period but does in the second. In such a case, it is easy to show that the demand in the first period comes from a subset of the consumers in the subsegments \( LL \) and \( LR \)—only those who do not care much about the self-expressive attribute (i.e., those whose \( v_j \) is not too high). The consumers with strong enough preference for the identity are worried that the identity of the brand might be the opposite to the one that they desire. Thus, they prefer to wait and decide whether to buy in the second period as additional information might be available.

Considering again the case in which \( \hat{I} > 0 \) and \( q > \frac{1}{2} \), the demand in the second period comes only from the \( LL \) and \( RL \) subsegments. Notice that \( LR \) consumers who did not buy in the first period will certainly not buy in the second period, after learning that \( I \) is positive. Because the only consumers who purchase in the second period value the \( L \) identity, it is immediate that the second-period demand increases with the expected identity, \( \hat{I} \). Thus, we again have that if the firm advertises (in this case, in the second period), it prefers consumers to expect that the identity would be as homogeneous as possible.

These findings imply that if the functional attribute is more appealing to the \( L \)-identity consumers, the firm prefers that consumers would know that. In other words, if the ad would say “\( q > \frac{1}{2} \),” the consumers would believe it and update their priors accordingly. However, because the firm always prefers consumers to believe that the identity is as homogeneous as possible, consumers would not trust the magnitude reported in the ad.

The results from the first and the second periods imply that truthful advertising about whether the

\(^5\) In addition, the decrease in the market share among the \( LR \) consumers is smaller than or equal to the increase in the market share among the \( LL \) ones.
product is suitable for the L- or R-identity segment is better than a deceitful one. Because ads are directionality truthful, they are used by consumers in updating their beliefs. In other words, brand image ads are both effective and informative.\(^6\)

The following proposition summarizes this discussion.

**Proposition 1. Consumers believe the ad content about the sign of \((q - \frac{1}{2})\), but not of the value. In other words, advertising can only be informative about which consumers the brand is most suitable for but not of the magnitude of this relationship.**

Note that Proposition 1 holds as long as advertising is not a costly signal. It may be possible that if the firm were able to choose a more or less expensive advertising, the amount of advertising would be informative about the strength of the correlation.

Comparing the profits of the firm when it advertises or not and when it advertises in the first period versus the second leads to the following result.

**Proposition 2. The firm strictly prefers to advertise than to not advertise.**

Proposition 2 can be viewed as a corollary to Proposition 1 and is closely related to the rationale for the second part of that proposition, which is that the firm prefers to make as extreme consumer beliefs (in the right direction) as possible. Specifically, no advertising would mean the weakest beliefs (i.e., expected image is zero). Because the firm prefers to create the strongest image, and advertising is an effective instrument at its disposal (as consumers do find advertising to be informative about the direction of the correlation), the firm should advertise.

These results demonstrate the role of image advertising in the past when consumer communication was often not effective and not considered as an important factor by the firms. A caveat of image ads, though, is that they are not informative about the magnitude of the correlation between the functional and the self-expressive preferences. This is, as we will soon see, where consumer communication can play an important role.

3.2. Advertising Strategy When Consumers Communicate

This subsection examines the impact of consumer communication on the firm’s incentives to send image ads. The existence of consumer communication raises the question of how the firm should account for the effect of the ad on the first-period buyer composition and, as a result, on the content of the consumer communication. Thus, we start the analysis by exploring this effect—the impact of an ad on the content of consumer communication.

It turns out that by sending an ad the firm can (in some cases) decrease the informativeness of consumer communication. To see this, consider first the case in which the firm does not advertise. In this case, in the first period, consumers do not know which identity segment is more likely to buy the brand, and thus their expectation of \(I\) and of \(u\), is 0. However, consumers also know that if they delay the purchase till the second period, they may benefit from learning the brand image from first-period consumers’ communication. This benefit is higher for the consumers who value self-expression more. Accordingly, the demand in the first period comes from those consumers in subsegments LL and LR who do not care too much about the self-expressive attribute (i.e., those whose \(v_i\) is not too high). Let us denote by \(\bar{v}_i\) the threshold \(v_i\) between those LR consumers who buy in the first period and those who do not.

Now, let us consider the case in which the firm advertises in the first period and attach an L identity to the brand. Such ads would decrease the demand among the LR consumers (while increasing it among LL and RL consumers). Specifically, the threshold between those who buy in the first period and those who do not within that subsegment would be \(\bar{v}_i\), where \(\bar{v}_i < \bar{v}_i\). In other words, if the brand is known to have an L identity, only those LR consumers who care very little about identity would buy it.

When \(v_i \in (\bar{v}_i, \bar{v}_i)\), the drop in the demand among the LR consumers makes their voice in the consumer communication nonexistent. Specifically, since \(v_i < \bar{v}_i\), it means that when the firm does not advertise, some R-identity buyers care enough about the identity of the brand to express themselves via consumer communication. However, since \(v_i > \bar{v}_i\), when the firm advertises, none of the R-identity buyers cares enough about the identity to participate in the consumers’ communication. In such a case, the consumer communication is one-sided.

In this model a one-sided consumer communication is not informative about the strength of the correlation, whereas a two-sided communication is.\(^7\) The reason for this is our assumption that the second-period communication is always informative about whether the brand is more appealing to the L- or R-identity segments, for brevity, we will refer to consumer communication as uninformative when it does not provide information about the strength of the relationship, i.e., when it is less than fully informative. Note that when the firm advertises, if consumer communication does not provide information about the strength of the correlation, it provides no new information.

\(^6\) An equilibrium with uninformative advertising, where consumers ignore the advertisement, and the firm’s advertising decision is independent of the information it has, always exists. However, it seems natural that the agents would coordinate on the equilibrium with informative advertising if such an equilibrium exists.

\(^7\) Because in this model consumer communication is always informative about whether the brand is more appealing to the L- or R-identity segments, for brevity, we will refer to consumer communication as uninformative when it does not provide information about the strength of the relationship, i.e., when it is less than fully informative.
consumers do not observe the exact number of the consumer communication messages of each kind but only the fraction of $L$ and $R$ messages among all messages. It is easy to show that because the consumers know all the parameters of the model other than $q$, they can extract $q$ from the ratio of these fractions as long as both of them are positive. In other words, when the consumer communication is two-sided (i.e., the fraction of neither $L$ nor $R$ messages is 0), the consumer communication fully reveals $q$. On the other hand, if the consumer communication is one-sided, the fractions are either 0 or 1, and thus the consumer communication is uninformative.

How reasonable is our assumption that consumers do not observe the total number of messages of each kind but rather only their relative proportion? There are various ways to support this assumption, such as (a) consumers cannot be exposed to all messages because there are so many of them (e.g., YouTube videos) and thus only get exposed to a sample of them, and (b) even if they were able to “listen” to each message, they are still likely to use the relative proportion, because they do not know the exact size of the market—e.g., how many consumers like energy drinks. Given this assumption, the result that a one-sided consumer communication is uninformative is immediate (as discussed above). However, how reasonable is this result in a more general context—i.e., when it is not necessarily within the exact assumptions of this model?

There are at least two additional reasons to believe this result. First, a one-sided consumer communication implies that all communicating users are sharing the same perspective on the brand. Under such conditions the incentive for users to generate one more content diminishes quickly. Furthermore, the diminishing incentive does not necessarily depend on the popularity of the product but rather on the absolute number of messages. In other words, when “enough” people express their view and it is not controversial, others do not find it anymore attractive to communicate, and this “enough” is an absolute number (i.e., independent of the number of buyers). Therefore the volume of the one-sided consumer communication cannot be used to reveal the number of buyers and therefore cannot be used to deduce $q$. Second, consumers may not be aware of the very existence of consumer communication on a certain topic unless the topic is “controversial”—i.e., unless different opinions are present in the discussion.

Another way to think about the effect of advertising on the informativeness of the consumer communication is that under certain conditions, the firm can enhance the voice of the consumer communication by refraining from advertising. Under which conditions would the firm be more interested in keeping consumer communication informative? Recall that the information in the consumer communication is much more precise than in the ad. Specifically, whereas an ad can only provide information about the sign of $(q - \frac{1}{2})$, consumer communication can provide exact information about $q$. Because when $v_i > \bar{v}_i$, none of the $R$ buyers participate in consumers’ communication either with ads or without ads. Thus, for a high enough $v_i$, consumer communication is uninformative with or without ads. Accordingly, when $v_i < \bar{v}_i$, consumer communication is informative regardless of the firm’s advertising decision in the first period.

Before summarizing this result in a proposition, it should be noted that the above discussion does not account for the potential signaling role of advertising. In other words, although the content of the ad is uninformative about $|q - \frac{1}{2}|$ (as stated in Proposition 1), it is possible that, in equilibrium, the firm’s decision as to whether to advertise might depend on $|q - \frac{1}{2}|$, and as a result, it may serve as a signal of the correlation for consumers. Before accounting for this potential informative role of the advertising decision, we first summarize the direct effect of advertising on consumer communication as the following proposition.

**Proposition 3.** If consumers do not update their beliefs about $q$ based on the firm’s decision as to whether to advertise, advertising in the first period weakly decreases the informativeness of consumer communication.

By “weakly,” we mean that depending on the model parameters, advertising either does not change the informativeness of consumer communication or decreases it.

Conceptually, the above proposition is the main result of the model. It establishes the effect the firm has to take into account when adapting its advertising strategy in response to the presence of consumer communication. This effect of advertising on consumer communication then implies that (in the absence of consumer inference from the firm’s decision on whether to advertise) the firm has an extra incentive to advertise if it would prefer consumer communication to be less informative and an extra incentive not to advertise if it would prefer consumer communication to be more informative. Of course, consumers may (and should) infer what they can from the firm’s decision. But what they infer would depend on how the firm actually adjusts its behavior and, thus, on the direct incentives the firm has (we come back to this argument and formalize it in Proposition 4).

Another way to think about the effect of advertising on consumer communication is that under certain conditions, the firm can enhance the voice of the consumer communication by refraining from advertising. Under which conditions would the firm be more interested in keeping consumer communication informative? Recall that the information in the consumer communication is much more precise than in the ad. Specifically, whereas an ad can only provide information about the sign of $(q - \frac{1}{2})$, consumer communication can provide exact information about $q$. Because
the firm’s profit is higher the stronger the expected identity, the firm would prefer consumer communication to be informative only if it will lead to a stronger identity compared to advertising. This would be the case if and only if \( q = q_{h0} \), i.e., only when the correlation is higher than what consumers expect based on their prior beliefs.

It turns out that this incentive can be strong enough to change the firms’ advertising decision. Specifically, we have the following proposition.

**Proposition 4.** For some parameter values, without consumer communication, the firm chooses to advertise in the first period regardless of \( q \), but with consumer communication, it advertises (in the first period) only if \( q = q_0 \).

However, the reverse is not possible: there are no parameter values under which in the absence of consumer communication the firm would optimally choose to advertise in the first period regardless of \( q \), but when consumer communication is potentially possible, it would advertise in the first period only if \( q = q_{h0} \).

Notice that whereas Proposition 3 demonstrates the effect of ads on the informativeness of consumer communication conditional on consumer beliefs without accounting for the firm’s decision whether to advertise, for the parameter values in Proposition 4, this signaling effect exists also in equilibrium (i.e., for equilibrium beliefs, where the decision to send an ad affects the beliefs). Furthermore, notice that the proposition does not only demonstrate that under some conditions it is optimal for the firm to refrain from advertising, the second part of the proposition also claims that such disincentive to advertise only applies for \( q = q_{h0} \).

Interestingly, only when the firm is especially well positioned to create a strong identity (i.e., \( q = q_{h0} \) so that the demand from the matching identity segment is large relative to the size of the mismatching identity segment if consumers buy just looking for the functional attribute), the possibility of consumer communication may make it optimal for it to refrain from advertising. The reason, as pointed out above, is that in such a case, the firm would rather rely on consumers’ communications because they create a more informative and thus more effective form of communication. Furthermore, the firm realizes that its ad would make the consumer communication less informative and thus ineffective.

The informativeness of advertising in this equilibrium is quite unique. When a firm does not send an ad, the consumers cannot know which identity type is likely to be attracted to its brand. However, they know that the identity of this brand will end up being stronger (because the firm does not advertise if and only if \( q = q_{h0} \)).

On the other hand, when a firm advertises, in equilibrium, consumers are fully informed. The ad content informs them about the type of consumers attracted to the brand (i.e., \( L \) or \( R \) identity segments), and the decision to advertise informs them that the brand identity will not be strong (because the firm advertises only when \( q = q_{h} \)).

Before proceeding, it is important to note that a significant assumption in this analysis is the one of exogenous price. Allowing price to be optimally set by the brand would greatly complicate the model. But more importantly, it also raises the possibility that the firm may signal the strength of correlation through price, and thus refraining from advertising in order to facilitate consumer communication may not be the equilibrium outcome. In fact, this may indeed be the case in the model with its current setting. However, more realistically, consumers are not only uncertain with respect to the identity of the brand but also with respect to several other dimensions such as the quality of the product. Furthermore, it is also possible that price could be a function of (not fully observable by consumers) variable production costs or distribution issues. Therefore, one should not expect a firm to be able to signal the strength of the correlation perfectly. As far as some consumer uncertainty remains, consumer communication will continue to play an important role in the information consumers have—consequently, so too will the incentives the firm has in facilitating or restraining consumer communication through its advertising strategy, and thus the ideas we present will still be relevant. Some of the previous theoretical analysis of image advertising has adopted a similar modeling strategy. For example, in his “variety model,” Wernerfelt (1990) also abstracts from price considerations. However, especially because of the issues mentioned above, not formally modeling the pricing decision is a limitation of the current model.

Proposition 4 focuses on a subset of the parameter space. Figure 1 illustrates other equilibrium possibilities. For a particular set of values of \( \delta, p, \) and \( v_{f_{i}} \) chosen for illustration, it identifies which equilibria exist depending on the pair \((q_{f}, q_{h})\). Specifically, there are combinations of \( q_{f} \) and \( q_{h} \) under which only the separating equilibrium identified in Proposition 4 exists (dark gray area in Figure 1), but there are also combinations under which, in addition, a pooling equilibrium in which the firm advertises regardless of

\[ \text{Recall that Proposition 2 states that absent consumer communication, the firm always prefers to advertise, but that proposition does not establish whether the firm prefers to advertise in the first or in the second period. With consumer communication, the incentive to advertise in the second period vanishes, because consumer communication is a (weakly) better vehicle of information than advertising. Therefore, the only relevant problem for the firm with respect to advertising is whether to advertise in the first period and convey an image faster or let the image be revealed in the second period (and possibly, be stronger).} \]
To gain some insight on the relationship between \( q \) and the type of equilibrium, consider the comparative statics of a move on the straight line marked by \( A \) (which keeps constant the expected image, i.e., the average \( q \)). Moving down along this line, the gap between \( q_h \) and \( q_l \) diminishes. It starts in an area in which only a separating equilibrium exists, moves into an area in which in addition to the separating equilibrium, a pooling equilibrium with one-sided communications exists, and ends in an area with only such pooling equilibria. The intuition behind this is straightforward: the separating equilibrium is enabled by the distinction between the two types (\( l \) and \( h \)) and when the level of this distinction decreases, the forces that hold the separation weaken. Moving down along line \( A \) in Figure 1 reflects the trade-off faced by the high-type firm. One force is the desire of the firm not to be perceived as low type and to achieve this, it needs to refrain from advertising in the first period. Put differently, refraining from advertising leads consumers to find out (in the second period) whether the correlation is high or low (which is good news for a high \( q \) firm but bad news for a low \( q \) firm). On the other hand, if the firm does advertise, consumers would believe the correlation is low (in the separating equilibrium), and they will not find it to be otherwise in the second period (because consumer communication is one-sided in the case where the firm advertises). Thus the high-type firm has a stronger incentive to refrain from advertising. However, there is also a conflicting force: refraining from advertising in the first period is a costly investment (i.e., the firm loses consumers in the first period to gain some in the second). But as the gap between \( q_h \) and \( q_l \) diminishes, both the cost of refraining increases and the benefits from it decreases; the high \( q \) firm is not willing to bear the cost to separate, and in equilibrium, both \( q_h \) and \( q_l \) firms advertise. Finally, notice that the incentives are, obviously, exactly the opposite when moving up along line \( A \). As a result, for each value of \( q_l \), there is a \( q_h \) that for any value above it the incentive of the high \( q \) firm to deviate does not permit a pooling equilibrium. This “border” is represented by line 3 in Figure 1, which will soon become useful.

Another insight about the characteristics of these equilibria comes from considering how incentives to advertise are affected when the expected image is changing, e.g., moving up along the \( B \) line of Figure 1. Such a movement represents an increase in the expected \( L \) image, which implies lower consumer demand from \( R \)-identity consumers, especially from the ones who value self-expression more. Therefore, consumer communication is then more likely to be one-sided and thus uninformative. This effect explains why the equilibrium shifts from one with informative consumer communication (dotted area) to uninformative (light gray area) as the expected image increases and the firm advertises.

The above discussion gives some insight into the four areas in which there are pure-strategy equilibria.

\(^9\)To be consistent with the terminology of the signaling games, we will refer to the firm in the case when correlation is high as a high-type firm.
To understand the striped area in which no pure-strategy equilibrium exists, first recall that pooling is not possible above line 3 (which includes the entire striped area). This is because in a pooling equilibrium, consumers’ expected $q \left((q_l + q_h)/2\right)$ is high enough to lead to uninformative communications, which motivates the high $q$ firm to refrain from advertising in order to promote the informativeness of communication. It turns out that the separating equilibrium does not exist either in this area. The reason for this is related to the fact that in this area, $q_l$ (the expected $q$ in the separating equilibrium when the firm does advertise) is small. When $q_l$ is small, even $R$-identity consumers who care enough about self-expression buy the $L$-identity brand, and thus communication in the second period is two-sided and informative. As a result, the high $q$ firm does not need to refrain from advertising to promote communication and therefore would prefer to deviate and advertise in the first period. This decision will lead consumers to conclude that it is a low $q$ firm. But in the second period, given that consumer communication is informative, it will be revealed as a high $q$ firm and gain additional sales. Thus, in this area, if there is an equilibrium, it must be hybrid: the low $q$ firm advertises, whereas the high $q$ firm mixes between advertising and not so that the consumer expectation of $q$ given advertising is such that consumer communication is on the border of informative and not (by the same logic, a hybrid equilibrium may exist between lines 1 and 3 in addition to the pooling one).

Given that our focus is on how the ease of consumer communication affects a firm’s strategy, it is interesting to consider the impact of $v_c$ on the range of the separating equilibrium (dark gray area). In Figure 1, the effect is as follows. Because line 3 is defined by the benefit of signaling high $q$ (as we know, the firm prefers a stronger image) versus the cost of not advertising (not having an image in the first period), it does not change. On the other hand, line 1 does change. This line is defined by the necessary condition that consumer communication should not be informative when the firm advertises and consumers expect $q = q_h$. This is true for a wider range of $q_l$ when the cost of communication is higher.\(^{10}\). Therefore, line 1 moves to the left, expanding the range of separating equilibrium. Of course, when $v_c$ is high enough, consumer communication will cease being informative even when consumers expect $q = q_h$ and the firm does not advertise. In such a case, the firm can not enable consumer communication through not advertising. Therefore, the firm would always advertise. This constraint is a function of $q_h$ (and not of $q_l$); i.e., it is defined by a horizontal line that would start to bound the range of the separating equilibrium from above as $v_c$ increases (it is not binding for the parameters considered in Figure 1).

4. Conclusion

The story behind the “Marlboro Man” and the success of Philip Morris in changing the feminine identity of its cigarette to a manly one is well known. The days in which marketers and especially ad agencies were powerful enough to attach an identity to a brand via image advertising are celebrated in the television show Mad Men. Alas, those days are over. The consumers’ role in determining the identity of brands has increased steadily during the last part of the 20th century, and riding the power of consumer communication enabled by the social media, it has become highly significant in the beginning of the 21st century. Because of these changes, firms and marketers are sailing into new territories. This study presents a theoretical framework that might assist them in this new and challenging journey.

This paper presents a model that captures a major ingredient of this new environment and presents insights about the impact of consumer communication on the incentives to use image advertising. Specifically, we find that sometimes by refraining from image advertising, the firm can strengthen the image of its brand. This result is based on the following intermediate finding—image advertising can diminish the informativeness of consumer communication by homogenizing the early consumers. Furthermore, it turns out that refraining from image advertising may be optimal when the firm is especially well positioned to create a strong identity (i.e., $q = q_h$). The reason is that in such a case not only is the identity created by consumer communications clear and reliable, but if the firm would advertise, the consumer communication would become uninformative. However, the reverse cannot happen: that is, it is not possible that the firm having the private information that image is likely to be weak (i.e., knowing that $q = q_l$) abstains from advertising because of its consideration of the effect of advertising on consumer communication. It seems that these insights have been missing from the discussion of these issues by both academic scholars and practitioners.

The idea that consumer communication is not “all evil” from the firm’s point of view has sunk

\(^{10}\) When $q_l$ is low, consumers are not worried about having a brand with nonmatching identity, and when $v_c$ is low, even the mismatched consumers take part in the conversation. Thus, when both $q_l$ and $v_c$ are low, the mismatched consumers will take part in the conversation even if the firm advertises. This means the $h$ firm does not need to refrain from advertising in order to keep communication informative. When $v_c$ increases, the separating equilibrium no longer exists for high enough values of $q_h$, but it does exist for low enough values of $q_l$ that did not “allow” separating earlier.
in, and firms are even encouraging this type of communication in various ways, such as buzz marketing and brand communities. The results of our model can improve these practices in various ways. Advertising inhibits informativeness of consumer communication, and therefore it may be optimal for a firm to refrain from advertising in the diffusion of the brand (and instead encourage consumer communications) if the correlation between consumer preferences for functional and identity attributes is higher than consumer prior belief would suggest. For example, Red Bull refrained from advertising and encouraged buzz by delivering free samples to student parties.

Although some firms are already adopting such a strategy, common wisdom in the diffusion literature says that advertising should be used in the early days of a product’s life cycle to stimulate sales and word of mouth. There is no doubt that early advertising can be optimal when a product category is introduced and/or when the aim of the firm is to position its product as functional. However, this study questions the optimality of such a strategy when it comes to image advertising and building a brand.

Appendix
This appendix includes four subsections that provide the proofs of the lemma and propositions. Specifically, §A.2 includes the proof of Lemma 1, and §§A.3, A.5, and A.6 present the proofs of Propositions 1, 2, and 4, respectively. Proof of Proposition 3 is established in the text.

Sections A.1 and A.4 describe how we calculate the model solutions in two cases in which there is no consumer communication: (i) when the firm advertises in the first period (§A.1) and (ii) when it advertises in the second period (§A.4). The solution presented in §A.1 is then used in §§A.2 and A.5, and the one described in §A.4 is used in §A.5.

A.1. Model Solution When the Firm Advertises in the First Period and There Is No Consumer Communication
In this case, consumers will receive no new information in the second period. Because each consumer with positive utility faces declining utility from the first to the second period, all demand will be realized in the first period. Next, we present the demand from each of the four segments (LL, LR, RL, and RR) as a function of I, consumers’ expectation of the identity. Without loss of generality, consider \( q > \frac{1}{2} \). If \( I > 0 \), the demand from the RR subsegment is always 0, and the demand from the LL subsegment is

\[
D_{LL} = \frac{q}{V_I V_F} \int_{I_p - \frac{V_F}{q}}^{I_p} \int_{V_F - q V_I}^{V_I} dV_I dV_F. 
\]

The double integral above represents a trapezoid area, which simplifies to \( \int_0^{\frac{q}{V_I}} \int_{V_F - q V_I}^{V_I} dV_I dV_F \) for \( V_I < p \) and \( \int_0^{V_I} \int_{V_F - q V_I}^{V_I} dV_I dV_F \) otherwise. Thus, \( D_{LL} \) simplifies to

\[
D_{LL} = \begin{cases} 
4(2p + V_I) - 2 & \text{if } I < p, \\
4 & \text{otherwise}.
\end{cases}
\]

Similarly, demands from LR and RL segments can be written as, respectively,

\[
D_{LR} = \begin{cases} 
(1 - q)(2V_F - p) - \hat{I}V_I & \text{if } I < V_F - p, \\
\hat{I}V_I & \text{otherwise},
\end{cases}
\]

and

\[
D_{RL} = \begin{cases} 
0 & \text{if } I < p, \\
(1 - q)\hat{I}V_I + V_F & \text{if } I < V_F + p, \\
2V_F - p - V_F & \text{otherwise}.
\end{cases}
\]

The total demand is obtained as \( D_{LL} + D_{LR} + D_{RL} \). Consistency of consumer expectations of the identity implies the following equilibrium condition on \( \hat{I} \):

\[
\hat{I} = 2 \frac{D_{LL} + D_{RL} + D_{RL}}{D_{LL} + D_{RL} + D_{RL}} - 1.
\]

Moving all terms to one side and taking the numerator reduces this equation to a polynomial equation of the third degree in \( \hat{I} \). The solution of the demand in the case \( \hat{I} < 0 \) is symmetric to the one above.

A.2. Proof of Lemma 1
Differentiating the total demand derived above with respect to \( \hat{I} \), the consumer expectation of the image, and simplifying the derivative in each of the cases \( V_I < p, V_I \in [p, V_F - p], V_I \in [V_F - p, p], V_I \in [p, V_F - p], V_I \in [\max[p, V_F - p], V_F + p], V_I + p \), and \( V_I > V_F + p \), we obtain that the derivative is always positive. Furthermore, the demand is continuous when changing between these regions. Therefore, the total demand always increases in \( \hat{I} \), and the first claim is proven.

To prove that the demand is higher when beliefs are directionally consistent than when they are exactly the opposite, without loss of generality, assume \( q > \frac{1}{2} \). We need to show that the firm prefers consumers to have expectations \( \hat{I} > 0 \) of the identity rather than \( -\hat{I} \). Notice that if the consumer expectation of the identity is \( -\hat{I} < 0 \) and \( q > \frac{1}{2} \), the demand is equal to that when the consumer expectation is \( \hat{I} > 0 \), but instead, \( \hat{I} \) is replaced by \( 1 - q \). Furthermore, although we assumed \( \hat{I} > 0 \) in the derivation of the demand, we did not assume \( q > \frac{1}{2} \). In other words, the calculations in the previous subsection can give us the demand with \( \hat{I} > 0 \) as well as the demand with \( -\hat{I} \) (as long as we plug in \( 1 - q \) instead of \( q \)). Therefore, to show that the firm prefers \( \hat{I} > 0 \) over \( -\hat{I} \), all we need to show is that the total demand derived above is lower when \( q > \frac{1}{2} \) is replaced by \( 1 - q \), which is straightforward.
A.3. Proof of Proposition 1

Lemma 1 implies that if the firm advertises in the first period, it prefers to influence consumers’ beliefs that the identity is positive and the largest possible whenever \( q > \frac{1}{2} \), and it is negative and the smallest possible when \( q < \frac{1}{2} \). Therefore, the firm’s advertising in the first period is truth-ful in direction (i.e., to which identity segment is the brand most suitable) but always the most extreme in magnitude. Likewise, if the firm does not advertise in the first period, advertising positive identity in the second period brings demand only from the \( L \)-identity segment. This is because if a consumer preferring the \( L \) identity would buy in the second period given the information that the identity is positive, she should have bought in the first period when the identity was uncertain. In the second period, the demand from the \( L \)-identity segment increases in the consumer belief about how positive the identity is. Therefore, in the second period, the firm also prefers the most extreme advertising.

Again, because the size of the \( L \)-identity segment is larger than that of the \( R \)-identity segment if and only if \( q > \frac{1}{2} \), the firm prefers to advertise positive identity if and only if \( q > \frac{1}{2} \). Thus, advertising in the second period is truthful only in direction as well.

A.4. Model Solution When the Firm Advertises in the Second Period Only

As in the case of firm advertising in the first period only, there are a number of subcases to consider to expand the repeated integrals. Because in this case the number of regions is even larger, we will illustrate the solution only in the case of \( \delta \hat{I} V_i \in [\text{max}[p, (2 - \delta) V_f - p], \delta V_f + p] \), where \( \hat{I} > 0 \) is the equilibrium identity. The derivations in the other cases are similar. This condition affects only how the multiple integrals are resolved into repeated integrals, not which constraints are binding.

If the firm does not advertise in the first period, the expected value of identity by the first period consumers is 0. This is because \( q - \frac{1}{2} \) is symmetrically distributed around \( \frac{1}{2} \), and we assume that the firm’s advertising strategy as a function of \( q \) is symmetric around \( q = \frac{1}{2} \). In the second period, consumers expect to update the expectation of the identity to be either \( \hat{I} \) or \( -\hat{I} \) for some \( I \in [0, 1] \), which, in equilibrium, will turn out to be the actual value of the identity. Given the above, the consumer’s strategy is to (1) buy in the first period, (2) not buy in either period, or (3) buy in the second period if and only if the firm’s advertising message indicates a favorable identity to her. This is because if the consumer would buy in the second period regardless of the advertising, the consumer would be strictly better off buying in the first period instead. Therefore the marginal consumer in the first period is either (1) indifferent between buying and not buying in the first period, i.e., this consumer has \( u_f = p \); or (2) indifferent between buying in the first period or buying in the second period in the favorable identity case. The equation defining the latter marginal consumer is \( u_f = \frac{\delta (u_f + \hat{I} v) - p}{2} \), (8) where \( \frac{1}{2} \) represents the probability of the advertising indicating the favorable identity. Therefore, the demand in the first period comes from consumers with \((u_f, v_i)\) satisfying \( u_f \geq p \) and \( v_i \leq \frac{(2 - \delta) v_i - p}{\delta \hat{I}} \). In the case we consider, the mass of these consumers is

\[
D_1 = \frac{(V_F - p)(2 - \delta) V_F - \delta p}{2 \delta \hat{I} V_i V_f}. \tag{9}
\]

Furthermore, because consumers do not know in the first period whether the image is positive or negative, the buying strategies of \( L \) and \( R \)-functional segments are the same. In other words, fraction \( q \) of the first-period demand comes from \( L \)-identity segment, and fraction \((1 - q)\) comes from the \( R \)-identity segment.

In the second period, let us separately consider demands coming from \( LL \), \( LR \), \( RL \), and \( RR \) subsegments. Because, without loss of generality, we consider the actual realization of \( \hat{I} > 0 \), no consumers from \( RR \) segment buy. Furthermore, all consumers from \( LR \) subsegment who may want to buy in the second period should have bought in the first period, because these consumers’ utility decreased by \( \delta \), and they received unfavorable information about the identity. The demand from the \( LL \) subsegment comes from consumers with \( \delta (u_f + \hat{I} v_i) \geq p \) and who have not bought in the first period; i.e., \( v_i > \frac{(2 - \delta) v_i - p}{\delta \hat{I}} \). In the case we consider, the expression for this demand can be simplified to

\[
D_{LL} = \frac{2 p (V_F - p) - (2 - \delta) V_F^2 + 2 \delta \hat{I} V_i V_f}{2 \delta \hat{I} V_f}. \tag{10}
\]

Finally, demand from \( RL \) subsegment comes from consumers who have \(-v_f + \hat{I} v_i \geq p \). Note that none of the consumers in this segment could have bought in the first period because their expected utility from buying in the first period was \( -v_f - p < 0 \). Demand from these consumers can be written as

\[
D_{RL} = (1 - q) \frac{(\delta \hat{I} V_i - p)^2}{2 \delta^2 \hat{I} V_i V_f}. \tag{11}
\]

For consumer expectations of the identity to be correct, we must have

\[
\hat{I} = 2 \frac{q D_1 + D_{LL} + D_{RL}}{D_{LL} + D_{RL} + D_1} - 1. \tag{12}
\]

This equation is the equilibrium condition on \( \hat{I} \). Moving all terms to one side and taking the numerator results in a polynomial equation of the third degree in \( \hat{I} \).

A.5. Proof of Proposition 2

The claim immediately follows from observing that for all parameter values, the demand derived in §5.1 when the firm advertises in the first period is higher than the demand \( (V_f - p)/V_f \) given no advertising.

A.6. Proof of Proposition 4

The first claim is checked by examining the solution for the following parameter values: \( p = 0.8 \), \( V_f = V_f = 1, \delta = 0.99 \), \( q_1 = 0.57 \), \( q_6 = 0.95 \), and \( V^* = 0.21 \). In this case, without consumer communication, the firm would find it optimal to advertise in the first period regardless of \( q \) (i.e., for both \( q_1 \) and \( q_6 \)), and following the message of \( q \) > \( \frac{1}{2} \), consumer expectation of \( q \) would be \( (q_1 + q_6)/2 \). Can different consumer beliefs make a different advertising strategy optimal? If advertising in the second period rather than in the first signals that \( q \) is high, then the firm would indeed prefer to abstain from advertising in the first period and advertise...
in the second. However, it prefers to do so regardless of \( q \). Therefore, advertising in the second period cannot serve as a signal, and thus such beliefs are not consistent. Therefore, in the absence of consumer communication, refraining from advertising in the first period does not signal high \( |q - \frac{1}{2}| \), and the equilibrium is for both firms to advertise in the first period.

If consumer communication is possible, it turns out not to be informative (because no consumer from the \( R \) identity segment communicates) if the firm sends message \( q > \frac{1}{2} \) in the first period. However, consumer communication turns out to be informative if the firm does not advertise in the first period (regardless of whether consumers believe \( |q - \frac{1}{2}| \) is high or low). Therefore, consumers find out the exact value of \( q \) in the second period if and only if the firm does not advertise.

If the firm does not advertise in the first period and consumers find out the exact value of \( q \) in the second period, the demand is \( 0.388231 \) and \( 0.6407813 \) for \( q = q_h \) and \( q = q_l \), respectively. Recall that consumers believe that the firm advertises in the first period if and only if \( q = q_h \). Now, let us see what would be the result of deviations. If the firm advertises in the first period and consumers believe it to have \( q = q_h \), the profits would be \( 0.393204 \) and \( 0.633064 \), respectively. Thus, when \( q = q_h \), the firm is willing to refrain from advertising in the first period and obtain the demand of \( 0.6407813 \) instead of getting only \( 0.633064 \) if it deviated. When \( q = q_h \), the firm cannot benefit from this behavior because consumer communication would reveal \( q = q_h \), and the demand would be \( 0.388231 \) instead of \( 0.393204 \).

To show that the firm advertises if and only if \( q = q_h \), we need to rule out the possibility of a pooling equilibrium. To do this, we calculate the demand when (i) \( q = q_h \), (ii) the firm advertises in the first period, and (iii) consumers believe that \( q = q_h \) or \( q = q_l \) with equal probability (the most favorable beliefs given that when \( q = q_h \), the firm always advertises in the first period). We find that the demand is \( 0.640445 \), which is closer but still lower than the demand (when \( q = q_h \)) if the firm refrains from advertising and lets consumer communication reveal the exact value of \( q \). The first part of the proposition is proven.

To prove the second claim, consider a potential equilibrium where the firm advertises if and only if \( q = q_h \). In such equilibrium, consumers must infer from no advertising that the firm is \( q_l \) type. Consider then the deviation of the firm with \( q = q_l \) to advertise. Consumers then infer that \( q = q_h \) and know the direction of the image. If communication were informative when the firm did not advertise, and if the firm with \( q = q_h \) did not advertise, consumers would find out in the second period the direction of the image and that it is indeed weak, as expected. By assumption (of what happens if consumer communication is not possible), the firm would want to advertise in the first period if the belief about \( q \) would then be the average (and otherwise could be something, but no belief is worse for the firm than the belief \( q = q_h \)). The firm therefore also prefers to advertise if otherwise the belief is that \( q = q_h \) and when advertised the belief is updated to \( q = q_h \) (because increasing the belief is always good according to Proposition 1). If communication would not be informative, than the firm has even stronger incentive to advertise and make consumers believe \( q = q_h \). Note that if after the firm deviates and advertises consumers communicate and find out that \( q = q_h \), sales could only be increased, because otherwise, nobody buys in the second period (no new information). This completes the proof of the proposition.

References