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Abstract

This paper presents a theoretical foundation of the possibility that multimarket contact enhances firms' abilities to sustain implicit collusion. When firms operate in a single market and can not perfectly monitor the opponents' choices of supply, it is impossible to achieve efficiency among these firms in a self-enforcing way, even though these firms have the long-term strategic relationship. By using models of infinitely repeated game with discounting, we shows that when firms encounter each other in a number of distinct markets and the degree of multimarket contact is large enough, efficiency can be approximately sustained by a subgame perfect equilibrium. This efficiency theorem in the imperfect monitoring case holds under almost the same condition on the discount factor as the perfect monitoring case.

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1. Introduction

A large enterprise typically supplies multiple products. Even a single-product firm operates in a number of distinct geographic markets. Such firms come to contact with each other in a considerable number of markets. Corwin Edwards first raised the possibility that *multimarket contact* enhances firms' abilities to sustain *implicit collusion*. Edwards (1955) argued;

"The multiplicity of their contact may blunt the edge of their competition."

Several empirical studies following Edwards such as Mueller (1977) also have found a significant multimarket effect.[1] The present paper gives a theoretical foundation of the effect of multimarket contact on the possibility of implicit collusion among rival firms.

It is well-known in the game theory literature that in an infinitely repeated game with *discounting* players can achieve fully collusive, or *efficient*, allocations among them as self-enforcing subgame perfect equilibria when players can *perfectly* monitor the rivals' actions. Friedman (1971) applied this efficiency theorem to the study of an oligopolistic single market in which a small number of rival firms have a long-term strategic relationship, and showed that these firms can enforce implicit collusion in a self-enforcing way.

However, it is also well known at the present time that we can not extend this efficiency theorem to the more realistic, *imperfect monitoring case*. If the discount factor is less than unity and players can *not* directly observe the opponent's choices, it is impossible that efficiency can be sustained by a subgame perfect equilibrium, even though players can obtain some noisy information about the opponents' unobservable actions. In a quantity-setting oligopoly, the market demand may randomly fluctuate according to some unobservable exogenous factors, and therefore, the market-clearing price is regarded as a random variable which includes noisy information about the opponent's unobservable choices of supply. Stigler (1964) first raised that in such cases of imperfect monitoring it is impossible for firms to detect the opponents' secret price cuts which cause the failure of implicit collusion. Green and Porter (1984) modeled an infinitely repeated game with imperfect monitoring and discounting, and derived an inefficiency result on this line.

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[1] See Scherer (1980) also.

In this paper, we argue that when firms encounter each other in multiple markets and make the choices in a market dependent *not only* on the history of the prices realized in this market *but also* on the histories of the prices realized in the other markets, it is much easier for these firms to achieve implicit collusion than when they operate in a single market. Of particular importance, it is shown that when the degree of multimarket is large enough, efficiency can be approximately sustained by a subgame perfect equilibrium even in the imperfect monitoring case *under almost the same condition on the discount factor as the perfect monitoring case.*

We investigate the following two-person infinitely repeated game with discounting. The component game is defined by a combination of m identical prisoner-dilemma games. In every period, each firm simultaneously chooses an m-dimensional vector of supply each component of which is either "small supply" or "large supply". The resulting market price in a market is either "high" or "low", and is randomly determined according to some identical probability function which depends only on the current choices of supply in this market. Each firm can not observe the opponent's choice of supply, but can imperfectly monitor it through the realization of noisy market price. The probability that the "low" price occurs is positive when both firms' choosing "small supply", but this probability is larger when a firm's choosing "large supply".

The Nash equilibrium payoff vector in the one-shot game says that both firms choose "large supply" in every market, but is Pareto-dominated by the efficient payoff vector induced by both forms' choosing "small supply" in every market. When monitoring is perfect, the optimal subgame perfect equilibrium payoff vector is equivalent to the efficient payoff vector if and only if the discount factor is more than or equal to some *threshold* which is strictly less than unity and is independent of the degree of multimarket contact m. However, in the imperfect monitoring case, the optimal subgame perfect equilibrium payoff vector is Pareto-dominated by the efficient payoff vector irrespective of how the discount factor is specified.

Our main theorem says that, in the perfect monitoring case, if the discount factor is more than this threshold, the optimal subgame perfect equilibrium payoff vector per market approaches the efficient payoff vector per market as the degree of multimarket contact m increases.

In the proof of the main theorem, we construct a strategy profile which is regarded as a generalization of the *trigger strategy profile* by Friedman and is also related to the idea of *review strategy profile* by Radner (1986). According to this specified strategy profile, a firm has no incentive to deviate in *all* markets at one time: By observing the ratio between the number of all markets m and the number of markets in which the "low" price occurred, and by using the law of large number, the rival firm can almost certainly detect such an all-market deviation. A firm also has no incentive to deviate only in a *single* market: It might be difficult for the rival firm to detect such a single-market deviation in the imperfect monitoring case, but a firm nevertheless hesitates to deviate only in a single market for fear that the rival firm should *retaliate in all markets*. This point corresponds to the view that Edwards (1955) first raised and is commonly held:

"A prospect of advantage from vigorous competition in one market may be weighed against the danger of retaliatory forays by the competitor in other markets."

The difficulty that makes this paper non-trivial might be found in the proof that a firm has no incentive to deviate in *more than one but less than all* markets. We will show as Lemma 3 that there exists an integer k^+ such that the gain from deviation when a firm chooses "large supply" in some k markets and "small supply" in the other $m-k$ markets is concave with respect to k if k is less than k^+, whereas it is convex with respect to k if k is more than k^+. This guarantees, as is shown in Lemma 4, that if a firm has incentive to deviate, this firm always has incentive to deviate either only in a single market or in all markets. Hence, all we have to do is to show that a firm has no incentive to deviate either only in a single market or in all markets.

In spite of the unquestionable importance of the multimarket contact effect, there are few previous works on its theoretical foundations. Exceptions are Bulow, Geanakoplos, and Klemperer (1985), and Bernheim and Whinston (1990). Bulow, Geanakoplos, and Klemperer investigated a two-stage game of price-setting duopoly where one firm operates in another market as a monopolist and its production cost in the duopolistic market depends on the sum of its output in the two markets. They explained some effect of commitment in the monopolistic market which calms price competition in the duopolistic market.

Bernheim and Whinston (1990) is much more related to our work. They investigated an infinitely repeated game, and confined their attention to the perfect monitoring case. They explained the multimarket contact effect on enhancing implicit collusion, and referred to the relevancy to several empirical studies. However, it must be recognized that the multimarket effect in the perfect monitoring case is quite limited, because, as they have mentioned as "an irrelevance result", when markets and firms are identical, multimarket contact never enhances firms' abilities to enforce implicit collusion.

The efficiency theorem presented in this paper should be regarded as the remarkable contribution in the repeated game literature also. Radner, Myerson and Maskin (1986) presented an example in which, in the imperfect monitoring case, efficiency can not be approximated by a subgame perfect equilibrium even though the discount factor is close to unity. Matsushima (1989) pointed out that the logical core of this "uniform inefficiency" depends crucially on the specialty of the example. Fudenberg, Levine and Maskin (1994), Abreu, Milgrom and Pearce (1991), and Kandori and Matsushima (1997) presented their respective limit folk theorems in which efficiency can be approximated by a subgame perfect equilibrium. These works commonly assumed that the discount factor is close to unity. Hence, the present paper should be regarded as the first work to derive the efficiency theorem with *low* discount factor.

The organization of this paper is as follows. Section 2.1 defines a prisoner-dilemma game. Section 2.2 explains multimarket contact, and define infinitely repeated games with discounting and with imperfect monitoring. Section 2.3 presents a basic result in the perfect monitoring case, which gives a necessary and sufficient condition on the discount factor under which efficiency can be sustained by a subgame perfect equilibrium. Section 3 gives the main theorem in the imperfect monitoring case, and section 4 present the proof of this theorem. Section 5 concludes.

2. The Model

2.1. Prisoner-Dilemma Games

We define a *prisoner-dilemma game* $G \equiv (N, A_1, A_2, u_1, u_2)$ by

$$N = \{1,2\},$$
$$A_1 = A_2 = \{c, d\},$$
$$u_i: A_1 \times A_2 \to R,$$
$$u_i(c,c) = 1 \text{ and } u_i(d,d) = 0 \text{ for } i = 1,2,$$
$$u_1(c,d) = u_2(d,c) = -L,$$

and

$$u_1(d,c) = u_2(c,d) = 1 + K,$$

where $K > 0$, $L > 0$ and $1 > K - L$. We denote $a_i \in A_i$ for $i = 1,2$, $a = (a_1, a_2)$, and $u(a) = (u_1(a), u_2(a))$.

In the main part of this paper, we will assume *imperfect monitoring* in the following sense: Each player $i = 1,2$ can *not* observe the opponent's choice of action, but can observe a *public signal* $\omega \in \Omega \equiv \{B, G\}$ which is randomly determined and depends on the action profile chosen by the players. When an action profile $a \in A_1 \times A_2$ was chosen, each player $i = 1,2$ observes public signal B and obtains payoff $g(a_i, B)$ with probability $p(a)$, whereas she observes public signal G and obtains payoff $g(a_i, G)$ with probability $1 - p(a)$. We assume

$$p(c,d) = p(d,c),$$

and

$$p(c,c) < p(d,c) < p(d,d).$$

We must note that $u_i(a)$ is an *expected value*, i.e.,

$$u_i(a) = p(a)g(a_i, B) + \{1 - p(a)\}g(a_i, G).$$

An application is a *quantity-setting oligopoly* where each firm $i = 1,2$ simultaneously chooses either a small amount of supply $a_i = c$ ("cooperation") or a large amount of supply $a_i = d$ ("defection"). Public signal ω is regarded as the *market price*, where $\omega = B$ means the competitive, low price and $\omega = G$ means the collusive, high price. Because the market demand fluctuates randomly according to some exogenous factors, the market-clearing price is also randomly determined. Each firm can not observe the opponent's choice of supply. The realized market price gives a noisy information about it.

We must note that the action profile (d,d) is the *unique Nash equilibrium* in G, and the associated expected payoff vector $u(d,d)=(0,0)$ is Pareto-inferior to an *efficient* payoff vector $u(c,c)=(1,1)$.

2.2. Multimarket Contact and Repeated Games

We introduce the situation of *multimarket contact* in the following way. There exist two rival firms, i.e., firm 1 and firm 2, which produce commodities in m number of distinct quantity-setting duopolistic markets. Each market $h=1,\dots,m$ is modeled by the *identical* prisoner-dilemma game defined in subsection 2.1. In market h, firm $i=1,2$ chooses an amount of supply $a_{i,h}\in\{c,d\}$, and then observes a public signal, or a market price in market h, $\omega_h\in\Omega$.

These firms produce in every market infinitely many times. This repeated situation is modeled by an *infinitely repeated game with discounting* denoted by $G(m,\delta)$, where m is the number of markets, $\delta\in(0,1)$ is the *discount factor*, and the component game is defined by a combination of m identical prisoner-dilemma games. Let $S_i\equiv A_i^m$ denote the set of actions for firm i, and let $\Phi\equiv\Omega^m$ denote the set of signal profiles. In every period $t=1,2,\dots$, firm i chooses an action $s_i(t)\equiv(a_{i,1}(t),\dots,a_{i,m}(t))\in S_i$, and then observes a public signal profile $\phi(t)\equiv(\omega_1(t),\dots,\omega_m(t))\in\Phi$. Here, $a_{i,h}(t)\in\{c,d\}$ is the amount of supply which firm i chooses in market h in period t, and $\omega_h(t)\in\{B,G\}$ is the price realized in market h in period t.

A *pure public strategy*, or simply a *strategy*, for firm i is defined by

$$\sigma_i:\bigcup_{t=0}^{\infty}\Phi^t\to S_i,$$

where $\Phi_0\equiv\{\phi_0\}$, ϕ_0 is the null history, for every $t\geq 1$,

$$\phi_t\equiv(\phi(1),\dots,\phi(t))\in\Phi^t,$$
$$\phi(\tau)\equiv(\omega_1(\tau),\dots,\omega_m(\tau))\in\Phi \text{ for } \tau=1,\dots,t,$$
$$\sigma_i(\phi_t)=(\sigma_{i,1}(\phi_t),\dots,\sigma_{i,m}(\phi_t))\in S_i,$$

and

$\sigma_{i,h}(\phi_t)\in A_i$ is the amount of supply which firm i chooses in market h in period $t+1$ given history ϕ_t.

A firm's choice of supply in a market may depend on *not only* the history relevant to this market *but also* the histories relevant to the other markets.

Let $\sigma\equiv(\sigma_1,\sigma_2)$. Let $v_i(\sigma,\delta)$ denote the normalized expected payoff for firm i induced by strategy profile σ. Let $v(\sigma,\delta)\equiv(v_1(\sigma,\delta),v_2(\sigma,\delta))$. For every history ϕ_t, let $\sigma_i|_{\phi_t}$ denote the strategy for firm i after history ϕ_t occurs. Let $\sigma|_{\phi_t}\equiv(\sigma_1|_{\phi_t},\sigma_2|_{\phi_t})$. A strategy profile σ is said to be a *public perfect equilibrium* in $G(m,\delta)$ if for every $t=1,2,\ldots$, every $\phi_t \in \Phi_t$, every $i=1,2$, and every strategy σ_i',

$$v_i(\sigma|_{\phi_t},\delta) \geq v_i(\sigma_i',\sigma_j|_{\phi_t},\delta) \text{ where } j \neq i.$$

Remark 1: The terminology of public strategy and public perfect equilibrium is introduced by Fudenberg, Levine and Maskin (1994) and Radner, Myerson and Maskin (1986). In general, a strategy for a player may depend on not only public histories of signal profiles but also private histories of her own actions. However, Fudenberg, Levine and Maskin have shown that the public perfect equilibrium property is *robust* in the sense that there exists no strategy which depends on private histories and gives the higher payoff than this public equilibrium strategy. See Fudenberg and Tirole (1991, Chapter 5).

2.3. Perfect Monitoring: Basic Result

Before starting the analysis of the imperfect monitoring case, it might be helpful to consider the *perfect monitoring case*. Assume that each firm can directly observe the opponent's choice at the end of every period. Define a strategy for firm i, which is so-called the *trigger strategy*, as follows:

Choose $s_i(1)=(c,\ldots,c)$ in period 1.
In every period $t \geq 2$, choose $s_i(t)=(c,\ldots,c)$ if firm i observed $((c,\ldots,c),(c,\ldots,c)) \in S_1 \times S_2$ in all previous periods.
Choose $s_i(t)=(d,\ldots,d)$, otherwise.

The trigger strategy says that firm i chooses "cooperation in all markets" as long as no firm deviates from "cooperation in all markets", but chooses "defection in all markets" once a firm deviated from "cooperation in all markets".

We must note that if both firms conform to the trigger strategy profile, they obtain the *efficient* normalized expected payoff (m,m).

Proposition 1: *The trigger strategy profile is a perfect equilibrium if and only if these firms are patient enough to satisfy the inequality*

$$\delta \geq \frac{K}{1+K}. \tag{1}$$

Proof: Whenever a firm deviates in a market, then the opponent will choose d in all markets forever from the next period according to the trigger strategy. Hence, choosing d in all markets in all periods is the best strategy among all deviating strategies. This deviating strategy gives the deviant the normalized expected payoff $(1-\delta)m(1+K)$. It is clear that this value is less than or equal to the normalized expected payoff induced by the trigger strategy profile m, if and only if inequality (1) holds.

Q.E.D.

Remark 2: We must note that the one-shot Nash equilibrium payoff vector $(0,0)$ is the *minimax point*, and therefore, the infinite repetition of the choice of (d,d) in all markets is regarded as the *severest* equilibrium punishment. According to Abreu (1988), one gets that inequality (1) is not only sufficient but also *necessary* for the attainability of efficiency among these firms, even though all strategy profiles other than the trigger strategy profile are taken into account as possible equilibrium strategy profiles.

Remark 3: We must also note that the necessary and sufficient condition (1) does not depend on the number of markets m. This corresponds to the "irrelevance result" by Bernheim and Whinston (1990, section 3) which says that in the perfect monitoring case multimarket contact gives no influence on the possibility of implicit collusion when the model is symmetric.

3. Imperfect Monitoring: Main Theorem

From now on, we investigate the imperfect monitoring case. It is well known in game theory literature that if monitoring is imperfect and the discount factor is strictly less than unity, it is impossible to achieve efficiency as a perfect equilibrium payoff vector.[2] We show as the main theorem of this paper that as the number of markets m increases, the efficiency loss per market of public perfect equilibrium in the imperfect monitoring case approaches zero, and therefore, the efficient payoff vector per market (1,1) *can* be approximately sustained by a public perfect equilibrium when m is large enough. Of particular importance, this approximate efficiency holds *under almost the same condition on the discount factor as the perfect monitoring case*, i.e., this approximate efficiency holds if inequality (1), which is the necessary and sufficient condition in the perfect monitoring case, holds with *strict* inequality.

Theorem: *If inequality (1) holds with strict inequality, i.e.,*

$$\delta > \frac{K}{1+K}, \tag{2}$$

then there exists an infinite sequence of strategy profiles $(\sigma^{[m]})_{m=1}^{\infty}$ *which satisfies that for every* $i = 1,2$,

$$\lim_{m\to\infty} \frac{v_i(\sigma^{[m]},\delta)}{m} = 1, \tag{3}$$

and there exists $\overline{m}$ *such that for every* $m \geq \overline{m}$, $\sigma^{[m]}$ *is a public perfect equilibrium in* $G(m,\delta)$.

We present the proof of this theorem in the next section.

[2] See Pearce (1992) and Fudenberg and Tirole (1991, Chapter 5).

4. Proof of the Theorem

The proof of this theorem is constructive. Fix a positive integer $r(m) \in \{1,\dots,m\}$ arbitrarily, which is called the *threshold* for m. Define $\sigma_i^{[m]}$ as follows:

$$\sigma_i^{[m]}(\phi_0) = (c,\dots,c),$$

for every $t \geq 1$,

$\sigma_i^{[m]}(\phi_t) = (c,\dots,c)$ if $\#\{h \in \{1,\dots,m\} : \omega_h(\tau) = B\} < r(m)$ for all $\tau = 1,\dots,t$,

and

$\sigma_i^{[m]}(\phi_t) = (d,\dots,d)$ if $\#\{h \in \{1,\dots,m\} : \omega_h(\tau) = B\} \geq r(m)$ for some $\tau = 1,\dots,t$.

Let $\sigma^{[m]} = (\sigma_1^{[m]}, \sigma_2^{[m]})$.

According to $\sigma^{[m]}$, each firm continues to choose "cooperation in all markets" as long as the number of markets in which signal B is observed is less than the threshold $r(m)$. Once this number is more than or equal to $r(m)$, both firms immediately stop choosing "cooperation in all markets" and continue to choose "defection in all markets" from the next period.

For every $r \in \{0,\dots,m\}$ and every $k \in \{0,\dots,m\}$, let $f(r,m,k) \in [0,1]$ denote the probability that the number of markets in which signal B is observed is r, i.e., the probability that $\#\{h \in \{1,\dots,m\} : \omega_h(\tau) = B\} = r$ holds, provided that a firm chooses d in some k markets, chooses c in the other $m-k$ markets, and the opponent chooses c in all markets. By the standard calculation,

$$f(r,m,k) \equiv \sum_{h=\max[0,r-k]}^{\min[r,m-k]} D(h,r,m,k), \tag{4}$$

where

$$D(h,r,m,k) \equiv \frac{k!(m-k)!}{h!(m-k-h)!(r-h)!(k-r+h)!} \cdot p(c,c)^h \{1-p(c,c)\}^{m-k-h} p(d,c)^{r-h} \{1-p(d,c)\}^{k-r+h}.$$

From the definition of $f(r,m,k)$, we must note

$$f(r,m,k) = p(d,c) f(r-1,m-1,k-1) + \{1-p(d,c)\} f(r,m-1,k-1), \tag{5}$$

and

$$f(r,m,k-1) = p(c,c) f(r-1,m-1,k-1) + \{1-p(c,c)\} f(r,m-1,k-1). \tag{6}$$

The normalized expected payoff for firm i induced by $\sigma^{[m]}$ is written by

$$v_i(\sigma^{[m]},\delta)=(1-\delta)m+\delta\sum_{r=0}^{r(m)-1}f(r,m,0)v_i(\sigma^{[m]},\delta),$$

that is,

$$v_i(\sigma^{[m]},\delta)=\frac{(1-\delta)m}{1-\delta\sum_{r=0}^{r(m)-1}f(r,m,0)}.$$

Choose an infinite sequence of thresholds $(r(m))_{m=1}^{\infty}$ which satisfies

$$\lim_{m\to\infty}\sum_{r=0}^{r(m)-1}f(r,m,0)=0, \tag{7}$$

$$\lim_{m\to\infty}\frac{r(m)}{m}=p(c,c), \tag{8}$$

and

$$\lim_{m\to\infty}mf(r(m)-1,m-1,0)>\frac{(1-\delta)K}{\delta\{p(d,c)-p(c,c)\}}. \tag{9}$$

Lemma 1: *There exists an infinite sequence of thresholds $(r(m))_{m=1}^{\infty}$ which satisfies equalities (7) and (8) and inequality (9).*

Proof: See Appendix A.

Equality (7) says

$$\lim_{m\to\infty}\frac{v_i(\sigma^{[m]},\delta)}{m}=\lim_{m\to\infty}\frac{1-\delta}{1-\delta\sum_{r=0}^{r(m)-1}f(r,m,0)}=1,$$

that is, equality (3) holds. Hence, for every large enough m, $\sigma^{[m]}$ approximately achieves the efficient payoff vector per market $(1,1)$.

We show below that there exists $\overline{m}$ such that for every $m\geq\overline{m}$, $\sigma^{[m]}$ is a public perfect equilibrium in $G(m,\delta)$.

If firm i deviates from $\sigma_i^{[m]}$ by choosing d in some k markets and choosing c in the other $m-k$ markets and conforms to $\sigma_i^{[m]}$ afterwards, then it is penalized by the opponent with probability $\sum_{r=r(m)}^{m}f(r,m,k)$, and its normalized expected payoff is

$$w_i(k,m)\equiv(1-\delta)(m+kK)+\delta\sum_{r=0}^{r(m)-1}f(r,m,k)v_i(\sigma^{[m]},\delta). \tag{10}$$

We must note that $v_i(\sigma^{[m]},\delta)=w_i(0,m)$, and $\sigma^{[m]}$ is a public perfect equilibrium in $G(m,\delta)$ if for every $k\in\{1,\dots,m\}$,

$$w_i(k,m)\leq v_i(\sigma^{[m]},\delta), \tag{11}$$

i.e., if for every $k \in \{1, \ldots, m\}$,

$$(1+\frac{kK}{m})\{1-\delta\sum_{r=0}^{r(m)-1} f(r,m,0)\} \le 1-\delta\sum_{r=0}^{r(m)-1} f(r,m,k). \quad (12)$$

[3]

The following property of *single-peakedness* of $f(r,m,k)$ with respect to k will simplify the proof for public perfect equilibrium of $\sigma^{[m]}$.

Lemma 2: *There exists an integer* $k^*(r,m)$ *such that*

$$f(r,m,k) \ge f(r,m,k-1) \text{ if } k \le k^*(r,m),$$

and

$$f(r,m,k) \le f(r,m,k-1) \text{ if } k > k^*(r,m).$$

Proof: See Appendix B.

By using Lemma 2, we prove a lemma as follows, which plays am important role in the proof of this theorem.

Lemma 3: *For every* m, $w_i(k,m) - w_i(k-1,m)$ *is nonincreasing with respect to* k *if* $k \in \{1, \ldots, k^*(r(m)-1, m-1)\}$, *whereas it is nondecreasing with respect to* k *if* $k \in \{k^*(r(m)-1, m-1)+1, \ldots, m\}$.

Proof: From equalities (5) and (6),

$$\sum_{r=r(m)}^{m} f(r,m,k) = \sum_{r=r(m)}^{m} f(r,m-1,k-1) + p(d,c) f(r(m)-1, m-1, k-1),$$

$$\sum_{r=r(m)}^{m} f(r,m,k-1) = \sum_{r=r(m)}^{m} f(r,m-1,k-1) + p(c,c) f(r(m)-1, m-1, k-1),$$

and therefore,

$$\sum_{r=r(m)}^{m} f(r,m,k) - \sum_{r=r(m)}^{m} f(r,m,k-1)$$
$$= \{p(d,c) - p(c,c)\} f(r(m)-1, m-1, k-1). \quad (13)$$

From equalities (10) and (11),

$$w_i(k,m) - w_i(k-1,m)$$

[3] Theorem 3 in Kandori and Matsushima (1997) is based on the main theorem of the present paper. Kandori and Matsushima cited in p.646, l.1 this inequality as "inequality (14) in Matsushima (1995)".

$$= (1-\delta)K - \delta\{\sum_{r=r(m)}^{m} f(r,m,k) - \sum_{r=r(m)}^{m} f(r,m,k-1)\} v_i(\sigma^{[m]},\delta)$$

$$= (1-\delta)K - \delta\{p(d,c) - p(c,c)\} f(r(m)-1, m-1, k-1) v_i(\sigma^{[m]},\delta). \quad (14)$$

From Lemma 2 and inequality $p(d,c) > p(c,c)$, value (14) is nonincreasing with respect to $k \in \{1,\ldots,k^*(r(m)-1,m-1)\}$, whereas it is nondecreasing with respect to $k \in \{k^*(r(m)-1,m-1)+1,\ldots,m\}$.

Q.E.D.

Lemma 3 implies that there exists an integer k^+ such that the gain from deviation when a firm chooses d in k markets and c in the other $m-k$ markets is concave with respect to k if k is less than k^+, whereas it is convex with respect to k if k is more than k^+. By using Lemma 3, we show that all we have to do is to prove that a firm has no incentive to deviate only in a *single* market and has no incentive to deviate in *all* markets. That is;

Lemma 4: *$\sigma^{[m]}$ is a public perfect equilibrium in $G(m,\delta)$ if*

$$w_i(1,m) \le v_i(\sigma^{[m]},\delta) \text{ and } w_i(m,m) \le v_i(\sigma^{[m]},\delta).$$

Proof: Lemma 3 says that $w_i(k,m) - w_i(k-1,m)$ is nonincreasing with respect to $k \in \{1,\ldots,k^*(r(m)-1,m-1)\}$, and therefore, there is $\hat{k} \in \{0,\ldots,k^*(r(m)-1,m-1)\}$ such that

$w_i(k,m)$ is nondecreasing with respect to k in $\{0,\ldots,\hat{k}\}$,

whereas

$w_i(k,m)$ is nonincreasing with respect to k in $\{\hat{k},\ldots,k^*(r(m)-1,m-1)\}$.

This implies that if

$$w_i(1,m) \le w_i(0,m) = v_i(\sigma^{[m]},\delta),$$

then

$$w_i(k,m) \le v_i(\sigma^{[m]},\delta) \text{ for all } k \in \{1,\ldots,k^*(r(m)-1,m-1)\}. \quad (15)$$

Hence, if firm i does not prefer to deviate in a single market, firm i does not prefer to deviate in any number of markets less than or equal to $k^*(r(m)-1,m-1)$ also.

Lemma 3 says that $w_i(k,m) - w_i(k-1,m)$ is nonincreasing with respect to $k \in \{k^{\circ}(r(m)-1,m-1)+1,\ldots,m\}$, and therefore, there is $\tilde{k} \in \{k^*(r(m)-1,m-1),\ldots,m\}$ such that

$w_i(k,m)$ is nonincreasing with respect to $k \in \{k^*(r(m)-1,m-1),\ldots,\tilde{k}\}$,

whereas

$w_i(k,m)$ is nondecreasing with respect to $k \in \{\tilde{k},\ldots,m\}$.

This implies that either

$$w_i(m,m) \geq w_i(k,m) \text{ for all } k \in \{k^*(r(m)-1, m-1), \ldots, m\},$$

or

$$w_i(k^*(r(m)-1, m-1), m) \geq w_i(k,m) \text{ for all } k \in \{k^*(r(m)-1, m-1), \ldots, m\},$$

and therefore, one gets that if

$$w_i(1,m) \leq v_i(\sigma^{[m]}, \delta) \text{ and } w_i(m,m) \leq v_i(\sigma^{[m]}, \delta),$$

then

$$w_i(k,m) \leq v_i(\sigma^{[m]}, \delta) \text{ for all } k \in \{k^*(r(m)-1, m-1), \ldots, m\},$$

because inequalities (15) imply $w_i(k^*(r(m)-1, m-1), m) \leq v_i(\sigma^{[m]}, \delta)$.

From these observations, we have proven this lemma.

Q.E.D.

We show that a firm has no incentive to deviate only in a *single* market.

Lemma 5: *There exists an integer* $\overline{m}$ *such that for every* $m \geq \overline{m}$,

$$w_i(1,m) \leq v_i(\sigma^{[m]}, \delta).$$

Proof: From equalities (3) and (14) and inequality (9),

$$\begin{aligned}
&\lim_{m\to\infty}\{w_i(1,m) - v_i(\sigma^{[m]}, \delta)\} = \lim_{m\to\infty}\{w_i(1,m) - w_i(0,m)\} \\
&= \lim_{m\to\infty}\left\{(1-\delta)K - \delta\{p(d,c) - p(c,c)\}mf(r(m)-1, m-1, 0)\frac{v_i(\sigma^{[m]}, \delta)}{m}\right\} \\
&= (1-\delta)K - \delta\{p(d,c) - p(c,c)\}\lim_{m\to\infty} mf(r(m)-1, m-1, 0) \\
&> (1-\delta)K - \delta\{p(d,c) - p(c,c)\}\frac{(1-\delta)K}{\delta\{p(d,c) - p(c,c)\}} \\
&> 0.
\end{aligned}$$

Hence, we have proven this lemma.

Q.E.D.

We show that a firm has no incentive to deviate in *all* markets.

Lemma 6: *If inequality (2) holds, then there exists an integer* $\overline{m}$ *such that for every* $m \geq \overline{m}$,

$$w_i(m,m) \leq v_i(\sigma^{[m]}, \delta).$$

Proof: The law of large number says that

$$\lim_{m\to\infty}\left\{\sum_{r\ge m\{p(d,c)-\varepsilon\}} f(r,m,m)\right\}=1 \text{ for all } \varepsilon>0.$$

Let $\varepsilon>0$ satisfy $\varepsilon<p(d,c)-p(c,c)$. From equality (8),

$$\lim_{m\to\infty}\left\{\sum_{r=0}^{r(m)-1} f(r,m,m)\right\}=1-\lim_{m\to\infty}\left\{\sum_{r\ge r(m)} f(r,m,m)\right\}$$

$$=1-\lim_{m\to\infty}\left\{\sum_{r\ge m\{p(d,c)-\varepsilon\}} f(r,m,m)\right\}=0.$$

This, together with equalities (3) and (10), implies

$$\lim_{m\to\infty}\frac{w_i(m,m)}{m}=(1-\delta)(1+K)+\delta\lim_{m\to\infty}\left\{\frac{v_i(\sigma^{[m]},\delta)}{m}\sum_{r=0}^{r(m)-1} f(r,m,m)\right\}$$

$$=(1-\delta)(1+K),$$

which is less than 1 because of inequality (2). From equality (3),

$$\lim_{m\to\infty}\frac{w_i(m,m)-v_i(\sigma^{[m]},\delta)}{m}<0,$$

and therefore, we have proven this lemma.

Q.E.D.

From Lemmata 4, 5 and 6, we have proven that there exists $\bar{m}$ such that for every $m\ge\bar{m}$, $\sigma^{[m]}$ is a public perfect equilibrium in $G(m,\delta)$ under strict inequality (2). Hence, we have completed the proof of this theorem.

5. Conclusion

In this paper, we have investigated the long-run strategic relationship between two rival firms which come into contact with each other in multiple distinct markets. We have modeled infinitely repeated games with discounting the component game of which is defined by a combination of multiple identical prisoner-dilemma games. We have shown that, in the imperfect monitoring case, multimarket contact significantly enhances firms' abilities to sustain implicit collusion.

Since the model studied in this paper is restrictive, we might be asked about whether the main theorem can be extended to more general environments. My conjecture is that its essential point can be extended to some general class in which public signals are *independent* each other. However, this might not be true in the case that public signals are *correlated* each other, or more specially, there exists a *common* random shock in all markets. In the latter case, we might not use the law of large number to detect a firm's all-market deviation.

The study in more general environments might be important, might have some new substance, and should be examined as the next step on this line of research.

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Appendix A: Proof of Lemma 1

The law of large number says that for every $\varepsilon > 0$,

$$\lim_{m\to\infty} \sum_{r:\left|\frac{r}{m}-p(c,c)\right|<\varepsilon} f(r,m,0) = 1,$$

and therefore, there exists an infinite sequence of positive real numbers $(\varepsilon(m))_{m=1}^{\infty}$ such that

$$\lim_{m\to\infty} \varepsilon(m) = 0, \tag{A1}$$

and

$$\lim_{m\to\infty} \sum_{r:\left|\frac{r}{m}-p(c,c)\right|<\varepsilon(m)} f(r,m,0) = 1. \tag{A2}$$

Lemma A-1: *There exists* $(r(m))_{m=1}^{\infty}$ *which satisfies that there exists* $\overline{m}$ *such that for every* $m \geq \overline{m}$, $r = r(m)$ *is the maximal integer among* $\{0,\ldots,m\}$ *satisfying inequalities*

$$mf(r-1,m-1,0) > \frac{(1-\delta)K}{\delta\{p(d,c)-p(c,c)\}}, \tag{A3}$$

and

$$r \leq m\{p(c,c)-\varepsilon(m)\}. \tag{A4}$$

Proof: Suppose that there exists no such $(r(m))_{m=1}^{\infty}$. Then, we can choose an infinite sequence of positive integers $(m(x))_{x=1}^{\infty}$ such that $\lim_{x\to\infty} m(x) = \infty$, and for every x, there is no r which satisfies both inequalities (A3) and (A4) for $m = m(x)$. From equality (4),

$$f(r,m,0) = \frac{m!}{r!(m-r)!} p(c,c)^r \{1-p(c,c)\}^{m-r},$$

and therefore,

$$f(r,m,0) = \frac{p(c,c)}{r} mf(r-1,m-1,0). \tag{A5}$$

For every x and every r satisfying inequality (A4) for $m = m(x)$, inequality (A3) does not hold for $m = m(x)$, and therefore, one gets from equality (A5) that

$$f(r,m,0) \leq \frac{(1-\delta)Kp(c,c)}{\delta r\{p(d,c)-p(c,c)\}}.$$

Hence,

$$\lim_{x\to\infty} \sum_{r:\left|\frac{r}{m(x)}-p(c,c)\right|<\varepsilon(m(x))} f(r,m(x),0)$$

$$\leq \lim_{x\to\infty} \sum_{r:m(x)\{p(c,c)-\varepsilon(m(x))\}<r<m(x)\{p(c,c)+\varepsilon(m(x))\}} \frac{(1-\delta)Kp(c,c)}{\delta r\{p(d,c)-p(c,c)\}}$$

$$\leq \lim_{x\to\infty} \sum_{r:m(x)\{p(c,c)-\varepsilon(m(x))\}<r<m(x)\{p(c,c)+\varepsilon(m(x))\}} \frac{(1-\delta)Kp(c,c)}{\delta m(x)\{p(c,c)-\varepsilon(m(x))\}\{p(d,c)-p(c,c)\}}$$

$$\leq \lim_{x\to\infty} \frac{2\varepsilon(m(x))(1-\delta)Kp(c,c)}{\delta m(x)\{p(c,c)-\varepsilon(m(x))\}\{p(d,c)-p(c,c)\}}$$

$$= 0,$$

which is a contradiction of equality (A2).

Q.E.D.

It is clear from inequality (A3) that $(r(m))_{m=1}^{\infty}$ specified in Lemma A-1 satisfies inequality (9).

We show below that $(r(m))_{m=1}^{\infty}$ satisfies equalities (7) and (8) also. Since $r = r(m)$ is the maximal integer among $\{0,\ldots,m\}$ satisfying inequalities (A3) and (A4),

$$\lim_{m\to\infty} \sum_{r=0}^{r(m)-1} f(r,m,0)$$

$$\geq \lim_{m\to\infty} \sum_{r:|r-mp(c,c)|<m\varepsilon(m)} \left\{ f(r,m,0) - \frac{(1-\delta)Kp(c,c)}{\delta rm\{p(d,c)-p(c,c)\}} \right\}$$

$$\geq \lim_{m\to\infty} \left\{ \sum_{r:|r-mp(c,c)|<m\varepsilon(m)} f(r,m,0) \right\} - \lim_{m\to\infty} \left\{ \frac{2\varepsilon(m)(1-\delta)Kp(c,c)}{\delta m\{p(c,c)-\varepsilon(m)\}\{p(d,c)-p(c,c)\}} \right\}$$

$$= 1 - 0 = 1,$$

which implies equality (7).

Next, suppose that equality (8) does not hold. Since inequality (A4) holds for $r = r(m)$, there exist an infinite sequence of positive integers $(m(x))_{x=1}^{\infty}$ and $\eta > 0$ such that $\lim_{x\to\infty} m(x) = \infty$, and for every x,

$$r(m(x)) \leq m(x)\{p(c,c) - \eta\}.$$

However, the law of large number says that

$$\lim_{x\to\infty} \sum_{r=0}^{r(m(x))-1} f(r,m(x),0) \leq \lim_{x\to\infty} \sum_{r:r<m(x)\{p(c,c)-\eta\}} f(r,m(x),0)$$

$$= 1 - \lim_{x\to\infty} \sum_{r:r\geq m(x)\{p(c,c)-\eta\}} f(r,m(x),0) = 1 - 1 = 0,$$

which is a contradiction of equality (7).

From these observations, we have proven Lemma 1.

Appendix B: Proof of Lemma 2

The following lemma will be helpful for the proof of Lemma 2.

Lemma B-1: *$f(r,m,k)$ is single-peaked with respect to r, that is, there exists an integer $r^*(m,k)$ such that*

$$f(r,m,k) \geq f(r-1,m,k) \text{ if } r \leq r^*(m,k), \tag{B1}$$

$$f(r,m,k) \leq f(r+1,m,k-1) \text{ if } r \geq r^*(m,k), \tag{B2}$$

where $r^*(m,k)$ is nondecreasing with respect to k.

Proof: Consider $k=0$. From equality (4),

$$f(r,m,0) = \frac{m!}{r!(m-r)!} p(c,c)^r \{1-p(c,c)\}^{m-r},$$

and therefore,

$$\frac{f(r+1,m,0)}{f(r,m,0)} = \frac{(m-r)p(c,c)}{(r+1)\{1-p(c,c)\}},$$

which is less than or equal to 1 if and only if

$$r \leq (m+1)p(c,c) - 1.$$

This implies that $f(r,m,0)$ is single-peaked with respect to r.

Suppose that $k \geq 1$, and $f(r,m,k-1)$ is single-peaked with respect to r. Equality (5) says that for every $r \in \{1,\ldots,r^*(m-1,k-1)\}$,

$$\begin{aligned} f(r,m,k) &= p(d,c)f(r-1,m-1,k-1) \\ &+ \{1-p(d,c)\}f(r,m-1,k-1) \\ &\geq p(d,c)f(r-2,m-1,k-1) + \{1-p(d,c)\}f(r-1,m-1,k-1) \\ &= f(r-1,m,k), \end{aligned}$$

where $f(-1,m-1,k-1) \equiv 0$. Moreover, equality (5) says that for every $r \in \{r^*(m-1,k-1)+1,\ldots,m-1\}$,

$$\begin{aligned} f(r,m,k) &= p(d,c)f(r-1,m-1,k-1) \\ &+ \{1-p(d,c)\}f(r,m-1,k-1) \\ &\geq p(d,c)f(r,m-1,k-1) + \{1-p(d,c)\}f(r+1,m-1,k-1) \\ &= f(r+1,m,k), \end{aligned}$$

where $f(m,m-1,k-1) \equiv 0$. Hence, $f(r,m,k)$ is single-peaked with respect to r.

From these observations, we have proven that there exists $r^*(m,k)$ which satisfies inequalities (B1) and (B2).

We show below that $r^*(m,k)$ is nondecreasing with respect to k. Suppose that $f(r+1,m,k-1) \geq f(r,m,k-1)$. Equality (6) says

$$
\begin{aligned}
0 &\le f(r+1,m,k-1) - f(r,m,k-1) \\
&= p(c,c)\{f(r,m-1,k-1) - f(r-1,m-1,k-1)\} \\
&\quad + \{1-p(c,c)\}\{f(r+1,m-1,k-1) - f(r,m-1,k-1)\},
\end{aligned}
$$

which, together with the single-peakedness with respect to r, implies

$$f(r,m-1,k-1) \ge f(r-1,m-1,k-1). \tag{B3}$$

If $f(r+1,m-1,k-1) \ge f(r,m-1,k-1)$, then equality (5) and inequality (B3) say

$$
\begin{aligned}
& f(r+1,m,k) - f(r,m,k) \\
&= p(d,c)\{f(r,m-1,k-1) - f(r-1,m-1,k-1)\} \\
&\quad + \{1-p(d,c)\}\{f(r+1,m-1,k-1) - f(r,m-1,k-1)\} \\
&\ge 0.
\end{aligned}
$$

If $f(r+1,m-1,k-1) < f(r,m-1,k-1)$, then equalities (5) and (6) and inequalities (B3) and $p(d,c) > p(c,c)$ say

$$
\begin{aligned}
& f(r+1,m,k) - f(r,m,k) \\
&= p(d,c)\{f(r,m-1,k-1) - f(r-1,m-1,k-1)\} \\
&\quad + \{1-p(d,c)\}\{f(r+1,m-1,k-1) - f(r,m-1,k-1)\} \\
&> p(c,c)\{f(r,m-1,k-1) - f(r-1,m-1,k-1)\} \\
&\quad + \{1-p(c,c)\}\{f(r+1,m-1,k-1) - f(r,m-1,k-1)\} \\
&= f(r+1,m,k-1) - f(r,m,k-1) \\
&\ge 0.
\end{aligned}
$$

From these observations, we have proven that if $f(r+1,m,k-1) \ge f(r,m,k-1)$, then $f(r+1,m,k) \ge f(r,m,k)$. This implies that $r^*(m,k)$ is nondecreasing with respect to k.

Q.E.D.

By using Lemma B-1, we prove Lemma 2 in the following way. Fix (r,m,k) arbitrarily.

Suppose that $0 \le r \le r^*(m-1,k-1)$. Lemma B-1 says $r^*(m-1,k) \ge r^*(m-1,k-1)$, and therefore, $0 \le r \le r^*(m-1,k)$, i.e.,

$$f(r,m-1,k) \ge f(r-1,m-1,k).$$

Equalities (5) and (6) and inequalities $p(d,c) > p(c,c)$ say

$$
\begin{aligned}
& f(r,m,k) - f(r,m,k+1) \\
&= \{p(d,c) - p(c,c)\}\{f(r,m-1,k) - f(r-1,m-1,k)\} \\
&\ge 0.
\end{aligned} \tag{B4}
$$

Suppose that $r^*(m-1,k-1) < r \le m$. Then,

$$f(r,m-1,k-1) \le f(r-1,m-1,k-1).$$

Equalities (5) and (6) and inequalities $p(d,c) > p(c,c)$ say

$$
\begin{aligned}
&f(r,m,k-1)-f(r,m,k)\\
&=\{p(d,c)-p(c,c)\}\{f(r,m-1,k-1)-f(r-1,m-1,k-1)\}\\
&\leq 0. \qquad \text{(B5)}
\end{aligned}
$$

From inequalities (B4) and (B5), one gets that for every (r,m,k),

either $f(r,m,k)\geq f(r,m,k+1)$ or $f(r,m,k-1)\leq f(r,m,k)$,

which means the single-peakedness with respect to k.

From these observations, we have proven Lemma 2.