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Public Finance in an Overlapping Generations Economy

by

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Preface

This book, Public Finance in an Overlapping Generations Economy, presents a theoretical-based comprehensive analysis of macroeconomic consequences of fiscal policy on capital accumulation and welfare using the overlapping generations growth model; a particularly well-suited model for providing a dynamic general equilibrium framework from which to explore many aspects related the realm of public finance, as well as those of macroeconomics. Use of a reader-friendly approach provides a fresh outlook on theoretical and applied problems in public-sector economics, while simultaneously allowing rich descriptions which cover a wide-range of important public finance issues that are successively analyzed under this general though easy-to-focus model possessing an integrated intertemporal optimization framework. Issues considered in detail include the effects of tax reform on dynamic efficiency, positive and normative effects of public spending, considerations of taxes on fixed assets and monetary holdings, sustainability of deficits, conflicts between the younger and older generations, and spillover effects of tax reform on the rest of the open-economy world. The insights gained from these analyses lead to an enhanced understanding of how implementation of fiscal policies ultimately influences saving, capital formation, and intergenerational welfare. In addition, as an aid to forecasting the implications of fiscal policy implementation, particular emphasis is directed at developing tools that can be applied to theoretically clarify essential modern-day economic concerns such as generational incidence of tax reform and a growing dependence on government bonds for covering financial deficits.

The theoretical foundations constructed throughout the text are rigorous enough to comprehensively treat recent developments in the field of public finance, yet much effort has been expended to adequately balance academic theory with practical application value. The knowledge acquired for this analytical and practical presentation can easily assist in developing a proficiency in modern public finance theory and intertemporal macroeconomic analysis, while also being beneficial in evaluating intergenerational incidence of fiscal reforms employed by the United States, Japan, and many other OECD countries as an economic measure for reducing their government deficits and stimulating medium-term capital accumulation and economic growth.

The first half of the book (Chapters 2 -- 6) is devoted to formulating and explaining the basic theoretical framework of the employed overlapping generations model, and then examining how the general equilibrium of the economy is influenced by fiscal policy implementation of consumption taxes, labor income taxes, capital income taxes, and public spending. Towards this end, the closed and open versions of the overlapping generations model are both utilized as a means to fully elucidate the impact of such policies on capital accumulation and welfare. In the remaining half (Chapters 7 -- 11), attention is focused on applying the presented methodology to investigate alternative finance methods that can cover public spending; namely, taxes on monetary holdings, land taxes, and debt finance. Also considered are the effects of intergenerational transfers made possible by bequests and social security subsidies.

My main objective in writing this book lies in a desire to provide a brief but thorough, informal but academic, introduction to the fundamental concepts upon which the overlapping generations model is constructed. Naturally, the principal results derived by its utilization must be presented, though equal consideration is given to showing their practical applications and how they can be applied to a variety of related fiscal issues. Consequently, emphasis is placed on geometric and economic intuition rather than on rigorous development of general results. In fact, I have purposely limited the complexity such that the book can easily be read and understood by anyone with an understanding of economics equivalent to that of a first-year graduate student. Although I assume a sound comprehension of modern microeconomics and good familiarity with basic calculus, I use no sophisticated mathematical analyses, instead using detailed explanations to supplement the understanding of more technical sections. This strategy is intended to make most sections of interest to a broader range of readers, and I hope you will agree with me.

During the long gestation period of this book, I have incurred a multitude of graciousness, and it is now with pleasure that I acknowledge these.

In 1975, my teacher Mitsugu Nakamura introduced me to some public debt problems treated under the framework of the overlapping generations model, and it was through the course of these problems that I became interested in the power of this unique model to analyze government policies in a growing economy. Since then I have received support and assistance from many friends and colleagues. I would like to thank all my professors and friends at University of Tokyo, Johns Hopkins University, Tokyo Metropolitan University, Osaka University, and Washington State University, where I have studied, visited, and taught. In particular, I would like to express my gratitude to my teachers and advisers, to whom I owe much for my intellectual development in various stages of my study: Carl Christ, Koichi Hamada, Tatsuo Hatta, Masaaki Homma, Tsuneo Ishikawa, Ryutaro Komiya, Louis Maccini, Takashi Negishi, Hugh Rose, Hirofumi Shibata, and Hirofumi Uzawa.

With regard to the specific topic of this book, I have gained much from discussions and cooperative work with Raymond Batina ever since I met him in 1989. I would like to thank Takashi Awasawa, Takero Doi, Hiroyuki Kawakatsu, Toshihiro Shimizu, and Masatoshi Yoshida, all of whom provided valuable comments on earlier drafts. I would also like to thank Peter Werp and William Schrade for the excellent editing. I am also grateful to all at Macmillan for their collaborative effort in producing the final product.

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All of the people mentioned above have helped at one stage or another as this manuscript evolved from an idea to a reality. But my greatest thanks go to my wife and daughter, Nami Ihori and Kumi Ihori, who believed in me and supported me at every stage.

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Chapter 1

Introduction

1. Overview

1.1 Basic Theoretical Framework

To facilitate the understanding of *Public Finance in an Overlapping Generations Economy,* we start by providing in this chapter a brief summery of the entire text. This is followed by a short review of the basic concepts to be used in these chapters. The choice of overlapping generations growth model as the vehicle of analysis in this text is based on the fact that for over three decades it has become a standard framework for analyzing dynamic economic phenomena. It has facilitated novel insights which help our understanding of aggregate economic behavior, while also maintaining simplicity and analytical tractability.

Chapter 2 begins by describing in detail the fundamental characteristics of the basic theoretical model, namely, a two-period life-cycle overlapping generations growth model. This model is formed under the assumption that (i) individual families maximize private utilities over finite life spans and (ii) separate optimization efforts of families, together with market-clearing conditions, determine the economy's overall growth. However, to incorporate the important aspect of heterogeneity among households, family member dates of birth are utilized. This type of two-period growth was particularly selected because its mathematical properties greatly simplify much of the subsequent analyses. Although most results could be obtained under a longer-horizon model, a two-period framework has the comparative advantage since the main theoretical issues can be analyzed much more easily.

Further, Chapter 2 also presents the dynamic properties of the analytical framework based on Diamond's work (1965) among others. Then, they are applied to elucidate the effect of capital accumulation on each generation's welfare during the ensuing transition and in the long run. Policy implications of the modified golden rule and golden rule are explained. In the overlapping generations model the first optimality theorem of welfare economics does not necessarily hold. Therefore, we investigate the reason as to why the competitive market does not always lead to a Pareto-optimal allocation of resources. Following this, applicability of the basic framework is extended in scope by including an endogenous labor supply, bequests, and a multi-period setting.

1.2 Fiscal Policy Implementation

Chapters 3 through 6 are devoted to examining how the general equilibrium of the economy is influenced by fiscal policy implementation of consumption taxes, labor income taxes, capital income taxes, and public spending.

The set of commodity taxes minimizing the deadweight loss is called Ramsey Taxes, where the Ramsey rule is a useful criterion for maintaining static efficiency, as shown later. On the other hand, as was mentioned when describing the golden rule, it is a useful criterion for dynamic efficiency. Such contrast is of interest, and, therefore, the relationship between these rules is investigated in a growing economy with the first- and second-best solutions being presented.

In chapter 3, to investigate this relationship, we examine the first best solution and then the optimal combination of consumption taxes, labor income taxes, and capital income taxes by incorporating an endogenous supply of labor. The (modified) golden rule is shown to hold at the first best solution. The subsequent analysis shows that it also holds at the second best solution when neither lump-sum taxation nor debt policy is available, although in this case the (modified) Ramsey rule is also needed. It is then demonstrated that when consumption taxes are not available, the optimal condition is given by a mixed Ramsey-Golden rule: the Ramsey tax condition that represents second-period consumption using a divergence from the (modified) golden rule criterion. It is also pointed out that this rule cannot be separated into the (modified) golden rule and the (modified) Ramsey rule unless all the effective non-lump sum taxes are available.

Chapter 3 also presents a simple analytical framework for understanding intergenerational incidence from tax reform. The principal concern of this study is the implications of changing the timing of tax payments on different generations for intergenerational incidence. It is posited that the difference between consumption and labor income taxation is not the exemption from taxation of capital income or the incentive effect but the different timing of tax payments. Therefore, the tax reform concerning consumption and labor income taxation may well be evaluated within the framework of lump-sum tax reform. It is shown that the direct tax reform effect, the tax postponement effect, and the tax timing effect are important for the evaluation of tax reform.

In chapter 3, we also examine the welfare effect of a piecemeal change in capital income taxes. Such an analysis is useful because it is difficult to implement the optimal tax structure in the real world, whatever it is. For this would require us to estimate the precise levels of own and the cross elasticities among all the relevant goods in the economy. Furthermore, the structure of the optimal tax system is very sensitive to the precise values of the relevant elasticities. If the exact optimum is out of reach, we may still hope that we can improve welfare by making the present tax structure somewhat closer to the optimum. It is proved that an increase in capital income taxation with a reduction in labor income taxation will enhance capital accumulation and hence is desirable in some cases where the initial capital stock is well below the golden rule level.

There have been several attempts to address quantitative issues in the tax reform area. Chapter 4 summarizes these simulation studies using multi-period overlapping generations models in which agents live for many periods. After formulating a bench mark model, we summarize the more recent simulation studies which incorporate an endogenous labor supply, bequests, and human capital investment. The main results are as follows.

The conversion from wage income taxation to consumption taxation will normally stimulate savings and capital formation. This result would hold in a variety of cases even if we incorporate endogenous labor supply, bequests, or human capital investment. Since consumption tends to occur later in life than income, the current elderly population pays more, while subsequent generations pay less by having their tax payments deferred to old age. Those cohorts old at the time of the tax change lose, while all younger cohorts gain. Finally, chapter 4 concludes that although qualitative implications of tax reform for savings and welfare are robust, the quantitative results in most cases depend crucially on model parameters.

Chapter 5 investigates positive and normative effects of public spending. Most of the previous literature on government expenditure have investigated the effect of public spending financed by lump sum taxes (or wage income taxes with an exogenous labor supply). Chapter 5 first examines the conventional view that an increase in public spending, which will not stimulate production, has a negative impact on capital formation. Chapter 5 also provides several counterarguments to conventional wisdom by showing that if substitution effects are strong, an increase in public spending financed by non-lump sum taxes (labor income taxes on an endogenous labor supply, consumption taxes) may raise the capital intensity of production.

Chapter 5 then examines the normative aspect of public spending by investigating the optimal combination of distortionary taxes and government spending. This chapter derives the optimal public spending rule as well as the optimal tax rule and it argues that the method of financing the public good will also generally affect the dynamic efficiency of the economy. The conventional wisdom suggests that the social cost of a public good necessarily increases if a distorting tax system is used to finance the public good. This chapter provides a counter-example to the conventional wisdom. This chapter also considers the optimal role of public spending on stimulating production. It is shown that the public rate of return is equal to the social rate of time preference even if neither lump sum taxes nor debt policy is available.

Chapter 6 considers fiscal policy in a two country framework. First, this chapter develops a two-country open economy model and demonstrates the results of the conventional wisdom that a switch from the income tax to the consumption tax increases capital accumulation, reduces the interest rate, and improves welfare. It is also shown that the long-run welfare effect of the tax reform can be decomposed into the three components: the golden rule effect, the interest payment effect, and the tax revenue effect.

Chapter 6 then provides counterarguments to the conventional wisdom by showing that under certain circumstances (initially high consumption taxes in the foreign country) imposition of (and raising) the consumption tax may create a negative international spillover effect due to the tax revenue effect.

Chapter 6 also develops a general equilibrium model of how tax treatment of capital income affects the amount and form of international flow of capital. We consider both 'territorial' and 'residence' tax systems and examine a revenue-neutral tax reform: conversion from capital income to consumption income taxes. It is shown that in the territorial system the tax reform will normally lead to a negative comovement between capital accumulation in two countries, while in the residence system it will lead to a positive comovement.

Finally, Chapter 6 explores the normative aspect of taxation in an open economy by extending the optimal taxation problem in chapter 3 into the two country model. In the territorial system a reduction in capital income tax rates will induce capital inflow and hence raise tax revenue from capital income. This is the tax competition effect. It is shown that this may lead to lower capital income taxation in a noncooperative Nash solution than in the cooperative solution. The tax competition effect does not work in the residence system. It is also shown that the noncooperative Nash solution in the residence system is the same as the cooperative solution when both countries are identical. In this sense, the residence system is more desirable than the territorial system.

1.3 Alternative Finance Methods

In the remaining half of the text (Chapters 7 -- 11), attention is focused on applying the presented methodology to investigate alternative finance methods that can cover public spending, e.g., taxes on monetary holdings, land taxes, and debt finance.

Also considered are the effects of intergenerational transfers made possible by bequests and social security subsidies.

Chapter 7 introduces money into the basic model. There are two views of money. In the "bubbly" view money is a pure store of value. This view implies that price of money grows at the real rate of interest and that money is held entirely for speculation. In the "fundamentalist" view, money is held to finance transactions. Only the fundamentalist view can explain the rate of return dominance of money by other assets. We explain the bubbly view of money by summarizing Samuelson's 1958 framework, in which he first introduced money into the overlapping generations model without capital accumulation[1]. We also explain the fundamentalist view of money by introducing money into the Diamond model with capital accumulation and examine dynamic properties of the economy as well as its steady state nature.

We then explore concepts of inflationary taxes and welfare costs of inflation. The revenue yield from increases in inflation has received much attention in the literature. In general the optimal rate of inflation will be determined implicitly by a combination of preferences and technology. Since it is difficult to say anything explicit about the optimal rate of inflation, this chapter employs an alternative approach to explore the welfare cost of inflation by starting with the expenditure function and comparing the amount by which a consumer must be compensated to restore him to a reference utility level to the gain in revenue.

Chapter 7 also considers the welfare implications of indexing capital income taxation in an inflationary economy. Because we currently tax the nominal income from investment and allow borrowers to deduct nominal interest costs, the real net rate of returns to debt and equity will be directly altered by a change in the rate of inflation. In a steady state economy in which individuals anticipate inflation correctly, it is by no means clear whether the inflation-induced change in the real net rate of return on capital is always undesirable. It is shown that it is generally desirable to relate the net real rate of return to the rate of inflation. We also provide an example where the optimal indexation of the tax system is fractional and increasing with the rate of inflation.

The basic model in chapter 2 assumes that consumption goods and investment goods are perfect substitutes as outputs of the production technology. In reality, however, a significant fraction of savings is invested in assets that has a very long life and are not easily substitutable for consumption. Feldstein (1977) modified the standard model to include both capital and a fixed factor (land). He obtained the "surprising" result that a land tax may raise the land price in the long run. Chapter 8 investigates the relationship among land, taxes, and capital formation. We first consider in detail the dynamic effects of tax financing on capital accumulation and land values where revenues are used for a temporary increase in expenditure.

Chapter 8 then introduces money in order to produce capital gains in the long run and explicitly considers the general equilibrium effects of land rent taxes, land value taxes, and capital gains taxes. Positive inflation will lead to positive capital gains for the holder of land. Using this device, we can investigate long-run economic effects of a capital gains tax on land. It is shown that land taxes may affect the real equilibrium only through changes in the total revenue from the land taxes. We may have the "surprising" result, depending on how the revenue from land taxes is used. It is, however, also shown that the nominal price of land will normally be reduced by land taxes and that the existing old generation suffers from the unexpected capital losses from land holding although it gets capital gains from money holding.

Chapter 9 explicitly introduces government debt. We first show that the tax-financed transfer payments and Diamond's debt may have the same effect on the real equilibrium. In other words, this national debt can be regarded as a device which is used to redistribute income between the younger and the older generations. The government debt policy becomes meaningless if lump sum taxes are appropriately adjusted among generations. We also examine the role of government debt in the altruism model. The altruistic model means that households can be represented by the dynasty who would act as though they were infinitely lived. Barro (1974) showed that public intergenerational transfer policy becomes ineffective once we incorporate altruistic bequests into the standard overlapping generations model. We explain his idea intuitively.

The events of the 1970s and 1980s suggest that when a government becomes strapped for funds, it will tend to borrow from the world credit market rather than raise taxes to finance additional public spending. Indeed, many governments either will not raise broadly based taxes, e.g., the Thatcher government in Great Britain or the Reagan and Bush Administrations in the United States, or simply cannot raise taxes without possibly causing riots, e.g., countries in Latin American, Eastern Europe, and, arguably, France in the reign of Louis XVI. This phenomenon is theoretically formulated as the so-called chain-letter mechanism, which involves a situation in which the future time path of taxes is fixed and debt finance is used to pay for any additional public spending. Debt issuance is thus endogenously determined by the government's budget constraint. Chapter 9 thus studies the sustainability of such Ponzi games and the dynamic effects of various policy alternatives available to a government confronting a potential debt crisis such as a decrease in the level of the public good and a decrease in the marginal cost of providing the public good.

Finally, Chapter 9 investigates the dynamic implications of future tax reform in a debt-financed economy. Since the extent the debt burden transferred to the next generation depends on the possibility of future tax reform, we consider how expectations of the future tax reform affects the efficacy of fiscal policy.

In unfunded social security systems, the contributions of the younger generation earn a return which is composed of the rates of growth of population (biological rate of interest) and wages. In contrast, funded social security systems earn the market rate of interest and thus the marginal productivity of capital is relevant. From this perspective, it comes as no surprise that many industrial countries introduced or expanded pay-as-you-go unfunded public pension schemes in the years following the post-war baby boom. Considering the recent decline in the birth rates, however, a reverse of policy would be inevitable.

Chapter 10 investigates intergenerational incidence effects of social security in an aging economy and considers the welfare implications of changing the social security system from unfunded to funded schemes. This chapter also investigates welfare effects of unfunded system when labor supply is endogenous. It is shown that under certain conditions a gradual abolition of unfunded pensions - using appropriate changes in lump-sum contributions in the transition phase - can lead to an intergenerational Pareto improvement.

Recently, several papers have considered endogenous economic growth by allowing for the presence of constant or increasing returns in factors that can be accumulated. Following them, chapter 11 develops an endogenous growth model with bequests. It investigates the rate of economic growth when intentional bequests are operative. We consider three motives: the altruistic bequest motive, the bequest-as-consumption motive, and the bequest-as-exchange motive. It is shown that the relationship between the growth rate and the bequest motive is qualitatively the same among the three bequest motives. Namely, we show that the engine of growth consists of two effects. First, a higher rate of generation preference implies a higher degree of intergenerational transfers. This is called the intergenerational transfer effect. Second, a sufficiently high marginal product of private capital leads to long-run positive growth. This is called the intertemporal incentive effect.

We also explore the role of public capital in the model of economic growth by presenting interesting choices about the relations among the size of government, the saving and bequest behavior, the social security program, and the rate of economic growth. Chapter 11 finally incorporates human capital investment as the engine of growth and investigates the effect of taxation on two types of capital accumulation (intergenerational transfer and life cycle capital accumulation). It is shown that the analytical results depend on whether physical bequests are operative or not.

2. Review Of Basic Concepts

This section provides a brief review of basic concepts used in the following chapters.[2]

2.1 Constrained Maximization

Consider the following constrained maximization problem

$$\begin{aligned} &\textit{Maximize } u(x_1, x_2) \\ &\textit{subject to } g(x_1, x_2) = 0 \end{aligned} \tag{1}$$

The corresponding Lagrangian function is given as

$$L = u(x_1, x_2) - \lambda g(x_1, x_2), \tag{2}$$

where the variable λ is called a Lagrange multiplier.

Differentiating the Lagrangian with respect to each of its arguments, the first order conditions lead to

$$\frac{\partial L}{\partial x_1} = \frac{\partial u}{\partial x_1} - \lambda \frac{\partial g}{\partial x_1} = 0, \tag{3-1}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial u}{\partial x_2} - \lambda \frac{\partial g}{\partial x_2} = 0, \tag{3-2}$$

$$\frac{\partial L}{\partial \lambda} = -g(x_1, x_2) = 0. \tag{3-3}$$

These three equations determine three unknown variables, x_1, x_2, and λ. The Lagrange multiplier at the solution measures the sensitivity of the optimal value of the objective function.

2.2 Pareto Optimality

The modern approach to welfare economics is based on the concept of "Pareto Optimality", a necessary condition for an economic optimum. A Pareto optimum is a situation in which no feasible reallocation of outputs and/or inputs in the economy could increase the level of utility of one or more individuals without lowering the level of utility of any other individuals. An efficient social state is often called Pareto-optimal.

For example, suppose that there is fixed amount X, Y of the two goods (x, y) and there are only two individuals A and B. For simplicity assume that each individual's utility u_i is respectively given as a quasi-linear function.

$$u_A = u_A(x_A) + y_A, \tag{4}$$

$$u_B = u_B(x_B) + y_B, \tag{5}$$

where x_i is agent i's consumption of good x and y_i is agent i's consumption of good y. i=A, B. A Pareto optimal allocation under this setting is one that maximizes the utility of agent A, while holding agent B's utility fixed at some given level of $\overline{u}$.

$$\text{Maximize } u_A(x_A) + y_A \qquad (6)$$
$$\text{subject to } u_B(X - x_A) + Y - y_A = \overline{u}$$

Substituting the constraint into the objective function, we have the unconstrained maximization problem

$$\text{Maximize } u_A(x_A) + u_B(X - x_A) + Y - \overline{u}. \qquad (6)'$$

The optimality condition is given as

$$\frac{du_A}{dx_A} = \frac{du_B}{dx_B}. \qquad (7)$$

Now, we consider the relationship between the optimality condition (7) and the competitive equilibrium. At an equilibrium price p*, each consumer adjusts his consumption of good x to have

$$\frac{du_A}{dx_A} = \frac{du_B}{dx_B} = p^*. \qquad (8)$$

This equation means that the necessary condition for Pareto optimality is satisfied. The market equilibrium can produce a Pareto optimal allocation of resources. This proposition is usually referred to as the first optimality theorem.

First Optimality Theorem: Resource allocation is Pareto optimal if there is perfect competition and no market failure.

The first basic theorem of welfare economics states that a competitive equilibrium is a Pareto optimum: i.e., the equilibrium is one for which no utility level can be increased without decreasing some other utility level.

Also, any allocation that is Pareto optimal must satisfy (7), which determines p*. This implies that this Pareto optimal allocation would be generated by a competitive equilibrium. We have:

Second Optimality Theorem: Any specified Pareto-optimal resource allocation that is technically feasible can be established by free market and an appropriate pattern of factor ownership.

The second basic theorem of welfare economics states that any Pareto optimum can be realized as a particular competitive equilibrium; i.e., for each Pareto optimum there is an associated price system and a system of resource ownership which would attain, as a competitive equilibrium, this solutions with differing distributions of utility. It says that every Pareto-efficient allocation can be attained by means of a decentralized market mechanism.

2.3 Dual Approach

Consider a standard utility maximization problem of a consumer.

$$\text{Maximize } u(x_1, x_2) \qquad (9)$$
$$\text{subject to } p_1x_1 + p_2x_2 = M$$

where x_i is his consumption of good i, p_i is a consumer price of good i, and M is his income (i=1,2). Then, the maximum utility is a function of M and the price vector p = (p_1, p_2).

The indirect utility function indicates the maximum utility attainable at given prices and income.

$$u = U(p, M). \tag{10}$$

From this equation solving for M, we may derive the expenditure function:

$$M = E(p, u), \tag{11}$$

where E( ) indicates the minimum money cost at which it is possible to achieve a given utility at given prices. The expenditure function has following properties.

(1) E(p,u) is nondecreasing in p.
(2) E(p,u) is homogeneous of degree 1 in p.
(3) E(p,u) is concave in p.
(4) E(p,u) is continuous in p.
(5) The compensated demand curve is $x_1(p_1, p_2^0, u^0) = \dfrac{\partial E(p_1, p_2^0, u^0)}{\partial p_1}$.

2.4 The Ramsey Rule

The Ramsey rule is a basic criterion for any optimal taxation problem. Suppose in the economy that there is only one consumer, who consumes leisure and two goods. Producers produce two goods and a public good, g, by applying leisure (labor). The variable indexed by 3 is associated with leisure and the variables indexed by 1 and 2 with goods. Prices he faces are called consumer's prices and are denoted by the vector $q=(q_1, q_2, q_3)$. Denote the consumer's net demand vector by $x=(x_1, x_2, x_3)$. Then his budget equation is given as

$$q_1x_1 + q_2x_2 + q_3x_3 = 0 \ . \tag{12}$$

Note that the consumer's net demand for leisure, x_3, is negative and his demand for other goods is positive on the relevant domain of the prices.

The production possibility frontier is of the constant cost type. The frontier is given as

$$p_1x_1 + p_2x_2 + p_3x_3 + g = 0, \tag{13}$$

where producer's prices $p=(p_1, p_2, p_3)$ are constants.

Specific excise taxes and a wage tax are imposed. Thus, we have

$$q_i = t_i + p_i, \ i = 1,2,3. \tag{14}$$

When a positive wage tax is imposed, the consumer's after-tax pay is less than what his employer pays. This implies $q_3 < p_3$ and $t_3 < 0$. An increase in t_3 implies a decrease in the wage tax.

The tax revenue collected is spent on the public good. The government budget constraint is given as

$$t_1x_1 + t_2x_2 + t_3x_3 = g. \tag{15}$$

Equation (15) may be derived from (12)(13) and (14). Thus, this equation will not explicitly be considered below.

The consumer's optimizing behavior may be summarized in terms of the expenditure function:

$$E(q, u) = 0. \tag{16}$$

The production possibility frontier (13) may be rewritten as

$$p_1E_1(q,u) + p_2E_2(q,u) + p_3E_3(q,u) + g = 0, \tag{17}$$

where $E_i = \dfrac{\partial E}{\partial q_i} = x_i(q,u)$ (i=1,2,3) is the compensated demand function for good i.

The maximization problem is to maximize u subject to (16) and (17). The associated Lagrangian function is given as

$$V = u - \lambda_1 E(q,u) - \lambda_2 [p_1 E_1(q,u) + p_2 E_2(q,u) + p_3 E_3(q,u) + g], \tag{18}$$

where λ_i (i=1,2) is a Lagrange multiplier.

The first-order conditions are given as

$$\frac{\partial V}{\partial q_i} = -\lambda_1 E_i - \lambda_2 [p_1 E_{1i} + p_2 E_{2i} + p_3 E_{3i}] = 0, \tag{19}$$

where $E_{ij} = \frac{\partial^2 E}{\partial q_i \partial q_j}$. Considering (14), (19) may be rewritten as

$$\frac{(q_1 - t_1)E_{1i} + (q_2 - t_2)E_{2i} + (q_3 - t_3)E_{3i}}{E_i} = -\frac{\lambda_1}{\lambda_2}.$$

Considering the homogeneity condition $\sum q_i E_{ij} = 0$ and the symmetrical cross effects $E_{ij} = E_{ji}$, we obtain the Ramsey rule as

$$\frac{t_1 E_{i1} + t_2 E_{i2} + t_3 E_{i3}}{E_i} = \frac{\lambda_1}{\lambda_2}. \tag{20}$$

Or in the elasticity term we have

$$e_1 \sigma_{i1} + e_2 \sigma_{i2} + e_3 \sigma_{i3} = \frac{\lambda_1}{\lambda_2} \text{ (i=1,2,3)}, \tag{20)'$$

where $e_i = t_i / q_i$ is the effective tax rate and $\sigma_{ij} = q_j E_{ij} / E_i$ is the compensated elasticity. The Ramsey rule means that under an optimal tax structure, the marginal deadweight burden of a unit increase in each tax rate is proportional to the demand for that good.

From the Ramsey rule (20) or (20)', we may derive some special propositions.

The Inverse Compensated Elasticity Rule: Assume that the cross-substitution terms among the commodities are all zero ($E_{ij} = 0$ *for* $i \neq j$). The intrinsic tax rate of a commodity is inversely related to its demand elasticity.

The Uniform Tax Structure Rule: A uniform tax structure is optimal if and only if wage elasticities of demand are equal for all commodities, i.e. $\sigma_{13} = \sigma_{23}$.

2.5 The Samuelson Rule

The Samuelson rule is a basic criterion of the optimal public good problem. There are two agents in the economy. Each agent i(=A, B) consumes the private good x_i ($X = x_A + x_B$) and the public good g. His utility function is given as

$$u_i = u_i(x_i, g) \quad \text{(i=A, B)}. \tag{21}$$

The production possibility frontier is given as

$$F(x_A + x_B, g) = 0. \tag{22}$$

A Pareto optimal allocation is given as the solution of maximizing the utility of agent A, while holding agent B's utility fixed at some given level of $\bar{u}$. The associated Lagrangian is

$$L = u_A(x_A, g) - \mu_1[\bar{u} - u_B(x_B, g)] - \mu_2 F(x_A + x_B, g). \tag{23}$$

The first order conditions are given as

$$\frac{\partial L}{\partial x_A} = u_{A1} - \mu_2 F_x = 0, \tag{24-1}$$

$$\frac{\partial L}{\partial x_B} = \mu_1 u_{B1} - \mu_2 F_x = 0, \tag{24-2}$$

$$\frac{\partial L}{\partial g} = u_{A2} + \mu_1 u_{B2} - \mu_2 F_g = 0, \tag{24-3}$$

where μ_i is a Lagrange multiplier (i=1,2), $u_{ij} = \frac{\partial u_i}{\partial j}$ (i=A,B, j=x,g), and $F_i = \frac{\partial F}{\partial i}$ (i=x,g).

From (24-1,2,3) we finally get

$$\frac{u_{A2}}{u_{A1}} + \frac{u_{B2}}{u_{B1}} = \frac{F_g}{F_x}. \tag{25}$$

Equation (25) is called the Samuelson rule. The left-hand side of (25) is the sum of the marginal rates of substitution between the public good and the private good over all individuals and the right-hand side of (25) is the marginal rate of transformation between the public good and the private good. The public good is efficiently supplied when the sum of the marginal rates of substitution of the public good is equal to the marginal rate of transformation of the public good.

2.6 Phase Diagram

A phase diagram is widely used to explore dynamic properties of overlapping generations models. Consider the first-order system of difference equations with respect to x_t and y_t.

$$x_{t+1} = a + bx_t + cy_t, \tag{26-1}$$

$$y_{t+1} = d + ex_t + fy_t. \tag{26-2}$$

Subtracting x_t from both sides of (26-1) and y_t from both sides of (26-2), we have

$$\Delta x_t = a + (b-1)x_t + cy_t, \tag{26-1)'}$$

$$\Delta y_t = d + ex_t + (f-1)y_t, \tag{26-2)'}$$

where $\Delta z_t = z_{t+1} - z_t$ $(z = x, y)$.

Setting the left-hand side of (26-1)' and (26-2)' = 0, we obtain phaselines, which are combinations of variables x and y for which the vector field vanishes in one direction. We have

$$y = -\frac{a}{c} - \frac{b-1}{c}x, \tag{27-1}$$

$$y = -\frac{d}{f-1} - \frac{e}{f-1}x. \tag{27-2}$$

In Figure 1.1 we draw the two phaselines in the case of $a < 0$, $b > 1$, $c > 0$, $d < 0$, $e > 0$, and $f < 1$. Then, line xx given as (27-1) is downward sloping and line yy given as (27-2) is upward sloping. x is increasing above line xx and decreasing below it. y is increasing above line yy and decreasing below it.

Figure 1.1 suggests the existence of a convergent saddlepath through the upper and lower quadrants and that all other paths diverge. If y is a predetermined variable, at time t=0 y_0 is given. Corresponding to y_0, there is a single value for x, for which the system converges to the steady state. x is a free variable and jumps at t=0 to the value needed to put the system on the saddle path.

Consider now a nonlinear first-order system of difference equations.

$$x_{t+1} = \Gamma(x_t, y_t) \tag{28-1}$$

$$y_{t+1} = \Omega(x_t, y_t) \tag{28-2}$$

Steady states of the dynamic system (28-1,2) are solutions to the system of equations

$$x = \Gamma(x, y) \tag{29-1}$$

$$y = \Omega(x, y) \tag{29-2}$$

These two equations (29-1,2) give two phaselines as in Figure 1.1.

We may approximate the nonlinear system (28-1,2) in the neighborhood of the steady state (x*,y*) by the following linear system

$$x_{t+1} = x^* + \frac{\partial\Gamma(x_t, y_t)}{\partial x_t}(x_t - x^*) + \frac{\partial\Gamma(x_t, y_t)}{\partial y_t}(y_t - y^*) \tag{30-1}$$

$$y_{t+1} = y^* + \frac{\partial\Omega(x_t, y_t)}{\partial x_t}(x_t - x^*) + \frac{\partial\Omega(x_t, y_t)}{\partial y_t}(y_t - y^*) \tag{30-2}$$

In some neighborhood of (x*,y*), the phase diagram for equations (28-1,2) can be approximated by equations (30-1,2).

Note that discrete dynamic system can, and do, jump across boundaries of regions in phase diagrams. Any useful information in a phase diagram is already contained in the Jacobian matrix at the steady state. Qualitative information drawn from discrete phase diagrams is quite tentative and should be confirmed from the local information contained in Jacobian matrices.

2.7 Saddle Point Stability

Saddlepath solutions are very popular in economic models. Suppose that x is a free variable (for example, consumption) and y is a predetermined state variable (for example, capital). Consider again a nonlinear system of (28-1,2). The stability type of a steady state depends on the eigenvalues of the Jacobian matrix of partial derivatives

$$J(x, y) = \begin{bmatrix} \Gamma_x & \Gamma_y \\ \Omega_x & \Omega_y \end{bmatrix} \tag{31}$$

The steady state solution is called a saddle if one eigenvalue is inside the unit circle in the complex plane and the other is outside.

The eigenvalues of the Jacobian matrix are obtained by solving the following equation:

$$\varphi(\lambda) = \begin{vmatrix} \Gamma_x - \lambda & \Gamma_y \\ \Omega_x & \Omega_y - \lambda \end{vmatrix} = \lambda^2 - (\Gamma_x + \Omega_y)\lambda + \Gamma_x\Omega_y - \Gamma_y\Omega_x = 0 \tag{32}$$

The eigenvalues of (32) are real if and only if

$$(\Gamma_x + \Omega_y)^2 - 4(\Gamma_x\Omega_y - \Gamma_y\Omega_x) \geq 0$$

Let λ_1 and λ_2 be the eigenvalues of (32). If $\varphi(1) < 0$ and $\varphi(-1) > 0$, the steady state is a saddle. If $\varphi(1) > 0$ and $\varphi(-1) < 0$, again the steady state is a saddle.

[1] . McCandless with Wallace (1991) and Azariadis (1993) provided a useful explanation of Samuelson's model.

[2] . For more detailed explanations, see Varian (1992) for sections 2.1, 2.2. and 2.3, Atkinson and Stiglitz (1980) for sections 2.4 and 2.5, and Azariadis (1993) for sections 2.6 and 2.7.

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Chapter 2

Model

1. Introduction

The overlapping generations growth model is a very general analytical framework from which one can launch a multitude of economic studies. In this chapter, we will establish how we will employ it throughout all our subsequent analyses. Perhaps, one of its most important aspects is its economic time frame . Here, we apply a two-period life cycle growth model. This model was originally proposed by Diamond (1965), and since that time , has been employed by many researchers due to its well-suited generalization enabling many simplifications. In this chapter, the dynamic properties of the model are focused upon as they are pivotal in investigating the effects of capital accumulation on each generation's welfare during the ensuing transition and in the long run. Policy implications of the modified golden rule and golden rule are explained, and following this, the applicability of the basic framework is extended in scope by including an endogenous labor supply, bequests, and a multi-period setting.

The chapter is organized in the following manner. Section 2 presents the basic model by defining the competitive equilibrium and discussing stability of the dynamic system. Section 3 investigates dynamic efficiency at the first best solution using the concept of the modified golden rule and then explores dynamic properties using a phase diagram after which the normative meaning of the golden rule is explained. Section 4 extends the basic model of Section 2 by incorporating an endogenous labor supply. Section 5 extends the basic model by incorporating bequests. Section 6 describes how the overlapping generations approach may be used in a multi-period framework, and section 7 concludes the chapter.

2. Basic Model

2.1 Consumer

Consider a closed economy populated by overlapping generations of two-period-lived consumers as well as firms. In this model one young and old generation exist at any point in time. The young have no nonhuman wealth, and the lifetime resources of the young correspond to the labor earnings they receive. There may be growth in population. Output is durable and may be accumulated as capital. For simplicity it is assumed that there is no capital depreciation. The physical characteristics of the endowment are important in overlapping generations economics since durable goods represents an alternative technology for transferring resources through time[1].

An agent of generation t is born at time t, considers itself "young" in period t, "old" in period t+1, and dies at time t+2. When young an agent of generation t supplies one unit of labor inelastically and receives wages w_t out of which the agent consumes c^1_t, and saves s_t in period t. An agent who saves s_t receives $(1+r_{t+1})s_t$ when old, which the agent then spends entirely on consumption, c^2_{t+1} in period t+1. r_t is the rate of interest in period t. There are no bequests, gifts, or other forms of net intergenerational transfers to the young[2]. Each period two generations are alive, young ones and old ones. The pattern of consumption is summarized in Table 2.1.

A member of generation t faces the following budget constraints

$$c_t^1 = w_t - s_t, \quad (1)$$

$$c_{t+1}^2 = (1 + r_{t+1})s_t. \quad (2)$$

From (1) and (2) his lifetime budget constraint is given as

$$c_t^1 + \frac{1}{1 + r_{t+1}} c_{t+1}^2 = w_t. \quad (3)$$

His lifetime utility function is given as

$$u_t = u(c_t^1, c_{t+1}^2). \quad (4)$$

The utility function u( ) is increasing in the vector (c^1, c^2), twice continuously differentiable and strictly quasi-concave.

$$\frac{\partial u}{\partial c_t^1} = u_1(c_t^1, c_{t+1}^2) > 0 \; for \; (c^1, c^2) > 0,$$

$$\frac{\partial u}{\partial c_{t+1}^2} = u_2(c_t^1, c_{t+1}^2) > 0 \; for \; (c^1, c^2) > 0.$$

Future consumption is a normal good,

$u_1u_{12} > u_2u_{11}$ for (c^1, c^2) > 0,

where $u_{12} = \partial^2 u / \partial c^1 \partial c^2$ and $u_{11} = \partial^2 u / \partial c^1 \partial c^1$. Starvation is avoided in both periods,

$$\lim_{c^1 \to 0} u_1(c^1, c^2) = \infty \; for \; c^2 > 0,$$

$$\lim_{c^2 \to 0} u_2(c^1, c^2) = \infty \; for \; c^1 > 0.$$

A consumer born in period t solves the following problem:

Maximize his lifetime utility (4) subject to the lifetime budget constraint (3) for given w_t and r_{t+1}. The agent is capable of predicting the future course of the economy and he adopts this prediction as his expectation. Such rational or perfect foresight expectations are independent of past observations and must be self-filling. Otherwise, expectations do not coincide with the actual course of the economy unless the economy is in long-run steady state.

In Figure 2.1 AB is the lifetime budget line and point E where an indifference curve is tangent to line AB is the optimal point. Solving this problem for s_t yields the optimal saving function of the agent:

$$s_t = s(w_t, r_{t+1}), \quad (5)$$

where $\partial s / \partial w = s_w > 0$ as follows from the normality of second period consumption. The sign of $\partial s / \partial r = s_r$ is ambiguous since the substitution effect and the income effect offset each other.

2.2 Production Technology

The aggregate production function is

$Y_t = F(K_t, N_t)$,

where Y_t is total output, K_t is capital stock, and N_t is labor supply. We assume the constant returns to scale technology, so that the production function may be rewritten as

$$y_t = f(k_t), \; f' > 0, \; f'' < 0, \quad (6)$$

where $y_t = Y_t/N_t$ and $k_t = K_t/N_t$. y_t is per capita output and k_t is the amount of capital per worker in period t. The production function is well behaved and satisfies the Inada condition; $f(0) = 0, f'(0) = \infty, f'(\infty) = 0$[3].

The population grows at the rate of n (> -1),

$N_t = (1+n)N_{t-1}$. (7)

Competitive profit maximization and a neoclassical technology require that firms hire labor and demand capital in such a way that

$$f'(k_t) = r_t , \quad (8)$$

$$f(k_t) - f'(k_t)k_t = w_t . \quad (9)$$

Equations (8) and (9) imply that the marginal product of capital is equal to the rate of interest and the marginal product of labor is equal to the wage rate. Constant returns to scale and atomistic competition mean that payments to factors of production will exhaust every profit-maximizing producer's revenue, leaving nothing for profit. Since the markets for renting and purchasing physical capital are competitive, the opportunity cost of owning capital for one period should equal the rental rate.

From (8) and (9), w_t may be expressed as a function of r_t.

$$w_t = w(r_t), \quad w'(r_t) = -k_t < 0 , \quad w'' > 0, \quad (10)$$

where w( ) is called the factor price frontier. See Figure 2.2. The elasticity of substitution between capital and labor σ is defined by

$$\sigma = -w''wr/(w'f) = -w'' \mu r/w' = \mu \varepsilon ,$$

where μ = w/f is labor share of income and ε = -rw''/w' is the elasticity of capital. Thus, high σ corresponds to high -w''/w'.

In an equilibrium, agents can save by holding capital. In this economy, equilibrium in the financial market requires

$$s_t N_t = K_{t+1},$$

Or

$$s_t = (1+n)k_{t+1} . \quad (11)$$

2.3 Equilibrium And Stability

In summary substituting (5), (8) and (9) into (11), the economy may be described by the following difference equation.

$$s[f(k_t) - f'(k_t)k_t, f'(k_{t+1})] = (1+n)k_{t+1} . \quad (12)$$

We can now define a competitive equilibrium in terms of either the equilibrium sequence of the rate of interest $\{r_t\}$ or the equilibrium sequence of capital intensity $\{k_t\}$. Each sequence is a state variable that completely describes the system. For example, if the capital intensity sequence is known, equilibrium interest rates, wage rates, and factor incomes are defined uniquely from equations (8) and (9). And individual consumption vectors are also known from the budget constraints (1) and(2), and the saving function (5). Therefore, a competitive equilibrium is a sequence of consumption and saving demand $\{c^1_t, c^2_{t+1}, s_t\}$ and a sequence of prices $\{w_t, r_{t+1}\}$ such that

(i) $\{c^1_t, c^2_{t+1}, s_t\}$ solves the representative young agent's decision problem at time t for all t, taking $\{w_t, r_{t+1}\}$ as exogenous;

(ii) firms maximize profits taking $\{w_t, r_{t+1}\}$ as exogenous;

(iii) (12) holds with equality for all t.

Further, a dynamic equilibrium is summarized by a sequence $\{k_t\}_{t=0}^{\infty}$ under equation (12) where k_0 is exogenously given.

If savings are a non-decreasing function of the interest rate, from (12) we have

$$k_{t+1} = \phi(k_t), \quad (13)$$

where ϕ is a single-valued function, $\phi(0) = 0$. Notice that

$$\frac{dk_{t+1}}{dk_t} = \phi'(k_t) = \frac{-s_w k_t f''(k_t)}{1+n-s_r f''(k_{t+1})} . \quad (14)$$

Galor and Ryder (1989) analyzed the existence, uniqueness, and stability of steady-state equilibrium in the same overlapping generations model with productive capital. They derived a set of sufficient conditions, which includes nonnegative s_r, for the existence of a steady-state equilibrium as well as for the existence of a unique and globally stable equilibrium. From which we have (Galor and Ryder 1989):

Proposition 1: The overlapping generations economy experiences a unique and globally stable (non-trivial) steady-state equilibrium if $k_0 > 0$ and

$$\text{(a)} \lim_{k\to 0} \frac{-s_w k f''(k)}{1+n-s_r f''(k)} > 1,$$
$$\text{(b)} \lim_{k\to\infty} f'(k) = 0,$$
$$\text{(c)} \phi'(k) \geq 0 \textit{ for all } k > 0,$$
$$\text{(d)} \phi''(k) \leq 0 \textit{ for all } k > 0,$$
$$\text{(e)} s_r \geq 0 \textit{ for all } (w,r) \geq 0. \quad (15)$$

In Figure 2.3 curve $k_{t+1} = \phi(k_t)$ satisfies (13). Uniqueness and global stability of non-trivial stationary equilibrium are satisfied if (i) a single valued function ϕ exists, (ii) the curve ϕ (k) is strictly concave, (iii) $\lim_{k\to 0}\phi'(k) > 1$, and (iv) the curve intersects the 45 degree line at k > 0. Condition (e) is sufficient for (i). Noting that ϕ (k) < f(k), condition (b) is sufficient for (iv). Furthermore, (a) implies (iii) and (c) and (d) imply (ii). Thus the proposition follows. The economy will converge to the long run equilibrium point E monotonously.

Under conditions (a)-(e)

$$\phi' < 1$$

at the non-trivial steady state equilibrium E. This inequality may be rewritten as[4]

$$\frac{(1-\mu)e_w}{\mu\varepsilon} < 1 + \frac{e_r}{\varepsilon}$$

where e_w is the saving elasticity with respect to wages and e_r is the saving elasticity with respect to the rate of interest. This inequality holds if saving increases sufficiently fast when the interest rate rises in the neighborhood of the steady state. It will also more likely happen if the elasticity of capital ε is high.

The competitive long run capital labor ratio k_L is given as a solution of (16).

$$s[f(k_L) - f'(k_L)k_L, f'(k_L)] = (1+n)k_L. \quad (16)$$

The competitive long run rate of interest r_L is given by $r_L = f'(k_L)$.

3. Capital Accumulation And Efficiency

3.1 Efficiency

3.1.1 Modified Golden Rule

The feasibility condition in the aggregate economy is given as

$$N_t c^1_t + N_{t-1} c^2_t + K_{t+1} = K_t + Y_t.$$

Or, in per capita term

$$c_t^1 + \frac{1}{1+n} c_t^2 + (1+n)k_{t+1} = k_t + f(k_t). \quad (17)$$

The left-hand side of (17) means consumption and investment, while the right-hand side of (17) means the available resource in period t.

Suppose that a central planner wants at time t=0 to maximize social welfare. We now analyze the growth path which would be chosen by a central planner who maximizes an intertemporal social welfare function expressed as the sum of generational utilities discounted by the social time preference factor, β <1.[5]

$$W = \sum_{t=0}^{\infty} \beta^t u_t \tag{18}$$

In other words, the first-best problem is to maximize the Lagrange function

$$W = \sum_{t=0}^{\infty} \beta^t \{u_t + \lambda_{t+1}[k_t + f(k_t) - c_t^1 - \frac{c_t^2}{1+n} - (1+n)k_{t+1}]\}, \tag{18'}$$

where λ is the current shadow price of k and the Lagrange multiplier of the resource constraint at time t is $\beta^t \lambda_{t+1}$.

The optimality conditions with respect to c^1_t, c^2_t, and k_t are given by

$$u_{1t} = \lambda_{t+1}, \tag{19-1}$$

$$u_{2t} = \frac{\beta \lambda_{t+1}}{1+n}, \tag{19-2}$$

$$(1+n)\lambda_t = \beta[f'(k_t) + 1]\lambda_{t+1}, \tag{19-3}$$

along with the transversality condition[6]

$$\lim_{t \to \infty} \beta^t \lambda_{t+1} k_t = 0, \tag{19-4}$$

where $u_{1t} = \partial u_t / \partial c_t^1$ and $u_{2t} = \partial u_{t-1} / \partial c_t^2$. The transversality condition means that the social welfare function is maximal when the terminal present value of the capital stock is zero.

Equations (19-1,2,3,4) imply that the economy moves towards a path of balanced growth. In the steady state c^1, c^2, s, r, and k are constant. We have from (19-3):

Proposition 2: The optimal long-run capital-labor ratio k* and the optimal long-run rate of interest r* are given as

$$r^* = f'(k^*) = (1+n)(1+\rho) - 1, \tag{20}$$

where ρ is the rate of time preference; $\beta = 1/(1+\rho)$.

(20) is called the modified golden rule[7].

3.1.2 Phase Diagram

Suppose for simplicity the utility function is logarithmic. Then from (19-1) and (19-2), c^1_t and c^2_t are decreasing function of λ_{t+1}. Thus, (17) may be rewritten as

$$c(\lambda_{t+1}) + (1+n)k_{t+1} = k_t + f(k_t), \tag{17'}$$

where $c = c^1 + c^2/(1+n)$ and $c'(\lambda) < 0$. Thus, the dynamic system can be summarized by (17)' and (19-3).

Let us investigate the dynamic properties of this economy using a phase diagram in Figure 2.4[8]. From (19-3) we have

$$\lambda_{t+1} = R(\lambda_t, k_t), \tag{21}$$

where

$$R_\lambda = \frac{\partial \lambda_{t+1}}{\partial \lambda_t} = \frac{1+n}{\beta(1+f')} > 0, \tag{22-1}$$

$$R_k = \frac{\partial \lambda_{t+1}}{\partial k_t} = -\frac{\lambda f''}{1+f'} > 0. \tag{22-2}$$

To analyze the behavior of λ_t, we first find the locus of (λ, k) where $\lambda_{t+1} = \lambda_t$. We call this locus the $\lambda\lambda$ curve. From (21) this locus is given as (20).

$$f'(k^*) = (1+n)(1+\rho) - 1.$$

Thus, the $\lambda\lambda$ curve is a vertical line. From (22-2), $\partial\lambda_{t+1} / \partial k_t$ is positive. Hence, on the right-hand side of the $\lambda\lambda$ curve $\lambda_{t+1} > \lambda_t$, and on the left-hand side of this locus $\lambda_{t+1} < \lambda_t$. If k were unchanged, on the right-hand (left-hand) side of this locus λ will increase (decrease).

Next, consider the dynamic behavior of k_t. From (17)' we have

$$k_{t+1} = \hat{B}(\lambda_{t+1}, k_t) \ . \tag{23)'$$

Substituting (21) into (23)', we have

$$k_{t+1} = \hat{B}[R(\lambda_t, k_t), k_t] = B(\lambda_t, k_t), \tag{23}$$

where

$$B_\lambda = \frac{\partial k_{t+1}}{\partial\lambda_t} = -\frac{R_\lambda c'(\lambda)}{1+n} > 0, \tag{24-1}$$

$$B_k = \frac{\partial k_{t+1}}{\partial k_t} = \frac{-R_k c'(\lambda) + 1 + f'}{1+n}. \tag{24-2}$$

From (23) the locus of (k, λ) where $k_{t+1} = k_t$, the kk curve, is given as

$$k = B(\lambda, k). \tag{25}$$

Totally differentiating (25), we have the slope of the kk curve as

$$\frac{d\lambda}{dk} = \frac{1 - B_k}{B_\lambda} = -\frac{n - f' + R_k c'}{R_\lambda c'}. \tag{26}$$

If $r \geq n$, the kk curve is downward sloping. If $r < n$, the kk curve may be upward sloping. From (24-1) we know that above the kk curve, $k_{t+1} > k_t$, and below the kk curve, $k_{t+1} < k_t$. If λ were unchanged, above (below) the kk curve k will increase (decrease).

The dynamic properties of the system are depicted in the phase diagram of Figure 2.4. Given the level of the capital-labor ratio at time 0, k_0, consistency with perfect-foresight equilibrium requires (k_0, λ_0) to be an element of the global stable manifold. Given the monotonicity of the steady state manifolds and their nonoverlapping feature, there exists a unique shadow price level at time 0, $\hat{\lambda}_0$, that is consistent with perfect foresight equilibrium. The perfect-foresight equilibrium path converges in a discrete fashion along this stable manifold to (k^*, λ^*). Namely, for given tastes and technology, there is a unique steady state equilibrium point E. From a stability point of view, point E is a saddle-point and hence unstable except along one convergent path aa. The transversality condition (19-4) implies that the optimal path is the convergent path aa.

3.1.3 Golden Rule

The competitive long-run capital-to-labor ratio, k_L, is not necessarily equal to the optimal capital labor ratio, k^*. In order to investigate the dynamic efficiency of competitive equilibrium, the golden rule criterion is useful. When r is equal to r_G (= n), the long run growth path is called the golden rule path[9]. The golden rule maximizes the long run utility but it is not concerned with the well-being of generations during transition. Consequently, we have:

Proposition 3. If the social welfare is only concerned with the well-being of generations living in the steady state, then the golden rule, r = n, is the optimal solution.

This rule simply maximizes the total per-capita consumption of steady states, $c = c^1 + c^2/(1+n)$. In Figure 2.5 c = f(k) -nk corresponds to the difference between f(k) and nk, which is maximized at $k = k_G$, where $f'(k_G) = r_G = n$.

The modified golden rule given as (20) is the optimal growth path that maximizes the discounted sum of all generations which includes generations living in the transition. So long as the rate of time preference ρ is positive, r* is greater than r_G. In order to attain the golden rule, the economy initially accumulates more capital, which would sacrifice generations in the transition. Thus, r* is greater than r_G.

The golden rule has an important normative meaning in assessing the competitive equilibrium. If $r < r_G$, the market economy is called *dynamically inefficient* and if $r > r_G$, the economy is called *dynamically efficient*. If $r < r_G$, the market economy can move to the golden rule path by eating capital during transition. This will benefit generations in the transition. A Pareto improvement can be achieved in a dynamically inefficient economy by allowing the current generation to devour a portion of the capital stock and then holding constant the consumption of all future generations. In other words, such a movement will benefit all generations, and hence the initial state is regarded as dynamically inefficient. If $r > r_G$, the economy has to move to the golden rule by hurting generations during transition. There is a trade-off between the well-being of generations in the transition and the well-being of generations living in the long run. In other words, the initial economy may be regarded as dynamically efficient.

In the competitive economy there is no reason to believe that $r_L = r^*$ or r_G. In the real economy most empirical studies suggest $r_L > r_G$; the actual economy is dynamically efficient[10]. Thus, in some of the following chapters we may assume n=0 for simplicity, which means $r_L > r_G = 0$.

3.2 Cobb Douglas Case

Suppose the economy is described by the Cobb-Douglas technology and preference.

$$f(k) = k \quad (0 < \delta < 1), \tag{27}$$

$$u = (1-\alpha)\log c^1 + \alpha \log c^2 \quad (0 < \alpha < 1). \tag{28}$$

Then we have

$$s = \alpha w, \tag{29}$$

where $w = (1-\delta)k$ and $r = \delta k^{\delta-1}$. The dynamic equation (12) will be rewritten as

$$\alpha(1-\delta)k_t^{\delta} = (1+n)k_{t+1}. \tag{30}$$

As shown in Figure 2.6, the system is dynamically stable and the economy will monotonously move to the unique non-trivial long run equilibrium. See also Proposition 1.

The long run equilibrium is now given as

$$\alpha(1-\delta)k_L^{\delta} = (1+n)k_L. \tag{31}$$

Thus, we have

$$r_L = \frac{\delta(1+n)}{(1-\delta)\alpha}, \tag{32}$$

which is not always equal to r* or r_G. The competitive rate of interest r_L may be greater (or less) than r_G. The competitive economy may be dynamically efficient (or inefficient). We conclude:

Proposition 4: In the competitive economy there is no reason to believe that the growth path will attain the modified golden rule or the golden rule.

Remark 1: The competitive equilibrium is dynamically inefficient when the saving rate α is too high. Economies with higher saving rates may not necessarily be better off than economies with lower saving rates. A high propensity to save does not always mean dynamic efficiency in the steady state.

Remark 2 : In the overlapping generations model the competitive equilibrium is not necessarily Pareto efficient; the first optimality theorem does not hold. Agents who face a single lifetime budget constraint can be thought of as trading in a market that takes place at the beginning of time. Looked at in this way the overlapping generations model is an Arrow-Debreu economy with the simple difference that the commodity space and the number of agents are infinite. This double infinity turns out to be the key to the failure of the first optimality theorem. The infinite sum of endowments weighted by Arrow-Debreu prices across all agents may not be finite. Society has 'infinite' resources, and as a consequence one may no longer conclude that a Pareto-dominating allocation is unattainable[11].

3.3 Diagrammatic Exposition

The long run equilibrium may be described in the (c^1, c^2) plane like Figure 2.7[12]. Considering the budget constraints (1) & (2), the factor price frontier (10) and the capital accumulation equation (11), in the steady state the consumer's each-period budget constraint is reduced to

$$c^1 = w(r) + (1+n)w'(r), \tag{33-1}$$

$$c^2 = -(1+n)(1+r)w'(r). \tag{33-2}$$

These two equations give combination of c^1 and c^2 for given r. An increase in r will raise c^1 for high r (low k) and reduce c^1 for low r (high k). An increase in r will reduce c^2 under the stability condition. Eliminating r from these equations, we can express c^1 as a function of c^2. Curve OT in Figure 2.7 represents the locus.

Line AB is the lifetime budget constraint when r = n.

$$c^1 + \frac{1}{1+n}c^2 = w(n)\,. \tag{34}$$

(34) is the same as the feasibility condition when r = n. Curve OT is tangent to line AB at point G where r = n is satisfied. In the region of GO of curve OT, r > n ;and in the region of GT of curve OT, r < n. At each point of curve OT we can draw the lifetime budget constraint. The competitive equilibrium is the point where the associated lifetime budget constraint is tangent to the indifference curve. Since the competitive equilibrium is unique, only one point can satisfy this condition. In Figure 2.7, point E is such an equilibrium point. If the indifference curve happens to be tangent to line AB at point G, the competitive equilibrium is realized at the golden rule path. However, there is no reason to assume this, as shown in section 3.2.

4. Endogenous Labor Supply

In this section we incorporate an elastic supply of labor. Suppose the utility function (4) is replaced with

$$u_t = u(c_t^1, c_{t+1}^2, H - l_t)\,, \tag{4)'$$

where H is the initial endowment of labor supply in the first period, and l is the amount of labor supply in the first period. The lifetime budget constraint (3) is rewritten as

$$c_t^1 + \frac{1}{1+r_{t+1}} c_{t+1}^2 = w_t l_t, \qquad (3)'$$

The labor supply function is now given as

$$l_t = l(w_t, r_{t+1}). \qquad (35)$$

We assume that the substitution effect outweighs the income effect so that $\partial l / \partial w = l_w >$ 0. We also assume that $\partial l / \partial r = l_r > 0$, which means that the intertemporal substitution effect is strong. As a result, an increase in r raises c^2 in place of c^1 since the consumer price of c^2 in terms of c^1 is lowered. It will also reduce the consumer price of c^2 in terms of H-l, the first-period leisure. Then the first-period leisure H-l is also reduced and hence l is stimulated[13].

Denote by k the capital-labor ratio (=K/Nl). Here K is the total stock of capital and N is the population of the younger generation. Then, per-capita capital is given by kl (=K/N). We still have the same factor price frontier (10). However, the capital accumulation equation (11) becomes

$$s_t = (1+n)k_{t+1}l_{t+1}. \qquad (11)'$$

In summary, the endogenous labor supply system may be described by the following dynamic equation.

$$s[w(r_t), r_{t+1}] = -(1+n)w'(r_{t+1})l[w(r_{t+1}), r_{t+2}]. \qquad (12)'$$

From (12)' when labor supply is endogenous, the dynamic system can be rewritten as

$$S(r_t, r_{t+1}, r_{t+2}) = 0. \qquad (36)$$

This is a second order difference equation.

The stability analysis of equation (36) follows straightforwardly by using the following substitution:

$$r_{t+1} = p_t. \qquad (37)$$

Substituting (37) into (36) we obtain

$$p_{t+1} = P(r_t, p_t). \qquad (38)$$

In a neighborhood of the initial equilibrium

$$\begin{bmatrix} dp_{t+1} \\ dr_{t+1} \end{bmatrix} = \begin{bmatrix} P_p & P_r \\ 1 & 0 \end{bmatrix} \begin{bmatrix} dp_t \\ dr_t \end{bmatrix},$$

where $P_p = \partial P / \partial p_t$ and $P_r = \partial P / \partial r_t$. In the above expression

$$P_p = \frac{S_{r1} + (1+n)w' l_w w'}{(1+n)l_r k}, \qquad (39)$$

$$P_r = \frac{S_{r0}}{(1+n)l_r k}, \qquad (40)$$

where $S_{r1} = s_r + (1+n)w'' l$, $S_{r0} = w' s_w$.

Denote the characteristic polynomial of the system composed of (37) and (38) as $\varphi(\lambda) = \lambda^2 - P_p\lambda - P_r$. If φ (1) < 0 and φ (-1) > 0, then the steady-state equilibrium will be a saddle. $S_{r0} < 0$. Where $S_{r1} > 0$, this corresponds to the stability condition with an exogenous supply of labor. Thus, if $l_r > 0$, then φ (-1) > 0. We have

$$\varphi(1) = -A / (1+n)kl_r, \qquad (41)$$

where $A = S_{r0} + S_{r1} - (1+n)k(l_r + l_w w')$. It is straightforward to show that if $l_r > 0$ and A > 0, then φ (1) < 0. In such a case the initial steady-state equilibrium will be a saddle and consequently the dynamic equilibrium is uniquely determined[14]. We will assume that this is true. In this extended framework, we still have propositions 1, 2, and 3.

5. Bequests

5.1 Bequest Motives

We now extend the basic model to allow for bequests. There are several models of bequeathing behavior that have appeared in the literature, (i) the altruistic bequest model, where the offspring's indirect utility function enters the parent's utility function as a separate argument, (ii) the bequest-as-consumption model, where the bequest itself enters the parent's utility function as a separate argument, (iii) the bequest-as-exchange model, where the parent gives a bequest to his offspring in exchange for a desirable action undertaken by the offspring, and (iv) the accidental bequest model, where a parent may leave an unintended bequest to his offspring because lifetimes are uncertain and annuities are not priced in an actuarially fair way[15].

The altruistic model means that households can be represented by the dynasty who would act as though they were infinitely lived. Other intentional bequest models mean that their behavior can be described by the life cycle framework where overlapping generations are concerned with a finite number of periods. In this section we will formulate the first motive in the overlapping generations framework.

5.2 Bequest And Capital Accumulation

When we incorporate bequests, a representative individual born at time t has the following budget constraints.

$$c_t^1 + s_t = w_t + \frac{1}{1+n} b_t , \tag{1)'$$

$$c_{t+1}^2 = (1+r_{t+1})s_t - b_{t+1} , \tag{2)'$$

where b_t is the inheritance received when young, b_{t+1} is his bequests which is determined when old. Then the lifetime budget constraint (3) can be rewritten as

$$c_t^1 + \frac{1}{1+r_{t+1}}(c_{t+1}^2 + b_{t+1}) = w_t + \frac{1}{1+n} b_t . \tag{42}$$

5.3 Altruistic Model

In the altruism model the parent cares about the welfare of his offspring. The parent's utility function is given as

$$U_t = u_t + \sigma_A U_{t+1} . \tag{43}$$

σ_A is the parent's marginal benefit of his offspring's utility. $0 < \sigma_A < 1$. An individual born at time t will solve the following problem of maximizing.

$$W_t = u[w(r_t) - s_t + \frac{1}{1+n} b_t , (1+r_{t+1})s_t - b_{t+1}] + \sigma_A \{u[w(r_{t+1}) - s_{t+1} + \frac{1}{1+n} b_{t+1} , (1+r_{t+2})s_{t+1} - b_{t+2}] + \sigma_A U_{t+2}\} \tag{44}$$

The optimality conditions with respect to s_t and b_{t+1} are

$$u_{1t} = (1+r_{t+1})u_{2t+1} , \tag{45-1}$$

$$(1+n)u_{2t+1} = \sigma_A u_{1t+1} . \tag{45-2}$$

(45-1,2) gives the long run capital labor ratio in the altruism model r_A :

$$n = \sigma_A (1+r_A) - 1 , \tag{46}$$

which must be compared with r* . For $\beta = \sigma_A$, r* $= r_A$. In such a case the maximization problem (44) is essentially the same as the first best maximization problem (18). We have

Proposition 5: If the altruistic bequest motive is operative, the market solution can attain the first best optimum.

Remark: From (43) we have

$$U_0 = u_0 + \sigma_A U_1 = u_0 + \sigma_A (u_1 + \sigma_A U_2) = u_0 + \sigma_A \{u_1 + \sigma_A (u_2 + \sigma_A U_3)\}$$
$$= \sum_{t=0}^{\infty} (\sigma_A)^t u_t$$

if $\lim_{t\to\infty}(\sigma_A)^t U_t = 0$. The above objective function is the same as (18) when $\beta = \sigma_A$.

We will discuss the plausibility of altruistic intergenerational transfers in the context of debt neutrality in chapter 9. We will examine the effects of fiscal policy on capital accumulation and welfare when bequests are operative in chapters 9 and 11.

6. Multi-Period Framework

The main use of the basic model is to provide theoretical results of what might be possible in an equilibrium model, based on simple examples in which one assumes that agents live for only two periods. Many of the key features of the multi (say, 55)-period life cycle model used to study dynamic fiscal policy can be illustrated within the two-period framework. Although the two-period model is a useful analytical framework, however, it obviously provides little insight into economic outcomes within a period that corresponds roughly to 30 years. There have been some attempts to address quantitative issues in the public finance area, using overlapping generations economies in which agents live for many periods. We will discuss these developments in chapter 4.

We now briefly describe how the standard two-period framework may be extended to the n period framework. (n > 2). At any given time the household sector comprises n overlapping generations. Each year a member of one generation dies and another takes it place. An individual born at t works in the first m periods and then retires at the end of m-th period. For simplicity labor supply during working time is fixed. His lifetime utility function is given as

$$u_t = u(c_t^1, c_{t+1}^2, \ldots, c_{t+n-1}^n), \tag{47}$$

where c^s_{t+s-1} is per capita consumption of generation t in period t+s-1.

His budget constraint is given as

$$c_{t+j-1}^j = w_{t+j-1}^j - s_{t+j-1}^j + (1+r_{t+j-1})s_{t+j-2}^{j-1} \quad \text{(j=1,2,...n)}, \tag{48}$$

where $w_{t+j-1}^j = 0$ *for* $j = m+1, \ldots n$ and $s_{t+j-1}^j = 0$ *for* $j = 0, n$. From these equations the lifetime budget constraint is given as

$$c_t^1 + \sum_{j=2}^n \Pi_{s=2}^j (1+r_{t+s-1})^{-1} c_{t+s-1}^s = w_t^1 + \sum_{j=2}^m \Pi_{s=2}^j (1+r_{t+s-1})^{-1} w_{t+s-1}^s. \tag{49}$$

Maximization of the utility function (47) subject to the lifetime budget constraint (49) yields the consumption and saving demand

$$c_{t+j-1}^j = c^j(w_t, \ldots, w_{t+m-1}, r_{t+1}, \ldots, r_{t+n-1}) \quad \text{(j=1,...n)}, \tag{50}$$

$$s_{t+j-1}^j = s^j(w_t, \ldots, w_{t+m-1}, r_{t+1}, \ldots, r_{t+n-1}) \quad \text{(j=1,...n-1)}. \tag{51}$$

Total labor supply , L_t, in period t is given as

$$L_t = \sum_{j=1}^{m} N_{t-j+1}, \tag{52}$$

where N_t is the number of generation t.

The aggregate production function is given as

$$Y_t = F(K_t, L_t), \tag{53}$$

and capital accumulation is formulated as

$$S_t = \sum_{j=1}^{n-1} s_t^j N_{t-j+1} = K_{t+1}, \tag{54}$$

where S_t is total saving in period t. From (54), we can formulate the dynamic system as in the two-period model. However, the dynamic system here would involve high-order nonlinear functions of K_{t+1} and the solutions to such problems would require numerical computation.

Since the multi-period overlapping generations framework is hard to investigate analytically, it is useful to conduct simulation analysis. Simulation analysis is usually conducted as follows. First, one must specify explicitly the key parameters, such as the elasticity of substitution in production of capital and labor and the intertemporal elasticity of substitution between consumption in different years. Second, given such parameterization, one can obtain an exact numerical solution for the equilibrium of the economy for any given fiscal policy. Finally, one can compare the results for different fiscal policies.

Auerbach and Kotlikoff (1987a, b) presented an intensitive simulationanalysis to illustrate concretely how fiscal policy operates in such a model. See also Laitner (1984, 1987). He developed a methodology for investigating the dynamic behavior of continuous-time decentralized growth models composed of overlapping generations of finite-lived families.

7. Conclusion

The overlapping generations model developed here can easily be extendedinto an open economy. Chapter 6 investigates a version of the overlapping generations model in the two country open economy framework. Recent models of endogenous economic growth can generate long-run growth without relying on exogenous changes in technology or population. A general feature of these models is the presence of constant or increasing returns in the factors that can be accumulated. Chapter 11 investigates an endogenous growth model of overlapping generations, where public capital and human capital play an important role.

A classical version of overlapping generations model, which we have not examined in this chapter, is Samuelson's 1958 model, which does not have productive capital. The existence, uniqueness, and stability of equilibria in the pure-exchange context have been addressed by Gale (1973), Balasko, Cass, and Shell (1980), Balasko and Shell (1980), Kehoe and Levine (1985), as well as others. Further, McCandless with Wallace (1991) and Azariadis (1993)provided a useful explanation of the Samuelson model. In chapter 7 we will briefly explain the characteristics of such a model and will explore the role ofmoney in the overlapping generations model. Recently, a continuous version of the overlapping generations model has also been popular. See Yaari(1965) and Blanchard (1985) and Weil (1989). We will not explore such a model in the following chapters.

Table 2.1: Two Period Overlapping Generations Model

generation \ period	t-1	t	t+1	t+2
t-1	c_{t-1}^1	c_t^2		
t		c_t^1	c_{t+1}^2	
t+1			c_{t+1}^1	c_{t+2}^2
t+2				c_{t+2}^1

[1]. As for the case of 100% capital depreciation, see McCandless with Wallace (1991).

[2]. Section 5 investigates bequest motives.

[3]. Unlike the standard infinite-horizon optimal growth model this will notbe sufficient to assume the existence of a non-trivial steady-state. See Galor and Ryder (1989).

[4]. See Azariadis (1993) p.202.

[5]. When $\beta = 1$, the government intends to maximize long-run utility in the steady state.

[6]. This additional equation is needed because the first-order conditions (19-1,2,3) describe a second-order difference equation in the capital stock. Given an initial condition the second -order equation has a different solution sequence for each terminal value of the capital stock. In other words, (19-4) is needed to put the system on the saddle path.

[7]. See Blanchard and Fischer (1989).

[8]. The local stability of the steady-state equilibria can also be analyzedmathematically. A phase diagram is widely used to explore dynamic properties of overlapping generations models. See Blanchard and Fischer (1989) and Azariadis (1993). Galor (1992) developed the similar geometrical technique that facilitates the global analysis of the dynamic system of a two-sector overlapping generations model. The economy will not move continuously along one of the trajectories, but rather it will jump from point to point on that trajectory. Thus we must check whether the system is indeed saddle point stable around E by computing the roots of the system linearized around E. This check is left to the reader.

[9]. The label "golden rule" was used by Phelps (1961) to characterizestationary planning optima in economic growth problems.

[10]. In an uncertain world, there is no obvious metric for economic growth;nor is there a single rate of return. Abel, Mankiw, Summers, and Zeckhauser (1989) showed that the appropriate indicator of dynamic efficiency is the rate of growth of the value of the capital stock, as measured in consumption goods. If the rate of return on any asset dominates this rate, the economy is dynamically efficient. They concluded that its application to the United States economy and the economies of other major OECD nations suggests that they are dynamically efficient.

[11]. See Farmer (1993).

[12]. This figure was first used by Ihori (1978) and Buiter (1981).

[13]. As for the intertemporal substitution effect, see Barro (1984).Azariadis (1993) discusses the possibility of indeterminacy and cycles under different conditions. He shows that cycles are possible in theelastic-labor-supply version of the Diamond model even if consumption goods are gross substitutes but not probable because they require consumption goods to be moderately substitutable and factor inputs to be highlycomplementary.

[14]. As shown in Azariadis (1993), the steady state is necessarily a saddle in the Cobb-Douglas case.

[15]. Barro (1974) and Becker (1974) first studied the altruism model. Seealso Lord and Rangazas (1991). On the bequest-as-consumption model see Yaari (1965), Becker (1981), Menchik and David (1982), Seidman (1983), and Gravelle (1991). Bernheim, Shleifer, and Summers (1985) proposed the bequest-as-exchange model. And considerable work has been done on the accidental bequest model.

Chapter 3

Tax Policy

1. Introduction

In Chapter 3, we investigate normative aspects of tax policy in an overlapping generations growth model. The set of commodity taxes that minimizes the deadweight loss is called Ramsey taxes. The Ramsey rule has a simple form. See Chapter 1. Under certain simplifying conditions, Ramsey taxes are proportional to the sum of the reciprocal of the elasticity of demand and supply. The tax rate should be set so that the increase in deadweight loss per extra dollar raised is the same for each commodity. The Ramsey rule is a useful criterion for static efficiency. In Chapter 2, on the other hand, we have shown that the golden rule is a useful criterion for dynamic efficiency. Thus, this chapter investigates the relationship between the Ramsey rule and the golden rule when lump sum taxes are not available in the overlapping generations growing economy.

In Section 2, after investigating the first best solution, we examine the optimal combination of consumption taxes, labor income taxes, and capital income taxes. It is first shown that the (modified) golden rule holds at the first best solution. This rule also holds at the second best solution where neither lump-sum taxation nor debt policy is available although we also need the (modified) Ramsey rule at the second best solution. It is then shown that when consumption taxes are not available, the optimality condition is given by the mixed Ramsey-Golden rule; the Ramsey tax condition with respect to the second-period consumption includes the divergence from the golden rule criterion. The mixed Ramsey-Golden rule can be separated into the (modified) golden rule and the (modified) Ramsey rule only if all the effective consumer prices are optimally chosen.

This chapter also presents a simple analytical framework for understanding intergenerational incidence caused by tax reform. To investigate this , Section 3 presents a simple analytical framework for understanding intergenerational incidence from tax reform. The principal findings of this study concern the implications of changing the timing of tax payments on different generations for intergenerational incidence along the transitional growth path of life cycle economies.

Finally, in this chapter we examine the welfare effect of a piecemeal change in capital income taxes. It may be difficult to implement the optimal tax structure in the real world. This would require us to estimate the precise levels of own and the cross elasticities among all the relevant goods in the economy. Furthermore, the structure of the optimal tax system is very sensitive to the precise values of the relevant elasticities. If the exact optimum is out of reach, we may still hope that we can improve welfare by making the present tax structure somewhat closer to the optimum. Such a movement is called tax reform. In Section 4, we examine the welfare effect of a piecemeal change in capital income taxes by allowing for the efficiency loss involving the distortion in the work-leisure choice. It will be shown that an increase in capital income taxation is, in some cases, desirable where the initial capital stock is below the golden rule level.

2. Optimal Tax Rule

2.1 Model

The theory of optimal taxation is one of the oldest topics of public finance. Originally, the studies centered on the theory of optimal consumption taxation. The first theoretical result appeared in Ramsey (1927), which was the starting point of a great deal of studies on this subject and is well known today as the Ramsey tax rule. This section examines the optimal combination of consumption taxes, labor income taxes, and capital income taxes in an overlapping generations growth model. In the context of economic growth we have to consider dynamic efficiency, namely, the golden rule as well. After investigating the first best solution, this section intends to clarify the relationship between the Ramsey rule and the golden rule when lump sum taxes are not available.

We apply the overlapping generations model of Chapter 2 in which every individual lives for two periods. We extend this basic model by incorporating several distortionary taxes and by allowing for endogenous labor supply because, otherwise, a labor income tax becomes a lump sum tax.

An individual living of generation t has the following utility function

$$u_t = u(c_t^1, c_{t+1}^2, x_t), \qquad (1)$$

where c^1 is his first-period consumption, c^2 is his second period consumption, and x = (H- l) - H = - l is his first-period net leisure. H is the initial endowment of labor supply.

His consumption, saving and labor supply programs are restricted by the following first- and second-period budget constraints:

$$(1+\tau_t)c_t^1 = (1-\gamma_t)w_t l_t - s_t - T_t^1, \qquad (2)$$

$$(1+\tau_{t+1})c_{t+1}^2 = [1+r_{t+1}(1-\theta_{t+1})]s_t - T_{t+1}^2, \qquad (3)$$

where τ is the consumption tax rate, γ is the tax rate on labor income, w is the real wage rate, s is his real savings, r is the real rate of interest, and θ is the tax rate on capital income. T^1_t is the lump-sum tax levied on the young in period t and T^2_t is the lump-sum tax levied on the old in period t.

From (2) and (3) his lifetime budget constraint reduces to

$$q_{1t}c_t^1 + q_{2t+1}c_{t+1}^2 + q_{3t}x_t + T_t = 0, \qquad (4)$$

where $q_t = (q_{1t}, q_{2t+1}, q_{3t})$ is the consumer price vector for generation t.

$$q_{1t} = 1+\tau_t, \qquad (5\text{-}1)$$

$$q_{2t+1} = \frac{1+\tau_{t+1}}{1+r_{t+1}(1-\theta_{t+1})}, \qquad (5\text{-}2)$$

$$q_{3t} = (1-\gamma_t)w_t. \qquad (5\text{-}3)$$

The present value of lifetime lump-sum tax payment on the individual of generation t (T_t) is given as

$$T_t = T_t^1 + \frac{T_{t+1}^2}{1+r_{t+1}(1-\theta_{t+1})} \qquad (5\text{-}4)$$

Equilibrium in the capital market is the same as in Chapter 2 Section 4 and simply

$$s_t = (1+n)k_{t+1}l_{t+1}, \qquad (6)$$

where n is the rate of population growth and k is the capital-labor ratio.

The feasibility condition is in period t

$$c_t^1 + \frac{c_t^2}{1+n} + g + (1+n)k_{t+1}l_{t+1} = w_t l_t + r_t k_t l_t + k_t l_t, \tag{7}$$

where g is the government's expenditure per individual of the younger generation.

The government budget constraint in period t is given as

$$\tau_t c_t^1 + \frac{\tau_t c_t^2}{1+n} + \theta_t r_t k_t l_t + \gamma_t w_t l_t + T_t^1 + \frac{T_t^2}{1+n} = g. \tag{8}$$

(8) may be rewritten as

$$t_{1t} c_t^1 + t_{2t} c_t^2 + t_{3t} x_t + T_t^1 + \frac{T_t^2}{1+n} = g, \tag{8'}$$

where t_i is a tax wedge and is given as

$$t_{1t} = \tau_t = q_{1t} - 1, \tag{9-1}$$

$$t_{2t} = \frac{\tau_t}{1+n} + \frac{\theta_t r_t k_t l_t}{c_t^2} = \frac{q_{2t}(1+r_t) - 1}{1+n} + \frac{\theta_t r_t T_t^2}{(1+n)[1+r_t(1-\theta_t)]}, \tag{9-2}$$

$$t_{3t} = -\gamma_t w_t = q_{3t} - w_t. \tag{9-3}$$

The tax wedge t_i is the difference between the consumer price q_i and the producer price. Note that labor income taxation means $t_3 < 0$ since $x < 0$. Capital income taxation ($q_2 > 1/(1+r)$) means that the consumer price of c^2 is greater than the producer price ($t_2 > 0$). $1/q_2 - 1$ is the after-tax net rate of return on savings. Thus, for $T^2 = 0$ t_2 is given as

$$(1+n)t_2 = \frac{[(1+r) - \frac{1}{q_2}]s}{c^2} = q_2(1+r) - 1.$$

On the left-hand side, we multiply (1+n) because t_2 is an effective tax rate on the second-period consumption ,and hence, relevant for the older generation.

Observe that the government budget constraint (8) is consistent with production feasibility condition (7). Namely, one of the three equations (4)(7) and (8) is not an independent equation, which can be derived by the other two equations.

2.2 Dual Approach

It would be useful to formulate the optimal tax problem by using the dual approach[1]. Let us write $E(q_t, u_t)$ for the minimum expenditure necessary for attaining the utility level u_t when prices are $q_t = (q_{1t}, q_{2t+1}, q_{3t})$. Then, we have

$$E(q_t, u_t) + T_t = 0, \tag{10}$$

which implicitly defines the utility level of generation t as a function of consumer prices; q_t and lump sum taxes T_t. (10) corresponds to the lifetime budget constraint (4).

On the other hand, as in chapter 2 the factor price frontier is written as

$$w_t = w(r_t), \tag{11}$$

where

$w'(r_t) = -k_t < 0$ and $w'' > 0$.

From (3)(5-4)(6) and (11) we can express the second-period budget constraint in terms of compensated demands as

$$q_{2t+1} E_2(q_t, u_t) + T_t - T_t^1 = (1+n) w'(r_{t+1}) E_3(q_{t+1}, u_{t+1}). \tag{12}$$

E_i denotes the partial derivatives for the expenditure function with respect to price q_i (i=1,2,3). Note that $E_3 = x = -l < 0$. We call (12) the compensated capital accumulation equation.

The production feasibility condition (7) is also rewritten in terms of compensated demands as

$$E_1(q_t,u_t)+\frac{E_2(q_{t-1},u_{t-1})}{1+n}+(1+n)w'(r_{t+1})E_3(q_{t+1},u_{t+1})+ \\ [w(r_t)-(1+r_t)w'(r_t)]E_3(q_t,u_t)+g=0 \tag{13}$$

2.3 First Best Solution

First of all, let us investigate the first best solution where two types of lump sum taxes T^1 and T^2 are available. The government does not have to impose any distortionary taxes; $\tau=\theta=\gamma=0$. As in Chapter 2, the government's objective at time 0 is to choose taxes to maximize an intertemporal social welfare function W expressed as the sum of generational utilities discounted by the factor of social time preference, β.

The associated Lagrange function is given as

$$W=\sum_{t=0}^{\infty}\beta^t\{u_t-\lambda_{1t}[E(q_t,u_t)+T_t]-\lambda_{2t}[E_1(q_t,u_t)+\frac{E_2(q_{t-1},u_{t-1})}{1+n}+g \\ +(1+n)w'(r_{t+1})E_3(q_{t+1},u_{t+1})+(w(r_t)-(1+r_t)w'(r_t))E_3(q_t,u_t)]- \\ \lambda_{3t}[q_{2t+1}E_2(q_t,u_t)+T_t-T_t^1-(1+n)w'(r_{t+1})E_3(q_{t+1},u_{t+1})]\} \tag{14}$$

where λ_{1t}, λ_{2t}, and λ_{3t} are Lagrange multipliers for the private budget constraint (10), the resource constraint (13), and the capital accumulation equation (12).

Differentiating the Lagrangian (14) with respect to T_t, T^1_t, and r_{t+1} respectively, we have

$$\frac{\partial W}{\partial T_t}=-\beta^t(\lambda_{1t}+\lambda_{3t})=0, \tag{15-1}$$

$$\frac{\partial W}{\partial T_t^1}=\beta^t\lambda_{3t}=0, \tag{15-2}$$

$$\frac{\partial W}{\partial r_{t+1}}=\beta^t\{-\lambda_{2t}(1+n)w''(r_{t+1})E_3(q_{t+1},u_{t+1})+\lambda_{2t+1}\beta(1+r_{t+1})w''(r_{t+1})E_3(q_{t+1},u_{t+1}) \\ +\lambda_{2t}\{E_{12}(q_t,u_t)+[w(r_t)-(1+r_t)w'(r_t)]E_{32}(q_t,u_t)\}\frac{\partial q_{2t+1}}{\partial r_{t+1}}+ \\ \lambda_{2t+1}\frac{E_{22}(q_t,u_t)\beta}{1+n}\frac{\partial q_{2t+1}}{\partial r_{t+1}}+\lambda_{2t-1}\frac{E_{32}(q_t,u_t)(1+n)w'(r_t)}{\beta}\frac{\partial q_{2t+1}}{\partial r_{t+1}} \\ +\lambda_{2t}\{E_{13}(q_t,u_t)+[w(r_t)-(1+r_t)w'(r_t)]E_{33}(q_t,u_t)\}\frac{\partial q_{3t}}{\partial r_{t+1}}+ \\ \lambda_{2t+1}\frac{E_{23}(q_t,u_t)\beta}{1+n}\frac{\partial q_{3t}}{\partial r_{t+1}}+\lambda_{2t-1}\frac{E_{33}(q_t,u_t)(1+n)w'(r_t)}{\beta}\frac{\partial q_{3t}}{\partial r_{t+1}}\}=0 \tag{15-3}$$

Considering the homogeneity condition ($\sum_{j=1}^{3}q_jE_{ij}=0$) and $\tau=\theta=\gamma=0$, in the steady state (15-3) reduces to

$$1+n=\beta(1+r). \tag{16}$$

This is the modified golden rule discussed in chapter 2. The optimal levels of T^1 and T^2 are solved to satisfy the government budget constraint (8) and the modified golden rule (16). We have:

Proposition 1: When two types of lump sum taxes T^1 and T^2 are available, the optimality condition is given by the modified golden rule (16) at the first best solution.

2.4. Second Best Solution

2.4.1 The Case Where All The Consumer Prices Are Optimally Chosen

We are ready to investigate normative aspects of distortionary tax policy. From now on in this section we do not impose lump sum taxes; $T_t^1 = T_t^2 = 0$. First of all, let us investigate the case where all the consumer prices q_1, q_2, and q_3 are optimally chosen. In other words, we assume that the government can choose consumption taxes, wage income taxes, and capital income taxes optimally.

The maximization problem may be solved in two stages. In the first stage, one can choose $\{r_{t+1}\}$ and $\{q_t = (q_{1t}, q_{2t+1}, q_{3t})\}$ (t = 0, 1, ...) so as to maximize W. In the second stage one can choose (τ, γ, θ) to satisfy (5-1)-(5-3). Thus, our main concern here is with the first stage problem. We propose:

Proposition 2: The optimization problem is solved in terms of the consumer price vector. The actual tax rates affect the problem only through the consumer price vector.

In other words, the problem is to maximize

$$W = \sum_{t=0}^{\infty} \beta^t \{u_t - \lambda_{1t} E(q_t, u_t) - \lambda_{2t}[E_1(q_t, u_t) + \frac{E_2(q_{t-1}, u_{t-1})}{1+n} + g$$
$$+ (1+n)w'(r_{t+1})E_3(q_{t+1}, u_{t+1}) + (w(r_t) - (1+r_t)w'(r_t))E_3(q_t, u_t)] - \qquad (17)$$
$$\lambda_{3t}[q_{2t+1}E_2(q_t, u_t) - (1+n)w'(r_{t+1})E_3(q_{t+1}, u_{t+1})]\}$$

(10) and (13) are homogeneous of degree zero with respect to the q vector, but (12) is not. If we consider the problem: Max W subject to (10) and (13), q_t is uniquely determined up to a proportionality. Then (12) will give the level of q_{2t+1} which is consistent with the solution of our main problem. Thus, we obtain

$$\lambda_{3t} = 0. \qquad (18)$$

Differentiating the Lagrangian (17) with respect to q_{1t}, q_{2t+1}, and q_{3t}, respectively, we have

$$\frac{\partial W}{\partial q_j} = \beta^t \{-\lambda_{1t} E_j(q_t, u_t) - \lambda_{2t} E_{1j}(q_t, u_t) - \lambda_{2t+1} \frac{E_{2j}(q_t, u_t)\beta}{1+n} -$$
$$\lambda_{2t-1} \frac{E_{3j}(q_t, u_t)(1+n)w'(r_t)}{\beta} - \lambda_{2t}[w(r_t) - (1+r_t)w'(r_t)]E_{3j}(q_t, u_t)\} = 0.$$
$$(j = 1,2,3) \qquad (19)$$

Differentiating with respect to r_{t+1}, we obtain

$$\frac{\partial W}{\partial r_{t+1}} = -\beta^t \{\lambda_{2t}(1+n)w''(r_{t+1})E_3(q_{t+1}, u_{t+1}) - \qquad (20)$$
$$\lambda_{2t+1}\beta(1+r_{t+1})w''(r_{t+1})E_3(q_{t+1}, u_{t+1})\} = 0$$

In a steady state (20) means

$$1+n = \beta(1+r), \qquad (16)$$

which is the modified golden rule in section 2.3. Considering (20) and the homogeneity condition ($\sum_{j=1}^{3} q_j E_{ij} = 0$), (19) in the steady state reduces to

$$\lambda_1 E_j - \lambda_2\{(q_{1j}-1)E_{1j} + (q_{2j} - \frac{\beta}{1+n})E_{2j} + [-\frac{(1+n)w'}{\beta} + q_{3j} - w +$$

$$(1+r_t)w']E_{3j}\} = 0$$

Or

$$\frac{t_1 E_{1i} + \beta t_2 E_{2i} + t_3 E_{3i}}{E_i} = \frac{\lambda_1}{\lambda_2} \quad (i = 1, 2, 3), \tag{21}$$

which is the (modified) Ramsey rule. Hence, we have (Ihori (1981))

Proposition 3: When all the consumer prices are available, the optimality condition is given by the modified golden rule (16) and the modified Ramsey rule (21).

Remark 1: Note that if $\beta = 1$, (21) will be reduced to the standard Ramsey rule[2]. In other words, if the government is concerned with the steady state utility only, we have the standard Ramsey rule as well as the golden rule. The standard Ramsey rule describes the static efficiency point. This rule implies that less elastic goods should be taxed more. See Atkinson and Stiglitz (1980). There is an important difference between our (modified) Ramsey rule and the standard (static) Ramsey rule even if we are only concerned with long-run welfare ($\beta = 1$); our rule is derived under the assumption that all the effective taxes are available in the sense that the government can choose all the consumer prices (q_1, q_2, q_3). It is because one cannot normalize q in a dynamic system (unless lump sum taxes are available).

Remark 2: Atkinson and Sandmo (1980) derived the Ramsey rule in the case where debt policy is employed to achieve a desired intertemporal allocation. This rule (and hence the golden rule) is, however, also relevant to the case of the second best solution where neither lump-sum taxation nor debt policy is available. This is because changes in consumption taxes and labor income taxes have lump-sum timing effects. As explained in the next section, an increase in consumption taxes with a reduction in labor income taxes is equivalent to an increase in lump sum taxes in the second period of life with a reduction in lump sum taxes in the first period of life.

Remark 3: Assume for simplicity of interpretation that the cross-substitution effects are zero ($E_{ij} = 0$ *for* $i \neq j$). Then from (21) we have in the elasticity form

$$e_1\sigma_{11} = \beta e_2\sigma_{22} = e_3\sigma_{33}, \tag{22}$$

where e_i is the effective tax rate (t_i/q_i) and σ_{ij} is the compensated elasticity ($q_j E_{ij}/E_i$). If labor supply is completely inelastic (along the compensated supply curve), the optimal tax on the second-period consumption is zero, while the tax on labor income is equivalent to a lump sum tax and could be set arbitrarily high. If, on the other hand, the demand for future consumption is inelastic, the argument is reversed, and future income is the ideal tax base from an efficient view. In general, the optimal rate of effective tax e_i depends on the relative magnitudes of the elasticities, and there is no particular reason to believe that the optimal rate should be the same for the three sources of tax base. This interpretation carries over, with appropriate modifications, to the case of non-zero cross-elasticities.

Remark 4: Considering the homogeneity condition in elasticity terms $\sum_{j=1}^{3}\sigma_{ij} = 0$ (i = 1, 2, 3), (21) may be reduced to

$$-e_1(\sigma_{12}+\sigma_{13}+\sigma_{21})+\beta e_2(\sigma_{21}+\sigma_{23}+\sigma_{12})=e_3(\sigma_{23}-\sigma_{13}). \tag{23}$$

If $\sigma_{13}=\sigma_{23}$, (23) is reduced to

$$(\sigma_{12}+\sigma_{13}+\sigma_{21})(\beta e_2-e_1)=0, \tag{24}$$

which implies $\beta e_2 = e_1$. Considering (9-1)(9-2) and (16), we obtain $q_1=(1+r)q_2$. Substituting (5-1) and (5-2) into the above equation, we finally have $\theta = 0$; the optimal tax on interest income is zero. $\sigma_{13}=\sigma_{23}$ is called the implicit separability condition[3].

Remark 5: In this economy the government intends to realize two objectives; to realize the intertemporal efficiency and to finance the public good, g. When two types of lump sum taxes on the young T^1 and the old T^2 are available, as shown in section 2.3, the government can attain these two objective at the same time. We call this the first best solution. On the other hand, if the government cannot control the total amount of lump sum taxes, T, but can control the combination of T^1 and T^2, it can realize the intertemporal efficiency at the modified golden rule, but it cannot finance the public good without imposing static efficiency costs. We call this the second best solution. Such a case is essentially the same as distortionary taxes with ideal debt policy. The modified Ramsey rule will be relevant as in the case where all the consumer prices are optimally chosen.

2.4.2 The Case Where Consumption Taxes Are Not Available

Suppose that consumption taxes are not available; $t_1=\tau=0$. How will the results in the previous section 2.4.1 be altered? The government may choose $\{q_{2t+1}\}$ and $\{q_{3t}\}$ so as to maximize W. In such a case λ_3 is not necessarily equal to zero and we have to consider the capital accumulation equation as well.

Differentiate the Lagrangian (17) with respect to q_{2t+1} and q_{3t}, respectively to obtain

$$\frac{\partial W}{\partial q_{2t+1}}=\beta^t\{-\lambda_{1t}E_2(q_t,u_t)-\lambda_{2t}E_{12}(q_t,u_t)-\lambda_{2t+1}\frac{E_{22}(q_t,u_t)\beta}{1+n}-$$
$$\lambda_{2t-1}\frac{(1+n)w'(r_t)E_{32}(q_t,u_t)}{\beta}-\lambda_{2t}[w(r_t)-(1+r_t)w'(r_t)]E_{32}(q_t,u_t)- \tag{25-1}$$
$$\lambda_{3t}[q_{2t+1}E_{22}(q_t,u_t)+E_2(q_t,u_t)]+\lambda_{3t-1}\frac{(1+n)w'(r_t)E_{32}(q_t,u_t)}{\beta}\}=0$$

$$\frac{\partial W}{\partial q_{3t}}=\beta^t\{-\lambda_{1t}E_3(q_t,u_t)-\lambda_{2t}E_{13}(q_t,u_t)-\lambda_{2t+1}\frac{E_{23}(q_t,u_t)\beta}{1+n}-$$
$$\lambda_{2t-1}\frac{(1+n)w'(r_t)E_{33}(q_t,u_t)}{\beta}-\lambda_{2t}[w(r_t)-(1+r_t)w'(r_t)]E_{33}(q_t,u_t)- \tag{25-2}$$
$$\lambda_{3t}q_{2t+1}E_{23}(q_t,u_t)+\lambda_{3t-1}\frac{(1+n)w'(r_t)E_{33}(q_t,u_t)}{\beta}\}=0$$

Differentiating (17) with respect to r_{t+1}, we obtain

$$\frac{\partial W}{\partial r_{t+1}}=\beta^t\{-\lambda_{2t}(1+n)w''(r_{t+1})E_3(q_{t+1},u_{t+1})+$$
$$\lambda_{2t+1}\beta(1+r_{t+1})w''(r_{t+1})E_3(q_{t+1},u_{t+1})+\lambda_{3t}(1+n)w''(r_{t+1})E_3(q_{t+1},u_{t+1})\}=0 \tag{26}$$

We now have the last term related to the capital accumulation equation in (25-1,2) and (26), respectively.

In a steady state (26) reduces to

$$\frac{\lambda_3}{\lambda_2} = \frac{1+n-\beta(1+r)}{1+n}. \tag{27}$$

Considering (27), (25-1) and (25-2) are rewritten in elasticity terms as

$$\beta e_2\sigma_{22} + e_3\sigma_{23} = \frac{\lambda_1}{\lambda_2} + \frac{1+n-\beta(1+r)}{1+n}, \tag{28-1}$$

$$\beta e_2\sigma_{32} + e_3\sigma_{33} = \frac{\lambda_1}{\lambda_2}. \tag{28-2}$$

Here the optimal tax rule (28) cannot be separated into the modified golden rule (16) and the modified Ramsey rule (21). Using (28) to solve for e_2 and e_3, we have the mixed Ramsey-Golden rule.

Proposition 4: The optimal tax rule where consumption taxes are not available is given by the mixed Ramsey-Golden rule;

$$e_2 = \frac{1}{H}\{\lambda(\sigma_{33}-\sigma_{23}) + \sigma_{33}\frac{1+n-\beta(1+r)}{1+n}\}, \tag{29-1}$$

$$e_3 = \frac{\beta}{H}\{\lambda(\sigma_{22}-\sigma_{32}) - \sigma_{32}\frac{1+n-\beta(1+r)}{1+n}\}, \tag{29-2}$$

where $\lambda = \frac{\lambda_1}{\lambda_2}$ and $H = \beta(\sigma_{22}\sigma_{33} - \sigma_{32}\sigma_{23}) < 0$.

Remark 1: (29-1) and (29-2) were first derived explicitly by Batina (1990). See his (7a) and (7b). The first term in each equation of (29) captures the effect of static efficiency, while the second term captures the effect of dynamic efficiency. The optimal solution when the modified golden rule does not hold might be called the third best solution.

Remark 2: From (14) an increase in T^1 at given T^2 means $\frac{\partial W}{\partial T^1} + \frac{\partial W}{\partial T} = \lambda_3 - (\lambda_1 + \lambda_3) = -\lambda_1$. Therefore, λ_1 corresponds to the marginal benefit of lump sum transfer for each individual of the younger generation financed by distortionary taxes. λ_1 (?) is normally negative in the static model but may be positive in the present dynamic model. It is because an increase in the disposable income in the first period of his life will stimulate saving and capital accumulation, which may improve the dynamic efficiency of the economy. λ_2 corresponds to the marginal benefit of a decrease in government revenues, which is positive. $\sigma_{22} < 0$, $\sigma_{33} > 0$. $\sigma_{23} >$ (<) 0 and $\sigma_{32} <$ (>) 0 if c^2 and x are substitutes (complements). If γ and θ are positive, $e_2 > 0$ and $e_3 < 0$. Hence, $\sigma_{33} - \sigma_{23} > 0$ will imply $e_2 > 0$. $\sigma_{22} - \sigma_{32} < 0$ will imply $e_3 < 0$ in the first term of (29) if λ_1 is negative. This is exactly what one would expect from the static optimal efficiency point.

Remark 3: If (1+n)-β(1+r) < 0, from the second term of (29), an increase in θ (e_2) will more likely be desirable. The intuition behind this result is the following. As will be explained in the next section, taxation of interest income imposes a tax liability later in the life cycle than taxation of labor income. As a result, taxpayers will tend to increase their savings early in the life cycle. They do this in order to meet the additional tax liability later in the life cycle when interest income is taxed instead of labor income. An increase in capital accumulation in the aggregate moves the economy closer to the optimal level of capital when the initial level of capital is below the optimal level and tends to improve steady state welfare on average as a result. (29-1,2) show that the before-tax rate of return r (not the after-tax rate of return (1-θ)r) is relevant to the dynamic efficiency criterion. When 1+n<β(1+r), capital accumulation is desirable even if θ is high and 1+n>$\beta[1+(1-\theta)r]$.

Remark 4: Suppose that consumption taxes and labor income taxes are available but capital income taxes are not available. How will the result in this section be altered? In this case we still have (27) but (28-1,2) may be rewritten as

$$e_1\sigma_{11}+e_3\sigma_{13}=e_1\sigma_{31}+e_3\sigma_{33}=\lambda .$$

We have the Ramsey rule but we do not have the modified golden rule (16).

Remark 5: Suppose that a lump sum tax on the old T^2 is also available in addition to labor income taxes and capital income taxes. Then, we have $\lambda_1+\lambda_3=0$. Considering (27) and (28-1,2), (29-1,2) may be rewritten as

$$e_2=-\frac{\lambda\sigma_{23}}{(\sigma_{22}\sigma_{33}-\sigma_{32}\sigma_{23})\beta} \quad \text{(29-1)'}$$

$$e_3=\frac{\lambda\sigma_{22}}{\sigma_{22}\sigma_{33}-\sigma_{32}\sigma_{23}} \quad \text{(29-2)'}$$

H<0. If λ <0, (29-2)' is always negative. Then, it is desirable to impose labor income taxes. If $\sigma_{23}=0$, (29-1)' is zero; it is not optimal to impose capital income taxes in such a case. If c^2 and x are complements ($\sigma_{23}<0$), it is desirable to impose capital income taxes. Since (1+n)=β(1+r) does not necessarily hold, the modified golden rule (16) does not hold in this case.

2.5 Heterogeneous Individuals And Distributional Objectives

We have been concerned with the generality of the (modified) golden rule and the (modified) Ramsey rule in a growing economy. It has been shown that even at the second best solution the golden rule and the Ramsey rule hold if all the effective non-lump sum taxes are available. We have then shown that when consumption taxes are not available the mixed Ramsey-Golden rule holds. Here, the optimal formulae include the divergence from the golden rule at the third best solution.

There have been a few studies of the optimal tax mix for an economy with heterogeneous individuals and distributional objectives. As pointed out before, if debt policy is chosen optimally, the intuition of the static results provides the correct guidance for tax policy in a dynamic economy. The standard separability result suggests that labor income taxes may be superior on efficiency counts than capital income taxes at least in some circumstances. Atkinson and Stiglitz (1976) and Stiglitz (1985) showed that if, with an optimal nonlinear income tax, the utility function is weakly separable between labor and all goods together, there is no need to employ differential indirect taxation to achieve an optimum. Furthermore, Deaton (1981) has shown that where there are many consumers and only a linear income tax and proportional commodity taxes are allowed, weak separability between goods and leisure, together with linear Engel curves for goods, removes the need for differential commodity taxation. Applying directly to the taxation of savings, the optimal capital income tax rate may be reduced to zero. Atkinson and Stiglitz (1980) suggested that the reason for the asymmetry between labor income and capital income is not because labor income is taxed in a nonlinear fashion; but because of the difference between people is based on their wages, not the rates of return on savings.

If the government could perfectly tell who has what ability, it could impose lump-sum redistributive taxes. Obviously, the government cannot tell, so the more able individuals have no incentive to reveal their greater ability. The government, in its choice of tax structure, must recognize these limitations on its information. These constraints are called self-selection constraints. The self-selection constraint has an important role to determine optimal progression. Differences in the progressivity of the tax rates are driven by differences in the source of income inequality between agents and the degree of inequality.

The assumption of fully nonlinear taxation may be unrealistic. In most developed countries income tax schedules are progressive and take, undoubtedly for simplicity, the form of continuous piecewise linear functions. Following the seminal analysis of general income tax structure by Mirrlees (1971), most work focused on the optimal undifferentiated linear income tax. Ordover and Phelps (1979) discussed the optimal mix of linear taxes of wealth and wages that maximize a maximin social welfare function. Recently, Park (1991) analyzed steady-state solutions of optimal tax mixes in an overlapping generations model of heterogeneous individuals with a utilitarian social welfare function. He showed that an uneven distribution of the innate abilities leads to high rates of consumption and wage-income taxes, and a high level of a lump-sum transfer.

There are a few papers which analyze the normative aspects of differentiated linear income taxation. Following Akerlof (1978) and Baumol and Fischer (1979), Bennett (1982) examined optimal linear labor income taxation when the government has the ability to differentiate marginal tax rates across individuals. Alesina and Weil (1992) demonstrated that any fiscal system with a continuous linear tax schedule can be Pareto improved by the introduction of a second-tax schedule, and by letting the taxpayers select their preferred tax function on the menu of linear schedule presented to them. In a two-type-two-period optimal linear income taxation model, Dillen and Lundholm (1992) investigated the case where the second-period tax system can be differentiated on the observations from the first period.

Using a two-type-two-period framework, Ihori (1992) showed that if differentiated lump sum taxes are available, the optimal marginal tax rates on the efficient household are zero. The government can impose redistributive lump sum taxes on him. However, it is necessary to use marginal taxes on labor and capital income of the less efficient household if the self-selection constraint is binding. When differentiated lump sum taxes are not available, it is desirable to use differentiated labor and capital income taxation. In such a case if the source of inequality is in labor income, optimal labor income taxation will normally be more progressive. On the other hand, when the source of inequality is in capital income, optimal capital income taxation may or may not be more progressive. The greater the degree of inequality, the more progressive the optimal capital income tax structure will be. The intuition is as follows. When the self-selection constraint is binding, the government can use the information about the source of inequality to discriminate among individuals. Thus, the optimal tax structure of the income which is the source of inequality may be more progressive than the optimal tax structure of the other income.

3. Tax Reform And Timing Of Tax Payments

3.1 Tax Postponement Effect

The previous section characterized tax structures that maximize the sum of generational utilities discounted by the social time preference. For purposes of analytical tractability, section 2 examined the steady state properties of the optimal rule. As the incentive effects are complicated and sensitive to parametric structure, theory alone cannot provide a clear-cut guidance to efficient dynamic tax structures. See the modified Ramsey rule (21) and the modified Ramsey-Golden rule (29). With the general model the rates of taxes would be highly sensitive to the compensated elasticities and covariances. Unfortunately, we have little empirical data on some of these parameters.

At this stage we have two alternatives. One is to address the quantitative issues of the incentive effects using numerical simulation models in which agents live for many periods. Summers (1981b) and Auerbach, Kotlikoff and Skinner (1983) investigated the effects of switching from a proportional income tax with the average rates similar to those in the United States to either a proportional tax on consumption or a proportional tax on labor income[4]. We will discuss this important approach in the next chapter.

The other is to eliminate the incentive effects. It should be stressed that the impact on intergenerational incidence of converting an income tax to either a consumption or wage tax does not depend solely on the difference in such incentive effects on a representative person. Consumption taxes and labor income taxes are equivalent from the viewpoint of the household budget constraint. Both taxes affect the relative price of c^2/c^1 in the same way.

In a two-class, disposable income growth model that eliminates the incentive effects of distortionary taxes, Seidman and Maurer (1982) showed that tax reform may alter capital intensity by shifting disposable income from low to high savers. The present section employs an alternative approach that eliminates the incentive effects. Namely, within the framework of lump-sum taxation, this section intends to analyze theoretically the effect of timing of tax payments on the welfare of earlier generations during the transition process.

The rationale for this approach is not that we believe that such incentive effects of distortionary taxes discussed in section 2 are unimportant. We will explore quantitative issues of incentive effects in chapter 4. Rather, the aim of this section is to demonstrate that even if there were no incentive effects, the three taxes (γ, τ, θ) would generate different intergenerational incidence because consumers differ in their timing of payments of taxes. This is called the tax timing effect. This point is worth demonstrating because much of the literature comparing the three taxes may lead the reader to believe that the impact on intergenerational incidence depends solely on the differing incentives on the representative person. The difference between consumption and labor income taxation is not the incentive effect. The tax reform concerning consumption and labor income taxation may well be evaluated within the framework of lump-sum tax reform. It is useful to analyze the implications of lump sum tax reform for intergenerational incidence more fully.

Essentially, if the rate of interest is greater than the rate of population growth, the effect of consumption tax is to reduce the lifetime present value of taxation by postponing tax payments to later in life. This is called the tax postponement effect[5]. Based on Ihori (1987a), this section theoretically investigates under what circumstances the tax postponement effect would be relevant and how the timing of tax payments would affect intergenerational incidence. This section, therefore, is intended as a complement to the incentive effect analysis in the previous section that has been performed within the distortionary taxation framework.

3.2 Analytical Framework

For simplicity, it is now assumed that labor supply is exogenous. We now incorporate lump sum taxes instead of distortionary taxes. Therefore, a person born in period t has the following saving function:

$$s_t = s(w_t, r_{t+1}, T_t^1, T_t). \tag{30}$$

Assuming consumption to be normal, $0 < s_w < 1$, $0 > s_{T^1} = \partial s / \partial T^1 > -1$, and $0 < s_T = \partial s / \partial T < 1$. However, the sign of s_r depends on the relative magnitude of income and substitution effects. For simplicity s_t is assumed to be independent of r_{t+1}.

Hence, the economy may be summarized by the following equation, where T^1_t and T_t are policy variables.

$$s[w(r_t), T_t^1, T_t] = -(1+n)w'(r_{t+1}). \tag{31}$$

In order to analyze the welfare aspect of tax reform on each generation, it will be useful to explore the dynamic properties of the economy. As discussed in Chapter 2 (Proposition 1), under the stability condition r will monotonously converge to the long-run equilibrium level, r_L. This implies

$$0 < \frac{-s_w w'}{(1+n)w''} < 1, \tag{32}$$

at the steady state equilibrium.

The government budget constraint for period t is simply

$$T_t^1 + \frac{T_t^2}{1+n} = g. \tag{33}$$

From (5-4) and (33), we have

$$T_t = \frac{(r_{t+1} - n)T_t^1}{1+r_{t+1}} + \frac{(1+n)g}{1+r_{t+1}} + \frac{T_{t+1}^2 - T_t^2}{1+r_{t+1}}. \tag{34}$$

Obviously, $T^1_t = T^1_{t+1} = T^1$ and $T^2_t = T^2_{t+1} = T^2$ when the tax structure is time invariant. T^2_t, T^2_{t+1}, and the third term appear only when the tax structure is time variant.

3.3 Lump Sum Tax Reform

Suppose that the government will change the combination of lump-sum taxes (T^1, T^2) in period j+1. T^2 is raised, and T^1 is reduced. This yields:

$T^2_j < T^2_{j+1} = T^2_{j+2} = T^2$ and $T^1_j > T^1_{j+1} = T^1_{j+2} = T^1$.

First of all, let us investigate the partial equilibrium effect of tax reform on the present value of the lifetime tax payment T. If r > n, postponing tax payments to later in life ($T^1 \to T^2$) means a reduction of the lifetime present value of taxation. This is so called the tax postponement effect. For future generations j+1+i (i = 1,2,..) (34) means that the present value of tax payments T decreases if and only if r > n. If r > n, this gives an extra benefit to the future generation. If r < n, the tax postponement effect is unfavorable for the future generation.

For the existing younger generation j+1, the tax postponement effect works in the same way as in the case of the future generation. The tax postponement effect is relevant to the steady state as well as the transition process. For the existing older generation j, T^2_{j+1} is increased, while T^1_j is not reduced. Therefore, the lifetime present value of taxation T_j is raised. This corresponds to the third term of (34). This gives an extra burden to generation j. This may be called the direct tax reform effect or the time horizon effect. During the transition the earlier generation may suffer significant reductions in welfare by the tax reform. Note that this effect works irrespective of whether r is greater than n or not. In this sense, this effect should be distinguished from the tax postponement effect[6].

Let us investigate the impact of tax reform on savings. A reduction of T^1 directly increases an individual's savings. On the other hand, if r > n, the decrease in T^1 will reduce T and hence indirectly reduce his savings. However, considering (34), we have

$$\frac{\partial s}{\partial T^1} = s_{T^1} + \frac{(r-n)s_T}{1+r} < -1 + \frac{r-n}{1+r} = -\frac{1+n}{1+r} < 0. \tag{35}$$

Hence, the direct effect of T^1 is always greater than the indirect effect of T^1; an individual's saving is raised irrespective of the sign of r-n. We have:

Proposition 5: The lump sum tax reform ($T^1 \rightarrow T^2$) will increase saving of the existing younger generation j+1 and the future generation.

This may be called the (permanent) tax timing effect. This tax reform imposes a tax liability later in the life cycle. As a result, taxpayers will tend to increase their savings early in the life cycle in order to meet the additional tax liability later in the life cycle.

The impact of this tax reform on generation j's saving is dependent on whether a member of generation j anticipates this tax reform in period j or not. If an individual of generation j does not anticipate, his saving is unaffected by the tax reform. If he anticipates, an increase in T^2_{j+1} will raise T_j and hence increase s_j. This may be called the (temporary) tax timing effect.

The impact of tax reform on capital accumulation is illustrated in Figure 3.1. Curve S_0 represents the initial saving function before the tax reform and curve S_1 represents the new saving function after the tax reform. By the tax reform the saving function of future generations will shift upwards. Hence, the tax reform stimulates capital accumulation during the transition path. The new long-run equilibrium capital-labor ratio k_{L1} is greater than the initial long run equilibrium ratio k_{L0}. The tax reform ($T^1 \rightarrow T^2$) will stimulate capital accumulation in the long run.

Let us then illustrate the temporary tax timing effect. If a member of generation j anticipates the tax reform, generation j's saving is greater than the level indicated by the initial saving function. This extra saving is represented by AA'. This will lead to an extra initial capital endowment to generation j+1, which is denoted by BB'. Therefore, generation j's extra saving will stimulate capital accumulation during the earlier transition process. Note that this temporary tax timing effect will disappear in the long run.

We now explore the welfare aspect of tax reform during the growth process. Let us examine the effect of tax reform on utility of each generation j+i, u_{j+1} (i = 0,1,2..). If the tax reform is to increase T^2 and to reduce T^1 from period j+1 on, u_j will definitely be reduced. This is due to the direct tax reform effect. Moreover, if a member of generation j does not anticipate the tax reform, u_j will be reduced more. The effect on the future generation j+i (i = 1,2..) depends on the tax postponement effect and the temporary and permanent tax timing effects. If r > n, the tax postponement effect is favorable for the future generation.

3.4 Welfare Aspects Of Tax Timing Effect

Let us investigate the welfare aspect of the tax timing effect. In order to analyze the welfare of each generation explicitly, it is useful to employ the expenditure function approach as in the previous section. The system will be summarized by

$$E[\frac{1}{1+r_{t+1}}, u_t] = w(r_t) - T_t, \tag{36}$$

$$E_2[\frac{1}{1+r_{t+1}}, u_t] = -(1+n)(1+r_{t+1})w'(r_{t+1}) - T^2_{t+1}, \tag{37}$$

where E[.] denotes the expenditure function and E_2[.] denotes the compensated demand function for the second-period consumption. Differentiating (36) and (37) totally, we have

$$\begin{bmatrix} E_u, & E_2 \frac{-1}{(1+r_{t+1})^2} \\ E_{2u}, & E_{22}[\frac{-1}{(1+r_{t+1})^2}] + (1+n)(w'+(1+r_{t+1})w'') \end{bmatrix} \begin{bmatrix} du_t \\ dr_{t+1} \end{bmatrix} = \begin{bmatrix} w' \\ 0 \end{bmatrix} dr_t, \tag{38}$$

where $E_u = \partial E / \partial u_t$, $E_{2u} = \partial E_2 / \partial u_t$, and $E_{22} = \partial E_2 / \partial (\frac{1}{1+r_{t+1}})$. Hence,

$$\frac{du_t}{dr_t} = \frac{1}{\Delta} w' \{E_{22}[\frac{-1}{(1+r_{t+1})^2}] + (1+n)[w'+(1+r_{t+1})w'']\}, \tag{39}$$

where Δ is the determinant of the matrix of the left-hand side of (38). And, we have

$$\frac{dr_{t+1}}{dr_t} = -\frac{E_{2u}w'}{\Delta}. \tag{40}$$

Under the global stability condition, $0 < dr_{t+1}/dr_t < 1$ at the steady state solution. Hence, $\Delta > 0$. The sign of [.] in (39) will be positive if the elasticity of substitution between labor and capital is large, which is consistent with the stability condition (32); in such a case higher capital endowment given to his generation makes his lifetime utility higher. An increase in k_t raises w_t and lowers r_{t+1}. The former effect will increase u_t, while the latter effect will decrease u_t. If the elasticity of substitution is large, a decrease in r_t raises w_t much. The net effect is likely to increase u_t under the stability condition.

Therefore, on the transitional growth process where capital accumulation is monotonously increased, each generation's lifetime utility is monotonously increased[7]. Note that this favorable tax timing effect works, irrespective of the sign of r-n. Therefore, generation j's extra saving will be favorable for the near future generation who are close to generation j. For the distant future generation, generation j's extra saving is not important. In this sense, the temporary tax timing effect is relevant only to the near future generation. Utility of the distant generation is dependent on whether the long-run equilibrium is closer to the golden rule by the tax reform than before. Hence, we have:

Proposition 6: If r > n, the tax reform ($T^1 \rightarrow T^2$) is favorable for the distant future generation from the viewpoint of the tax postponement effect and the permanent tax timing effect.

Our analysis of tax reform and intergenerational incidence may be summarized in Table 3.1, which shows that if r > n, tax reform has different impacts on the existing older generation and the existing younger and future generations. Namely, the tax reform ($T^1 \rightarrow T^2$) hurts the existing older generation and benefits the future generation. On the other hand, the reverse tax reform ($T^1 \leftarrow T^2$) benefits the existing older generation and hurts the future generation. This is a trade-off relationship between the existing older generation's welfare and the future generation's welfare.

As explained in chapter 2, if $r > n$, the growth path is efficient in the sense that no generation is better off unless some generations are worse off. On the contrary, suppose the growth path is inefficient; $r < n$. Then, tax reform will affect the welfare of the existing older generation and the distant future generation in the same direction. However, even in this case if a member of the existing older generation anticipates the tax reform, the temporary capital accumulation effect will produce a trade-off relationship between the existing older generation and the near future generation.

3.5 Some Remarks

So far we have considered the case where taxes are lump sum. Our analysis suggests that the direct tax reform effect, the tax postponement effect, the temporal tax timing effect, and the permanent tax timing effect are important for the evaluation of tax reform. When taxes are distortionary, how would the results of this section be affected? As for the timing of tax payments, a wage tax corresponds to T^1 and a capital income tax corresponds to T^2. A consumption tax may be regarded as a combination of T^1 and T^2. Among the three taxes, an individual pays wage taxes the earliest in life. In this sense, converting a wage tax to consumption tax is associated with the tax reform $(T^1 \rightarrow T^2)$. It should be stressed that the difference between consumption and labor income taxation is not the exemption from taxation of capital income or the incentive effect but the different timing of tax payments. Therefore, the tax reform concerning consumption and labor income taxation may well be evaluated within the framework of lump-sum tax reform[8].

As far as the income effect is concerned, the implications of distortionary tax reform would be the same as in this section. For example, if the tax reform $(T^1 \rightarrow T^2)$ is desirable, then a capital income tax is better than a wage or consumption tax. However, a change in the tax rate on capital income would also have an incentive effect. If the interest elasticity of saving is large, a reduction of the capital tax is desirable during the efficient growth process[9]. We will investigate this aspect in the next section.

The lump sum tax reform model developed in this section should be regarded as a complement to the incentive analysis that has been used to compare income, wage, and consumption taxes. The standard incentive and simulation analyses are better suited to capture the differing incentive effect of each tax. The lump sum tax approach is better suited to explore qualitatively the consequences of the differing timing of tax payments, an aspect of reality that has not been systematically analyzed in most of the literature comparing consumption, wage and income taxes. This approach shows clearly that even with the incentive effects ignored, the differing timing of tax payments would cause consumption, wage, and income taxes to achieve different intergenerational incidence during the transition process when tax rates are set to achieve identical tax revenue per worker.

4. Capital Income Taxation

4.1 Tax Reform

The way in which capital income taxation affects economic welfare has recently attracted attention - both theoretical and empirical. Developments in the theory of optimal taxation have stimulated thinking in this area. Section 2 has examined the optimal taxation of capital and labor income in a simple two-period overlapping generations growth and derived formulae for the optimality tax rates.

It may be difficult, however, to implement the optimal tax structure in the real world, whatever it is. This would require us to estimate the precise levels of own and the cross elasticities among all the relevant goods in the economy. Furthermore, the structure of the optimal tax system is very sensitive to the precise values of the relevant elasticities. If the exact optimum is out of reach, we may still hope that we can improve welfare by making the present tax structure somewhat closer to the optimum. Such a movement is called tax reform. Restructuring the taxation of income from capital may well be the top priority for tax reform. See Auerbach and Hines (1988) among others.

Feldstein (1978) and Summers (1981b) indicated the potential for large gains from eliminating capital income taxes. See Chapter 4. Boskin and Shoven (1980) suggested that moving towards an expenditure tax offers potential gains in allocational efficiency. But, as such these works do not necessarily mean that it is always desirable to reduce capital income taxation. The second best theory tells us that some piecemeal changes that may appear to move in the correct direction turn out to be wrong. See Atkinson and Stiglitz (1980) and Hatta (1986).

In this section we examine the welfare effect of a piecemeal change in capital income taxes, using the model developed in Section 2, which allows for the efficiency loss involved in the distortion of the work-leisure choice. It will be shown that an increase in capital income taxation is desirable in some cases where the initial capital stock is below the golden rule level[10].

4.2 Model

We allow for the work-leisure choice and consumption taxes are assumed away. In other words, the model is the same as in section 2.4.2 where $\tau = T^1 = T^2 = 0$ and $\gamma, \theta > 0$. For simplicity we concentrate on the steady state property.

Remember that equilibrium in the capital market is given in terms of compensated demands as:

$$q_2 E_2(q,u) = (1+n)w'(r)E_3(q,u). \tag{12'}$$

Or, r may be solved as a function of q and u.

$$r = r(q, u). \tag{41}$$

The government budget constraint in terms of compensated demands, is rewritten as

$$t_2 E_2(q,u) + t_3 E_3(q,u) = g \ . \tag{42}$$

An increase in q_2 will be regarded as an increase in capital income taxation. It may correspond to an increase in an interest income tax or an increase in a corporate income tax. As long as such a movement raises q_2 (lowers the net rate of return on savings), we can regard it as an increase in capital income taxation[11].

4.3 Piecemeal Change

As formulated in (41), r is a function of q and u. Considering this relationship, we totally differentiate the individual's lifetime budget constraint (10) and the government budget constraint (42). Then, we have

$$\begin{bmatrix} E_u, & E_3 \\ t_2 E_{2u} + t_3 E_{3u} + r_u\left(\frac{q_2 E_2}{1+n} + kE_3\right), & t_2 E_{23} + t_3 E_{33} + E_3 + r_3\left(\frac{q_2 E_2}{1+n} + kE_3\right) \end{bmatrix} \begin{bmatrix} du \\ dq_3 \end{bmatrix}$$

$$= -\begin{bmatrix} E_2 \\ t_2 E_{22} + t_3 E_{32} + \frac{1+r}{1+n} E_2 + r_2\left(\frac{q_2 E_2}{1+n} + kE_3\right) \end{bmatrix} \begin{bmatrix} dq_2 \end{bmatrix} \tag{43}$$

where $E_{ij} = \partial E_i / \partial q_j$, $r_i = \partial r / \partial q_i$, and $r_u = r / \partial u$ (i, j = 2,3). Note that using (12)', the effect of (compensated) capital accumulation has been canceled out.

Therefore,

$$\frac{du}{dq_2} = -\frac{1}{\Delta^*}[E_2(t_2E_{23} + t_3E_{33} + E_3) - E_3(t_2E_{22} + t_3E_{32} + \frac{1+r}{1+n}E_2)]$$

$$= -\frac{1}{\Delta^*}E_2E_3[e_2(\sigma_{32} - \sigma_{22}) - (\sigma_{23} - \sigma_{33})e_3 + \frac{n-r}{1+n}]$$

$$= -\frac{1}{\Delta^*}E_2E_3[e_2(\sigma_{32} - \sigma_{22} + \sigma_{23} - \sigma_{33}) - (\sigma_{23} - \sigma_{33})(e_2 + e_3) + \frac{n-r}{1+n}] \quad (44)$$

where Δ^* is the determinant of the matrix on the left-hand side of (43). It is reasonable to assume that the denominator Δ^* is negative[12]. σ_{32} is compensated elasticity of leisure with respect to q_2. σ_{22}, σ_{33}, and σ_{23} will be defined similarly. We know $\sigma_{33} > 0$ and $\sigma_{22} < 0$. By assumption, $t_2 > 0$ and $t_3 < 0$. Thus, we have:

Proposition 7: In the case of $\sigma_{23} - \sigma_{33} \le 0$, $\frac{du}{dq_2} > 0$ if

$$B = e_2(\sigma_{32} - \sigma_{22}) + \frac{n-r}{1+n} < 0. \quad (45)$$

In the case of $\sigma_{23} - \sigma_{33} > 0$, $\frac{du}{dq_2} > 0$ if

$$C = e_2(\sigma_{32} - \sigma_{22} + \sigma_{23} - \sigma_{33}) + \frac{n-r}{1+n} < 0 \quad (46)$$

and $e_2 + e_3 > 0$.

Feldstein and Summers (1977) presented evidence that the U.S. capital stock is well below the golden rule level. The marginal product of capital (r) is estimated to about 0.1, in contrast to the growth rate (n) of about 0.03 per year. Suppose one period is 25 years. Then $r = (1.1)^{25} - 1 = 9.8$ and $n = (1.03)^{25} - 1 = 1.1$. The net impact of the tax treatment of capital income is to tax the real return on capital (r) at rate θ. Recall $e_2 = t_2/q_2 = \theta r/(1+n)$. A plausible value of θ is 0.3. Table 3.2 ($\sigma_{23} - \sigma_{33} \le 0$) presents the value of (1+n)B for various values of σ_{22}, σ_{32}, and θ. Table 3.3 ($\sigma_{23} - \sigma_{33} > 0$) presents the value of (1+n)C for various values of σ_{22}, σ_{23}, σ_{32}, σ_{33}, and θ. In a wide variety of cases B, C < 0. Remember that both (45) and (46) are sufficient conditions. Even if B, C > 0, it is still possible to have $du/dq_2 > 0$. Therefore, there will be wide class of cases in which more capital income taxation is desirable from the viewpoint of piecemeal policy[13].

The expresion, $e_2+e_3=t_2/q_2+t_3/q_3 > 0$, implies that capital income is initially taxed more heavily than labor income. Our results suggest that an increase in capital income taxation may be desirable even in such a case. Several remarks will be useful for the economic intuition of the seemingly paradoxical result.

Remark 1. Conditions (45) and (46) say that if the difference between r and n is greater than the difference between r and the net rate of return on savings ($1/q_2$-1) multiplied by the elasticity terms, then an increase in capital income taxation is desirable. When 1+r-($1/q_2$) is large, either (45) or (46) is unlikely to be satisfied. As far as the direct effect is concerned, du/dq_2 is a decreasing function of t_2 if $\sigma_{32} - \sigma_{22} > 0$. The larger t_2, the more likely it is that $du/dq_2 < 0$. Namely, when capital income taxation is extremely heavy, we cannot recommend more taxation of capital income. On the other hand, if $1+r-(1/q_2)$ is negative, it may well be that $du/dq_2 > 0$. In this sense, (44) (and hence (45) and (46)) is consistent with our intuition.

Remark 2. Under conditions (45) and (46) it is likely to have $(1/q_2)-(1+n) > 0$. Incidentally, all of our examples (Tables 3.2 and 3.3) imply $(1/q_2)-(1+n) > 0$. As is well known, at the first best optimum $1/q_2 = 1+n$. Hence, an increase in q_2 may appear to move in the correct direction as far as the difference between $1/q_2$ and $1+n$ is concerned.

Remark 3. Let us investigate the effect of an increase in q_2 on r. From (12)' and (41), we have

$$\frac{dr}{dq_2} = r_2 + r_3 \frac{dq_3}{dq_2} + r_u \frac{du}{dq_2},$$

where

$$r_2 = \frac{E_2 + q_2 E_{22} - (1+n)w' E_{32}}{(1+n)w'' E_3} = \frac{E_2(1+\sigma_{22} - \sigma_{32})}{(1+n)w'' E_3},$$

$$r_3 = \frac{q_2 E_{23} - (1+n)w' E_{33}}{(1+n)w'' E_3} = \frac{q_2 E_2 (\sigma_{23} - \sigma_{33})}{(1+n)w'' E_3 q_3},$$

$$r_u = \frac{q_2 E_{2u} - (1+n)w' E_{3u}}{(1+n)w'' E_3}.$$

We know $w'' > 0$, $E_{2u} > 0$, and $E_{3u} > 0$. Hence, $r_u < 0$. It seems plausible to assume $dq_3/dq_2 > 0$; an increase in capital income taxation will reduce labor income taxation. If $\sigma_{23} - \sigma_{33} > 0$, then $r_3 < 0$. If $1+\sigma_{22} - \sigma_{32} > 0$, then $r_2 < 0$. The smaller $|\sigma_{22}|$ and the larger $|\sigma_{32}|$, the more likely it is that $r_2 < 0$. Note that such a situation is consistent with (45) and (46). Therefore, in the case of $du/dq_2 > 0$, for various values of the relevant parameters it is likely that $dr/dq_2 < 0$; an increase in q_2 deepens capital intensity. This corresponds to the tax timing effect. As far as the difference between r and q_2 is concerned, our piecemeal changes may appear to move in the correct direction. As shown in section 2.4.2, the before-tax rate of return r (not the after-tax rate of return $(1-\theta)r$) is relevant to the dynamic efficiency criterion. When $1+n<\beta(1+r)$, capital accumulation is desirable even if θ is high and $1+n>\beta[1+(1-\theta)r]$.

Remark 4. Both (45) and (46) suggest that the smaller $|\sigma_{22}|$ and σ_{23} and the larger σ_{33} and $|\sigma_{32}|$, the more likely it is that $du/dq_2 > 0$. Small values of $|\sigma_{22}|$ and σ_{23} mean that the compensated changes in c^2 are small. Large values of σ_{33} and $|\sigma_{32}|$ mean that the compensated changes in l are large. The properties of c^2 and l may be consistent with the welfare improving changes in q_2 from the viewpoint of the Ramsey rule.

Remark 5. It has been assumed that a wage income tax is levied on the whole labor income (wl), while capital income taxes are levied on return on savings (rs) only, and hence the principle of savings (s) is not taxed. If, in addition to capital income, the principle is taxed, then we may not recommend more capital income taxation. For example, suppose

$$c^2 = (1-\theta)(1+r)s$$

where θ is now the overall tax rate on the whole capital income (1+r)s. Then, $1+r-(1/q_2) = (1+r)\,\theta$. In this case, for $\theta > 0.1$ plausible values of the relevant elasticities will not justify further taxation of the whole capital income[14].

Remark 6. We have illustrated the possibility that more capital income taxation is likely to be desirable when in the initial economy the capital accumulation is below the golden rule level and capital income taxes are more heavily imposed than labor income taxes. Academic debate over the size of a single elasticity (i.e. the interest elasticity of savings, or dr/dq_2) is only a part of the set of considerations that are relevant to the tax reform. By presenting a counter-example to the earlier conjectures, more insight is gained into the important issues in analyzing partial welfare improvements of the current tax treatment of capital income.

Remark 7. The analysis here is confined to the conventional two-period formulation. It would be useful to consider partial welfare improvements of capital income taxation in a multi-period formulation. The other important extension is to examine the transition from an existing growth path to a new growth path which would result from a change in the rate of tax. The welfare effect on the existing older generation can be analyzed as in Section 2. Namely, converting a wage tax to capital income tax is associated with the tax reform ($T^1 \rightarrow T^2$). Thus, the existing older generation would lose by an increase in capital income taxation although it is favorable for the future generation. Chapter 4 discusses these aspects.

5. Further Topics

There are several other topics which we have not discussed in tax policy here. The first topic is about the time consistency of the government's dynamic optimal tax policy. Since Kydland and Prescott (1977, 1980) and Fischer (1980), it has been well understood that as time passes and the economy's initial state changes, so does the policy that seems most desirable. If governments systematically change tax policy in order to take advantage of changes in the state of the economy, then it is rational for individuals to expect these changes, and it is appropriate to think of tax reforms as endogenous. In other words, time-inconsistency may be a reason for tax reform. Rogers (1991) showed how a switch from wage to consumption taxation might be motivated by time-inconsistency. Batina (1993) derived the time consistent income tax policy and showed that the main results derived in a static model will only hold for the last period of the government's planning horizon in the time consistent equilibrium.

Another important topic is about a wedge between borrowing and lending rates. In the real economy the interest rate on consumption loans exceeds the rate of return to savings, so that households face a kink in their intertemporal budget constraint. Altig and Davis (1992) showed that the tax treatment of household interest payments has powerful effects on capital intensity and aggregate savings in life-cycle and especially altruistic linkage models.

Table 3.1: Tax Reform and Intergenerational Incidence

Tax Reform		Existing older generation	Near future generation	Distant future generation
$T^1 \rightarrow T^2$	r>n	DTR(-)	TPP(+) TTT(+)	TPP(+) PTT(+)
	r<n	DTR(-)	TPP(-) TTT(+)	TPP(-) PTT(-)
$T^1 \leftarrow T^2$	r>n	DTR(+)	TPP(-) TTT(-)	TPP(-) PTT(-)
	r<n	DTR(+)	TPP(+) TTT(-)	TPP(+) PTT(+)

Notes:
(i) (+) means a favorable effect, and (-) means an unfavorable effect.
(ii) DTR denotes the direct tax reform effect. TPP denotes the tax postponement effect. TTT denotes the temporary tax timing effect. PTT denotes the permanent tax timing effect.

Table 3.2: Piecemeal Change in Capital Income Taxation

$(\sigma_{23} - \sigma_{33} \leq 0)$

σ_{22}	σ_{32}	σ_{23}	σ_{33}	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$
-1.2	0	0	0.4	-5.17	-3.99	-2.82
-1.2	-0.3	0.25	0.4	-6.05	-5.17	-4.29
-0.8	0	0	0.4	-6.34	-5.56	-4.78
-0.8	-0.3	0.25	0.4	-7.23	-6.74	-6.25

(r=9.8, n=1.1)

Table 3.3: Piecemeal Change in Capital Income Taxation

$(\sigma_{23} - \sigma_{33} > 0)$

σ_{22}	σ_{32}	σ_{23}	σ_{33}	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$
-1.2	-0.3	0.25	0.2	-5.90	-4.97	-4.04
-1.2	-0.3	0.4	0.2	-5.46	-4.38	-3.31
-0.8	-0.3	0.4	0.2	-6.64	-5.95	-5.27
-1.2	0	0.4	0.2	-4.58	-3.21	-1.84

(r=9.8, n=1.1)

[1] . As for the dual approach, see for example Varian (1992).

[2] . As for the standard Ramsey rule, see Atkinson and Stiglitz (1980).

[3] . See Deaton (1981). This condition may be associated the uniform tax rule presented in chapter 1. Recently Alvarez, Burbidge, Farrell, and Palmer (1992) extended Deaton's atemporal optimal commodity tax model to a life-cycle environment in which the individual may choose to work in many periods.

[4] . Auerbach and Kotlikoff (1987a, b) presented a very useful simulation analysis of tax reform. See chapter 4.

[5] . The tax postponement effect was pointed out by Summers (1981b) and Evans (1983). However, as their main interest was the interest elasticity of the saving rate, they did not clarify the full implication of the timing of tax payments for intergenerational incidence. See Ihori (1987a).

[6] . Note that the tax postponement effect is relevant to the existing younger generation and the future generation, not to the existing older generation.

[7] . See Calvo (1979).

[8] . If $r > n$, the tax reform of converting the labor income tax to the consumption tax is always desirable from the viewpoint of steady state comparison. This result is consistent with simulation results of Summers (1981b). See chapter 4. Our analysis shows that this result is in fact general, not a consequence of the choice of parameters.

[9] . If $r > n$, converting the capital income tax to either the consumption tax or the labor income tax will produce an unfavorable tax postponement effect to the existing younger and the future generations. Evans (1983) explored the possibility that converting the income tax to the labor income tax is not desirable. Our analysis suggests that in such a case the tax postponement effect and the permanent capital accumulation effect outweigh the effect of the interest elasticity of saving.

[10] . This section is based on Ihori (1984b).

[11] . Since q_2 includes r as well as capital income taxes, a piecemeal change in q_2 is not exactly the same as a piecemeal change in a particular capital income tax parameter, θ. Remember that the main concern of this subject is with the level of the net rate of return on savings, $1/q_2 - 1$. Therefore, it is meaningful to examine the welfare effect of a piecemeal change in q_2. Qualitative results would be the same if a piecemeal change in θ is considered.

[12] . As in Dixit (1975), $\Delta^* < 0$ holds if and only if an increase in the consumer's endowment at given prices increases welfare. $\Delta^* < 0$ is compatible with the stability condition of the system.

[13] . The empirical estimates are drawn from models which are specified differently from the present model. These values are used only for illustrative purposes.

[14] . We do not argue that the taxation of both interest and principle has good reality. In a multiperiod model the distinction between taxes on savings and taxes on capital income may be obscured because the present capital income will be used for savings. It suggests that it is less likely to have the paradoxical result in a multiperiod model than in the two-period model.

Chapter 4

Simulation Studies

1. Introduction

This chapter summarizes several simulation studies on tax reform using multi-period overlapping generations models. Based on Summers (1981b), we first formulate a bench mark model in which many generations coexist at any instant. The quantitative relationship between savings and the interest rate is complex and depends on all of the other parameters in the model. Section 2 compares the simulation results of Summers with Evans (1983).

We then summarize more recent simulation studies which incorporate endogenous labor supply, bequests, and human capital investment. First, Section 2 also examines Seidman's 1983 analysis which includes bequests and inheritances, based on utility maximization. Section 3 summarizes Auerbach and Kotlikoff's simulation model (1987a, b), in which an important extension was to incorporate endogenous labor supply.

The intergenerational redistribution of the tax burden during transition is one of the most important issues in the tax reform. Rather than tracing the transition explicitly, Gravelle (1991) introduced an alternative method of separating the efficiency and redistributional components of steady-state gains. Section 4 discusses this issue.

Finally, Section 5 addresses human capital investments by simulating the transitional responses of both human and conventional (nonhuman) savings following the replacement of wage income with consumption taxes.

2. Basic Model And Tax Reform

2.1 Analytical Framework

Although the two-period model is a useful analytical framework, it obviously provides little insight into economic outcomes within a period that corresponds roughly to 30 years. There have been some attempts to address quantitative issues in the public finance area, using overlapping generations economies in which agents live for many years. Based on Summers (1981b), in this section we formulate a bench mark model in which many generations coexist at any instant.

Let us describe how the standard two-period framework may be extended to the T period framework. (T>2). At any given time the household sector comprises T overlapping generations. Each year a member of one generation dies and another takes it place. An individual works in the first T' periods and then retires at the end of T'-th period. For simplicity labor supply during working time is fixed in this section. First of all, let us formulate the behavior of representative agent of generation 1. The utility function is assumed to take some simple functional form, namely,

$$U = \frac{1}{1-\frac{1}{\rho}} \sum_{t=1}^{T} \frac{U(c_t)}{(1+\delta)^{t-1}}, \qquad (1)$$

$$U(c_t) = c_t^{1-\frac{1}{\rho}} \text{ for } \rho \neq 1 \text{ or } U(c_t) = \log c_t \text{ for } \rho = 1,$$

where T is length of economic life, known with certainty, c_t is consumption in year t, δ

is the rate of pure time preference, and ρ is the intertemporal elasticity of substitution between consumption in different years. $1/\rho$ means the elasticity of marginal utility.

It is also assumed in this subsection that the individual receives no inheritance and leaves no bequests, that the real interest rate r is constant and certain, and that labor supply is exogenous. His labor supply in the first period is normalized to one. Individual labor supply in the efficiency term and hence wage income grow at some constant rate g (> 0) per period until retirement, after T' years in work force. This yields a budget constraint of the following form:

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = w\sum_{t=1}^{T'} \left[\frac{1+g}{1+r}\right]^{t-1}, \tag{2}$$

where w is the individual's wage income in the first period of economic life.

With these assumptions the associated Lagrangian for the individual maximization problem becomes

$$\Omega = \frac{1}{1-\frac{1}{\rho}} \sum_{t=1}^{T} \frac{U(c_t)}{(1+\delta)^{t-1}} - \lambda[\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} - w\sum_{t=1}^{T'} \left[\frac{1+g}{1+r}\right]^{t-1}]. \tag{3}$$

Solving the individual maximization problem (3) yields the first order condition for consumption in each year that must be satisfied by the optimum values of consumption:

$$(1+\delta)^{-(t-1)} c_t^{-\frac{1}{\rho}} = \lambda(1+r)^{-(t-1)}, \tag{4}$$

where λ is the shadow price of the lifetime budget constraint and represents the utility value of an additional income in present value.

The first order condition (4) yields the following result:

$$c_{t+1} = \left[\frac{1+r}{1+\delta}\right]^{\rho} c_t . \tag{5}$$

This equation defines the slope of the individual consumption profile. (2) and (5) can be solved jointly for values of c_{t+1} and c_1 after which everything else follows.

$$c_{t+1} = \left[\frac{1+r}{1+\delta}\right]^{\rho t} c_1 , \tag{6-1}$$

$$c_1 = w \frac{\sum_{t=1}^{T'} \left[\frac{1+g}{1+r}\right]^{t-1}}{\sum_{t=1}^{T} \left[\frac{1+r}{1+\delta}\right]^{\rho(t-1)} (1+r)^{-(t-1)}}, \tag{6-2}$$

2.2 Aggregate Economy

In order to make aggregate computations, we need further assumptions.

(1) There are T overlapping cohorts alive at any time point, differing only in age.

(2) All individuals in the work force at a given time are paid the same wage w on the efficiency labor unit, irrespective of age differences.

(3) Population grows at a steady state of n (> 0) percent per year.

With this additional structure, one can aggregate over cohorts and examine aggregate magnitudes.

From aggregate consumption, it is possible to calculate the savings rate out of labor income using the steady state assumption. Steady state growth implies

$$(n+g)K = wL + rK - C = S, \tag{7}$$

where K is aggregate capital, L is aggregate labor supply in the efficiency unit, C is aggregate consumption and S is aggregate savings. From (7) we have

S/wL = (n+g)(C/wL -1)/(r-n-g). (8)

We can calculate C/wL from (6). Substituting this into (8) yields derivation of a savings to labor income ratio (S/wL) in terms of r and model parameters.

Under the steady state assumption (7), we have

S/wL = (n+g)K/wL. (8)'

Hence S/wL can be readily converted into a capital-labor income ratio by dividing into the growth rate. Thus, the capital-labor income ratio may be represented in terms of r and model parameters. This is represented by the upward-sloping SS curve in Figure 4.1 as long as savings respond positively to the interest rate.

An alternative expression for that same ratio may be derived from an analysis of the production sector. We have the conventional neoclassical production function

Y = F(K, L),

where Y is aggregate output. Considering $\partial Y / \partial K = F_K = r$ and $\partial Y / \partial L = F_L = w$, the production function also implies a relationship between the capital-labor income ratio and the interest rate. This is also plotted as the downward-sloping PP curve in Figure 4.1. Equilibrium occurs where both curves cross at point E. Namely, those household and production expressions are equated at E. Using a computer implemented procedure, one can solve for r; given r, one readily solves for all relevant steady state characteristics.

2.3 Interest Elasticity Of Savings

The quantitative relationship between savings and the interest rate is complex and depends on all of the other parameters in the model. In Table 4.1, values of the interest elasticity of aggregate savings calculated by Summers (1981b) and Evans (1983) are reported for plausible parameter values. The interest elasticities of saving computed are partial equilibrium, as the feedback effect of the capital stock on the interest rate is not considered. Hence, r is exogenously given at this stage.

Both Summers (1981b) and Evans (1983) assumed the following values for parameters. Population grows at a 1.5 percent per year (n = 0.015), productivity increases by 2 percent per year (g = 0.02), and individuals live 50 year economic lives with retirement at age 40 (T' = 40, T = 50). Real rate of return is 4 percent per year (r = 0.04).

The intertemporal elasticity of substitution in consumption ρ is more problematic. Summers used values from 0.17 to 2.0. Evans used values of ρ ranging from 0.25 to 1.0. It can be seen from Table 4.1A and Table 4.1B that the higher the value of the risk aversion coefficient $1/\rho$, the lower are the interest elasticity and the saving rate.

Based on Table 1A, Summers emphasized that the simulation results support a high interest elasticity. In the plausible logarithmic utility case (ρ=1), the interest elasticity of saving rate varies from 3.36 at 4 percent r to 1.87 at 8 per cent r. Table 4.1A also generates the unimportance of the intertemporal elasticity of substitution between present and future consumption, ρ so long as ρ is between 1 and 0.5.

When the interest rises, the human wealth endowment declines as future income is more heavily discounted. Even in the Cobb-Douglas case where the consumption propensity out of wealth is independent of the interest rate, consumption will fall as the interest rate rises. With income constant, an increase in savings is implied. This may be called a human wealth effect. Since savings represent only a small fraction of income, even a small effect on consumption can translate into a large effect on savings. Summers stressed that the human wealth effect is much more important than the substitution effect of interest changes, so that the interest elasticity of saving is very high.

On the contrary, based on Table 4.1B, Evans emphasized that the lower the time preference rate δ, the lower is the interest elasticity of saving. He also showed that the lower is the productivity growth rate, the smaller is the reduction of human wealth effect. The smaller the rates of population and productivity growth, the greater the relative size of the older cohorts, the larger is the aggregate income effect, and the smaller is the interest elasticity of saving. His simulation results suggest that the interest elasticity of saving could be low in some cases.

2.4 Effect Of Tax Reforms

Let us examine the effect of replacing capital income taxes with wage and/or consumption taxes by comparing steady states. First of all, consider the effect of imposing a tax on capital income θ as well as a tax on labor income γ. This drives a wedge between the gross interest rate determined by the production function and the net rate received by savers. The new equilibrium occurs where the difference between the interest rate along the PP and SS curves is equal to the tax in Figure 4.1. As long as savings respond positively to the interest rate, imposition of a capital tax raises the gross return and reduces the net return on capital. Hence capital taxes will be partially but not completely shifted. Under the existence of taxes (8) may be rewritten as

$$S/wL = S[F_L(1-\gamma), F_K(1-\theta)]/wL=(n+g)\ K/F_LL \qquad (8)''$$

Once the steady state value of r is found from (8)", the levels of output and consumption as well as factor prices can be found from steady state condition and the production function.

The decrease in capital income taxation reduces the tax wedge, and hence r is reduced and K/wL is increased. The high interest elasticity of savings leads to a large increase in capital intensity. A shift to consumption taxes has more positive effect on savings than a shift to wage taxes. The reason for the more substantial results under a shift to consumption taxation is that such a system postpones tax payments, thus reducing their present value at the start of the individual's economic life, and so effectively increases his lifetime resources. This is the tax postponement effect discussed in Chapter 3. We will also expect the tax timing effect. This tax reform imposes a tax liability later in the life cycle. As the result, taxpayers will tend to increase their savings early in the life cycle in order to meet the additional tax liability later in the life cycle. See Section 3 of Chapter 3.

In calculating the quantitative effects of tax changes, it is necessary to make assumptions about parameter values of taxes. The appropriate value of the tax rates is not easy to determine. The capital income tax is presumed to represent the combined effect of corporate taxes, individual income taxes on dividends and interest income, and property taxes. Summers (1981b) assumed a value of 0.5 for the capital income tax rate (θ = 0.5) and a tax rate of 0.2 on labor income (γ = 0.2).

The model is now solved for the steady state under this tax regime. The steady state is then recalculated with exactly equal revenue yield and with the capital income tax replaced by a consumption tax or labor income tax. Thus, all the analysis here is carried out within a differential incidence framework, in which alternative sources of the same amount of government revenue are contrasted. Representative results are presented in Table 4.2.

Since the gross rate of return is assumed to be well above the golden rule level, the economy is dynamically efficient and hence steady-state consumption is increased as capital intensity rises. When capital income taxes are replaced by wage income taxes, consumption rises by 13.3 percent, while it rises by 17.4 percent with consumption taxes for the case of $\rho = 0.5$. See Table 4.2A. The results come from high interest elasticities of savings with respect to r. Summers emphasized the following two points. First, the multiperiod model used here suggests that a very high interest elasticity of savings is likely to obtain for almost any reasonable parameter values. Second, a large increase in gross wages results from the increased capital intensity arising from eliminating capital taxation.

On the other hand, as shown in Table 4.2B, Evans examined the case where the elasticity of substitution in consumption is low and there are substantial bequests. In such a case, a partial equilibrium interest elasticity becomes negative. Thus, the results of Table 4.2B are dramatically different from those of Table 4.2A. Namely, the effect of a consumption tax in reducing the lifetime present value of taxation very slightly outweighs the effect of the negative partial equilibrium interest elasticity. When capital income taxes are abolished by a shift to wage taxation in case 2 of Table 2B, steady-state consumption falls by 3 percent, and the steady state capital stock falls by 12 percent.

From this result, Evans concluded that the alleged theoretical presumption by Summers in favor of elimination of taxes on capital income as an appropriate way to stimulate capital formation, and their replacement by wage or consumption taxes, does not necessarily extend to cases in which bequests generate a significant fraction of capital formation. When private intergenerational transfers were introduced, negative interest elasticities of saving became entirely plausible. However, Evans did not explicitly formulate the utility function with bequest motives. In his ad hoc bequest formulation, the bequest final-period consumption ratio is exogenously given and not derived from utility maximization.

2.5 Inclusion of Bequests

Seidman (1983) extended Summers' analysis to include bequests and inheritances, based on utility maximization. In this sense, his bequest formulation is more plausible than Evans' ad hoc formulation. The utility function of generation 1 is now given as

$$U = \frac{1}{1-\frac{1}{\rho}} \sum_{t=1}^{T} \frac{U(c_t)}{(1+\delta)^{t-1}} + \frac{bK_T}{(1+\delta)^{T-1}}, \qquad (1)'$$

where parameter b is the taste for bequests and K_T is bequests of generation 1 who dies at period T. Here b (> 0) indicating a bequest motive does not imply that the individual cares about the welfare of his heirs. This is not the altruistic bequest motive. A person may derive utility from planning a bequest and accumulating the corresponding wealth, because of the security, prestige, or ego gratification that may accompany such wealth accumulation. This is called the bequest-as-consumption model.

A representative sample of results is presented in Table 4.3. Seidman obtained three main results.

First, a bequest motive undermines the neutrality of a consumption tax. With b > 0, a consumption tax that exempts a bequest is not neutral, since both a bequest and consumption now yield utility, but are not taxed symmetrically.

Second, a consumption tax that exempts bequests usually achieves a higher steady-state k than a consumption tax that taxes bequests, when tax rates are set to equate steady-state revenue per effective labor. However, the reverse is possible.

Third, when tax rates are set to equate steady-state revenue per unit of effective labor, for all parameters tried yielding a plausible steady-state interest rate and bequest-consumption ratio, the ranking of taxes, beginning with the tax achieving the highest capital intensity is (1) a consumption tax that exempts a bequest (C-Tax), (2) a consumption tax that taxes a bequest (CB-Tax), (3) a wage tax that exempts an inheritance (W-Tax), (4) a wage tax that taxes an inheritance (WH-Tax), (5) an income tax (Y-Tax). As pointed out in Section 2.4, the tax timing effect and the tax postponement effect are the reason that a consumption tax achieves a higher steady-state capital than a wage tax when tax rates are set to equate steady-state revenue per effective labor. Seidman showed that these effects continue to apply when $b > 0$.

3. Dynamic Simulation Model With Endogenous Labor Supply

3.1 Analytical Framework

Auerbach and Kotlikoff (1987a, b) present a dynamic general equilibrium numerical simulation model. Their important extension was to incorporate endogenous labor supply. Households live for 55 periods (T=55; age 20 to 75). The agents have rational expectations and maximize a CES lifetime utility function of consumption and leisure subject to the budget constraint that the present value of consumption not exceed the present value of after-tax labor income plus transfers. There are nonnegativity constraints on the labor supply of each individual in each cohort at each age. When the shadow wages associated with these constraints are positive, the individual is retired.

The CES utility function has constant intertemporal elasticities of substitution. It also has a time preference rate δ and a leisure share parameter α. The budget constraint depends not only on the interest rate r_t and the wage profile w_t but also on the average tax rates on capital income, labor income, and consumption, the payroll tax used to finance social security, and the level of social security benefits. In cases when the tax system is progressive, the average tax rates vary with the size of the tax base. In their formulation, this dependence is considered in the optimization decision, with both marginal and average tax rates affecting the household's choices.

Firms behave competitively and have a CES production function in labor and capital. In the base case Auerbach and Kotlikoff use a Cobb-Douglas production function with capital's income share equal to 0.25. The production function is normalized so that the wage in the base case is 1.

Labor is a variable factor of production, leading firms to set the marginal product of labor equal to gross wage. Changes in the capital stock are subject to quadratic adjustment costs. This convex cost of adjustment leads to the smoothing of investment, so that outside of the steady state, the marginal product of capital will not necessarily equal the interest rate, and the value of the firm of an additional unit of capital may diverge from its replacement cost[1].

The government raises taxes to pay for government spending on goods and a separable unfunded social security system. The government budget constraint is that the present value of taxes equals the present value of government spending plus the initial stock of debt.

After solution for the initial and final steady states of the economy are found, the economy's transition path is calculated in the following way. (1) To provide the economy with 150 years to reach the new steady state.(2) To solve for behavior during those 150 transition years fixing expectations for years after 150 at the final steady-state values that will, in fact, obtain. They used the rational expectations approach to simulate the transition path. Variations in initial guesses and the number of years permitted for transition to take place have never produced changes in the solutions obtained.

3.2 Results

Among many results Auerbach and Kotlikoff have derived several issues are of particular interest.

3.2.1. Intergenerational Equity

Steady state efficiency calculations in section 2 ignore what is probably the most important issue in the switch from income to consumption taxation: the intergenerational redistribution of the tax burden during transition. The current elderly people cannot enjoy a reduction in wage income taxes because they do not earn labor income. Since consumption tends to occur later in life than income, for example a switch to consumption taxation shifts each year's tax burden toward the elderly. The result is that the current elderly population pays more, while subsequent generations pay less by having their tax payments deferred to older age. This may be called the time horizon effect.

Auerbach and Kotlikoff explicitly examined the welfare effect of tax reform during the transition. Removing capital income taxation directly from the proportional income tax base, that is, switching to a wage income tax, while equivalent in a static model to adopting a proportional consumption tax, has quite different results in a dynamic model, since there is an opposite tax windfall. The very different intergenerational transfer effects of these two tax policies are shown in Figure 4.2.

Their dynamic analysis showed that the impacts of switching to consumption vs. wage income taxation are quite different during transition. So too are the efficiency impacts of the switches to consumption and wage income taxation. Along the consumption tax transition path, young and future cohorts achieve utility gains, partly at the expense of older generations. In contrast, the wage income tax transition involves increased levels of welfare for initial elderly generations and reductions in welfare for initial young generations as well as for all future generations. Under the consumption tax, the break-even (experiencing no change in utility) cohort is age 13 at the time the consumption tax is introduced. The break-even cohort under the wage tax is age 10 at the initiation of the wage income tax.

To analyze the efficiency gains of switching tax bases their model introduces a concept of a Lump Sum Redistribution Authority (LSRA) that transfers resources across generations in a lump sum fashion. In their efficiency transition calculations, the LSRA maintains the preexisting utility levels of generations initially alive at the time of the tax change, and any efficiency gains (loses) are allocated across subsequent generations so that they may enjoy a uniform increase (decrease) in utility. The LSRA approach measures efficiency gains or losses from dynamic tax reform as a wealth equivalent.

In switching from 15 percent income taxation to consumption taxation, the non-LSRA steady state welfare gain is 2.32 percent, which is eight times larger than the corresponding LSRA efficiency gain of 0.29 percent. Similarly, in switching to wage taxation from an initial 15 percent income tax, the non-LSRA steady state welfare loss is 0.90 percent, 3.6 times larger in absolute value than the corresponding LSRA efficiency loss of 0.25 percent. Auerbach and Kotlikoff found that 60 percent of the difference between the non-LSRA changes in long-run welfare under labor income taxation and consumption taxation is attributable to intergenerational transfers. Therefore, the most of the long-run gain to future generations in switching to consumption taxation is attributable to the policy's intergenerational redistribution rather than to its improvement in economic efficiency.

3.2.2. The Impact Of Investment Incentives

Auerbach and Kotlikoff stressed that the impact of investment incentives may be analyzed as the impact of consumption taxes. The introduction or enhancement of investment incentives not only encourages investment, but it also lowers the present value of taxes on new investment, while leaving unchanged the present value of taxes on old capital. Because old capital is at a tax disadvantage, its market value must fall. In the case of an investment tax credit, for example, the effect will be to drive the value of old capital down to the cost of new capital net of the investment tax credit, for which only new capital qualifies.

The major findings of their simulation studies are as follows.

(1) A drop in the value of capital, combined with a cut in the tax burden on new investment, is good for savings of young people but bad for old people, just like a consumption tax.

(2) Investment incentives can dramatically alter stock market values. Such reevaluations are dampened somewhat by assuming significant adjustment costs.

(3) Investment incentives, even those financed by short-run increases in the stock of debt, significantly increase capital formation in life cycle economies.

(4) Deficit-financed investment incentives can be self-financing for particular, but not unreasonable, parameterizations of overlapping generations growth models.

(5) The windfalls associated with a move to investment expensing may be quite large. For an adjustment cost parameter of b = 10 (on the low end of empirical estimates, but by no means small), a move from a 15 percent income tax to the same tax with complete expensing (i.e., a consumption tax) reduces the value of the existing capital stock by nearly two-thirds the size of the tax rate cut on new investment, or about 9.5 percent.

4. Separation Of Efficiency And Redistribution

4.1 Compensation Method

As discussed in Section 3.2.1, the intergenerational redistribution of the tax burden during transition is one of the most important issues in the tax reform. Rather than tracing the transition explicitly, Gravelle (1991) introduced an alternative method of separating the efficiency and redistributional components of steady-state gains. It uses a system of transfers which ensures that changes in welfare in the steady state do not arise from redistribution of income to or from transitional generations.

His compensation method, explained in this section, corrects for the distributional effects, allowing efficiency gains to be estimated without calculating the transition explicitly. The compensation scheme both isolates the excess burden from distributional effects and determines how to divide the Harberger triangles among individuals.

First of all, let us consider a three-period overlapping generations model with fixed labor supply, no bequests and no growth, similar to Gravelle (1991). There is a representative older retired worker, a middle-aged worker, and a young worker, denoted by superscripts 3, 2, and 1. The individual budget constraints in the steady state can be written as

$$C^3* = [1+r^*(1-\theta^*)]K^{3*}, \qquad (9\text{-}1)$$

$$C^2* = [1+r^*(1-\theta^*)]K^2* + (w^*L/2)(1-\gamma^*) - K^3*, \qquad (9\text{-}2)$$

$$C^1{}^* = (w^*L/2)(1-\gamma^*) - K^2{}^*, \qquad (9\text{-}3)$$

where C refers to consumption, r is the rate of return, w is the wage, L is total labor supply, θ is the tax rate on capital income, γ is the tax rate on labor income, K refers to capital, and values in the original steady state are denoted by asterisks.

With a switch to a consumption tax, these budget constraints are

$$C^3_t = (1+r_t)K^3_t(1-\tau_t) + H^3_t, \qquad (10\text{-}1)$$

$$C^2_t = [(1+r_t)K^2_t + (w_tL/2) - K^3_{t+1}](1-\tau_t) + H^2_t, \qquad (10\text{-}2)$$

$$C^1_t = [(w_tL/2) - K^2_{t+1}](1-\tau_t) + H^1_t. \qquad (10\text{-}3)$$

The consumption tax is imposed at rate τ. The H terms are lump sum compensation payments.

Set the compensation payments, H^3_t, H^2_t, and H^1_t such that

$$C^3_t = C^{3*} + (1+r_t)(K^3_t - K^{3*}) + s^3_t[Y_t - Y^* - r_t(K_t - K^*)], \qquad (11\text{-}1)$$

$$C^2_t = C^{2*} + (1+r_t)(K^2_t - K^{2*}) - (K^3_{t+1} - K^{3*}) + s^2_t[Y_t - Y^* - r_t(K_t - K^*)], \qquad (11\text{-}2)$$

$$C^1_t = C^{1*} - (K^2_{t+1} - K^{2*}) + (1 - s^3_t - s^2_t)[Y_t - Y^* - r_t(K_t - K^*)], \qquad (11\text{-}3)$$

where the s terms refer to shares and Y refers to total output.

This compensation scheme restores old consumption levels, which is equivalent to giving individuals their old wage rates and their old rates of return on their old levels of capital, correcting both for changes in factor prices. It also pays them the new rates of return on their changes in investment. The last term in brackets is the excess of the average return on the new discrete change in capital over the marginal return to the last unit of capital. The method Gravelle suggests is to distribute the term in brackets in proportion to change in capital.

$$s^3_t = (K^3_t - K^{3*})/(K_t - K^*) \qquad (12\text{-}1)$$

$$s^2_t = (K^2_t - K^{2*})/(K_t - K^*) \qquad (12\text{-}2)$$

This distribution gives nothing directly to the young in their initial period since $K^2_t + K^3_t = K_t$ and $K^{3*} + K^{2*} = K^*$.

The welfare gain is divided between the initial young and the initial middle aged in proportion to their own change in capital. This treatment is continued throughout the transition and in the steady state[2].

4.2 Applications Of The Compensation Scheme

Gravelle applied the above compensation scheme to the Summers-fixed-labor-supply model and the Auerbach-Kotlikoff-endogenous-labor-supply model. In these models, he replaced the income tax with either an equal-yield consumption tax or an equal-yield wage tax. He adopted the bequest-as-consumption model as in Seidman (1983). Bequests are a final argument in the utility function, but are modified so that the elasticities for bequests and own consumption can be different. Bequests are received at age 20 and are given at death. The stock of bequests accounts for 50 percent of the capital stock. Gravelle used the 0.15 rate for labor income taxes (γ = 0.15) but applied a 0.30 capital income tax rate (θ = 0.30). The after-tax rate of return is 0.05 ((1-0.3)r = 0.05).

Table 4.4 reports the result from the Summers-exogenous-labor model. If no compensation is introduced, there is a steady-state welfare gain of 5.03 percent with a consumption tax and a loss of 4.09 percent with a wage tax in the case of ρ = 0.25. The welfare gain with the compensation scheme is 0.61 percent (for both cases, since the labor tax is a lump sum tax when labor supply is fixed.) These results show that welfare effects in the uncompensated steady state are primarily the result of redistribution.

Bequests make little difference to the compensated welfare gains when they are insensitive to price or when the intertemporal substitution elasticity ρ is low in general. But they do alter the uncompensated estimates. This result is compatible with Seidman and supports the basic result of Summers.

When labor supply is endogenous, the welfare gains from replacing the income tax by a wage or consumption tax are ambiguous. As explained in Section 3, Auerbach and Kotlikoff correct for redistribution by lump sum payments. They do this in order to keep taxpayers at a fixed level of utility, using the LSRA approach, when the tax is instituted. Auerbach and Kotlikoff report a steady-state gain of 0.29 percent with the consumption tax and a steady state loss of 0.25 percent with a wage tax. Applying the compensation scheme in this section to the same model and parameters results in a gain of 0.17 percent for the consumption tax and a loss of 0.02 percent for the wage tax.

These differences in efficiency gains with compensation arise from the different distribution of efficiency gains. In the consumption tax case Auerbach and Kotlikoff's reported steady state gain is larger than Gravelle's estimate because the small gains of those initially alive are totally transferred to new individuals. In the wage-tax case, Gravelle's compensation method yields virtually no steady state change, but Auerbach and Kotlikoff's method produces a large loss, because initial generations have lost utility due to the time-horizon effect of intertemporal gains. This loss is transferred to new generations in their formulation. On the other hand, in the compensation scheme suggested by Gravelle, the transitional generations share in efficiency gains and losses.

Table 4.5 reports the effects in the Auerbach and Kotlikoff model, which is calibrated to the same tax rates and other values as in the fixed-labor case reported in Table 4.4, whose values Gravelle regards as more representative of U.S. income taxes and savings rates. In this endogenous labor model, switching tax bases produces either a welfare gain or loss with compensation. The results are quite sensitive to the choice of intertemporal elasticity, ρ. A steady state loss is quite plausible for the wage-tax substitution, and it cannot be ruled out for the consumption-tax substitution.

The compensation method only ensures that gains or losses do not reflect redistribution of original incomes; that is, individuals have the resources to obtain their original consumption set.

5. Human Capital Investment

5.1 Tax Reform And Human Capital

The above reported simulation studies of overlapping generations models emphasize the fact that private saving is normally higher under consumption taxes than wage income taxes. These analyses however have abstracted from any potential effects on human saving. Human capital investments in the United States, for example, comprise roughly half of aggregate investment. Rates of return on human capital are sensitive to net interest rates, wages, and other input prices. Tax reform may change the rate of return on human capital and hence aggregate investment. Based on Lord (1989), this section will address that issue by simulating the transitional responses of both human and conventional (i.e. nonhuman) savings following the replacement of wage income with consumption taxes.

It is now well recognized that the more human capital is accumulated, the steeper the wage income profile, the greater are outlays when young, and the lesser is conventional saving[3]. Consequently, if the switch from wage income taxation to consumption taxation does increase human saving, the increase in nonhuman savings may be less than when abstracting from human capital. Less clear is how endogenizing human capital decisions, following increased reliance on consumption taxes, might affect total savings, output and welfare.

This section first summarizes a framework of simulating the transition path from an income tax base to a consumption tax base using an overlapping generations growth model featuring both human and nonhuman savings. The reform retains the initial steady rate of capital income taxation in order to focus on the differences between wage and consumption taxation.

Let us briefly review the framework of Lord (1989). In any year t, the household sector consists of T overlapping generations. In a steady state, households differ only in age; cohorts differ in size. Population grow at 100 n % annual rate. For the representative individual, utility maximization is separable into two stages. First, the individual makes human capital investment decisions during the T' working years in order to maximize life cycle disposable earnings (E), which is the present value of wage earnings minus expenditures on human capital goods inputs. Then, subject to that endowment, a consumption profile is chosen to maximize lifetime utility.

The effective labor supply depends on individuals' human capital decisions. The aggregate effective labor supply is that portion of the human capital stock devoted to current labor supply. During adult years, the production function for gross additions to the stock of human capital follows a Cobb-Douglas specification with decreasing returns to scale and constant output elasticities. Human capital produced during adulthood is assumed to depreciate at a constant rate d. The technology constraining a child's human capital production is distinct from that of adults, although of the same basic Cobb-Douglas form. The human capital production function for children is

$$Q_t = v(s_t H_t)^{\mu_t} (D_t)^{\xi_t}, \tag{13}$$

$$\mu_t = \frac{\mu_0}{(1+\alpha)^{t-t_0}}, \tag{14}$$

$$\xi_t = \frac{\xi_0}{(1+\beta)^{t-t_0}}, \tag{15}$$

where Q_t is gross additions to the stock of human capital at age t. v is an efficiency scalar, and s is the proportion of time. Of the current stock of human capital (H), allocated to the production of human capital at age t, D are goods inputs. ξ and μ are productivity parameters. (13) allows the output elasticities of parental time and goods inputs to vary with the age of the child, as shown in (14) and (15).

Lifetime utility is given by a CES representation. Weighting the representative households' age-related profiles by cohort size scalars yields cohort profiles. Summing across cohorts, one obtains aggregate human capital input demands, effective labor supply, savings, and consumption. This modeling of human capital decisions with taxes creates wedges between gross and net of depreciation human capital savings, and between gross and net potential income.

Let us briefly summarize the simulation results of Lord. Immediately following the tax reform there is a large increase in conventional nonhuman savings, S_c. The relatively numerous young save more from a larger net wage income for retirement expenditures which now includes consumption taxes,. Although those retired consume less, their (before-tax) expenditures are unaffected. As a consequence, in the first post-reform year S_c from net potential income, Y^n_p increases more than 60%. With initial savings rates so low, a modest effect on consumption translates into a large effect on saving.

The additional conventional saving increases the capital intensity. This lowers the interest rate and raises the wage rate. The lower interest rate flattens the consumption profile, reducing the increase in S_c. Both factor price movements increase the magnitude and valuation of human saving, further dampening nonhuman saving,

S_c. Namely, the falling interest rate increases the discounted to present benefits of human investments. Also, the relative wage in terms of the effective price of goods inputs w/P_g rises, further stimulating human investment. Consequently, the continuing increases of capital intensity and w/P_g raise the benefits of human investment by a greater percentage than the costs.

A striking result is that every cohort in the labor force experiences an increase in gross life cycle human capital production. Figure 4.3 plots those life cycle gains for individuals of each year of birth, where H (L) means a high (low) elasticity case. These gains are trivial for individuals nearing retirement at the time of the tax reform, but are appreciable for those beginning their economic lives a few years before, and any time after, the tax reform. These increases result from higher w/P_g and lower r.

The general pattern of welfare changes is similar to that found by Auerbach and Kotlikoff (1987) and Seidman (1983, 1984) in their analyses of conversion to complete reliance on consumption taxes. As explained in the previous sections, that pattern is an appreciable welfare reduction for those cohorts old at the time of the tax change, but an increase for all cohort younger than some age t'. The well-known reason for old generations experiencing a reduction in welfare is that their unplanned payment of consumption taxes increases during retirement years, although they are unable to benefit from the elimination of wage income taxation.

The simulation results of Lord indicate that replacing wage with consumption taxes has important implications for the magnitude and form of capital accumulation. Policies that increase conventional savings without raising the return to capital may encourage human capital accumulation. These effects on human savings are another implication of the differences between consumption and wage taxes when the same amount of revenues are raised under each regime. Fortunately, the simulation results of Lord also suggest that analyses abstracting from adult human capital accumulation discussed in the previous sections bias only the quantitative, and not qualitative implications of this reform for savings, income and intergenerational incidence.

5.2 Human And Physical Bequests

More recently economists have begun to distinguish between human and physical bequests as well as the difference between human and nonhuman savings. Outlays on children as human bequests affect the shape of the family's labor supply and expenditure profiles, on which the level of aggregate savings crucially depends. If these outlays are interest-sensitive substitutes for physical bequests, they will also influence the interest elasticity of savings.

This subsection summarizes a model of Lord and Rangazas (1991). Their model makes explicit examination of these issues possible[4]. They extended Lord's (1989) multiperiod model of life cycle savings and adult human capital investment examined in the previous subsection by adding altruistically motivated human and physical bequests. The model is calibrated using microeconomic data on earnings, time and goods expenditures on children's human capital, and physical bequests. They then used the model to examine the contribution of bequests to wealth accumulation and the level of savings.

They showed that if a pure life cycle model generates low aggregate savings rates, then augmenting the model with altruistic bequests that mimic available microdata will not necessarily cause the savings rate to rise significantly. The utility function is the familiar CES utility function augmented with altruistic preferences toward the new generation. They adopted the altruistic bequest motive. They considered six baselines in total, with a high (H) and low (L) response case for each of three assumptions made about the share of lifetime wealth devoted to bequests. Their review of the available evidence yields three different estimates of the bequest share: 1, 2.5, and 4 percent. Table 4.6 presents the simulations for a number of variables under each case.

The most surprising feature of their results is that the aggregate savings rates show little variation across bequest-share assumptions. This finding contradicts the commonly held belief that overlapping generations models augmented with bequest motive would produce significantly higher savings rates. The impact of financial and human bequests has very little effect on physical capital accumulation. There appears to be a substitution between the different types of savings and the different types of expenditures, so as to leave total savings rates essentially unchanged.

Adding or subtracting human and financial bequests in a overlapping generations model causes indirect wealth effects on lifetime consumption, which serve to offset the direct impact on savings. The offsetting wealth effect is relatively large when the difference between the steady-state interest rate and the effective population growth rate is relatively large. For a given population growth rate, the size of this gap is driven by the underlying level of life cycle savings. The lower the level of life-cycle savings, the larger is the gap and the smaller is the effect on savings from altering the level of bequests.

6. Concluding Remarks

We have discussed several attempts to address quantitative issues in the tax reform area, using overlapping generations models in which agents live for many periods. The main results may be summarized as follows.

First of all, the conversion from wage income taxation to consumption taxation will normally stimulate savings and capital formation. This result would hold in a variety of cases even if we incorporate endogenous labor supply, bequests, or human capital investment.

Secondly, since consumption tends to occur later in life than when income is earned, the current elderly population pays more, while subsequent generations pay less by having their tax payments deferred to old age. That pattern is an appreciable welfare reduction for those cohorts old at the time of the tax change, but an increase for all cohort younger than some age t'. The quantitative analysis of intergenerational redistribution of the tax burden during transition is one of the most important issues of the tax reform.

Finally, although qualitative implications of tax reform for savings and welfare are robust, quantitative results in most cases depend crucially on model parameters.

Table 4.1: Interest Elasticities of Saving

Table 1A (Summers)

ρ	r=0.04	r=0.06	r=0.08
2	3.71	2.26	2.44
1	3.36	1.89	1.87
0.67	3.09	1.71	1.54
0.5	2.87	1.59	1.37
0.33	2.38	1.45	1.22
0.17	0.74	1.09	1.18

Note: n = 0.015, g = 0.02, T = 50, T' = 40, and δ = 0.03.

Table 4.1B (Evans)

	δ=0.00			δ=0.03		
ρ	saving elasticity	saving rate	wealth/ income	saving elasticity	saving rate	wealth/ income
1	1.11	0.34	9.83	3.55	0.06	1.64
0.5	0.74	0.11	3.25	3.24	0.04	1.09
0.33	0.56	0.08	2.21	2.97	0.03	0.82
0.25	0.40	0.06	1.67	1.90	0.01	0.40

Notes: T = 50, T' = 40, g = 0.02, n = 0.015, r = 0.04. Saving rates and wealth/income ratios are computed relative to total income (including interest).

Table 4.2 Steady State Welfare Cost Of Capital Income Taxation

Table 4.2A (Summers)

	ρ=2	ρ=1	ρ=0.5
	percentage change in steady state consumption		
payroll taxation	12.7	13.1	13.3
consumption taxation	14.0	15.9	17.4
	welfare gain (expressed as a percentage of lifetime income)		
payroll taxation	7.0	4.9	1.4
consumption taxation	11.2	11.7	11.6

Note: T = 50, T' = 40,n = 0.015, g = 0.02, δ= 0.03, γ =0.2, θ = 0.5.

Table 4.2B (Evans)

new regime	percentage change in consumption	percentage change in wealth	percentage change in saving	final interest rate	wage tax rate	consumption tax rate
Case 1						
consumption tax	+67.2	+158.5	+158.5	4.92	0.0	21.6
wage tax	+48.8	+101.0	+101.0	5.62	26.7	0.0
Case 2						
consumption tax	+0.2	+0.9	+0.9	7.95	0.0	31.6
wage tax	-3.0	-12.3	-12.3	8.82	39.3	0.0

Notes: T = 50, T' = 40, g = 0.02, n = 0.015, δ = 0.01. Case 1: ρ = 1.0, no bequests. Case 2: ρ = 0.2; individual in initial steady state receives inheritance equal to three times his initial net wage in the start of economic life.

Table 4.3: Inclusion of Bequests

ρ	δ		No-Tax	C-Tax	CB-Tax	W-Tax	WH-Tax	Y-Tax
1	0.10	r	11.89	11.89	11.93	12.06	12.10	13.17
		s	7.57	7.57	7.55	7.46	7.44	6.84
2	0.10	r	10.85	10.82	10.87	10.93	10.95	12.00
		s	8.30	8.32	8.28	8.24	8.22	7.50
0.5	0.10	r	13.90	13.94	13.96	14.21	14.30	15.40
		s	6.48	6.46	6.45	6.33	6.30	5.84
0.1	0.01	r	20.32	20.65	20.64	21.62	22.11	22.56
		s	4.43	4.36	4.36	4.16	4.07	3.99

Notes: All entries in the table are percentages. The production function is Cobb-Douglas, with the capital share, 30 percent; $n = 0.01$, $g = 0.02$, $T' = 45$, $T = 60$, $b = 1.6$.

Table 4.4: Effects Of Changing To A Wage Or Consumption Tax, Fixed-Labor Supply Model

K_T/K	ρ	ϕ	σ	consumption	wage	compensated
0.0	0.25	NA	1.00	5.03	-4.09	0.61
0.0	1.00	NA	1.00	5.66	0.21	1.19
0.5	0.25	0.00	1.00	4.28	-6.41	0.58
0.5	0.25	0.25	1.00	4.99	-4.68	0.66
0.5	1.00	0.00	1.00	5.34	-0.25	1.16
0.5	1.00	1.00	1.00	5.90	-1.46	1.94
0.0	0.25	NA	0.50	5.31	-4.30	0.44
0.0	1.00	NA	0.50	4.57	0.18	0.67
0.5	0.25	0.00	0.50	5.71	-9.29	0.42
0.5	0.25	0.25	0.50	5.23	-5.73	0.57
0.5	1.00	0.00	0.50	4.52	-0.08	0.66
0.5	1.00	1.00	0.50	4.25	0.85	0.79

Note: K_T/K is the fraction of the capital stock accounted for by bequests, ρ is the intertemporal substitution elasticity, and σ is the factor substitution elasticity. ϕ is a term that determines the substitution of bequests for direct consumption.

Table 4.5: Effects Of Changing To A Wage Or Consumption Tax, Endogenous Labor

Tax scheme	ρ	ϕ	σ	capital*	labor*	welfare*
Uncompensated						
consumption tax	0.25	0.8	1	34.1	-1.5	2.02
wage tax	0.25	0.8	1	11.1	-4.6	-1.10
Compensated						
consumption tax	0.25	0.80	1.00	20.5	-2.0	0.39
	0.25	0.80	0.50	12.9	-0.6	0.29
	0.01	0.80	1.00	1.6	-3.0	-0.19
	1.00	0.80	1.00	31.8	-1.2	0.73
	0.25	1.50	1.00	20.5	-1.6	0.39
	0.25	0.01	1.00	20.5	-2.3	0.34
wage tax	0.25	0.80	1.00	18.9	-4.1	0.15
	0.25	0.80	0.50	11.4	-2.4	0.10
	0.01	0.80	1.00	1.5	-5.6	-0.48
	1.00	0.80	1.00	29.5	-3.1	0.51
	0.25	1.50	1.00	17.3	-5.8	-0.07
	0.25	0.01	1.00	20.5	-2.3	0.35

Note: The symbols ρ, ϕ, and σ refer respectively to the intertemporal substitution elasticity, the intratemporal substitution elasticity, and the factor substitution elasticity in production. * means percentage change.

Table 4.6: Steady State Baselines

	bequest share					
	1%		2.5%		4%	
variable	H	L	H	L	H	L
saving rates[a]	4.0	3.5	4.2	3.8	4.3	4.0
interest rate[b]	5.0	5.7	4.8	5.3	4.6	4.9
marginal propensity to bequeath[c]	0.27	0.25	0.29	0.27	0.30	0.28
expenditure share[d]	5.2	3.9	5.4	4.3	5.6	4.6
Modigliani wealth share[e]	0.14	0.27	0.32	0.49	0.47	0.62
wealth share including interest[f]	0.26	0.54	0.59	0.96	0.83	1.15
flow share[g]	0.01	0.01	0.01	0.02	0.02	0.03
saving rate[h]	4.2	3.6	4.3	3.9	4.5	4.2
saving rate[i]	4.0	3.3	4.0	3.2	3.9	3.1

Note: H and L are the high- and low-response cases.

a: aggregate savings divided by aggregate output

b: the net of tax interest rate, with a tax rate of 20%

c: increase in the present value of financial bequests following a one-unit increase in parents' wealth

d: the present value cost of time and goods inputs allocated to children's human capital divided by human wealth

e: total inheritance wealth divided by total wealth

f: adds interest income from inheritance to Modigliani wealth share

g: the flow of bequests divided by total wealth

h: aggregate savings divided by aggregate output when the efficient level of young human capital is costlessly provided

i: aggregate savings divided by aggregate output when there are no human or financial bequests

[1] . The ratio of this market value to the replacement cost of capital is Tobin's q.

[2] . The method can be generalized to account for an endogenous labor supply, many sectors, and bequests.

[3] . Considerable attention has been devoted to examining qualitative implications of taxation for adult human capital investment. In a partial equilibrium setting, Heckman (1976) noted that if expenditures on human capital goods inputs may be written off immediately for tax purposes, human capital accumulation will be independent of taxes on wage income. If not all outlays on goods inputs are tax deductible, taxes on expenditures or payrolls would deter human capital accumulation. In a general equilibrium context, Kotlikoff and Summers (1979) examined the incidence of a compensated wage income tax in a two-period life cycle model in which human capital is produced without goods, using own time alone. They also found human capital to be independent of a tax on wage income.

[4] . Chapter 11 investigates theoretically the role of human and physical bequests to generate economic growth.

Chapter 5

Public Spending

1. Introduction

In this chapter, we investigate positive and normative effects of public spending. Most of the previous literature on government expenditure have investigated the effect of public spending financed by lump sum taxes (or wage income taxes with exogenous labor supply). The conventional view is that an increase in public spending, which will not contribute to stimulating production, has a negative impact on capital formation due to the resource withdrawal effect.

In the conventional IS-LM model, for example, as shown in Blinder and Solow (1973), public spending has a negative impact on capital formation. In the Diamond-type overlapping generations model (1965), as shown by Hamada (1986) and Kehoe (1987), public spending reduces capital formation in the long run. In the Blanchard-type uncertain lifetime model (1985), as shown by Blanchard and Fischer (1989), we would expect the same result. Marini and Ploeg (1988) have shown using the Blanchard-type model that an increase in government spending leads to a fall in capital. In the standard neoclassical infinite horizon model, as pointed out by Mankiw (1987b), Aschauer (1988) and Ihori (1990c), permanent increases in government purchases should not affect real interest rates while temporary increases in government purchases should increase real interest rates. Later in Section 2, we will briefly explain the conventional view using the Diamond model.

This chapter also provides several counterarguments to the conventional wisdom by showing that, under certain plausible circumstances, an increase in public spending financed by non-lump sum taxes may raise the capital intensity of production. When an increase in public spending is financed by non-lump sum taxes, we would expect the addition of a substitution effect as well as the resource withdrawal effect. Incorporating endogenous labor supply into the model may serve to reverse the usual effect of the labor income tax due to the disincentive effect on labor supply.

Mankiw (1987b) has shown that a simple neoclassical model, incorporating a durable consumer good, can generate a reverse dynamic response caused by changes in government spending. Section 3 shows that, without incorporating durable consumer goods, increases in government spending financed by non-lump sum taxes may cause reductions in real interest rates. This result is consistent with data for the United States. As Barro (1984) documents, wars are not associated with high real interest rates. Section 3 also considers the case where government spending is financed by consumption taxes.

This chapter finally examines the normative effect of public spending. Namely, Section 4 investigates the optimal combination of distortionary taxes and government spending. We argue that the method of financing the public good will generally affect the dynamic efficiency of the economy. If the government can perfectly adjust the economy's aggregate intertemporal allocation through the use of its debt policy, then the tax system will only reflect static efficiency considerations and the effect of the tax system on the social cost of the public good will have the usual sign. If the government does not have this much control over the economy, the dynamic efficiency effect on the social cost of the public good will work in the opposite direction of the static efficiency.

Section 5 then examines the optimal public investment policy when distortionary taxes are employed. It is shown that the public rate of return is equal to the social rate of time preference at the second-best or third-best solution.

2. Fiscal Spending Financed By Lump Sum Taxation

2.1 Analytical Framework

The analytical framework applied here is almost the same as the basic model in Chapter 2. We incorporate lump sum taxes. Individuals are identical within and across generations. People live for two periods, work in the first period of their lives, and retire in the second. Population is stationary[1] and labor supply is perfectly inelastic in the basic model. An individual living in periods t and t+1 faces the following budget constraints.

$$c_t^1 = w_t - s_t - T, \tag{1}$$

$$c_{t+1}^2 = (1+r_{t+1})s_t, \tag{2}$$

where c^1_t is her first-period consumption, c^2_{t+1} is her second-period consumption, s is her savings, w is the wage rate, r is the rate of interest, and T is the lump sum tax levied when young.

Then, an individual's lifetime budget constraint is given as

$$c_t^1 + \frac{1}{1+r_{t+1}} c_{t+1}^2 = w_t - T. \tag{3}$$

The agent's decision problem is to maximize $u(c^1_t, c^2_{t+1})$ subject to (3). Thus, her saving function is given as

$$s_t = s(w_t - T, r_{t+1}), \tag{4}$$

where $0 < s_w = \partial s / \partial (w - T) < 1$ and $s_r = \partial s / \partial r$ is non-negative[2].

Capital accumulation is given as

$$s_t = k_{t+1}, \tag{5}$$

where k_{t+1} is per capita capital at the beginning of period t+1.

Competitive profit maximization and a neoclassical technology require that firms hire labor and demand capital so that

$$w_t = w(r_t), \quad w'(r_t) = -k_t. \tag{6}$$

The government budget constraint is given as

$$T = g, \tag{7}$$

where g is government spending.

Considering (6), the system may be summarized by

$$s[w(r_t) - g, r_{t+1}] = -w'(r_{t+1}). \tag{8}$$

We will focus attention on the effect of an increase in public spending, g, on the real equilibrium. Thus, the endogenous variable is r, and the exogenous variable is g. This equation will be the basis of our analysis of this section.

2.2 Effect Of Public Spending

Let us investigate the effect of an increase in public spending g on r. Equation (8) can be written as

$$S(r_t, r_{t-1}; g) = 0. \tag{9}$$

Notice that $\partial r_{t+1} / \partial r_t = -S_{r0} / S_{r1}$, $S_{r0} = \partial S / \partial r_t = w' s_w < 0$, and $S_{r1} = \partial S / \partial r_{t+1}$

$= s_r + w'' > 0$. Since we assume that the initial equilibrium is locally stable,

$$S_{r0} + S_{r1} > 0. \tag{10}$$

To derive the comparative statics of the basic model, differentiate the steady state version of (9) to obtain

$$\frac{dr}{dg} = \frac{s_w}{S_{r0} + S_{r1}} > 0. \tag{11}$$

(11) shows :

Proposition 1: An increase in public spending financed by lump sum taxes on the younger generation reduces capital formation.

The intuition is as follows. An increase in public spending and hence taxation will decrease saving. This is the resource withdrawal effect. A decrease in the savings reduces capital accumulation and raises the interest rate during transition and in the long run.

3. Fiscal Spending Financed By Non-Lump Sum Taxation

3.1 Labor Income Taxes And Endogenous Labor Supply

Let us introduce a wage income tax, γ. Since a wage income tax becomes a lump sum tax in the case of exogenous labor supply, in this section we incorporate an elastic supply of labor. The model is almost the same as in chapter 3 section 2. We assume $\theta = \tau = 0$. Suppose l (= -x) is the amount of labor supply in the first period. The lifetime budget constraint (3) is rewritten as

$$c_t^1 + \frac{1}{1+r_{t+1}} c_{t+1}^2 = (1-\gamma_t) w_t l_t. \tag{12}$$

The labor supply function is given as

$$l_t = l[(1-\gamma_t) w_t, r_{t+1}], \tag{13}$$

where $\partial l / \partial w = l_w (1-\gamma)$, $\partial l / \partial \gamma = -l_w w$, and $l_w = \partial l / \partial [(1-\gamma) w]$. As in chapter 2 section 4, we assume that the substitution effect outweighs the income effect so that $l_w > 0$. In order to explore the disincentive effect of labor income taxation, this assumption seems useful. We will also assume that $\partial l / \partial r = l_r > 0$. This is due to the intertemporal substitution effect.

Denote by k the capital-labor ratio. Then, the capital accumulation equation (5) becomes

$$s_t = k_{t+1} l_{t+1}. \tag{5'}$$

In summary the system may be described by the following two equations.

$$\gamma_t w(r_t) l[(1-\gamma_t) w(r_t), r_{t+1}] = g, \tag{7'}$$

$$s[(1-\gamma_t) w(r_t), r_{t+1}] = -w'(r_{t+1}) l[(1-\gamma_{t+1}) w(r_{t+1}), r_{t+2}]. \tag{8'}$$

In the initial equilibrium, we can write (7)' as $a_t = G(r_{t+1}, r_t)$ where $a = 1-\gamma$. Substituting this into (8)', we obtain

$$S[G(r_{t+1}, r_t), r_t, r_{t+1}, G(r_{t+1}, r_{t+2}), r_{t+2}] = 0. \tag{14}$$

This is a second order difference equation. We can investigate the dynamic property of (14) as in Chapter 2. See equation (36) of Chapter 2.

To obtain the comparative statics of the model with endogenous labor supply, we differentiate the steady state version of (7)' and (8)' and evaluate at $\gamma = 0, l = 1$. Then we have

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} dr \\ da \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} dg, \quad (15)$$

where $A = w's_w + s_r + w'' - k(l_r + l_w w')$, $C = 0$, $D = -wl$, $B = s_w w - kl_w w$. It is tedious but straightforward to show that if $l_r > 0$ and $\Delta =$ AD-CB < 0, then the initial equilibrium will be a saddle, and hence the dynamic equilibrium is uniquely determined. We will assume this is true.

Solving (15) yields

$$\frac{dr}{dg} = -\frac{w[s_w - kl_w]}{\Delta}. \quad (16)$$

If $l_w > s_w / k$, then the numerator of (16) is negative and an increase in public spending leads to lowering the interest rate. Thus, we have:

Proposition 2: If labor supply increases to a large degree with the effective after-tax wage ($l_w > s_w / k$), government spending, financed by labor income taxes, stimulates capital accumulation.

This is not as unlikely as it may appear. $l_w - s_w / k = l(\varepsilon^l - \varepsilon^s) / w > 0$ if the elasticity of labor supply with respect to w (ε^l) is greater than the elasticity of saving with respect to w (ε^s). Incorporating endogenous labor supply into the model reduces the likelihood that public spending will raise the interest rate. It actually creates the possibility that public spending may lower the interest rate.

Intuitively, if labor supply decreases with labor income taxes enough, then the capital intensity of production increases and the interest rate falls. This is the disincentive effect of labor income taxes on labor supply. If this disincentive effect dominates the resource withdrawal effect, an increase in government spending, financed by labor income taxes, raises the capital labor ratio and, hence, reduces the rate of interest.

3.2 Consumption Taxation

As explained in chapter 3, the incentive effect of consumption taxation is essentially the same as that of labor income taxes. When the disincentive effect of consumption taxes on labor supply is strong enough, public spending increases the capital labor ratio. Furthermore, as discussed in chapter 3, the tax timing effect now works to stimulate savings when consumption taxes are raised. An increase in consumption taxes shifts some of the tax payer's tax liability into the future near the end of the life cycle. A rational tax payer increases his savings to pay the increase in his future taxes and capital accumulation increases as a result.

3.3. Remarks

In many countries' recent tax reforms, the changes in public spending have been accompanied by changes in consumption, labor income, and/or capital income tax rates. Our analysis contributes to an understanding of the structure of fiscal interdependence under such circumstances. In the framework of section 2, it has been assumed that public spending is financed by lump sum taxes (or labor income taxes with exogenous labor supply). An increase in public spending (and hence taxation) will decrease saving and, therefore, will decrease the supply of capital. This is the conventional resource withdrawal effect.

When an increase in public spending is financed by non-lump sum taxes, we would expect an additional substitution effect as well as the resource withdrawal effect. When an increase in labor income taxes affects labor supply, the disincentive effect on labor supply may raise the capital labor ratio. When public spending is financed by consumption taxes, in addition to the substitution effect, we also have the tax timing effect, which may stimulate capital formation too. This section has shown that, if public spending is financed by non-lump sum taxes, the substitution effect may dominate the resource withdrawal effect and may produce a positive effect of capital formation.

We have assumed that the economy is efficient, namely, that the real rate of interest is higher than the rate of population growth and hence, capital accumulation is always desirable. If an increase in public spending is financed by lump sum taxes, it has a negative welfare effect. On the other hand, if an increase in public spending is financed by non-lump sum taxes, it may be desirable from the viewpoint of dynamic efficiency. This suggests that the optimal level of public spending is dependent on how it is financed. The optimal level of public spending financed by non-lump sum taxes may be higher than the optimal level of public spending financed by lump sum taxes.

4. Optimal Spending Of Public Goods

4.1 Pigou's Argument

In this section, we investigate the normative aspect of public spending. Samuelson's rule (1954) involves the socially optimal provision of a pure public good conferring consumption benefits at the first best solution. His insight, that the sum of the marginal rates of substitution between a pure public good and a numeraire private good be equal to the marginal rate of transformation, i.e., $\sum MRS = MRT$, has greatly increased our understanding of the normative aspect of public spending and has led to a tremendous amount of useful research.

If the first-best rule cannot be implemented, what are the characteristics of the rule in the second-best situation, i.e., how should Samuelson's rule be modified when the full range of policy instruments is not available? Samuelson's rule must be modified to take into account the excess burden associated with the tax system used to finance the public good at the second best optimum.

The conventional wisdom suggests that the social cost of a public good necessarily increases if a distorting tax system is used to finance the public good at the second best solution. Pigou (1947) argued that the 'indirect damage' caused by a tax system which distorts economic decision-making at the margin would tend to increase the social cost of the public good. The greater the deadweight loss involved in financing the public good, the larger the social cost and the smaller the possibility that society would find the public good acceptable[3]. If the government can perfectly adjust the economy's aggregate intertemporal allocation through the use of its debt policy, then the tax system will only reflect static efficiency considerations and the effect of the tax system on the social cost of the public good will have the usual sign. If, on the other hand, the government does not have this much control over the economy, we have to consider the third best situation.

This section shows that the method of financing the public good also generally affects the dynamic efficiency of the economy. It provides a counter-example to the conventional wisdom in such a case. If the government does ignore the dynamic efficiency effect and rejects more projects at the margin as a result, then the actual magnitude of government activity, as measured by the number of projects being undertaken, will be lower than would be optimal.

4.2 The Model

In this section, we introduce labor income taxes (γ) and capital income taxes (θ). Thus, the model is almost the same as in Chapter 3 Section 2.3.2. We explicitly introduce the benefit of public goods into the utility function. Suppose that each generation has n individuals and there is no population growth. The total amount of the public good provided by the government is ng_t in period t. Due to the spillover nature of the public good, each young individual can enjoy ng_t[4].

In other words, using the expenditure function, the consumer's optimizing behavior is summarized as follows

$$E(q_t, u_t, ng_t) = 0, \tag{17}$$

where $q_t = (1, q_{2t+1}, q_{3t})$ is the consumer price vector for generation t. $q_{2t+1} = 1/[1+(1-\theta)r_{t+1}]$ and $q_{3t} = (1-\gamma)w_t$.

The feasibility condition per capita of the younger generation t is given as

$$\begin{aligned} &E_1(q_t, u_t, ng_t) + E_2(q_{t-1}, u_{t-1}, ng_{t-1}) + cg_t + w'(r_{t+1})E_3(q_{t+1}, u_{t+1}, ng_{t+1}) \\ &+ [w(r_t) - (1+r_t)w'(r_t)]E_3(q_t, u_t, ng_t) = 0 \end{aligned} \tag{18}$$

where c, the marginal cost of the public good, is assumed constant for simplicity. The public good can be produced at a constant marginal cost of c so that ng_t units can be produced at a total cost of cng_t at time t.

The compensated capital accumulation equation is given as

$$q_{2t+1}E_2(q_t, u_t, ng_t) = w'(r_{t+1})E_3(q_{t+1}, u_{t+1}, ng_{t+1}). \tag{19}$$

As in chapter 3 the tax wedge t_i (i = 2, 3) is given as

$$t_{2t} = \frac{\theta_t r_t k_t l_t}{c_t^2} = q_{2t}(1+r_t) - 1, \tag{20-2}$$

$$t_{3t} = -\gamma_t w_t = q_{3t} - w_t. \tag{20-3}$$

The government's objective is to choose taxes to maximize an intertemporal social welfare function W. This is expressed as the sum of generational utilities discounted by the social time preference factor β, subject to the private budget constraint (17), the resource constraint (18), and the capital accumulation equation (19). The associated Lagrange function is given as

$$\begin{aligned} W = \sum \beta^t \{&u_t - \lambda_{1t}E(q_t, u_t, ng_t) - \lambda_{2t}[E_1(q_t, u_t, ng_t) + E_2(q_{t-1}, u_{t-1}, ng_{t-1}) + cg \\ &+ w'(r_{t+1})E_3(q_{t+1}, u_{t+1}, ng_{t+1}) + (w(r_t) - (1+r_t)w'(r_t))E_3(q_t, u_t, ng_t)] \\ &- \lambda_{3t}[q_{2t+1}E_2(q_t, u_t, ng_t) - w'(r_{t+1})E_3(q_{t+1}, u_{t+1}, ng_{t+1})]\} \end{aligned}$$

λ_1 is the Lagrange multiplier for the private budget constraint, λ_2 is the Lagrange multiplier for the resource constraint, and λ_3 is the Lagrange multiplier for the capital accumulation equation.

Differentiating the Lagrangian with respect to g_t, we obtain

$$\begin{aligned} \frac{\partial W}{\partial g_t} = \beta^t \{&-n\lambda_{1t}E_g(q_t, u_t, ng_t) - \lambda_{2t}\{nE_{1g}(q_t, u_t, ng_t) + c + \\ &n[w(r_t) - (1+r_t)w'(r_t)]E_{3g}(q_t, u_t, ng_t)\} - \lambda_{2t+1}nE_{2g}(q_t, u_t, ng_t)\beta - \\ &\lambda_{2t-1}\frac{nw'(r_t)E_{3g}(q_t, u_t, ng_t)}{\beta} - n\lambda_{3t}q_{2t+1}E_{2g}(q_t, u_t, ng_t) + \\ &n\lambda_{3t-1}\frac{w'(r_t)E_{3g}(q_t, u_t, ng_t)}{\beta}\} = 0 \end{aligned} \tag{21}$$

where $E_g = \partial E / \partial(ng)$ and $E_{ig} = \partial E_i / \partial(ng)$.

Differentiating the Lagrangian with respect to r_{t+1}, we derive

$$\frac{\partial W}{\partial r_{t+1}} = \beta^t \{-\lambda_{2t} w''(r_{t+1}) E_3(q_{t+1}, u_{t+1}, ng_{t+1}) +$$
$$\lambda_{2t+1}\beta(1+r_{t+1}) w''(r_{t+1}) E_3(q_{t+1}, u_{t+1}, ng_{t+1}) + \lambda_{3t} w''(r_{t+1}) E_3(q_{t+1}, u_{t+1}, ng_{t+1})\} = 0 \tag{22}$$

In a steady state, (22) will be reduced to

$$\frac{\lambda_3}{\lambda_2} = 1 - \beta(1+r). \tag{23}$$

Considering $\sum q_i E_{ig} = E_g$ and (20-2)(20-3)(23), (21) can be rewritten as (24). Our main result involves the following version of the mixed Ramsey-Samuelson rule (24), which governs the optimal provision of the public goods in a steady state. We have (Batina (1990)):

Proposition 3: The optimal rule is given by

$$\beta t_2 E_{2g} + t_3 E_{3g} = \frac{c}{n} + (\lambda + 1) E_g, \tag{24}$$

where $\lambda = \lambda_1 / \lambda_2$.

From the first-order conditions with respect to q_2 and q_3, we also have the mixed Ramsey-Golden rule,

$$\beta e_2 \sigma_{22} + e_3 \sigma_{23} = \lambda + 1 - \beta(1+r) \tag{25-1}$$
$$\beta e_2 \sigma_{32} + e_3 \sigma_{33} = \lambda \tag{25-2}$$

in which e_i is the effective tax rate (t_i/q_i) and σ_{ij} is the compensated elasticity ($q_j E_{ij} / E_i$) (i, j = 2, 3). These two equations are essentially the same as (28-1,2) in Chapter 3.

Remark 1: As discussed in Chapter 3, λ_1 corresponds to the marginal benefit of lump sum transfer for each young individual financed by distortionary taxes. λ_1 is negative[5] in the static model but may be positive in the present dynamic model. λ_2, which is positive, corresponds to the marginal benefit of a decrease in government revenues. Note that an increase in tax burden itself reduces utility. $-E_g$ is the agent's marginal willingness to pay for the public good, i.e., the amount of wealth the agent would be willing to give up in order to receive an additional unit of the public good, which is positive.

Remark 2: When the government controls two types of lump sum taxes T^1 and T^2, the first best can be attained. In such a case, as shown in Chapter 3 Section 2.3, we can easily see that $\lambda = 0$ and distortionary taxes are zero. Thus, from (23) we have the modified golden rule, and (24) reduces to

$$-E_g = \frac{c}{n} \tag{24'}$$

which is the Samuelson rule.

Remark 3: As first noted by Diamond and Mirrlees (1971), provision of the public good may alter taxed economic behavior. Whether or not savings and labor supply respond to the provision of a particular public good is an empirical question that must be decided on case-by-case basis. In general, however, theory predicts that both labor supply and savings will directly respond to the provision of the public good. As a result, tax revenues will also respond and this will alter the social cost of the public good. This is captured by the term $(\beta t_2 E_{2g} + t_3 E_{3g})$ in (24).

Remark 4: The term $-(\lambda + 1)E_g$ captures the efficiency effect associated with the method of financing the public good. From (25-1), we know

$$\lambda = \beta(r-\rho) + (\beta e_2 \sigma_{22} + e_3 \sigma_{23}), \tag{26}$$

ρ is the rate of time preference; $\beta = 1/(1+\rho)$.

The first term on the right hand side in (26) captures the effect of the tax system on the dynamic efficiency of the economy, while the second term captures the effect on the static efficiency. It is shown (see fotenote 5) that this second term will be non-positive in a static model. Therefore, if debt policy can be chosen optimally or if the consumption tax is chosen optimally, the tax system will only reflect static efficiency (see the results of Chapter 3 Section 2.4.1). The loss in the static efficiency associated with the tax system will serve to increase the social cost of the public good. This is the static efficiency effect mentioned by Pigou and studied in the subsequent literature as the second-best case.

However, it does not seem realistic to assume that the dynamic efficiency of the economy is completely unaffected by the tax system. In the third-best case, where the government cannot completely control the dynamic efficiency of the economy through its debt or lump sum tax policy, it is possible that an appropriate choice of its tax policy may improve the dynamic efficiency of the economy and possibly lower the social cost of the public good.

To see this assume that debt or lump sum tax policy cannot be chosen optimally. In that case, it is entirely possible that $r > \rho$ at the third-best optimum[6]. The first term in (26) then is positive while the second term may be negative. Therefore, the effect of the tax system on the dynamic efficiency of the economy will tend to offset the effect of static efficiency on the social cost of the public good from (25). If the dynamic efficiency effect outweighs the static efficiency effect, the social cost of the public good in the third-best case may actually be lower than in the first-best case when debt policy is unavailable even though a distorting tax system is being used to finance the public good. Recall that, as pointed out in chapter 3, the before-tax rate of return r (not the after-tax rate of return $(1-\theta)$r) is relevant to the dynamic efficiency criterion even when $\theta > 0$.

Remark 5: The intuition behind this result is the following. If debt policy is unavailable, the effect of public spending on capital accumulation will become important. An increase in the disposable income in the first period of his life stimulates saving (see equation (30) in Chapter 3). As shown in the previous section, when government spending is financed by distortionary labor income taxes, an increase in public spending may well stimulate capital accumulation. An increase in capital accumulation in the aggregate moves the economy closer to the optimal level of capital when the initial level of capital is below the optimal level. Consequently, it tends to improve steady state welfare on average as a result. The improvement in the economy's dynamic efficiency will, in turn, serve to reduce the social cost of a public good at the margin.

Remark 6: Clearly, if the dynamic efficiency effect does serve to reduce the social cost of the public good, and this is ignored in calculating the social cost of the public good, it is also entirely possible for the government to reject a project when it would have been accepted under the more general criterion. If the government does reject more projects at the margin, ultimately the size of the government will tend to be smaller than would otherwise have been the case, ceteris paribus. This would provide a counter-example to those who would argue that governments generally tends to be too large because too many projects are being accepted at the margin.

4.3 Summary

Pigou (1947) pointed out that, at the second best optimum, any 'indirect damage' or deadweight loss associated with the tax system, used to finance a public good, must also be included in calculating the social cost of the public good at the margin. Samuelson's rule governing the optimal provision of a pure public good, conferring consumption benefits at the first best optimum, must then be modified to include this additional social cost. The literature has focused attention on the excess burden associated with the static efficiency of the tax system. Presumably, the greater the excess burden, the larger the social cost of a public good.

If the government has less than perfect control, the tax system certainly affects the dynamic efficiency of the economy. This allows us to infer the logical conclusion that the tax system may improve the dynamic efficiency of the economy at the third best optimum. The dynamic efficiency effect on the social cost of the public good then works in the opposite direction of the static efficiency effect mentioned by Pigou. In that case it is possible for the social cost of the public good in the third best case to be lower than in the first-best case, even though distorting taxes are being used to finance the public good.

If the social cost of a public good is generally lower when dynamic efficiency is taken into account and this is ignored in calculating the social cost of a potential project, the government will tend to reject more projects at the margin than otherwise. In that case, the overall size of the government in a steady state will be smaller as a result.

5. Public Investment

5.1 Analytical Framework

So far we have not considered the role of public spending on stimulating production. There have been several studies of the optimal public investment policy in an intertemporal framework. Using the standard overlapping generations model, Diamond (1970), Hamada (1972), and Pestieau (1974) have studied the optimal public investment problem of a government striving to maximize intertemporal social welfare subject to the demand and supply relations of a decentralized private sector. Let us investigate the optimal public investment policy using our analytical framework.

The technology of the economy is specified by a production function of the type:

$$y_t = f(k_t, g_t) \tag{27}$$

where y_t is per labor output, k_t is per labor capital stock in the private sector, and g_t is now per labor capital stock in the public sector in period t. Public investment here yields production benefits but not consumption benefits, at least not directly.

Competitive profit maximization means that the rate of interest, r, equals the marginal product of private capital, f_k.

$$f_k(k_t, g_t) = r_t \tag{28}$$

We assume for simplicity that all of the residuals are received by workers. Thus, the wage rate, w, is given as

$$y_t - r_t k_t = w_t \tag{29}$$

From the above three equations (27)(28) and (29), w and k are solved by a function of r and g, respectively.

$$w_t = w(r_t, g_t),\ w_r = -k,\ w_g = f_g \tag{30}$$

$$k_t = k(r_t, g_t),\ k_r = \frac{1}{f_{kk}},\ k_g = -\frac{f_{kg}}{f_{kk}} \tag{31}$$

where $w_r = \frac{\partial w}{\partial r}$, $w_g = \frac{\partial w}{\partial g}$, $f_g = \frac{\partial f}{\partial g}$, $k_r = \frac{\partial k}{\partial r}$, $k_g = \frac{\partial k}{\partial g}$, $f_{kk} = \frac{\partial^2 f}{\partial k \partial k}$, $f_{kg} = \frac{\partial^2 f}{\partial k \partial g}$.

As in the previous section, we consider labor income taxes and capital income taxes with an elastic supply of labor. The consumer's optimizing behavior is given as

$$E(q_t, u_t) = 0 \tag{32}$$

Recall that we do not incorporate public spending into the utility function in this section. The feasibility condition is given as

$$\begin{aligned} &E_1(q_t, u_t) + E_2(q_{t-1}, u_{t-1}) - g_{t+1}E_3(q_{t+1}, u_{t+1}) + g_t E_3(q_t, u_t) \\ &+w_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1}) + [w(r_t, g_t) - (1+r_t)w_r(r_t, g_t)]E_3(q_t, u_t) = 0 \end{aligned} \tag{33}$$

The compensated capital accumulation equation is now given as

$$q_{2t+1}E_2(q_t, u_t) = w_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1}) \tag{34}$$

Note that (32) and (33) imply the government budget constraint

$$t_{2t}E_2(q_{t-1}, u_{t-1}) + t_{3t}E_3(q_t, u_t) = -g_{t+1}E_3(q_{t+1}, u_{t+1}) + g_t E_3(q_t, u_t).$$

This expression governs the public capital accumulation.

5.2 Optimal Investment Rule

The government's objective is to choose taxes to maximize an intertemporal social welfare function W. This is expressed as the sum of generational utilities discounted by the social time preference factor β. It is subject to the private budget constraint (32), the resource constraint (33), and the capital accumulation equation (34). The associated Lagrange function is given as

$$\begin{aligned} W = \sum \beta^t \{&u_t - \lambda_{1t}E(q_t, u_t) - \lambda_{2t}[E_1(q_t, u_t) + E_2(q_{t-1}, u_{t-1}) \\ &-g_{t+1}E_3(q_{t+1}, u_{t+1}) + g_t E_3(q_t, u_t) + w_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1}) \\ &+(w(r_t, g_t) - (1+r_t)w_r(r_t, g_t))E_3(q_t, u_t)] \\ &- \lambda_{3t}[q_{2t+1}E_2(q_t, u_t) - w_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1})]\} \end{aligned} \tag{35}$$

We differentiate the Lagrangian with respect to g_t to obtain

$$\begin{aligned} \frac{\partial W}{\partial g_t} = \beta^t \{&-\lambda_{2t-1}\{-\frac{E_3(q_t, u_t)}{\beta} - \frac{k_g(r_t, g_t)E_3(q_t, u_t)}{\beta}\} \\ &+\lambda_{2t}\{-E_3(q_t, u_t) - [w_g(r_t, g_t) + (1+r_t)k_g(r_t, g_t)]E_3(q_t, u_t)\} \\ &-\lambda_{3t-1}\frac{k_g(r_t, g_t)E_3(q_t, u_t)}{\beta}\} = 0 \end{aligned} \tag{36}$$

Differentiating the Lagrangian with respect to r_{t+1}

$$\begin{aligned} \frac{\partial W}{\partial r_{t+1}} = \beta^t \{&\lambda_{2t}k_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1}) - \lambda_{2t+1}\beta(1+r_{t+1})k_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1}) \\ &- \lambda_{3t}k_r(r_{t+1}, g_{t+1})E_3(q_{t+1}, u_{t+1})\} = 0 \end{aligned} \tag{37}$$

In a steady state (37) reduces to (23).

$$\frac{\lambda_3}{\lambda_2} = 1 - \beta(1+r) \tag{23}$$

Considering (23), in a steady state from (36) we have

$$1 = \beta(1 + w_g)$$

Or

$$f_g = \rho \tag{38}$$

(38) governs the rate of return on public investment. In other words, we have Pestieau (1974):

Proposition 4: The public rate of return is equal to the social rate of time preference.

Remark 1: The government's choice of a discount rate is, in the long run, the same as it would be if it were able to control the economy as in a first-best solution. When two types of lump sum taxes T^1 and T^2 are available, it is easy to see $\lambda_3 = 0$ and from (23) and (38) we have

$$f_k = f_g = \rho \tag{39}$$

The condition for intertemporal first best optimality would imply equality among the market rate of interest and the marginal productivities of private and of public capital. The condition $f_k = \rho$ is the modified golden rule. Thus, (39) holds at the second best solution when all the distortionary taxes are available as in Chapter 2.4.1.

Remark 2: At the third best solution where two types of lump sum taxes as well as consumption taxes are not available, the marginal productivities of private and of public capital are not necessarily equalized. The inequality of these rates is an indication of the failure to achieve the optimum. From the first-order conditions with respect to q_2 and q_3, we again obtain the mixed Ramsey-Golden rule (25-1,2). Thus, we have

$$f_g - f_k = e_2(\sigma_{22} - \sigma_{32}) + (1+\rho)e_3(\sigma_{23} - \sigma_{33}) \tag{40}$$

This equation shows how the financing of the public investment affects the various parts of the private sector, present consumption, future consumption, and leisure, so as to minimize the total deadweight loss measured by the elasticity terms. The discrepancy between the rates of return on public and private investment varies positively with the elasticity of leisure (σ_{33}) and varies negatively with the (absolute) elasticity of future consumption ($-\sigma_{22}$).

Recently, Kanemoto (1987) introduced market imperfection into the analysis of social discount rates. He obtained the optimal discount rate for the two-state case in which returns on private projects can take only two values.

Aschauer (1989) argued that public capital is a potential factor in the productivity slowdown in the U.S. economy. Gramlich (1994) provides a useful review essay on public investment. There have been some attempts to incorporate public capital into models of endogenous growth. See Chapter 11.

[1] . Even if population growth is positive, the qualitative results would be the same so long as r is greater than n, the rate of population growth.

[2] . For simplicity we assume that public spending does not directly affect savings until Section 4. See also Chapter 9.

[3] . Pigou's argument has been clarified and extended by Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), and more recently Wildasin (1984, 1985) and King (1986), within the context of a well-known static model.

[4] . For simplicity we assume that ng_t does not benefit the old generation.

[5] . It is always desirable for the economy to change its financing from the optimally chosen distortionary taxes to lump sum taxes in the static model. See Atkinson and Stiglitz (1980).

[6] . We can call the case where neither dynamic efficiency nor static efficiency is attained the third best case.

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Chapter 6

Open Economy

1. Introduction

This chapter extends the basic model into a two country framework. Since the work of Feldstein (1978), Bradford (1980a,b), Mieskowski (1980), and Summers (1981b) among others, the consumption tax policy has gained considerable support in the last decade or so. Section 2 demonstrates the results of the conventional wisdom. In an open economy, a switch from the income tax to the consumption tax increases capital accumulation, reduces the interest rate, and improves welfare in the world-wide economy. Next, this section investigates the positive spillover welfare effect of tax reform in the home country on the foreign country. Since our main concern in this chapter is to extend the results in previous chapters into a two-country open-economy framework, for simplicity we will mainly consider the long-run properties.

Section 3 provides a counterargument to the conventional wisdom and explores possible negative international spillover effects. Suppose we follow the conventional theoretical framework where (i) labor supply is exogenous, (ii) bequests are assumed away, (iii) only long-run welfare is considered, and (iv) the interest payment effect is assumed away. Still, the world-wide positive effect on capital accumulation does not necessarily imply the desirable spillover effect on the foreign country's welfare if initial consumption taxes are large in the foreign country.

Section 4 develops a general equilibrium model of how the tax treatment of capital income affects the amount and form of international flow of capital[1]. The model is then used to investigate the effects of tax reforms involving changes in capital income taxes on capital accumulation in a two country framework. We consider both 'territorial' and 'residence' tax systems. In a territorial system capital income tax burdens depend on where the income is earned, but not on the owner's country of residence. Conversely, under a residence system tax burdens depend on country of residence, not on where income earned. Based on Ihori (1991), we consider a revenue-neutral tax reform: conversion from capital income to consumption income taxes. It is shown that in the territorial system the tax reform will normally lead to a negative co-movement between capital accumulation in two countries, while in the residence system it will lead to a positive comovement.

Finally, Section 5 explores the normative aspects of taxation in a two country open economy. In the territorial system, a reduction in capital income rates will induce capital inflow and hence raise tax revenues from capital income. This is the tax competition effect. This may lead to lower capital income taxation in the non-cooperative Nash solution than in the cooperative solution.

2. Two Country Model

2.1 Capital Mobility

We will use a two-country version of the overlapping generations model developed by Buiter (1981). Consider a one-good world economy consisting of two countries: the home country and the foreign country. Each country is populated by overlapping generations of two-period-lived consumers as well as firms and a government. There is no growth in population[2] and the size of population is the same between countries. For each country the analytical framework is the same as the basic model in Chapter 2. An agent of generation t supplies inelastically one unit of labor and receives wages w_t out of which the agent consumes c^1_t, and saves s_t. An agent who saves s_t receives, when he's old, $(1+r_{t+1})s_t$ which the agent then spends entirely on consumption, c^2_{t+1}. All variables associated with the foreign country will be distinguished by an asterisk, if necessary.

Capital is perfectly mobile across countries but labor is not. In a world equilibrium with perfect capital mobility, domestic agents can save by holding either domestic or foreign capital.

$$s_t = k_{t+1} + z_{t+1}, \tag{1}$$

where z_t is net foreign capital in period t owned by domestic agents. If $z_t > 0$, the home country is a capital exporting country. If $z_t < 0$, the home country is a capital importing country. In the two country framework $z_t + z^*_t = 0$. In other words, in this integrated world economy, equilibrium in the financial market requires

$$s_t + s_t^* = k_{t+1} + k_{t+1}^*. \tag{2}$$

World capital mobility implies that the rates of return on capital are equalized across countries, that is, $r_t = r^*_t$.

In this two country model, several assumptions are useful.

First, suppose production technology is the same between two countries. We have $w^* = w$ and $k = k^*$. From (1) we know that if $s > s^*$, then $z > z^*$. In other words, a high saving country is a capital exporting country. A country which has a lower rate of time preference is a capital exporting country.

Second, recall national income of the home country is given by w+rk+rz. Thus, an increase in the rate of interest raises the return on capital exports, leading to benefiting a capital exporting country. Even if $s = s^*$, there is international dependence due to the perfect capital mobility. Fiscal policy in the home country can have a spillover effect on the foreign country.

Finally, international trade and international lending and borrowing are part and parcel of the same transaction, and hence the only way to pay for an extra unit of output today is with a promise of future output. It follows that, in the case of no international capital mobility, it means the autarky exits in this framework[3].

2.2 Tax Reform And Capital Accumulation

2.2.1 Tax Reform

The government imposes consumption taxes and wage income taxes. Therefore, the agent pays consumption taxes $\tau_t c^1_t$ and wage income taxes $\gamma_t w_t$ in period t and pays consumption taxes $\tau_{t+1} c^2_{t+1}$ in period t+1. For simplicity the government fixes consumption tax rates; $\tau_t = \tau_{t+1} = \tau$.

The optimal consumption and saving behavior for the agent in the home country is summarized by

$$c_t^1 = c^1(a_t w_t, r_{t+1}), \tag{3-1}$$

$$c_{t+1}^2 = c^2(a_t w_t, r_{t+1}), \tag{3-2}$$

$$s_t = \frac{1}{1+r_{t+1}} c^2(a_t w_t, r_{t+1})(1+\tau), \tag{3-3}$$

where $a = (1-\gamma)/(1+\tau)$, $\partial c^1 / \partial w = c_{1w} a > 0$, $\partial c^2 / \partial w = c_{2w} a > 0$, $\partial c^1 / \partial a = c_{1w} w > 0$,

$\partial c^2 / \partial a = c_{2w} w > 0$ and $\partial c^i / \partial (aw) = c_{iw}$ (i=1,2).

Competitive profit maximization and a neoclassical technology require that firms hire labor and demand capital in such a way that

$$w_t = w(r_t),\ \ w'(r_t) = -k_t, \tag{4}$$

where w( ) is the factor price frontier and k_t is the amount of capital per worker in the home country in period t.

The home government collects taxes on consumption, $\tau\,(c^1_t+c^2_t)$, and on wages, $\gamma_t w_t$, which it uses to finance (exogenously given) government spending, g, yielding the following government budget constraint:

$$\tau(c_t^1 + c_t^2) + \gamma_t w_t = g. \tag{5}$$

The foreign country's budget constraint is given as

$$(1 - a_t^*) w_t^* = g_t^*, \tag{5)'$$

where $a^* = 1 - \gamma^*$ and γ^* is the wage tax rate in the foreign country. τ^* is assumed to be zero.

In the initial equilibrium, both countries use a wage income tax to finance an exogenously determined level of government spending. The level of government spending and the wage tax rate may differ across countries. The policy experiment we will consider throughout section 2 involves the following. The home country switches to the consumption tax at the margin and the wage tax rate in both countries adjusts endogenously so as to maintain the level of government spending in each country. We will evaluate our results in the initial equilibrium where $\gamma > 0$, $\gamma^* > 0$, and $\tau = 0$.

In summary the world economy may be described by the following three equations.

$$c^2[a_t w(r_t), r_{t+1}](1+\tau) + c^{2*}[a_t^* w^*(r_t), r_{t+1}] = -w'(r_{t+1})(1+r_{t+1}) - w^{*\prime}(r_{t+1})(1+r_{t+1}) \tag{6}$$

$$\tau\{c^1[a_t w(r_t), r_{t+1}] + c^2[a_{t-1} w(r_{t-1}), r_t]\} + \{1 - a_t(1+\tau)\} w(r_t) = g, \tag{7}$$

$$(1 - a_t^*) w^*(r_t) = g^*. \tag{8}$$

(6) is the equilibrium condition in the world financial market. (7) is the home country's government budget constraint. (8) is the foreign country's government budget constraint.

2.2.2 Capital Accumulation

In order to investigate local stability, evaluate (7) at the initial equilibrium where $\tau = 0$. Then, we can write (7) as

$$a_t = G(r_t), \tag{7)'$$

where $G_r = da_t / dr_t = w'(1-a)/w < 0$. Equation (8) can be written as

$$a_t^* = 1 - \frac{g^*}{w_t^*(r_t)} = G^*(r_t), \tag{8)'$$

where $da^*/dr = G_r^* = w^{*\prime}(1-a^*)/w^* < 0$, since $w^{*\prime} < 0$. We substitute (7)' and (8)' into (6) and write the resulting equation as

$$S[G(r_t), G^*(r_t), r_t, r_{t+1}] = 0. \tag{9}$$

Notice that $\partial r_{t+1} / \partial r_t = -(BG_r + B^* G_r^* + S_{r0}) / S_{r1}$, where $B = \partial S / \partial G = c_{2w} w > 0$, $S_{r0} = \partial S / \partial r_t = w' c_{2w} a + w^{*\prime} c_{2w}^* a^*$ and $S_{r1} = \partial S / \partial r_{t+1} = c_{2r} + c_{2r}^* + w''(1+r)$ $+ w^{*\prime\prime}(1+r) + w' + w^{*\prime}$. S_{r1} is most likely positive for plausible values of the elasticity of substitution between labor and capital. As in Chapter 2, we will assume that S_{r1} is positive.

Local stability requires $0 < \partial r_{t+1} / \partial r_t < 1$ at the steady state solution. Since $S_{r1} > 0$, this condition means

$$S_{r0} + S_{r1} + BG_r + B^*G^*_r > 0. \tag{10}$$

We will assume that (10) holds.

To derive the comparative statics of the basic model, considering (8)', differentiate the steady state version of (6) and (7) to obtain

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} dr \\ da \end{bmatrix} = -\begin{bmatrix} c^2 \\ c^1 + c^2 - aw \end{bmatrix} d\tau, \tag{11}$$

where $A = S_{r0}+S_{r1} > 0$, $B = c_{2w}w > 0$, $C = w'(1-a) < 0$, and $D = -w < 0$. Solving (11) yields

$$\frac{dr}{d\tau} = -\frac{1}{\Delta}[c^2 D - (c^1 + c^2 - aw)B] < 0, \tag{12}$$

where $\Delta = AD-BC < 0$ from (10). $[c^2D-(c^1+c^2-aw)B]$ is negative. We have:

Proposition 1: An increase in the consumption tax rate in the home country will stimulate world capital accumulation, given the level of government spending.

Intuitively, a reduction in γ, coupled with an increase in τ, shifts some of the tax payer's tax liability to the future and near the end of the life cycle. A rational tax payer will increase his savings to pay the increase in his future taxes and capital accumulation will increase as a result. This is known as the tax timing effect[4].

From (11), we also have

$$\frac{da}{d\tau} = -\frac{1}{\Delta}[(c^1 + c^2 - aw)A - c^2 C] > 0, \tag{13}$$

Hence, considering (8)' and (12),

$$\frac{da^*}{d\tau} = \frac{w^{*\prime}(1-a^*)}{w^*}\frac{dr}{d\tau} > 0. \tag{14}$$

Let us denote the indirect utility function for the agent in the steady state in the home country as

$$u = U(aw(r), r), \tag{15}$$

with the familiar properties that

$$U_1 = \lambda, \tag{16-1}$$

$$U_2 = \frac{\lambda c^2}{(1+r)^2}, \tag{16-2}$$

where $U_1 = \partial U / \partial(aw)$ and $U_2 = \partial U / \partial r$. $\lambda > 0$ is the marginal utility of lifetime income of the consumer in the home country.

The welfare effect of τ on U evaluated at an equilibrium where consumption taxes are zero is given as

$$\begin{aligned}\frac{dU}{d\tau} &= (U_1 aw' + U_2)\frac{dr}{d\tau} + U_1 w \frac{da}{d\tau} \\ &= \frac{\lambda}{1+r}\{-[r(1-\gamma) - \gamma]k + z\}\frac{dr}{d\tau} + \lambda w \frac{da}{d\tau}\end{aligned} \tag{17-1}$$

where z (= s-k) is the foreign net asset holding of the home country. Similarly, the welfare effect of τ on U* is given by

$$\frac{dU^*}{d\tau} = \frac{\lambda^*}{1+r}\{-[r(1-\gamma^*)-\gamma^*]k+z^*\}\frac{dr}{d\tau}+\lambda^* w^{*\prime}\gamma^*\frac{dr}{d\tau}$$
$$= \frac{\lambda^*}{1+r}(-rk+z^*)\frac{dr}{d\tau} \qquad (17\text{-}2)$$

Thus, from (17-1) we have

Proposition 2: The long-run welfare effect of the own tax reform can be decomposed into the three components; the golden rule effect , $-\frac{\lambda}{1+r}[r(1-\gamma)-\gamma]k\frac{dr}{d\tau}$, the interest payment effect, $\frac{\lambda z}{1+r}\frac{dr}{d\tau}$, and the tax revenue effect, $\lambda w\frac{da}{d\tau}$.

Remark 1 (the golden rule effect): Since the first term, $-[r(1-\gamma)-\gamma]kdr/d\tau$ is normally positive, an increase in τ will have a favorable welfare effect on the home country. As shown in chapter 2, if the economy is efficient, namely, $r > n$ (the population growth rate), capital accumulation raises long run utility. This is the golden rule effect. Since we have assumed $r > n = 0$, capital accumulation due to the tax reform normally raises long-run welfare of the home country.

Remark 2 (the interest payment effect): Let us investigate the second term, $zdr/d\tau$. Suppose initially $z > 0$. As shown in Buiter (1981) and Hamada (1986), a country with a lower pure rate of time preference or a lower level of government spending normally becomes a net creditor in the long run. Then, an increase in τ will reduce r and hurt the net creditor living in the home country. This is the interest payment effect. Since $z+z^* = 0$, the interest payment effect always has the opposite effect on welfare between the two countries[5]. In the debtor country the golden rule effect and the interest payment effect work in the same direction.

Remark 3 (the tax revenue effect): The impact on *a* and a^* may be called the tax revenue effect. If *a* were fixed, $ad\tau = -d\gamma$. The first-round effect of an increase in τ on tax revenue is $(c^1+c^2)d\tau + wd\gamma = (c^1+c^2-aw)d\tau > 0$. Therefore, in order to maintain the same amount of revenue, γ must reduce further, which implies an increase in *a*. An increase in τ will raise the disposable income of the agent in the home country, aw, which is favorable to the home country.

Remark 4: In the foreign country, the tax revenue effect comes from positive γ^* (see (14)). Substituting (14) into (17-2), γ^* will disappear in the overall golden rule effect (see (17-2)). Thus, the tax reform will have a positive spillover effect on the foreign country unless the interest payment effect hurts it enough.

To sum up, if the golden rule effect dominates the interest payment effect, the tax reform in the home country will improve welfare in both countries. In particular, the tax reform will not have a negative spillover effect on the foreign country. This provides a strong argument for switching to the consumption tax in an open economy since the home country could switch to the tax, improve welfare at home, and would not have to worry about retaliation from the foreign country.

3. Negative International Spillover Effects Of Consumption Taxation

3.1. Consumption Taxes

As explained in Chapters 3, 4, and Section 2 of this chapter, it would seem fair to say that the conventional wisdom would suggest that the consumption tax is superior to the income tax on efficiency grounds. It would also have a significant beneficial impact on the economy since capital formation would be favorably affected. The purpose of this section is to provide a counterargument to the conventional wisdom by showing that under certain circumstances imposition of the consumption tax may create a negative international spillover effect[6]. For example, we will show that the spillover welfare effect depends critically on the foreign economy's initial rates of consumption tax.

Table 6.1 reports the current rates of consumption taxes applied by European countries. The comparison of consumption taxes across countries is complicated by progression in the rates, which of course differs from country to country. Nevertheless, it is worth noticing that the actual rates of consumption taxes are not close to zero. Even if an introduction of value-added taxes (VAT) is desirable in the home economy[7], it may not be desirable in a two-country open economy. The reason is that it may hurt the foreign country which has already introduced high VAT rates in its system.

3.2 Model

The analytical framework is almost the same as in section 2. We now introduce $\tau^* > 0$ here. For simplicity technology is the same between two countries; w( ) = w*( ). The world economy in the steady state may be described by the following three equations.

$$c^2[aw(r),r](1+\tau)+c^{2*}[a^*w(r),r](1+\tau^*) = -2w'(r)(1+r) \tag{6'}$$

$$\tau\{c^1[aw(r),r]+c^2[aw(r),r]\}+\{1-a(1+\tau)\}w(r) = g \tag{7'}$$

$$\tau^*\{c^{1*}[a^*w(r),r]+c^{2*}[a^*w(r),r]\}+\{1-a^*(1+\tau^*)\}w(r) = g^* \tag{18}$$

As before we consider the tax reform of the home country; an increase in τ at a given level of g. From (7)' a and hence γ will be endogenously adjusted. The foreign country does not change its own expenditure g* as well as its own tax rate on consumption τ^*. a^* (or a labor income tax rate, γ^*) will be endogenously adjusted. It is assumed that in the home country wage income and consumption taxes are zero at the initial equilibrium; the effects of tax reform will be evaluated at $\gamma = \tau = 0$. (g = 0). It is also assumed that in the foreign country γ^* is zero but τ^* is positive at the initial equilibrium. (g* > 0)[8].

3.3 Tax Reform

We now investigate the comparative static effect of a change in τ on r, a, and a^*. Totally differentiating (6)' (7)' and (18), we have

$$\begin{bmatrix} A & B & B^* \\ C & D & 0 \\ C^* & 0 & D^* \end{bmatrix} \begin{bmatrix} dr \\ da \\ da^* \end{bmatrix} = -\begin{bmatrix} c^2 \\ c^1+c^2-w \\ 0 \end{bmatrix} d\tau, \tag{19}$$

where C=0, $C^* = -G^*_{r0}-G^*_{r1}$, $B^* = c^*_{2w}w(1+\tau^*) > 0$, $D^* = \tau^* w(c^*_{1w}+c^*_{2w})-(1+\tau^*)w$. $G^*_{r0} = -w'\tau^* a^*(c^*_{1w}+c^*_{2w})$, $G^*_{r1} = -\tau^*(c^*_{1r}+c^*_{2r})$. We know B > 0 and D < 0. $G^*_{r0} > 0$ and $G^*_{r1} < 0$. The sign of C* is generally ambiguous. If $c^*_{1w}+c^*_{2w}$ is large and/or $c^*_{1r}+c^*_{2r}$ is small, C* is likely to be negative. We assume this. If τ^* is high, D* may well be positive, which means that an increase in γ^* for given r and τ^* will reduce the total tax revenue by reducing consumption and hence tax revenue from consumption taxation. This paradoxical Laffer effect may happen if the initial level of τ^* is very high. Since we are interested in the spillover welfare effect when τ^* is initially positive and high, from now on we assume D* > 0. This may be called the cross-Laffer effect assumption in the sense that an increase in γ^* will reduce tax revenues at $\gamma^*=0$ if τ^* is large.

Then, we have

$$\frac{dr}{d\tau}=-\frac{D^*}{\Delta}\{c^2D-(c^1+c^2-w)B\}, \quad (20)$$

$$\frac{da}{d\tau}=-\frac{1}{\Delta}(c^1+c^2-w)(AD^*-B^*C^*), \quad (21)$$

$$\frac{da^*}{d\tau}=-\frac{C^*}{\Delta}\{-c^2D+(c^1+c^2-w)B\}, \quad (22)$$

where Δ is the determinant of the matrix of the left-hand side of (19).

$$\Delta = D^*AD-C^*DB^*. \quad (23)$$

$S_{r0}=\partial S/\partial r_t=w'[s_w+s_w^*a^*(1+r^*)] < 0$, and $S_{r1}=\partial S/\partial r_{t+1}=s_r+s_{r^*}(1+\tau^*)+2[w''(1+r)+w']$. As in section 2, A > 0, and hence $\Delta<0$. We assume that the system is stable and the elasticity of substitution between capital and labor is large. Inequality $\Delta<0$ corresponds to the saddle-point stability[9].

Under these assumptions, the sign of dr/dτ is negative as in section 2.

Proposition 3: An increase in the consumption tax rate in the home country at a given level of its government expenditure will stimulate world capital accumulation even if τ^* is large and the cross-Laffer effect is valid.

This is the positive supply-side effect on capital accumulation. An intuitive explanation is the same as in Section 2.

Let us investigate the impact of an increase in τ on *a*. We have (21) > 0. If *a* is fixed, adτ = -dγ. The first-round effect of an increase in τ on the government revenue is

$$(c^1+c^2)d\tau + wd\gamma = (c^1+c^2-aw)d\tau > 0$$

Therefore, in order to maintain the same amount of expenditure, γ is required to be reduced further, a fact which implies an increase in *a*. This is the direct tax revenue effect[10]. This term is always positive. (20) <0 and (21) >0 are conventional results, which provide a strong argument for switching to the consumption tax when $\tau=0$ in the home country.

Finally, we investigate the impact of an increase in τ on *a**. This impact, the sign of (22), is negative under the above assumptions.

Proposition 4: An increase in the consumption tax rate in the home country will reduce $a^*=\frac{1-\gamma^*}{1+\tau^*}$ of the foreign country if τ^* is initially large.

Remark: Capital accumulation due to the tax reform negatively affects $a^*=\frac{1-\gamma^*}{1+\tau^*}$ of the other country. An intuitive explanation is as follows. Capital accumulation will raise w^* and total consumption $c^{1*}+c^{2*}$, hence raising tax revenues from consumption taxation in the foreign country, $\tau^*(c^{1*}+c^{2*})$. In order to meet the government budget, the foreign government has to adjust wage taxes γ^* so as to reduce tax revenues. If τ^* is high ($D^* > 0$), *an increase* in γ^* will reduce the tax revenue by reducing consumption and hence tax revenues from consumption taxation. Therefore, capital accumulation will lead to an increase in γ^* and hence a reduction in a^*. Thus, the cross-Laffer effect assumption ($D^* > 0$) is crucial for (22). Note that if τ^* is initially zero, (22) will be reduced to (14), which is positive.

We are now ready to investigate the long run welfare effects of the tax reform. Since $da/d\tau > 0$, an increase in τ will further reduce γ and raise disposable income of the agent in the home country, aw, which is favorable to the home country. As for the foreign country we now have

$$\frac{dU^*}{d\tau} = \frac{\lambda^*}{1+r}(-rk+z^*)\frac{dr}{d\tau} + \lambda^* w^* \frac{da^*}{d\tau}. \qquad (17\text{-}2)'$$

The spillover indirect tax revenue effect on the foreign country, $da^*/d\tau$ is likely to be negative when τ^* is large.

In the previous literature on the spillover effect of tax reform in an open economy, the main concern has been with the interest payment effect due to the existence of current account imbalances (see Hamada (1986), Frenkel and Razin (1988, 1989), and Sibert (1990)). The main purpose of this section is to show that the indirect tax revenue effect may have an undesirable spillover effect on the foreign country's welfare if τ^* is initially high.

3.4 The Cobb-Douglas Preference

Suppose the economy is described by the Cobb-Douglas preference.

$$u = (1-\alpha)\log c^1 + \alpha \log c^2 \qquad (0 < \alpha < 1) \qquad (24)$$

and $\alpha = \alpha^*$. Note that an increase in α corresponds to a decrease in the rate of time preference, ρ. Then we have

$$s = s^* = \alpha aw,\ c^1 = (1-\alpha)aw,\ c^2 = (1+r)a\alpha w, \qquad (25)$$

$$c_{1w} = 1-\alpha,\ c_{2w} = (1+r)\alpha,\ c_{1r} = 0,\ c_{2r} = a\alpha w.$$

In this case $z = z^* = 0$ and the interest payment effect will disappear. The Cobb-Douglas preference is useful since we can explore the importance of the tax revenue effect in the simplest form. In the home country, the tax reform is beneficial because of the golden rule effect and the tax revenue effect. In the foreign country, if τ^* is high, the spillover-indirect-tax-revenue effect may be negative. Let us compare the overall size of the golden rule effect and the indirect tax revenue effect in the foreign country.

From (20) and (22) we have

$$-\frac{C^*}{D^*}\frac{dr}{d\tau} = \frac{da^*}{d\tau}, \qquad (26)$$

where $C^* = -k\tau^*(a^* + r\alpha^* a^* - 1) < 0$ and $D^* = w(\alpha r \tau^* - 1)$. Substituting this into (17-2)', we have

$$\frac{dU^*}{d\tau} = \frac{\lambda^*}{(1+r)(1+\tau^*)} \frac{\{\tau^* r\alpha(1-r\tau^*) + r(1+\tau^*) - (1+r)\tau^{*2}\}k^*}{r\alpha^*\tau^* - 1}\frac{dr}{d\tau}. \qquad (27)$$

Therefore, if $\tau^* > 1/r\alpha^*$ ($D^* > 0$), $dU^*/d\tau$ may be negative. When the initial tax rate of τ^* is high and the indirect tax revenue effect is negative, the indirect tax revenue effect may outweigh the golden rule effect. In this case, the tax reform may reduce the welfare of the foreign country.

For example, suppose $\alpha = 0.60$, one period is 25 years, and the annual rate of interest is 0.10. Then $r = (1.10)^{25} - 1 = 9.8$ and $\alpha^* r = 0.59$. In such a case if $0.59 > \tau^* > 0.17$, the overall spillover effect is negative. When τ^* is high, it is likely to have the negative spillover welfare effect on the foreign country. We have

Proposition 5: Although the tax reform produces the positive supply-side effect on capital accumulation, the spillover welfare effect may be negative because of the existence of high consumption taxes in the foreign country.

Remark: The assumptions that labor is inelastically supplied and that the consumption tax rate is time invariant imply that taxes are in a sense lump sum. Let us compare the present tax reform analysis with the lump sum tax case in Section 2. As far as the home country is concerned, there is no difference between our tax reform and the lump sum tax reform because we assume that wage income and consumption taxes are initially zero in the home country. It should, however, be stressed that the spillover welfare effects on the foreign country are not necessarily the same between the present case in Section 3 and the lump sum tax case in Section 2.

The lump sum tax case will produce the positive spillover effect on the foreign country's welfare (see (17-2)). The indirect tax revenue effect disappears because capital accumulation will not affect the tax base of lump sum taxes in the foreign country. The present case may produce the negative spillover effect because of the existence of initial consumption taxes and the cross-Laffer effect assumption (see (17-2)' and (27)).

Therefore, as far as the spillover effect on the foreign country's welfare is concerned, there is qualitatively a large difference between the present case where initial consumption tax rates may be high and the lump sum tax case where the tax base is independent of capital accumulation.

3.5 Transition

Let us briefly discuss the welfare effect of the tax reform during transition. A member of the existing old generation in the home country will suffer from the tax reform because he has to pay an extra consumption tax in the second period of his life. The existing old generation in the foreign country will also suffer from the tax reform because the rate of return on its savings will be reduced by the positive supply-side effect. As far as the existing younger generation and the future generation are concerned, the welfare effect of the tax reform will qualitatively be the same as in the steady state case. The golden rule effect, the interest payment effect, and the tax revenue effect hold in their case.

Even if there are social welfare concerns about the current generation as well as future generation, the main result of our analysis remains unchanged. It is because the tax reform will hurt both current generation and future generation if the spillover welfare effect is negative. The main purpose of this section is to show that, even in comparison to the steady state welfare, such a tax reform may not be desirable. In order to highlight the results clearly, we have concentrated on the steady state comparison. When the steady state comparison does not support such a tax reform, switching to the consumption tax will not be desirable in a real world where the transitional effects are also important.

Although an introduction of value-added taxes (VAT) is desirable in a closed economy, it may not be desirable in a two-country economy where it may hurt the foreign country which has already introduced the VAT system at high rates. By highlighting the negative spillover consequences of domestic tax reforms and by identifying the channels through which the effects of such reforms are transmitted to the rest of the world, this section provides a clear rationale for studying the international coordination of tax policies. However, additional research will be required before the profession will be able to conclude that the consumption tax system is preferred to the income tax, or vice versa. Hopefully, the analysis of this section is a step in the right direction.

4. Capital Income Taxation

4.1 International Taxation System

In a territorial system, capital income tax burdens depend on where the income is earned, but not on the owner's country of residence. Conversely, under a residence system, tax burdens depend on country of residence, not on where income earned. In the territorial system the after-tax rate of return will be equalized, while in the residence system the before-tax rate of return will be equalized.

When capital income taxation is reduced, we normally expect two effects; the supply side effect and the arbitrage effect. A reduction in the capital income tax rate in the home country will increase supply of savings in the world wide capital market. If a tax reform stimulates the world supply of savings, this is called the supply side effect. In order to attain the arbitrage condition, capital will move from the lower rate-of-return country to the higher rate-of-return country. If a tax reform alters the arbitrage condition and hence international capital movement is induced, this is called the arbitrage effect. In the territorial system, if the elasticity of substitution between capital and labor is large, the arbitrage effect will dominate the supply side effect, so that the tax reform will lead to a negative comovement of capital accumulation between two countries. In the residence system there is no arbitrage effect, so that the tax reform will affect capital accumulation in each country in the same way. We will explore these aspects by incorporating capital income taxes into the basic model.

4.2. Territorial System

4.2.1 The Model

We introduce capital income taxation into the model. We assume $\tau > 0$ and $\gamma = 0$. $\theta > 0$ is the tax rate on capital income. Income earned by an individual outside his or her nation of citizenship is potentially of interest to the tax authorities of the citizen's home and host governments. Tax treatment of capital income may follow either a residence or territorial system. Under a residence system, the total tax due is independent of where it is earned. Under a territorial system, taxes are paid to the country in which the income is earned and total taxes depend on the geographical distribution of earnings.

We first formulate the territorial system in this subsection 4.2. As the arbitrage condition we have

$$r(1-\theta) = r^*(1-\theta^*). \tag{28}$$

The net rate of return will be equalized by the arbitrage behavior.

Thus, consumption and saving equations are given as

$$c^1 = c^1(aw, q_2), \tag{3-1)'$$

$$c^2 = c^2(aw, q_2), \tag{3-2}'$$

$$s = q_2 c^2(1+\tau) = s(aw, q_2, \tau), \tag{3-3}'$$

where

$$a = \frac{1}{1+\tau}.$$

$$q_2 = \frac{1}{1+(1-\theta)r}. \tag{29}$$

The world capital market is perfect and capital moves freely across the nation border in such a way to attain the arbitrage condition (28). Thus, $z + z^* = 0$. Or, we have

$$(1+\tau)q_2c^2[aw(r), q_2] + (1+\tau^*)q_2c^{2*}[a^*w(r^*), q_2] = -w'(r) - w'(r^*). \tag{30}$$

For simplicity technology is the same; w( ) = w*( ).

The capital income tax base is given by rk (not by rs) in the territorial system. Then, tax revenue from the capital income tax is given by $\theta rk = (1+r-1/q_2)k$. Therefore, the government budget constraint in each country is given as respectively

$$\tau\{c^1[aw(r), q_2] + c^2[aw(r), q_2]\} - (1 + r - \frac{1}{q_2})w'(r) = g, \tag{31}$$

$$\tau^*\{c^{1*}[a^*w(r^*), q_2] + c^{2*}[a^*w(r^*), q_2]\} - (1 + r^* - \frac{1}{q_2})w'(r^*) = g^*. \tag{32}$$

4.2.2 Tax Reform

The system will be summarized by the three equations (30), (31), and (32). Endogenous variables are r, r*, and q_2. Fiscal variables are τ, τ^*, a, a*, g, and g*. As for the revenue-neutral tax reform which intends to reduce capital income taxation, we consider a reduction in θ with an increase in τ. Since q_2 and r* may vary due to the tax reform, the foreign country cannot maintain all the fiscal variables at the initial levels. We assume that the foreign country will change θ^* to keep the initial g* and fix the tax parameters τ^* and γ^*, because this section investigates the effects of changes in capital income taxation in a two country model[11].

For simplicity we assume that in the initial state all taxes are zero; $\tau = \tau^* = 0$ and $q_2(1+r) = 1$. Totally differentiating (30), (31), and (32) and evaluating at $\tau = \tau^* = 0$ and $q_2(1+r) = 1$, we have

$$\begin{bmatrix} \hat{A} & \hat{B} & \hat{B}^* \\ \hat{C} & \hat{D} & 0 \\ \hat{C}^* & 0 & \hat{D}^* \end{bmatrix} \begin{bmatrix} dq_2 \\ dr \\ dr^* \end{bmatrix} = -\begin{bmatrix} q_2(c^2 - c_{2w}w) \\ c^1 + c^2 \\ 0 \end{bmatrix} d\tau, \tag{33}$$

where

$$\hat{A} = (c^2 + q_2c_{22}) + (c^{2*} + q_2c^*_{22}),$$

$$\hat{B} = q_2c_{2w}w' + w'',$$

$$\hat{C} = \frac{k}{q_2^2},$$

$$\hat{D} = k,$$

and $c_{i2} = \partial c^i / \partial q_2$ and $c_{iw} = \partial c^i / \partial(aw)$ (i = 1, 2).

Both countries are identical in every respect in the initial state; $\hat{B} = \hat{B}^*$, $\hat{C} = \hat{C}^*$, and $\hat{D} = \hat{D}^*$. Then, $\hat{C} = \hat{C}^* > 0$ and $\hat{D} = \hat{D}^* > 0$. $\hat{C} > 0$ means that an increase in τ will raise tax revenues at the initial level of r. Similarly, $\hat{D} > 0$ means that a direct impact of an increase in r will raise capital income tax revenues. $\hat{B} > 0$ is a local stability condition in the closed economy. See chapter 2. When the elasticity of substitution between capital and labor is large, $\hat{B}$ is likely to be positive. The sign of $\hat{A}$ is dependent on the saving behavior with respect to the net rate of return, s_r. Since s_r is non-negative, $\hat{A}$ is non-positive. Under these assumptions, the determinant of the matrix of LHD of (33), $\hat{\Delta} = \hat{A}\hat{D}^2 - 2\hat{B}\hat{C}\hat{D}$, is negative.[12]

4.2.3 Comparative Static Effects

From (33), we have

$$\frac{dq_2}{d\tau} = -\frac{1}{\hat{\Delta}}\hat{D}^* \{q_2(c^2 - c_{2w}w)\hat{D} - (c^1 + c^2)\hat{B}\}, \tag{34}$$

$$\frac{dr}{d\tau} = -\frac{1}{\hat{\Delta}}\{\hat{D}^*[\hat{A}(c^1 + c^2) - \hat{C}(c^2 - c_{2w}w)q_2] - \hat{B}^*\hat{C}^*(c^1 + c^2)\}, \tag{35}$$

$$\frac{dr^*}{d\tau} = -\frac{1}{\hat{\Delta}}\hat{C}^* \{\hat{B}(c^1 + c^2) - q_2(c^2 - c_{2w}w)\hat{D}\}. \tag{36}$$

$c^2 - c_{2w}w = c^2(1-\varepsilon)$, where ε is the income elasticity of c^2, is normally close to zero. Thus, it is likely to have $dr/d\tau < 0$. If the elasticity of substitution between capital and labor is large and hence $\hat{B}$ is large, $q_2c^2(1-\varepsilon)\hat{D} - (c^1 + c^2)\hat{B}$ is negative. In this case, $dq_2/d\tau < 0$ and $dr^*/d\tau > 0$.

Proposition 6: Under the territorial system a revenue-neutral increase in τ with a reduction in θ will stimulate capital accumulation in the home country.

As shown in chapter 3, it is well known that relative to a labor income tax, consumption taxation increases savings by postponing life cycle tax payments. This is the tax timing effect. On the other hand, since taxes are now paid somewhat earlier in life, the present value of life cycle taxes increases (due to a reverse tax postponement effect), causing a decrease in savings. If s_r (> 0) is large, the supply side effect is large due to the substitution effect (the first term of the left-hand side of (35)). However, even if $s_r = 0$, (35) is negative when $c^2 - c_{2w}w > 0$. This means that the reverse tax postponement effect is dominated by the tax timing effect. Total supply of capital increases in the international financial market. This is the supply side effect.

An increase in τ has another effect. In the home country an increase in τ will produce consumption tax revenues, which makes it possible to reduce a tax rate on capital income, θ. The net rate of return increases in the home country at the initial allocation of capital between countries. In order to attain the arbitrage condition, capital will move from the lower rate-of-return country to the higher rate-of-return country. This effect is the arbitrage effect.

While the positive supply side effect will stimulate capital accumulation in both countries, the arbitrage effect will stimulate capital accumulation in the home country but will depress capital accumulation in the foreign country, offsetting the supply side effect. Thus, we have

Proposition 7: If the elasticity between capital and labor is large, the arbitrage effect dominates under the territorial system, so that an increase in τ with a reduction in θ will depress capital accumulation in the foreign country.

4.3. Residence System

Under the residence system, tax burdens depend on country of residence, not where income earned. Thus, as the arbitrage condition we have

$$r = r^*. \tag{37}$$

The before-tax rates of return are equalized in this system. Therefore, in place of (30), we have

$$(1+\tau)q_2c^2[aw(r),q_2] + (1+\tau^*)q_2^*c^{2*}[a^*w(r),q_2^*] = -2w'(r). \tag{38}$$

Note that in this system q_2 (or the after tax rate of return) is not equalized across the countries.

The capital income tax base is now rs (not rk). Then, tax revenue from the capital income tax is given by $\theta rs = [q_2(1+r)-1]c^2$. Therefore, in place of (31) and (32), we have respectively

$$\tau\{c^1[aw(r),q_2] + c^2[aw(r),q_2]\} + \{q_2(1+r)-1\}c^2[aw(r),q_2] = g, \tag{39}$$

$$\tau^*\{c^{1*}[a^*w(r),q_2^*] + c^{2*}[a^*w(r),q_2^*]\} + \{q_2^*(1+r)-1\}c^{2*}[a^*w(r),q_2^*] = g^* \tag{40}$$

The system will be summarized by the three equations (38), (39), and (40). As in Section 4.2, we consider a revenue-neutral tax reform which intends to reduce θ; a reduction in θ with an increase in τ. Totally differentiating the system (38)(39) and (40), we have

$$\begin{bmatrix} E & F & F^* \\ G & H & 0 \\ G^* & 0 & H^* \end{bmatrix}\begin{bmatrix} dr \\ dq_2 \\ dq_2^* \end{bmatrix} = -\begin{bmatrix} q_2c^2(1-\varepsilon) \\ c^1+c^2 \\ 0 \end{bmatrix} d\tau, \tag{41}$$

where

$$E = q_2c_{2w}w' + q_2^*c_{2w}^*w' + 2w'',$$
$$F = (c^2 + q_2c_{22}),$$
$$G = q_2c^2,$$
$$H = (1+r)c^2.$$

As before, $E > 0$, $F = F^* \leq 0$, $G = G^* > 0$, and $H = H^* > 0$. The determinant of LHD of (41), $\Delta^* = H^*(EH-FG) - G^*HF^*$, is positive. From (41) we have

$$\frac{dr}{d\tau} = -\frac{H^*}{\Delta^*}\{q_2c^2(1-\varepsilon)H - (c^1+c^2)F\}, \tag{42}$$

$$\frac{dq_2}{d\tau} = -\frac{1}{\Delta^*}\{H^*[E(c^1+c^2) - Gc^2(1-\varepsilon)q_2] - F^*G^*(c^1+c^2)\}. \tag{43}$$

$$\frac{dq_2^*}{d\tau} = -\frac{1}{\Delta^*}G^*\{F(c^1+c^2) - q_2c^2(1-\varepsilon)H\}. \tag{44}$$

From (42) and (44), an increase in τ will reduce r and increase q_2^*. If the elasticity of substitution between capital and labor is large, (i.e., if E is large), $dq_2/d\tau$ is negative.

As in subsection 4.2, a revenue-neutral increase in τ will raise saving in the home country, which will produce the positive supply side effect on capital accumulation. In the residence system r is always equal to r*, independent of θ. Thus, changes in θ will not produce the arbitrage effect.

Proposition 8: Under the residence system the revenue-neutral increase in τ with a reduction in θ will stimulate capital accumulation in each country due to the positive supply side effect.

4.4. Remarks

We have shown that the effect on capital accumulation is very different between two tax systems. Our results are summarized in Table 6.2. In the residence system a tax reform in the home country will affect capital accumulation in each country in the same way. On the other hand, in the territorial system conversion from capital income to consumption taxes may lead to a negative comovement of capital accumulation between two countries. In the residence system, there exists the supply side effect only, so that an increase in τ will stimulate the world-wide capital accumulation by raising the home country's saving. However, in the territorial system, if the arbitrage effect is dominant, a reduction in θ will hurt the capital accumulation in the foreign country by inducing a capital movement from the foreign country to the home country.

We can analyze the effects of conversion from capital to labor income taxes in a similar way. The difference between labor income taxes, γ, and consumption taxes, τ, arises from an implicit change in government debt, since as explained in Chapter 3 the only difference between them is in the timing of tax payments. Conversion from consumption to labor income taxes would reduce savings. Thus, the supply side effect of an increase in γ would be weaker than that of an increase in τ. If the elasticity of substitution between capital and labor is large, the arbitrage effect is likely dominant in the case of an increase in γ with a reduction in θ. In such a case an increase in γ in the territorial system will stimulate capital accumulation in the home country by inducing a capital movement from the foreign country to the home country. In the foreign country the arbitrage effect always dominates the supply side effect.

We can also investigate the mixed system of territorial and residence tax principles. Suppose the home country imposes the territorial system and the foreign country imposes the residence system. If tax payments to the home country are deductible as costs in calculating taxable income in foreign country, after-tax rate of return from located in the home country will be $r(1-\theta)(1-\theta^*)$. Thus, the arbitrage condition is now $(1-\theta)(1-\theta^*)r = (1-\theta^*)r^*$ or $(1-\theta)r = r^*$. In this case, a reduction in θ would produce the arbitrage effect. Comparative static effects of conversion from τ to θ would qualitatively be the same as in subsection 4.2. On the other hand, if the home country imposes the residence system and the foreign country imposes the territorial system, the arbitrage condition is $r = r^*(1-\theta^*)$. Here, a reduction in θ would not produce the arbitrage effect. Comparative static effects of conversion from τ to θ would qualitatively be the same as in subsection 4.3. For more recent study on the mixed system see Frenkel, Razin, and Sadka (1991).

5. Optimal Tax And Spending Policy

5.1 Territorial System

5.1.1 Model

In this section we intend to explore the normative aspect of taxation in an open economy. We extend the optimal taxation problem of Chapter 3 into the two country model with identical technology and preferences. We consider the optimal combination of labor income taxes and capital income taxes by allowing for an endogenous supply of labor. The government does not impose consumption taxes.

First of all, let us investigate the case where both countries impose the territorial system. The world economy in the territorial system may be summarized by the following equations.

$$E(q_t, u_t) = 0 \tag{45}$$

$$E(q_t^*, u_t^*) = 0 \tag{46}$$

$$q_{2t+1}E_2(q_t, u_t) + q_{2t+1}E_2(q_t^*, u_t^*) = \tag{47}$$
$$w'(r_{t+1})E_3(q_{t+1}, u_{t+1}) + w'(r_{t+1}^*)E_3(q_{t+1}^*, u_{t+1}^*)$$

$$E_1(q_t, u_t) + [w(r_t) - (1 + r_t - \frac{1}{q_{2t}})w'(r_t)]E_3(q_t, u_t)$$
$$+ g_t + q_{2t+1}E_2(q_t, u_t) = 0 \tag{48}$$

$$E_1(q_t^*, u_t^*) + [w(r_t^*) - (1 + r_t^* - \frac{1}{q_{2t}})w'(r_t^*)]E_3(q_t^*, u_t^*)$$
$$+ g_t^* + q_{2t+1}E_2(q_t^*, u_t^*) = 0 \tag{49}$$

where

$$q_{2t} = q_{2t}^* \tag{50-1}$$

$$q_{2t+1} = \frac{1}{1 + r_{t+1}(1-\theta)} \tag{50-2}$$

$$q_{3t} = (1-\gamma)w_t \tag{50-3}$$

(45) and (46) describe the individual's optimizing behavior using the expenditure function for both countries. (47) means the world-wide capital accumulation equation, which corresponds to (30). (48) and (49), are respectively the equilibrium conditions of a good market for both countries. The feasibility condition (48) is different from the counterpart in the closed economy in that

$$q_{2t+1}E_{2t} - w'_{t+1}E_{3t+1} - \frac{(q_{2t}E_{2t-1} - w'_t E_{3t})}{q_{2t}}$$

means net exports of the home country to the foreign country. Namely, net export in period t, EX_t, is given as

$$EX_t = z_{t+1} - \frac{z_t}{q_{2t}} \tag{51}$$

Recall that z_t is net capital exports in period t. See equation (1). Note that the current account surplus, z_{t+1}-z_t, is equal to net exports, EX_t, plus net return on capital exports, $(1-\theta^*)r^*_t z_t$.

5.1.2 Nash Equilibrium

The home government may choose $\{q_{2t+1}\}$ and $\{q_{3t}\}$ so as to maximize its intertemporal social welfare function W by assuming that the foreign country's fiscal policy is fixed. Namely, the home government regards $\{q_{3t}^*\}$ as fixed by the foreign government.[13] Recall that the home government can control q_{2t}^* in the territorial system (see (50-1)).

The home government's objective function is

$$W = \sum_{t=0}^{\infty} \beta^t u_t \tag{52}$$

Differentiating the associated Lagrange function with respect to q_{2t+1} and q_{3t} as done in chapter 3 section 2, it is tedious but straightforward to derive the following first-order conditions in the steady state.

$$2[\beta e_2 \sigma_{22} + e_3 \sigma_{23} - \frac{\rho - r}{1+\rho}] = \frac{\lambda_1}{\lambda_2} \tag{53-1}$$

$$\beta e_2 \sigma_{32} + e_3 \sigma_{33} = \frac{\lambda_1}{\lambda_2} \tag{53-2}$$

where e_i is the effective tax rate (t_i/q_i) and σ_{ij} is the compensated elasticity ($q_j E_{ij}/E_i$). $t_2=q_2(1+r)-1$ and $t_3=q_3-w$. $\beta = 1/(1+\rho)$. λ_1 corresponds the marginal benefit of lump sum transfer to each young individual financed by distortionary taxes. λ_2 corresponds to the marginal benefit of a decrease in government revenues, which is positive. Note that in a symmetric Nash equilibrium $\sigma_{ij} = \sigma_{ij}^*$ and $e_i = e_i^*$.

From (53-1,2) we have

Proposition 9. The optimal tax rule in the territorial system is given as

$$e_2 = \frac{1}{H}\{\lambda(\frac{\sigma_{33}}{2} - \sigma_{23}) + \sigma_{33}\frac{\rho - r}{1+\rho}\} \tag{54-1}$$

$$e_3 = \frac{\beta}{H}\{\lambda(\sigma_{22} - \frac{\sigma_{32}}{2}) - \sigma_{32}\frac{\rho - r}{1+\rho}\} \tag{54-2}$$

where $\lambda = \lambda_1 / \lambda_2$ and $H = \beta(\sigma_{22}\sigma_{33} - \sigma_{32}\sigma_{23})<0$.

Remark: It is useful to compare the optimal tax rule (54-1,2) with the optimal tax rule, the mixed Ramsey-Golden rule (29-1,2) in Chapter 3. As explained in Chapter 3 Section 2, the first term of (54-1) corresponds to the static efficiency point. Compared with (29-1) in Chapter 3, σ_{33} here is divided by 2, the number of countries. This means that a low value of θ is more likely to be desirable if λ < 0. In the territorial system, a reduction in θ will induce capital inflow and hence raise tax revenues from capital income. This is the tax competition effect. The second term of (54-1) corresponds to the dynamic optimal efficiency point, which implications are the same as in Chapter 3 Section 2.

5.2 Residence System

5.2.1 The Model

In the residence system the model will be summarized by

$$E(q_t, u_t) = 0 \tag{45}$$

$$E(q_t^*, u_t^*) = 0 \tag{46}$$

$$\begin{aligned} &q_{2t+1}E_2(q_t, u_t) + q_{2t+1}^* E_2(q_t^*, u_t^*) = \\ &\quad w'(r_{t+1})E_3(q_{t+1}, u_{t+1}) + w'(r_{t+1})E_3(q_{t+1}^*, u_{t+1}^*) \end{aligned} \tag{47'}$$

$$E_1(q_t,u_t)+E_2(q_{t-1},u_{t-1})+[w(r_t)-(1+r_t)w'(r_t)]E_3(q_t,u_t)$$
$$+g_t+q_{2t+1}E_2(q_t,u_t)+\frac{q_{2t}E_2(q_{t-1},u_{t-1})-w'(r_t)E_3(q_t,u_t)}{q^*_{2t}}=0 \quad (48)'$$
$$E_1(q^*_t,u^*_t)+E_2(q^*_{t-1},u^*_{t-1})+[w(r_t)-(1+r_t)w'(r_t)]E_3(q^*_t,u^*_t)$$
$$+g^*_t+q^*_{2t+1}E_2(q^*_t,u^*_t)+\frac{q^*_{2t}E_2(q^*_{t-1},u^*_{t-1})-w'(r_t)E_3(q^*_t,u^*_t)}{q_{2t}}=0 \quad (49)'$$

This model is almost the same as in the territorial system. (47)' means the world-wide capital accumulation equation. (48)' and (49)' are the feasibility conditions for both countries. Recall that in this system r=r* but q_2 is not necessarily equal to q_2*. Thus, (51) is rewritten as

$$EX_t=z_{t+1}-\frac{z_t}{q^*_{2t}} \quad (51)'$$

5.2.2 Nash Equilibrium

Conducting the similar differentiation of the associated Lagrange function with respect to q_{2t+1} and q_{3t} as in subsection 5.1.2, we have in the steady state

$$\beta e_2\sigma_{22}+e_3\sigma_{23}-\frac{\rho-r}{1+\rho}=\lambda \quad (53\text{-}1)'$$
$$\beta e_2\sigma_{32}+e_3\sigma_{33}=\lambda \quad (53\text{-}2)$$

From (53-1)' and (53-2) we may derive (54-1)' and (54-2)' at the symmetric Nash solution:

Proposition 10: The optimal tax rule in the residence system is given as the mixed Ramsey-Golden rule.

$$e_2=\frac{1}{H}\{\lambda(\sigma_{33}-\sigma_{23})+\sigma_{33}\frac{\rho-r}{1+\rho}\} \quad (54\text{-}1)'$$
$$e_3=\frac{\beta}{H}\{\lambda(\sigma_{22}-\sigma_{32})-\sigma_{32}\frac{\rho-r}{1+\rho}\} \quad (54\text{-}2)'$$

Remark 1: Let us compare (54) with (54)'. In the residence system, the static optimal efficiency effect (the first term of the optimal tax rule) is the same as in the closed economy in Chapter 3 Section 2.4.2. This is because in the residence system a change in θ will not induce international capital movement. A decrease in θ cannot directly induce capital inflow into the home country. The second term of (54-1)' is the same as in (54-1). This dynamic efficiency effect works, irrespective of the international tax principle. The optimal tax rule (54)' in the residence system is the same as the mixed Ramsey-Golden rule on the closed economy (29) in Chapter 3 Section 2.4.2. This means that when both countries are identical, the noncooperative Nash solution in the residence system is the same as the cooperative solution. In this sense, the residence system is more desirable than the territorial system.

Remark 2: In the territorial system, a reduction in capital income tax rates will induce capital inflow and hence raises tax revenues from capital income. This is the tax competition effect. This may lead to lower capital income taxation in the non-cooperative Nash solution than in the cooperative solution. On the other hand, the tax competition effect does not work in the residence system. Frenkel, Razin, and Sadka

(1991) showed that the residence system is desirable for the small country. Bucovetsky and Wilson (1991) presented a model where a wage tax and both source- and residence-based capital taxes are available. They showed that the absence of residence-based taxes on capital income, not taxes on wage income, is responsible for the under-provision of public goods.

5.3 Further Study

Capital exporting countries often allow taxes paid to foreign governments to be credited against domestic tax liabilities by firms investing abroad rather than simply deducted as costs from taxable income. Bond and Samuelson (1989) examined the welfare of a capital exporting or source country and a capital importing or host country under tax credit and tax deduction systems. They showed that both countries receive higher welfare under a deduction rather than a credits scheme at the non-cooperative Nash solution. Mutti and Grubert (1985) developed a simulation model of the United States and the rest of the world to demonstrate how international capital mobility alters the incidence and capital formation incentives of taxes on capital income. Tabellini (1990) investigated the desirability of international fiscal policy coordination in the presence of a domestic political distortion.

There are several papers which investigate the optimal provision of public goods under cooperative and non-cooperative tax policies in a world of high capital mobility.

Ghosh (1991) discussed the optimal provision of public goods when governments do not have non-distortionary taxes at their disposal. He showed that, if tax policies are chosen non-cooperatively, then relative to the coordinated regime, there will be an appropriate provision of public goods. He also showed that a non-cooperative equilibrium may result in a too low or too high tax rate for both economies.

Krelove (1992) showed that the competitive equilibria in the territorial system are not constrained efficient in general: there is another set of distortionary taxes and associated public expenditures for which all individuals in the economy are better off. He pointed out that the source of the failure can be interpreted as a missing market, and the form of the best decentralized remedy was derived.

Sorensen (1990) showed that, if the marginal source of public finance is a capital income tax based on the territorial principle, countries can almost certainly make a long run gain by undertaking a coordinated increase in their level of taxation and public expenditure.

Chari and Kehoe (1990) examined the limiting behavior of cooperative and non-cooperative fiscal policies as countries' market power goes to zero. They showed that if countries raise revenues through distortionary taxes, there can be gains to coordination even when a single country's policy cannot affect world prices. All of these papers suggest that strategic aspects of public finance in the lump sum tax regime are very different from these of public finance in the non-lump sum tax regime. It would be useful to investigate strategic aspects of fiscal policy when distortionary taxes are imposed.

Table 6.1 Taxation Of Consumption

Country	VAT standard tax rate
Belgium	19
Denmark	22
France	20.6
Germany	15
Greece	16
Ireland	23
Italy	19
Luxembourg	12
Netherlands	18.5
Portugal	17
Spain	12
UK	17.5

Table 6.2: Comparative Static Results

	Territorial System	Residence System
r	-	-
r*	(+)*	-
q_2	(-)*	(-)*
q_2*	(-)*	+
SS	+	+
AB	$k^* \rightarrow k$	absent

Notes
* means "if the elasticity of substitution between capital and labor is large". SS means the supply side effect. AB means the arbitrage effect.

[1]. Frenkel and Razin (1989) showed that the domestic and international consequences of tax reforms characterized by revenue-neutral conversions from income to consumption taxes depend critically on the trade-balance position. They highlighted the significance of open-economy considerations in the analysis of tax reforms. However, they did not explore the formal analysis of alternative capital taxation rules. See also Gorden (1986) and Gorden and Varian (1989).

[2]. Even if population growth is positive, the qualitative results would be the same so long as r is greater than n, the rate of population growth.

[3]. Ihori (1987b) extended a one-commodity version of the open economy model by grafting the two-commodity structure of international trade. This allows for potential gains from trade through specialization in production. Matsuyama (1988) examined the effects of terms-of-trade changes on the external adjustment of a small open economy where each consumer has a life-cycle saving function.

[4]. The effect on saving due to changes in the timing of tax payments holding the present value of taxes fixed is called the tax timing effect. See Chapter 3.

[5]. Hamada (1986) studied the welfare effect of a debt-financed increase in government expenditures in a similar Buiter model and showed that when the home country is a debtor country, capital accumulation is likely to have a negative spillover effect on the foreign country's welfare.

[6]. Batina and Ihori (1991) showed that incorporating endogenous labor supply into the model may serve to reverse the usual effect of the consumption tax. Labor supply may increase with the tax and thus lower the capital intensity of production thus raising the interest rate. They further showed that if bequests are included in the model and bequests are taxed at the consumption tax rate, capital accumulation for the purpose of making a bequest may decrease with the tax. This will also raise the world interest rate.

[7]. The move towards value-added taxes (VAT) is now wide spread and has been stimulated especially by the decision in the United Kingdom in 1979 to nearly double its VAT rate. Such a move has subsequently been put on the tax-reform agenda of other countries, including Australia, Canada, Greece, Japan, Korea, New Zealand, Portugal, Spain, and Turkey.

[8]. The assumption that the government budget is balanced at each point in time may be strong. In fact, all the real effects of the tax experiment follow from intergenerational distributional effects. Hence, the government could neutralize these effects by using debt policy. However, in order to present the results in the simplest way and in their strongest form, we assume (7) and (18).

[9]. The saddle-point stability condition will be reduced to

$$S_{r0} + S_{r1} + B^*(G^*_{r0} + G^*_{r1}) > 0$$

Then, under the above saddle-point stability condition the economy will move to the long-run equilibrium monotonically.

[10]. The direct tax revenue effect is due to the assumption of efficient growth. Since r is greater than the rate of population growth, the present value of life cycle taxes decreases when taxes are paid somewhat later in life, holding the government budget fixed. The effect due to changes in the present value of taxes is called the tax postponement effect.

[11]. In this section we formulate a reduction in θ with an increase in τ in the following way. The home country exogenously raises consumption taxation and maintains other fiscal policy variables γ and g. θ is adjusted (normally reduced) to keep the

government budget constraint. We can consider an alternative formulation where θ is exogenously reduced and τ is endogenously adjusted at given γ and g. The qualitative results would be the same.

[12]. $\hat{\Delta} < 0$ corresponds to the stability condition of the present model, although we do not explicitly analyze the stability property in the text.

[13]. For simplicity, we examine the static Nash solution as in Razin and Yuen (1993). Ha and Sibert (1992) examined the dynamic Nash solution as well as the static Nash solution in a similar setting. They showed that qualitative results are the same in both cases: Nash equilibrium corporate taxes are zero for identical countries and strictly positive (negative) for countries which discount the future more than the rest of the world.

Chapter 7

Money

1. Introduction

This chapter introduces money into the basic model. There are two views of money. In the "bubbly" view, money is a pure store of value. This view implies that price of money grows at the real rate of interest and that money is held entirely for speculation. In the "fundamentalist" view money is held to finance transactions. Only fundamentalist view can explain the rate of return dominance of other assets over money. Samuelson (1958) first introduced money into the overlapping generations model. Section 2 summarizes his framework without capital accumulation[1] and explains the bubbly view of money. We also introduce money into the Diamond model with capital accumulation and explain the fundamentalist view of money by examining dynamic properties of the economy as well as its steady state nature.

Then, we explore in Section 3 the concepts of inflationary taxes and welfare costs of inflation. The revenue yield from increasing inflation has received much attention in the literature. In general the optimal rate of inflation will be determined implicitly by a combination of preferences and technology. It is now recognized how difficult it is to say anything explicit about the optimal rate of inflation. An alternative approach is to explore the welfare cost of inflation. Section 3 examines welfare costs by starting with the expenditure function and comparing the amount by which a consumer must be compensated to restore him to a reference utility level to the gain in revenue.

Section 4 considers the welfare implications of indexing capital income taxation in an inflationary economy. In a steady state economy in which individuals anticipate inflation correctly, it is by no means clear whether the inflation-induced change in the real net rate of return on capital is always undesirable. One cannot take the desirability of the foregoing neutrality for granted. This issue should be subjected to prior economic analysis. Based on Ihori (1984a), this section intends to explore welfare implications of indexing capital income taxation. It will be shown that it is generally desirable to relate the net real rate of return to the rate of inflation. Using the specification of Feldstein, Green, and Sheshinski's corporate tax system (1978), we provide an example where the optimal indexation of the tax system is partial and increasing with the rate of inflation.

2. Overlapping Generations Models With Money

2.1 Samuelson's Model

2.1.1 Analytical Framework

Samuelson (1958) was the first to introduce money into the overlapping generations model with perishable goods. It is useful to summarize his framework. Population grows at rate n (> 0).

$$N_t = (1+n)^t . \tag{1}$$

Each individual is endowed with one unit of a consumption good when young but receives no endowment when old. Since the good received by the young is perishable, there is no capital accumulation.

The utility of an individual born at time t is given as

$$u_t = u(c_t^1, c_{t+1}^2). \tag{2}$$

Note that money is not introduced into the utility function.

The consumption possibilities in period t are given as

$$c_t^1 + \frac{1}{1+n} c_t^2 = 1. \tag{3}$$

If all the goods are given to the young, the young can consume only one unit each at point A in Figure 7.1. If, instead, all the goods were given to the old, they could each consume 1+n units because there are only 1/(1+n) of them per each young person at point B. Equation (3) is drawn as line AB in Figure 7.1. Point E is the optimal point where utility of generation t is maximized.

The young would like to exchange goods this period against goods next period to attain point E. But they can only trade this period with the old who will not be there next period and will therefore not be able to deliver goods next period. Thus, no trade can take place, and the decentralized outcome is given by point A with individuals consuming all of their endowment when young and consuming nothing when old. The decentralized equilibrium is clearly not Pareto optimal.[2]

2.1.2 Introduction Of Money

Suppose that at time zero the government gives to the old completely divisible pieces of paper called money; H. Suppose also that the old and every generation thereafter believe that they will be able to exchange money for goods, at price p_t in period t. p_t is referred to as the price level. If this is the case, the maximization problem of an individual born at t is given by maximizing $u(c^1_t, c^2_{t+1})$ subject to the first-period and second-period budget constraints

$$p_t = p_t c_t^1 + M_{t+1}, \tag{4}$$

$$p_{t+1} c_{t+1}^2 = M_{t+1}, \tag{5}$$

where M_{t+1} is the individual's (per capita) demand for money at the end of period t (at the beginning of period t+1).

From (4) and (5) the lifetime budget constraint is given as

$$c_t^1 + \frac{p_{t+1}}{p_t} c_{t+1}^2 = 1. \tag{6}$$

From the utility maximizing behavior c^1_t and c^2_{t+1} are given by a function of p_{t+1}/p_t. Therefore, real money demand $M_{t+1}/p_t = 1-c^1_t$ is also given by a function of p_{t+1}/p_t. In other words, the demand function for money is given as

$$\frac{M_{t+1}}{p_t} = L\left(\frac{p_{t+1}}{p_t}\right). \tag{7}$$

The demand for money is just a saving function. Let us define the rate of inflation by $\pi_{t+1} = p_{t+1}/p_t - 1$. The rate of return on money is given by $1/(\pi_{t+1}+1) = p_t/p_{t+1}$.

The old generation supplies inelastically the money it has saved. This must be equal to H, an exogenously given supply of money. The money market equilibrium condition is therefore given by

$N_t M_{t+1} = H$.

Or

$$(1+n)^t L(\pi_t + 1) p_t = H. \tag{8}$$

In the steady state π must be constant so that, from (8), $(1+n)^t p_t$ is fixed. We have $1+\pi = 1/(1+n)$, from which we can solve for $\pi = -n/(1+n)$. The rate of deflation must be equal to the rate of growth of population: prices must decrease at a rate such that the real money supply grows at the same rate as the aggregate demand for money, which is itself growing at the rate of population growth, n. Thus, we have:

Proposition 1: In the steady state if money supply is constant, the rate of deflation is given by the rate of population growth (π = -n/(1+n)).

With a rate of deflation equal to n/(1+n) (or $p_{t+1}/p_t = 1/(1+n)$), the budget line for the individual coincides with the frontier AB in Figure 7.1 and individuals will choose a point given by E. Money can have positive value. If money is valued, the introduction of money allows for new trades. Thus, we have derived the following result.

Proposition 2: Assuming that the economy reaches steady state, the introduction of money can lead to a Pareto optimal allocation of resources across generations.

2.2 Money In An Economy With Durable Goods

Suppose now that durable goods (capital) are available and that the rate of return from holding capital is fixed and given by r. In this case, trade between the younger generation and the older generation is possible. Individuals can now save by holding either capital or money. As in section 2.1, we still have proposition 1. Since two assets (money and capital) are available, individuals would compare the rate of return of holding money $p_t/p_{t+1} - 1 = n$ and the rate of return of holding capital r.

If r is less than n, storage is not too productive. The individual would prefer money to capital. The individual, in the absence of money, can choose in Figure 7.2 any point on AG that has slope -(1+r). But the social possibility frontier is still given by AB, with slope -(1+n), which lies above AG. The barter economy equilibrium is still not a Pareto optimum.

If r is larger than n, the barter equilibrium is a Pareto optimum. The individual would prefer capital to money. There cannot be a monetary equilibrium with a constant money stock. Because the rate of return on money is less than the return on storage, nobody wants to hold money; thus money is not valued.

In summary we have:

Proposition 3: If the barter equilibrium is not a Pareto optimum, there exists a monetary equilibrium that leads to a Pareto optimum; if the barter equilibrium is already a Pareto optimum, there cannot be a monetary equilibrium.

There is a close relation between this result and that of dynamic efficiency discussed in chapter 2. If the economy is dynamically inefficient (if r is less than n), the introduction of money can make everybody better off. This is not the case if the economy is dynamically efficient. Money provides a way in which transfers of resources, if they are Pareto improving, can be achieved voluntarily rather than through government programs.

2.3 Money In The Diamond Model

2.3.1 Analytical Framework

We now introduce money into the Diamond model developed in chapter 2. The rate of return of holding capital r is now endogenous. The initial endowment in each period is also endogenous and there is capital accumulation. We will examine dynamic properties of the economy as well as its steady state nature.

If individuals hold capital at time t, they earn a gross rate of return of $1+f'(k_{t+1})$. If money sells at price $1/p_t$ in terms of goods, the gross rate of return on money is equal to p_t/p_{t+1}. Arbitrage between the two assets implies that

$$1+f'(k_{t+1})=\frac{p_t}{p_{t+1}}. \tag{9}$$

Let m_t be the per capita real value of money held by generation t-1 in period t so that

$$m_t=\frac{H}{p_t N_{t-1}}. \tag{10}$$

Substituting (10) into (9), we have

$$m_{t+1}=\frac{m_t[1+f'(k_{t+1})]}{1+n}.$$

Or, the above equation can be rewritten as

$$m^*_{t+1}=\frac{m^*_t[1+f'(k_t)]}{1+n}, \tag{10)'}$$

where $m^*_t \equiv (1+\pi_{t+1})m_{t+1}$ is per capita first-period money holdings of generation t. Real money will grow in per capita terms if $f'(k)$ exceeds n.

In real terms the individual's budget constraints are given as

$$c^1_t=w_t-s_t-(1+\pi_{t+1})m_{t+1}, \tag{11-1}$$

$$c^2_{t+1}=(1+r_{t+1})s_t+m_{t+1}. \tag{11-2}$$

where w is the wage rate and s is savings in the form of capital. Arbitrage condition (9) implies that money and capital are perfect substitutes and individuals are indifferent between holding capital and holding money. The capital market equilibrium condition in the presence of money in per capita terms is hence given as

$$k_{t+1}=\frac{A(w_t,f'(k_{t+1}))-m^*_t}{1+n}, \tag{12}$$

where A( ) is the total saving function: A = s+m*.

2.3.2 Phase Diagram

Considering the factor price frontier w = w(r) and w'(r) = -k, equations (10)' and (12) give the dynamics of the system in r and m*. Rewrite (12) as

$$r_{t+1}-r_t=\Gamma(r_t,m^*_t). \tag{13}$$

In Figure 7.3 we draw the locus where r_{t+1}-r_t equals zero as rr. Since we have

$$\frac{dm^*}{dr}=(1+n)w''+A_w w'+A_r>0,$$

from the stability condition in the Diamond moneyless economy, curve rr is upward sloping.

On the other hand, (10)' gives

$$m^*_{t+1}-m^*_t=\frac{m^*_t(r_t-n)}{1+n}. \tag{14}$$

We draw the locus where m^*_{t+1}-m^*_t =0 as MM. Curve MM is given by r=n and crosses curve rr at the steady state point E.

The dynamics of r and m* will be as follows. If, starting on a point on Γ (r_t, m^*_t) = 0, we increase m^*_t, savings in the form of capital decrease, and capital accumulation decreases. The capital stock is therefore decreasing and hence r is increasing at all points above rr. Starting from a point on MM, if we increase r, the marginal product of capital increases and the rate at which the real value of money is required increases. The real money per capita is therefore increasing at all points to the right of MM and decreasing at all points to the left. The long run equilibrium with a positively valued money is saddle point stable if it exists. The Diamond moneyless equilibrium point F is stable.

In Figure 7.3 the economy has initial capital $k_0 = -w'(r_0)$. Let us look at possible trajectories. Consider first a point above the stable arm, say, C. At that point the real value of money is large and increasing. As it increases, capital accumulation decreases, until eventually the capital stock starts decreasing. All along, the interest rate increases, making the value of money grow even faster. At some point the real money becomes so large that capital decumulation exceeds the existing capital stock. Of course, this is impossible, and this rules out any paths above the stable arm.[3]

Consider now a point below the stable arm, say, D. The real value of money per capita eventually decreases. Asymptotically, the real value of money per capita becomes so small that the economy converges to the Diamond equilibrium.

If the rate of interest at F, r_F, is greater than n, the real value of money eventually starts increasing and the capital stock eventually starts decreasing, independent of the initial state. As stated before, at some point the real money becomes so large that the economy goes bankrupt. This means that if the economy without money is dynamically efficient ($r_F > n$), money cannot exist.

The case where the real value of money is just such that the economy is on the saddle point path is called the knife-edge case. This case is interesting. In this case the real value of money is just such as to make the interest rate asymptotically equal to n. Thus not only does the real value of money remain large compared to the economy, but money solves the dynamic inefficiency problem by driving the steady state interest rate to the level n. Consequently, We have shown:

Proposition 4: Money cannot exist if the economy is already dynamically efficient without money. If the economy is dynamically inefficient, money can exist. In the knife-edge case money solves the dynamic inefficiency problem by driving the steady state interest rate to the level of population growth rate.

Tirole (1985) employed an overlapping generations model with capital accumulation and various types of rents, and gave necessary and sufficient conditions for the existence of an aggregate bubble. He showed that if the economy is dynamically efficient, the equilibrium is bubbleless and the interest rate converges to point F. If the equilibrium is dynamically inefficient, bubble can exist. There exists a unique bubbly equilibrium. It is asymptotically bubbly and the interest rate converges to n. There is a close relation between his results on the existence of a bubble and those on dynamic properties in this section.

2.4 Money In The Utility Function

So far money is valued in this simple overlapping generations model only when it is not dominated in the rate of return by any other asset. Tirole (1985) called it the "bubbly view" of money. Namely, money is a pure store of value. It does not serve any transactional purpose at least in the long run. This view implies that the price of money (bubble) grows at the real rate of interest, and that money is held entirely for speculation. Recall the arbitrage condition between money and capital, equation (9). Thus, if storage yields a higher return than money, money is no longer valued. Existence of a monetary equilibrium is tenuous: money disappears when the rate of inflation is too high.

But, in practice, money is dominated in the rate of return by many assets and continues to be used even during the most extreme hyperinflations. This suggests that the role of money should be studied in models in which money is indeed dominated in the rate of return. And many of the dramatic results obtained in this section are likely to disappear. Money is held to finance transactions. To this purpose, money must be a store of value. Tirole called it the "fundamentalist" view of money.

Only the fundamentalist view can explain the rate of return dominance of money by other assets. There are two approaches in which a monetary equilibrium is achieved even though money is dominated in the rate of return by other assets. First, we may impose the transaction constraint such that money is necessary for transaction in advance. This is called the cash-in-advance approach.[4] Second, we may assume that money produces utility by saving time for transaction. This is called the money-in-utility function approach. From now on we use the latter device of putting money into the utility function[5]. Feenstra (1986) showed that maximization problems subject to a Baumol-Tobin transaction saving technology can be approximately rewritten as maximization problems with money in the objective function. Tirole showed that the market fundamental of money is equal to the present discounted value of transaction savings.

3. Money And Inflationary Taxes

3.1 Seigniorage

Here, we develop an overlapping generations model which consists of identical individuals and which allows for money holdings in the utility function. Savings are held in the form of money and capital. The effect is to introduce money into the basic model of chapter 2.

A member of generation t has a standard utility function.

$$u_t = u(c_t^1, c_{t+1}^2, x_t, m_{t+1}). \tag{15}$$

where m_{t+1} is real money balances that he holds at the beginning of the second period of his life. As explained in section 2.4, it is assumed that the larger m_{t+1}, the smaller time is needed to transact in the second period of his life, leading to more leisure available.

The total supply of money was assumed fixed in section 2. From now on we consider the case where the government increases money supply to finance its spending. The government is assumed to purchase in each period a required quantity, g, of the consumption good per worker. Its expenditure is financed by means of monetary expansion. Real values are obtained by deflating p_t.

The government budget constraint in per worker terms for period t is

$$(1+\pi_{t+1})m_{t+1} - \frac{m_t}{1+n} = g. \tag{16}$$

Monetary expansion has the same effect of financing its expenditure as taxes. In this sense monetary finance may be referred as inflationary taxes.

In the steady state we have from (16)

$$g = m[1+\pi - \frac{1}{1+n}]. \tag{17}$$

Inflationary taxes are called seigniorage. How will g change as π increases? Recall money demand m is a decreasing function of π. When π is low, an increase in π will raise g, but when π is high, an increase in π may reduce g. It is because m decreases much when π is high. We can draw this relationship in Figure 7.4. This curve is a Laffer curve in the tax literature. Namely, m may be regarded as the tax base and π may be regarded as the tax rate. When the tax rate is low, an increase in the tax rate raises tax revenue. However, when the tax rate is too high, an increase in the tax rate reduces the tax base so that tax revenue may decrease. There is a revenue-maximizing π. This is π_{max} in Figure 7.4. Friedman (1971) treated the expected inflation rate as a policy variable and obtained his elasticity formula. So if the elasticity of the real money demand with respect to the expected inflation rate π is $-\pi/\eta$, and η is a positive parameter, $\pi_{max} = \eta$. We have:

Proposition 5: Generally we have two inflation rates which raise the same revenue. The smaller rate of inflation is desirable to raise the same revenue.

If the economy is not in the steady state, the relationship between inflation and revenue is very complicated. This is because in period t+1 m_{t+1} is given. By expecting π_{t+1} , the private sector holds m_{t+1} in period t. When the government increases π_{t+1} in period t+1, m_{t+1} does not change. Hence, the government can raise any revenue by simply increasing π_{t+1}. In other words, the revenue-maximizing rate of inflation π_{max} would be ∞. However, if the private sector anticipates this, nobody would like to hold money in period t. It would seem that the revenue-maximizing strategy would be to promise not to increase the money supply in the future (period t+1 and onward) and to issue money in period t.

The fact that the promised policy is not the best strategy for the government to follow in the future raises an important question about the credibility of the government when it makes promises about future policy. If the optimal policy today is for the government to promise to do something in the future that is not optimal to do once the future has been reached, the initial policy is called time inconsistent. As an example, a revenue-maximizing policy might be to promise to keep the future inflation rate constant and to keep breaking the promise[6].

Calvo (1978) stressed that the Friedman elasticity solution is not time-consistent and would be relevant only if the government could bind itself to fulfill its commitment regarding future inflation rates. Grossman and Huyck (1986) investigated the maximal-seigniorage question within a more general model of the determination of expected inflation and the relation between expected inflation and actual money issue. They showed that the objective of maximal seigniorage produces an equilibrium inflation rate equal either to a generalization of the Friedman elasticity solution or to the rate at which the government discounts future seigniorage adjusted for the growth rate, whichever is larger. Bordo and Redish (1992) extended the theory of the revenue maximizing rate of monetary growth to the case of a temporary suspension of convertibility. There they showed that the price level must drop at the point of suspension of convertibility, so that there is no discontinuity at the date of resumption.

3.2 Welfare Cost Of Inflation

3.2.1 Optimal Rate Of Inflation

The revenue yield from increases in inflation has received much attention in the literature. Since Phelps (1973), most work on inflation and welfare relies on some variant of the optimal taxation framework. By the straightforward application of the static optimal taxation literature, Helpman and Sadka (1979) attempted to present sufficient conditions under which the optimal rate of inflation is positive in a second-best world, although they did not discuss the level of optimal inflation. As stressed by Siegel (1978), in the static problem there is an infinite set of tax rates that minimize welfare costs, and hence a value of optimal inflation cannot be determined. All we can say is that Friedman's optimum quantity of money rule is generally not optimal in a second-best world.

In an intertemporal framework, however, an optimal inflation rate can be determined. Summers (1981a) presented an explicit expression for the optimal rate of inflation under ad hoc formulations of savings behavior. In general the optimal rate of inflation will be determined implicitly by a combination of preferences and technology. The difficulty in saying anything explicit about the optimal rate of inflation is now generally recognized. See also Feldstein (1979), Kimbrough (1986) and Lucas (1986).

An alternative approach is to explore the welfare cost of inflation using the growth model. The basic idea that capital-accumulation could be beneficial on the tax revenue collected via inflation was set forth in Drazen (1979,1981). If the welfare cost of inflationary finance is relatively small compared with other distortionary taxes, the optimal rate of inflation may well be high. The welfare effects are examined by starting with the expenditure function and comparing the gain in revenue to the amount by which a consumer must be compensated to restore him to a reference utility level.

3.2.2 Analytical Framework

Suppose that public expenditure is financed by means of (a) consumption taxes (τ), (b) capital income taxes (θ), (c) labor income taxes (γ) and (d) monetary expansion. We allow for an endogenous supply of labor as formulated by (15). The representative individual's consumption, saving, and labor supply programs are restricted by the following budget constraints.

$$(1+\tau)c_t^1 = (1-\gamma)w_t l_t - (1+\pi_{t+1})(1+\tau)m_{t+1} - s_t \quad (11\text{-}1)'$$

$$(1+\tau)c_{t+1}^2 = (1+\tau)m_{t+1} + [1+(1-\theta)r_{t+1}]s_t \quad (11\text{-}2)'$$

From these two equations we have

$$q_1 c_t^1 + q_{2t+1}c_{t+1}^2 + q_{3t}x_t + q_{4t+1}m_{t+1} = 0 \quad (18)$$

where

$$q_1 = 1+\tau \quad (19\text{-}1)$$

$$q_{2t+1} = \frac{1+\tau}{1+(1-\theta)r_{t+1}} \quad (19\text{-}2)$$

$$q_{3t} = (1-\gamma)w_t \quad (19\text{-}3)$$

$$q_{4t+1} = (1+\tau)[1+\pi_{t+1} - \frac{1}{1+(1-\theta)r_{t+1}}] \quad (19\text{-}4)$$

q_4 is the opportunity cost of holding money instead of holding real capital. In section 2 q_4=0 is the arbitrage condition (9). Here q_4 is not necessarily zero since money has utility which capital does not.

The government budget constraint in per worker terms for period t is

$$\tau c_t^1 + \frac{1}{1+n}\tau c_t^2 + \gamma w_t l_t + \theta r_t k_t l_t + (1+\tau)[(1+\pi_{t+1})m_{t+1} - \frac{1}{1+n}m_t] = g \quad (20)$$

Equilibrium in the capital market is simply

$$s_t = (1+n)k_{t+1}l_{t+1} \quad (21)$$

We can express the steady state saving function in terms of compensated demands

$$s(q,u) = q_2[E_2(q,u) - E_4(q,u)] \quad (22)$$

where E( ) is the expenditure function and E_i is the compensated demand function for good i (i=c^1, c^2, x, m). Then, the compensated capital accumulation equation is given as

$$q_2[E_2(q,u) - E_4(q,u)] = (1+n)w'(r)E_3(q,u) \quad (23)$$

where w( ) is the factor price frontier and w' = -k.

In the steady state the government budget constraint (20) may also be rewritten in terms of compensated demands as

$$\sum_{i=1}^4 t_i E_i(q,u) = g \quad (24)$$

where

$$t_1 = \tau = q_1 - 1 \quad (25\text{-}1)$$

$$t_2 = \frac{\tau + \theta r q_2}{1+n} = \frac{q_2(1+r)-1}{1+n} \quad (25\text{-}2)$$

$$t_3 = -\gamma w = q_3 - w \quad (25\text{-}3)$$

$$t_4 = (1+\tau)[1+\pi - \frac{1}{1+n}] - \frac{\theta r q_2}{1+n} = q_4 + q_2 \frac{n-r}{1+n} \qquad (25\text{-}4)$$

We call t_1, t_2, t_3, and t_4 effective tax wedges.

3.2.3 Welfare Cost

We now evaluate the long run effect of inflation on an individual's welfare. An unanticipated change in the rate of inflation would, of course, benefit debtors and harm creditors. In order to abstract from such temporary effects and from the problems of the transition from one equilibrium to another, this section focuses on the comparative steady-state equilibria of a growing economy.

The question being asked is how to measure the loss to the economy of a permanent increase in the rate of inflation. We define the welfare loss as the excess of the extra income we must give an individual to restore him to his initial utility level over the extra tax revenue collected from him. Following Diamond and McFadden (1974), we measure the tax revenue for this definition as the level collected at the competitive equilibrium after the individual has been restored to his original utility level.

Considering the government budget constraint (24), let us define the compensated excess tax revenue function by

$$R = \sum_{i=1}^{4} t_i E_i(q, u^0) - g \qquad (26)$$

where u^0 is the level of utility before the permanent increase in the rate of inflation. The welfare loss function, L, is

$$L = E(q, u^0) - R \qquad (27)$$

Note that at the initial state the welfare loss is zero. We consider small changes from an arbitrary initial equilibrium and use its utility level as the standard.

The deadweight loss shows how much the consumer would have to be paid to induce him to accept the permanent increase in inflation. Thus, when we consider how it varies with the permanent increase in inflation, we do not have to maintain the fixed government budget constraint. Otherwise, the welfare cost of an increase in π would be obscured by the welfare costs associated with alternative distorting taxes.

Given a concept of welfare cost, let us ask how it varies with a permanent increase in the rate of inflation. Differentiating (27) with respect to the rate of inflation, we have

$$\frac{dL}{d\pi} = (\frac{\partial E}{\partial \pi} - \frac{\partial R}{\partial \pi}) + \frac{\partial E}{\partial r}\frac{dr}{d\pi} - \frac{\partial R}{\partial r}\frac{dr}{d\pi} \qquad (28)$$

The inflation change has three impacts on the welfare cost.

(1) The static effect

The first term measures the welfare cost of a permanent increase in the rate of inflation at given level of r. Considering (19), we have

$$\frac{\partial E}{\partial \pi} - \frac{\partial R}{\partial \pi} = (1+\tau)E_4 - (1+\tau)E_4 - (1+\tau)\sum_{i=1}^{4} t_i E_{i4} \qquad (29)$$

$$= -(1+\tau)\sum_{i=1}^{4} t_i E_{i4}$$

From (29), we obtain

Proposition 6: If there are no other distortions in the economy ($\tau = \theta = \gamma = 0$ and r=n), this static measure is exactly equal to the traditional measure of the welfare loss by Friedman (q_4E_{44}); the value of the liquidity services that society forgoes because of inflation, that is, the area under the portion of the compensated demand curve that corresponds to the induced reduction in real money balances.

An increase in inflation raises t_4 and hence the tax revenue. Phelps (1973) has emphasized this direct revenue gain. However, this effect is completely offset by an increase in q_4. As the opportunity cost of inflation is raised, the government has to pay all the direct revenue gain to the consumer to induce him accept an increase in inflation. Therefore, (29) means that the induced revenue-enhancement effect of higher inflation on the private sector is relevant in terms of the welfare loss. This is the factor stressed by Drazen (1979), that is, the resource freed for government use to a change in inflation. This effect corresponds to the excess burden of the Ramsey rule.

(2) The golden rule effect

In addition to the static effect, there are dynamic effects. An increase in the rate of inflation may well affect the capital accumulation. The second and third terms in (28) correspond to this intertemporal aspect.

First, let us examine the second term. Considering (19) and (23),

$$\begin{aligned}\frac{\partial E}{\partial r} &= (E_2 - E_4)\frac{\partial q_2}{\partial r} + E_3\frac{\partial q_3}{\partial r} \\ &= kE_3\frac{(1-\theta)(1+n) - [1+(1-\theta)r](1-\gamma)}{1+(1-\theta)r}\end{aligned} \tag{30}$$

As explained in chapter 2, the golden rule criterion has important normative implications. Equation (30) is consistent with the golden rule criterion. Namely, if the economy was saving previously to a point where the marginal product of capital exceeded the rate of population growth, then (30) > 0; an increase in the capital intensity has a desirable welfare effect on the long run utility (and vice versa).

(3) The tax revenue enhancement effect

We next consider the tax-revenue-enhancement effect due to capital accumulation. Considering (25) and (26), we have

$$\begin{aligned}\frac{\partial R}{\partial r} &= \gamma kE_3 + [\frac{\theta q_2}{1+n} + \frac{\theta r}{1+n}\frac{\partial q_2}{\partial r}](E_2 - E_4) + \sum_{i=1}^4 t_i\frac{\partial E_i}{\partial r} \\ &= \gamma kE_3 + \frac{\theta(1+\tau)(E_2 - E_4)}{(1+n)[1+(1-\theta)r]^2} + \sum_{i=1}^4 t_i[(E_{i2} - E_{i4})\frac{\partial q_2}{\partial r} + E_{i3}\frac{\partial q_3}{\partial r}]\end{aligned} \tag{31}$$

The first term is negative for $\gamma > 0$; an increase in r will reduce w. Consequently, taxes from wage income will be reduced. The second term is positive for $\theta > 0$; an increase in r will raise rq_2. Taxes from the second-period consumption increase, and taxes from money holdings reduce. Since the second-period consumption is always greater than the money holdings, the overall effect of an increase in r enhances tax revenues.

Let us investigate the third term in (31). $\sum_{i=1}^4 t_i(E_{i2} - E_{i4})\frac{\partial q_2}{\partial r}$ is the effect of an increase in r on tax revenues through a change in q_2 (and hence q_4). We know $\frac{\partial q_2}{\partial r} - \frac{\partial q_4}{\partial r} = 0$. The positive effect of q_2 is associated with the negative effect of q_4. Suppose $\tau = \theta = 0, \gamma > 0$ and $1 + \pi - \frac{1}{1+n} > 0$ ($t_1=t_2=0$, $t_3<0$, $t_4>0$). If $E_{32}-E_{34}<0$ and $E_{42}>0$, then $\sum_{i=1}^4 t_i(E_{i2} - E_{i4})\frac{\partial q_2}{\partial r} < 0$. An intuitive explanation of this result is as follows. Because $\frac{\partial q_4}{\partial r} > 0$, an increase in capital accumulation lowers q_4. The opportunity cost of holding money is reduced and hence m is increased. For given t_4, taxes on m increase. Moreover, if $E_{24}>0$, a higher q_2 increases m. Hence, for given t_4, taxes on m increase. Similarly, if $E_{34}>0$, a lower q_4 increases labor supply. For given t_3, taxes on labor income increase by capital accumulation.

The sum $\sum t_i E_{i3} \frac{\partial q_3}{\partial r}$ denotes the effect of an increase in r on tax revenues through a change in q_3. As $\frac{\partial q_3}{\partial r} < 0$, the sign of this term is dependent on $\sum t_i E_{i3}$. Suppose $\tau = \theta = 0$, $\gamma > 0$ and $1 + \pi - \frac{1}{1+n} > 0$. We know $E_{33}<0$; an increase in q_3 (an increase in the after-tax real wage) will reduce the compensated consumption of leisure. It follows that an increase in capital accumulation will increase taxes on labor income. If leisure and money are net substitutes ($E_{34}>0$), an increase in q_3 will reduce the money holdings. Therefore, an increase in capital accumulation will raise tax revenues from money holdings.

In the case of $t_1=t_2=0$, $t_3<0$, and $t_4>0$, the third term of (31) is negative if $E_{23}-E_{34}<0$, $E_{24}>0$, and $E_{34}>0$. Note that in this case the second term is zero, and we have $\frac{\partial R}{\partial r} < 0$. In such a case an increase in capital accumulation has a desirable welfare effect on the long-run utility. As a result, the tax revenue enhancement effect holds. Logically, we conclude

Proposition 7: Suppose labor income taxes and money finance are available. If both leisure and the second-period consumption are substitutes with money and if money is more substitutable with leisure than the second period consumption, then an increase in capital accumulation is desirable.

Remark: The representative person chooses to both hold money and save because larger money holdings at the beginning of the second period of his life save on transacting and would be associated with more leisure (after transacting) in the second period. The second period leisure and money holdings at the beginning of the second period would be complements. Since the first-period leisure and the second-period leisure are normally net substitutes, it seems natural to think of the first-period leisure and money as net substitutes; that would make the term E_{34} positive.

(4) The compensated Tobin effect

The importance of the golden rule effect and the intertemporal tax-revenue-enhancement effect depends on the sign and magnitude of $dr / d\pi$. If money is more substitutable than the second-period consumption with respect to leisure ($E_{32}-E_{34}<0$), if leisure and money are net substitutes ($E_{34}>0$), and if the elasticity of substitution between capital and labor is sufficiently high ($w'(1-\theta)+w''[1+(1-\theta)r]>0$), then a compensated increase in the rate of inflation will have a positive effect on capital accumulation, and the compensated Tobin effect holds[7].

An intuitive explanation of this phenomenon is as follows. An increase in inflation raises the opportunity cost of holding money and hence increases savings ($E_{24}>0$). It reduces labor supply ($E_{34}>0$). Therefore, if the elasticity of substitution between capital and labor is large, capital intensity rises. At the same time an increase in capital intensity reduces the opportunity cost of holding money and raise the consumer price of second-period consumption. Hence, it increases labor supply ($E_{34}>0$) and taxes from labor income, and raises money holdings ($E_{42}>0$) and taxes from money holdings.

Ihori (1985) has shown that under some simple conditions, a compensated increase in inflation has a positive effect on capital accumulation. When this "compensated Tobin effect" holds, higher inflation (and with it, higher capital intensity) will raise the consumer price of c^2 and the after-tax real wage, but will reduce the opportunity cost of m. Consequently, l and m are likely to increase. Moreover, in this case, it is likely that an increase in inflation may be desirable from the viewpoint of the intertemporal tax-revenue-enhancement effect. For given tax rates on l and m, higher inflation has a beneficial effect on the tax revenue collected. If this effect is greater than the static distortionary effect, inflation will enhance welfare.

3.2.4 Further Study

The tax smoothing theory of inflation has been considered more fully in the recent literature. If the marginal social cost of raising revenue is increasing in the tax rate, optimal tax policy entails the smoothing of tax rates over time (see Barro (1979)). This smoothing principle applied to the case of seigniorage implies that nominal interest rates and inflation should be smoothed as well and that such smoothing makes these series approximately random walks. This implication of the theory has been subject to recent empirical work by Mankiw (1987a) and Poterba and Rotemberg (1990), who reported mixed results about its empirical validity. Calvo and Leiderman (1992) expressed the inflation-rate smoothing implications in the form of an empirically testable orthgonality condition. They showed that there are several data points characterized by higher rates of inflation than the optimal rates under precommitment.

4. Welfare Implications Of Indexing Capital Income Taxation

4.1. The Model

The purpose of this section is to consider the welfare implications of indexing capital income taxation in an inflationary economy. Because we currently tax the nominal income from investment and allow borrowers to deduct nominal interest costs, the real net rate of returns to debt and equity will be directly altered by a change in the rate of inflation. Feldstein (1976), Green and Sheshinski (1977), and Feldstein, Green, and Sheshinski (1978) investigated how the rates of inflation, which can be expected in the future, will affect the effective rates of return on capital income in a steady state economy. They suggested that indexation is desirable because it would eliminate the dependency of real rate of return on inflation.

We now consider several taxes on capital income. The government is assumed to purchase in each period a required quantity of the consumption good gN_t. Its expenditure is financed by means of (a) labor income taxes (γ), (b) several capital income taxes, and (c) deficit finance. For simplicity consumption taxes are zero; τ = 0.

4.1.1 Consumer Behavior

Therefore, the representative individual's consumption, saving, and labor supply programs are restricted by the following first- and second-period budget constraints.

$$c_t^1 = (1-\gamma)w_t l_t - (\pi_{t+1}+1)m_{t+1} - s_t, \qquad (11\text{-}1)''$$

$$c_{t+1}^2 = m_{t+1} + (1+r_{Dt+1})s_t, \qquad (11\text{-}2)''$$

where r_{Dt+1} is the real net rate of interest on debt from period t to period t+1.

By definition, we have

$$1+r_{Dt+1} = \frac{(1-\theta)i_{t+1} + 1 + \theta\lambda\pi_{t+1}}{1+\pi_{t+1}}, \qquad (32)$$

where i_{t+1} is the nominal rate of interest in one period debt from period t to period t+1,

θ is the interest income tax rate, and λ is a parameter of indexation. If $\lambda = 0$, nominal interest is taxed. If $\lambda = 1$, real interest is taxed[8]. For small rates of π and r_D, the nominal net rate of interest $(1-\theta)i+\theta\lambda\pi$ is approximately equal to the real net rate of interest r_D plus the rate of inflation, π.

4.1.2 Producer Behavior

Let us describe the effect of the corporate tax on the profit-maximizing problem. In order to make the results easier to compare with the earlier literature, our specification of the corporate tax system is similar to Feldstein and Green and Sheshinski. We do not make explicit distinction between a corporate and non-corporate sector. Also, following them, we assume that all capital is financed by debt denominated in monetary units.

In period t-1, firms issue nominal debt by D_t and buy K_t units of capital at price p_{t-1};

$$p_{t-1}K_t = D_t . \tag{33}$$

This equation means

$$(1+\pi_{t+1})d_{t+1} = s_t ,$$

where d_{t+1} is the total stock of real debt at the beginning of period t+1.

As the interest payments are tax deductible, the firm's nominal cash flow in period t is equal to $p_t y_t N_t - p_t w_t N_t l_t - i_t D_t$. In addition, the stock of capital, which is assumed not to depreciate, is increasing in nominal values, $(p_t - p_{t-1})K_t$.

These two components of profits can be taxed at differential rates, ϕ for cash flow and α for the inventory revaluation. Therefore, after tax profits are

$$(1-\phi)[p_t f(k_t)N_t l_t - p_t w_t N_t l_t - i_t p_{t-1}K_t] - \phi\lambda\pi_t p_{t-1}K_t + (1-\alpha)(p_t - p_{t-1})K_t.$$

Note that if firms are allowed to deduct only their real interest expense ($\lambda=1$), this is equivalent to allowing a deduction of the nominal interest payment and taxing the real gain that results from the decline in the real value of debt.

Maximum profits are attained and eliminated by competition when

$$f(k_t) - f'(k_t)k_t = w_t , \tag{34}$$

$$\frac{(1-\alpha)\pi_t + (1-\phi)(1+\pi_t)f'(k_t) - \phi\lambda\pi_t}{1-\phi} = i_t . \tag{35}$$

Substituting (35) into (32), the real net rate of interest on debt r_{Dt} is rewritten as

$$r_{Dt} = (1-\theta)f'(k_t) - \frac{\pi_t b}{1+\pi_t}, \tag{36}$$

where

$$b = \frac{(\phi-\theta)(\lambda-1) + \alpha(1-\theta)}{1-\phi}. \tag{37}$$

4.1.3: Capital Income Taxes

Tax revenue from nominal profits is

$$p_t\phi[f'_t - \frac{i_t}{1+\pi_t} + \frac{\lambda\pi_t}{1+\pi_t}]K_t = \frac{p_t K_t \pi_t \phi[\lambda - (1-\alpha)]}{(1+\pi)(1-\phi)}. \tag{38}$$

Tax revenue from nominal capital gains is

$$\frac{p_t K_t \alpha\pi_t}{1+\pi_t}. \tag{39}$$

Thus, total tax revenue from the corporate sector is

$$\frac{p_t K_t \pi_t(\phi\lambda + \alpha - \phi)}{(1-\phi)(1+\pi_t)}. \tag{40}$$

Tax revenue from interest income is $(\theta i_t - \lambda\theta\pi_t)D_t$. Thus, the total nominal tax revenue from capital income in period t, R, is

$$R = \{\frac{(\phi\lambda + \alpha - \phi)\pi_t}{(1-\phi)(1+\pi_t)} + \frac{\theta i_t - \lambda\theta\pi_t}{1+\pi_t}\}p_t K_t$$
$$= \{\theta f'_t + \frac{[\phi\lambda + \alpha - \phi + \theta(1-\alpha) - \lambda\theta]\pi_t}{(1-\phi)(1+\pi_t)}\}p_t K_t \quad (41)$$

Therefore, substituting (36) and (37) into (41), R is finally expressed as

$R = (f' - r_{Dt})p_t K_t$. (42)

Real per-capita capital income taxes may be summarized by $(f'-r_{Dt})k_t l_t$. Thus, the government budget constraint is given as

$$[f'(k_t) - r_{Dt}]k_t l_t + (1+\pi_{t+1})m_{t+1} - \frac{m_t}{1+n} = g \quad (20)'$$

(36) shows that the direct effect of inflation on the real rate of interest is dependent on b. Inflation changes the real rate of interest directly. When $\phi - \theta > 0$, then $b < 0$ only if $\lambda < 1$, and b is increasing with λ. The definitions of taxable income and expenses can be varied to eliminate the direct effect of inflation on equilibrium real yields (b=0). The most obvious adjustments are to end the taxation of nominal capital gains ($\alpha = 0$) and to index capital income fully ($\lambda = 1$). Does there generally exist any optimal level of b? If so, what is the optimal level of b (and hence the optimal level of indexation, λ)? We investigate such normative questions in the following sub-section.

4.2. Optimal Level Of Indexation

4.2.1 Optimization Problem

We now consider long-run optimality. In steady states the representative individual's maximization problem is

Choose c^1, c^2, x and m
to maximize $u(c^1, c^2, x, m)$,
subject to $q_1c^1+q_2c^2+q_3x+q_4m = 0$, (18)

where

$$q_1 = 1, \quad (19\text{-}1)'$$

$$q_2 = \frac{1}{1+r_D} = \frac{1}{1+(1-\theta)f' - \frac{\pi b}{1+\pi}}, \quad (19\text{-}2)'$$

$$q_3 = (1-\gamma)w, \quad (19\text{-}3)$$

$$q_4 = 1+\pi - \frac{1}{1+(1-\theta)f' - \frac{\pi b}{1+\pi}}. \quad (19\text{-}4)'$$

Let us define the real rental price of capital, r, by

$$r = \frac{(1-\phi)i - (1-\alpha)\pi + \pi\phi\lambda}{(1-\phi)(1+\pi)} (= f') \quad (43)$$

Then we have the conventional factor price frontier with respect to w and r;

$w = w(r)$, $w'(r) = -k$.

Therefore, we may still derive the compensated capital accumulation equation (23). From (23), r can be expressed as a function of q and u.

$r = r(q, u)$. (23)'

The government budget constraint reduces to

$$t_2E_2(q,u) + t_3E_3(q,u) + t_4E_4(q,u) = g, \quad (24)'$$

where

$$t_2 = \frac{r - r_D}{(1+n)(1+r_D)} = \frac{q_2(1+r)-1}{1+n}, \tag{25-2)'}$$

$$t_3 = -\gamma w = q_3 - w, \tag{25-3}$$

$$t_4 = 1 + \pi - \frac{1}{1+n} - \frac{r - r_D}{(1+n)(1+r_D)} = q_4 + q_2 \frac{n-r}{1+n}. \tag{25-4)'}$$

The nature of the solution may depend critically on the range of instruments assumed to be at the disposal of the government. It is useful to distinguish two cases; case (i) where the rate of inflation is optimally chosen and case (ii) where the rate of inflation is exogenously given.

Nevertheless, there are some interesting properties which hold in general. That is, as explained in chapter 3, our optimization problem can be solved in two stages, irrespective of whether π is optimally chosen or not. In the first stage, one can choose q, r, and w so as to maximize u. In the second stage, one can choose $\gamma, \theta, \phi, \alpha, \lambda$ and b to satisfy (19-2)'(19-3) and (19-4)' for the optimal values of the q's, r, and w. The second stage has a solution, if for every vector q, r, and w, there exist ($\gamma, \theta, \phi, \alpha, \lambda$, b) which satisfy (19-2)'-(19-4)'. From (19-3), γ = 1-q_3/w. Thus, the optimal level of γ is uniquely determined. It is interesting to note, however, that the optimal levels of $\theta, \phi, \alpha, \lambda$ and b cannot be uniquely determined. Only one of the capital income taxes is sufficient for attaining the optimal state. Put it another way, we have:

Proposition 8: If capital income taxes are optimally chosen, indexing capital income taxes is a redundant policy instrument. Complete indexation can be replaced by any partial indexation with no change in welfare.

It is inappropriate to regard the level of indexation as a target rate if capital income taxes are optimally chosen. The system of the economy is described in terms of consumer prices q; the individual's budget constraint (18), the capital accumulation equation (23)', and the government budget constraint (24)' are all functions of q and u. Capital income taxes affect u only through changes in the relative price of the second-period consumption.

It should be also noted that if θ, ϕ, and α are exogenously given and are not necessarily chosen at the optimal levels, then the level of indexation, λ, is an important policy variable for the optimization problem. Let us now consider the optimal level of indexation in that situation.

4.2.2 The Case Where π Is Optimally Chosen

Suppose the government can choose optimally the wage income tax rate, γ, and the rate of inflation, π. Given realistic parameters, what should b and λ be (approximately)? The government's first stage problem in case (i) is to choose q_2, q_3, q_4, and r so as to maximize u subject to (18)(23)' and (24)'. Then, from (19-2)' the optimal level of b will be determined.

$$b = \frac{(1 + r - \frac{1}{q_2} - \theta r)(1+\pi)}{\pi}. \tag{44}$$

It will be useful to divide into two possibilities. First, let us consider the case where it is optimal to have no taxation of capital income. 1+r=1/q_2 and hence t_2 =0. This will be the case in which debt policy is employed to achieve a desired intertemporal allocation. In such a situation, as was shown in chapter 3's non-monetary economy, if in addition U is directly additive, unitary expenditure elasticities imply that the optimal level of q_2 is given by $1/(1+r)$ with t_2 equal to zero. It is easy to show that this result holds in our monetary economy as well; it is then optimal to have no taxation of the second-period consumption in that situation[9].

Let us consider the optimal level of λ in that situation. Substituting $1+r = 1/q_2$ into (44), we have

$$b = -\frac{\theta r(1+\pi)}{\pi}. \tag{45}$$

As shown in (45), b is an increasing function of π (> 0). A higher rate of inflation will reduce the optimal (absolute) value of b so as to keep $\pi b/(1+\pi)$ constant. Remember that when $\phi - \theta > 0$, b is increasing with λ. Therefore, λ is increasing with π. As capital income taxes are set at some positive levels, the optimal level of indexation, λ, is given by the level that would exactly offset the effect of interest income taxes on the relative price of the second-period consumption.

Another possibility is $1+r \neq 1/q_2$ and so $t_2 \neq 0$. From (44) if $1+r-1/q_2 < 0$ and $\pi > 0$, then $b < 0$ and $\lambda < 1$. If it is optimal to subsidize capital income in an inflationary economy, partial indexation is desirable. Even if capital income taxation is optimal, it is still desirable to have partial indexation at a relatively higher value of θ (so long as $\phi - \theta > 0$)[10].

4.2.3 The Case Where π Is Exogenously Given

Let us assume that for political or any other reasons the government cannot control or change a given positive rate of inflation. The government's first stage problem in case (ii) is to choose q_2, q_3, q_4 and r so as to maximize u subject to (18)(23)'(24)' and $q_4 = 1+\pi-q_2$ for $\pi > 0$. Generally the optimal levels of q_2, q_3 and q_4 are functions of π. The (optimal) real rate of return, r_D, will therefore change as the rate of inflation changes.

Let us explicitly solve the value of $1+r-1/q_2$. We can formulate the first stage problem in terms of a Lagrange function.

$$H = u - \mu_1 E(1,q_2,q_3,1+\pi-q_2,u) - \mu_2\{\frac{[q_2(1+r)-1]E_2(1,q_2,q_3,1+\pi-q_2,u)}{1+n}$$

$$+(q_3 - w(r))E_3(1,q_2,q_3,1+\pi-q_2,u) + [1+\pi-\frac{(1+r)q_2}{1+n}]E_4(1,q_2,q_3,1+\pi-q_2,u)$$

$$-g\} - \mu_3[r - r(1,q_2,q_3,1+\pi-q_2,u)]$$

The necessary conditions are

$$\frac{\partial H}{\partial q_2} = -\mu_1(E_2 - E_4) - \mu_2[\sum t_i(E_{i2} - E_{i4}) + \frac{1+r}{1+n}(E_2 - E_4)] + \mu_3(r_2 - r_4) = 0, \tag{46}$$

$$\frac{\partial H}{\partial q_3} = -\mu_1 E_3 - \mu_2(\sum t_i E_{i3} + E_3) + \mu_3 r_3 = 0, \tag{47}$$

$$\frac{\partial H}{\partial r} = -\mu_2(\frac{q_2(E_2 - E_4)}{1+n} + kE_3) - \mu_3 = 0. \tag{48}$$

Considering (23), (48) implies $\mu_3 = 0$. Substituting $\mu_3 = 0$ and $t_4 = 1+\pi-1/(1+n)-t_2$ into (46) and (47), we have

$$-\mu_1(E_2 - E_4) - \mu_2(E_2 - E_4) - \mu_2[t_2(E_{22} - E_{24} - E_{42} + E_{44}) + t_3(E_{32} - E_{34})$$

$$+(1+\pi-\frac{1}{1+n})(E_{42} - E_{44}) - \frac{(n-r)(E_2 - E_4)}{1+n}] = 0$$

and

$$-\mu_1 E_3 - \mu_2 E_3 - \mu_2[t_2(E_{23} - E_{43}) + t_3 E_{33} + (1+\pi - \frac{1}{1+n})E_{43}] = 0$$

Or

$$t_2(E_{22} - 2E_{24} + E_{44}) + t_3(E_{32} - E_{34}) = \\ [-\mu + \frac{n-r}{1+n}](E_2 - E_4) - (1+\pi - \frac{1}{1+n})(E_{24} - E_{44}) \quad (49)$$

$$t_2(E_{23} - E_{43}) + t_3 E_{33} = -\mu E_3 - (1+\pi - \frac{1}{1+n})E_{43}, \quad (50)$$

where $\mu = \frac{\mu_1 + \mu_2}{\mu_2}$. From (49) and (50), the optimal level of t2 is given as

$$t_2 = \frac{\{(\frac{n-r}{1+n} - \mu)(E_2 - E_4) - \Pi(E_{24} - E_{44})\}E_{33} + (E_{23} - E_{34})(\Pi E_{34} + \mu E_3)}{(E_{22} - 2E_{24} + E_{44})E_{33} - (E_{23} - E_{34})^2} \quad (51)$$

where $\Pi = (1+\pi - \frac{1}{1+n})$. μ expresses the benefits in terms of revenue from being able to switch from the optimal indirect tax system to lump sum taxation. $\mu > 0$ in a static model, but μ might be negative in a dynamic model. Generally, the sign of t_2 cannot be assessed on a priori reason. If $E_{23} = E_{34}$, $E_{24} > 0$, and $n < r + \mu(1+n)$, then $t_2 > 0$.

The optimal level of indexation will generally depend on the rate of inflation. From (44), we have

$$\frac{db}{d\pi} = -\frac{1+r-\frac{1}{q_2}-\theta r}{\pi^2} + \frac{1+\pi}{\pi}[(1-\theta)\frac{dr}{d\pi} + \frac{1}{(q_2)^2}\frac{dq_2}{d\pi}]. \quad (52)$$

When $b < 0$ and $\lambda < 1$, $1+r-1/q_2-\theta r < 0$. Therefore, the first term of (52) is positive. The sign of the second term is ambiguous. If $E_{23} = E_{34}$, the direct effect of an increase in π on t_2 is from (51)

$$\frac{\partial t_2}{\partial \pi} = -\frac{E_{24} - E_{44}}{E_{22} - 2E_{24} + E_{44}} \quad . \quad (53)$$

If, in addition, $E_{24} > 0$, then $\partial t_2 / \partial \pi > 0$ and $\partial q_2 / \partial \pi > 0$[11]. In the case where debt policy is employed to achieve a desired intertemporal allocation ($r = n$), we have $dr/d\pi = 0$. Therefore, the sign of the second term might well be positive. As far as $db/d\pi > 0$, we can say that a higher rate of inflation will increase the optimal level of b; the optimal indexation, λ, will be increasing with the rate of inflation. The second stage problem will give the optimal value of $-\pi b/(1+\pi)$ for an arbitrary given π. So long as an increase in π does not raise $-\pi b/(1+\pi)$ rapidly, b will increase.

Thus, we have:

Proposition 9. The optimal indexation may well be increasing with the rate of inflation.

4.3 Remarks

It is important to formulate explicitly the analytical framework for welfare evaluations of indexing capital income taxation. As in the standard optimal taxation problem, the welfare implications of indexation may depend critically on the range of instruments assumed to be at the disposal of the government. It has been shown that if capital income taxes are optimally chosen, tax indexing for inflation is not a meaningful policy target[12].

If, on the other hand, capital income taxes are exogenously given, the level of indexation does matter. When the rate of inflation is optimally chosen, partial indexation may be desirable. When the rate of inflation is exogenously given, the real net rate of return is generally a function of the rate of inflation. In some cases, it would indeed be undesirable to let the rate of inflation affect the net real rate of inflation. However, such a case means that the exogenously given interest income tax is desirable. When the interest income tax is not levied optimally, partial indexation may also be desirable. We illustrate a possibility that the optimal indexation of the tax system is partial and increasing with the rate of inflation. All of our results shows that the full tax indexing for inflation is not necessarily desirable in a steady state economy.

Finally, a strong note of caution concerns the whole concept of optimality. We have limited ourselves to steady state optimality. Other meanings are possible, and optimality of indexation could be examined in terms of a number of government policy objectives. See, for example, Bohn (1988), who studied the choice between nominal and indexed debt in a stochastic macroeconomic model with discretionary monetary and fiscal policy. We would argue that if one is to use any concept of optimality in choosing a level of indexation, the analytical framework for welfare evaluations should be formulated explicitly. This section has cleared up some of the conceptual and theoretical issues in the problem of indexing tax system.

[1] . McCandless with Wallace (1991) and Azariadis (1993) provided a useful explanation of Samuelson's model.

[2] . If there are any difference of preferences and/or initial endowment among individuals, trade can take place. See McCandless with Wallace (1991).

[3] . If some policy action is expected to bring the economy on the stable arm in future, it is possible that the economy will be above the stable arm for a while. We will investigate such policy programs in Chapter 9.

[4] . For further research see Blanchard and Fischer (1989).

[5] . See Sidrauski (1967) and Weiss (1980).

[6] . Kydland and Prescott (1977) first showed that optimal macroeconomic policies could well be dynamically inconsistent. Cukierman, Edwards, and Tabellini (1992) showed that countries with a more unstable and polarized political system will have more inefficient tax structures and, thus, will rely more heavily on seigniorage.

[7] . Tobin (1965) argued that relative to money, inflation makes capital seem more attractive that it would otherwise be and hence increases the capital stock. We examine the compensated effect of inflation on capital accumulation and call this the compensated Tobin effect.

[8] . Real saving s_t is given by $(1+\pi_{t+1})d_{t+1}$.

[9] . In the first best economy where lump sum taxes are available, it is also optimal to have no taxation of capital income. However, this case is not relevant for the present study in the sense that the resulting optimal rate of inflation is negative.

[10] . This observation is consistent with Summers (1981a). He showed that complete indexing is not optimal, using a variant of the neoclassical monetary growth model. However, his model relied on ad hoc formulation of saving behavior.

[11] . It is assumed that λ, r, E_i and E_{ij} (i, j = 2,4) are independent of π. This assumption is relevant for the direct effect of $\partial t_2 / \partial \pi$.

[12] . We do not argue that tax indexing for inflation is not a meaningful policy target in a non-steady state economy. If we allow for any sort of uncertainty, indexing may well be desirable. What we are saying here is that indexing capital income could be redundant in a steady state economy.

Chapter 8

Land

1. Introduction

It has been assumed that consumption goods and investment goods are perfect substitutes as the outputs of production technology. In reality, however, a significant fraction of savings is invested in assets that has a very long life and are not easily substitutable with consumption.

Feldstein (1977) modified the standard stylized framework of a two-period overlapping generations model to include both capital and a fixed factor (land). He obtained the "surprizing" result that a land tax may raise the land price in the long run. Recently, under the assumption of perfect foresight, Chamely and Wright (1987), Eaton (1988), and Fried and Howitt (1988) asked if the inclusion of capitalization effects through fixed assets such as land may significantly affect the inclusion of fiscal policy. Their dynamic approach is a necessary complement to a general comparative statics analysis of Feldstein's model. Their approach furnishes the correct stability conditions which are different from those in the conventional literature.

Fried and Howitt analyzed the effects of fiscal deficits on welfare, interest rates and the balance of payments by supposing capital is fixed. Chamley and Wright analyzed the impact on capital accumulation and the price of land by using the technology of a fixed factor (land), malleable capital, and labor. They showed that (permanent) public spending financed by lump-sum wage taxes will crowd out capital formation, while public spending financed by land taxes will crowd in capital formation. They also showed that a land tax may initially raise land values, but the upper bound is less than one-half of the tax revenues. Using a small open economy, Eaton showed that a permanent increase in net foreign investment can reduce steady-state welfare if a consequence is higher land value.

In this chapter based on their work and Ihori (1990a,b), we investigate the relationship among land, taxes, and capital formation. Section 2 considers in detail the dynamic effects of tax financing on capital accumulation and land values where revenues are used for temporary and permanent increases in public expenditure.

In Section 3, we consider the dynamic effects of tax financing on capital accumulation and land values in an inflationary economy. We examine the effects of land taxes on the price of land and the rate of inflation when the government expenditure is financed by monetary expansion as well. We explicitly consider the general equilibrium effects of land rent taxes, land value taxes, and capital gains taxes in an inflationary situation. Land taxes may affect the real equilibrium only through changes in the total revenue from the land taxes.

2. Land And Lump Sum Taxes

2.1 Analytical Framework

2.1.1 Technology

Consider technology that requires land, labor, and capital to produce a composite capital/consumption good. Output is produced by the technology:

$$y_t = f(K_t, N_t, L_t), \quad (1)$$

where y is output, K is capital, N is labor, and L is land. Land is fixed. Land does not have a finite maturity date. It is an infinity lived asset. There is a population of constant size. N=L=1.

From the factor price frontier the marginal returns on land and labor, z and w, and capital stock, K, are respectively given by a function of the rate of interest, r, which is equal to the marginal product of capital, f_K.

$$w_t = w(r_t), \quad (2)$$

$$z_t = z(r_t), \quad (3)$$

$$K_t = K(r_t). \quad (4)$$

K'<0. The sign of z' (= $-K - \frac{f_{LK}}{f_{KK}} L$) and w' (= $-K - \frac{f_{NK}}{f_{KK}} N$) is ambiguous. However, unless $f_{iK} = \frac{\partial^2 f}{\partial i \partial K}$ (i=L,N) is large enough, w' and z' are likely to be negative.

As in the basic model, each individual lives for two periods. When young, he supplies one unit of labor, and consumes c^1 unit of output. With his savings he purchases assets that he sells in his second period to purchase consumption goods of the amount of c^2. The budget constraints are

$$c_t^1 = w_t - (K_{t+1} + P_t) \quad (5)$$

$$c_{t+1}^2 = (1 + r_{t+1}) K_{t+1} + P_{t+1} + z_{t+1} \quad (6)$$

The price of a unit of land is P_t.

2.1.2 Price Of Land

We do not introduce land into the utility function. Holding land does not produce utility. Thus, given perfect foresight without uncertainty, capital and land are perfect portfolio substitutes and have the same rate of return:

$$1 + r_{t+1} = \frac{z_{t+1} + P_{t+1}}{P_t}. \quad (7)$$

Suppose that r_{t+1}, z_{t+1}, and P_{t+1} are given in period t. If the left-hand side of (7) is greater than the right-hand side of (7) at time t, individuals would like to hold more capital than land, so that the price of land in period t, P_t, will decrease to satisfy (7), and vice versa. Changes in the asset prices are a measure of the capital gains or losses incurred by current asset holders. In a steady state, $r_t = r_{t+1} = r$, $z_t = z_{t+1} = z$, and hence $P_t = P_{t+1} = P$. We do not have capital gains in the long run in a non-monetary economy.

Suppose for simplicity r and z are fixed. Then, from (7) P_t is given as

$$P_t = \frac{z + P_{t+1}}{1+r} = \frac{z}{1+r} + \frac{z + P_{t+2}}{(1+r)^2}$$
$$= \frac{z}{1+r} + \frac{z}{(1+r)^2} + \frac{z}{(1+r)^3} + \quad = \frac{z}{r} \quad (8)$$

P_t is given as the discounted present value of future returns. Or, substituting $P_t = P_{t+1} = P_t$ into (7), we also get the above equation. (8) is the perfect foresight price (or the fundamental price) of land. (8) implies that the price of land is a forward-looking price and depends on what will happen in the future. Since r and z are endogenous in a general equilibrium setting, the price of land is determined by a forward-looking way in the overlapping generations model. We now investigate how P_t is determined using a simple dynamic general equilibrium model where r and z are also endogenous.

2.1.3 Dynamics Of Model

The asset market clearing condition is given as

$$s(w_t, r_{t+1}) = K_{t+1} + P_t, \tag{9}$$

where s( ) is the total saving function for real capital and land.

Considering (2),(3), and (4), the dynamics of the model is entirely determined by (7) and (9). Substituting (2) and (4) into (9), r_{t+1} may be regarded as a function of r_t and P_t.

$$r_{t+1} = R(r_t, P_t), \tag{10}$$

where $R_1 = \frac{\partial R}{\partial r_t} = \frac{s_w w'}{K' - s_r}$ and $R_2 = \frac{\partial R}{\partial P_t} = -\frac{1}{K' - s_r}$. Substituting (10) into (7) and considering (3), P_{t+1} may be also regarded as a function of r_t and P_t.

$$1 + R(r_t, P_t) = \frac{z[R(r_t, P_t)] + P_{t+1}}{P_t}. \tag{11}$$

In the initial period t=0 r_0 is given, but P_0 is not given by the history. Since P is a jumping variable, we have to determine P_0 by some means. A plausible restriction is to concentrate on the perfect foresight path of P_t. If P_t does not converge as $t \to \infty$, such a path should be excluded. However, when the system is dynamically stable, the price of land is indeterminate because for any value there is a dynamic path that converges to the steady state. From now on we focus on saddle-point equilibria; so P can jump instantanenously in period 0 to the convergent path. In other words, P_0 is uniquely determined since the convergent path is unique.

2.1.4 Phase Diagram

Let us investigate dynamic properties of this economy using a phase diagram. To analyze the behavior of r_t, we find the locus of (P, r) where $r_{t+1}=r_t$. We call this locus the rr curve. To find the behavior of P_t, we investigate the locus of (P, r) where $P_{t+1}=P_t$. We call this locus the PP curve.

From (9) the rr curve is given as

$$s[w(r), r] = K(r) + P. \tag{9'}$$

The slope of this curve is given as

$$\frac{dP}{dr} = s_r + s_w w' - K'. \tag{12}$$

If s_r-K'<0, then this curve is downward sloping, but if s_r-K'>0, this curve may be upward sloping.

Substituting $P_{t+1}=P_t$ into (11), the PP curve is given as

$$P[1 + R(r, P)] = z[R(r, P)] + P. \tag{11'}$$

The slope of this curve is given as

$$\frac{dP}{dr} = -\frac{(P - z')R_1}{r + (P - z')R_2}. \tag{13}$$

It is plausible to assume P-z'>0. If K'-s_r<0, R_1>0 and R_2>0. Then this curve is downward sloping. If K'-s_r>0, this curve may be upward sloping.

From (10), if K'-s_r>0, then $R_2 = \frac{\partial r_{t+1}}{\partial P_t} < 0$, which implies that above the rr curve $r_{t+1}<r_t$, and below the curve $r_{t+1}>r_t$. If P were not changed, above (below) this locus r will decrease (increase). If K'-s_r<0, vice versa. From (11),

$$\frac{\partial P_{t+1}}{\partial r_t} = (P - z')R_1 .$$

Thus, if $K' - s_r < 0$, $\frac{\partial P_{t+1}}{\partial r_t} > 0$. In other words, on the right-hand side of the PP curve $P_{t+1} > P_t$, and on the left-hand side of the curve $P_{t+1} < P_t$. If r were not changed, on the right (left) hand side of the PP curve P will increase (decrease). If $K' - s_r > 0$, vice versa.

The phase diagram will be divided to the following three cases, depending on the sign of $K' - s_r$. The equilibrium is always a saddle-point and hence the economy is unstable except only one convergent path. Perfect foresight means that the economy always chooses this convergent path.

Figure 8.1-(i) corresponds the case of $K' - s_r < 0$. Actually, this figure assumes $s_r + s_w w' - K' > 0$; the rr curve is upward sloping. In this case an increasing saving function with respect to the rate of interest may well be compatible with the dynamic property. Figures 8.1-(ii)(iii) correspond to the case of $K' - s_r > 0$. When $K' - s_r > 0$, the rr curve is downward sloping but the PP curve may be upward sloping.

2.2 Dynamic Impact Of Policy Action

2.2.1 Tax-Financed Temporary Government Spending

The conceptual experiment through which we investigate the effects of fiscal action is the following. Suppose the economy before period 1 is in a stationary equilibrium with no government expenditures. In period 1 only the government unexpectedly makes a positive expenditure g. This requires the government to levy the amount g of lump sum wage taxes on generation 1. After the initial fiscal action, there will be no more surprises, so that the economy will return to an undisturbed equilibrium from period 2 forward.

We consider the tax finance of temporary government spending. The asset market clearing equation is for period 1 and period 2 respectively

$$s[w(r_1) - g, r_2] = K(r_2) + P_1 \tag{14}$$

$$s[w(r_2), r_3] = K(r_3) + P_2 \tag{15}$$

g enters in (14) because lump sum taxes reduce disposable income. Hence, in period 1 the rr curve shifts downward by an introduction of taxes on generation 1. Since the asset market clearing condition is dependent on g in period 1, the PP curve shifts as well. It is, however, difficult to see how the PP curve shifts.

It is now useful to introduce the LP curve defined by

$$1 + r = \frac{z(r) + P}{P}$$

Or,

$$P = \frac{z(r)}{r} \tag{16}$$

The equilibrium point may be given by the intersection of the rr curve and the LP curve. The slope of the LP curve is negative if the elasticity of z with respect to r is less than unity. As $z'(r)$ is likely negative, we assume this. The LP curve is useful for the comparative statics in that this curve is independent of the fiscal action.

By introducing the LP curve, it is easy to know how the convergent path to the new equilibrium will shift after the fiscal action. The intersection would be attained if the r'r' curve of period 1 were maintained forever. Put another way, this intersection will be associated with the convergent path in period 1. Hence, once we know the convergent path in period 1, it is easy to describe the transitional movement from E_1 to

E_2. Similarly, the intersection of the LP curve and the rr curve will determine the equilibrium point and the associated convergent path in period 2. From this information we can easily describe the transitional movement from E_2 to E_0.

2.2.2 Crowding-Out Case

Based on these dynamic properties, the impact of tax finance is described in Figure 8.1. In Figure 8.1-(i) the rr curve is upward sloping. In this case, in period 1, P jumps from E_0 to E_1, so that in period 2, the economy is at point E_2, which is on the convergent path toward E_0. During the transition P rises from period 1, and r is greater than the initial level. Intuitively, in period 1 disposable wage income decreases by taxation. Real saving declines, and hence P_1 and K_2 decline. In period 2, disposable income rises to the stationary level and hence saving rises. Thus P_2 and K_3 increase. Government spending will crowd out private capital accumulation to some extent. We have:

Proposition 1: If s_r is not so small ($K' < s_r$), government spending financed by lump sum taxes has a temporary crowding-out effect.

2.2.3 Crowding-In Case

Figure 8.1-(ii) describes the case in which the rr curve is downward sloping and the absolute slope of curve rr is less than that of curve LP. In this case saving is decreasing with r, but the negative elasticity of saving with respect to r is not so strong compared with case (iii). Saving in period 1 increases due to a reduction of r_2, but P_1 decreases. In period 2 as the government expenditure returns to the original level, saving is greater than investment on capital and land at the initial r and P. r rises to lower saving. r and P will increase to the original levels. During the transition r is less than the original level. We have a temporary crowding-in effect.

In Figure 8.1-(iii) the rr curve is downward sloping and the absolute slope of curve rr is greater than that of curve LP. When the negative elasticity of savings with respect to the rate of interest is large, saving in period 1 rather increases due to a reduction in r_2 (an increase in K_2), and hence P_1 rises. We also have the temporary crowding-in effect.

Thus, we have:

Proposition 2: If s_r is negative enough ($K' > s_r$), government spending financed by lump sum taxes has a temporary crowding-in effect.

During the transition we may have higher or lower asset prices than the equilibrium level in the crowding-in case. This is due to the dynamic behavior with the fixed asset. With productive capital and fixed land, perfect foresight requires saddlepoint equilibria, so P can jump instantaneously from the old convergent path to the new convergent path.

3. Money, Land, And Taxes

3.1 Analytical Framework

3.1.1 Introduction Of Money

We now consider the possibility that savings are held in the form of money, land, and capital, based on Ihori (1990b). This section considers the dynamic effects of tax financing on capital accumulation and land values in an inflationary economy.

For simplicity we assume that wage w and rent z are constant. A member of generation t now has a standard utility function with fixed labor supply

$$u_t = u(c_t^1, c_{t+1}^2, m_{t+1}). \qquad (17)$$

where m_{t+1} is the per capita real value of money held by generation t.

We now introduce capital gains taxes and land taxes. Namely, the government's expenditure is financed by (a) a land rent tax, τ, (b) a land value tax, β, (c) a capital gains tax, ε, and (d) monetary expansion. These land taxes are levied on the base of nominal income.

3.1.2 Arbitrage Between Capital And Land

The assets are titled to land, capital, and money. A unit of fixed asset, land, is a promise to pay z units of the consumption good as a rent each period forever. In this section we assume for simplicity that z and w are fixed, independent of r. The real price of a unit of land is a_t. p_t is the nominal price of goods and $P_t = a_t p_t$. The nominal (after-tax) interest factor on an asset (land) purchased in period t and sold t+1, R_{t+1}, is defined as

$$\frac{(1-\tau)zp_{t+1} + (1-\beta)p_{t+1}a_{t+1} - \varepsilon(p_{t+1}a_{t+1} - p_t a_t)}{a_t p_t} = 1 + R_{t+1}. \qquad (18)$$

Using the definition of the rate of inflation, $\pi_{t+1} = p_{t+1}/p_t - 1$, (18) may be rewritten as

$$(1+\pi_{t+1})\left[\frac{(1-\tau)z + (1-\beta)a_{t+1}}{a_t}\right] - \varepsilon\left[(1+\pi_{t+1})\frac{a_{t+1}}{a_t} - 1\right] = 1 + R_{t+1} \qquad (18)'$$

The real rate of interest from period t to t+1, r_{t+1}, is defined as

$$(1+r_{t+1})(1+\pi_{t+1}) = 1 + R_{t+1}. \qquad (19)$$

Given perfect foresight without uncertainty, capital and land are perfect portfolio substitutes and have the same rate of return r.

Thus, in the long run, substituting $\pi_{t+1} = \pi$ and $a_{t+1} = a_t = a$ into (18)' and considering (19), we have as the arbitrage condition

$$\frac{(1-\tau)z + (1-\beta)a}{a} - \frac{\varepsilon\pi}{1+\pi} = 1 + r. \qquad (20)$$

We have a positive capital gains tax in the long run so long as the rate of inflation is positive. This is because a capital gains tax is imposed on nominal capital gains (π a). Changes in the nominal prices (nominal price of land ap and nominal price of good) are a measure of the capital gains or losses incurred by current asset holders.

From (20), we have

$$a = \frac{(1-\tau)z}{r + \beta + \dfrac{\varepsilon\pi}{1+\pi}}. \qquad (20)'$$

An increase in a capital gains tax will increase the discount rate and hence reduce the price of land for given r and $\pi > 0$. In this sense, the effect of ε is the same as the effect of β.

3.1.3 Consumers' Behavior

Considering the arbitrage condition (18), the budget constraints (5) and (6) are rewritten as

$$c_t^1 = w - (1+\pi_{t+1})m_{t+1} - s_t \qquad (21\text{-}1)$$

$$c_{t+1}^2 = m_{t+1} + (1+r_{t+1})s_t \qquad (21\text{-}2)$$

and then the demand functions are respectively

$$c_t^1 = c^1(r_{t+1}, \pi_{t+1}), \tag{22-1}$$
$$c_{t+1}^2 = c^2(r_{t+1}, \pi_{t+1}), \tag{22-2}$$
$$m_{t+1} = m(r_{t+1}, \pi_{t+1}), \tag{22-3}$$

and the real saving function for capital and land is

$$s_t = s(r_{t+1}, \pi_{t+1}), \tag{23}$$

where $c_{2r} = \partial c^2 / \partial r > 0$, $m_r = \partial m / \partial r < 0$, $m_\pi = \partial m / \partial \pi < 0$ from the normality assumption. By the budget constraint, we have $c_{1r} + c_{2r} > 0$. An increase in r leads to an increase in c^2, and hence the aggregate demand for consumption c^1+c^2 increases.

3.1.4 Government's Behavior

The government budget constraint is, for period t,

$$T_t + [(1+\pi_{t+1})m_{t+1} - m_t] = g, \tag{24}$$

where $T_t = \tau z + \beta a_t + \varepsilon(a_t - \frac{1}{1+\pi_t}a_{t-1})$ means the total real revenue from the three land taxes.

In the long run we have

$$T = \tau z + \beta a + \varepsilon \frac{\pi}{1+\pi} a.$$

Thus, considering (20), we have

$$z - ra = T. \tag{25}$$

Real land tax revenues are equal to rental income minus net returns on land. Land taxes (T > 0) imply that the price of land (a) is lower than the discounted value of future rental income (z/r). For given z, ar is a decreasing function of T in the long run. However, *a* might rise by an increase in T if r would decline sufficiently.

3.1.5 Equilibrium

Given a sequence of fiscal action and an initial stock of money supply, an equilibrium is a sequence in which markets clear under perfect foresight starting at t=1. More formally, it is a sequence $\{a_t, r_{t+1}, \pi_{t+1}\}_{t=1}^{\infty}$ that satisfies the arbitrage condition (18), the government budget constraint (24), and the goods-market-clearing condition:

$$c_t^1 + c_t^2 = Y(r_t, r_{t+1}) - g \text{ for all } t > 1, \tag{26}$$

where c^1_t, c^2_{t+1}, and m_{t+1} are given by the demand functions (22-1), (22-2), and (22-3) for t > 1[1], and $Y(r_t, r_{t+1}) \equiv y(r_t) + K(r_t) - K(r_{t+1})$. y is output and K is capital. Both y and K are negatively related to r as shown in (1) and (4).

The consumption c^2_t of the initial old is not given by the demand function (22-2) because our definition of equilibrium does not require that that generation's expectations be fulfilled. Unexpected movements in a_1 and p_1 confer capital gains or losses on the initial old generation. In equilibrium the asset-market-clearing condition

$$s_t = a_t + K_{t+1}, \tag{27}$$

holds for all t > 1.

3.1.6 Stability

The dynamic system may be summarized by

$$c^1(r_{t+1}, \pi_{t+1}) + c^2(r_t, \pi_t) = Y(r_t, r_{t+1}) - g, \tag{28}$$
$$(1+\pi_{t+1})m(r_{t+1}, \pi_{t+1}) - m(r_t, \pi_t) + T = g. \tag{29}$$

From (28) r_{t+1} is given by a function of π_{t+1}, r_t, and π_t.

$$r_{t+1} = R(\pi_{t+1}, r_t, \pi_t). \qquad (30)$$

From (29) π_{t+1} is given by a function of r_{t+1}, r_t, and π_t.

$$\pi_{t+1} = M(r_{t+1}, r_t, \pi_t). \qquad (31)$$

Substituting (31) into (28), we have

$$c^1[r_{t+1}, M(r_{t+1}, r_t, \pi_t)] + c^2(r_t, \pi_t) = Y(r_t, r_{t+1}) - g. \qquad (32)$$

Substituting (30) into (29), we have

$$(1+\pi_{t+1})m[R(\pi_{t+1}, r_t, \pi_t), \pi_{t+1}] - m(r_t, \pi_t) + T = g. \qquad (33)$$

The dynamic evolution of the model is entirely determined by (32) and (33).

The local stability of the equilibria can be analyzed mathematically. From (32) and (33), we obtain at the neighborhood of an equilibrium

$$\begin{bmatrix} d\hat{r}_{t+1} \\ d\hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} d\hat{r}_t \\ d\hat{\pi}_t \end{bmatrix}$$

where $d\hat{r} = \hat{r} - r^*$, $d\hat{\pi} = \hat{\pi} - \pi^*$, and ($r^*$, π^*) is an equilibrium. In the above expression

$$A = -\frac{c_{1\pi}M_{r0} + c_{2r} - Y_{r0}}{c_{1r} + c_{1\pi}M_{r1} - Y_{r1}},$$

$$B = -\frac{c_{1\pi}M_{\pi} + c_{2\pi}}{c_{1r} + c_{1\pi}M_{r1} - Y_{r1}},$$

$$C = -\frac{(1+\pi)m_r R_{r0} - m_r}{m + (1+\pi)(m_\pi + m_r R_{\pi 1})},$$

$$D = -\frac{(1+\pi)m_r R_{\pi 0} - m_\pi}{m + (1+\pi)(m_\pi + m_r R_{\pi 1})},$$

where $c_{ir} = \partial c^i / \partial r$, $c_{i\pi} = \partial c^i / \partial \pi$ (i = 1, 2), $M_{r0} = \partial M / \partial r_t$, $M_{r1} = \partial M / \partial r_{t+1}$, $M_\pi = \partial M / \partial \pi_t$, $R_{r0} = \partial R / \partial r_t$, $R_{\pi 0} = \partial R / \partial \pi_t$, $R_{\pi 1} = \partial R / \partial \pi_{t+1}$, $Y_{r0} = \partial Y / \partial r_t$, and $Y_{r1} = \partial Y / \partial r_{t+1}$.

Following Chamley and Wright (1987), as explained in Section 2, we disregard all the stable equilibria and concentrate on the saddle-point unstable case. Let us analyze the sign of the characteristic polynominal of the matrix

$$\psi(\lambda) = \lambda^2 - (A+D)\lambda + (AD - BC). \qquad (34)$$

There exists a unique dynamic path near the steady state if $\psi(1) < 0$ and $\psi(-1) > 0$.

Suppose for simplicity $c_{1\pi} = 0$. Then in the long run we have

$$1 - A = \frac{c_{1r} + c_{2r} - y'}{c_{1r} + K'},$$

$$B = -\frac{c_{2\pi}}{c_{1r} + K'},$$

$$C = -\frac{(1+\pi)m_r(y' + K' - c_{2r}) - m_r(K' + c_{1r})}{(c_{1r} + K')[m + (1+\pi)m_\pi]},$$

$$1 - D = \frac{(c_{1r} + K')(m + \pi m_\pi) - (1+\pi)m_r c_{2\pi}}{(c_{1r} + K')[m + (1+\pi)m_\pi]}.$$

Then, we have

$$\psi(1) = (1-A)(1-D) - BC$$
$$= \frac{(c_{1r} + c_{2r} - y')(m + \pi m_\pi) - c_{2\pi} m_r \pi}{(c_{1r} + K')[m + (1+\pi)m_\pi]}$$
$$\psi(-1) = (1+A)(1+D) - BC$$
$$= \frac{(c_{1r} + 2K' + y' - c_{2r})[m + (2+\pi)m_\pi] + (2+\pi)m_r c_{2\pi}}{(c_{1r} + K')[m + (1+\pi)m_\pi]}$$

We know y' < 0, K' < 0, $c_{1r} + c_{2r} > 0$, $m_r < 0$, and $m_\pi < 0$. Then the following set of conditions will satisfy the saddle-point instability condition:

$m+(2+\pi)m_\pi > 0$

$c_{1r} < 0$

$c_{1\pi} = 0$

$0 < c_{2\pi}$

Such a set of conditions will be satisfied if m is more substitutable with c^2 than with c^1. The set of stability condition implies $s_r > 0$. Under the saddle-point stability condition the economy is unstable except only one convergent path. Perfect foresight means that the economy always chooses this convergent path.

From (28) and (29) it is easy to see that the dynamic paths of r_t and π_t are independent of land tax parameters (τ, β, ε) so long as the total revenue from land taxes is fixed. We have:

Proposition 3: Changes in the land rent tax, the land value tax, or the capital gains tax will not affect the real equilibrium of the economy so long as the total tax revenue from the land taxes is constant.

An increase in the land rent tax will have the same effect on the real economy as an increase in the land value tax or the capital gains tax if the resulting increase in the land tax revenue is the same. Note that Proposition 3 holds in a non-monetary economy too. This is because the arbitrage condition (18) can be expressed in terms of the total tax revenue from land taxes, T.

$$\frac{zp_{t+1} + p_{t+1}a_{t+1}}{a_t p_t} - T_{t+1}\frac{p_{t+1}}{a_t p_t} = 1 + R_{t+1}$$

We are now ready to consider the impact of fiscal action which would change the total revenue from the land taxes. We consider two cases: (1) the government will increase its government expenditure (the balanced budget incidence) or (2) the government will reduce lump sum taxes (the differential incidence)[2]. After the initial fiscal action, there will be no more surprises, so that the economy will be on a new undisturbed convergent path.

3.2. An Increase In Government Expenditure

3.2.1 Long Run Effect

The long run equilibrium is given as

$$c^1(r,\pi) + c^2(r,\pi) = y(r) - g, \qquad (28)'$$

$$\pi m(r,\pi) + T = g. \qquad (29)'$$

We are interested in the case where T < g and hence $\pi > 0$. The balanced budget incidence means dT = dg > 0. Totally differentiating (28)' and (29)' with respect to r and

π, we have

$$\begin{bmatrix} c_{1r}+c_{2r}-y' & c_{1\pi}+c_{2\pi} \\ \pi m_r & m+\pi m_\pi \end{bmatrix}\begin{bmatrix} dr \\ d\pi \end{bmatrix}=\begin{bmatrix} -1 \\ 1 \end{bmatrix}dg+\begin{bmatrix} 0 \\ -1 \end{bmatrix}dT. \tag{35}$$

Considering dg = dT, we have

$$\frac{dr}{dT}=-\frac{1}{\Delta}(m+\pi m_\pi), \tag{36}$$

where Δ is the determinant of the matrix of the left-hand side of (35).

Let us investigate the sign of Δ. We know that $c_{1r}+c_{2r}-y'>0$. From the saddle point stability condition we have $m+\pi\, m_\pi>0$ and $c_{1\pi}+c_{2\pi}=c_{2\pi}>0$. Thus, $\Delta>0$. If dg = 0, from (35) we know $d\pi/dT<0$ if and only if $\Delta>0$. In such a case an increase in land tax revenues is associated with a decrease in inflationary taxes. Thus, $\Delta>0$ is intuitively plausible. Therefore, $dr/dT<0$. In other words, we have

Proposition 4: An increase in the government expenditure financed by land taxes will reduce the real rate of return on land and capital, and hence it will stimulate capital accumulation.

Similarly, we have

$$\frac{d\pi}{dT}=\frac{1}{\Delta}\pi m_r<0. \tag{37}$$

A decrease in r means an increase in real money demand m. Inflationary taxes would rise at the initial π, so that the rate of inflation must decrease to maintain the government budget. This effect is induced by the substitution from land to money in the portfolio of savings. In other words, we have:

Proposition 5: An increase in the government expenditure financed by land taxes will reduce the long run rate of inflation (i.e., the expansion rate of monetary growth) and hence the nominal price of land.

What is the long run effect on the real price of land *a*? Remember that $s=a+K=w-c^1-(1+\pi)m$. Considering the government budget constraint, the balanced budget incidence means $d(\pi m)=0$. Hence, we have

$$da=-dK-dc^1-dm=-K'dr-(c_{1r}dr+c_{1\pi}d\pi)-(m_r dr+m_\pi d\pi). \tag{38}$$

We know $K'<0$, $m_r<0$, $m_\pi<0$, $dr<0$ and $d\pi<0$. From the stability condition $c_{1r}<0$, $c_{1\pi}=0$. Therefore, $da<0$. It follows:

Proposition 6: An increase in the government expenditure financed by land taxes will reduce the real value of land in the long run.

In the partial equilibrium analysis, r and π are supposed to be given. Then from (25) an increase in land taxes will reduce *a*. Our general equilibrium approach produces the similar qualitative result concerning on changes in *a*[3].

3.2.2 Transitionary Effect

We now consider the transitionary effects. Let us investigate dynamic properties of this economy using a phase diagram like Figure 8.2. To analyze the behavior of r_t, we find the locus of (π, r) where $r_t=r_{t+1}$ in (30). We call this locus the rr curve. To find the behavior of π_t, we investigate the locus of (π, r) where $\pi_t = \pi_{t+1}$ in (31). We call this locus the $\pi\pi$ curve. It is easy to see that curve rr is downward sloping, while curve $\pi\pi$ is upward sloping under the saddle-point stability condition. π_0 can jump instantaneously to the convergent path so that we can solve the indeterminacy problem.

From the long run comparative statics analysis we know that the new equilibrium point E* is to the south-west of the initial equilibrium point E_0 in Figure 8.2. Thus, in period 1 π jumps from E_0 to E_1, so that in period 1 the economy is on the new convergent path toward E*.

How are a_1 and c^2_1 affected? In period 1, we have $c^1_1 + c^2_1 = y_1 + K_1 - K_2 - g$ and $(1+\pi_2)m_2 - m_1 + T = g$. Since r_2 and π_2 are less than the initial long run equilibrium values, c^1_1 and m_2 of generation 1 are greater than the initial equilibrium values. In other words, c^1_1 is higher than the initial equilibrium value c^1_0. Furthermore, g rises. Hence, c^2_1 of generation 0 must be lower than the (expected) old equilibrium value. The existing old generation will suffer from unexpected capital losses.

Note that m_1 is defined by M_1/p_1, and M_1 is exogenously given by the history. Changes in m_1 are associated with changes in p_1. As π_1 is lower than the old equilibrium value, p_1 is lower and m_1 is higher than the old equilibrium value. Intuitively, an increase in m_2 means an increase in inflationary taxes at the initial m_1. In order to maintain the government budget, the government must "transfer" these revenues to the existing older generation as a form of unexpected capital gains on money holding.

From (20)', when π_1 decreases, a_1 must decrease so long as $1 > \beta + \varepsilon$. Thus, a_1 will normally decline. We have:

Proposition 7: The existing old generation gets unexpected capital gains from money holding, but it suffers from unexpected capital losses from land holding.

As far as the effect on the second-period consumption is concerned, the latter effect is stronger than the former effect: the second-period consumption decreases unexpectedly.

3.3 A Decrease In Lump Sum Taxes

3.3.1 Long Run Effect

In this case the long run equilibrium is given as

$$c^1(r,\pi,L) + c^2(r,\pi,L) = y(r) - g, \tag{39}$$

$$\pi m(r,\pi,L) + T + L = g, \tag{40}$$

where L is a lump sum tax in the younger period. The differential incidence means dT+dL = 0.

Totally differentiating (39) and (40), we can easily derive $d\pi/dT < 0$. In other words,

Proposition 8: A tax reform from lump sum taxes to land taxes (an increase in land taxes with a decrease in lump sum taxes) will reduce the long run rate of inflation.

Intuitively, a decrease in lump sum taxes will raise the demand for money holdings and hence inflationary taxes as well. In order to maintain the government budget, a decrease in monetary expansion is necessary, and hence the rate of inflation will decline.

The sign of dr/dT is ambiguous. A decrease in lump sum taxes (dT = -dL > 0) will lead to an excess demand for the good market for given r. If this happens, dr/dT is likely to be negative. However, as the rate of inflation declines, the good market might be in a state of excess supply if $c_{1\pi} + c_{2\pi}$ is large. If c^2 and m are highly substitutable, this would be the case, and hence dr/dT > 0.

Proposition 9. If $0 < \frac{(c_{1L} + c_{2L})(m + \pi m_{\pi})}{\pi m_L} < c_{2\pi}$, then dr/dT will be positive: we have the 'crowding-out' case where land taxes will increase the real rate of return on land and capital.

This 'crowding-out' case will occur only in the inflationary economy. This case is induced by the substitution from land and capital to money in the portfolio of savings.

3.3.2 Transitionary Effect

We now consider the transitionary effects. π_1 will decline. As in subsection 3.2 we may derive that c^2_1 decreases. The existing old generation will suffer from unanticipated capital losses on land holding. The effect on m_1 is the same as before. π_1 decreases, p_1 decreases, and hence m_1 increases. The existing old generation will get unexpected capital gains from money holding. Our results in this section are summarized in Table 8.1.

3.4 Remark

We have examined the effects of land taxes on the price of land and on the rate of inflation when factors of production are fixed in supply. We explicitly considered the general equilibrium effects of land rent taxes, land value taxes, and capital gains taxes in an inflationary situation. Land taxes may affect the real equilibrium only through changes in the total revenue from the land taxes.

Our dynamic analysis of the overlapping generations model shows that the inclusion of a fixed asset, such as land, is important in determining the incidence of fiscal policies. As explained by Chamley and Wright (1987), public spending financed by lump-sum wage taxes will normally reduce capital accumulation, while public spending financed by land taxes will stimulate capital accumulation. Feldstein (1977) and Chamley and Wright (1987) obtained the "surprising" results; that if $s_r < 0$, land taxes may increase the price of land in a non-monetary economy.

Our dynamic incidence results in section 3's monetary economy are quite different from the results obtained in a non-monetary economy. First, by incorporating monetary considerations into a model of land, we have shown that the nominal price of land will normally decrease by land taxes, while the real price of land might rise. Second, our stability condition implies that real saving is an increasing function of the real rate of return. Nevertheless, the real price of land may rise if the income effect due to a reduction in lump sum taxes in the younger generation is dominant. Third, it seems likely that the real price of land will initially decrease even if the long-run real price of land will rise in the "surprising" case. Finally, we may have the crowding-out case where the differential land taxes will increase the real rate of return on capital. This effect is induced by the substitution from land and capital to money in the portfolio of savings, which will occur only in the three-asset inflationary economy.

Table 8.1: Dynamic Incidence Results

	balanced budget incidence	*differential incidence*
r	-	-*
π	-	-
a	-	?**
c^1	+	+
c^2	-	-
m	+	+
c^2_0	-	-
m_0	+	+
π_1	-	-
a_1	-	-

*. If $0 < \frac{(c_{1L} + c_{2L})(m + \pi m_\pi)}{\pi m_L} < c_{2\pi}$, this sign will be positive.

**. If the income effect due to a reduction of lump sum taxation is stronger than the substitution effect due to changes in r and π, this sign may be positive.

[1] . We implicitly assume that money supply is endogenous so as to meet the government budget constraint. As is well known, one of the policy instruments is given by the government budget constraint. Since we control government expenditures g and the total tax revenue from land taxes T, money supply must be endogenous.

[2] . Our fiscal action assumes $M_2 = M_1$, although M_t $(t > 3)$ changes so as to satisfy the government budget constraint.

[3] . We implicitly assume that an increase in g will not affect the marginal choice of the consumption-saving behavior. This is satisfied if the utility function is additively separable between consumption and government expenditures.

Chapter 9

Government Debt

1. Introduction

Through Chapter 9 we introduce government debt into the basic model. We first show that the tax-financed transfer payments and Diamond's debt have the same effect on the long-run equilibrium. We also show that if lump sum taxes are appropriately adjusted, debt policy is not effective and hence the government deficit is a meaningless policy indicator. We then examine the burden of debt and show that an increase in a constant amount of government debt per worker will crowd out capital accumulation in the long run. Section 2 also analyzes economic activities of government by introducing government capital.

Section 3 then investigates the role of government debt in the altruism model. Barro (1974) extended Ricardian neutrality to the strongest proposition of Barro's debt neutrality. Under certain conditions debt policy is meaningless even if lump sum taxes are not adjusted appropriately among generations. Further, Barro studied the effect of debt policy in the altruism model of overlapping generations. The altruistic model means that households can be represented by the dynasty who would act as though they were infinitely lived. He showed that public intergenerational transfer policy becomes ineffective once we incorporate altruistic bequests into the standard overlapping generations model. This section explains his idea intuitively.

The events of the 1970s and 1980s suggest that when a government becomes strapped for funds, it will tend to borrow from the world credit market rather than raise taxes to finance additional public spending. Indeed, many governments will either not raise broadly based taxes, e.g., the Thatcher government in Great Britain or the Reagan and Bush Administrations in the United States, or simply cannot raise taxes to prevent causing riots, e.g., countries in Latin American and Eastern Europe, and, arguably, France in the reign of Louis XVI[1].

The so-called chain-letter mechanism (or a Ponzi debt game) involves a situation where the future time path of taxes is fixed and debt finance is used to pay for any additional public spending; debt issuance is thus endogenously determined by the government's budget constraint. If the mechanism is successful, increased taxation need not necessarily be required in order to finance increased government spending as the economy converges to the steady state equilibrium. If the mechanism is unsuccessful, the government will eventually go bankrupt in the sense that it will be unable to raise enough revenue to finance public spending and debt repayment. As debt crowds out private capital formation, the economy will also eventually go bankrupt if the mechanism fails. This suggests that studying the chain-letter mechanism and associated austerity measures is quite important in terms of understanding the effects of government activity on the economy.

Therefore, Section 4 of this chapter therefore studies the dynamic effects of various policy alternatives. These alternatives are the ones available to a government confronting a potential debt crisis such as a decrease in the level of the public good and a decrease in the marginal cost of providing the public good.

Finally, Section 5 investigates the dynamic implications of future tax reform in a debt-financed economy. The young hold government debt to provide for old-age consumption. This requires confidence and trust. But no one can guarantee to the young that the rate of return on debt is the same as that of real capital. Put another way, it is not sure to what extent the debt burden will be transferred to the next generation. This depends on the possibility of future tax reform. Valuation of an intrinsically useless and unbacked asset performing intergenerational transfers from the young to the old requires enough confidence that this asset will not be worthless in the future. Section 5 considers how expectations of the future tax reform affect the efficacy of fiscal policy.

2. Government Debt And Intergenerational Transfer

2.1 Transfer Program

We shall assume that the government issues debt b_t to the younger generation in period t. This debt has a one-period maturity and will be repaid in the next period with interest at the same rate of return as on capital. b can be negative, in which case b means "negative debt"; that is, the government lends b to each individual of the younger generation and will redeem this debt with interest.

Let us denote the (per-capita) lump sum tax levied on the younger generation and the older generation in period t by T^1_t and T^2_t, respectively. Suppose for simplicity that the government does not spend any public expenditures. Then the government budget constraint in period t is

$$b_{t-1}N_{t-1}(1+r_t) - b_tN_t = T^1_tN_t + T^2_tN_{t-1}. \tag{1}$$

where N_t is the number of generation t.

The following cases are of considerable interest:

(a) $T^2 = 0$: The tax collected to finance interest costs minus new debt issuance is lump sum taxes on the younger generation. This debt issue corresponds to Diamond's internal debt.

(b) $T^1 = 0$: The tax collected to finance interest costs minus new debt issuance is lump sum taxes on the older generation.

(c) $b = 0$: The government does not issue debt. The government levies the lump sum tax T^1 on the younger generation and transfers it to the older generation in the same period. This corresponds to the unfunded pay-as-you-go system.

The private budget constraints of generation t are as follows:

$$c^1_t = w_t - s_t - b_t - T^1_t, \tag{2}$$

$$c^2_{t+1} = (s_t + b_t)(1+r_{t+1}) - T^2_{t+1}. \tag{3}$$

Each individual's disposable income ($\hat{w}_t$) is given by (w_t-T^1_t), his disposable income in the younger period t minus ($T^2_{t+1}/(1+r_{t+1})$) the present value of the tax in the older period t+1. Thus, the lifetime budget constraint is given as

$$c^1_t + \frac{1}{1+r_{t+1}}c^2_{t+1} = \hat{w}_t, \tag{4}$$

where $\hat{w}_t = w_t - T^1_t - \frac{1}{1+r_{t+1}}T^2_{t+1}$.

The optimizing behavior of each individual is represented by maximization of $u(c^1_t, c^2_{t+1})$, substituting $\hat{w}_t$ for w_t in the basic model of Chapter 2. The capital accumulation equation is

$$N_ts_t = N_{t+1}k_{t+1}. \tag{5}$$

Considering (2), (5) may be rewritten as

$$w_t - c^1(\hat{w}_t, r_{t+1}) - b_t - T_t^1 = (1+n)k_{t+1}. \tag{6}$$

where n is the rate of population growth.

Let us define effective taxes by

$$\tau_t^1 = b_t + T_t^1, \tag{7-1}$$

$$\tau_{t+1}^2 = -(1+r_{t+1})b_t + T_{t+1}^2. \tag{7-2}$$

τ^1 and τ^2 are net receipts from the young and old. Thus, the dynamic equilibrium will be summarized by the following two equations;

$$\tau_t^1 + \frac{1}{1+n}\tau_t^2 = 0, \tag{1'}$$

$$w(r_t) - c^1[w(r_t) - \tau_t^1 - \frac{\tau_{t+1}^2}{1+r_{t+1}}, r_{t+1}] - \tau_t^1 = -(1+n)w'(r_{t+1}). \tag{6'}$$

In other words, fiscal action is completely summarized by a sequence of effective taxes $\{\tau^1_t\}$ and $\{\tau^2_t\}$. One of b, T^1, and T^2 is redundant to attain any fiscal policy. The three cases (a), (b), and (c) are equivalent so long as two of T^1, T^2, and b are adjusted to attain the same $\{\tau^1_t\}$ and $\{\tau^2_t\}$. In cases (a) and (b), the government budget is not balanced. But in case (c), the government budget is balanced since b = 0. This means that the government deficit is not a useful policy indicator to summarize the fiscal action. We have (Kotlikoff (1992)):

Proposition 1: If lump sum taxes are appropriately adjusted, debt policy is not effective and the government deficit is a meaningless policy indicator.

Remark 1: The tax-financed transfer payments (case (c)) and Diamond's internal debt (case (a)) have the same effect on the competitive equilibrium. In other words, this national debt can be regarded as a device which is used to redistribute income between the younger and the older generations. Any intergenerational redistribution that can be supported by debt and taxes can also be supported just with taxes and without debt.

Remark 2: As Auerbach and Kotlikoff (1987a) and Buiter and Kletzer (1992) stressed, unfunded social security could easily be run as an explicit government debt policy. The government can label its social security receipts from young workers either 'borrowing' or 'taxes'. It can label benefit payments to retired people as either 'principal plus interest payments' on the government's borrowing or 'transfer payments'. The economy's real behavior is not altered by the relabeling. This makes one wary of relying on official government debt numbers as indicators of the government's true policy with respect to intergenerational redistribution. As is stressed by Kotlikoff (1992), generational accounting is a relatively new tool of intergenerational redistribution. It is based on the government's intertemporal budget constraint which requires that the government's bill be paid by current or future generations. Fehr and Kotlikoff (1995) show how changes in generational accounts relate to the generational incidence of fiscal policy.

Remark 3: Proposition 1 shows that if lump sum taxes are appropriately adjusted among generations, debt policy is meaningless. The government deficit is not a useful policy indicator. This result is due to Ricardian debt neutrality; the agent is concerned only with the lifetime budget constraint and the period-to-period budget constraint is meaningless. However, as shown in Section 2.2, this result does not necessarily deny the effectiveness of fiscal policy with respect to intergenerational redistribution. As shown in (1)' and (6)', changes in $\{\tau^1_t\}$ and $\{\tau^2_t\}$ have the real effect.

2.2 Burden Of Debt

If there is no freedom to adjust lump sum taxes appropriately, then changes in government debt has real effects. This case has been investigated as the topic of debt burden[2]. Let us define the relative burden ratio v by

$$v_t = \frac{T_t^2}{T_t^1(1+n)}, \tag{8}$$

which is assumed to be constant. When v is exogenously fixed, changes in b has real effects.

Suppose a constant amount of debt per worker (b) is maintained. Considering equation (6), the long-run competitive capital-labor ratio with debt policy $\bar{k}$(b, v) is determined by

$$a(\hat{w},r) = (1+n)k + b, \tag{9}$$

where a = s + b. From equation (1) and (8) we have

$$\hat{w}_t = w_t - \{\frac{r_t - n}{(1+v)(1+n)} + \frac{v(r_{t+1}-n)}{(1+r_{t+1})(1+v)}\}b. \tag{10}$$

Substituting (10) into (9) and taking the total derivative of k with respect to b in a steady state, we have

$$\frac{dk}{db} = \frac{\frac{\partial a}{\partial b} - 1}{1+n-\frac{\partial a}{\partial k}}. \tag{11}$$

As in Chapter 2, from the assumption of the stability of the system, the denominator is positive and from the assumption of the normality of the utility function the numerator is negative. Therefore dk/db is definitely negative. This result is referred to the burden of debt. We have similar to Diamond (1965):

Proposition 2. An increase in a constant amount of government debt per worker will crowd out capital accumulation in the long run.

Recall in Chapter 2 Section 3.3 that the OT curve summarizes the steady state consumption behavior as in Figure 9.1. Now from (1)(2)(3)(8) the steady state consumption possibilities with government debt may be rewritten as

$$c^1 = f(k) - (1+n+f')k - \frac{1+f'+v(1+n)}{(1+n)(1+v)}b, \tag{12-1}$$

$$c^2 = (1+n)(1+f')k + \frac{1+f'+v(1+n)}{1+v}b. \tag{12-2}$$

The two equations (12-1) and (12-2) imply that the steady state consumption-possibility curve now depends on b and v as well as k. The last terms in (12) $-\frac{1+f'+v(1+n)}{(1+n)(1+v)}b$ and $\frac{1+f'+v(1+n)}{1+v}b$ reflect transfer programs of the government.

The government collects the amount of $\frac{1+f'+v(1+n)}{(1+n)(1+v)}b$ per capita from the younger generation and transfers it to the older generation.

When $b > 0$ ($b < 0$), curve OT will shift upward and to the left (downward and to the right). Suppose that the government is only concerned with long-run welfare. The government can attain the golden rule by choosing b appropriately. In other words, in Figure 9.1 on the left where H is on GB, b needs to be positive, and in Figure 9.1 on the right where H is on GA, b needs to be negative. H means the golden rule point with government debt, while G means the golden rule point without government debt.

Proposition 3: If the long-run competitive capital-labor ratio without debt k_L is greater than the golden rule ratio k_G , the government debt b needs to be positive, and vice versa.

The slope of the indifference curve at the long-run competitive capital-labor ratio with debt policy $\bar{k}$ (b,v) equals the slope of the budget constraint $-(1+f'(\bar{k}))$, while the slope of the indifference curve at H equals -(1+n). Thus, if the government is only concerned with long-run welfare, the optimal level of b is such that $\bar{k}$ (b, v) is equal to k_G, the golden rule capital-labor ratio. The utility of each individual living in the long run will be increased by increasing b whenever $\bar{k}$ (b,v) $> k_G$, and by decreasing b whenever $\bar{k}$ (b,v) $< k_G$. The government can shift curve OT so that H is just associated with the competitive capital-labor ratio $\bar{k}$ (b,v).

2.3 Economic Activities Of Government

We now examine economic activities of government. Government capital investment will be financed from debt issue. We assume that the technology of government production is the same as that of private production. Since receipts from the debt issue in period t will be invested in production as government capital in period t+1, we have

$$N_t b = N_{t+1} k_{s.t+1}, \tag{13}$$

where $k_{s.t}$ denotes per worker government capital in period t. Rewriting per worker private capital in period t as $k_{p.t}$, per worker total capital in period t is defined by

$$k_t = k_{s.t} + k_{p.t}. \tag{14}$$

Because of its economic activities, the government need not levy taxes: $T^1 = T^2 = 0$. Hence, the disposable income is w_t, and the lifetime private budget constraint (4) will be the same as in the basic model. From (13) and (14) in a steady state we have

$$k = k_p + \frac{b}{1+n}. \tag{15}$$

In this case (9) may be rewritten as

$$a(w_t, r_{t+1}) = (1+n)k_{p\,t+1} + b \tag{9'}$$

Substituting (15) into (9)', we have

$$a(w_t, r_{t+1}) = (1+n)k_{t+1}$$

which means that government capital does not affect the real economy. It is easy to find that the consumption-possibility curve corresponding to government capital is nothing more than the original OT curve in Chapter 2. It is true that the long-run private capital labor ratio $\bar{k}_p$ depends on b, but the long-run capital labor ratio $\bar{k}$ is independent of b.

This means that public investment would exactly replace private investment. In other words, the long-run effects of public investment are null. The government neither levies a tax nor grants a subsidy, and promises to pay the same interest rate on the debt as could be earned on the purchase of real capital. Since the budget constraint is expressed by the same equation in the market process, the government cannot redistribute income between generations. This case is equivalent to the fully funded social security system[3].

3. Debt Neutrality With Altruistic Bequests

3.1 Barro's Model

Barro (1974) extended the neutrality result of Proposition 1 (Ricardian neutrality) to the strongest proposition of Barro's debt neutrality. Under certain conditions debt policy is meaningless even if lump sum taxes are not adjusted appropriately among generations.

Barro studied the effect of debt policy in the altruism model of overlapping generations. The altruistic model means that households can be represented by the dynasty who would act as though they were infinitely lived. He showed that public intergenerational transfer policy becomes ineffective once we incorporate altruistic bequests into the standard overlapping generations model. Let us explain intuitively his idea in this section.

A representative individual born at time t has the following budget constraints.

$$c_t^1 = w_t - s_t - b_t - T_t^1 + \frac{e_t}{1+n}, \tag{16}$$

$$c_{t+1}^2 = (1+r_{t+1})(s_t + b_t) - e_{t+1} - T_{t+1}^2, \tag{17}$$

where $e_t/(1+n)$ is the inheritance received when young, e_{t+1} is his bequests which is determined when old.

In the altruism model the parent cares about the welfare of his offspring instead of the bequest itself. The parent's utility function is given as

$$U_t = u_t + \sigma_A U_{t+1}, \tag{18}$$

where u_t is utility from his own consumption: $u(c^1_t, c^2_{t+1})$. σ_A is the parent's marginal benefit of his offspring's utility.

An individual born at time t will solve the following problem of maximizing.

$$W_t = u[w(r_t) - s_t - b_t - T_t^1 + \frac{e_t}{1+n}, (1+r_{t+1})(s_t + b_t) - T_{t+1}^2 - e_{t+1}] +$$

$$\sigma_A\{u[w(r_{t+1}) - s_{t+1} - b_{t+1} - T_{t+1}^1 + \frac{e_{t+1}}{1+n}, (1+r_{t+2})(s_{t+1} + b_{t+1}) - T_{t+2}^2 - e_{t+2}] \tag{19}$$

$$+ \sigma_A U_{t+2}\}$$

The optimal conditions with respect to s_t and e_{t+1} are

$$\frac{\partial u}{\partial c_t^1} = (1+r_{t+1})\frac{\partial u}{\partial c_{t+1}^2}, \tag{20-1}$$

$$(1+n)\frac{\partial u}{\partial c_{t+1}^2} = \sigma_A \frac{\partial u}{\partial c_{t+1}^1}. \tag{20-2}$$

Since the first order conditions are independent of government debt, the public intergenerational policy due to debt issuance is completely neutral. It would not affect the real equilibrium. We have (Barro 1974):

Proposition 4: If the altruistic bequest motive is operative, the public intergenerational policy is neutral.

(20-1,2) give the long run rate of interest, r_A, in the altruism model r_A :

$$n = \sigma_A(1+r_A)-1, \tag{21}$$

which is independent of b. (21) is the same as (46) in chapter 2, where government debt was not incorporated.

Remark: Let us define effective bequests by

$$e_t^* = \tau_t^2 + e_t. \tag{22}$$

Recognizing (1)'(7-1)(7-2) and (22), (16) and (17) may be rewritten as

$$c_t^1 = w_t - s_t + \frac{1}{1+n}e_t^*, \tag{16)'$$

$$c_{t+1}^2 = (1+r_{t+1})s_t - e_{t+1}^*. \tag{17)'$$

and substituting (16)' and (17)' into (20-1,2), it is easy to see that (20-1,2) will determine the optimal path of $\{e^*_t\}$. Public intergenerational transfer through changes between τ^1 and τ^2 (or b, T^1, and T^2) is completely offset by appropriate changes in private transfer, e. When the government changes b, the private sector will change bequests so as to maintain the optimal path of effective bequests, which is determined by (20-1,2).

3.2 Theoretical Debate

The debt neutrality proposition requires a number of key assumptions about the economic environment and the behavior of economic agents. These assumptions include (1) perfect capital markets with no borrowing constraints on consumers, (2) nondistortionary taxes, (3) full certainty about the path of future taxes, government budget policies, and earnings, and (4) equal planning horizon for private and public sectors. Ricardian neutrality needs (1)-(3), while Barro's neutrality needs (1)-(4)[4]. We will discuss each of these cases.

(1) Borrowing Constraints

There is substantial evidence that at least a modest fraction of the population is liquidity constrained at a given point in time. Liquidity constraints raise the marginal propensity to consume out of temporary tax changes to a large multiple of the small amount predicted under perfect capital markets. Altig and Davis (1989) showed that borrowing constraints imply the nonneutrality of government debt irrespective of whether transfer motive operates. On the other hand, Hayashi (1987) provided examples from the literature on imperfect capital market in which debt neutrality holds despite the existence of borrowing constraints. His examples suggest that it is important to identify how the exact nature of imperfections in loan markets is identified.

(2) Distortionary Taxes

Distortionary taxes in general imply that financial policy may not be neutral. Changes in the timing of distortionary taxes can affect private sector and economy-wide allocation through their induced wealth, redistribution, and intertemporal substitution effects. They lead to deviations from debt neutrality. For example, Abel (1983) showed how a different type of non lump-sum tax, a progressive tax on bequests or capital, changes the relative cost of current consumption and bequests, and thus introduces an incentive to consume more at the present time.

(3) Uncertainty About Future Taxes And Earnings

Although the current tax cuts may indeed be associated with future increases in taxes, the exact timing, the type of tax to be increased, and the incidence of the tax across individuals are all uncertain. This uncertainty may lead to deviations from neutrality. Feldstein (1988) also showed that when earnings are uncertain the substitution of deficit finance for tax finance or the introduction of an unfunded social security program will raise consumption even if all bequests reflect intergenerational altruism. The uncertainty of future income means that bequests are also uncertain. This uncertainty of future bequests means that an individual will not generally be indifferent between receiving an additional dollar of income when he is young and his children later receiving an equivalent amount with a present value of one dollar.

(4) Different Planning Horizons For Private And Public Sectors

A necessary condition for Barro's debt neutrality to obtain is that households and government have the same planning horizons and they use the same discount factor in their present-value calculations. This condition is satisfied if the altruistic bequest motive is fully operative. Weil (1987b) addressed questions about the operativeness of an altruistic transfer motive in overlapping generations economies. His numerical analysis of a parametric version of his model indicates that parent must 'love their children' very much for the transfer motive to operate. On the other hand, Altig and Davis (1989) obtained a different conclusion: for reasonable lifetime productivity profiles and a modest desire to smooth consumption intertemporally, parents need love their children only a little bit for the transfer motive to operate in the loan economy. In any case the extent of altruistic transfer motives is a key determinant of the long-run and short-run savings response to government deficits.

Bernheim and Bagwell (1987) showed that if families were interconnected via altruism in complicated networks, any change in relative prices would be completely neutralized. They would completely rob the price system of its ability to allocate resources. This conclusion is simply untenable. It tends to cast serious doubt on the Barro model of altruism. In that case changes in the stock of debt will have real effects on the economy and a model, where the agent experiences a finite horizon can capture those effects in a reasonably tractable way. In this sense, Ricardian neutrality seems more plausible than Barro's neutrality.

Leiderman and Blejer (1990) and Seater (1993) presented a useful survey of empirical evidence on the impact of government budget variables on private consumption and on the debt neutrality hypothesis.

4. Chain Letter Problem

4.1. Theoretical Framework

The purpose of this section is to study the dynamic effects of various policy alternatives available to a government confronting a potential debt crisis[5]. The so-called chain-letter mechanism (or a Ponzi debt game) involves a situation in which the future time path of taxes is fixed and debt financing is used to pay for any additional public spending. Debt issuance is thus endogenously determined by the government's budget constraint. If the mechanism is successful, increased taxation need not necessarily be required in order to finance increased government spending as the economy converges to the steady state equilibrium. If the mechanism is unsuccessful, the government will eventually go bankrupt in the sense that it will be unable to raise enough revenue to finance public spending and debt repayment. As debt crowds out private capital formation, the economy will also eventually go bankrupt if the mechanism fails. This suggests that studying the chain-letter mechanism and associated austerity measures is quite important in terms of understanding the effects of government activity on the economy.

In this section we study the dynamic behavior of the model, where a public good is explicitly incorporated. We show that there is a unique convergent path with positive debt being issued. We also study the long run feasibility of the chain-letter mechanism given a change in the marginal cost of the public good and a change in the level of the public good itself.

As formulated in previous sections, saving is invested partly in real capital which does not depreciate and is a perfect substitute for the consumption good, and partly in government bonds purchased from the older generation. In the second period, the agent, leaving no bequests, consumes all of his accumulated wealth.

Consider the maximizing problem facing a consumer representative of generation t. He chooses c^1_t, c^2_{t+1}, s_t, and b_t to maximize $u(c^1_t, c^2_{t+1}, g)$ subject to $c^1_t = w_t - T - s_t - b_t$ and $c^2_{t+1} = (1+r_{t+1})(s_t+b_t)$, where T is an exogenously given lump sum tax levied in the first period of his life. We now include g into the utility function explicitly since we consider the effect of a change in public spending in this section as in Chapter 5.

Since government debt and real capital are both safe assets and perfect substitutes, the agent's portfolio composition is indeterminate. Then, the total amount of asset accumulation a (= s + b) will be a function of w, r, T, and g. Thus,

$$a = a(w, r, T, g). \tag{23}$$

As in the previous chapters, $1 > a_w = \partial a/\partial w > 0$, $a_r = \partial a/\partial r \geq 0$ and $-1 < a_T = -a_w = \partial a/\partial T < 0$. What is the effect of g on *a*? This depends on the effect of g on the marginal utility of consumption in each period. From the first order condition of the utility maximization problem, it is also easy to show that $a_g = \partial a/\partial g$ is positive if and only if $u_{1g}u_2 < u_{2g}u_1$, where $u_{ig} = \partial u_i/\partial g$. It is said that if u_{ig} is positive (negative), c^i and g are Edgeworth complements (substitutes). Thus, we establish

Proposition 5: If c^1 and g are Edgeworth substitutes (complements) and if c^2 and g are Edgeworth complements (substitutes), the total saving, *a*, is increasing (decreasing) with g.

The important point to notice is that the agent may respond differently to a change in g relative to a change in T.

There are a number of public goods which may affect asset accumulation. For example, police, firemen, and in a broad sense, national defense protect the accumulation of private property and would serve to increase the formation of capital. On the other hand, maintaining national forests and recreational areas might cause private agents to reduce the amount of capital they choose to accumulate.

A chain letter mechanism (or a Ponzi debt game) of debt finance means that taxes are predetermined and government debt issuance is endogenously determined by the government budget constraint. The government budget constraint in period t+1 is;

$$b_{t+1} = \frac{(1+r_{t+1})b_t}{1+n} + cg - T, \tag{24}$$

where c is the marginal cost of the public good assumed constant for simplicity. We will assume that T > cg and r > n in a steady state.

The bond and capital markets clear:

$$a(w_t, r_{t+1}, T, g) = -(1+n)w'(r_{t+1}) + b_t. \tag{25}$$

A government is solvent if it does not pursue policies that force the private sector into bankruptcy when there exists an alternative policy that would not do so. The private sector is bankrupt when the non-negativity constraints on consumption by the young, consumption by the old, or the capital stock become binding. The stock of public debt is limited by the condition that the total amount of resources taken by the government from the young, whether through borrowing or through taxes, cannot exceed the wage income of the young. As shown by Buiter and Kletzer (1992), if the government can make net transfer payments to a generation when it is young and impose net taxes on that generation when it is old, and if these transfer payments and taxes can grow at least at the rate of interest, Ponzi finance is possible. This is true regardless of the relationship between the interest rate and the growth rate, and regardless of whether or not the economy is dynamically inefficient or Pareto efficient.[6] If either of these assumptions is violated, the solvency constraint implies that the sequence of public debt discounted at the rate of interest converges to zero.

4.2 Dynamics Of Model

The dynamic system can be summarized by (24) and (25). Let us investigate the dynamic properties of this economy using a phase diagram in Figure 9.2. From (25) we have

$$r_{t+1} = R(r_t, b_t; T, g), \tag{26}$$

where

$$R_r = \frac{\partial r_{t+1}}{\partial r_t} = -\frac{a_w w'}{a_r + (1+n)w''}, \tag{27-1}$$

$$R_b = \frac{\partial r_{t+1}}{\partial b_t} = \frac{1}{a_r + (1+n)w''}. \tag{27-2}$$

To analyze the behavior of r_t, we first find the locus of (b, r) where $r_{t+1} = r_t$. We call this locus the rr curve. From (26) this locus is given as

$$r = R(r, b; T, g). \tag{28}$$

Totally differentiating (28), we have the slope of the rr curve as

$$\frac{db}{dr} = \frac{1-R_r}{R_b} = (1+n)w'' + a_w w' + a_r. \tag{29}$$

(29) is likely to be positive when the elasticity of substitution between capital and labor is large. Inequality $(1+n)w'' + a_w w' + a_r > 0$ is a local stability condition in the basic economy of Chapter 2. We will assume this condition holds.

Hence, the rr curve is upward sloping. From (27-2), $\partial r_{t+1} / \partial b_t$ is positive. Hence, above the rr curve $r_{t+1} > r_t$, and below this locus, $r_{t+1} < r_t$. If b were unchanged, above (below) this locus r will increase (decrease).

Next, consider the behavior of b_t. From (24) we have

$$b_{t+1} = \hat{B}(r_{t+1}, b_t; T, g, c).$$

Substituting (26) into the above equation, we have

$$b_{t+1} = \hat{B}[R(r_t, b_t; T, g), b_t; T, g, c] = B(r_t, b_t; T, g, c), \tag{30}$$

where

$$B_r = \frac{\partial b_{t+1}}{\partial r_t} = \frac{R_r b}{1+n}, \tag{31-1}$$

$$B_b = \frac{\partial b_{t+1}}{\partial b_t} = \frac{R_b b + 1 + r}{1+n}, \tag{31-2}$$

Totally differentiating (30) in the steady state, we have the slope of the bb curve as

$$\frac{db}{dr} = -\frac{R_r b}{r - n + R_b b} = \frac{a_w w' b}{(r-n)[a_r + (1+n)w''] + b}, \tag{32}$$

which is negative at the neighborhood of an equilibrium. The bb curve is downward sloping. From (31-1) we know that if b > 0 on the right-hand side of the bb curve, $b_{t+1} > b_t$, and on the left-hand side of the bb curve, $b_{t+1} < b_t$. If r were unchanged, on the right (left) hand side of the bb curve b will increase (decrease).

The dynamic properties of the system are depicted in the phase diagram in Figure 9.2. For given tastes, technology, and policy, there is a unique steady state equilibrium point E. From a stability point of view, point E is a saddle-point and hence unstable except along one convergent path aa.

There are three possibilities for the government's policy:

(i) eventual bankruptcy

Above the convergent path, b and r will eventually approach infinity. As b increases, savings of real capital will decrease. The economy ends up in a vicious circle where the government borrows to finance the interest payments on its continually increasing stock of public debt. As soon as public borrowing completely absorbs private saving, the stock of capital is exhausted, and all economic activities stop. It seems reasonable to call such a situation bankruptcy of the economy. The paths above the convergent path will not be equilibrium paths. The government cannot roll its debt over forever in dynamically efficient economies. If the future austerity programs are expected, however, it is possible for the economy to be above the convergent path for a while[7].

(ii) saddle-point equilibrium

If the economy is initially on the aa path, the economy eventually approaches point E; the chain-letter mechanism will be feasible in the long run. In the steady state, T = cg + (r-n)b and the government is able to issue new debt to finance the interest and redemption of its inherited stock of debt as the economy converges to the steady state equilibrium.

(iii) balanced budget policy:

If the economy is initially below the aa curve, it will eventually approach the point where b = 0. In that case, the government can attain the balanced budget of T = cg by reducing taxes or raising spending, both of which are most likely politically acceptable policies and such policies would in general be Pareto-improving.

4.3. Public Goods And Dynamics

4.3.1 Changes In The Marginal Cost Of The Public Good

Consider the effect of a change in the marginal cost of providing the public good, c. The interest rate and the stock of debt will both be higher in the new steady state equilibrium when c increases. Thus, an increase in the marginal cost of providing the public good will make the chain-letter mechanism less viable.

Consider the dynamic adjustment of the economy. The rr curve is independent of c while the bb curve will shift upwards when c decreases and downward when c increases. Suppose the economy is initially at point A in Figure 9.3 and thus above the convergent path. Then, a reduction in the marginal cost of the public good is required to attain point B on the new convergent path a'a'.

Proposition 6: A reduction in the marginal cost of the public good is effective to avoid bankruptcy.

Why is this particular experiment important? Most models of public goods are highly abstract. They usually assume a very simple cost structure for providing the public good. Embedded in the cost structure of providing the public good is a set of institutional arrangements, some of which involve procurement practices. If the government must impose an austerity program, one potential source of savings is the procurement process itself. An institutional reform which can lower the marginal cost of providing the public good can be made a part of the austerity program. Indeed, it may actually take a severe crisis to convince government bureaucrats and elected officials of the need for reforming the procurement process.

4.3.2 Changes In The Level Of Public Spending

Let us next investigate the implications of a change in the level of the public good. In general, the steady state response will depend on the response of capital accumulation to the provision of the public good. In terms of the dynamic adjustment of the economy, since g appears in both (26) and (30), a change in g will generally shift both the rr curve and the bb curve. From (26) and (30) we have

$$R_g = \frac{\partial r_{t+1}}{\partial g} = -\frac{a_g}{a_r + (1+n)w''}, \tag{27-3}$$

$$B_g = \frac{\partial b_{t+1}}{\partial g} = \frac{R_g b}{1+n} + c = \frac{-a_g b + (1+n)c[a_r + (1+n)w'']}{(1+n)[a_r + (1+n)w'']}. \tag{31-3}$$

As for the sign of a_g, we have three possibilities:

(i) $a_g = 0$

First, suppose that a change in public spending does not affect private saving. In this case from (27-3) $R_g = 0$. The rr curve is independent of g. From (31-3) $B_g > 0$, and an increase in g will shift the bb curve downwards, while a decrease in g will shift it upwards.

This case is essentially the same as 4.3.1. Suppose the government is following a path like A in Figure 9.3. Eventually, public debt will begin crowding out private capital accumulation and the economy will go bankrupt. The government can alleviate the possible bankruptcy of the economy by reducing the level of the public good. A once and for all reduction in g at point B will shift the bb curve upwards and it can shift the economy to a new convergent path converging to the new steady state equilibrium E_1. If the government is reluctant to reduce the level of the public good and postpones the once and for all reduction in g, it will become necessary at a later time to reduce the level of the public good by a larger amount in order to attain the new convergent path . The longer the postponement, the larger both r and b will be in the new steady state.

(ii) $a_g < 0$

From (27-3), $R_g > 0$. An increase in g will shift the rr curve to the right. From (31-3) $B_g > 0$: an increase in g will shift the bb curve downwards. And a decrease in g will shift the rr curve to the left and the bb curve upwards. When the economy is above the convergent path, the government can alleviate the possible future bankruptcy by reducing the level of the public good.

(iii) $a_g > 0$

Finally, consider the case of $a_g > 0$. From (28-3), $R_g < 0$: an increase in g will shift the rr curve to the left. From (31-3) the sign of B_g is ambiguous if $a_g > 0$. If $a_g > (1+n)c[a_r + (1+n)w'']/b$, then $B_g < 0$: an increase in g will shift the bb curve upwards. In such a case an increase in g will shift the convergent path upwards.

As before suppose the government is following a path like A in Figure 9.4. Assume first that $a_g < (1+n)c[a_r + (1+n) w'']/b$. If the government lowers the level of the public good at point A, then the convergent path will shift to a'a'. The economy will follow the path A-B-E_1. However, if $a_g > (1+n)c[a_r + (1+n)w'']/b$ and the government reduces the level of the public good, then the convergent path will shift to a"a" and the crisis will become much worse as the economy moves toward bankruptcy. In this case, the government can alleviate the possible bankruptcy of the economy by raising the level of the public good.

An increase in the public good has two real effects. First, it may stimulate private saving. This will reduce the interest rate and consequently, reduce the interest payments on the government's outstanding stock of debt. Second, an increase in g itself requires financing. The first effect is stabilizing, while the second effect is destabilizing. If the former effect dominates the latter, ($a_g > (1+n)c[a_r+(1+n)w'']/b$), then the future bankruptcy of the economy can be avoided by raising the level of the public good.

Proposition 7: A reduction in public spending may not be effective if public spending stimulates saving much.

4.4. Remarks

Many governments prefer to rely on the issuance of debt rather than explicit taxation in financing expenditures. Recent experience suggests that a number of countries are facing potential bankruptcy as a result of issuing too much debt. Our analysis of the chain-letter mechanism has shown that the response of the private sector to an austerity program introduced by the government in order to alleviate the future bankruptcy of the economy will be crucial in determining the success of the program.

The chain-letter mechanism would most likely be feasible when the initial interest rate and stock of government debt are smaller or when the propensity to save and the growth rate are higher. If the government chooses a level of debt that is too large, it will follow a divergent path and the economy will eventually go bankrupt.

When the government goes eventually bankrupt, austerity measures will be required. Serious mistakes, which will possibly exacerbate the bankruptcy problem, may occur if the wrong action is taken. This will depend critically on the response of the private sector to the specific austerity policy and more specifically the response of capital accumulation. The conventional wisdom suggests that either the government must raise taxes or dramatically reduce spending. We provide an example whereby it is possible for the government to alleviate the future bankruptcy problem by raising the level of public good. This is contingent on an increase in capital accumulation taking place in response to the change in policy.

We discussed the possibility of institutional reform in altering the marginal cost of providing a public good. The cost structure for providing a public good contains a number of institutional details which are generally ignored in analyzing the effects of a public good on the economy. This cost structure, however, will generally depend on the process whereby government officials procure the public good, e.g., the process of negotiating contracts with private firms, the actual provisions of the contracts, and so on. Institutional reform of the procurement process can possibly lower the marginal cost of providing a public good. If this is possible, it should certainly be made a part of any austerity program a government may consider. Indeed, it may take a severe crisis before government officials are willing to actually consider such a reform.

The central message of this section is that some of the actual provisions of the government's austerity program may induce a response which worsens the crisis. In such a case the austerity program is not effective. If the austerity program is regarded as unreliable by the private sector, the paths above the convergent path cannot be the equilibrium, so such paths will be excluded from the beginning.

Several important papers investigated debt Ponzi games under uncertainty. The average riskless rate may be a poor guide as to whether permanent rollover of debt is feasible when economies are stochastic. Blanchard and Weil (1992) showed that whether or not governments can rollover debt in dynamically efficient economies depends on whether the issuance of public debt can partially substitute missing markets. Bohn (1991) showed that the sustainability even of simple policy rules like balanced budgets or tax rate smoothing should not be taken for granted in a stochastic economy and that sustainability is often sensitive to assumptions about debt management. The sustainability question in stochastic models is an aspect of fiscal policy that deserves more attention in future research and in policy-making.

5. Future Tax Reforms In A Debt Financed Economy

5.1 Tax Reform For Debt Repudiation

The previous section has investigated the so called chain-letter mechanism of debt finance where taxes are predetermined and government debt issuance is endogenously determined. In reality it is argued that if the current deficits seem not sustainable, governments in such countries will be forced to in effect repudiate their debt. They do this either explicitly through an introduction of new taxes or through inflation depreciation (inflationary taxes). We may call such a policy change the tax reform for debt repudiation. The more likely the current deficits seem not sustainable, the higher the subjective probability of the future tax reform. The consequent taxation postponement is not free from credibility problems: Will the additional debt be paid off in full, or will the government find it optimal to resort to higher inflation or currency devaluation to diminish the burden of the debt, etc?

In the previous section, government debt and real capital have been perfect substitutes. Accordingly, all incidence effects occur through changes in the size of the stock of government bonds rather than through changes in the relative prices. It should be stressed, however, that if the private sector recognizes such possibilities of future tax reforms for debt repudiation, government bonds and real capital may no longer be regarded as perfect substitutes.

Bearing these aspects in mind, Ihori (1989b) attempted to formalize one such psychological phenomenon: confidence. Holding government debt to provide for old-age consumption requires confidence and trust since no one can guarantee the young that the rate of return on debt is the same as that of real capital. Put another way, it is not sure to what extent the debt burden will be transferred to the next generation. This fact depends on the possibility of future tax reform. Valuation of an intrinsically useless and unbacked asset performing intergenerational transfers from the young to the old requires enough confidence that this asset will not be worthless in the future. Expectations of the future tax reforms may play a crucial role for the efficacy of fiscal policy[8].

5.2 Theoretical Framework

5.2.1 Lump Sum Tax Reform

The theoretical framework is basically the same as before. Let us denote by 1-θ a subjective probability that all of the debt burden will be transferred to the next generation. In other words, θ is the subjective probability of the future tax reform. We assume that utility is time separable and that consumers are Von Neumann-Morgenstern expected utility maximizers. A young born at t chooses

$$c_t^1, c_{t+1}^1, s_t, b_t$$

to maximize

$$E_t\{u(c_t^1) + \sigma u(c_{t+1}^2)\}$$

subject to

$$c_t^1 = w_t - s_t - b_t \tag{33}$$

$$c_{t+1}^{2+} = (1+r_{t+1})s_t + (1+i_{t+1})b_t \quad for\ pb. = 1-\theta \tag{34-1}$$

$$c_{t+1}^{2-} = (1+r_{t+1})s_t + (1+i_{t+1})b_t - T_{t+1} \quad for\ pb. = \theta \tag{34-2}$$

where σ is the subjective discount factor. E( ) denotes the expected value conditional on information available to the young, i is the rate of return on government debt, and T is now a tax levied in the second period of his life.

The government budget constraint is now given as

$$b_{t+1} = \frac{1+i_{t+1}}{1+n} b_t - \frac{qT_{t+1}}{1+n} \tag{35}$$

Suppose that the government introduces a random tax T. It imposes a tax T on the fraction of q of the old generation. If q=0, the government does not actually introduce tax reforms for debt repudiation. If q=1, the government introduces tax reforms to all the consumers. If q=θ, the expectation of the private sector is fulfilled.

When the tax reform is lump sum, there is no uncertainty about the net rate of return on government debt. Government debt and real capital are both safe assets and perfect substitutes. Portfolio composition would be indifferent.

Then, the first order condition is simply

$$E_t u'(c_t^1) = (1+r_{t+1})\sigma E_t u'(c_{t+1}^2) \tag{36}$$

The total saving a (=s+b) will be a function of w, r, θ, and T. An increase in θ will reduce the expected disposable lifetime income and hence the first period consumption. It follows that the total savings will increase with θ.

An increase in θ has two effects. First, the real saving of the private sector increases because the expected lifetime income and the desired first-period consumption decrease. Second, the rate of interest on government debt reduces due to capital accumulation. The more the value of $\partial a / \partial \theta$, the more likely that the chain letter mechanism is feasible in the long run. The economy may avoid bankruptcy if θ is raised enough. An increase in the subjective probability of the future lump-sum tax reform has a desirable effect on the sustainability of the system and long run welfare.

5.2.2 Debt Holding Tax Reform

When the tax reform means a debt holding tax, α, we have

$$T_{t+1} = \alpha b_t \tag{37}$$

If a tax, t_r, is levied on the interest of government bonds, $\alpha = t_r i$. If α =i-n and q=1, the perfect debt repudiation is realized. From now on for simplicity α is assumed to be fixed.

Now consumers believe that the net rate of return $\hat{i}$ is a random variable, depending on the subjective probability of the tax reform. The first order conditions for an interior maximum are given as

$$E[u'(c_t^1) - \sigma(1+r_{t+1})u'(c_{t+1}^2)] = 0 \tag{38}$$

$$E[u'(c_t^1) - \sigma(1+\hat{i}_{t+1})u'(c_{t+1}^2)] = 0 \tag{39}$$

As for the comparative static results, all income effects are positive. An increase in w will raise both s and b. It is possible to increase both present consumption and future consumption from the levels enjoyed before the change in yield. As for substitution effects, direct substitution effects are positive, while the signs of the cross-substitution effects are indeterminate. If debt burden is more likely transferred to the next generation (i.e. the subjective probability of the introduction of the tax reform is lowered), the demand for debt, the risky asset, increases.

An increase in the subjective probability of an introduction of a debt-holding tax has two real effects. First, the gross rate of return on government debt increases because debt is now perceived as more risky by the private sector. Second, the real savings of the private sector may or may not increase because capital is more attractive than debt to the private sector, while the gross rate of return on government debt is raised. The former effect is destabilizing, while the latter effect may or may not be stabilizing. In the special case of the Cobb-Douglas utility function the latter effect is assumed away because the total savings are dependent only on the lifetime labor income. It follows that the higher the subjective probability of the debt-holding tax reform, the more likely the economy eventually goes bankrupt.

Ihori (1989b) showed that the debt-holding tax reform is not effective in the long run because the gross rate of return on debt is adjusted to offset changes in the debt tax rate. The size of the debt-holding tax reform does not matter once the reform has been introduced. Taxes on labor income and consumption, or uniform taxation on capital will not have such an offsetting effect. Thus, taxes on labor income and consumption or uniform taxation on capital income may well be better than a differential tax on debt holdings so as to avoid bankruptcy.

5.3 Remarks

Tirole (1985) and Weil (1987a) examined in the overlapping generations framework deterministic and speculative bubbles which are, like government debt, intergenerational schemes based on trust. Weil considered a two-state model with real capital and a bubble. The bubble has probability θ of bursting every period. The main result in Weil is that the highest sustainable bubble (the equivalent of the highest sustainable debt in the present chapter) decreases with the probability of bursting (debt repudiation).

Finally, economic theory has begun to catch up with political reality. It has done this by not only studying the optimality of fiscal policy in a context where explicit account is taken of the government's budget constraint but it has gone a step further by examining the time consistency of optimal policy. Here, it is the issue of whether it is optimal to keep promises that were optimal to make in the past. The latter lies at the heart of the credibility dilemma faced by any serious politician[9].

The fiscal regime prevailing in an economy, as well as the type of fiscal relationships expected to arise from a such a regime, is an important factor in determining the response of private agents to fiscal signals. Fiscal regimes differ across countries and change over time. At each point in time there is uncertainty about the regime that will prevail from then on. A high government deficit financed by debt can be regarded as unsustainable and therefore may be taken to signal future contractions in the deficits. However, whether these contractions will be effected through cuts in spending or increases in explicit tax collections, and when these actions will be taken is in general unknown. Expectations of future policy changes are crucial in understanding seemingly counterintuitive macroeconomic dynamics. Bertola and Drazen (1993) argued that expectations about the discrete character of future fiscal adjustments can help explain the effects of current fiscal policy. They showed that if government spending follows an upward-trending stochastic process which the public believes may fall sharply when it reaches specific 'trigger' points, then optimizing consumption behavior and simple budget-constraint arithmetic imply a nonlinear relationship between private consumption and government spending. This theoretical relation is consistent with the experience of several countries.

[1] . "Carlos Saul Menem became President of a virtually bankrupt Argentina today, promising tough economic adjustments, ..." (The New York Times, July 9, 1989).

[2] . As for the effect of government debt on capital accumulation, see Diamond (1965), Ihori (1978), and Okuno (1983).

[3] . In Chapter 11 we will show that such generational redistribution policy with public capital may have real effects in an endogenous growth model.

[4] . Ricardian neutrality is often regarded as the same as Barro's neutrality. However, it seems useful to distinguish Ricardian neutrality from Barro's neutrality. Subsection 2.2 is consistent with Ricardian neutrality in the sense that the individual is only concerned with the lifetime budget constraint, not the period-to-period budget constraint, and still debt matters.

[5] . Burbidge (1983) contrasted the results of Samuelson (1958) and Gale (1973) with those of Diamond (1965) on debt policy and argued that the stock of debt is endogenous in the Samuelson-Gale model but exogenous in Diamond's model. Thus, we follow Samuelson and Gale by assuming that the stock of debt is endogenous. McCallum (1984) investigated the chain letter mechanism in a maximizing model that incorporates the crucial components of the Ricardian view, namely, infinite-lived agents who correctly take account of the government budget constraint. Ihori (1988) and Schmid (1988) examined the chain letter mechanism in a finite horizon setting. See also Batina and Ihori (1993) and Carlberg (1994).

[6] . This result corresponds to Proposition 1.

[7] . Although we have to impose the long run solvency constraint, the economy can be off the stable manifold in the short run. People believe that sooner or later the policy will be changed so as to satisfy the solvency constraint. However, they do not think that such a change will happen before they die. The purpose of this section is to investigate what kind of policy changes will be effective to satisfy the solvency condition in the long run. Thus, even if the economy is off the stable manifold initially, people would not expect that the government goes bankrupt. It is because the government is assumed to change the policy sooner or later. It seems that this sort of story is plausible in the real economy. In some countries such as Italy or LDCs the government does not clearly satisfy the solvency condition if the present policy remains unchanged. People still hold the government bonds, which means that they anticipate a future change in fiscal policy so as to satisfy the solvency condition. We think that it is important to have a framework where it appears that the government will eventually go bankrupt but people still hold the debt.

[8] . Such a situation might be relevant for the recent Japanese economy. A recent line of economic research suggests that private agents realize that current bond-financed deficits carry with them future tax obligations. Anticipating higher future taxes, private agents change current spending behavior to smooth consumption intertemporally. Although the econometric study of this issue is still in its infancy, some recent research indicates that private Japanese behavior has partially offset recent changes in fiscal policy (see Homma et al. (1986) and Ihori (1987c)(1989a)).

[9] . See Kydland and Prescott (1977) among others. Recently Calvo (1988) studied models in which debt repudiation is possible and showed that expectations may play a crucial role in the determination of equilibrium. See also Chari and Kehoe (1993), Bulow and Rogoff (1989), and Atkenson (1991).

Chapter 10

Social Security

1. Introduction

It is well known that in unfunded social security systems, the contributions of the younger generation earn a return which is composed of the rates of growth of population (biological rate of interest) and wages. Whereas for funded social security systems, the market rate of interest, and thus, the marginal productivity of capital are relevant. From this perspective, it comes as no surprise that many industrial countries introduced or expanded pay-as-you-go unfunded public pension schemes in the years following the post-war baby boom. Considering the recent decline in the birth rates in an aging economy, however, a reverse transition would be inevitable.

Whereas considerable research has been conducted on the saving impact of government fiscal policies, few studies have investigated the effect of demographic change on saving. In Section 2, we investigate the macroeconomic effects of social security in an aging economy and we consider the welfare implications of changing the social security system from unfunded to funded schemes as fertility changes.

Section 3 summarizes Aurerbach and Kotlikoff (1987a)'s simulation analysis, which examined the economic effects of a demographic transition, particularly the interaction of demographics and social security in a multi-period overlapping generations model.

Finally, Section 4 investigates welfare effects of unfunded system when labor supply is endogenous. We will see that a mandatory unfunded pension system can lead to welfare losses if the contributions are levied in such a way that the labor supply decision of the individuals is heavily distorted. Under certain conditions a gradual abolition of unfunded pensions - using appropriate changes in lump-sum contributions in the transition phase - can lead to an intergenerational Pareto improvement.

2. Overlapping Generations Model When Fertility Changes

2.1 The Baby Boom Generation

Consider a standard two-period overlapping generations model developed in Chapter 2, in which the working period and the retirement period are of equal length. There are an infinite number of generations, but following Hatta and Oguchi (1992), we focus on first six, which we refer to as 0-V. Among them, generation III is the baby boom generation; we assume that its population consists of two people, while all other generations contain only one person.

In each period, the generation in its working period and another in its retirement period live concurrently. Figure 10.1 depicts the population size of each generation in each period. The horizontal axis measures the period and the vertical axis the generations. The white boxes show the size of the working-age population and the shaded boxes the size of the population in retirement.

In order to concentrate on the effect of changing fertility, we employ a very simple linear technology. Namely, we assume that one worker produces A units of durable outputs when he is young but does not work in old age. Accordingly, we have

$$Y_t = AN_t, \quad A > 0 \ , \tag{1}$$

where Y_t is the output level of the economy and N_t is the population size of the working generation in period t. $N_0 = N_1 = N_2 = N_4 = N_5 = 1,\ N_3 = 2$.

We will assume the interest rate to be zero in this section. We further assume that an individual saves half his expected lifetime disposable income when he is young and dissaves it when he retires. This amounts to assuming a Cobb-Douglas utility function.

The government has no expenditures other than pensions, and there are no taxes other than the social security tax. Social security taxes are imposed only on the generation working. There are no inheritances or bequests. Thus, the aggregate consumption function for the working generation in period t is written as

$$C_t^1 = C_{t+1}^2 = \frac{1}{2}(Y_t + B_{t+1} - T_t), \tag{2}$$

where C^1_t ($=c^1_tN_t$) is the aggregate consumption level of the working generation in period t, C^2_{t+1} ($=c^2_{t+1}N_t$) is that of the retired generation in period t+1, B_{t+1} is the aggregate public pension benefit that the retired generation in period t+1 receives, and T_t is the total tax that the working generation pays in period t. The right-hand side of (2) is one-half the aggregate lifetime disposable income.

When no public pension system exists (B=T=0), a person in any generation consumes A/2 units during his working years and another A/2 units during his retirement years. The consumption of any person in any period is equal. Define national saving, S_t, by

$$S_t = Y_t - C_t^1 - C_t^2 . \tag{3}$$

There is no investment in this economy, and the macro saving gap is adjusted by the balance of trade. Positive national saving implies a surplus in the balance of payments, while positive cumulative national saving implies an accumulated positive net foreign asset position.

Once N_t, B_{t+1}, and T_t are given, (1) determines the output level, (2) the consumption levels, and (3) the national saving.

2.2. Intergenerational Redistribution

Let b_t ($= B_t/N_{t-1}$) be the (per-capita) social security benefit one receives in period t and τ_t ($=T_t/N_t$), the (per-capita) social security tax that a working person pays.

2.2.1 A Fully Funded System

Now suppose that in period 2 an actuarially fair fully funded public pension system is introduced. By definition we have

$$B_{t+1} = T_t, \ b_{t+1} = \tau_t, \ \ t > 1 \ , \tag{4}$$

and hence

$$C_t^1 = C_{t+1}^2 = \frac{1}{2}Y_t, \ c_t^1 = c_{t+1}^2 = A, \ \ t > 1.$$

We have:

Proposition 1: The fully funded pension system will not affect the consumption pattern of any generation and hence will not redistribute income among generations.

2.2.2 A Pay-As-You-Go System

Now suppose that a pay-as-you-go unfunded system is implemented in period 2. A pay-as-you-go (or just pay-go) social security program is defined to be a program in which social security tax revenues equal benefits in each period. The social security program transfers income from the young to the old. By definition, the benefits are financed by the social security taxes paid by the currently working generation. Thus,

$$B_t = T_t,\ t > 1. \qquad (5)$$

This yields $\tau_1 = 0$ and $bN_{t-1} = \tau_t N_t$, $\tau_t > 0$, $t > 1$. For simplicity we assume $b_t = b$.

Thus, the per capita net benefit of generation t, g_t can be written as

$$g_1 = b,$$

$$g_t = b(1 - \frac{N_{t-1}}{N_t}),\ t > 1. \qquad (6)$$

After the system is introduced, the net benefit of generation t (g_t; t>1)) is positive if and only if the population of generation t-1 is smaller than its own.

Remember that the introduction of the funded system does not affect consumption patterns. The introduction of a pay-as-you-go unfunded public pension system in period 2, however, does affect consumption patterns and therefore national saving. The retired in period 2 will consume all the unexpected benefit in this period, which yields

$$C_2^2 = \frac{1}{2}Y_1 + B_2. \qquad (7)$$

Also, from (2) and (3) we have

$$C_t^1 = C_{t+1}^2 = \frac{1}{2}(Y_t + B_{t+1} - B_t),$$

$$c_t^1 = c_{t+1}^2 = \frac{1}{2}(A + g_t),\ t > 1$$

$$c_2^2 = b + \frac{A}{2}. \qquad (8)$$

Table 10.1 summarizes per capita pension benefits received, contributions, and net benefits of each generation. This table reveals two features of a pay-as-you-go system. First, the aggregate net benefit of generation III is equal to the aggregate net loss of generation IV since the population size of the former is twice the latter. Hence, the economy as a whole gains by the net benefit of generation I, that is by b. Since consumption corresponds to lifetime disposable income, the introduction of the system increases the sum of consumption of all generations. Thus, the introduction of a pay-as-you-go system creates a net increase in the consumption for the economy as a whole. This case corresponds to the chain-letter issuance of national debt in Chapter 9.

Second, an introduction of the pay-as-you-go unfunded system creates inequality among generations. Generation I, which is in retirement when the pension system is introduced, receives the most net benefits from the system. The baby boomers receive net benefits to some extent because the tax rate they face when young is low due to temporary positive population growth in period 3. Generation IV, which comes immediately after the baby boom generation, receives negative net benefits due to temporary negative population growth in period 4; it has to support the retired baby boomers.

Proposition 2: The pay-as-you-go system creates income inequity among generations;

it benefits the first and the baby boomer generations, while a net burden is borne by the generation that comes immediately after the baby boomers due to temporary negative population growth in that period.

This is the reason why an unfunded system was easily introduced in the years following the post-war baby boom in many countries[1]. And, considering the recent decline in the fertility rate, a reverse transition seems inevitable.

Third, in view of (3)(7) and (8), we have as for the change in savings the following;

$$S_2 = \frac{1}{2}[A(N_2 - N_1) - (N_2 + N_1)b],$$

$$S_t = \frac{1}{2}[A(N_t - N_{t-1}) - (N_t - N_{t-2})b],\ t > 2. \qquad (9)$$

Thus, national saving in a given period is influenced by the population size of the current and possibly two preceding generations.

As for movements of national saving, two factors are useful to note. The first relates to the consumption surge by generation I in period 2, which receives the unexpected free-ride benefits from the newly started social security system. This creates a negative saving balance in the second period. If the population size did not change thereafter, national saving in each period after 2 would remain zero permanently.

The second relates to the existence of the baby boomers, or generation III. In period 3, the baby boomers, who are then in their working years, consume more under the pay-as-you-go unfunded system than under the fully funded system because they receive positive net benefits during their lifetime. This is the reason why national saving is smaller under the pay-as-you-go system than under the fully funded system. Moreover, national saving in period 5 is positive because the post-baby boomer generation, with reduced per capita lifetime disposable income, is dissaving in this period at a lower rate than under the fully funded system due to negative population growth[2].

2.2.3 Summary Of Results

The observations in this subsection may be summarized as follows.

First, an introduction of a fully funded system does not affect consumption patterns of any generation. Hence, it causes no intergenerational transfer of income. Nor does it affect the national saving in any periods. However, the introduction does create a positive cumulative balance of government budget surplus. The cumulative balance of private saving reduces exactly to offset the budget surplus of the government. Government saving and private saving are perfect substitutes.

Second, an introduction of a pay-as-you-go system creates a negative cumulative balance of national saving, which is permanently carried forward[3]. Also, it creates income inequity among generations; it benefits the first and the baby boomer generations, while a net burden is borne by the generation that comes immediately after the baby boomers due to temporary negative population growth in that period. Moreover, the introduction increases the sum of the present value of consumption of all generations while reducing the cumulative balance of saving at the steady state by exactly the same amount.

In other words, the consumption increase caused by the pay-as-you-go unfunded system is made possible by a reduction in the cumulative saving; the apparent welfare improvement is a result of a Ponzi game. Namely, this is equivalent to the chain-letter finance of national debt. As discussed in Chapter 9, if n > r, this is feasible in the long run[4].

Thus, an introduction of a pay-as-you-go unfunded system creates inequality among generations. But it does not create static efficiency gain or loss within this model of exogenous output. If the model is expanded to incorporate endogenous labor supply, then the price distortions created by the pay-as-you-go system will cause inefficiency, so that every generation could be better off by suitable conversion to the funded system. See Section 4.

2.3. Evaluation Of The Reform Plans

Based on Hatta and Oguchi (1992), we now investigate the policy change of converting unfunded to funded schemes within the basic model of exogenous labor supply. Let us now assume that a pay-as-you-go system was introduced in period 2 and that the system is reformed in period 3 in order to attain an actuarial fairness eventually.

The reform makes the system fully funded in period 3 and afterward, a fact which we will call a switch to the fully funded. The reform will be attained by (i) raising the tax rate on generation III so as to finance not only the current benefit payment for generation II but also the future benefit payment for generation III (as the change to the funded system) and (ii) imposing taxes on generation IV and subsequent generations by the amount equal to the benefits received (as the maintenance of the funded system).

Thus, the taxes and benefits satisfy the following:

$$T_3 = B_3 + B_4, \qquad (10)$$

$$T_t = B_{t+1},\ t > 3$$

From (10) the per capita tax rates after the reform are

$$\tau_3 = b + \frac{b}{2}, \qquad (11)$$

$$\tau_t = b,\ t > 3$$

The net benefits of each generation after the switch to the fully funded are depicted in Table 10.2. Generation IV and all subsequent generations receive zero net benefits because of the introduction of the actuarially fair funded system. But the switch turns the net benefit of generation III from positive to negative. This is because taxes to redeem "implicit debt" (B_3) are levied on generation III.

Proposition 3: The switch to the fully funded system turns the net benefit of the baby boomers from positive to negative.

This reform is politically difficult to accomplish. It puts a large burden on the baby boom generation - the working and the decision-making generation when the switch is made. Thus, Hatta and Oguchi proposed another reform; a switch to the actuarially fair system by making the tax rate on generation III and subsequent generations exactly equal to the present value of the benefit each of them receives. This reform is essentially the same as issuing government debt of B_3 in period 3, which will be transferred to future generations forever. The switch to an actuarially fair system has advantages that the switch to a fully funded one does not have. The net benefit of generation III is no longer negative. It is zero. Since generation III gets net benefits in the unfunded system, however, this switch still hurts generation III as the switch to the fully funded system. In this sense the original unfunded system is Pareto efficient so long as labor supply is exogenous. See Section 4.

3. Multi-Period Overlapping Generations Model

Auerbach and Kotlikoff (1987a) examined the economic effects of a demographic transition, particularly the interaction of demographics and social security in a multi-period overlapping generations model[5]. They first look at the impact of demographic change on savings and other economic variables in the absence of social security. They obtain simulation results for two types of demographic transitions; (i) a sudden and permanent reduction in the birth rate (bust) and (ii) a cycle of decline and increase in the birth rate followed by a permanent drop (bust-boom-bust). They also consider a variety of social security policy responses to such demographic changes. These include reductions in benefit replacement rates, advances in social security retirement age, taxation of social security benefits, and the accumulation of a significant social security trust fund.

Their main simulation results are as follows;

(1) Major swings in fertility rates such as those currently under way in the United States can have considerable effects on long-run factor returns and produce precipitous changes in short-term saving rates. The simulated long-run changes in factor returns and capital-labor ratios from major fertility declines are of the same order of magnitude as the simulated effect of entirely abolishing unfunded social security.

(2) Although social security policy has important effects on the simulated demographic transitions, these effects are of secondary importance to the long-run level of economic welfare.

(3) Baby busts require large changes in social security finances. These must take the form of significant payroll tax increases, sizable benefit cuts, substantial advances in the social security retirement age, or the accumulation of a large social security trust fund.

(4) Even if payroll tax rates rise dramatically, long-run welfare is nonetheless substantially higher in the case of a sustained drop in the fertility rate; while a sustained decline in fertility eventually means a larger ratio of elderly per capita, the concomitant decline in children per capita means an eventual overall decline in the ratio of dependents to prime-age workers in the economy. Long run welfare is also greater because of the capital deepening associated with lower population growth rates.

(5) In comparison with simply allowing payroll taxes to adjust upward to meet required benefit payments, major reductions in replacement rates, major increases in the retirement age, or the accumulation of a significant trust fund can all raise the long-run level of welfare by an amount equivalent to almost 4 percent of lifetime expenditure on consumption and leisure.

(6) The potential long-run welfare gain is not, however, freely obtained; rather, such long-run welfare gains come at the price of reductions in the welfare of transition cohorts, typically those alive at the time of the demographic change as well as those born within 25 years of the initial date of the change. Hence the choice of social security policy in the midst of the demographic transition is of considerable importance to the intergenerational distribution of welfare.

Their simulation results suggest that plausible conversion from an unfunded to a funded pension system, even with variable labor supply, is bound to hurt at least one of the transition generations.

4. Welfare Effects Of Unfunded System When Labor Supply Is Endogenous

4.1 Analytical Framework

We now investigate theoretically the welfare effects of unfunded system when labor supply is endogenous by using the basic model developed in Chapter 2 Section 4. The utility function is given as

$$u_t = u(c_t^1, c_{t+1}^2, H - l_t) \tag{12}$$

where H is the initial endowment of labor supply in the first period and l is the amount of labor supply in the first period.

In a pay-as-you-go pension system we have

$$\tau_t = \theta w_t l_t \tag{13}$$

$$b_t = \theta w_t l_t (1+n) \tag{14}$$

where θ is the contribution rate. Therefore, the budget constraint in each period is respectively given as

$$c_t^1 = (1-\theta) w_t l_t - s_t \tag{15-1}$$

$$c_{t+1}^2 = (1 + r_{t+1}) s_t + \theta(1+n) w_{t+1} l_{t+1} \tag{15-2}$$

where s is saving and r is the rate of interest.

4.2 Long Run Welfare

Let us investigate the long run welfare effect of the unfunded system. In a steady state we have

$$q_1 c^1 + q_2 c^2 + q_3 x = -\frac{\theta w x (1+n)}{1+r} \tag{16}$$

where

$$q_1 = 1 \tag{17-1}$$

$$q_2 = \frac{1}{1+r} \tag{17-2}$$

$$q_3 = (1-\theta) w(r) \tag{17-3}$$

x $(=-l)$ is net leisure. w(r) is the factor price frontier.

As before, from (16) the consumer's optimizing behavior is summarized as the expenditure function.

$$E(q,u) + \frac{1+n}{1+r} \theta w(r) E_3(q,u) = 0 \tag{18}$$

From (15-2), the compensated capital accumulation equation is given as

$$q_2[E_2(q,u) + \theta(1+n) w(r) E_3(q,u)] - (1+n) w'(r) E_3(q,u) = 0 \tag{19}$$

Totally differentiating (18) and (19) with respect to θ, we have

$$\begin{bmatrix} E_u + \frac{1+n}{1+r}\theta w E_{3u}, & D \\ q_2 E_{2u} + (1+n)(q_2 \theta w - w') E_{3u}, & J \end{bmatrix} \begin{bmatrix} du \\ dr \end{bmatrix} = \begin{bmatrix} \frac{(r-n)w}{1+r} E_3 + \frac{(1+n)\theta w^2}{1+r} E_{33} \\ w\{q_2 E_{23} - q_2(1+n) E_3 + (1+n)(q_2 \theta w - w') E_{33}\} \end{bmatrix} d\theta \tag{20}$$

where

$$D = -\frac{E_2}{(1+r)^2} + \{(1-\theta) w' + \theta(1+n)[\frac{w'}{1+r} - \frac{w}{(1+r)^2}]\} E_3 + \frac{1+n}{1+r} \theta w \{-\frac{E_{32}}{(1+r)^2} + (1-\theta) w' E_{33}\}$$

$$J=-\frac{E_2+\theta(1+n)wE_3}{(1+r)^2}+q_2\{-\frac{E_{22}+\theta(1+n)wE_{32}}{(1+r)^2}+(1-\theta)w'E_{32}+$$

$$\theta(1+n)w'[E_3+(1-\theta)wE_{33}]\}-(1+n)\{w''E_3-w'\frac{E_{32}}{(1+r)^2}+w'E_{33}(1-\theta)w'\}$$

The sign of D is ambiguous. $J>0$ corresponds to a local stability condition. When the elasticity of substitution between capital and labor is large, J is likely to be positive. We assume this.

From (20) we have

$$\frac{dr}{d\theta}=\frac{1}{\Delta}\{[E_u+\frac{1+n}{1+r}\theta wE_{3u}][q_2E_{23}-q_2(1+n)E_3+(1+n)(q_2\theta w-w')E_{33}]w-$$

$$[q_2E_{2u}+(1+n)(q_2\theta w-w')E_{3u}][\frac{(r-n)w}{1+r}E_3+\frac{(1+n)\theta w^2}{1+r}E_{33}]\} \tag{21}$$

$$\frac{du}{d\theta}=\frac{1}{\Delta}\{-Dw[q_2E_{23}-q_2(1+n)E_3+(1+n)(q_2\theta w-w')E_{33}]+$$

$$J[\frac{(r-n)w}{1+r}E_3+\frac{(1+n)\theta w^2}{1+r}E_{33}]\} \tag{22}$$

where $\Delta=[E_u+\frac{1+n}{1+r}\theta wE_{3u}]J-[q_2E_{2u}+(1+n)(q_2\theta w-w')E_{3u}]D$. Δ is likely to be positive. We assume this. Since the term $[q_2E_{23}-q_2(1+n)E_3+(1+n)(q_2\theta w-w')$ $E_{33}]w$ is ambiguous, the sign of (21) is generally ambiguous. Suppose the absolute value of this term is not so large. Then, if r>n, (21) will be positive.

Proposition 4: An increase in the contribution rate in the unfunded system will raise the long-run rate of interest and reduce the long-run capital accumulation if the rate of interest is greater than the rate of population growth.

In such a case, it is easy to see that the sign of (22) is negative.

Proposition 5: An increase in the contribution rate in the unfunded system will reduce welfare in the long run if the rate of interest is greater than the rate of population growth.

Remark: If labor supply is exogenously given, it is tedious but easy to find that an increase in the contribution rate always reduces capital accumulation; (21) is always positive. In such a case Proposition 5 still holds due to the golden rule effect.

4.3 Pareto Optimality

A sequence of consumption, savings, and labor, $S=(c_{t+1}^2,s_t,l_t)_{t=0,..}$ is called short-run Pareto efficient in the interval [T,V] if there is no other feasible sequence $S'=(c_{t+1}^2{}',s_t',l_t')_{t=0,..}$ with

(a) $u(c_t^1{}',c_{t+1}^2{}',l_t')\geq u(c_t^1,c_{t+1}^2,l_t)$ for t=T,...,V

(b) $u(c_t^1{}',c_{t+1}^2{}',l_t')>u(c_t^1,c_{t+1}^2,l_t)$ for at least one t

(c) $(s_{T-1},c_T^2)=(s_{T-1}',c_T^2{}')$

(d) $(s_V, c_{V+1}^2) = (s_V{}', c_{V+1}^2{}')$

S is called short-run Pareto efficient if it is short-run Pareto-efficient in every finite interval.

Consider a sequence S which is short-run Pareto efficient in the interval [1,2] at given values of (s_0, c_1^2) and (s_2, c_3^2). We may choose l_1, l_2, s_1, c_2^2 to maximize the following Lagrangian

$$W = u[\frac{F(N_1 s_1, N_2 l_2)}{N_2} - s_2 - \frac{c_2^2}{1+n} + \frac{s_1}{1+n}, c_3^2, H - l_2] - \lambda\{u[\frac{F(N_0 s_0, N_1 l_1)}{N_1} - s_1 - \frac{c_1^2}{1+n} + \frac{s_0}{1+n}, c_2^2, H - l_1] - \bar{u}\} \tag{23}$$

where F(K, L) is the aggregate production function. The first-order conditions are given as

$$\frac{\partial W}{\partial l_1} = -\lambda\{\frac{\partial u}{\partial c_1^1}\frac{\partial F}{\partial L_1} - \frac{\partial u}{\partial x_1}\} = 0, \tag{24-1}$$

$$\frac{\partial W}{\partial l_2} = \frac{\partial u}{\partial c_2^1}\frac{\partial F}{\partial L_2} - \frac{\partial u}{\partial x_2} = 0, \tag{24-2}$$

$$\frac{\partial W}{\partial s_1} = \frac{\partial u}{\partial c_2^1}\{\frac{1}{1+n}\frac{\partial F}{\partial K_1} + \frac{1}{1+n}\} - \lambda\frac{\partial u}{\partial c_1^1} = 0, \tag{24-3}$$

$$\frac{\partial W}{\partial c_2^2} = -\frac{\partial u}{\partial c_2^1}\frac{1}{1+n} + \lambda\frac{\partial u}{\partial c_2^2} = 0. \tag{24-4}$$

(24-1,2) mean

$$\frac{\frac{\partial u}{\partial x_t}}{\frac{\partial u}{\partial c_t^1}} = \frac{\partial F}{\partial L_t}. \tag{25}$$

The marginal rate of substitution between leisure and present consumption equals the marginal productivity of labor. (24-3,4) mean

$$\frac{\frac{\partial u}{\partial c_t^1}}{\frac{\partial u}{\partial c_{t+1}^2}} = 1 + \frac{\partial F}{\partial K_t}. \tag{26}$$

The marginal rate of substitution between present and future consumption equals the marginal productivity of capital plus one. These two equations (25) and (26) are conditions of short-run Pareto optimality.

Since the way in which the contributions to an unfunded system are levied distorts the labor/leisure choice, the unfunded system is not short-run Pareto efficient. Breyer and Straub (1993) investigated theoretically welfare effects of unfunded pension systems when labor supply as well as capital accumulation is endogenous. A mandatory unfunded pension system can lead to welfare losses if the contributions are levied in such a way that the labor supply decision of the individuals is heavily distorted. They show that under certain conditions a gradual abolition of unfunded pensions - using lump-sum transfers in the transition phase - can lead to an intergenerational Pareto improvement[6]. When labor supply is endogenous, we have (Breyer and Straub (1993)):

Proposition 6: If the long run welfare is monotone decreasing in the contribution rate, there is a time path which constitutes a Pareto-improving transition from the unfunded to a funded system in finite time.

Remark 1: When labor supply is exogenously given as in the basic model of Section 2, the way in which the contributions to an unfunded pension system are levied does not distort the labor/leisure choice. Then it is impossible to improve intergenerational welfare, as measured by the Pareto criterion, by changing the pension system in a finite number of periods. Breyer and Straub showed that Pareto efficiency is violated unless (i) either there is no utility attached to leisure, (ii) or the contributions are levied in a lump-sum fashion, (iii) or the benefits are actuarially fair, given the contributions.

Remark 2: If, on the other hand, labor supply was assumed to react to net wages and - as is generally true - contributions to the pension system are levied in the form of payroll taxes with fixed rates, then unfunded social security may no longer be Pareto efficient. Static distortions in the labor market could be sufficient for the existence of a Pareto-improving transition from an unfunded to a funded pension system. More precisely, replacing contributions in one period by government debt would reduce distortions in the labor supply decision and thus could raise the welfare level of the generation that was active in that period without hurting any other generation if the rate of interest is greater than the rate of population growth.

Remark 3: As explained in the previous sections, plans of the transition from an unfunded to a funded pension system have been analyzed in several simulation studies, but none of them has demonstrated the existence of a transition path which would improve the welfare of every generation. Breyer and Straub showed that the avoidance of the deadweight loss implied by an unfunded pension system would by itself suffice to build up the fund required to replace the unfunded by a funded system. Namely, the avoidance of (static) deadweight loss inherent in the labor-supply distortions due to the payroll tax nature of social security contributions would suffice to build up a capital stock big enough to fund the existing level of pensions. It contradicts results from simulation studies which suggest that any conversion from an unfunded to a funded pension system, even with variable labor supply, is bound to hurt at least one of the transition generations. Thus, their theoretical argument deserves more attention in future simulation research.

5. Further Study

Marchand, Michel, and Pestieau (1992) compare the relative merits of a pay-as-you-go social security scheme and those of a tax induced retirement saving plan. This comparison is conducted in a setting of endogenous growth and changing population. They show that the case for a pay-as-you-go social security scheme is weak as opposed to a funded scheme with interest subsidy. However, within the transition period following a fertility decline, a transfer from the younger to the older generations appears to be desirable.

Table 10.1: Pay-As-You-Go-System

generation	*public pension benefit*	*public pension tax*	*net benefit*
0	0	0	0
I	b	0	b
II	b	b	0
III	b	b/2	b/2
IV	b	2b	-b
V	b	b	0

period	*savings*	*accumulated savings*
1	0	0
2	-b	-b
3	(A-b)/2	A/2-3b/2
4	-A/2	-3b/2
5	b/2	-b
6	0	-b

Table 10.2: Switch to the Fully Funded

generation	*public pension benefit*	*public pension tax*	*net benefit*
0	0	0	0
I	b	0	b
II	b	b	0
III	b	3b/2	-b/2
IV	b	b	0
V	b	b	0

[1] . The existing older generation is better off, and the future generation gets benefits if the growth path is inefficient.

[2] . This was called the 'over-shooting' of saving by Auerbach et al. (1989).

[3] . In the absence of a public pension system, a positive cumulative balance of national saving is created when the baby boomers are of working period. But the cumulative balance returns to zero. Generally, the social security may reduce private saving in two ways. First, if a member of generation I anticipates the introduction of a pay-go plan, generation I's saving will be reduced. Second, if $r > n$, the future generation's saving will be reduced by the tax postponement effect. The former effect is temporary, while the latter effect is permanent.

[4] . A correct evaluation of the welfare increase must be based on a combined consideration of the utility increase and the change in the cumulative saving, while embodies the potential capital formation.

[5] As for the general properties of their simulation model, see Chapter 4.

[6] . Thus, they extend a result derived by Homburg (1990) for a small open country to the closed economy case.

Chapter 11

Intergenerational Transfers

1. Introduction

It seems possible to argue that the class of infinitely lived models is too narrow to accommodate some forms of intergenerational heterogeneity. In particular, it is impossible to understand the effect of intergenerational income redistribution on growth. This is one of the main reasons why we have used the overlapping generations model in this book. However, it is also impossible to understand the long-run effect on the growth rate, where the long-run growth rate is exogenously given, in the standard overlapping generations model as developed in the previous chapters.

Recently, several papers have considered endogenous economic growth by extending the framework of the standard overlapping generations model. Jones and Manuelli (1990) showed that an income tax-financed redistributive policy can be used to induce positive endogenous growth. Azariadis and Drazen (1990) and Caballe (1991) presented models of endogenous growth in which the accumulation of human capital is subject to externalities. Caballe (1991) showed that intergenerational transfer policies will be ineffective when altruistic bequests are fully operative. He then investigated the effect of an unfunded social security system on economic growth. But he only investigated the case where bequests are not operative.

Bequests and human capital investment appear to be relatively prevalent in the real economy. Several studies have applied the methodology of Kotlikoff and Summers (1981) to the case of the United States and Japan in order to estimate the shares of life cycle and transfer wealth (wealth deriving from intergenerational transfers). As summarized by Horioka (1991) in Table 11.1, the share of transfer wealth in the United States and Japan appears to be significant.

The nature of bequest motives is a key determinant of the long-run and short-run response to fiscal policy. There have been however few analyses on the normative role of pay-as-you-go social security when bequests are prevalent. Since both forms of transfer are large in the real economy, it is important to analyze the relationship between them. The conventional wisdom is that the normative role of social security is dependent on the bequest motive. Namely, as shown in Chapter 9 Section 2, when the households are fully altruistic and have a dynastic behavior a la Barro (1974), social security has no real effects. The market solution is always efficient and coincides with the optimal solution in an exogenous growth setting. On the other hand, social security may have real effects and hence have some normative role when bequests are not due to the dynastic behavior.

This chapter develops an endogenous growth model, which is a natural extension of the conventional overlapping growth model. Using such a model, Section 2 attempts to analyze the dynamic properties of endogenous growth in which private transfers are operative and crucial for positive economic growth. It is shown that the effect of the bequest motives on the growth rate is qualitatively the same in all of the three major bequest motives (the altruistic model, the bequest-as-consumption model, and the bequest-as-exchange model).

Section 3 incorporates a public sector into the model of Section 2 and presents interesting choices about the relations among the size of government, the saving and bequest behavior, the social security program, and the rate of economic growth. By incorporating altruistic bequests and public capital into the model, we explore the role of public capital in the model of economic growth.

Finally, Section 4 explores the role of human capital formation in endogenous growth model. This section incorporates three types of taxes on capital (a tax on physical capital, a tax on human capital, and a tax on physical bequests) into an endogenous growth model with altruistic bequests. The analytical results depend on whether physical bequests are operative or not. When physical bequests are not operative and the externality effect of human capital is small, the laissez faire growth rate may well be too high. An increase in the tax on human capital may raise the rate of economic growth, while an increase in the tax on life-cycle physical capital will reduce the growth rate. If physical bequests are operative, the laissez faire growth rate is too low. A tax on life-cycle capital will not affect the growth rate, while an increase in the tax on transfer capital will reduce the growth rate. We will also consider how to attain the first best solution.

2. Intergenerational Transfers with Bequests

2.1. Optimality Conditions

2.1.1 An Endogenous Growth Model

We present an overlapping-generations model, which generates endogenous growth. After deriving the optimality conditions, we consider the market solutions with alternative bequest motives and compare each of them with the corresponding optimality solution.

Recent models of endogenous economic growth can generate long-run growth without relying on exogenous changes in technology or population. A general feature of these models is the presence of constant or increasing returns in the factors that can be accumulated. This section employs the simplest version of endogenous growth models.

Firms act competitively and use a constant returns-to-scale technology.

$$Y_t = AK_t, \tag{1}$$

where Y_t is output, K_t is a broad concept of capital which includes human capital as well as physical capital. A is a productivity parameter which is taken here to be multiplicative and to capture the idea of endogenous growth a la Rebelo (1991)[1].

In this subsection 2.1, we study the normative properties of a model in which individuals live for a finite number of periods. To make the point clear consider an endogenous growth version of the two-period overlapping generations model similar to Jones and Manuelli (1990), and Marchand, Michel and Pestieau (1992). An individual born at time t consumes c^1_t in period t and c^2_{t+1} in period t+1 and derives utility from his own consumption.

$$u_t = \varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2, \quad 0 < \varepsilon_i < 1. \tag{2}$$

For simplicity, we assume a log-linear form throughout this chapter.

Individuals work only in the first period of their life and supply inelastically a given amount of time using human capital. There is no growth in population and the number of individuals of each generation is normalized to one.

Considering (1), the feasibility condition in the aggregate economy is given as

$$c_t^1 + c_t^2 + K_{t+1} = (A+1)K_t. \tag{3}$$

2.1.2 Optimization Problem

We now analyze the growth path which would be chosen by a central planner who maximizes an intertemporal social welfare function expressed as the sum of generational utilities discounted by the social generation preference factor, β. $0 < \beta < 1$.

$$W = \beta^{-1}\varepsilon_2 \log c_0^2 + \sum_{t=0}^{\infty} \beta^t \{\varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2\}, \tag{4}$$

subject to the feasibility condition (3). In other words, the problem is to maximize the following Lagrangian

$$L = \beta^{-1}\varepsilon_2 \log c_0^2 + \sum_{t=0}^{\infty} \beta^t \{\varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2 + \lambda_{t+1}[(A+1)K_t - c_t^1 - c_t^2 - K_{t+1}]\}, \tag{5}$$

where λ_t is the current shadow price of K_t and the Lagrange multiplier of the resource constraint at time t is $\beta^t \lambda_{t+1}$.

The optimality conditions with respect to c^1_t, c^2_t, and K_t are given as

$$\frac{\varepsilon_1}{c_t^1} = \lambda_{t+1}, \tag{6-1}$$

$$\frac{\varepsilon_2}{c_t^2} = \beta\lambda_{t+1}, \tag{6-2}$$

$$\lambda_t = \beta(A+1)\lambda_{t+1}, \tag{6-3}$$

along with the transversality condition

$$\lim_{t\to\infty} \beta^t \lambda_{t+1} K_t = 0. \tag{7}$$

(6) and (7) imply that the economy moves right from the first period on a path of balanced growth. Denote by γ the growth rate of an economic variable, X. $\gamma_t = X_t / X_{t-1}$. If $\gamma > 1$, then X grows. We have:

Proposition 1: The optimal growth rate at the first best solution γ^* is given as

$$\gamma^* = \beta(1+A). \tag{8}$$

Substituting (6-1) and (6-2) into (6-3) yields the evolution of c^1_t and c^2_t along the balanced growth path

$$c_t^1 = c_0^1 \gamma^{*t}, \tag{9-1}$$

$$c_t^2 = c_0^2 \gamma^{*t}, \tag{9-2}$$

where $\gamma^* = \beta(1+A)$ and

$$c_0^1 \varepsilon_2 = c_0^2 \varepsilon_1 \beta. \tag{9-3}$$

Substituting (9-1)(9-2) and (9-3) into (3), we have

$$K_{t+1} = (1+A)K_t - c_0^1\left(1 + \frac{\varepsilon_2}{\varepsilon_1 \beta}\right)\gamma^{*t}. \tag{10}$$

The solution of this difference equation is generally

$$K_t = \kappa_1 (1+A)^t + \kappa_2 \gamma^{*t}. \tag{11}$$

To satisfy the transversality condition (7), as shown in Marchand, Michel, and Pestieau (1992), we have $\kappa_1 = 0$. Hence,

$$K_t = K_0 \gamma^{*t}. \tag{12}$$

Considering (3) we have

$$c_0^1 + c_0^2 = K_0(1 + A - \gamma^*). \tag{13}$$

Substituting (9-1)(9-2) and (9-3) into (13), we finally get

$$c_t^1 = (\gamma^*)^t K_0 (1+A)(1-\beta)\frac{\beta\varepsilon_1}{\beta\varepsilon_1+\varepsilon_2}, \tag{14-1}$$

$$c_t^2 = c_t^1 \frac{\varepsilon_2}{\beta\varepsilon_1}. \tag{14-2}$$

The higher the social discount factor (β) (the lower the rate of social discount rate ρ = $1/\beta$ - 1) and the productivity factor (A), the higher the optimal growth rate (γ *). The higher the productivity factor (A) and the lower the social discount factor (β), the larger the initial consumption ($c^1{}_0$, $c^2{}_0$).

2.2. Market Solutions With Alternative Bequest Motives

2.2.1 Intergenerational Transfer

In a market economy we introduce voluntary intergenerational transfers to generate endogenous growth. It is assumed that the inheritance from the parent determines the lifetime income. A representative individual born at time t has the following budget constraints.

$$c_t^1 + s_t + b_t = (1+r_t)b_{t-1}, \tag{15-1}$$

$$c_{t+1}^2 = (1+r_{t+1})s_t, \tag{15-2}$$

where b_{t-1} is the inheritance received when young, b_t is his bequests which is determined when young, r_t is the market rate of interest, and s_t is his savings. Although the inheritance could be assumed entirely financial or physical, we follow the interpretation suggested by Becker and Tomes (1979), under which b includes transfers in support of human capital accumulation as well. Here labor income of the younger generation includes wage income and equals (1+r)b because K now includes human capital. Physical capital and human capital are perfect substitutes. Private capital is held by the older generation in the form of their savings and is held by the younger generation in the form of their inheritance received.

Capital accumulation is given as

$$s_t + b_t = K_{t+1}. \tag{16}$$

The market rate of interest is given as

$$r = A. \tag{17}$$

2.2.2 Alternative Bequest Motives

As explained in Chapter 2, Section 5, there are several theoretical models of bequeathing behavior that have appeared in the literature: (i) the altruistic bequest model, where the offspring's indirect utility function enters the parent's utility function as a separate argument, (ii) the bequest-as-consumption model, where the bequest itself enters the parent's utility function as a separate argument, (iii) the bequest-as-exchange model, where the parent gives a bequest to his offspring in exchange for a desirable action undertaken by the offspring, and (iv) the accidental bequest model, where a parent may leave an unintended bequest to his offspring because lifetimes are uncertain and annuities are not priced in an actuarially fair way.

The altruistic model means that households can be represented by the dynasty who would act as though they were infinitely lived. Other bequest models mean that their behavior can be described by the life cycle framework where overlapping generations are concerned with a finite number of periods. In this section, we will consider the first three intentional motives. The main concern here is with the difference and/or similarity between the altruistic and nonaltruistic motives. It is shown that the market solution is qualitatively the same in all of the three bequest motives under the log-linear utility function.

2.2.3: Altruistic Model

As explained in Chapter 2, in the altruism model the parent cares about the welfare of his offspring. The parent's utility function is given as

$$U_t = u_t + \sigma_A U_{t+1} \qquad (18)$$
$$= \varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2 + \sigma_A U_{t+1}$$

$0 < \sigma_A < 1$. σ_A reflects the parent's concern for the child's well-being. Namely, σ_A is the parent's marginal benefit of his offspring's utility and may be regarded as the private rate of generation preference or the private discount factor of the future generation. The higher σ_A, the greater the parent cares about his offspring.

An individual born at time t will solve the following problem of maximizing.

$$W_t = \varepsilon_1 \log[(1+r)b_{t-1} - b_t - s_t] + \varepsilon_2 \log(1+r)s_t + \sigma_A\{\varepsilon_1 \log[(1+r)b_t - b_{t+1} - s_{t+1}] + \varepsilon_2 \log(1+r)s_{t+1} + \sigma_A U_{t+2}\} \qquad (19)$$

The optimality conditions with respect to s_t and b_t are

$$\frac{\varepsilon_1}{c_t^1} = \frac{(1+r)\varepsilon_2}{c_{t+1}^2}, \qquad (20\text{-}1)$$

$$\frac{1}{c_t^1} = \frac{\sigma_A(1+r)}{c_{t+1}^1}. \qquad (20\text{-}2)$$

(20-2) gives the laissez-faire growth rate in the altruism model:

Proposition 2: The laissez-faire growth rate in the altruism model is given as

$$\gamma\,(A) = \sigma_A\,(1+r) = \sigma_A\,(1+A). \qquad (21)$$

(21) must be compared with γ^* given by (8). For $\beta = \sigma_A$, $\gamma^* = \gamma$ (A). Suppose initial values of b_{-1} and s_{-1} are such that (9-1) and (9-2) hold here for t = 0. Then, when the private discount factor is equal to the social discount factor, the laissez-faire solution is identical to the optimal solution. This result corresponds to the conventional wisdom in the exogenous growth model.

2.2.4: Bequest-As-Consumption Model

In the bequest-as-consumption model the parent cares about the bequest itself instead of the welfare of his offspring. The parent's utility function is given as

$$U_t = u_t + \sigma_B v(b_t) \qquad (22)$$
$$= \varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2 + \sigma_B \log b_t$$

σ_B is the parent's marginal benefit of his bequeathing. $1 = \varepsilon_1 + \varepsilon_2 + \sigma_B$. v() may be regarded as a proxy of his offspring's utility. Hence, σ_B represents the private discount factor of the future generation as in Section 2.2.3.

An individual born at time t will solve the following problem of maximizing.

$$W_t = \varepsilon_1 \log[(1+r)b_{t-1} - b_t - s_t] + \varepsilon_2 \log(1+r)s_t + \sigma_B \log b_t. \qquad (23)$$

The optimality conditions with respect to s_t and b_t are

$$\frac{\varepsilon_1}{c_t^1} = \frac{(1+r)\varepsilon_2}{c_{t+1}^2}, \qquad (24\text{-}1)$$

$$\frac{\varepsilon_1}{c_t^1} = \frac{\sigma_B}{b_t}. \qquad (24\text{-}2)$$

From (15) and (24) the consumption and bequest functions are respectively given as

$$c_t^1 = \varepsilon_1(1+r)b_{t-1}\ , \qquad (25\text{-}1)$$

$$c_{t+1}^2 = \varepsilon_2(1+r)(1+r)b_{t-1}, \qquad (25\text{-}2)$$

$$b_t = \sigma_B(1+r)b_{t-1}. \qquad (25\text{-}3)$$

(25-3) gives[2]

Proposition 3: The laissez-faire growth rate in the bequest-as-consumption model is given as

$$\gamma\ (\mathrm{B}) = \sigma_B\ (1+\mathrm{r}) = \sigma_B\ (1+\mathrm{A})\ . \qquad (26)$$

For $\beta = \sigma_B$, $\gamma^* = \gamma$ (B). If the social discount factor is equal to the private discount factor, the optimal solution can be realized in the laissez-faire economy. If $\sigma_A = \sigma_B$, (26) is the same as (21). The higher σ_B , the higher γ .

2.2.5: Bequest-As-Exchange Model

In the bequest-as-exchange model of strategic bequests, the parent cares about some service or action undertaken by the offspring and the bequest given to the offspring is the payment for the service or action. Undertaking the action reduces utility. The action is defined in terms of time. Namely, the endowment of time is divided between leisure and action.

The parent's preferences are represented by a utility function given as

$$U_t = u_t + \sigma_C v(a_{t+1}) - \eta_C v(a_t) \qquad (27)$$
$$= \varepsilon_1 \log c_t^1 + \varepsilon_2 \log c_{t+1}^2 + \sigma_C \log a_{t+1} - \eta_C \log a_t$$

where a_t is the action the parent undertakes for his parent and a_{t+1} is the action the parent would like his offspring to undertake for him. σ_C / η_C is the ratio of the parent's benefit of his offspring's action to the parent's cost of undertaking action for his parent. An increase in a_{t+1} will raise U_t by σ_C / a_{t+1} and reduce U_{t+1} by η_C / a_{t+1}. Hence, σ_C / η_C may be regarded as the private discount factor of the future generation as in Sections 2.2.3 and 2.2.4.

In the bequest-as-exchange model the parent chooses the bequest subject to his budget constraint and, in addition, a self-selection constraint. Namely, the offspring will undertake the action more than $\bar{a}$ if

$$U_{t+1}[(1+r)b_t - b_{t+1} - s_{t+1}, (1+r)s_{t+1}, a_{t+2}, a_{t+1}] \geq \qquad (28)$$
$$U_{t+1}[\bar{b} - b_{t+1} - s_{t+1}, (1+r)s_{t+1}, a_{t+2}, \bar{a}]$$

where $U_{t+1}(\ )$ is the offspring's utility function. The utility on the right hand side of inequality (28) is the amount of utility the offspring receives if he refuses to undertake the action more than $\bar{a}$ and the parent refuses to inherit him more than $\bar{b}$. $\bar{a}$ and $\bar{b}$ correspond the exogenously given threat point[3].

Solving the budget constraints (15-1) and (15-2) for c^1_t and c^2_{t+1} and substituting the parent's utility function (27) and the self-selection constraint (28), we have the corresponding Lagrangian

$$L = \varepsilon_1 \log[(1+r)b_{t-1} - b_t - s_t] + \varepsilon_2 \log(1+r)s_t + \sigma_C \log a_{t+1} - \eta_C \log a_t$$
$$+q_{t+1}\{\varepsilon_1 \log[(1+r)b_t - b_{t+1} - s_{t+1}] + \varepsilon_2 \log(1+r)s_{t+1}$$
$$+\sigma_C \log a_{t+2} - \eta_C \log a_{t+1} - [\varepsilon_1 \log(\bar{b} - b_{t+1} - s_{t+1}) +$$
$$\varepsilon_2 \log(1+r)s_{t+1} + \sigma_C \log a_{t+2} - \eta_C \log \bar{a}]\} \tag{29}$$

where q_{t+1} is the Lagrange multiplier for the self-selection constraint at time t+1. The parent may choose (s_t, b_t, a_{t+1}) subject to the self-selection constraint. Hence, the first order conditions for the parent's problem are as follows:

$$\frac{(1+r)\varepsilon_2}{c^2_{t+1}} = \frac{\varepsilon_1}{c^1_t}, \tag{30-1}$$

$$\frac{q_{t+1}(1+r)}{c^1_{t+1}} = \frac{1}{c^1_t}, \tag{30-2}$$

$$\sigma_C = q_{t+1}\eta_C. \tag{30-3}$$

Assuming the constraint is binding, we have

Proposition 4: The laissez-faire growth rate in the bequest-as-consumption model is given as

$$\gamma(C) = \frac{\sigma_C(1+r)}{\eta_C} = \frac{\sigma_C(1+A)}{\eta_C}. \tag{31}$$

which must be compared with γ^*. For $\beta = \sigma_C / \eta_C$, $\gamma^* = \gamma$ (C). When the social discount rate is equal to the private discount rate, the optimal solution is realized in the laissez-faire economy.

Remark 1: We have investigated three intentional bequest motives. The qualitative results are almost the same. If $\sigma_C / \eta_C = \sigma_A = \sigma_B$, the bequest-as-exchange model and the altruism model have the same consumption and bequest functions (25-1, 2, 3) as in the bequest-as-consumption model.

Remark 2: The engine of growth consists of two effects, the intertemporal incentive effect (r) and the intergenerational transfer effect ($\sigma_{A,B,C}$). The former effect means that when the marginal product of capital is high, economic growth is promoted. A high level of the real rate of interest is a key to promote high growth. The latter effect means that the intergenerational transfer from the old to the young induces positive growth within the overlapping generations framework. The higher the private discount factor, the higher the growth rate.

Remark 3: The optimal solution is realized in the laissez-faire economy with intentional bequest motives if the social discount factor is equal to the private factor. Strictly speaking, the meaning of the private discount factor is different, depending on the bequest motive. However, an increase in $\sigma_{A,B,C}$ may be regarded as an increase in the care of future generations in all of the three bequest motives.

2.2.6 Pay-As-You-Go Social Security

Ihori (1994a) investigated the effect of pay-as-you-go social security on economic growth when intentional bequests are operative by using the model developed in this section. When the government employs a pay-as-you-go social security program, an increase in the public transfer may not affect the long-run growth rate in the three cases. Namely, if the social security contribution is levied on bequests and the social security benefit is regarded as a lump sum, the public intergenerational transfer policy does not affect the growth rate, irrespective of the bequest motives. If, on the other hand, the social security system is a lump sum form including contributions, in cases of the altruistic motive and the bequest-as-exchange motive, we still have the neutrality result. But in the case of the bequest-as-consumption motive, an increase in the contribution will reduce the growth rate.

The normative role of social security is qualitatively different among the three bequest motives. For example, in the altruistic model and the bequest-as-exchange model, anticipated lump-sum public transfer is completely neutral; it cannot affect welfare of generations. The growth rate of consumption is unaffected by the policy change. In the bequest-as-consumption model, public transfer will benefit the initially old generation and hurt the initially young generation and future generations. The growth rate of consumption is lowered only in the period of policy change, although the growth rate of bequests is unaffected. This result may be useful when we try to judge which bequest motive is most relevant in an empirical study.

3. Public Capital And Economic Growth

3.1 Analytical Framework

3.1.1 Public Capital

We now incorporate a public sector of production as the social factor of production into the endogenous growth model of Section 2. Let G be the quantity of public capital owned by the government. We assume that the services of public capital are provided to the private sector with user charges[4].

We only consider the role of public stock as an input to production. Production exhibits constant returns to scale in private capital K and public capital G together but diminishing returns in K and G separately. Even with a broad concept of private capital K, which includes nonhuman capital and human capital, production involves decreasing returns in production of outputs if the complementary government capital G does not expand in a parallel manner.

Given constant returns to scale, the production function (1) can be rewritten as

$$Y_t = AK_t^{1-\alpha}G_t^{\alpha}, \tag{32}$$

where $0 < \alpha < 1$ and in order to concentrate our study on the steady state it is assumed that the production function is Cobb-Douglas.

Thus, we have

$$q_k = \frac{\partial Y}{\partial K} = Ag^{\alpha}(1-\alpha), \tag{33}$$

$$q_g = \frac{\partial Y}{\partial G} = Ag^{\alpha-1}\alpha, \tag{34}$$

where q_k and q_g are the marginal productivity of private capital and public capital, respectively. g is the ratio of public to private capital (G/K). Neither private nor public capital will depreciate. Hence, the net rate of return on private capital r, which is equal to the market rate of interest, is now endogenous and given as

$$r = q_k. \tag{35}$$

The long run optimality condition of the allocation of capital between private and public sectors is given as the arbitrage condition.

$$q_k = q_g, \quad or \quad g^* = \frac{\alpha}{1-\alpha}. \tag{36}$$

The optimal growth rate at the first best solution is now given as

$$\gamma^* = \beta[Ag^{*\alpha}(1-\alpha)+1] = \beta[A\alpha^{\alpha}(1-\alpha)^{1-\alpha}+1] \tag{8'}$$

3.1.2 Growth Rate

As in Section 2, the relationship between the growth rate and the bequest motive is qualitatively the same among the three bequest motives. The engine of growth consists of two effects in this model. First, a higher $\sigma_{A,B,C}$ implies a higher degree of intergenerational transfers. This is called the intergenerational transfer effect. Second, a sufficiently high marginal product of private capital leads to long-run positive growth. This is called the intertemporal incentive effect.

In order to determine the ratio of public to private capital g = G/K in a market economy, it is necessary to specify how the government employs public capital policy and intergenerational transfer policy. We will not investigate either the bequest-as-consumption model or the bequest-as-exchange model in this section. The reader could conduct the similar comparative statics as in the altruism model.

3.2 Fully Funded System

3.2.1 The Model

The model can be used to study the effect of a fully funded social security program on the growth rate[5]. When we incorporate a funded system, the private budget constraints (15-1) and (15-2) are rewritten as

$$c_t^1 + s_t + b_t + \theta(1+r_t)b_{t-1} = (1+r_t)b_{t-1}, \tag{37-1}$$

$$c_{t+1}^2 = (1+r_{t+1})s_t + (1+q_{g_{t+1}})\theta(1+r_t)b_{t-1}, \tag{37-2}$$

where θ is the contribution rate. It seems plausible to assume that contributions are dependent on the first-period labor income.

We assume that contributions are invested to public capital in the fully funded system.

$$\theta(1+r_t)b_{t-1} = G_{t+1}. \tag{38}$$

Hence, the rate of return on the contribution is given as the net rate of return on public capital. This funded system is equivalent to debt-financed public investment.

In the fully funded social security system, an individual's maximization problem is rewritten as

$$\begin{aligned} W_t = {} & \varepsilon_1 \log[(1-\theta)(1+r_t)b_{t-1} - b_t - s_t] + \varepsilon_2 \log[(1+r_{t+1})s_t + \\ & (1+q_{g_{t+1}})\theta(1+r_t)b_{t-1}] + \sigma_A\{\varepsilon_1 \log[(1-\theta)(1+r_{t+1})b_t - b_{t+1} - s_{t+1}] + \\ & \varepsilon_2 \log[(1+r_{t+2})s_{t+1} + (1+q_{g_{t+2}})\theta(1+r_{t+1})b_t] + \sigma_A U_{t+2}\} \end{aligned} \tag{39}$$

The optimal conditions with respect to s_t and b_t are

$$\frac{\varepsilon_1}{c_t^1} = \frac{(1+r_{t+1})\varepsilon_2}{c_{t+1}^2}, \tag{40-1}$$

$$\frac{\varepsilon_1}{c_t^1} = \sigma_A\left[\frac{\varepsilon_1(1-\theta)(1+r_{t+1})}{c_{t+1}^1} + \frac{\varepsilon_2(1+q_{g_{t+2}})\theta(1+r_{t+1})}{c_{t+2}^2}\right]. \tag{40-2}$$

Therefore, when the funded social security system is imposed, the long-run growth rate in the altruism model is given as

$$\gamma = \sigma_A[(1-\theta)(1+r)+\theta(1+q_g)] = \sigma_A[1+r-\theta(r-q_g)]. \tag{41}$$

The marginal benefit of an increase in b_t is affected by the funded social security system in two ways: first, the social security contribution reduces c^1_{t+1} by (1+r). Secondly, the social security benefit raises c^2_{t+2} by (1+r)(1+q_g), which is equal to (1+q_g) in the first period. Thus, the overall effect of θ on γ is dependent on the sign of r-q_g.

3.2.2 Optimal Public Policy

Suppose θ (and hence g) is adjusted to satisfy the optimal allocation (36). Then r = q_g. From (33) and (36), $r = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Substituting this into (41) and considering r=q_g, we have

Proposition 5: When the optimal allocation between public capital and private capital is realized, the growth rate is given as

$$\gamma = \sigma_A[A\alpha^{\alpha}(1-\alpha)^{1-\alpha}+1]. \tag{42}$$

Note that γ is independent of ε_2. The saving motive for bequests σ_A stimulates the growth rate, but the saving motive for the old age ε_2 is irrelevant for the long run growth rate. When θ is endogenously determined to realize the optimal allocation of capital (36), an increase in ε_2 will be offset by an increase in θ, so that the ratio of public to private capital g remains fixed as g*. Since the optimal allocation (36) is always realized, only the intergenerational transfer effect is relevant.

(42) must be compared with (8)'. When the private discount factor is equal to the social discount factor, the laissez-faire solution is identical to the optimal solution. This result corresponds to the conventional wisdom in the exogenous growth model.

3.2.3 Exogenously Given Contribution Rate

If the contribution rate θ is exogenously given, r is not necessarily equal to q_g. From (37-1)(37-2) and (40-1), the private saving function is given as

$$s_t = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}(1+r_t)E_t b_{t-1} - \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} b_t - \frac{1+q_{g t+1}}{1+r_{t+1}}\theta(1+r_t)b_{t-1}, \tag{43}$$

where $E_t = 1 - \frac{r_{t+1}-q_{g t+1}}{1+r_{t+1}}\theta$. $E_t(1+r_t)b_{t-1}$ is lifetime income. Thus,

$$b_t + s_t = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} b_{t-1}(1+r_t)E_t + \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2} b_t - \frac{1+q_{g t+1}}{1+r_{t+1}}\theta(1+r_t)b_{t-1} = K_{t+1}. \tag{44}$$

Considering (38) and $b_t = \sigma_A E_t(1+r_t)b_{t-1}$, from (44) g is given as

$$g = \frac{\theta(\varepsilon_1+\varepsilon_2)(1+r)}{(\varepsilon_2+\sigma_A\varepsilon_1)(1-\theta)(1+r)-(1+q_g)\theta\varepsilon_1(1-\sigma_A)}. \tag{45}$$

(45) means that the sign of dg/dθ is normally positive. An increase in θ will raise g. From (41) we also have

$$\frac{d\gamma}{d\theta} = -\sigma_A(r-q_g) + \frac{\partial\gamma}{\partial g}\frac{dg}{d\theta}$$

where the sign of $\partial\gamma / \partial g$ is determined by the sign of $\theta - g/(1+g)$. Namely, we have $\partial\gamma / \partial g > 0$ if and only if $\theta < g/(1+g)$.

Proposition 6: If r < q_g and $\theta < g/(1+g)$, $d\gamma / d\theta > 0$. On the other hand, if r> q_g and $\theta > g/(1+g)$, $d\gamma / d\theta < 0$.

In other words, if public capital is under-accumulated and the contribution rate is very low, an increase in the fully funded social security program will stimulate the growth rate. If public capital is over-accumulated and the contribution rate is high, an increase in θ will reduce the growth rate. When θ is very low (high), it is likely that public capital is under-(over-) accumulated.[6] In this sense a low (high) level of θ will be associated with the positive (negative) effect of θ on γ.

Note than if r = q_g, we may still have $d\gamma / d\theta > 0$. In the conventional overlapping generations model (see Chapter 9 Section 2.3 and Blanchard and Fischer (1989) Chapter 3), it is well known that the fully funded program has no effect on capital accumulation. However, in the present model an increase in the ratio of public to private capital g always raises the marginal productivity of the private capital and hence it has a positive effect on the rate of economic growth. Since this intertemporal incentive effect always stimulates economic growth, the increase in the contribution rate may raise the growth rate even if r = q_g initially.

When θ is exogenously given, as shown in (45), ε_2 will affect γ. It is easy to see $dg/d\varepsilon_2 < 0$. Thus, if $\theta > g/(1+g)$, then an increase in ε_2 will stimulate γ and vice versa. The effect of σ_A on γ through changes in g is the same as that of ε_2. In addition, σ_A directly stimulates γ. Thus, an increase in σ_A will normally raise γ.

3.3 Remarks

Our analysis has shown how the long run growth rate is related to the bequest motives, social security program, and public capital. As in the standard infinite horizon model, the overlapping generations model with voluntary intergenerational transfers can produce the long-run positive growth rate. It should be stressed that the effect on the rate of growth is different from the effect on capital accumulation in the endogenous growth model.

The engine of growth consists of two effects, the intertemporal incentive effect and the intergenerational transfer effect. The first effect means that when capital is allocated between the public and private sectors to produce a high level of the net marginal product of private capital, economic growth is promoted. The second effect means that the intergenerational transfer from the old to the young induces positive growth.

Since the intergenerational transfer effect induces positive growth, the saving motive for bequests σ_A stimulates the growth rate for all the cases. On the other hand, the saving motive for the old age ε_2 may not necessarily stimulate the growth rate although an increase in ε_2 will normally stimulate private capital accumulation. See Table 11.2. In this sense, the bequest motive is more important than the preparation motive for the old age to attain high growth. It seems fair to say that intergenerational transfers are important in the real economy (see Kotlikoff and Summers, (1981)), which suggests that σ_A is high. Thus, our analysis means that the high level of intergenerational transfers can be valuable in promoting high economic growth.

4. Human Capital And Endogenous Growth

4.1 Three-Period Overlapping Generations Model

Recently, there have been some attempts to explore the role of human capital formation in endogenous growth. King and Rebelo (1990) worked within a calibrated, two-sector endogenous growth model, which has its origins in the microeconomic literature on human capital formation. They showed that national taxation can substantially affect long-run growth rates. In particular, for small open economies with substantial capital mobility, national taxation can readily lead to "development traps" or to "growth miracles".

Glomm and Ravikumar (1992) presented an overlapping generations model with heterogeneous agents in which human capital investment through formal schooling is the engine of growth. They examined the implications of public investment in human capital on growth and the evolution of income inequality in an economy in which individuals have different income/skill levels. They found that public education reduces income inequality more quickly than private education, while private education yields higher per capita incomes.

We incorporate two types of capital; physical capital and human capital and investigate the effect of three types of taxes on capital (a tax on physical capital, a tax on human capital, and a tax on physical bequests). Firms act competitively and use a constant-returns-to-scale technology.

$$Y_t = AK_t^{1-\alpha}H_t^{\alpha}, \qquad (46)$$

where Y is output, K is physical capital, and H is human capital. Human capital plays a similar role as public capital in the previous section.

To make the point clear consider an endogenous growth version of the three-period overlapping generations model similar to Batina (1987), Jones and Manuelli (1990), Caballe (1991), Glomm and Ravikumar (1992), Marchand, Michel and Pestieau (1992), and Buiter and Kletzer (1993). The number of households of each generation is normalized to one.

In the first period of his life ("youth"), a consumer born in period t-1 has an endowment of time, m, which he can either choose to consume as leisure ℓ_{t-1} in period t-1 or to allocate to an alternative use, education e_{t-1}. We assume for simplicity that this choice is exogenously given; $e_{t-1} = e$. This educational process during the first period of the household's life adds to the endowment of labor time in efficiency units H_t (human capital) during the second period ("middle age") i.e., during period t for a household born in period t-1.

In period t-1 when the household of generation t is young the parent of generation t-1 can choose to spend private resources other than time on human capital formation of his child, B_{t-1}, and physical savings (bequests) for his child, M_{t-1}.

The amount of time measured in efficiency units (human capital) which the household of generation t is endowed with a birth, is given by the (average) amount of human capital achieved by the previous generation during middle age. The stock of human capital used in employment by generation t during period t, H_t, is assumed to be a sum of a function of the current inputs (life cycle input $e_{t-1} = e$ and transfer input B_{t-1}) and the inherited stock of human capital, which equals the average level of human capital achieved by the previous generation, $\hat{H}_{t-1}$.

$$\hat{H}_t = (1-\delta)H_t + \overline{H}_t, \qquad (47)$$

where $\delta = 1 - \frac{1}{n}$. $\overline{H}_t$ is the ratio of the others' human capital to the total number of people[7]. n is the total number of individuals of each generation. The first term reflects the effect of his own human capital on the average human capital and the second term reflects the effect of the others' human capital on the average level. When $n \to \infty$, he would not recognize the externality effect of his own capital. The externality effect of human capital is perfect. When $n = 1$, he considers his own capital and the average level as equivalent, and hence the externality effect is absent. Thus, δ may be regarded as the degree of externality. This extra term, $\hat{H}_{t-1}$, embodies the similar kind of externality as in Romer (1986) and Lucas (1988). The introduction of this externality reflects the fact that production is a social activity.

Thus we have

$$H_t = \hat{H}_{t-1} + B_{t-1}\eta e,\ \eta > 0.$$

Or

$$H_t = \hat{H}_{t-1} + B_{t-1}, \tag{48}$$

where technological parameter η is normalized to $1/e$. The external effect in the accumulation of human capital is not fully considered by parents when they decide how much to invest in their children's education.

During middle age, the household choice of generation t concerns how much to consume c_t^1, to save for the old age s_t, to save for his child, M_t, and to spend on human capital formation of his child, B_t. The entire endowment of labor time services in efficiency units H_t is supplied inelastically in the labor market and wage income $h_t H_t$ is obtained.[8] h is the wage rate. In the last period of life ("old age " or "retirement") households do not work or educate themselves. The old of generation t consume c_{t+1}^2.

The government imposes taxes on capital accumulation when private intergenerational transfers and life cycle savings are present. For simplicity we assume that taxes are levied on two types of capital as well as capital income. We also assume that tax revenues are returned as a lump sum transfer to the same generation. This is a standard assumption of the differential incidence. Otherwise, the tax policy would include the intergenerational redistribution effect such as debt issuance or unfunded social security.

Thus, the middle-age budget constraint is given as

$$\begin{aligned} &c_t^1 + s_t + B_t + M_t + \theta_B(\hat{H}_t + h_t H_t) + \theta_M(1+r_t)M_{t-1} = \\ &\quad h_t H_t + (1+r_t)M_{t-1} + R_t^1 \end{aligned} \tag{49-1}$$

Or, substituting (48) into (49-1), we have

$$\begin{aligned} &c_t^1 + s_t + M_t + H_{t+1} + \theta_B(\hat{H}_t + h_t H_t) + \theta_M(1+r_t)M_{t-1} = \\ &\quad (\hat{H}_t + h_t H_t) + (1+r_t)M_{t-1} + R_t^1 \end{aligned} \tag{49-1'}$$

The old-age budget constraint is given as

$$c_{t+1}^2 + \tau(1+r_{t+1})s_t = (1+r_{t+1})s_t + R_{t+1}^2, \tag{49-2}$$

where θ_B is a tax on human capital, θ_M is a tax on physical bequests, and τ is a tax on life-cycle physical capital. R_t^1 is a lump sum transfer on the young in period t, and R_t^2 is a lump sum transfer on the old in period t.

The government budget constraint is given as

$$R_t^1 = \theta_B(1+h_t)H_t + \theta_M(1+r_t)M_{t-1}, \tag{50-1}$$

$$R_{t+1}^2 = \tau(1+r_{t+1})s_t. \tag{50-2}$$

Taxes on human capital are represented by taxes on wage income plus human capital:

$\theta_B(1+h_t)H_t$ 9. Note that from (47) $H_t = \hat{H}_t$ holds in the aggregate economy.

The feasibility condition in the aggregate economy is given as

$$c_t^1 + c_t^2 + K_{t+1} + H_{t+1} = Y_t + K_t + H_t. \quad (51)$$

Physical capital accumulation is given as

$$s_t + M_t = K_{t+1}. \quad (52)$$

Recall that both life cycle saving and physical bequests provide fund for physical capital accumulation in the aggregate economy. Note that human capital accumulation is given by (47) and (48). The rates of return on two types of capital are respectively given as

$$r = \partial Y / \partial K = A(1-\alpha)k^{-\alpha}, \quad (53\text{-}1)$$

$$h = (Y - rK)/H = \partial Y / \partial H = A\alpha k^{1-\alpha}. \quad (53\text{-}2)$$

where $k = K/H$ is the physical capital-human capital ratio.

4.2. Economic Growth And Efficiency

4.2.1 Optimizing Behavior In The Market Economy

An individual born at time t-1 will solve the following problem of maximizing. He will choose s_t, H_{t+1}, and M_t given $\overline{H}_{t+1}$ in (47). Substituting (47)(49-1)' and (49-2) into (18), we have

$$U_t = \varepsilon_1 \log[(1-\theta_B)(\hat{H}_t + h_t H_t) + (1-\theta_M)(1+r_t)M_{t-1} - H_{t+1} - s_t - M_t + R_t^1] +$$
$$\varepsilon_2 \log[(1-\tau)(1+r_{t+1})s_t + R_{t+1}^2] + \sigma_A\{\varepsilon_1 \log[(1-\theta_B)((1-\delta)H_{t+1} + \overline{H}_{t+1} + h_{t+1}H_{t+1}) +$$
$$(1-\theta_M)(1+r_{t+1})M_t - H_{t+2} - s_{t+1} - M_{t+1} + R_{t+1}^1] + \varepsilon_2 \log[(1-\tau)(1+r_{t+2})s_{t+1} + R_{t+2}^2]$$
$$+\sigma_A U_{t+2}\} \quad (54)$$

The optimality conditions with respect to s_t, H_{t+1}, and M_t are respectively

$$1/c_t^1 = (1-\tau)(1+r_{t+1})\varepsilon / c_{t+1}^2, \quad (55\text{-}1)$$

$$1/c_t^1 = \sigma_A(1-\theta_B)(1-\delta+h_{t+1})/c_{t+1}^1, \quad (55\text{-}2)$$

$$1/c_t^1 \geq \sigma_A(1-\theta_M)(1+r_{t+1})/c_{t+1}^1 \text{ with equality if } M_t > 0, \quad (55\text{-}3)$$

where $\varepsilon = \varepsilon_2/\varepsilon_1$. s cannot be zero. Otherwise, c^2 would be zero, which is inconsistent with the optimizing behavior. H cannot be zero either. Otherwise, Y would be zero, which is inconsistent with the optimizing behavior. However, M could become zero. If the private marginal return of educational investment is higher than the private marginal return of physical bequests at $M=0$, the bequest is operated only in the form of human capital investment.

4.2.2 The Constrained Economy

Suppose the government does not levy any taxes; $\tau = \theta_B = \theta_M = 0$. If $1-\delta+h > 1+r$ at $M=0$, we have the corner solution where physical bequests are zero. In such a case we have from (49-2) and (55-1)

$$s = c^1\varepsilon.$$

Substituting this into (49-1)', we have

$$H_{t+1} + [\frac{1}{\varepsilon}+1]s_t = (1+h_t)H_t.$$

On the other hand, from (55-2) we have in the steady state

$$H_{t+1} = \sigma_A(1-\delta+h_t)H_t$$

Hence, considering the above two equations, the steady-state physical capital-human capital ratio $\hat{k}$ is given as a solution of (56).

$$1+(\frac{1}{\varepsilon}+1)k=\frac{1+h}{(1+h-\delta)\sigma_A}. \qquad (56)$$

As shown in Figure 11.1, the left-hand side of (56) is increasing with k, while the right-handside of (56) is decreasing with k. When ε increases, the left-hand side decreases, so that $\hat{k}$ increases. When σ_A decreases, the right-hande side increases, so that $\hat{k}$ increases.

On the other hand, considering (53-1) and (53-2), $1-\delta+h>1+r$ at $M=0$ if and only if

$$A\alpha\hat{k}^{-\alpha}[\hat{k}-\frac{1-\alpha}{\alpha}]>\delta.$$

Or

$$\hat{k}>\tilde{k},\ \ where\ \tilde{k}\ satisfies\ A\alpha k^{-\alpha}[k-\frac{1-\alpha}{\alpha}]=\delta. \qquad (57)$$

When there are less incentives to do physical bequests, we may well have the corner solution of $M=0$. The larger ε and the smaller σ_A, it is more likely to have inequality (57).

From (55-2) and (56) we have

Proposition 7: The laissez faire growth rate in the bequest constrained economy is given as

$$\gamma_{M=0}=\sigma_A(1-\delta+A\alpha\hat{k}^{1-\alpha}) \qquad (58)$$

where $\hat{k}$ is given by (56).

As shown in (56), an increase in the intragenerational preference for life cycle capital ε will raise the physical capital-human capital ratio k. Hence it will raise the rate of return on human capital and will promote economic growth. An increase in the intergenerational preference σ_A has two effects. It will stimulate the intergenerational transfer from the old to the young, which induces high growth. On the other hand, it will reduce k and the rate of return on human capital, h, which depresses economic growth.

Considering (56) and (58), we have

$$\frac{\partial\gamma}{\partial\sigma_A}=(1+h-\delta)\frac{(1+h-\delta)[(1+h)(1-\sigma_A)+\sigma_A\delta-(1-\alpha)h]}{\delta(1-\alpha)h+[(1+h)(1-\sigma_A)+\sigma_A\delta](1+h-\delta)} \qquad (59)$$

Thus, if $\frac{1+\alpha h}{1+h-\delta}>\sigma_A$, then $\frac{\partial\gamma}{\partial\sigma_A}>0$ (and vice versa). In other words, if α and δ are high, it is likely to have $\frac{\partial\gamma}{\partial\sigma_A}>0$. However, it should be stressed that $\frac{\partial\gamma}{\partial\sigma_A}<0$ is also possible. In the bequest constrained economy an increase in the parent's concern for the child's welfare does not necessarily raise the growth rate.

The laissez faire economy may not attain the first best solution due to two reasons. First, the externality effect in the accumulation of human capital is not considered by the parent. This means that the competitive growth rate becomes too low. Second, M_t cannot be negative because there is no institutional mechanism to enforce such a liability on future generations. Human capital is too little and the marginal return of human capital is too high, which means that the competitive growth rate becomes too high. The lower is δ, it is more likely that the second effect dominates and the laissez faire growth rate is too high.

4.2.3 The Unconstrained Economy

When $M > 0$, we have both (55-2) and (55-3) with equality. Hence,

$$1-\delta+h = 1+r. \tag{60}$$

k is given as $\tilde{k}$.

$$A\alpha k^{-\alpha}[k - \frac{1-\alpha}{\alpha}] = \delta. \tag{61}$$

From (55-2,3) and (61), we have

Proposition 8: The unconstrained growth rate is given as

$$\gamma_{M>0} = \sigma_A(1-\delta + A\alpha\tilde{k}^{1-\alpha}) = \sigma_A[1 + A(1-\alpha)\tilde{k}^{-\alpha}]. \tag{62}$$

In this case an increase in σ_A always raises the growth rate. ε cannot affect the growth rate. $\tilde{k}$ is independent of σ_A or ε. (62) shows that the life cycle saving motive ε does not affect the growth rate, while an increase in the transfer saving motive σ_A definitely raises the growth rate.

Since physical bequests are operative, the competitive economy is different from the first best solution only due to the externality effect of human capital. Thus, the laissez faire growth rate is always too low.

4. 3 Taxes And Economic Growth

4.3.1 The Constrained Economy

We now consider the effect of taxes on capital accumulation in the bequest constrained economy of $M = 0$. When taxes are incorporated, (56) may be rewritten as

$$1 + [\frac{1}{\varepsilon(1-\tau)} + 1]k = \frac{1+h}{(1+h-\delta)\sigma_A(1-\theta_B)} \tag{56'}$$

(58) may be rewritten as

$$\gamma_{M=0} = \sigma_A(1-\theta_B)(1-\delta + A\alpha\hat{k}^{1-\alpha}) \tag{58'}$$

An increase in the tax on life-cycle capital, τ, reduces k and hence will depress the growth rate.

However, the effect of an increase in the tax on transfer human capital, θ_B, on the growth rate is ambiguous. It will directly reduce the growth rate, while it will indirectly raise the growth rate by raising k and h. Namely, an increase in the bequest tax raises the physical capital-human capital ratio, and hence increases h. If this effect is dominant, an increase in the bequest tax raises the growth rate.

We have from (58)'

$$\frac{\partial\gamma}{\partial\theta_B} = -\sigma_A \frac{(1+h-\delta)[(1+h)(1-\sigma_A{*}) + \sigma_A{*}\delta - (1-\alpha)h]}{\delta(1-\alpha)h + [(1+h)(1-\sigma_A{*}) + \sigma_A{*}\delta](1+h-\delta)} \tag{63}$$

where $\sigma_A{*} = (1-\theta_B)\sigma_A$. Hence, if $(1-\theta_B)\sigma_A < \frac{1+\alpha h}{1+h-\delta}$, then $\frac{\partial\gamma}{\partial\theta_B} < 0$ (and vice versa). When $\theta_B = 1$, we always have $\frac{\partial \gamma}{\partial \theta_B} < 0$. When $\theta_B = 0$, the sign of (63) is just opposite to the sign of (59). Therefore, if α is high and $\frac{\partial \gamma}{\partial \sigma_A} > 0$, then we will have $\frac{\partial \gamma}{\partial \theta_B} < 0$. However, if $\sigma_A > \frac{1+\alpha h}{1+h-\delta}$, then $\frac{\partial \gamma}{\partial \theta_B} > 0$ at $\theta_B = 0$. Suppose $\delta = 0$, $1 = \varepsilon$, $\sigma_A = 0.8, \alpha = 0.5, A = 2\sqrt{6}$, then it is easy to see $\frac{\partial \gamma}{\partial \theta_B} > 0$ at $\theta_B = 0$ and $\frac{\partial \gamma}{\partial \theta_B} = 0$ at $\theta_B = 1/16$. As shown in Figure 11.2, the tax rate which maximizes the growth rate, θ_B^*, is given by 1/16 in such a case[10].

We have

Proposition 9. An increase in the tax on life-cycle capital, τ, will reduce k and hence will depress the growth rate. However, the effect of an increase in the tax on transfer human capital, θ_B, on the growth rate is ambiguous.

Finally, let us consider how to attain the first best solution by using capital taxes. The optimal levels of τ and θ_B are given as

$$\tau = 0, \tag{64}$$

$$(1-\theta_B)[1-\delta + A\alpha(\frac{\alpha}{1-\alpha})^{\alpha-1}] = 1 + A\alpha(\frac{\alpha}{1-\alpha})^{\alpha-1} \tag{65}$$

From (65) the optimal level of θ_B is negative so long as $\delta > 0$[11]. Furthermore, in order to attain the first best solution, an additional lump-sum intergenerational transfer from the young to the old such as debt issuance or unfunded social security is also needed. Such a policy can substitute negative physical bequests.

4.3.2 The Unconstrained Economy

When $M > 0$, we have both (55-2) and (55-3) with equality. Hence,

$$(1-\theta_B)(1-\delta+h) = (1-\theta_M)(1+r) \tag{66}$$

Suppose $\theta_B = \theta_M$. Then $1+r = 1-\delta + h$, or k is given by $\tilde{k}$. The growth rate is hence given as

$$\gamma_{M>0} = \sigma_A(1-\theta_B)(1-\delta + A\alpha\tilde{k}^{1-\alpha}) = \sigma_A(1-\theta_M)[1 + A(1-\alpha)\tilde{k}^{-\alpha}] \tag{62'}$$

An increase in $\theta_B = \theta_M$ does not affect k but reduces the growth rate. An increase in θ_B only raises k and reduce r. Hence from the second equation of (62)' it reduces the growth rate. An increase in θ_M only reduces k and h. Hence from the first equation of (62)' it also reduces the growth rate. In other words,

Proposition 10. An increase in any taxes on transfer capital will definitely reduce the growth rate when physical bequests are operative.

(62)' is independent of τ; the tax rate on the life cycle capital does not affect the growth rate.

The optimal level of θ_B is given as (65), which is the same as in the constrained case. Note that the optimal level of θ_M is zero. We also have $\tau = 0$ at the first best solution. In this case the market failure comes only from the externality effect of human capital. A subsidy to human capital accumulation raises the growth rate and can attain the first best solution.

4.4. Remarks

This section has incorporated the altruistic bequest motive and human capital accumulation into an endogenous growth model of overlapping generations. We have shown that the impact of taxes on capital accumulation on the growth rate is different, depending on whether physical bequests are operative. When physical bequests are zero, an increase in the tax on human capital accumulation may not reduce the rate of economic growth, while an increase in a tax on life-cycle physical capital will reduce the growth rate. If physical bequests are operative, a tax on life cycle capital accumulation will not affect the growth rate, while an increase in any taxes on transfer capital (educational investment or physical bequests) will reduce the growth rate. Our analysis explored the paradoxical possibility that taxes on capital accumulation may not reduce the rate of economic growth in several cases.

Finally, in the bequest constrained economy the laissez faire growth rate may be too high if the externality effect is small. A subsidy to human capital accumulation and a lump-sum transfer from the young to the old can attain the first best solution. In the unconstrained economy, the market failure comes only from the externality effect of human capital. A subsidy to human capital accumulation raises the growth rate and can attain the first best solution.

5. Further Study On Human Capital Formation

Galor and Zeira (1993) analyzed the role of wealth distribution in macroeconomics through investment in human capital. They developed an equilibrium model of open economies with overlapping generations and inter-generational altruism. A single good can be produced by either a skill-intensive or an unskilled-intensive process. Individuals live for two periods. In the first period they may either invest in human capital and acquire education or else work as unskilled. In the second period, they work as skilled or unskilled - according to their education level, consume and leave bequests. Individuals are assumed to be identical with respect to their potential skills and preferences, and differ only with respect to their inherited wealth. It is further assumed that there are enforcement and supervision costs on individual borrowers and hence the borrowing interest rate is higher than the lending rate. Consequently, the inheritance of each individual determines whether she invests in human capital or not.

There are two major assumptions in their model. One is that credit markets are imperfect, as the interest rate for individual borrowers is higher than that for lenders. The second is that investment in human capital is indivisible, namely that there is a technological non-convexity.

In their formulation, the distribution of wealth determines the aggregate levels of investment, of skilled and unskilled labor and of output. But the effect of wealth distribution is not only short run, as the different levels of investment in human capital in turn determine the distribution of income, which gradually changes the distribution of wealth through time. They showed that the economic dynamics of dynasties depends on initial wealth. There are rich dynasties, in which all generations invest in human capital, work as skilled, and leave a large bequest. There are poor dynasties, in which people inherit less, work as unskilled, and leave less to their children. Therefore, the initial distribution of wealth determines how big these two groups of dynasties are, and therefore what is the long-run equilibrium in the economy. Wealth distribution, therefore, carries long run as well as short-run implications.

Perroti (1993) analyzed the impact of income distribution on growth when investment in human capital is the source of growth and individuals vote over the degree of redistribution in the economy. Individuals can belong to one of three different income groups. Growth and changes in pre-tax income distribution are the effect of investment in education. The latter benefits the investor directly, and all the other agents indirectly through a production externality. As in Galor and Zeira (1993), in the absence of perfect capital markets, those individuals whose post-tax income is below the cost of acquiring education will be unable to invest in human capital, and in the next period, they will earn the same pre-tax income. In contrast, those who can afford the expenditure needed to obtain education will have a higher income.

His model has three main features. First, very different patterns of income distribution are conducive to high growth at different levels of per capita income. Second, growth is associated with an externality whereby investment in human capital by one group increases the productivity of other groups, thus potentially enabling them to invest in human capital. Third, the initial pattern of income distribution and the resulting political equilibrium are crucial in determining whether the transmission of this externality is promoted, in which case growth is enhanced, or prevented, in which case growth is stopped. The model implies an inverted-U relation between levels of inequality and levels of income in cross-sections, but not necessarily in time series, a result that seems consistent with a number of empirical studies. Human capital formation deserves more attention in future research of fiscal policy in the overlapping generations growth model.

Table 11.1: Estimates Of The Share Of Transfer Wealth

Author of study	*Year*	*Share of transfer wealth*
	Japan	
Cambell	1974-84	At most 28.1
Dekle	1966-83	3-27
Hayashi	1969-74	At least 9.6
Barthold and Ito		At least 27.7-41.4
Dekle	1983	At most 48.7
	U.S.	
Ando and Kennickell	1960-80	15.0-41.2
Kotlikoff and Summers	1974	20-67
Barthold and Ito		At least 25
Menchik and David	1946-64	18.5
Projector and Weiss		15.5
Barlow et al.	1964	14.3-20
Morgan et al.		less than 10

Source: Horioka (1991)

able 11.2: The effect on growth rate

	θ	ε_2	σ_A
3.3.2	endogenous	0	+
3.3.3	+ if θ is low	-	+?
	- if θ is high	+	+

1 . This section does not include the external contribution of investment to aggregate productivity such as Arrow (1962) and Marchand, Michel and Pestieau (1992). Section 4 investigates the external contribution of human capital accumulation.

2 . In the homothetic utility function the propensity to bequeath σ_B would be a function of r_{t+1}. The qualitative results would be the same as in the Cobb Douglas case. If the parent's utility depends on the lifetime wealth of his offspring instead of the bequest itself, we have the same bequest function (25-3) in the case of the Cobb-Douglas one.

3 . For simplicity we assume that the parent has the bargaining power such that he captures all of the gains from trade against the offspring. Bernheim, Shleifer, and Summers (1985) and Cremer, Kessler, and Pestieau (1992) showed that when a parent can successfully threaten to disinherit a potential child, he extracts the full surplus generated through interaction with the child. However, success in this regard requires him to specify an alternative use for his resources that is believable; in particular, he must have more than one potential children to whom he can credibly plan to leave the bulk of his estate.

4 . A number of questions arise concerning the specification of public services as input to production. Barro and Sala-i-Martin (1990, 1992, 1995) considered three versions of public services: publicly-provided private goods, which are rival and excludable; publicly-provided public goods, which are non-rival and non-excludable; and publicly-provided goods that are subject to congestion. The present model can be modified to include this aspect of public services without altering the general nature of the results.

5 . This section is based on Ihori (1994b).

6 . Nemoto, Kamada, and Kawamura (1990) assessed the optimality of Japanese public capital using the social discount rates for public investment. They found that actual levels of public capital stocks during the 1960-82 period had been persistently less than optimal levels. Their result implies the deficiency of public capital, which happens to be consistent with intuitive claims prevailing in Japan. They also found that public capital had been accumulated at a higher rate than the optimal level had grown and, hence, that the gap between actual and optimal levels of public capital had continuously diminished. If so, our analysis suggests that higher growth of public capital has stimulated the Japanese growth rate. Furthermore, an increase in the contribution rate in the Japanese funded social security system thus far has probably stimulated the growth rate as well.

7 . $\hat{H}_t = \frac{1}{n}\sum H_t^i = \frac{1}{n}H_t^j + \frac{1}{n}\sum_{i\neq j} H_t^i$, where H_t^j is household j of generation's human capital.

8 . For simplicity we do not incorporate unskilled labor.

9 . We could consider the case where the government imposes taxes on wage income and interest income. The qualitative results would be the same.

10 . Figure 11.2 is very close to a diagram of the effect of taxes on growth by Barro (1990). Barro incorporated public capital to produce such a figure. We can derive the similar relationship in the framework where tax revenues are not used for public input.

11 . The growth-maximizing tax rate, which is 1/16 in the example above, is not optimal. When $\delta = 0$, the welfare-maximizing tax rate should be zero.

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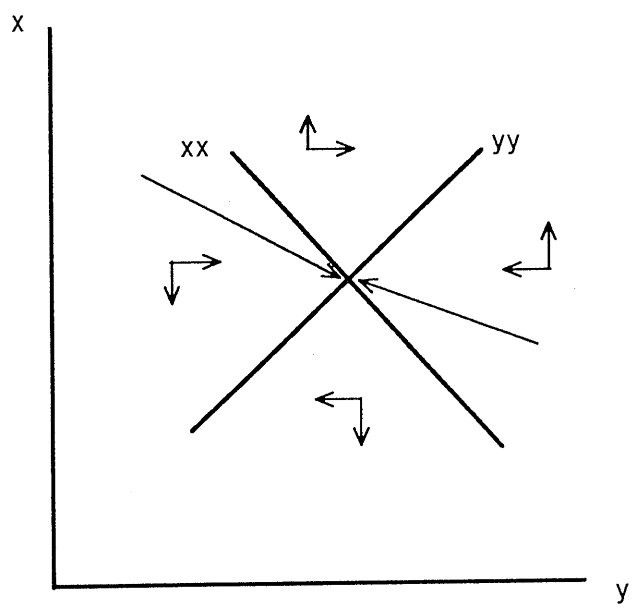


Figure 1.1 Phase Diagram

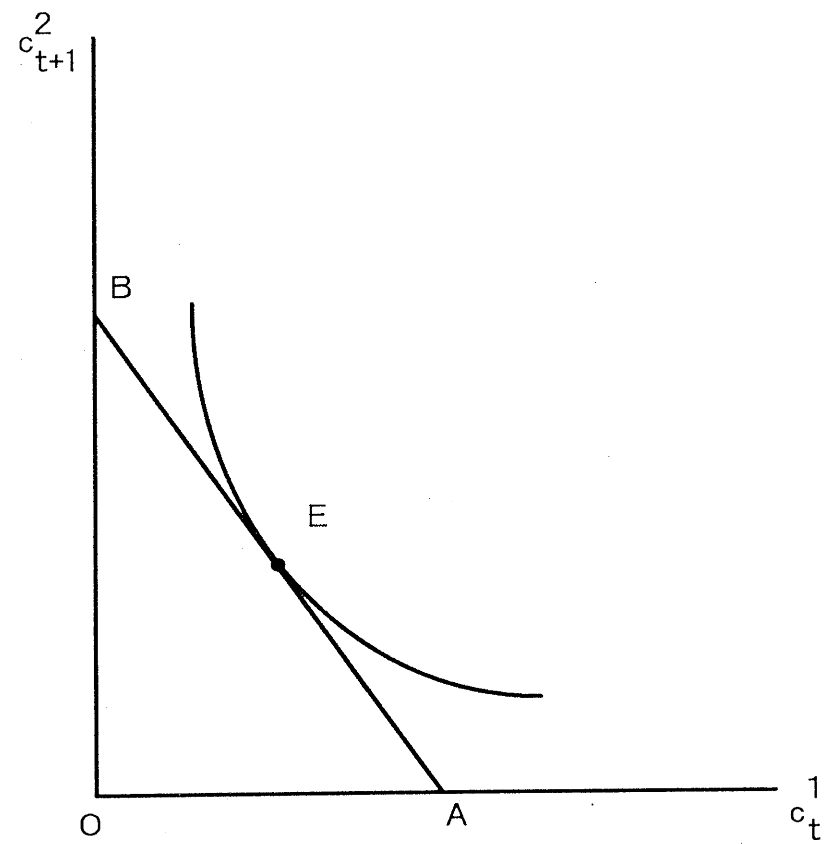


Figure 2.1 Consumer's Optimizing Behavior

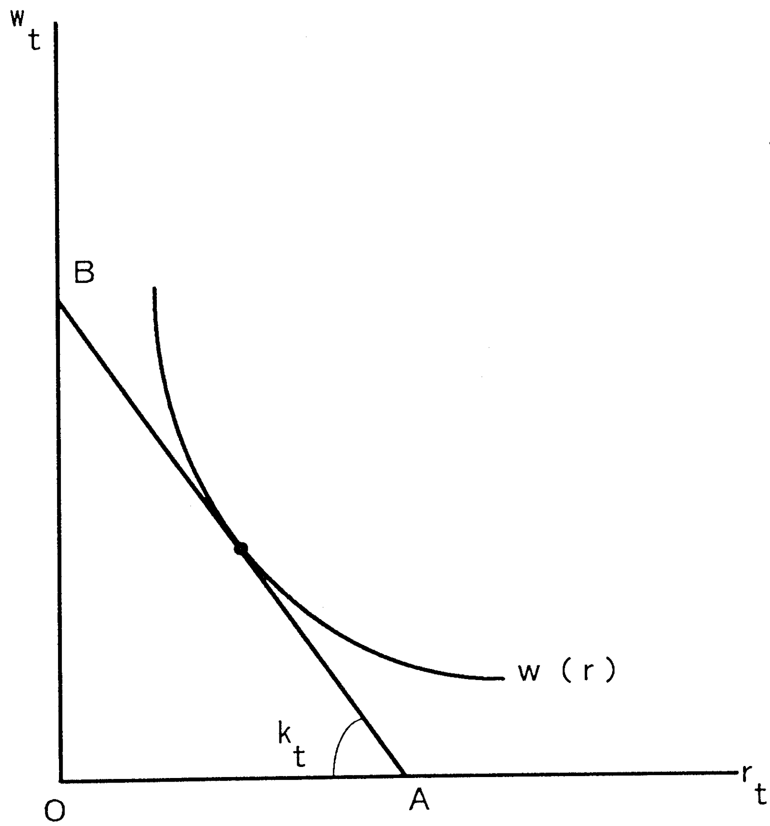


Figure 2.2 Factor Price Frontier

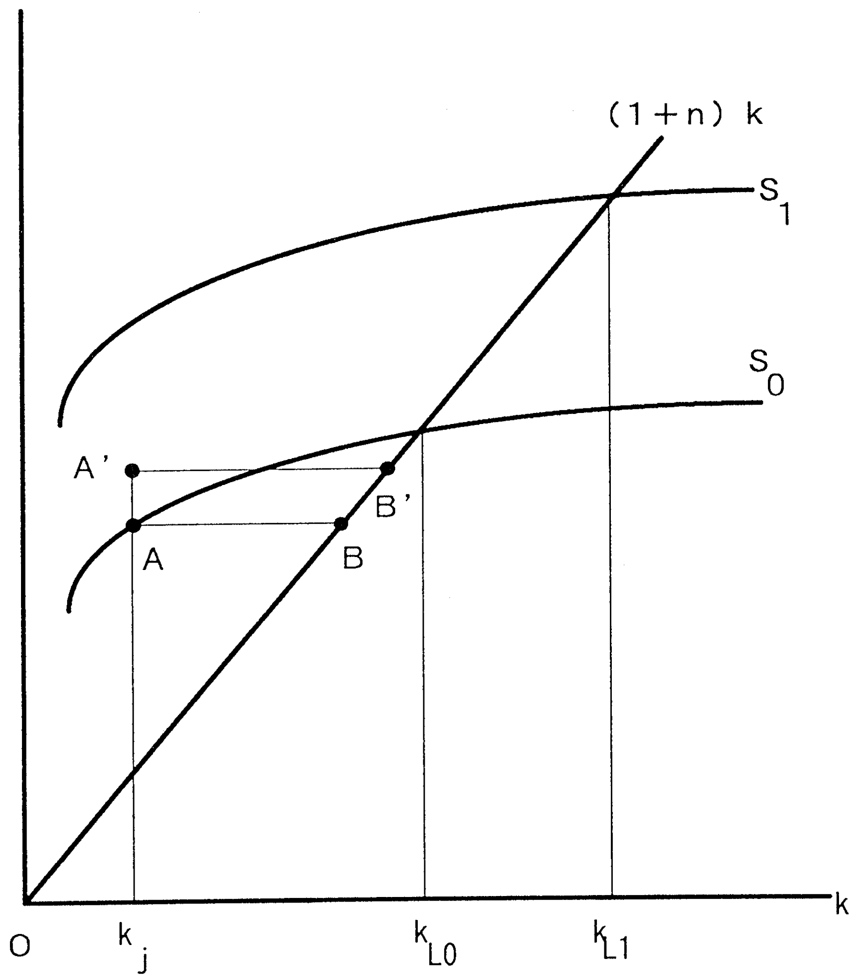


Figure 3.1 Tax Reform and Capital Accumulation

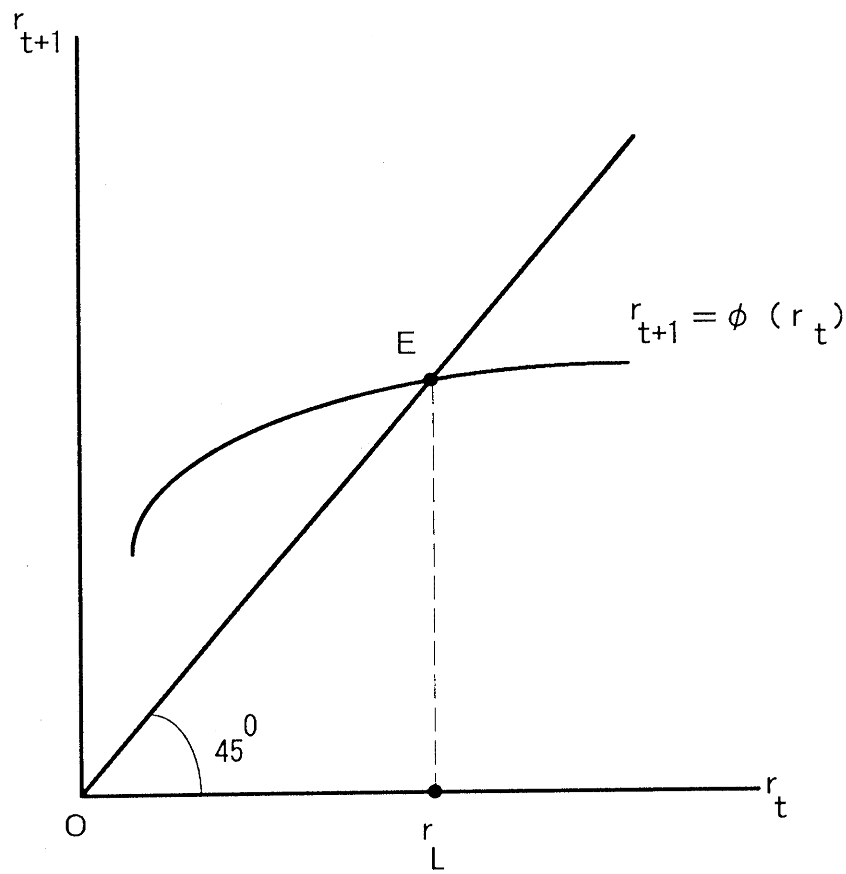


Figure 2.3 Uniqueness and Stability

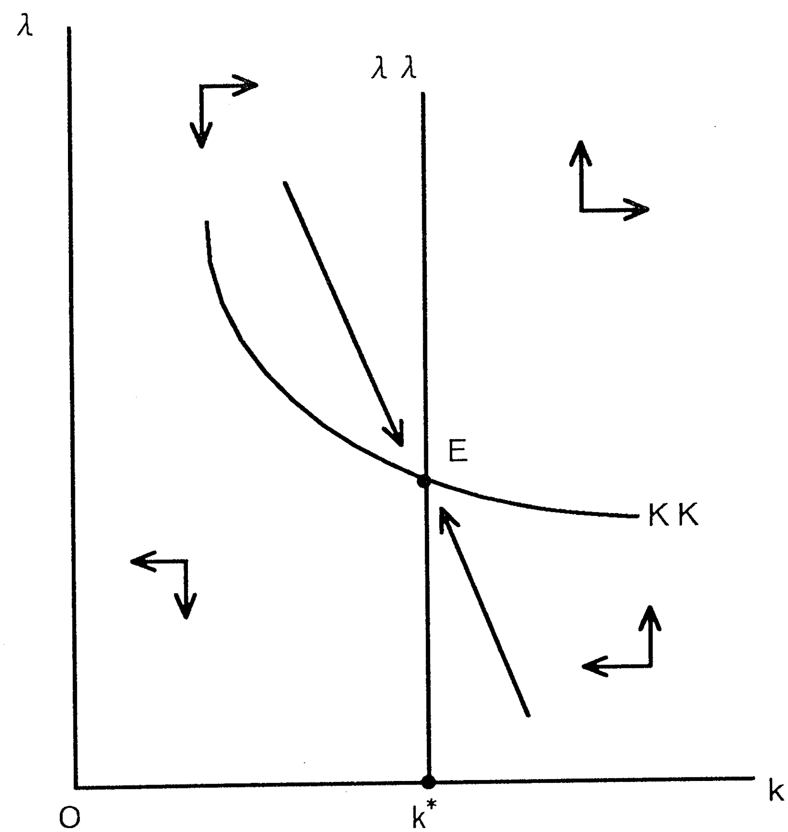


Figure 2.4 Phase Diagram of the Basic Model

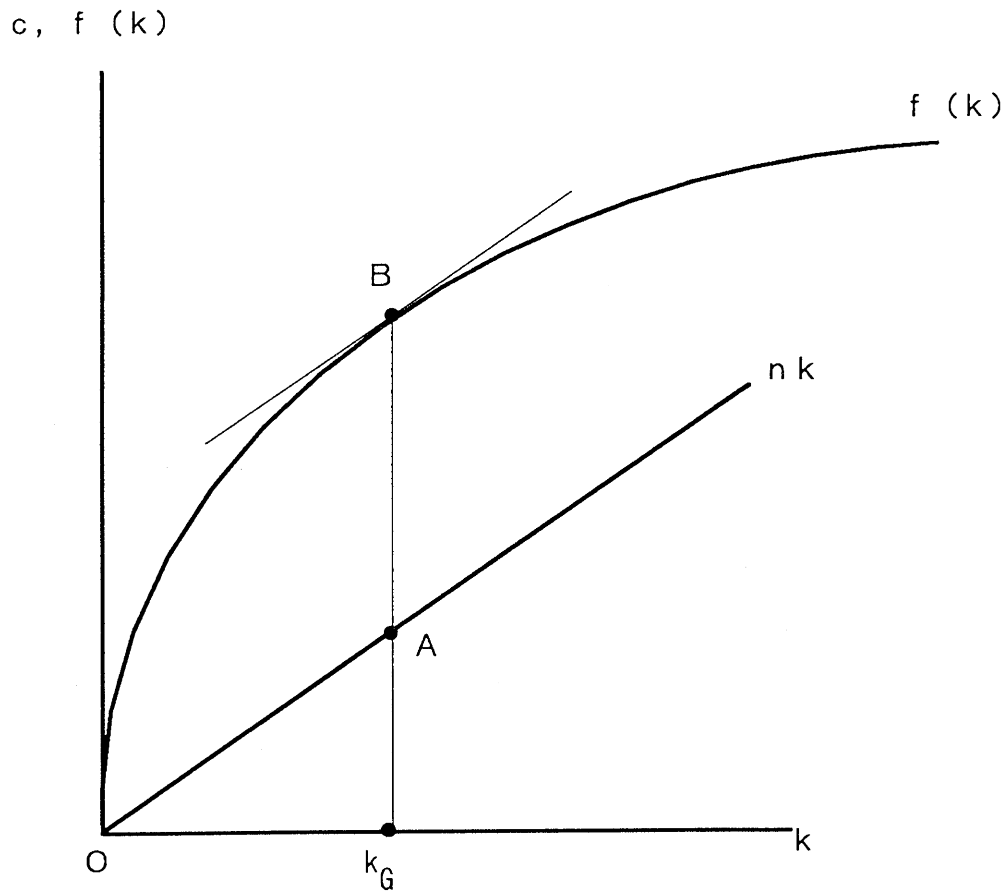
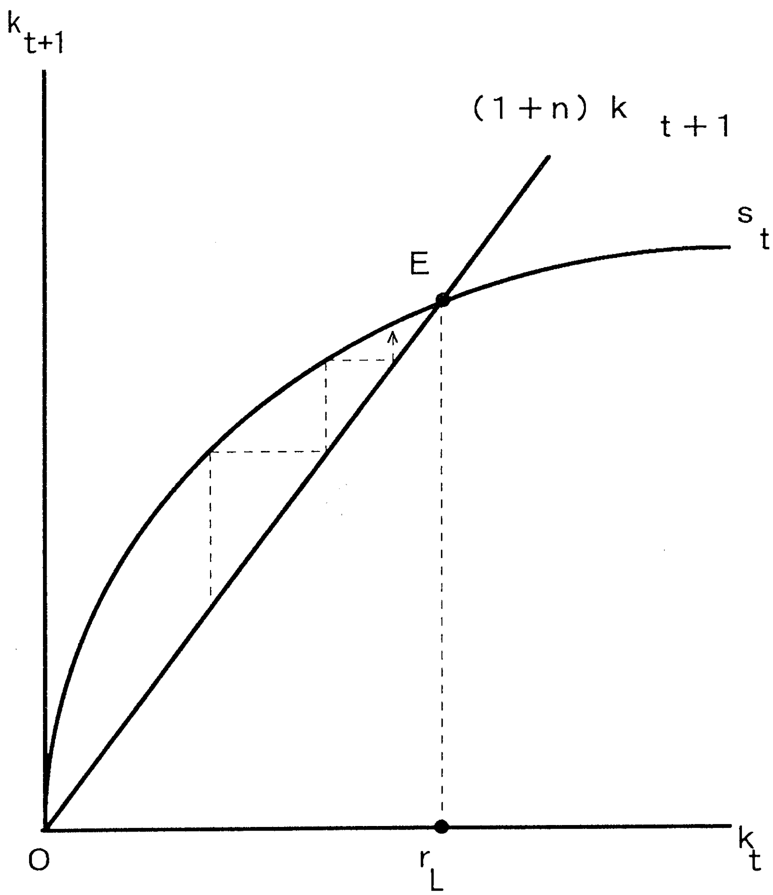


Figure 2.5 The Golden Rule



Fogire 2.6　Cobb-Douglas Case

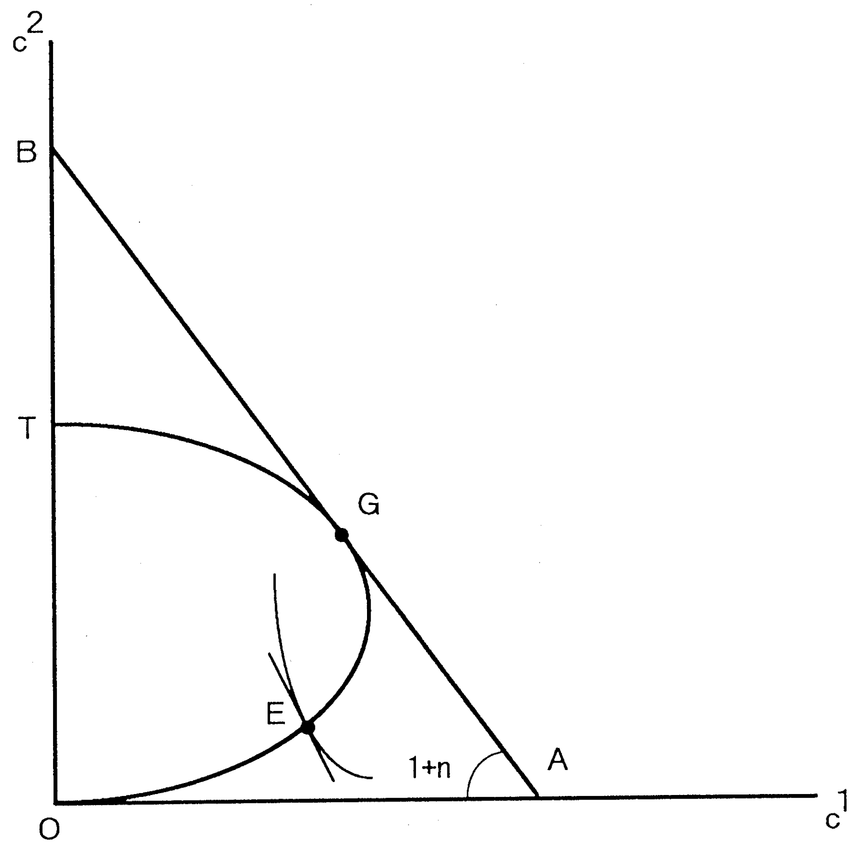


Figure 2.7 Long-Run Equilibrium

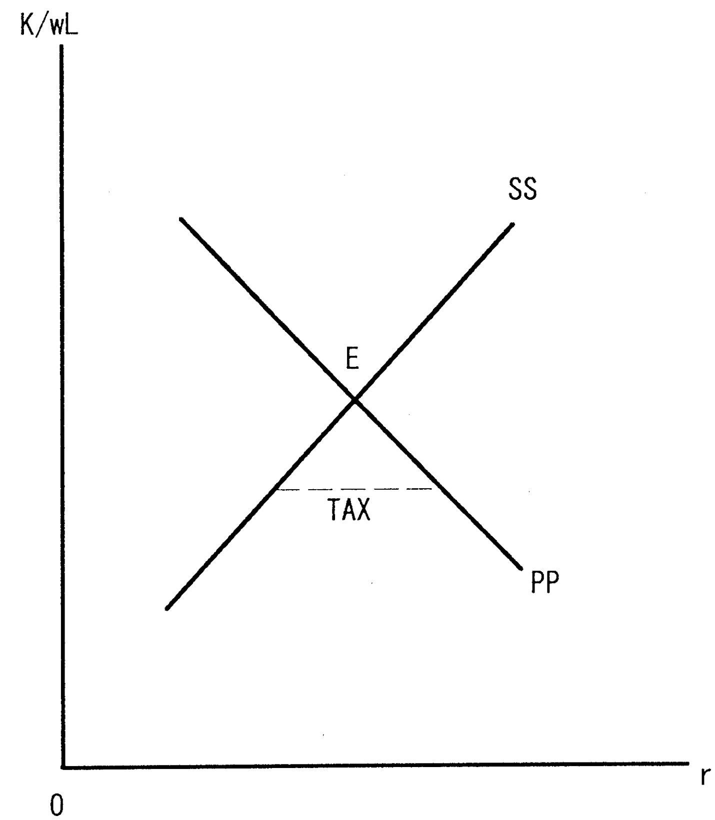


Figure 4.1 Equilibrium in the Bench Mark Model

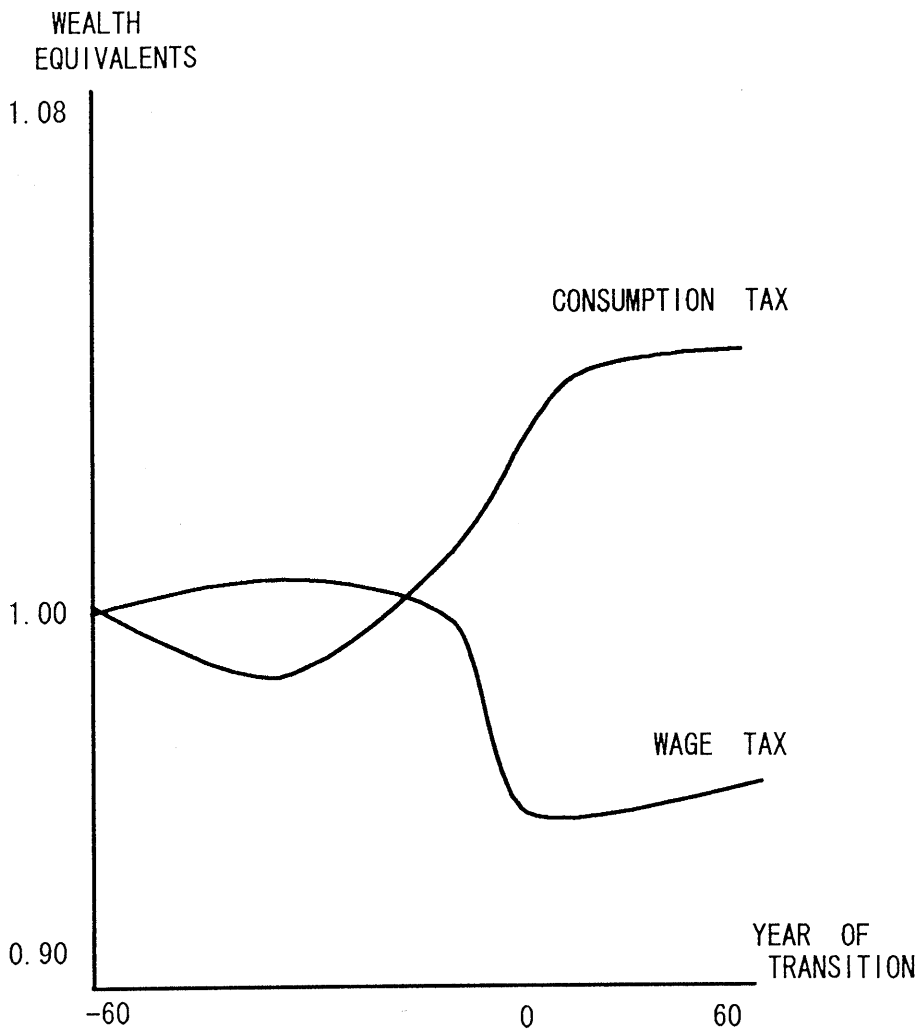


Figure 4.2 Welfare Effects of Tax Reform

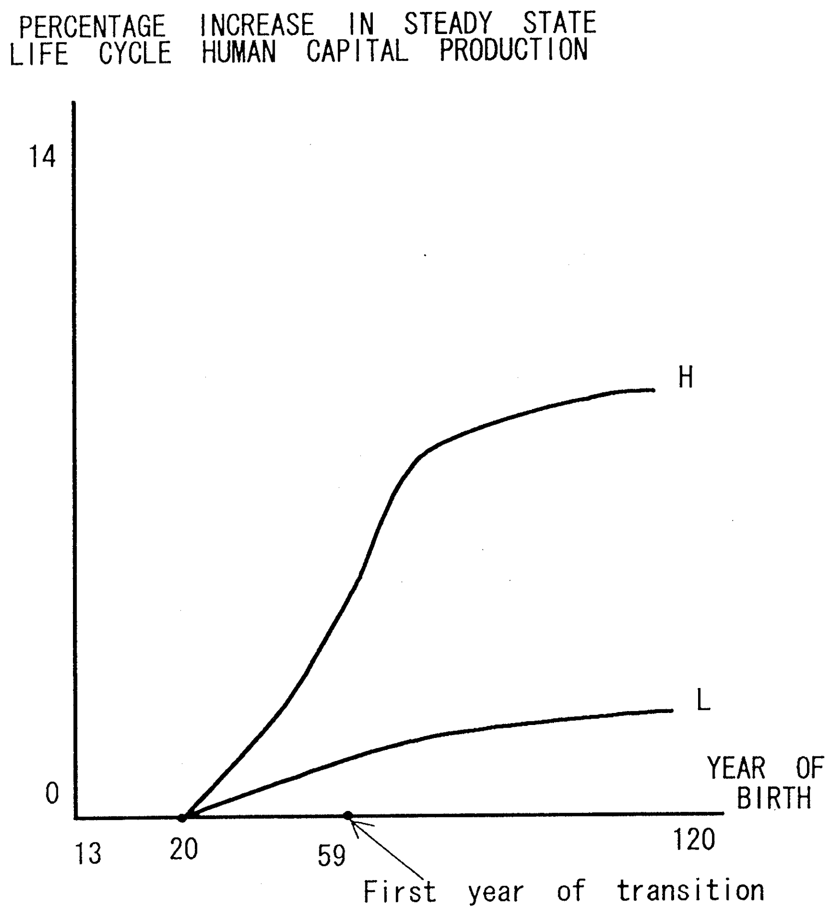


Figure 4.3 Human Capital Accumulation and Tax Reform

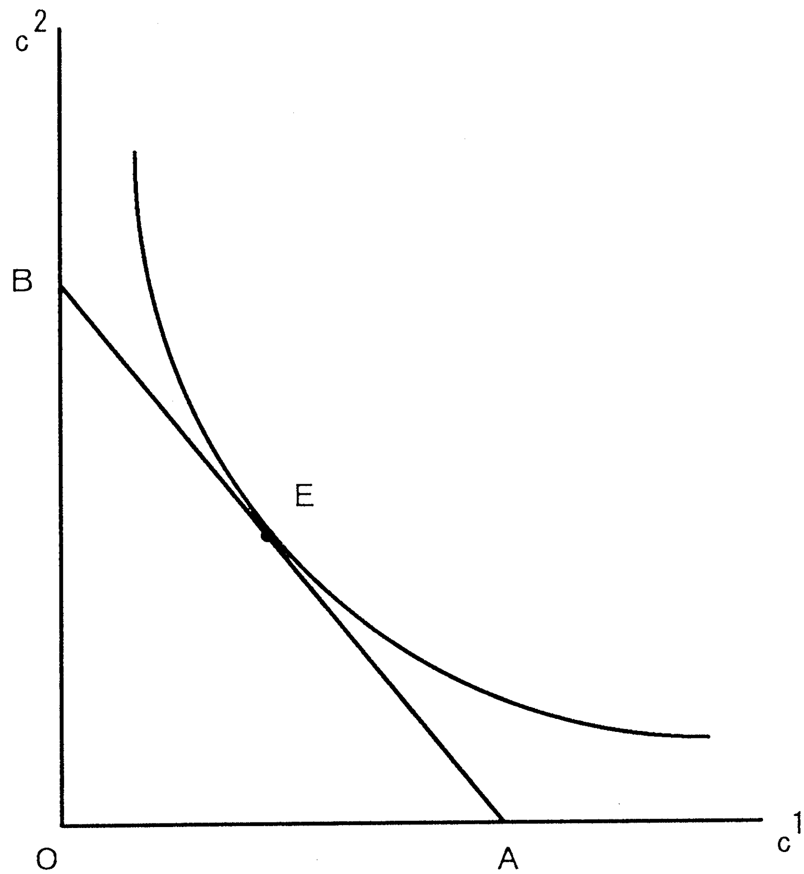


Figure 7.1 Samuelson's Overlapping Generations Model

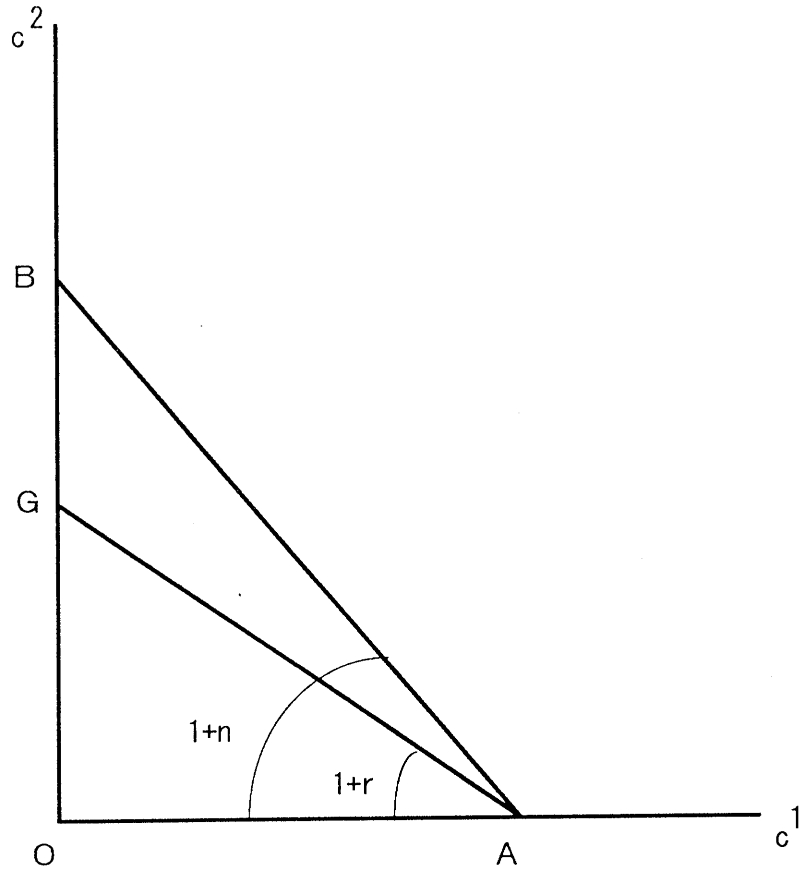


Figure 7.2 Money in an Economy with Durable Goods

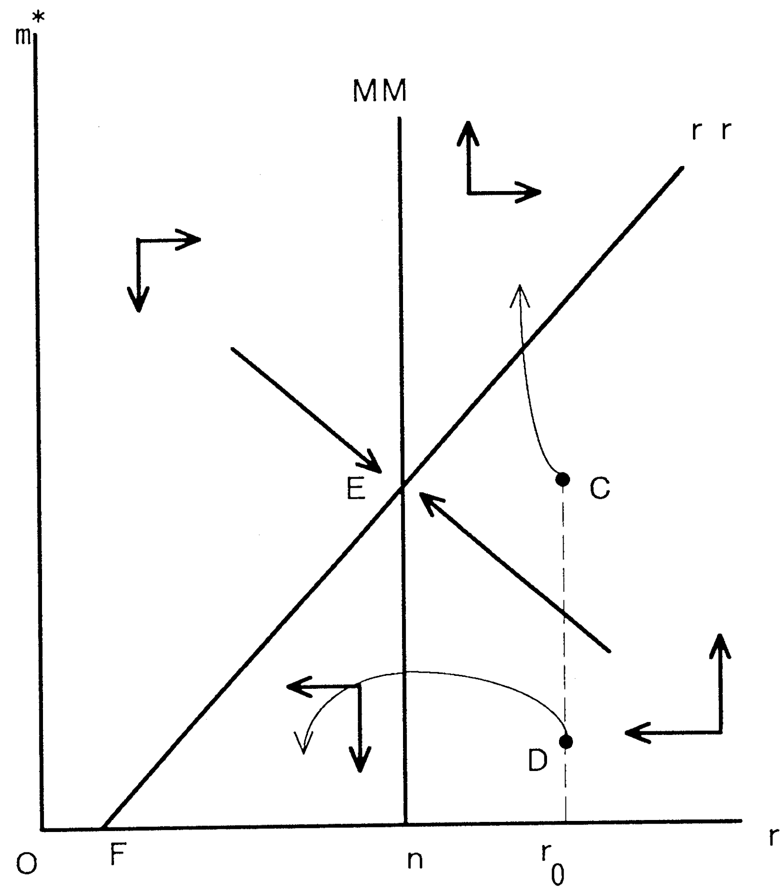


Figure 7.3 Dynamics of Diamond's Model

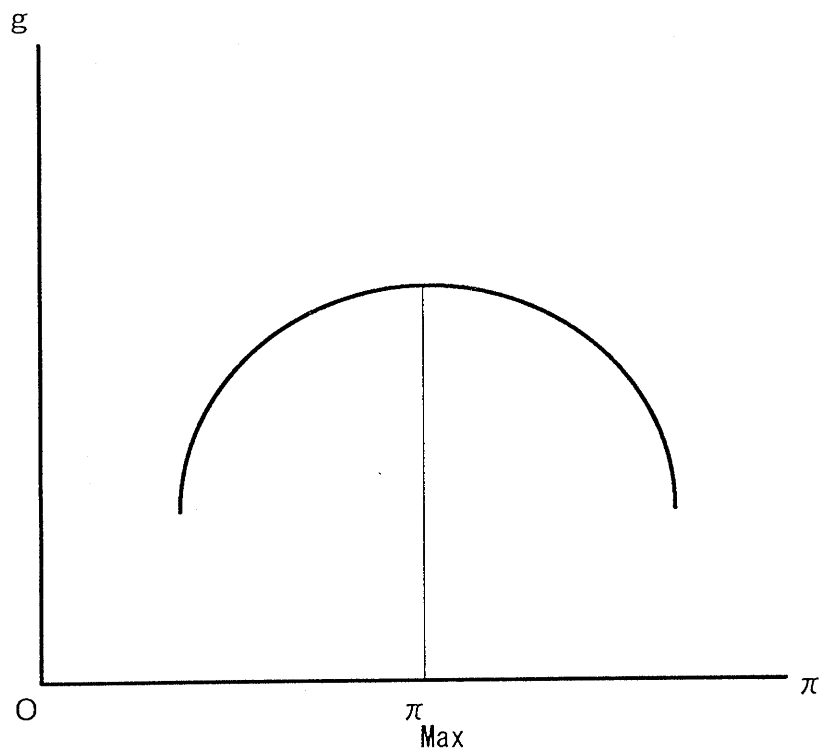


Figure 7.4 Seigniorage

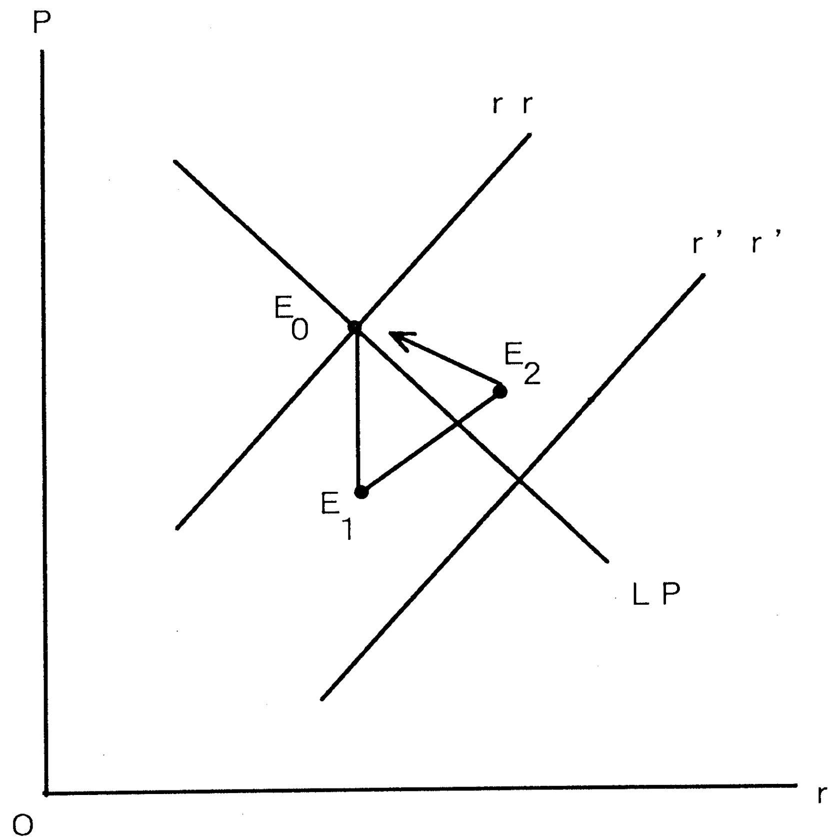


Figure 8.1 (I) Dynamics of Model with Land

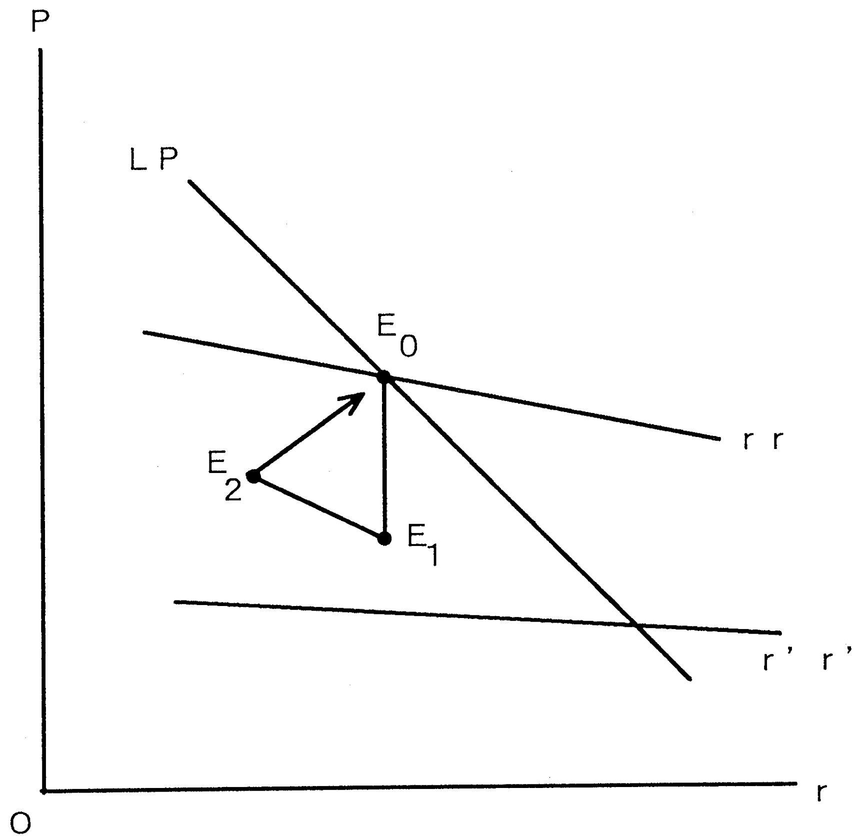


Figure 8.1 (ii) Dynamics of Model with Land

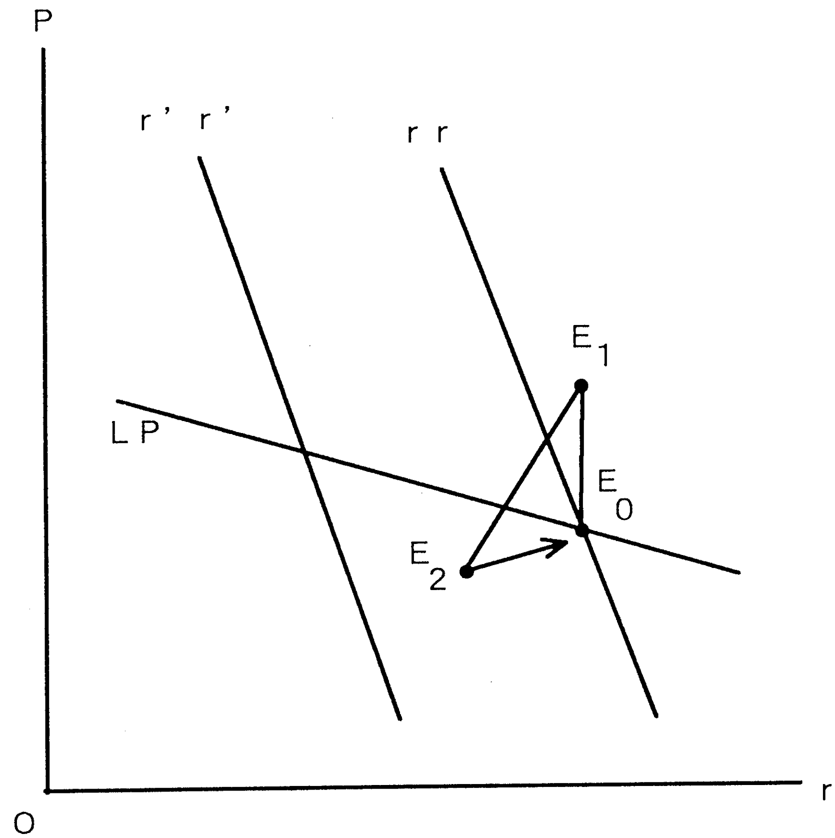


Figure 8.1 (iii) Dynamics of Model with Land

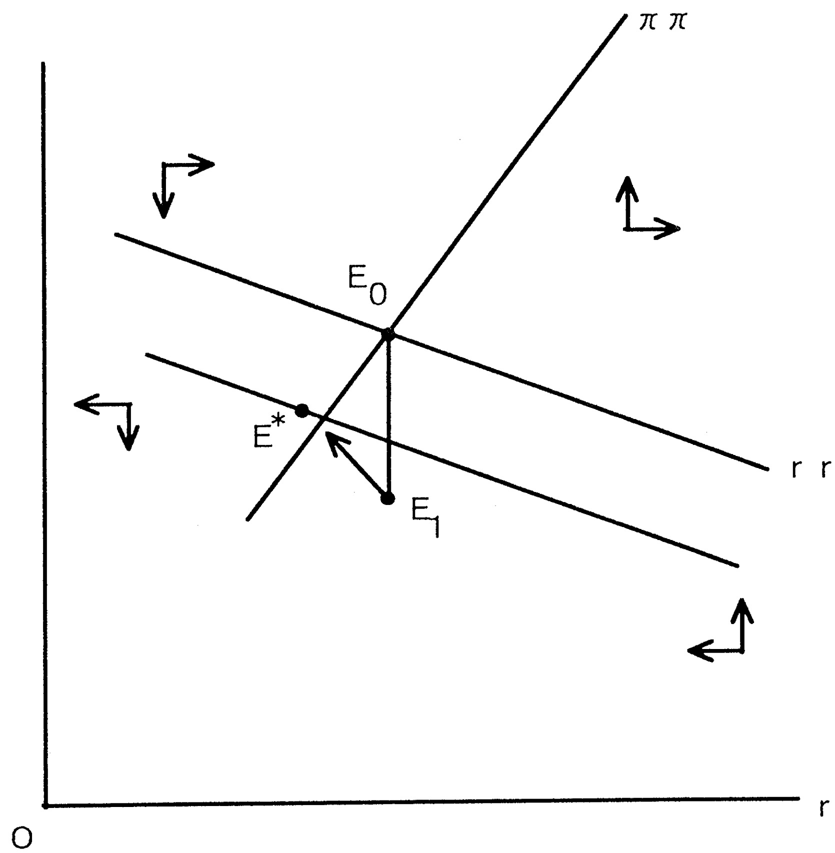


Figure 8.2 Transitionary Effect: An Increase in Government Spending

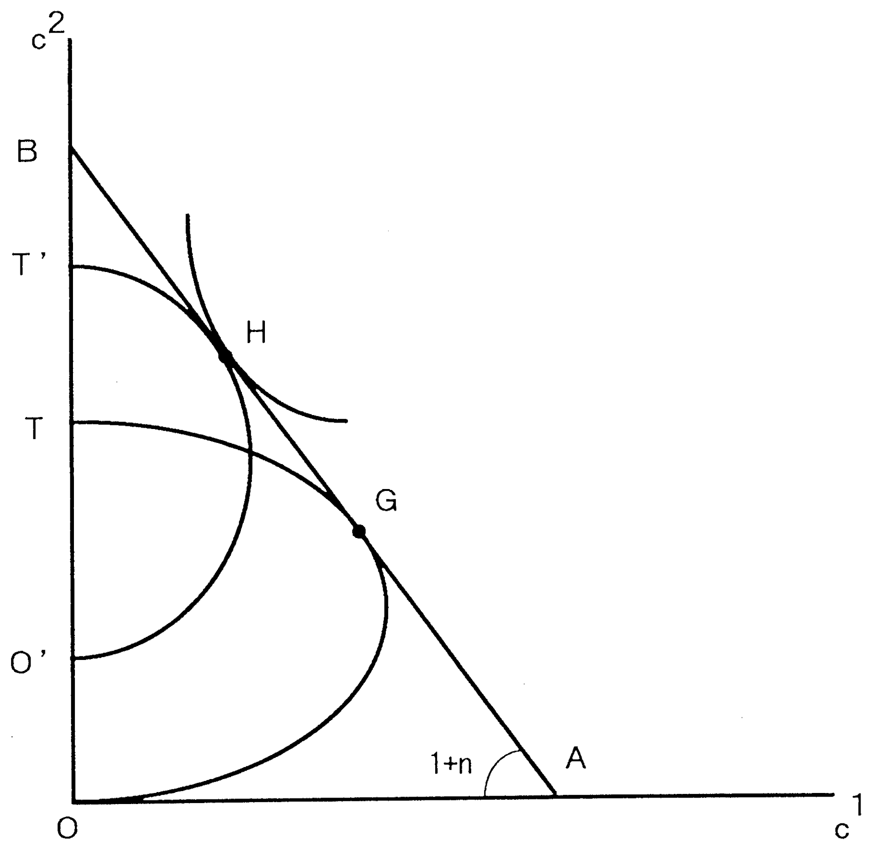


Figure 9.1 Burden of Debt

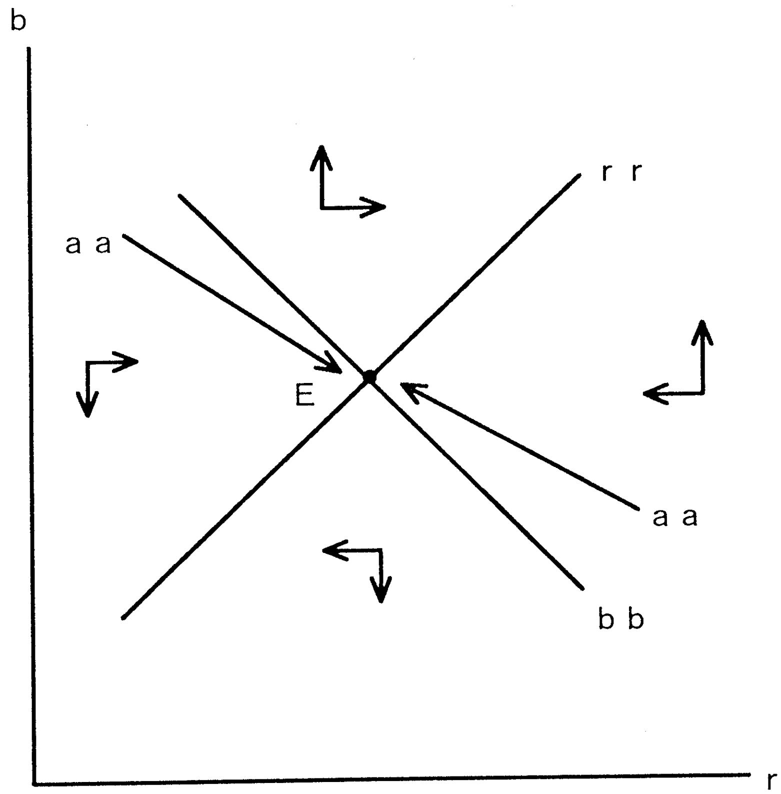


Figure 9.2 Dynamics of Model

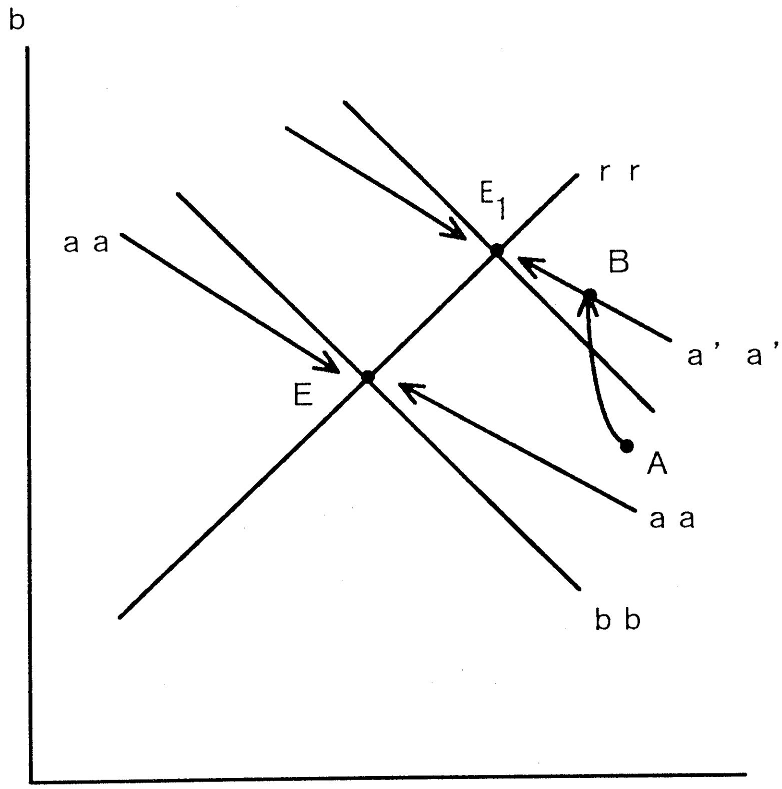


Figure 9.3 Changes in the Marginal Cost of Public Goods

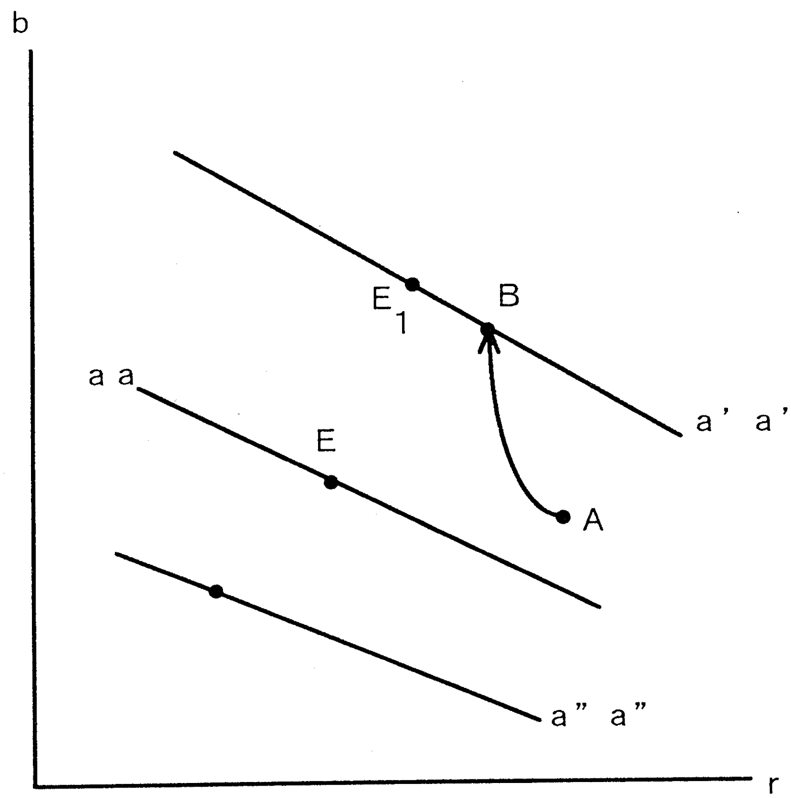


Figure 9.4 Changes in the Level of Public Spending

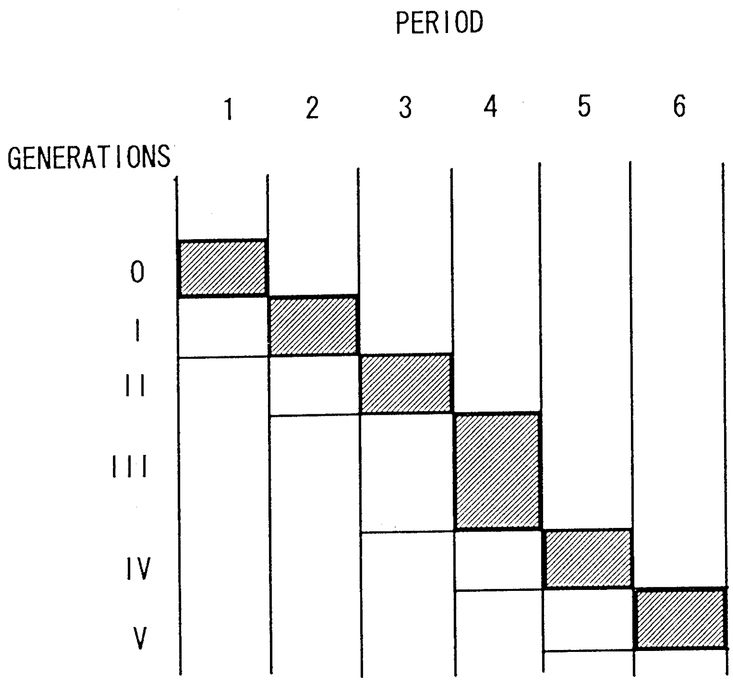


Figure 10.1 The Baby Boomer Generation

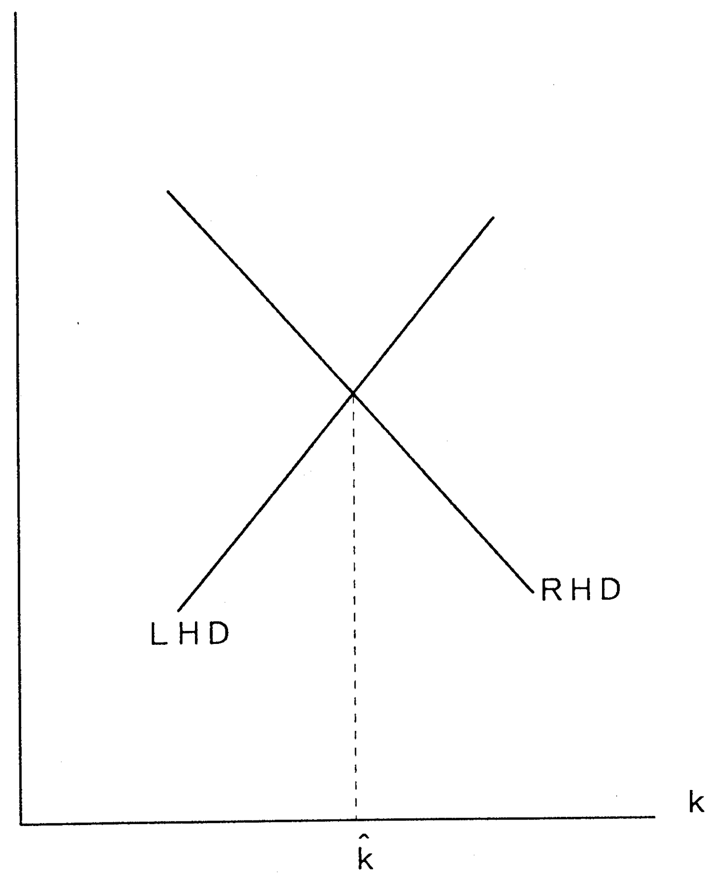


Figure 11.1 Steady State Physical Capital Human Capital Ratio

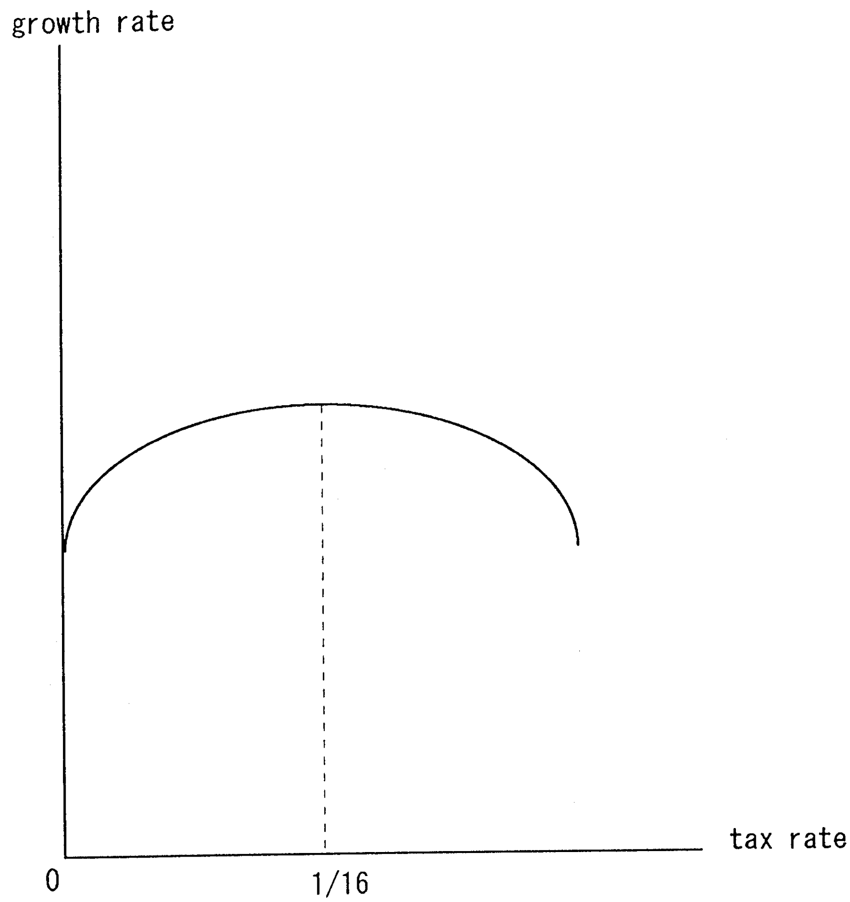


Figure 11.2 Taxes and Economic Growth