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Does Money Always Make People Happy?

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Abstract

This paper investigates the cause and consequence of the valuation of fiat money in an overlapping generations economy with private information. In this economy, the value of money and the extent of moral hazard are simultaneously determined: the valuation of fiat money worsens moral hazard; and the worsened moral hazard induces people to hold fiat money. People born in and after the first period may be worse off when money has positive value than they are when it does not.

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1. Introduction

In standard overlapping generations models [e.g. Samuelson(1958), Wallace(1980), Tirole(1985)], the valuation of the intrinsically useless asset, or fiat money, is benign to all agents: every agent in the economy becomes happier when money has positive value than when it does not. As is widely known, markets in some of those economies fail to achieve efficient allocation unless incomes are appropriately transferred across generations. If money is demanded as a store of value, it promotes such transfers and, as a result, improves economic welfare.

This paper, however, gives another picture of fiat money not so rosy as the above. In the model of this paper, the valuation of fiat money may be malign to the generations born in and after the first period. Roughly speaking, this result is brought about by the installation of the following externality in a Wallacian overlapping generations economy: the rate-of-return on physical investment is a decreasing function of the value of money; and dominates the rate of population growth when money is valueless. Owing to the externality, the valuation of money lowers the rate-of-return on physical investment. This implies that the interest rates when money is valued are lower than those when it is not, because arbitrage equalizes the rate-of-return on money with that on physical investment. As the result of this decline in interest rates, the agents except the initial old are worse off when money has positive value.

In this model, the valuation of fiat money is not due to the dynamic inefficiency of nonmonetary economy but to the above-mentioned externality. Notice that money and externality explain each other in this model: the externality is not activated unless money has positive value; at the same time, money cannot have positive value unless the externality is activated. That is, the valuation of fiat money activates the externality, which, in turn, causes the valuation of money by lowering the rate-of-return on physical investment. This is how fiat money is valued in this model. In the following sections the externality is not simply assumed but derived from uncertainty and asymmetric information (moral hazard).

There are some works closely related to this paper. Using a somewhat different model, Grossman and Yanagawa(1993) also reached a similar conclusion. In their model, money harms future generations by lowering their labor incomes, rather than interest rates. There are also some studies pointing out that things such as uncertainty and informational friction can give positive value to fiat money. For example, Azariadis and Smith(1993) showed how adverse selection in credit market makes people to demand fiat money. In Kitagawa(1994), fiat money is demanded as a riskless asset by the people to whom insurance is not available. In fact, the model of this paper is an extended version of Kitagawa's: like his model, money is demanded because of incomplete insurance; but, unlike his model, incomplete insurance is no assumption but the result of agents' money-holding.

The remainder of the paper is organized as follows. In the next section the basic model is presented. Although it conveys the central message, the results of this model depend on rather restrictive assumptions. Thus the model is a little bit generalized in Section 3. Section 4 concludes the paper.

2. The Model without Storage

A. Environment

An economy comprising an infinite sequence of two periods lived overlapping generations is considered. Time is indexed by t=1,2,... In the initial period (t=1), there is a unit mass of old generation, each member of which is endowed with q units of corn and M units of fiat money. No other agents have any endowment of fiat money and no other kinds of goods but corn exist in this economy. All agents of this generation have identical utility functions $v(c_1)$. where c_1 denotes a level of consumption of corn. The function v is assumed to be increasing. They die at the end of initial period.

In addition, a new generation is born at the beginning of each period. Each generation is identical in size, containing a continuum of agents of measure one. Although members of each generation derive utility only from the second period consumption, it is not in the first period when they receive e units of corn per capita. Thus they must save for their old age. Two ways of savings, distinguished by their extents of riskiness, are available. The first and more risky way is to plant the endowed corn: planting s units of corn brings in the next period as units of corn with probability θ and nothing with probability $1-\theta$. Parameters, a and θ, are positive constants satisfying:

$$\theta \in (0,1), \qquad \theta a > 1. \tag{1}$$

The risks associated with corn-planting are assumed to be not aggregate but idiosyncratic, so that it is possible to offset these risks by pooling them. The second and less risky one is to exchange the endowed corn for fiat money: exchanging m units of corn for money promises the purchase of $\frac{p}{p'}m$ units of corn in the next period with probability 1, where p (p') denotes money price of corn in the first period (that in the second period, respectively). In this section the storage of corn is assumed to be impossible for simplicity, but this assumption will be removed in the next section.

Newly born agents have the same expected utility functions:

$$\theta u(c^S) + (1-\theta)u(c^F). \tag{2}$$

where c^S (c^F) denotes a consumption when a good harvest is observed (a consumption when it is not, respectively). The function u is assumed to be concave, increasing, and continuously differentiable and to satisfy:

$$\lim_{c\to 0} u'(c) = +\infty. \tag{3}$$

These properties of u imply that agents in this model are risk-averse and demand insurance for uncertain harvests.

There are also countably infinite number of risk-neutral insurance companies. At the beginning of each period they offer insurance contracts to young agents, each of whom is allowed to accept only one of those offers. An insurance contract consists of an insurance premium α and an insurance money β: if a good harvest is observed, an insured agent must pay an amount α to the insurance company; in some, not necessarily all, of the other cases the company must pay an amount β to that agent. Each agent's realized state, good harvest or not, is assumed to be publicly observable.

To what extent those insurance contracts protect agents from their individual risks depends considerably on the observability of insured agents' corn-planting behaviors. In the following subsections we first consider the case that insurance companies can observe amounts of planted corn, and then the case that no companies can observe those amounts.

B. The Case of Full Information

Assume that insurance companies can observe and verify an amount of corn each agent plants. Since a good harvest of any agent is assumed to be publicly observable, this means that insurance companies can determine the cause of any observed poor harvest, that is, whether it occurred by accident or by intentionally planting no amount of corn. In this case the payment of insurance premium can be dependent on the amount of planted corn.

Competition among insurance companies give the following features to equilibrium contracts: equilibrium contracts (a) earn zero profit, i.e.,

$$\theta\alpha - (1-\theta)\beta = 0 \tag{4}$$

and (b) maximize each agent's indirect utility function, which is defined to be the maximal level of expected utility attainable under a fixed insurance contract. That is, equilibrium contracts are the most favorable ones to planting agents among those which are actuarially fair. Especially under full information, they fully insure the agents who planted all of the endowed corn from their individual risks, as will be shown in the proof of the next proposition.

Proposition 1: *Suppose that* a *and* θ *satisfy (1). Then, fiat money has no value under full information.*

(Proof) See Appendix.

Since equilibrium contracts get rid of the risks associated with corn-planting completely, the rate of return on insured corn-planting in this case is $\theta a (>1)$ with probability 1. As pointed out by Wallace(1980), no rational expectation equilibrium with valued money exists for such an environment. The rate of return on fiat money, 1 at its highest, is always dominated in rate-of-return by fully insured corn-planting, so that agents never demand it as a store of value.

Thanks to full insurance, each agent can consume the same amount of corn, θa, whether the harvest is rich or poor. Thus the level of their expected utilities is $u(\theta a)$. And each initial old cannot but consume q units of corn, so that the level of their utility is $v(q)$.

C. The Case of Private Information

In this subsection the information structure is modified as follows: nothing but whether each agent harvests and whether contracted payments occur is publicly observable. No one can observe either what amount of corn an agent planted or what amount of corn a successful agent harvested. This implies that no one can identify whether any observed poor harvest occurs by accident or as a result of planting no amount of corn. In this situation every enforceable insurance contract is subject to the following restrictions. Firstly, payments in an insurance contract cannot be made dependent on anything but the observation of whether the insured agent harvests or not. Thus, every contract prescribes that insurance companies must pay insurance money, β, to their insured agents who did not harvest, and that the insured must pay insurance premium, α, to their insurance companies if she harvested. Assume that the law allows insurance companies to forfeit the whole amount of harvest of a successful agent if she does not pay a promised premium. Obviously, successful agents have no incentive to default premium under this regulation. Secondly, every contract must satisfy the following incentive constraint:

$$\theta u[as^* + \frac{p}{p'}m^* - \alpha] + (1-\theta)u[\frac{p}{p'}m^* + \beta] > u[\frac{p}{p'}e + \beta] \qquad (5)$$

The inequality (5) states that the contract is required to make the honest planting behavior more profitable to the insured agents than the cheating behavior. Otherwise, fraud being more profitable, the agents choose to plant none of the endowed corn in order to ensure the receipt of insurance money, which ultimately leads the insurance companies offering such contracts to bankruptcy.[1] The pair (s^*, m^*) denotes the solution for the maximizing problem:

$$Max_{(s,m)} \theta u[as + \frac{p}{p'}m - \alpha] + (1-\theta)u[\frac{p}{p'}m + \beta], \qquad s.t. \qquad e = m + s. \qquad (6)$$

Note that:

[1]We neglect the moral hazard on the side of insurance companies. In this paper insurance companies are assumed to behave honestly at any time.

$$(s^*, m^*) = (e, 0), \quad if\ p' = +\infty \tag{7}$$

In equilibrium all offered contracts earn zero profit, so that the incentive constraint (5) is reduced to:

$$V(\alpha; \frac{p}{p'}) > u[\frac{p}{p'}e + \frac{\theta}{1-\theta}\alpha] \tag{8}$$

where the function V is the insured's indirect utility function defined as:

$$V(\alpha; \frac{p}{p'}) \equiv \theta u[as^* + \frac{p}{p'}m^* - \alpha] + (1-\theta)u[\frac{p}{p'}m^* + \frac{\theta}{1-\theta}\alpha]. \tag{9}$$

The equilibrium insurance premium α^* solves the maximizing problem:

$$Max_{\alpha} V(\alpha; \frac{p}{p'}) \quad s.t. \quad V(\alpha; \frac{p}{p'}) > u[\frac{p}{p'}e + \frac{\theta}{1-\theta}\alpha] \tag{10}$$

because, of all actuarially fair contracts constrained by (8), only the most favorable one to the agents is demanded in equilibrium. Especially if $p' = +\infty$, the problem (10) is reduced to:

$$Max_{\alpha}\, \theta u[ae - \alpha] + (1-\theta)u[\frac{\theta}{1-\theta}\alpha]$$
$$s.t. \quad \theta u[ae - \alpha] + (1-\theta)u[\frac{\theta}{1-\theta}\alpha] > u[\frac{\theta}{1-\theta}\alpha] \tag{11}$$

The solution of this problem, $\alpha^* = (1-\theta)ae$, means that the agents in this case are (almost) fully insured from individual risks by the equilibrium contract, just as in the case of full information.[2]

The money demand function can be expressed as $\phi(\frac{p}{p'})$, because the money demand of an insured agent, m^*, is dependent on endogenous variables, $\frac{p}{p'}$, α, and β, all of which are determined by $\frac{p}{p'}$. From eq.(7), the next relation is obtained:

$$\phi(\frac{p}{p'}) = 0, \quad if\ p' = +\infty \tag{12}$$

Definition: *A private information equilibrium is a sequence of money prices of corn,* $\{p_t\}_{t=1}^{\infty}$, *satisfying:*

$$\phi(\frac{p_t}{p_{t+1}}) = \frac{M}{p_t} \qquad 0 < p_t \qquad \forall t \geq 1. \tag{13}$$

We focus on two stationary equilibria: the "nonmonetary" equilibrium, i.e., $p_t = +\infty, \forall t \geq 1$; and the

[2]Exactly saying, the equilibrium insurance premium must be infinitesimally smaller than $(1-\theta)ae$ to motivate the agents to behave honestly. This is what the adverb 'almost' means.

"stationary monetary" one, i.e., $p_t = p < +\infty, \forall t \geq 1$.

The nonmonetary equilibrium always exists, because the price sequence, $p_t = +\infty, \forall t \geq 1$, apparently satisfies eq.(8). In this equilibrium the insurance contract induces each agent to plant the whole amount of the endowed corn, and also gives her (almost) full protection against individual risk. Thus the allocation is (almost) the same as that in the case of full information: every agent born in and after period 1 consumes an amount θae independent of her realized state, whereas each initial old consumes her endowment q.

The next proposition provides the necessary and sufficient condition for the existence of the stationary monetary equilibrium:

Proposition 2: *Let $\alpha^*(1)$ denote the solution for the maximizing problem:*

$$Max_{\alpha} \quad V(\alpha;1), \qquad s.t. \qquad V(\alpha;1) \geq u[e + \frac{\theta}{1-\theta}\alpha] \tag{14}$$

The stationary monetary equilibrium exists if and only if $\alpha^*(1)$ satisfies:

$$-\theta(a-1)u'[ae - \alpha^*(1)] + (1-\theta)u'[\frac{\theta}{1-\theta}\alpha^*(1)] > 0 \tag{15}$$

(Proof) See Appendix.

In this equilibrium money explains moral hazard and moral hazard explains money. When fiat money has a positive value, full insurance contracts are never offered because none of them satisfy the incentive constraint (5). Thus, the valuation of fiat money worsens the moral hazard problem and, as a result, the insured agents have to bear part of their own risks. But this is the very reason they demand money. The rate of return on fiat money being certain, money-holding reduces the uncertainty of future consumption, and even makes the agents better off when the condition (15) holds.[3] Then, they choose to exchange parts of the endowed corn for fiat money and, as a result, it has a positive value in each period.

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[3]This is the point of Kitagawa's paper [Kitagawa(1994)].

Notice that the reason fiat money has positive value in this model is quite different from those in the standard overlapping generations models of fiat money [e.g. Samuelson(1958), Wallace(1980), Tirole(1985)]. While it was explained by the dynamic inefficiency of nonmonetary economy in those models, the valuation of fiat money is here due to asymmetric information. Welfare comparison between the nonmonetary equilibrium and the stationary monetary one provides one evidence: unlike standard ones, this model shows that the utilities of some agents in the stationary monetary equilibrium are lower than those in the nonmonetary equilibrium. Figure 1 depicts the patterns of consumption in those equilibria: in the nonmonetary equilibrium each agent born in and after the initial period always consumes θae, so that her utility is $u(\theta ae)$; in the stationary monetary one, the consumption of the same agent varies according to her realized state, namely $ae-(a-1)\phi(1)-\alpha^*(1)$ in the good state and $\phi(1)+\frac{\theta}{1-\theta}\alpha^*(1)$ in the bad one. Her expected utility in the latter equilibrium is determined to be $V(\alpha^*(1);1)$. It is easy to establish the next inequality:

$$V(\alpha^*(1);1) < u(\theta ae) \tag{16}$$

That is, the valuation of fiat money may harm all generations except the initial old. Needless to say, the decline in their utilities results from the moral hazard valued money worsens. Also note that the initial old benefit from the valuation of money: their utilities increase from $v(q)$ in the nonmonetary equilibrium to $v(q+\phi(1))$ in the stationary monetary one. Therefore, two equilibria cannot be Pareto-ranked.

3. The Model with Storage

The model of the preceding section is a good example exhibiting that the valuation of fiat money does not always make people happy. But the structure of that model is restrictive in that it precludes the storage of corn. In this section, it is assumed that corn is storable: storing z units of corn brings δz units of corn in the next period, where δ is a positive constant satisfying $\delta \in (0,1)$. Public knowledge is still nothing but whether each agent harvested or not. Obviously, these assumptions do not alter the existence condition of the stationary monetary equilibrium.

Whether the stationary monetary equilibrium exists or not, there exist equilibria with time-invariant insurance premium $\alpha^*(\delta)$, the level of which is determined by:

$$V(\alpha^*(\delta);\delta) = u[\delta e + \frac{\theta}{1-\theta}\alpha^*(\delta)] \tag{17}$$

Proposition 3: *Consider an equilibrium with time-invariant insurance premium* $\alpha^*(\delta)$. *In this equilibrium, storage is used if and only if* $\alpha^*(\delta)$ *satisfies:*

$$-\theta(a-\delta)u'[ae-\alpha^*(\delta)] + (1-\theta)\delta u'[\frac{\theta}{1-\theta}\alpha^*(\delta)] > 0 \tag{18}$$

(Proof) See Appendix.

Only when there is an equilibrium where storage is used, there are also equilibria where fiat money is demanded as a perfect substitute for storage. In those equilibria the value of money decreases over time at the rate of δ, though the combined demand for riskless assets, money and storage, is constant.[4]

When the inequality (18) does not hold, there is an equilibrium where neither storage nor money is used. In that equilibrium the agents are not fully insured from their own risks, though they plant all of their endowed corn. Since the behavior of storing corn is not publicly observable, full insurance would just motivate the agents to cheat their contractual partners: they would choose to store all of their corn and in the next period require the payment of insurance money. Thus, the incentive constraint binds even when money is valueless and storage is not used. In this regard, this model contrasts finely with that of the preceding section: in that model, agents were (almost) fully insured when money was valueless.

Now, consider the following two examples: both examples exhibit the stationary monetary equilibrium and the nonmonetary/nonstorage one, but they are different from each other in implications for the valuation of fiat money.

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[4]Such money/storage equilibria can be Pareto-ranked: a money/storage equilibrium Pareto-dominates those with smaller initial values of money. Across those equilibria both total demand and rate-of-return on the riskless assets, i.e., money and storage, is equal and constant over time. Thus the higher initial value of money raise the utility of initial old without harming future generations.

Example 1: Suppose that $u(c) = \ln c$, $\theta = 0.5$, $a = 2.44$, and $\delta = 0.488$. In this case equilibrium insurance premia are determined as:

$$\alpha^*(0,488) = 0.488e$$

and

$$\alpha^*(1) = 0.020e.$$

Then, the utility of agent born in and after the initial period is equal to $\ln[0.976e]$ in the nonmonetary/nonstorage equilibrium; and $\ln[1.020e]$ in the stationary monetary one.[5]

In Example 1, the stationary monetary equilibrium Pareto-dominates the nonmonetary/nonstorage one. In other words, the valuation of fiat money makes all agents better off in a class of economies with storage. But the next example tells that this is not always true.

Example 2: Suppose that $u(c) = \ln c$, $\theta = 0.5$, $a = 3.25$, and $\delta = 0.65$. In this case equilibrium premia are:

$$\alpha^*(0.65) = 0.65e$$

and

$$\alpha^*(1) = 0.143e$$

Then, the utility of an agent born in and after the initial period is equal to $\ln[1.3e]$ in the nonmonetary/nonstorage equilibrium; $\ln[1,143e]$ in the stationary monetary one.[6]

When the storage is possible, there are some cases in which the valuation of money is benign to all agents. But there are also some cases exhibiting that money is still malign to future generations. This

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[5]The levels of expected utilities in monetary and nonmonetary/nonstorage equilibria are derived as follows. Since incentive constraints bind in both equilibria, equilibrium insurance premia are calculated from these constraints. Substituting obtained value of insurance premium into the RHS of incentive constraint, we can evaluate the level of expected utility in each equilibrium.

[6]The derivations of insurance premia and expected utilities are the same as in Example 1.

result reflects the fact that the relation between the expected utility of an insured agent and the rate-of-return on the riskless asset, i.e., money or storage, is not monotone. When the rate-of-return on the riskless asset is sufficiently low, the agents planting corn honestly never hold that asset. This implies that the higher return on the riskless asset makes honest agents worse off, because it just raises the gain from cheating behavior and, as a result, makes insurance contracts less favorable to those agents. When its rate-of-return is close to but less than 1, on the other hand, the honest agents may hold the riskless asset. In that situation the expected utility of an insured agent may be an increasing function of the rate-of-return on the riskless asset. The higher return on the riskless asset in this case raises both gains from honest behavior and from cheating behavior, which in turn changes the levels of insurance premium and insurance money. In general the total effect of these changes on the expected utility of an agent is obscure, but there are some cases where they make all agents better off, as shown in Example 1.

4. Concluding Remarks: Does Money Always Make People Happy?

The answer is NO. In the presence of private information, the valuation of money makes it difficult to provide insurance protecting people completely from their individual risks, which sometimes ends up to the decline in their utilities. This does not mean that the valuation of fiat money always harms future generations. As shown in Section 3, welfare consequence of valuation is, in general, hard to evaluate.

A promising extension of this model can be pointed out. Remember that the valuation of money reduced the size of physical investments by hindering risk-pooling. This reduction did not have any further effect in this model, but the situation changes if corn-planting is replaced with capital accumulation. In that world the reduction of physical investments would also reduce the incomes of future generations through capital accumulation. This situation is quite similar to that of Grossman and Yanagawa(1993). Although this extension seems rewarding, it is left to future researches.

Appendix

Proof of Proposition 1: Since the function u is concave, Jensen's inequality obtains:

$$\theta u[ae-\alpha]+(1-\theta)u[\frac{\theta}{1-\theta}\alpha]\le u[\theta ae]. \quad (A1)$$

The equality holds if and only if $\alpha=(1-\theta)ae$. The LHS of (A1) represents the expected utility of the insured agent who planted the whole amount of the endowed corn, and the RHS of (A1) is its maximum. Because of competition, insurance companies offer the contract $((1-\theta)ae, \theta ae)$ to this agent. This, in turn, implies that the agent demand money if and only if money-holding promises a utility higher than $u(\theta ae)$. Suppose that such an equilibrium exists. Define m^*_t as:

$$m^*_t \equiv \arg\max\{\theta u[ae-(a-\frac{p_t}{p_{t+1}})m_t-\alpha_t]+(1-\theta)u[\frac{p_t}{p_{t+1}}m_t+\frac{\theta}{1-\theta}\alpha_t]\}. \quad (A2)$$

In fact, m^*_t is the value of money in this equilibrium. It must satisfy $\forall t\ge 1, m^*_t \in (0,e)$. For all generations to demand money, the following inequality must hold:

$$\forall t\ge 1, \theta u[ae-(a-\frac{p_t}{p_{t+1}})m^*_t-\alpha_t]+(1-\theta)u[\frac{p_t}{p_{t+1}}m^*_t+\frac{\theta}{1-\theta}\alpha_t]>u(\theta ae). \quad (A3)$$

Using Jensen's inequality, one can also obtain:

$$\forall t\ge 1,$$

$$\theta u[ae-(a-\frac{p_t}{p_{t+1}})m^*_t-\alpha_t]+(1-\theta)u[\frac{p_t}{p_{t+1}}m^*_t+\frac{\theta}{1-\theta}\alpha_t]\le u[\theta ae-(\theta a-\frac{p_t}{p_{t+1}})m^*_t]. \quad (A4)$$

The inequalities (A3) and (A4) jointly imply:

$$\forall t\ge 1, \quad \frac{p_t}{p_{t+1}}>\theta a>1. \quad (A5)$$

The inequality (A5) means that in this equilibrium the value of money exceeds the endowment e in finite time, which is not feasible. Contradiction. Q.E.D.

Proof of Proposition 2: This proposition can be regarded as a special case of Proposition 3. Set $\delta=1$ in Proof of Proposition 3. Q.E.D.

Proof of Proposition 3: Consider an agent who made a contract $(\alpha, \frac{\theta}{1-\theta}\alpha)$ with an insurance company. If she stores z units of corn, her expected utility is:

$$EU(z) \equiv \theta u[ae-(a-\delta)z-\alpha]+(1-\theta)u[\delta z+\frac{\theta}{1-\theta}\alpha]. \tag{A6}$$

Of course, she chooses z maximizing (A6). The first- and the second derivatives of $EU(z)$ are:

$$EU'(z) = -(a-\delta)\theta u'[ae-(a-\delta)z-\alpha]+\delta(1-\theta)u'[\delta z+\frac{\theta}{1-\theta}\alpha] \tag{A7}$$

and

$$EU''(z) = (a-\delta)^2\theta u''[ae-(a-\delta)z-\alpha]+\delta^2(1-\theta)u''[\delta z+\frac{\theta}{1-\theta}\alpha]<0. \tag{A8}$$

Define $z^\$$ as:

$$z^\$ = \min[e, \frac{ae-\alpha}{a-\delta}]. \tag{A9}$$

The following is obvious:

$$EU'(z^\$)<0. \tag{A10}$$

The inequalities (A8) and (A10) jointly mean that the agent plants the whole amount of the endowed corn if:

$$EU'(0) = -(a-\delta)\theta u'[ae-\alpha]+\delta(1-\theta)u'[\frac{\theta}{1-\theta}\alpha]\leq 0 \tag{A11}$$

and stores part of the endowment, otherwise. Let $\alpha^\#$ denote the root of the equation:

$$-(a-\delta)\theta u'[ae-\alpha]+\delta(1-\theta)u'[\frac{\theta}{1-\theta}\alpha]=0 \tag{A12}$$

Using $\alpha^\#$, one can obtain:

$$\begin{aligned} EU'(0) &> 0 \quad \text{if } \alpha \in [0,\alpha^\#) \\ &\leq 0 \quad \text{if } \alpha\in[\alpha^\#, ae] \end{aligned} \tag{A13}$$

Then the amount of storage, z, is determined to be:

$$\begin{aligned} z = z^\#(\alpha) &> 0, \quad \text{if } \alpha \in [0,\alpha^\#) \\ &= 0, \quad \text{if } \alpha\in[\alpha^\#, ae] \end{aligned} \tag{A14}$$

where $z^\#:[0,\alpha^\#)\to R_{++}$ is a function implicitly defined by:

$$-(a-\delta)\theta u'[ae-(a-\delta)z^\#(\alpha)-\alpha]+\delta(1-\theta)u'[\delta z^\#(\alpha)+\frac{\theta}{1-\theta}\alpha]=0. \tag{A15}$$

It is easy to establish that $z^\#(\alpha)$ is continuous and that:

$$\lim_{\alpha\to\alpha^\#} z^\#(\alpha)=0. \tag{A16}$$

The indirect utility function is obtained from (A14):

$$V(\alpha;\delta) = \theta u[ae-(a-\delta)z^{\#}(\alpha)-\alpha]+(1-\theta)u[\delta z^{\#}(\alpha)+\frac{\theta}{1-\theta}\alpha], \quad if\ \alpha\in[0,\alpha^{\#})$$

$$= \theta u[ae-\alpha]+(1-\theta)u[\frac{\theta}{1-\theta}\alpha], \quad if\ \alpha\in[\alpha^{\#},ae] \tag{A17}$$

Note that the function $V(.;\delta)$ is continuous and that:

$$V(\alpha;\delta) \le u(\theta ae) \tag{A18}$$

because of Jensen's inequalities:

$$V(\alpha;\delta) \le u[\theta ae-(\theta a-\delta)z^{\#}(\alpha)] < u[\theta ae], \quad if\ \alpha\in[0,\alpha^{\#})$$

$$\le u[\theta ae], \quad if\ \alpha\in[\alpha^{\#},ae] \tag{A19}$$

The equality of (A18) holds if $\alpha=(1-\theta)ae$. That $(1-\theta)ae\in[\alpha^{\#},ae]$ is ensured by that $EU'(0)\le 0, if \alpha=(1-\theta)ae$. The inequality (A10) implies that the following relation never holds:

$$z^{\#}(\alpha)=e \tag{A20}$$

therefore:

$$V(\alpha;\delta) > u[\delta e+\frac{\theta}{1-\theta}\alpha], \quad if\ \alpha=0 \tag{A21}$$

The following is obtained from (A19):

$$V(\alpha;\delta) < u[\delta e+\frac{\theta}{1-\theta}\alpha], \quad if\ \alpha=(1-\theta)ae \tag{A22}$$

The inequalities (A21) and (A22) jointly imply that there exist αs satisfying:

$$V(\alpha;\delta) = u[\delta e+\frac{\theta}{1-\theta}\alpha]. \tag{A23}$$

Among these αs, the maximal one is the solution for the next maximizing problem, i.e., equilibrium insurance premium, $\alpha^*(\delta)$:

$$Max_{\alpha\in[0,ae]} V(\alpha;\delta), \quad s.t. \quad V(\alpha;\delta) \ge u[\delta e+\frac{\theta}{1-\theta}\alpha] \tag{A24}$$

Thus, insurance companies offer the contract $(\alpha^*(\delta),\frac{\theta}{1-\theta}\alpha^*(\delta))$. In response to this offer, the agent chooses the amount of storage as:

$$z = z^{\#}(\alpha(\delta))\in(0,e), \quad if\ \alpha^*(\delta)<\alpha^{\#}$$

$$=0, \quad if\ \alpha^{\#}\le\alpha^*(\delta) \tag{A25}$$

The inequality $\alpha^*(\delta)<\alpha^{\#}$ is equivalent to (18). Q.E.D.

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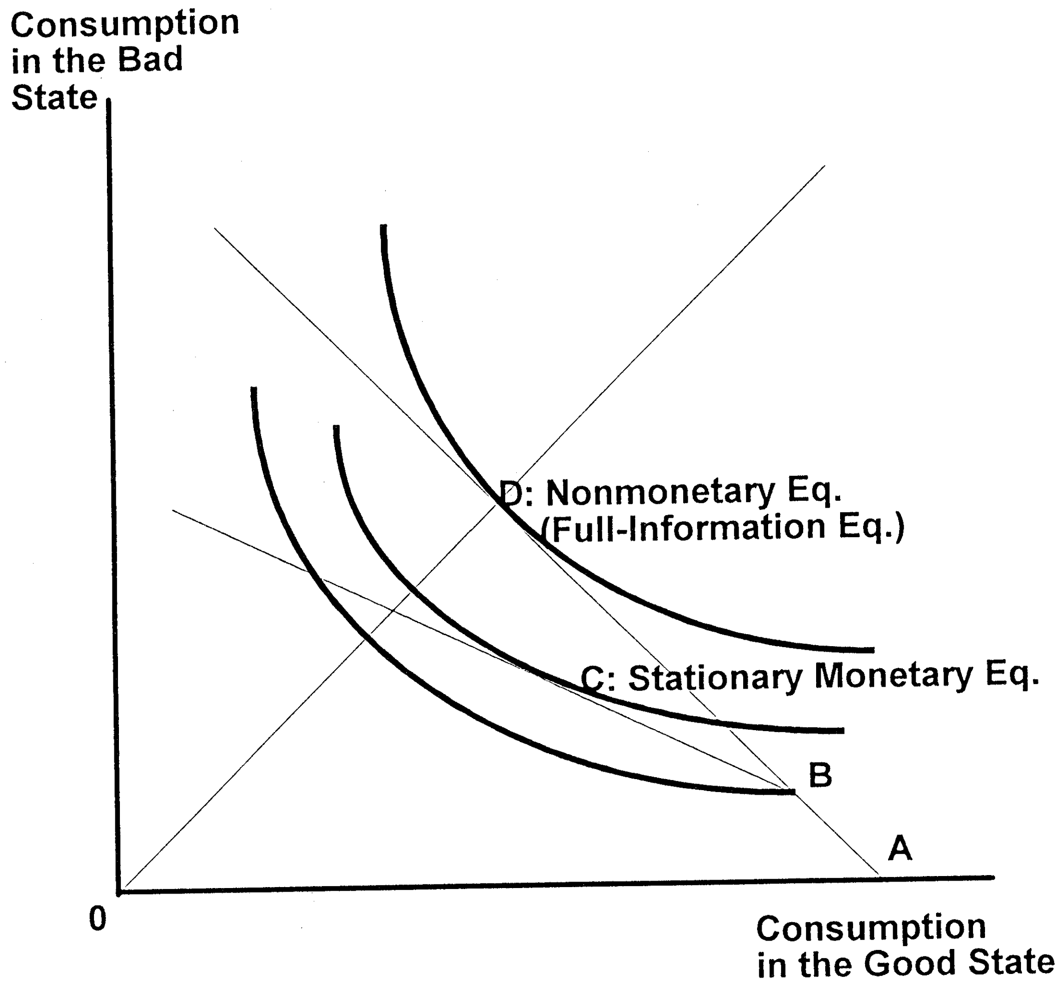

Figure 1: Patterns of Consumption

A: $(ae,0)$,

B: $(ae-\alpha^*(1), \frac{\theta}{1-\theta}\alpha^*(1))$,

C: $(ae-(a-1)\phi(1)-\alpha^*(1), \phi(1)+\frac{\theta}{1-\theta}\alpha^*(1))$,

D: $(\theta ae, \theta ae)$