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1. Introduction

It is sometimes pointed out that products' quality and their costs improve more effectively when a few, but not too many, firms compete each other head to head, being fully conscious of each other's relative quality and cost performance. In fact, Itoh and Matsui [1989] named the above type of competition "face-to-face competition" and illustrated it by the following story of paper suppliers supplying rolls of paper to a newspaper company in Japan.

For newspaper companies the quality of paper is essential as, when the paper is torn during the printing, the entire printing process must be stopped and considerable amount of time is wasted until a new paper roll is installed. This may lead to a significant loss, as delay in printing would result in delay in home deliveries, a critical part of newspaper service in Japan, causing subscribers' switch to rival newspapers. In order to assure better quality of papers supplied, the newspaper company that Itoh visited contracted with two paper suppliers. In addition, the relative performance of each supplier, i.e., a large table indicating the number of times each paper got torn during the printing in the previous month was shown in the wall of the office to which sales personnel of each paper supply company visits every week. With this device, each paper supply company gets to know fully its relative performance with its rival and, according to Itoh, it creates strong incentive to improve the quality of the paper supplied, so that the supplier can win relative to its rival.

A similar mechanism exists in the relationship between an assembler and parts suppliers in the Japanese automobile industry. As is well-known[1], there is a long-run relationship among an assembler and its associated parts suppliers in Japan. Assemblers,

[1]See, e.g., Asanuma [1989].

however, usually make sure that there are two potential suppliers (one may be the assembler's in-house production) for each parts and components (for each particular model of cars it produces). This practice can be justified on various grounds, such as insurance against the possibility of one supplier breaking relationship with the assembler, improving bargaining power of the assembler with respect to parts suppliers, etc.

An alternative justification, however, is to induce face-to-face competition between the two suppliers. In Japan, model change takes place every four years and assemblers allow about one-year lead time for designing and preparing production of each parts; the period called the "design-in". In this stage, the assembler normally asks two suppliers to design and to provide a sample of parts. When the actual productions starts to take place, only one is chosen to participate in the actual production based upon their relative performance in the design-in.

In this paper, we shall construct a model to explain the mechanism behind face-to-face competition. The model is based on two particular aspects of these transactions: relation-specific nature of suppliers' *ex ante* investment and incomplete nature of their transactions. It is extremely difficult to write a contract which is contingent on the relative quality of supplies, an attribute difficult to describe on paper and almost impossible to verify to a third party. It follows that the contract must be necessarily incomplete in nature. The return to suppliers are determined by the *ex post* (re-)negotiations which is itself influenced by many factors including the relative performance of *ex ante* investments. Such bargaining outcomes may provide a non-marginal difference between the better performer and the worse performer in terms of the *ex ante* investments. This difference in relative payoffs would provide an incentive to be a leader as in rank-order tournament models[2]. The important property of the proposed mechanism is the fact that this discrete difference (or prize) is incentive compatible in the sense that no party has an incentive to renegotiate *ex post*. Being created endogenously and incentive compatible, the principal needs not introduce an artificial exogenous prize nor design commitment mechanism to such a prize. Consequently, the level of *ex ante* investments becomes larger compared with the case of single supplier, where hold-up problem should arise. It may even exceed the first-best level if the incentive to be the first is sufficiently strong.

In section 2, we shall discuss our model in view of the existing relevant literature. In section 3, formal model will be presented. Section 4 will discuss the first best allocation and a sub-optimal outcome with a single supplier where the hold-up problem arises. Both results would play benchmark in the rest of the paper. In sections 5 and 6, we shall examine the cases of two suppliers and analyze how they mitigate the hold-up problem. In section 5 we assume the principal can commit *ex ante* that she will contract exclusively with the winner of the investment competition between the two agents. In section 6, we shall extend our analysis and examine how the principal can provide an incentive compatible endogenous prize for the winner if we use the Shapley Value as a solution concept for the *ex post* bargaining. Section 7 discusses the robustness of our result and concludes the paper.

2. Motivation

In the introductory section, we discussed two examples of a typical transactional form

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[2]For rank-order tournament models, see for example Lazear and Rosen [1981], and Nalebuff and Stiglitz [1983].

between suppliers and a buyer in Japan. There are two issues we should emphasize about this transactional form: incomplete contracts in transaction relations and the resulting hold-up problem on one hand, and rank-order tournament in dynamic competition among a few on the other. It was Williamson [1975, 1985] who first pointed out the problem of under-investment in relational contracts. In many transaction relations, such as those we discussed in the introduction, relation-specific investments by suppliers are required. However, contracts which specifies contingent actions regarding these investments are often too costly to write and/or implement because of the lack of verifiability. It follows that the division of revenues among the transaction parties is often determined by *ex post* renegotiations (or bargaining games). As the parties other than the supplier who has invested in this relation-specific assets will have bargaining power *ex post*, the returns from the investment will not be fully appropriated to the party who has invested. Since the supplier still has to bear the full cost of investments, he will choose the level of investment which is less than the first best; the under-investment or hold up problem occurs[3].

Grossman and Hart [1986] and Hart and Moore [1990] analyzed the hold-up problem and discussed ways to solve it by optimally allocating residual control rights. These studies, however, focused organizational forms such as vertical integrations which are prevalent in the U.S. and other Western economies. In this paper, however, we shall focus on inter-firm relationship prevalent in Japan and analyze how such transactional structures may help mitigate the hold-up problem[4].

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[3]For a simple explanation of the Hold-up problem, see Tirole [1988, p.24] as well as Milgrom and Roberts [1992, pp.136-137].

[4]An intuition that the existence of multiple agents (suppliers) should necessarily increase the incentive for investments is not always correct. Indeed, Hart and Moore [1990] showed that the problem of under-investment exists even with multiple agents using Shapley Value

One way, and the way we use below, to solve the problem of under-investment with multiple suppliers is to induce rank-order tournament by offering a prize for the winner of the investment competition. The issue we shall analyze in this paper is to investigate theoretically how one can endogenously introduce such a prize in incomplete contracts. There are existing literature which assume the principal being able to introduce an exogenous prize and can commit to its payment. For example, Garcia-Cestona [1993] analyzed, using such a setup, the inter-firm relationship of a car-assembler and multiple parts-suppliers we discussed in the introduction[5]. However, as he does, if one assumes that the principal can commit to offer any arbitrary exogenous prize, the mechanism to solve under-investment is basically the same as the well-known rank-order tournament.

In the world where only incomplete contracts could be written, however, a contract that offers an exogenous prize, for example in the form of giving the exclusive contract to the winner, would be renegotiated *ex post*. This is so because the principal would have an *ex post* incentive to provide additional contract to the loser in order to limit the bargaining power of the winner by avoiding bilateral monopoly situation. Alternatively, the loser may attempt to obtain the contract by bribing the principal. It follows that the *ex ante* contract with a clause of not giving *ex post* contract to the loser would not function as a credible threat and rank-order tournament incentive would not work.

An alternative form of contracts we analyze in this paper is to provide an endogenous incentive compatible prize with incomplete contracts, in the sense that the proposed contract is compatible with the *ex post* negotiation outcome. Even if the principal cannot commit to

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as the solution concept for outcome of the *ex post* bargaining game. As will be shown in the text, what matters is how the agents compete each other, and not how many agents compete.

[5]The first version of this paper was written independent of Garcia-Cestona [1993].

the exclusive contract to the winner *ex ante*, she may be able to produce an incentive compatible endogenous prize, if there are multiple equilibria for the game to be played *ex post* by the suppliers. This is so because the assembler can make the payoff outcomes for the winner and the loser different by switching from one equilibrium to the other equilibrium. Because such switch is compatible with (a proper selection of) equilibria and hence self-enforcing, the difference between two equilibrium payoffs would function as an endogenous and incentive compatible prize.

3. The model

Consider a principal assembling parts which only the parts-suppliers (agents) can manufacture and the principal herself cannot produce. One of the parts, which is specific to and indispensable for the production of the product of the principal, is produced by two homogeneous agents, *i* and *j*. The principal seeks to have the agents manufacture the part with quality as high as possible. To improve the quality, agents should undertake larger investment which is specific to the product.

Since two agents are homogeneous, hereafter, we shall describe our model from the point of agent *i*. He undertakes specific investments during two periods. He initially has some stock of the specific asset denoted by $\bar{K}_i$, which is known to everybody. He undertakes the specific investments, h_i in the first period and e_i in the second period. The first-period investment contributes to the stock at the end of that period by $h_i + \varepsilon_i$, where ε_i is a noise which is independent across agents. We assume that the support of the noise is $[-\bar{\varepsilon}, \bar{\varepsilon}]$, $\bar{\varepsilon} > 0$ and its density function $\phi_i(x)$ is symmetric, i.e., $\forall\ x \in [-\bar{\varepsilon}, \bar{\varepsilon}]$, $\phi_i(x) = \phi_i(-x)$. Needless to say, this assumption implies that the unconditional expectation of ε_i is nil, that is, $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} x$

$\phi_i(x)dx = 0$. We also write the cumulative distribution function of the noise as $Prob(x \leq \varepsilon) = \int_{-\bar{\varepsilon}}^{\varepsilon} \phi_i(x)dx \equiv \Phi_i(\varepsilon)$. Note that, to help readers not to confuse, we distinguish $\phi_i(x)$ from $\phi_j(x)$ in their notations, though they are the same functions, i.e., $\phi_i(x) = \phi_j(x) = \phi(x)$. Of course, it is also true for the cumulative distribution function, that is, $\Phi_i(x) = \Phi_j(x) = \Phi(x)$.

The first-period investment as well as the initial stock of the specific asset depreciates at the rate of α during the second period. To simplify the analysis, we assume that the quality of the product manufactured at the end of the second period is equal to the amount of the stock obtained at that time[6]:

$$X_i = \alpha K_i + e_i,$$

where K_i represents the stock level accumulated by agent i until the end of the first period, that is,

$$K_i = \bar{K}_i + h_i + \varepsilon_i.$$

Both agents have the same production technology of this type. The revenue from the sales of the final product (of the principal) is assumed as:

$$R(X_1, X_2) = max\ (X_1, X_2).$$

The cost functions of the first- and the second- period investments are denoted by g(h) and c(e), respectively, satisfying $c'(h), g'(e) > 0$ as well as $c''(h), g''(e) > 0$ hold for all $h > 0$ and $e > 0$. Finally, the revenues and the costs accruing during the second period are discounted by a factor, δ.

As for the time and informational structure, we shall assume the following. At the beginning of the first period, the principal presents a contract. In period 1 (period 2, respectively), both agents choose h_i (e_i, resp.) and the end of the period stock of specific

[6]Here we assume that the outcome of the second-period investment is certain and non-stochastic, but this does not affect any qualitative results of this paper.

assets K_i (X_i, resp.) will be determined. The levels of investments, h_i and e_i, are private information for agent i. The principal as well as agents observe stocks of specific assets, K_i and X_i, at the end of respective period. However, we assume they are not verifiable. It follows that it is not possible to enforce a contract which is contingent on K_i and X_i. At the end of each period, after all the parties observe K_i or X_i respectively, they can renegotiate. It follows that the contract is enforceable only if it is incentive compatible, i.e., the payoffs are consistent with the *ex post* bargaining outcomes in each period.

4. The Case of Single Agent: Complete Contract and Incomplete Contract

In this section, in order to provide a benchmark solution we consider the case of a single agent. For that purpose, suppose there is only one agent and the level of his investment in each period is verifiable for the outside parties such as a court. Then, the principal can commit to any contingent contract at the beginning of the first period, effectively specifying the investment levels that the agent should take in each period. Let h^C and e^C be the specified investment levels. In our setup, the principal can commit to the first-best contract that maximizes the expected net gain from producing the final product. Since the expectation of the noise term is equal to nil, the net expected gain calculated at the beginning of the first period becomes $\delta\{\alpha(\bar{K} + \mathrm{h}) + e\} - \delta c(e) - g(\mathrm{h})$[7]. Thus, h^C and e^C are determined to satisfy, respectively:

$$g'(h^C) = \alpha\delta, \tag{1}$$

and

[7]Since we assume there is only one agent in this section, we drop subscripts i and j.

$$c'(e^c) = 1. \tag{2}$$

Types of investments in this model are, however, often unverifiable for the outsiders due to their relation-specific nature, and the principal may not commit to the first-best contingent contract. It follows that the agent must take account of the possibility of being forced to accept disadvantageous terms once his investment is sunk. In such situations, only contracts that the principal can offer to the agent and make him accept must be incentive-compatible so that neither party can induce others to renegotiate about the division of the gains *ex post*. In other words, *ex ante* contracts should specify the division of the gains between the concerned parties in the same way as the *ex post* bargaining does.

Suppose the *ex post* bargaining leads to the Nash solution. Then, the total revenue from selling the final output is divided in half between the principal and the agent through the *ex post* Nash bargaining. Assuming the total revenue from the final product is equal to the stock of the specific asset of the agent, X, at the end of period 2, the principal should offer exactly $X/2$ to the agent employed at the beginning of the first period.

Let EW and EU be the expected payoffs for the principal and the agent, respectively. When the agent accepts this incentive compatible incomplete contract at the beginning of the first period, the expected payoffs, EW and EU, are written respectively as:

$$EW = \frac{\delta}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\bar{K} + h + \varepsilon) + e)\phi(\varepsilon)\, d\varepsilon, \tag{3}$$

and

$$EU = \frac{\delta}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\bar{K} + h + \varepsilon) + e)\phi(\varepsilon)\, d\varepsilon - \delta c(e) - g(h). \tag{4}$$

Agent chooses the first- and the second-period investments to maximize (4), anticipating the outcome through the *ex post* bargaining. Then, the equilibrium levels of those investments are h^H and e^H such that

$$g'(h^H) = \frac{\alpha\delta}{2} \tag{5}$$

and

$$c'(e^H) = \frac{1}{2}, \tag{6}$$

and the equilibrium expected payoffs are given as follows:

$$EW = \frac{\delta}{2}\{\alpha(\bar{K} + h^H) + e^H)\} \tag{7}$$

and

$$EU = \delta\bar{v} + \frac{\alpha\delta}{2}(\bar{K} + h^H) - g(h^H), \tag{8}$$

where $\bar{v} = \frac{1}{2}e^H - c(e^H)$.

Comparing (5) and (6) with (1) and (2), there arises the hold-up problem, $h^C > h^H$ and $e^C > e^H$. An incomplete contract specifying the division of the gains in *ex post* incentive-compatible fashion brings about the inefficiency embodied in the under-investment in the specific assets. The reason is clear; agent can acquire only a half of the marginal gains that his investments would generate through the *ex post* bargaining.

5. Rank-order Tournament for the second-period exclusive contract

The principal seeks to mitigate the hold-up problem and to induce agents to invest more by designing an appropriate incentive scheme, even though the first-best contract is not enforceable. To mitigate the hold-up problem, we consider the principal making two agents compete in their relative performance of the first-period investments, like a *rank-order tournament*. To be more precise, consider a scheme in which the principal offers contracts to both agents at the beginning of the first period, while postponing till the end of the first period the decision of which agent to be selected as the exclusive supplier of parts. This decision is made based on the agents' relative performance in the first period.

As we have stressed in section 2, the principal is not likely to be able to commit to such a contract, as the relative performance of the first period investments is often not verifiable. There are two reasons we want to analyze this case. First reason is to provide yet another benchmark, i.e., the case where she can commit to the contract of providing the exclusive contract based upon their relative performance. We shall postpone the discussion of the second reason until the concluding section.

Even after one of the agents can be picked as the eventual exclusive parts supplier, complete contingent contracts still cannot be written due to unverifiability of the investment levels. Thus, to pay a half of the total revenue is the only enforceable contract that the principal can offer to the winning agent selected at the beginning of the second period. In this respect, this tournament scheme does not affect the performance of the investment in the second period.

The performance of the first-period investment, however, will be improved compared with single agent contracts. This is so because the principal can commit to contract only with the agent who achieved higher stock level at the end of the first period. We shall call this agent as the "winner" and the other agent the "loser". With the *ex post* Nash bargaining, a half of the total revenue goes to the winner as a prize of the first-period tournament while the loser receives nothing. Both of the agents will be motivated to undertake more investment in the first period with this tournament scheme.

5.1 Rank-order tournament and prize for agents

We now suppose that there are two homogeneous agents, i and j. Under the tournament scheme explained above, agent i faces the following payoff schedule at the beginning of the first period:

$$U_i(h_i, h_j) = \begin{cases} \delta(\bar{v} + \frac{\alpha}{2}K_i) - g(h_i), & \text{if } K_i \geq K_j, \\ -g(h_i), & \text{if } K_i < K_j. \end{cases} \tag{9}$$

The payoff schedule for agent j is similarly defined.[8] These schedules give a positive prize for the winner, since it has a jump at $K_i = K_j$.

Under this payoff schedule, the expected payoff for agent i at the beginning of the first period is represented by:

$$\begin{aligned} EU_i(h_i, h_j) &= \delta\{\bar{v} + \frac{\alpha}{2}(\bar{K} + h_i)\} \cdot Prob(K_j \leq K_i) \\ &+ \frac{\delta\alpha}{2} E[\varepsilon_i | K_j \leq K_i] \cdot Prob(K_j \leq K_i) - g(h_i) \end{aligned} \tag{10}$$

Note that the probability that agent i becomes the winner is

$$Prob(K_j \leq K_i) = \int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \Phi_j(h_i - h_j + x)\phi_i(x)\,dx \tag{11}$$

and the joint distribution function of agent i's noise term and the event that agent i becomes the winner is:

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[8]In this formulation, we specify agent i as the winner when the two agents tie, i.e., $K_i = K_j$. Since this is a measure-zero event, however, our specification does not affect the following analyses.

$$Prob(K_j \le K_i, x \le \varepsilon_i) = Prob(\varepsilon_j \le h_i - h_j + x, x \le \varepsilon_i) = \int_{-\bar{\varepsilon}}^{\varepsilon_i} \Phi_j(h_i - h_j + x)\phi_i(x)\,dx. \tag{12}$$

Thus, the second term in the RHS of (10) is written as

$$\frac{\delta\alpha}{2} E[\varepsilon_i | K_j \le K_i] \cdot Prob(K_j \le K_i) = \frac{\delta\alpha}{2} \int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} x\phi_i(x)\Phi_j(h_i - h_j + x)\,dx. \tag{13}$$

Given agent j's first-period investment h_j, a simple calculation yields the first order condition for agent i's first-period investment that maximizes his expected payoff;

$$\begin{aligned} g'(h_i) = {} & \delta\{\bar{v} + \frac{\alpha}{2}(\bar{K} + h_i)\}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \phi_i(x)\phi_j(h_i - h_j + x)\,dx \\ & + \frac{\delta\alpha}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \phi_i(x)\Phi_j(h_i - h_j + x)\,dx \\ & + \frac{\delta\alpha}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} x\phi_i(x)\phi_j(h_i - h_j + x)\,dx. \end{aligned} \tag{14}$$

We assume the second order condition always holds. Together with the individual rationality condition, $EU_i(h_i, h_j) \ge 0$, holds if and only if $h_i \ge 0$, we can derive the first-period reaction function of agent i, whose graph is denoted in Figure 1.

[Insert Figure 1 about here]

In figure 1, the reaction curve consists of four different components. For sufficiently small h_j's, i.e., if $0 \le h_j < h^H - 2\bar{\varepsilon}$, agent i will undertake h^H, since this investment level ensures agent i to win with certainty. As h_j becomes larger, agent i aggressively responds to it by investing more in a Bertrand fashion. If h_j increases further, however, agent i starts to respond it less aggressively by investing less because his marginal cost increases. Finally, for sufficiently large h_j's, agent i is preempted and prefers to be the loser by not undertaking the first period investment at all. There is a critical value for h_j where agent i's reaction function is discontinuous[9].

5.2 Symmetric equilibrium and the comparison with one-agent case

In the rest of this section, we confine our attention to the symmetric equilibrium case, i.e., $h_i = h_j = h^T$ at the equilibrium. Then, the first-order condition (14) is simplified as:

$$g'(h^T) = \frac{\delta\alpha}{4} + \delta(\bar{v} + \frac{\alpha}{2}(\bar{K} + h^T))f(0), \quad (15)$$

where $f(\eta) = \int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \phi(x)\phi(\eta + x)dx$, i.e., the density of the difference between two noise terms, $\varepsilon_i - \varepsilon_j$, being equal to η. Note also that $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \phi(x)\Phi(x)dx = Prob(\varepsilon_j \leq x, \varepsilon_i = x) = Prob(\eta \leq 0) = 1/2$, and $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} x[\phi(x)]^2dx = 0$ since $x[\phi(x)]^2$ is symmetric because of the symmetricity of the original density function.

The RHS of (15) represents the marginal (expected) revenue from the first-period investment when two agents invest symmetrically: its first term captures the *hold-up* effect that generates the hold-up problem. Because the probability to win is one half under this scheme, expected average revenue from the first period investment is reduced to $\alpha/4$. In this

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[9]At the intersection with the 45° line, the reaction curve necessarily has a negative slope. Algebraically,

$$\frac{\partial^2 EU_i(h_i,h_j)}{\partial h_i \partial h_j}\Big|_{h_i=h_j} = -\delta\{\bar{v}+\frac{\alpha}{2}(\bar{K}+h_i)\}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}\phi(x)\phi'(x)dx$$
$$-\frac{\delta\alpha}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}[\phi(x)]^2dx - \frac{\delta\alpha}{2}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}x\phi(x)\phi'(x)dx.$$

Because the symmetricity of $\phi(x)$ ***ensures*** $\phi'(x) = -\phi'(-x)$ ***and*** $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}\phi(x)\phi'(x)dx = 0$, $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}x\phi(x)\phi'(x)dx > 0$ ***holds. As*** $\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}[\phi(x)]^2dx > 0$, $\frac{\partial^2 EU_i(h_i, h_j)}{\partial h_i \partial h_j}\Big|_{h_i=h_j} < 0$ ***follows.***

respect, this scheme reinforces the hold-up problem. The second term of the RHS of (15) represents the *strategic* effect which mitigates the hold-up effect by inducing a strategic investment motive to win the prize in the form of the exclusive contract. Recall the expected payoff, (10), which gives an expected prize of $\delta(\overline{v} + \frac{1}{2}\alpha(\overline{K} + h))$ to the winner investing h in the first period. By undertaking the first-period investment marginally more than the opponent, each agent can increase the probability to be the winner, and hence to win the prize, by the amount of $f(0)$.

At the symmetric equilibrium, the agent's expected payoff is equal to:

$$\begin{aligned} EU_i &= \frac{\delta}{2}\{\overline{v} + \frac{\alpha}{2}(\overline{K} + h^T)\} + \frac{\delta\alpha}{2}E[\varepsilon_i|\varepsilon_i \geq \varepsilon_j] - g(h^T) \\ &= \frac{\delta}{2}\{\overline{v} + \frac{\alpha}{2}(\overline{K} + h^T)\} + \frac{\delta\alpha}{2}\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} x\phi_i(x)\Phi_j(x)\,dx - g(h^T)\,. \end{aligned} \tag{16}$$

Note that $0 < \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} x\phi_i(x)\Phi_j(x)dx < \overline{\varepsilon}/2$ since the expectation of the noise term is nil and $\Phi(x)$ is nondecreasing in x. As long as $EU_i(h_i, h_j)$ is strictly convex in $h_i \in (0, +\infty)$, the existence of the symmetric equilibrium is assured if and only if the equilibrium satisfies the individual rationality constraint, i.e., $EU_i(h^T, h^T) \geq 0$ for agent i.

We further assume the symmetric equilibrium is locally stable. This is satisfied when the slope of the reaction curves at the equilibrium is less than unity, or algebraically:

$$g''(h^T) - \frac{\alpha\delta}{2}f(0) > 0. \tag{17}$$

We now compare the equilibrium investment levels under the two alternative schemes. Note that:

$$\frac{g'(h^T)}{g'(h^H)} = \frac{1}{2} + (\overline{K} + h^T)f(0) + \frac{2\overline{v}}{\alpha} \tag{18}$$

holds by the use of (5) and (15). Alternatively,

Proposition 1:

If the symmetric equilibrium exists, the first period effort h^T (under the tournament for the exclusive contract) is higher than h^H (with one-agent case) if and only if:

$$f(0) > [\frac{1}{2} - \frac{2\overline{v}}{\alpha}]\frac{1}{\overline{K} + h^T}.$$

It follows that, other things being equal, $\overline{v}$ increases (*i*) *as* α *becomes smaller*, or (*ii*) *as* $f(0)$ *becomes larger (i.e., support of* ε *becomes smaller)*, the ratio h^T/h^H becomes larger and the hold-up problem is mitigated. This is so because both of these factors make the strategic effect of the first-period investment relatively more important than the 'hold-up' effect, in deciding how much to invest during the first period.

In terms of the expected payoffs, there are two reasons that make the principal better off. Mitigating the hold-up problem would imply more efficient investment, while picking the winner implies she can avoid the supplier whose noise term fell in the lower-half of its support. More precisely, she will obtain:

$$EW = \frac{\delta}{2}\{\alpha(\overline{K} + h^T) + e^H\} + \delta\alpha\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} x\phi_i(x)\Phi_j(x)\,dx. \qquad (19)$$

Since the second term of the RHS of (19) is positive, as far as $h^T \geq h^H$, the expected payoff is larger with the tournament scheme compared with the single supplier case of (7). Comparing (16) with (8), agents are worse off as far as the second term of the RHS of (16) is sufficiently small or, equivalently, if the support of the noise term is sufficiently small.[10]

[10]If the support of the noise term is sufficiently large, each agent may be better off under the tournament scheme by undertaking relatively small investment, for the scheme enables each agent to avoid the lower-side uncertainty.

To sum up:

Proposition 2:

If the rank-order tournament for the exclusive second-period contract mitigates the hold-up problem (*i.e.,* $h^T \geq h^H$), compared with the one-agent case the tournament scheme provides: (a) unambigously higher payoff for the principal, and (b) lower expected payoff for agents if the support of ε (noise) is small.

6. Tournament with an endogeneous prize by the use of multiple equilibria

In the previous section, we analyzed a contract where the principal can offer an exclusive contract to one of the agents. The principal commits how she selects the agent at the end of the first period with respect to their relative performance. However, such a discretionary choice of supplier is generally not incentive compatible. This is so because, at the end of the first period, the principal may offer the loser an additional supply contract in order to contain the strong bargaining position of the winner. With two suppliers, her bargaining position would be strengthened compared with a single supplier with monopoly power. In such situations, the contract we analyzed in the previous section is not incentive-compatible.

In this section, we analyze an incentive-compatible contract in this sense, assuming the division of the total revenue is determined by the *ex post* bargaining at the end of the second period by the principal and both agents. In order to do so, total revenue achieved by these three players becomes relevant. As we defined in section 3, we shall assume the total revenue is:

$R(X_i, X_j) = max\ (X_i, X_j).$

This is a natural extension of the definition used in the previous sections as each agent invests both in the first and the second period to improve the quality of the same parts to be used in production of the final product.

6.1. *Shapley value bargaining and multiple equilibria in the second period*

At the negotiation to take place at the end of period 2 three players, the principal and two agents, have potentially positive bargaining power. To describe the outcome of such negotiations, we shall use the Shapley value as a solution concept[11]. Shapley value bargaining specifies payoff for the principal as:

$$W = \begin{cases} \frac{1}{2}(K_i+e_i) + \frac{1}{6}(K_j+e_j) & if\ K_i + e_i \geq K_j + e_j \\ \frac{1}{6}(K_i+e_i) + \frac{1}{2}(K_j+e_j) & if\ K_i + e_i < K_j + e_j \end{cases} \tag{20}$$

and that for agent i as:

$$R_i = \begin{cases} \frac{1}{2}(K_i + e_i) - \frac{1}{3}(K_j+e_j) & if\ K_i + e_i \geq K_j + e_j, \\ \frac{1}{6}(K_i + e_i) & if\ K_i + e_i < K_j + e_j, \end{cases} \tag{21}$$

Payoff for agent j is similarly defined.

Note that, according to (21), payoff to agent i, R_i, is continuous in K_i as well as in e_i, if $K_i + e_i = K_j + e_j$. Payoff being continuous, contracts with Shapley value bargaining do not seem to provide a non-marginal prize for an agent who manufactured the product with relatively higher quality. Put differently, the *ex post* bargaining among the three parties does not seem to generate an extra incentive to capture a prize, which the rank-order tournament

[11]Gul [1988] and Mas-Collel [1993] provided non-cooperative foundations of Shapley value bargaining. Hart and Moore [1990] used Shapley value bargaining to analyze property rights problem with incomplete contracts.

model in the pervious section does.

When K_i, K_j and e_j are given, however, the payoff schedule (21) is convex in e_i having a kink at $e_i = K_j + e_j - K_i$. This makes the reaction function discontinuous, which may lead to multiple Nash equilibria in the second period subgame. If the payoff outcomes differ among those multiple equilibria, the principal can offer a contract specifying different equilibrium outcome depending upon who is the winner in the first period investment competition, and create a prize endogenously and incentive compatibly.

In order to describe such a contract, we consider a second-period subgame starting from any given pair of (K_i, K_j). Let $e^L = argmax\ [e_i/6 - c(e_i)]$ and $\underline{v} = e^L/6 - c(e^L)$. Needless to say, e^L satisfies:

$$c'(e^L) = \frac{1}{6} \tag{22}$$

and, recalling (5), $e^H > e^L$ and $\overline{v} > \underline{v}$ hold. It follows that given rival's effort choice e_j, agent i's payoff is the maximum of the following two values:

$$U_i^W(e_j) = \frac{K_i}{2} - \frac{K_j + e_j}{3} + \overline{v} \tag{23}$$

and

$$U_i^L = \frac{K_i}{6} + \underline{v}. \tag{24}$$

Figure 2 graphically explains how these payoffs are determined.

[Insert Figure 2 about here]

Clearly agent i should play e^H if and only if $U_i^W(e_j) \geq U_i^L$, and from Figure 2, their relative magnitude should depend on that of the stock levels at the end of this period[12]. Thus, for any K_i and K_j, the second-period reaction function for agent i is defined as follows:

$$e_i = \begin{cases} e^H & \text{if } e_j < e(K_i, K_j), \\ \{e^H, e^L\} & \text{if } e_j = e(K_i, K_j), \\ e^L & \text{if } e_j > e(K_i, K_j), \end{cases} \qquad (25)$$

where $e(K_i, K_j) = K_i - K_j + 3\Delta v$ and $\Delta v = \bar{v} - \underline{v} > 0$. The graph of this reaction function has a jump at $e = e(K_i, K_j)$ which might generate multiple equilibria. Figure 3 depicts this graph for the case where multiple Nash equilibria exist. Second period subgames can be classified into three subcategories depending what type of Nash equilibrium they generate:

$$(e_i^N, e_j^N) = \begin{cases} (e^H, e^L) & \text{if } K_i - K_j < -\Delta K, \\ \{(e^H, e^L), (e^L, e^H)\} & \text{if } -\Delta K \le K_i - K_j \le \Delta K, \\ (e^L, e^H) & \text{if } K_i - K_j > \Delta K. \end{cases} \qquad (26)$$

where $\Delta K = \min\{e^H - 3\Delta v, 3\Delta v - e^L\}$ and Nash equilibrium is denoted by (e_i^N, e_j^N). Note that, we have $1/6 < (c(e^H) - c(e^L))/(e^H - e^L) < 1/2$ and hence $\Delta K > 0$.

[Insert Figure 3 about here]

The middle case of (26) represents the case of multiple Nash equilibria in the second-period subgame[13]. When this subgame takes place, contracts specifying either equilibrium as suggested action and the corresponding equilibrium payoff as the payment to each agent are incentive compatible. From the point of the principal's payoff, however, it is desirable

[12]Indeed, it is easily verified that $K_i + e_i \geq K_j + e_j$ when $U_i^W(e_j) \geq U_i^L$.

[13]There is also a mixed strategy equilibrium which we put aside from consideration.

for the agent with a larger stock level at the beginning of the second period to play e^H and the other to play e^L, since higher quality increases her payoff. Thus, when the principal observes any pair of (K_i, K_j) in the middle of (26), she will offer a contract that specifies such equilibrium actions, assigning e^H for the agent with a larger stock and vice versa.

6.2. *Incentive-compatible equilibrium selection and endogenous tournament structure*

In this subsection, we analyze incentive-compatible contracts that the principal can offer to the agent at the beginning of the first period. This contract specifies the actions to play in the second period as well as the payoff. As the tournament model in the pervious section, we still call agent i the winner when $K_i \geq K_j$ and the loser otherwise. Then, an contract specifying second-period action e^H for the agent who became a winner in the first period competition and e^L for the loser is incentive-compatible. More specifically, let the specified action for agent i be:

$$e_i = \begin{cases} e^H & if \quad K_i \geq K_j \\ e^L & otherwise \end{cases} \tag{27}$$

and his second-period payoff be:

$$u_i = \begin{cases} \overline{V} + \frac{1}{2}K_i - \frac{1}{3}K_j - \frac{1}{3}e^L & if \ K_i \geq K_j, \\ \underline{V} + \frac{1}{6}K_i & otherwise. \end{cases} \tag{28}$$

Contract for agent j is similarly defined.

This contract can endogenously and incentive compatibly produce the structure of rank-order tournament and a prize for the first-period investment competition; the payoff schedule has a jump at $K_i = K_j$. The amount of the prize for the winner (if it is agent i) is:

$$\Pi(K_i - K_j) = \Delta v - \frac{e^L}{3} + \frac{K_i - K_j}{3}, \tag{29}$$

which is increasing in the difference between the stock levels observed at the beginning of the second period.

6.3. *Mitigation of the hold-up problem*

With the incentive-compatible contract defined in the previous subsection, agent i's expected payoff at the first period becomes as follows:

$$EU_i(h_i, h_j) = \delta\{\underline{v} + \frac{\alpha}{6}(\overline{K} + h_i) + \int_{-2\overline{\epsilon}}^{h_i - h_j} \Pi(h_i - h_j + \eta) f(\eta) d\eta\} - g(h_i). \tag{30}$$

The first term in the brace represents the minimum payoff that agent i can acquire at the end of the second period, and the second term is the expected value of the prize that he can receive as the winner.

As in the previous section, we confine our attention to the symmetric equilibrium in terms of the first-period investment competition. When its existence is assured, the following first-order condition characterizes the symmetric equilibrium strategy h^S in the first period.

$$g'(h^S) = \frac{\delta\alpha}{3} + \delta(\Delta v - \frac{e^L}{3}) f(0). \tag{31}$$

Note that the symmetric equilibrium strategy should meet the individual rationality condition for its existence. Since agents can always choose to be the loser, they can guarantee for himself the loser's payoff:

$$\bar{U} = \max_{h_i} \delta\{\underline{v} + \frac{\alpha}{6}(\bar{K} + h_i)\} - g(h_i)$$
$$= \delta\{\underline{v} + \frac{\alpha}{6}(\bar{K} + h^L)\} - g(h^L). \tag{32}$$

where the maximand h^L satisfies $g'(h^L) = \delta\alpha/6$. Agent i obtains this when he is the loser with certainty. The individual rationality constraint, $EU_i(h^S, h^S) \geq \bar{U}$, may be revised in view of (32)[14], yielding the next proposition:

Proposition 3:

The symmetric equilibrium exists only if:

$$\delta\Omega(0) \geq (\frac{\alpha\delta}{6}h^L - g(h^L)) - (\frac{\alpha\delta}{6}h^S - g(h^S))$$
$$\text{where } \Omega(0) = \frac{1}{2}(\Delta v - \frac{e^L}{3}) - \frac{\alpha}{3}\int_{-2\varepsilon}^{0}\eta f(\eta)\,d\eta.$$

Note that $\Omega(0)$ is the expected value of the prize in the symmetric equilibrium.

Comparing (31) with the one-agent first-order condition (5), we find that the hold-up effect and the strategic effect work in the opposite direction. In particular, the hold-up effect (the first term in the RHS) is smaller in (31), but the strategic effect (the second term) exists only in (31). Comparing two tournament schemes, one with an endogenous prize (31) and the other with the exclusive contract (15), we find again the two effects work against each other. As simple calculations would verify, the former has an unambiguously larger hold-up effect while the latter has an unambiguously larger strategic effect. Further straightforward computation would establish that if $\bar{K}$ is sufficiently large (in particular if it exceeds 1/6), the

[14]To be precise, the individual rationality condition also requires that the expected payoff to exceed the reservation payoff (*i.e.,* 0). However as $\bar{U}$ is larger than 0, (32) subsumes this condition.

latter strategic effect dominates and $h^T \geq h^S$. It follows:

Proposition 4:

If the symmetric equilibrium exists for the model of this section,

(a) the first period effort h^S is larger than the one-agent effort h^H if and only if:

$$f(0) > \frac{\alpha}{2(3\Delta v - e^L)}.$$

(b) the first period effort h^S is smaller than h^T if:

$$\frac{\alpha\delta}{2}(\overline{K} - \tfrac{1}{6} + h^T) + [\delta(\underline{v} + \tfrac{1}{3}e^L)f(0)] > 0.$$

Compared with the one-agent case, the endogenous prize with Shapley bargaining will mitigate the hold-up problem if α is sufficiently small, or $f(0)$ is sufficiently large. Between the two tournament schemes, the difference in the hold-up effects arises from the difference in the assumed bargaining solutions. Under the Shapley value bargaining each agent obtains one-sixth for sure, with additional one-third if he wins (with the probability of one-half). The expected share of the total pie which each agent expects to obtain (when their *ex ante* positions are symmetric) is one-third of the total pie. Under the Nash solution, each agent expects to receive one-half of the total pie only when he wins. This makes the expected share for each agent one-quarter. It follows that under the endogenous prize (with Shapley value bargaining), the average expected payoff for the first-period investment is larger than that under the tournament for exclusive contract (with Nash bargaining). The expected amount of the winner's prize becomes smaller under the endogenous prize with Shapley value bargaining, however, as not only the winner but also the loser makes positive effort.

The expected payoffs for the principal and each agent are as follows:

$$EW = \delta\Big\{E[\tfrac{1}{2}X_i + \tfrac{1}{6}X_j | K_i \geq K_j]\cdot Prob(K_i \geq K_j) + E\left[\tfrac{1}{2}X_j + \tfrac{1}{6}X_i | K_i < K_j\right]\cdot Prob(K_i < K_j)\Big\} = \delta\Big\{\tfrac{2}{3}\alpha(\bar{K} + h^S) + \tfrac{1}{2}e^H + \tfrac{1}{6}e^L + \int_0^{2\bar{\varepsilon}} \eta f(\eta)\, d\eta\Big\}. \tag{33}$$

and

$$EU_i = \delta\Big\{(\underline{V} + \frac{\bar{K} + h^S}{6}) + \Omega(0)\Big\} - g(h^S) = \delta\Big\{\frac{\bar{V} + \underline{V}}{2} + \frac{\alpha(\bar{K} + h^S) - e^L}{6} - \frac{\alpha}{3}\int_{-\bar{\eta}}^{0} \eta f(\eta)\, d\eta\Big\} - g(h^S). \tag{34}$$

In comparison with (7) and (19), (33) implies that the Shapley bargaining scheme rewards the principal most, if $h^S \geq h^T$ and $h^S \geq h^H$ hold. The reason is three-fold. First, the hold-up problem is reduced by the face-to-face competition between the agents in the first period. Second, the bargaining power of the principal is strengthened under Shapley bargaining than under Nash bargaining. Third, competition between two agents enables the principal to avoid the lower-side uncertainty, for the principal can discard the product with the inferior quality, even if it occurs because of pure randomness, without incurring any costs.

Comparing (35) with (8), if the support of the noise term is not sufficiently large and if $h^S > h^H$ holds, each agent gains less through the tournament for endogenous prize because of their weakened bargaining power and the severe investment competition. Compared with (35) with (16), however, relative size of agents' payoffs are unambiguous between the two tournament schemes, because of the tradeoff between the *ex ante* (first period) competition and *ex post* bargaining powers. To sum up:

Proposition 5:

(a) If $h^S \geq h^H$ (if $h^S \geq h^T$, respectively), the principal obtains larger payoff under the tournament for the endogenous prize (with Shapley value bargaining) than the one-agent case (under the tournament for the exclusive contract, respectively).

(b) If $h^S \geq h^H$ and the support of ε is sufficiently small, agents' expected payoff is smaller under the tournament with endogenous prize than the one-agent case.

(c) Relative size of agents' expected payoffs between two tournament schemes depends upon the tradeoff between the *ex ante* and *ex post* factors.

7. Concluding Remarks

In this paper, we have examined face-to-face competition which is generated as an endogenous and incentive compatible mechanism. Even if goods transacted between the assembler and part suppliers are not easily definable in written contracts, and transactions are necessarily carried out through relational transactions with incomplete contracts, contracts which provide a better treatment to the supplier who has performed better relative to other suppliers would create a strong incentive for improvements of the transacted goods. This is essentially a well-known rank-order tournament mechanism, but the prize can be created endogenously and incentive compatibly. This mechanism sometimes contribute to improvement of quality and cost of goods transacted, but the analysis we provided showed that having multiple agents engage in face-to-face competition is profitable for the principal only for a certain set of parameters. For some other parameter values, it is better to engage with a single supplier, though it may cause hold-up problem.

Some readers may question about the robustness of our result, because our conclusion in section 6 heavily depends upon the special property of Shapley value. In this respect, we should provide an alternative interpretation of our model in section 5 with exclusive second-period contract as a prize. The discussion we provided in section 5 was based upon our interpretation that the principal can commit to the clause that the exclusive contract be given to the winner of the first period competition. However, consider the following situation where (a) the principal cannot commit to such a clause, (b) the division of the total revenue will be decided by the *ex post* bargaining at the end of period 2, and (c) this bargaining will produce the following division: The loser of the whole investment competition (i.e., the agent i if $X_i < X_j$) will have no bargaining power and receives nothing. The winner and the principal will divide the revenue in half. Faced with such a bargaining possibility, there are again multiple equilibria for subgames; one equilibrium where agent i invests a positive amount in e while agent j invests nothing, and the other equilibrium where agent j invests a positive amount in e while agent i invests nothing. Like the model of section 6, it is more profitable for the principal to give better equilibrium outcome for the winner of the first period competition. The resulting incentive compatible contract would generate exactly the same choice as the model of section 6.

Next remark is concerned with our choice of the shape of revenue function; $R(X_i, X_j) = max\ (X_i, X_j)$. This form gives complete substitutability of assets provided by the two agents. Whichever is higher in quality makes the other asset completely useless. In the reality, these assets would be complementary to some extent, reflecting buffer in the case of one agent going bankrupt, etc. We have assumed this particular form because, even without complementarity, the principal wants to employ multiple agents in order to induce face-to-face competition. If there is complementarity, her incentive to employ multiple agents increases.

The relationship between the assembler and part suppliers in Japan is long-run and repeated in nature. To be exact, we must model it as a repeated game with the model of the text as its stage game (hence each stage game consists of two sub-periods[15]). However, these stage games are *not* time independent as they depend upon the initial stocks of quality (capital) by two suppliers, and rigorously analyzing such repeated games are relatively difficult. Intuitively speaking, as is straightforward from the existing literature, there are two channels through which incentives for investment (carried out in the first sub-period) are affected; change in the size of the prize (to be given in the second sub-period) and change in the structure of the game after the next period (at the end of the second sub-period). It can be easily shown (in view of the envelope theorem) that, while two effects provide the same effects in terms of their direction, the former has the first-order effect but the latter provides only the second-order effect. It follows that, even if we extend our model to repeat itself indefinitely, our qualitative results should remain unaffected unless we allow such possibilities as collusion among agents.

[15]Alternatively, one may model the repeated game consists of two activities taking place in each period, investment activity for h and investment activity for e. Under this formulation, the size of h will affect the prize for e in the next period.

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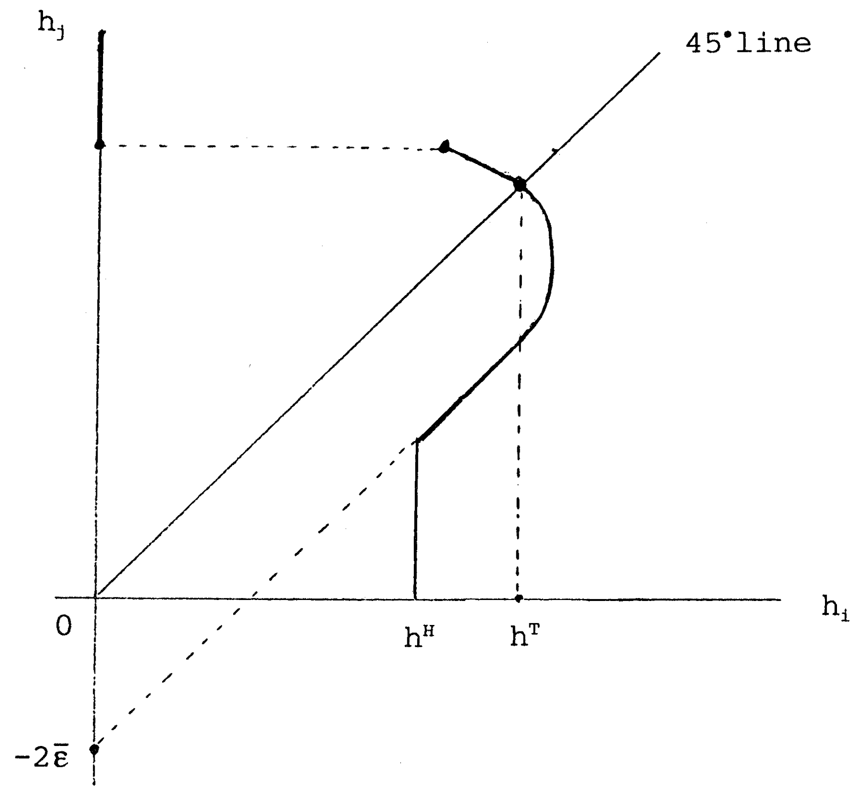
<u>Figure 1</u>



<u>Figure 2</u>

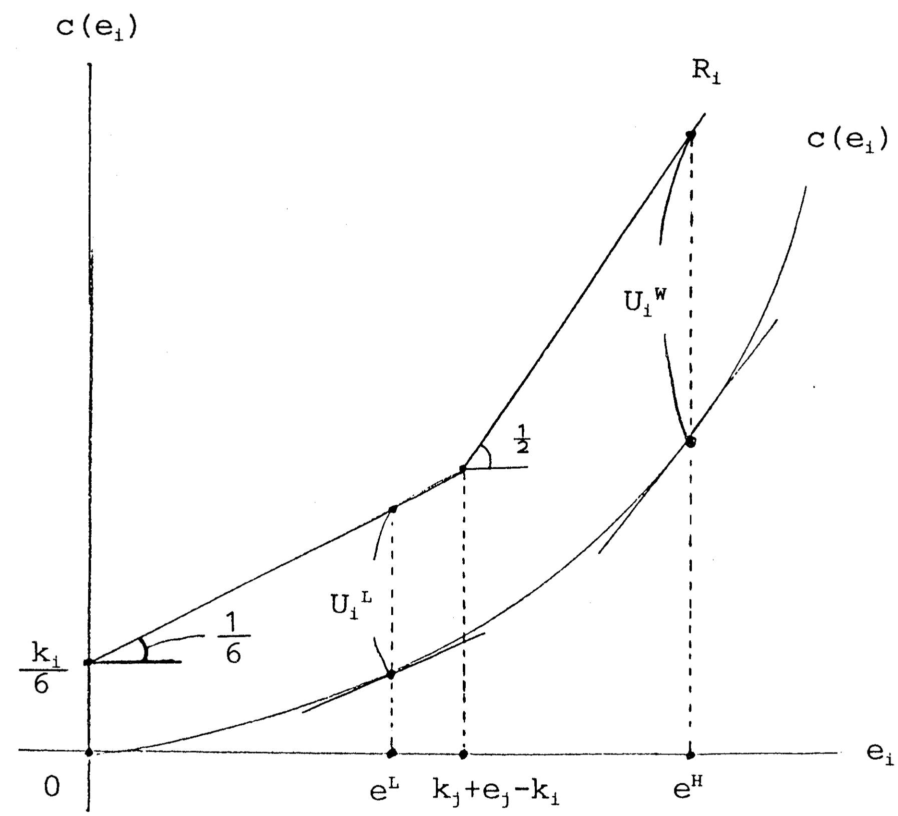


Figure 3