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GAINS FROM PRICE RIGIDITY

by

Kiyohiko G. Nishimura
University of Tokyo

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Faculty of Economics
University of Tokyo

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ABSTRACT

This paper shows that there are substantial gains from price rigidity in an imperfectly competitive economy. Firms can increase their profits by agreeing some markets as markets of long-term contracts, of which prices are determined in advance to other spot market prices. Although they determine prices non-cooperatively in both markets, the mutual commitment of making prices of some markets predetermined induces price-price spiral between firms, which results in substantial gains for both firms. These gains outweigh the cost of inflexibility due to price rigidity even though demand fluctuation is large and marginal cost is increasing.

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1. INTRODUCTION

In many U. S. industrial markets, prices do not fully reflect current market conditions (Carlton 1986). Imperfect price adjustment is not confined to isolated industries, but found throughout the U. S. and other industrialized economies, except for well-organized commodity markets. In fact, price "rigidity" is the core of macroeconomics.[1] A recent study by Blinder (1991) reports that there are substantial lags (mean lag of about 3.5 months) in price adjustment to changes in demand and cost in various product markets.

Although the majority of product prices may adjust only imperfectly to current conditions, it should be emphasized that there are a substantial number of "flexible" product prices incorporating market conditions quickly. The *coexistence* of rigid and flexible prices is often found even in the same industry. Prices between firms in the long-run relationship are usually slow to reflect changes in demand and cost conditions, while spot-market prices pick up them quickly. It is not uncommon that prices are rigid for some products but flexible for others in the same industry.[2]

Various explanations of the imperfect-adjustment aspect of price rigidity have been surfaced so far[3], and among them imperfect information and price-adjustment costs have been attracted much attention (Nishimura 1992). These explanations, however, tend to point out factors making prices *generally* not to adjust current conditions fully, and do not explain why rigid and flexible prices coexist in the same industry and in the same economy. Moreover, their explanation is based on the "cost" of price adjustment[4] which is greater than the gain from price adjustment, and the necessary price adjustment cost supporting price-rigidity becomes implausibly large when marginal cost is increasing and demand fluctuation is non-trivial.

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[1]There are two issues in price rigidity in macroeconomics: the failure of prices to adjust themselves fully to current market conditions, and the persistence of this imperfect adjustment. If firms determine prices based only on imperfect information about market conditions, then their prices fail to reflect current conditions fully. If the prices do not change for several periods, then the imperfect adjustment is persistent. In this paper, I am concerned mostly with the imperfect adjustment, since this issue has been the focus of lively debates over past decades (see Blanchard 1988 and Nishimura 1992).

[2]For example, rigidity of the luxury-model prices of automobiles is contrasted with volatility of the popular-model ones.

[3]Blinder (1991) identifies twelve such theories, and tentatively "test" their applicability using the survey data.

[4]Either a direct cost of changing menus or an indirect cost of gathering information.

The purpose of this paper is to present another explanation of imperfect price adjustment, which is based on the coexistence of rigid and flexible prices, and which is capable of supporting imperfect price adjustment even though the degree of increasing marginal cost is substantial and demand fluctuation is non-trivial. This explanation is motivated by the fact that in many industries firms produce various differentiated products, and compete with each other in several markets (or segments of the market). I will show that both firms gain substantially from agreed-on price rigidity in the form of *commitment in mutually pre-determining some prices before other prices*.

Consider two firms A and B competing both in the Product-I and Product-II markets, and compare the following two regimes. In the mutual-commitment regime, A and B pre-commit their Product-I price non-cooperatively in Period 1 before they determine their Product-II price non-cooperatively in Period 2, while in the non-commitment regime, Product-I and Product-II prices are determined simultaneously and non-cooperatively in Period 2.

In the mutual-commitment regime, if Firm A increases its pre-determined Product-I price in Period 1, Firm B is likely to respond by increasing its non-committed Product-II price in Period 2. This property can be called the "cross-market cross-strategic complementarity," which is a natural assumption if Product-I and Product-II are substitutes. Since an increase in Firm B's price increases Firm A's profit, the marginal benefit of Firm A's price increase is higher in the mutual-commitment regime where firms take account of this effect, than in the non-commitment regime where firms do not. Thus, for given Firm B's pre-determined Product-I price, Firm A's pre-determined Product-I price is higher than the corresponding price in the non-commitment regime, so that its profit is also higher.

Note that Firm B also has the same incentive to increase its pre-determined Product-I price for given Firm A's pre-determined Product-I price. Then, Firm B increases its price, and this price increase in turn triggers a further increase in Firm A's price, which again induces a further increase in Firm B's price, and so on. Thus, we have a "price-price spiral" or a "multiplier process" leading to substantially higher equilibrium predetermined prices, so long as the cross-market cross-strategic complementarity is strong. Since pre-determined prices are higher, non-committed prices are also higher through the substitution effect. Consequently, prices are higher and firms' profit is higher in the mutual-commitment regime than in the non-commitment regime. Thus, firms have strong incentive to agree on making some markets mutual commitment ones.

It is well-known that price pre-commitment in the form of unilateral price-leadership increases the leader's profit. However, the gain from unilateral pre-commitment in price is generally small, which is usually outweighed by the cost of inflexibility due to pre-commitment if there is fluctuations in demand and cost. This is because non-committed firm will "exploit" the inflexibility of the pre-committed firm. However, in the mutual commitment regime, *both* firms pre-commit on prices, and gains from mutual commitment are substantially higher than the unilateral pre-commitment.

The mutual-commitment regime is different from price collusion. Firms agree on pre-committing their price in some markets, but prices themselves are determined non-cooperatively. There is no legal restriction of preventing predetermined prices, while price collusion is *per se* illegal. It is perfectly legal and commonly observed that firms offer long-term contracts pre-specifying transaction prices and quantity in advance or offer forward contracts specifying price for future delivery of products. The result of this paper suggests that such instruments involving price rigidity may be used as *collusive device*, and that they are frequently observed in an imperfectly competitive economy, in which cross-market strategic complementarity between oligopolists is present.[5]

This paper is organized as follows. In Section 2, a model of multi-product duopoly is presented. In this section, I use a symmetric linear-demand constant-marginal-cost model in order to delineate conditions that make substantial gains from mutually pre-determining some prices before others. In Section 3, a general model of asymmetric multi-market duopoly is analyzed and the results obtained in Section 2 is shown to hold in the general case. In Section 4, I introduce uncertainty, and examine the robustness of the mutual commitment as a theory of price rigidity. I compare it with the menu-cost and the price-leadership arguments of price rigidity. In Section 5, I explore whether firms choose the mutual-commitment regime non-

[5]The importance of pre-determined prices in microfoundations of macroeconomics is emphasized by Nishimura (1992).

cooperatively when the collusion on the regime choice is not possible. Finally, in the concluding remark of Section 6, I argue that multi-market mutual commitment framework can explain business practices of which no good explanation is given previously. Proofs of main theorems in Section 4 are relegated to the APPENDIX.

2. PRE-DETERMINED PRICES AND NON-COMMITTED PRICES

2.1. Differentiated-Product Multi-Market Duopoly

In this paper, I assume a multi-market duopoly with differentiated products. There are two firms competing in two product markets. Products in both markets are differentiated and substitutes of one another. I consider the Bertrand price competition. I call the first market the *p*-market, in which the first firm's price is p_1 and the second firm's price is p_2. Similarly, the second market is called the *q*-market, in which the first firm's price is q_1 and the second firm's price is q_2. Collusion on prices is prohibited in both markets. Duopoly and two markets are assumed only for expository purposes. The basic result holds even if there are more than two firms and more than two markets.

There are two periods: 1 and 2. Period 2 is the actual transaction period, in which firms produce products, and customers consume the products. Firms can choose their prices simultaneously and non-cooperatively. In addition, I allow firms to "pledge" or "pre-commit" their prices by writing a "forward contract" or "long-term contract" in Period 1, in which the price and the quantity of transaction in Period 2 are specified. The contract is legally enforceable.[6] Although the length of Period 1 (contract-negotiation period) is not explicitly specified in the paper, it is by no means negligible since contract negotiation usually takes time. Moreover, when writing long-term contracts in Period 1, the market conditions in Period 2 may not be fully known. Thus, firms may have to pre-commit their prices in Period 1 under imperfect information about Period 2.

I compare two regimes, and examine which regime firms will choose. First, in the *mutual-commitment regime*, both firms pre-commit their *q*-market price non-cooperatively in Period 1, while they determine their *p*-market price non-cooperatively in Period 2 after observing the pre-determined *q*-market price. The second regime is the *non-commitment regime*, in which firms do not make pledge in Period 1, and they determine both *q*-market and *p*-market prices in Period 2 simultaneously and non-cooperatively. I compare the Nash (sub-game perfect) equilibrium of the two types of regimes. I assume that the agreement of mutual

[6]Thus, pre-commitment considered here is credible.

pre-commitment is enforceable. I will show that the mutual-commitment regime has substantially higher profit than the non-commitment regime under general conditions, so that it is likely to be adopted.

2.2. The Case of linear Demand and Constant Marginal Cost

To illustrate why mutual commitment increases both firms' profit substantially, let us consider the following symmetric duopoly model with linear demand and constant marginal cost. The general asymmetric case will be considered in Section 3. For simplicity, uncertainty is ignored until Section 4.

Let D_1^q be the demand for Firm 1's q-market product, and D_1^p be the demand for Firm 1's p-market product, such that

(1) $$D_1^q = a - bq_1 + cq_2 + dp_1 + ep_2;$$

(2) $$D_1^p = f - gp_1 + hp_2 + iq_1 + jq_2,$$

where a, b, c, f, g, and h are all positive constants. Parameters d, e, i, and j represent the cross-market effect. I have assumed that p-market products and q-market products are substitutes (such as luxury passenger car and popular one), thus d, e, i, and j are positive.

The marginal cost of production is constant, and set to be zero for expositional simplicity. Firm 1 maximizes its profit

(3) $$V^1 = q_1 D_1^q + p_1 D_1^p.$$

Firm 2's demand and profit functions are analogously defined.

I assume that equilibrium prices in both mutual commitment and non-commitment regimes are positive, which requires

(4) $$(2b-c)(2g-h)-(d+i+e)(d+i+j) > \frac{h(d+i)+2gj}{4g^2-h^2}\{h(d+i+j)+e(2g-h)\}$$

This condition implies that the own-price effects, b and g, are sufficiently large compared with the competitor's-price effects, c and h, and with cross-market effects, d, e, i, and j.

2.3. Market Equilibrium of the Two Regimes

Take the non-committed regime as a frame of reference. In the non-committed regime, all prices are simultaneously determined. Thus, equilibrium is determined by the four equations: $\partial V^1/\partial 0_1 = 0$, $\partial V^1/\partial p_1 = 0$, $\partial V^2/\partial q_2 = 0$, and $\partial V^2/\partial p_2 = 0$. To solve them, it is worthwhile to proceed in two steps.

First, solve for p_1 and p_2 as a function of q_1 and q_2 from $\partial V_1/\partial p_1 = 0$, and $\partial V_2/\partial p_2 = 0$. Then, we have for $i = 1, 2$,

(5) $p_i = \alpha + \beta\, q_i + \gamma\, q_j; j \neq i$, where $\alpha = \dfrac{f}{2g-h}$, $\beta = \dfrac{hj + 2g(d+i)}{4g^2 - h^2}$, and $\gamma = \dfrac{h(d+i) + 2gj}{4g^2 - h^2}$.

Since (5) determines the p-market price as a function of q-market price, I call (5) the *cross-market response function*. We have $2g > h$ from (4), so that α, β, and γ are positive.

The second step is to substitute (5) into $\partial V_1/\partial q_1 = 0$ and $\partial V_2/\partial q_2 = 0$ to get the following *reduced-form reaction function* such that

(6) $$q_i = \frac{A}{C} + \frac{B}{C} q_j \; i = 1, 2, \; j \neq i, \text{ where}$$

$$A \equiv a + (d+i+e)\alpha\,;\; B \equiv c + (d+i)\gamma + e\beta\;;\; C \equiv 2b - \{(d+i)\beta + e\gamma\}.$$

This reduced-form reaction function is "stable" in the usual sense ($-1 < B/C < 1$) since we have $2b - \{(d+i)\beta + e\gamma\} > c + (d+i)\gamma + e\beta$ from (4), and its response to the demand condition is "regular" (i.e., an increase in a increases the price) because we get $2b > (d+i)\beta + e\gamma$ from (4). Then, q-market prices in the non-committed equilibrium are

(7) $$q^{non} = \frac{a(2g-h) + f(d+i+e)}{(2b-c)(2g-h) - (d+i+e)(d+i+j)}$$

which is positive under our assumption (4).

Let us now consider the mutual-commitment regime, in which both firms pre-commit their q-market prices, while p-market prices are non-committed. In Period 2, q-market prices are already determined in Period 1, so that only p-market prices are determined. Thus, Period-2 equilibrium is determined by $\partial V^1/\partial p_1 = 0$, and $\partial V^2/\partial p_2 = 0$. Therefore, equilibrium non-committed prices are determined by the cross-market response function (5). Thus, firms in the mutual-commitment regime take account of the effect of their pre-determined price on their competitor's non-committed price through the equilibrium response function, whereas they do not in the non-commitment regime. Therefore, equilibrium q-market prices in the mutual-commitment regime are determined by

(8) $$\frac{\partial V^1}{\partial q_1}+\frac{\partial V^1}{\partial p_2}\frac{\partial p_2}{\partial q_1}=0 \text{ and } \frac{\partial V^2}{\partial q_2}+\frac{\partial V^2}{\partial p_1}\frac{\partial p_1}{\partial q_2}=0,$$

instead of $\partial V^1/\partial q_1 = 0$ and $\partial V^2/\partial q_2 = 0$. Here the envelope relation $\partial V^1/\partial p_1 = 0$, and $\partial V^2/\partial p_2 = 0$ are utilized.

Equilibrium determination in both regimes can easily be explained by using the reaction curve. Let us consider Figure 1. On the one hand, Firm 1's reduced-form reaction curve in the non-commitment regime is AA, which is (6). On the other hand, BB is Firm 1's reaction curve in Period 1 in the mutual-commitment regime, which takes account of the strategic effect due to pre-determined nature of q_1 in (8), such that

(9) $$q_1=\frac{A+h\gamma\alpha}{C-\gamma(e+h\beta)}+\frac{B+h\gamma^2}{C-\gamma(e+h\beta)}q_2$$

where A, B and C are defined in (6).

Since the model is symmetric, A'A' and B'B' represent Firm 2's corresponding reaction curves in the two regimes, and the equilibrium is determined by the intersection of the corresponding two reaction curves.

Cross-Market Cross-Strategic Complementarity and Equilibrium Price Levels

In this model, *cross-market cross-strategic interaction* between firms and the resulting *price-price spiral* shape the equilibrium. The model under consideration has two properties in the mutual commitment regime. First, if the firm increases its pre-committed price, this induces the *increase* in the competitor's non-committed price ($\partial p_1/\partial q_2 > 0$), which I call the *cross-market cross-strategic complementarity*. Second, an increase in the competitor's price is always *beneficial* to the firm ($\partial V^1/\partial p_2 = eq_1 + hp_1 > 0$).

Firms take account of cross-market cross-strategic complementarity ("my q-market price will increase my opponent's p-market price") shown in the cross-market response function (5) in the mutual commitment regime, while they do not in the non-commitment regime. Since the competitor's p-market-price increase is beneficial, the firm increases its q-market-price more in the mutual commitment regime than in the non-commitment regime. This represents the shift of AA to BB. Note that (9) with (6) shows that the mutual commitment regime's reaction function is higher and steeper than that in the non-commitment regime.

Since both firms' reaction curve is shifted upward, there is a price-price spiral between two firms' prices leading to substantially higher prices. Thus, it is now apparent that the equilibrium pre-committed price q^{mut} is higher than its counterpart q^{non} in the non-commitment regime:

$$(10) \qquad q^{mut} = \frac{a(2g-h)+f(d+i+e)+h\gamma f}{(2b-c)(2g-h)-(d+i+e)(d+i+j)-\gamma\{h(d+i+j)+e(2g-h)\}} > q^{non},$$

where condition (4) ensures that the denominator of (10) is positive. Then, (5) implies that the p-market price is also higher.

Cross-Market Own-Strategic Complementarity and Equilibrium Profits

If p-market prices and q-market prices are simultaneously determined, the competitor can choose p_i and q_i freely. However, if both firms agree to make q-market prices pre-determined, p_2 is determined by the cross-market response function $p_i = \alpha + \beta q_i + \gamma q_j$. This is the restriction on the competitor's price imposed by the mutual commitment to make q-market price predetermined.

Let us examine whether this restriction on the competitor's q-market and p-market price increases the firm's profit. Differentiating Firm 1's utility with respect to Firm 2's q-market price and utilizing the envelope relation ($\partial V^1/\partial p_1 = 0$), we have

$$(11) \quad \frac{\partial V^1}{\partial q_2}(x(q_1,q_2),y(q_1,q_2),q_1,q_2) = \frac{\partial V^1}{\partial p_2}\frac{\partial p_2}{\partial q_2} + \frac{\partial V^1}{\partial q_2} = (eq_1 + hp_1)\gamma + (cq_1 + jp_1) > 0.$$

Therefore, the restriction on the competitor *increases* Firm 1's profit.

The restriction is also imposed on the firm's own p-market price. However, this restriction has little effect on the firm's profit, since the firm determines its q-market price taking account of this restriction. Differentiating the firm's profit with respect to its q-market price and utilizing the envelope relation ($\partial V^1/\partial p_1 = 0$), we have

$$\text{(12)} \qquad \frac{\partial V^1}{\partial q_1}(x(q_1,q_2), y(q_1,q_2), q_1, q_2) = \frac{\partial V^1}{\partial p_2}\frac{\partial p_2}{\partial q_1} + \frac{\partial V^1}{\partial q_1} = 0$$

at the mutual-commitment equilibrium. Consequently, Firm 1's profit is higher in the mutual-commitment regime, than in the non-commitment regime.

To see this, note that when p and q are symmetric equilibrium prices, Firm 1's profit is

$$V^1 = aq + fp + (d + e + i + j)pq - (b - c)q^2 - (g - h)p^2.$$

Since both non-commitment and mutual-commitment equilibria satisfy (5), we have

$$V^1 = -(g-h)\alpha^2 + f\alpha + \{a + f(\beta+\gamma) + (d+e+i+j)\alpha - 2(g-h)\alpha(\beta+\gamma)\}q$$
$$+ \{(d+e+i+j)(\beta+\gamma) - (b-c) - (g-h)(\beta+\gamma)^2\}q^2.$$

Therefore, q maximizing V^1 is

$$q^{max} = \frac{a + f(b+g) + (d+e+i+j)a - 2(g-h)a(b+g)}{2\{(d+e+i+j)(b+g) - (b-c) - (g-h)(b+g)^2\}}.$$

Straightforward (though tedious) calculation shows that $q^{non} < q^{mut} < q^{max}$ and $\partial V^1/\partial q$ is positive for $q < q^{max}$. Therefore, Firm 1's profit is higher in the mutual-commitment regime than in the non-commitment regime.

The above discussion reveals that cross-market strategic complementarity (both cross-strategic and own-strategic), together with the property that the opponent's price increase is beneficial, yields higher prices and higher profit in the mutual-commitment regime than in the non-commitment regime. In the next section, I show that this result also holds in a general model of duopoly. Before proceeding analysis, however, remarks may be due about the determinants of the gains from mutual commitment, and an alternative way of mutual commitment.

2.4. Substitutability of Products and Gains from Mutual Commitment

Parameters of substitutability among products between the two markets, that is, d, e, i, and j, are among the most important determinants of the gains. To see this, let us first consider the case of no substitutability ($d = e = i = j = 0$). In this case, there is no cross-market strategic relationship, so that mutual commitment and non-commitment result in the same profit. In contrast, if products are substitutes, I have shown in the previous section that the gain from mutual commitment depends on the term (11), which is, by using (5),

$$\text{(13)} \qquad \frac{\partial V^1}{\partial p_2}\frac{\partial p_2}{\partial q_2} + \frac{\partial V^1}{\partial q_2} = (eq_1 + hp_1)\frac{h(d+i)+2gj}{4g^2 - h^2} + (cq_1 + jp_1) .$$

For given p_1 and q_1, this term is larger when the degree of substitutability (the absolute value of d, e, i, and j) is larger.

Figure 2 illustrates the relationship. In this figure, I take an example in which $a = f = 10$, $b = g = 0.5$, $c = h = 0.25$, and $d = e = i = j$. The profit ratio on the y-axis is the ratio of the mutual-commitment profit to the non-commitment profit. This figure shows that the profit ratio increases as the degree of substitutability, d, increases on the x-axis.

2.5. Cross-Commitment

In the preceding sections, I have assumed that both firms pre-determine the q-market price in the mutual commitment regime. However, this is not the only way to mutually pre-determine prices. Firms can adopt *cross-commitment*, in which Firm 1 offers a long-term contract in the q-market (thus predetermines the q-market price), while Firm 2 does the same in the p-market. In contrast, Firm 1's p-market price is non-committed, while Firm 2's q-market is non-committed.

Since there is cross-market strategic relationship, cross-commitment also increases both firms' profit in the same way as the mutual commitment does in the example of this section. In fact, in some cases, cross-commitment results in a higher profit that the mutual commitment considered here. In the following analysis, I consider cross-commitment as a variant of mutual commitment.

3. MUTUAL COMMITMENT AND ASYMMETRIC DUOPOLY

Symmetric duopoly analyzed in the previous section is seldom found in the economy, but asymmetry is the rule in the real-world oligopoly. Moreover, the model in the previous section assumed linear demand and constant marginal cost. In this section, however, I show that the main conclusion in Section 2 still holds true even in the general setting of asymmetric oligopoly. I show that two conditions are crucial in making the mutual commitment attractive: (i) the competitor's price increase must be beneficial to the firm, and (ii) there is cross-market strategic complementarity, both cross-strategic and own-strategic.

The result obtained in this paper is quite general. In order to illustrate its generality, the following discussion is based on firms' utility functions. If firms maximize their profits, then their utility functions are profit functions. However, the result of this paper holds even though firms' objective is not to maximize their profits, but to maximize, for example, their growth (see Baumol 1962 and Marris 1964), workers' utility in the case of worker-managed firms (see Vanek 1970), or the mixture of profits and workers' utility (see Komiya 1987, for a model of Japanese firms maximizing such a mixture).

Let the first firm's utility be V and the second firm's utility be W such that

$$V(p_1, p_2, q_1, q_2); \tag{14}$$

$$W(p_1, p_2, q_1, q_2). \tag{15}$$

where $(p_1, p_2, q_1, q_2) \in D^* \subset R_+^4$, where D^* is a bounded convex set.

I assume V and W are twice continuously differentiable in D^*. Hereafter, V_i is the first derivative of V with respect to the i-th argument. For example, $V_1 = \partial V/\partial p_1$. Similarly, W_j is the first derivative of W with respect to the j-th argument, such as $W_1 = \partial W/\partial p_1$.

The *non-commitment equilibrium* is a pair $(p_1^{non}, p_2^{non}, q_1^{non}, q_2^{non})$ satisfying

$$p_1^{non} = arg\ max\ V(p_1, p_2^{non}, q_1^{non}, q_2^{non});\ p_2^{non} = arg\ max\ W(p_1^{non}, p_2, q_1^{non}, q_2^{non});$$

$$q_1^{non} = arg\ max\ V(p_1^{non}, p_2^{non}, q_1, q_2^{non});\ q_2^{non} = arg\ max\ W(p_1^{non}, p_2^{non}, q_1^{non}, q_2).$$

The *mutual-commitment equilibrium* is a pair (q_1^{mut}, q_2^{mut}) satisfying

$$q_1^{mut} = arg\ max\ V(x(q_1, q_2^{mut}), y(q_1, q_2^{mut}), q_1, q_2^{mut});$$

$$q_2^{mut} = arg\ max\ W(x(q_1^{mut}, q_2), y(q_1^{mut}, q_2), q_1^{mut}, q_2),$$

where x and y are, respectively, the first firm's *cross-market response function*, $p_1 = x(q_1, q_2)$: $D^*_x \to R_+$, and the second-firm's *cross-market response function*, $p_2 = y(q_1, q_2)$: $D^*_y \to R_+$, defined as the solution of

$$p_1 = arg\ max\ V(p_1, p_2, q_1, q_2) \text{ and } p_2 = arg\ max\ W(p_1, p_2, q_1, q_2).$$

It is assumed that the firms' optimal prices are unique and lie in the interior of D^* in both regimes, and that cross-market response functions x and y are also uniquely determined and differentiable in (q_1, q_2). Then, cross-market response functions satisfy

(16) $$V_1(p_1, p_2, q_1, q_2) = 0; \text{ and}$$

(17) $$W_2(p_1, p_2, q_1, q_2) = 0.$$

Define first-firm's *reduced-form q-market reaction function* $q_1 = \text{u}(q_2)$: $D^*_u \to R_+$ as the solution of

(18) $$V_3(x(q_1, q_2), y(q_1, q_2), q_1, q_2) = 0: (q_1, q_2) \in D^*_x \cap D^*_y,$$

and the second firm's *reduced-form q-market reaction function* $q_2 = z(q_1)$: $D^*_z \to R_+$ is defined as the solution of

(19) $$W_4(x(q_1, q_2), y(q_1, q_2), q_1, q_2) = 0: (q_1, q_2) \in D^*_x \cap D^*_y.$$

The reduced-form reaction functions are also assumed to be differentiable.

It is evident that the non-commitment equilibrium is determined by the intersection the two reduced-form reaction functions. I make the following fairly standard assumptions on the reduced-form reaction functions.

REGULARITY. Let s_1 be a shift parameter in V such that $\partial V_3/\partial s_1 > 0$. Then, we have $\partial u/\partial s_1 > 0$. Similarly, let s_2 be a shift parameter in W such that $\partial W_4/\partial s_2 > 0$. Then, we have $\partial z/\partial s_2 > 0$.

STABILITY. $|u'(q_2)| < 1$ and $|z'(q_1)| < 1$.

STRATEGIC COMPLEMENTARITY. $0 < u'(q_2)$ and $0 < z'(q_1)$.

Regularity implies that if there is an upward shift in one firm's marginal profitability of q-market-price increase, then this firm's optimal q-market price increases for given q-market price of the other firm. Stability is a standard assumption in the analysis of duopoly. Prices are strategic complements in many cases, although there are exceptions.[7]

The following two assumptions are crucial.

ASSUMPTION 1: COMPETITOR'S PRICE INCREASE IS BENEFICIAL:

$$V_2 > 0,\ V_4 > 0,\ W_1 > 0,\ W_3 > 0.$$

ASSUMPTION 2: CROSS-MARKET STRATEGIC COMPLEMENTARITY.

(20) $\quad x_2(q_1,q_2) \equiv \dfrac{\partial x(q_1,q_2)}{\partial q_2} > 0$ and $y_1(q_1,q_2) \equiv \dfrac{\partial y(q_1,q_2)}{\partial q_1} > 0$. (cross-strategic)

(21) $\quad x_1(q_1,q_2) \equiv \dfrac{\partial x(q_1,q_2)}{\partial q_1} > 0$ and $y_2(q_1,q_2) \equiv \dfrac{\partial y(q_1,q_2)}{\partial q_2} > 0$. (own-strategic)

The following theorem generalizes the result of the previous section. Let equilibrium prices in the non-commitment regime be $(p_1^{non}, p_2^{non}, q_1^{non}, q_2^{non})$, and those in the mutual-commitment regime be $(p_1^{mut}, p_2^{mut}, q_1^{mut}, q_2^{mut})$.

THEOREM 1: If (1) the competitor's price increase is beneficial (ASSUMPTION 1), and (2) there is cross-market strategic complementarity (ASSUMPTION 2), then prices of both markets are higher in the mutual commitment regime than in the non-commitment regime, that is, $p_i^{non} < p_i^{mut}$, and $q_i^{non} < q_i^{mut}$.

Proof. See APPENDIX.

Note that by definition, q_1^{mut} maximizes $V(x(q_1, q_2^{mut}), y(q_1, q_2^{mut}), q_1, q_2^{mut})$ in the neighborhood of q_1^{mut}. Similarly, q_2^{mut} maximizes $W(x(q_1^{mut}, q_2), y(q_1^{mut}, q_2), q_1^{mut}, q_2)$ in the neighborhood of q_2^{mut}. The next assumption stipulates that the "neighborhood" is sufficiently

[7]See Tirole (1988: p.337) for cases where prices are not strategic complements.

large containing non-commitment equilibrium. This weak additional assumption ensures profits are higher in the mutual commitment than the non-commitment.

ASSUMPTION 3: "GLOBAL" NATURE OF INDIVIDUAL OPTIMUM IN THE MUTUAL COMMITMENT REGIME

$$V\left(x(q_1^{mut}, q_2^{mut}), y(q_1^{mut}, q_2^{mut}), q_1^{mut}, q_2^{mut}\right) > V\left(x(q_1^{non}, q_2^{mut}), y(q_1^{non}, q_2^{mut}), q_1^{non}, q_2^{mut}\right)$$

$$W\left(x(q_1^{mut}, q_2^{mut}), y(q_1^{mut}, q_2^{mut}), q_1^{mut}, q_2^{mut}\right) > W\left(x(q_1^{mut}, q_2^{non}), y(q_1^{mut}, q_2^{non}), q_1^{mut}, q_2^{non}\right)$$

THEOREM 2: If ASSUMPTION 3 holds in addition to ASSUMPTIONS 1 and 2, then both firms' utility is unambiguously higher in the mutual-commitment regime than in the non-commitment regime. Moreover, we have

(22) $$V(p_1^{mut}, p_2^{mut}, q_1^{mut}, q_2^{mut}) - V(p_1^{non}, p_2^{non}, q_1^{non}, q_2^{non})$$

$$> \text{Min } [V_2 y_2 + V_4]\ (q_2^{mut} - q_2^{non})$$

(23) $$W(p_1^{mut}, p_2^{mut}, q_1^{mut}, q_2^{mut}) - W(p_1^{non}, p_2^{non}, q_1^{non}, q_2^{non})$$

$$> \text{Min } [W_1 x_1 + W_3]\ (q_1^{mut} - q_1^{non}).$$

Proof. See APPENDIX.

As in the linear-demand constant-marginal-cost model (see (13)) the gains from mutual commitment depends on (1) how much the competitor's price-increase is beneficial (V_2 and V_4; W_1 and W_3) and (2) how strong the cross-market own-strategic complementarity is (y_2 and x_1).

4. MENU COSTS, UNILATERAL PRICE LEADERSHIP, AND MUTUAL COMMITMENT AS THEORY OF PRICE RIGIDITY

In the previous sections, I have investigated the pure gain from pre-determined prices under certainty. In this section, I introduce demand fluctuation and examine the robustness of the mutual commitment argument of price rigidity, and compare it with two alternative explanation of price rigidity, namely, menu costs and unilateral price pre-commitment (price leadership).

If there is uncertainty in demand, price rigidity implies inflexibility. Since inflexibility leads to inefficiency, there is a cost of pre-determined prices due to this inflexibility. The pure gain from price rigidity must be compared with this cost of inflexibility. The cost of inflexibility generally increases as demand fluctuates more and marginal cost increases more rapidly. I show in this section that menu-cost argument loses its ground quickly as demand fluctuates substantially and marginal cost is increasing. Similarly, price-leadership argument is easily upset when demand fluctuation is substantial. However, the mutual-commitment argument still holds true even if there is substantial demand fluctuation and the marginal cost is rapidly increasing. Here, cross-market cross-strategic complementarity and resulting price-price-spiral effect play a crucial role.

Since my concern is the robustness of the arguments of price rigidity in the presence of substantial demand fluctuation and increasing marginal cost, not the effect of uncertainty *per se*, I use specific numerical examples in this section in order to highlight the discussion.[8]

4.1. Problems in the Menu-Cost and Price-Leadership Explanation

In order to make analysis as simple as possible, I examine only one market and exclude the multi-market interaction. Specifically, I assume a symmetric duopoly with demand functions for Firm 1 (D_1) and Firm 2 (D_2) such that

(24) $D_1 = 10 - 0.5p_1 + 0.25p_2$; and $D_2 = 10 - 0.5p_2 + 0.25p_2$.

[8]As it is now well-known, the effect of uncertainty on price behavior is generally ambiguous. Thus, whether the demand uncertainty increases or decreases equilibrium prices depends on particular curvature of profit function and characteristics of uncertainty. Since the main motivation of this section is not the analysis of uncertainty, I do not delve into this issue.

Cost functions for Firm 1 (C_1) and Firm 2 (C_2) are such that

(25) $$C_1 = (1/2)rD_1^2; \text{ and } C_2 = (1/2)rD_2^2,$$

where r represents the degree of increasing marginal cost. I consider both positive r and negative r. If r is negative, this implies marginal cost is decreasing rather than increasing.[9]

Menu Cost Under Non-Constant Marginal Cost And Substantial Fluctuation

Menu cost argument for price rigidity assumes that there is a cost of changing prices, which may be small (as the term "menu cost" suggests). However, given that the competitor does not change its price, the firm's gain from adjusting its price to demand fluctuation is smaller than the menu cost, so that non-adjustment is the Nash equilibrium. I now examine the robustness of this argument.

At first there is no demand fluctuation and prices are determined optimally. Then, suppose that the demand intercept is now uncertain: k % higher than 10 with probability 1/2 and k % lower with probability 1/2. Thus, k % represents the magnitude of the demand fluctuation.

Table 1 shows the minimum menu cost supporting price rigidity for several pairs of (k, r), where the menu cost is measured as a percentage of the no-fluctuation profit. The minimum menu cost supporting price rigidity is the minimum cost of price adjustment that is sufficient to make the firm choose not to adjust its price, *given that the competitor does not adjust*.

As the menu cost argument suggests, the minimum menu cost is small if the magnitude is small. However, it becomes large when fluctuation is not trivial and the marginal cost is increasing as in the case of (k, r) = (40%, 1.5). Since it is generally hard to assume a sizable cost of price adjustment of this magnitude, this table reveals that the menu cost argument becomes implausible in realistic setting.

[9]This specification of demand and cost implicitly assumes that both firms do not ration their products. If firms ration their products, then the demand function depends on not only price but also the quantity produced by the competitor, in a non-linear way reflecting specific rationing scheme. The condition of no ration is always satisfied in the case of constant marginal cost, but it is not always so in the case of increasing marginal cost. The possibility of rationing complicates analysis greatly. See Bénassy (1989) for details. Following the literature of menu costs, I ignore the problem of rationing altogether in this section.

This table also reveals that the menu-cost argument loses ground again when marginal cost is *decreasing*, instead of increasing. The cost of inflexibility becomes large when fluctuation is large, regardless of whether marginal cost is increasing or decreasing. Thus, the menu-cost argument is plausible only if marginal cost is close to constant.

Price-leadership under uncertainty

The second explanation I consider is unilateral price pre-commitment, which is often called price leadership. Consider the same duopoly model as in the menu cost model. One firm is assumed to be the price leader and the other the follower. Now there are two periods. In Period 1, the price leader determines its price, and then in Period 2, the demand condition is determined and the follower sets its price. I assume the price leader cannot observe demand conditions when it determines its price in Period 1, while the follower can observe them before it sets its price in Period 2.

Table 2 shows (pure) gain from pre-determined price (that is, price leadership), the cost of inflexibility, and the net gain. Here, gain from price leadership is the gain of the price leader in excess of the non-commitment profit when it could determine its price *after* observing the demand condition. Cost of inflexibility is the difference in profit between the case in which the price-leader can observe the demand condition and the case in which it cannot. Both of them are measured as percentage of the non-commitment profit. The net gain is the gain of price leader under uncertainty, which is the difference between pure gain from pre-determined price and the cost of inflexibility.

This table reveals that the gain from price leadership is small for the price leader. The cost of inflexibility due to price rigidity dominates the gain if there is non-trivial uncertainty. In this table, if the magnitude of demand fluctuation is non-trivial or marginal cost is increasing (see (1) and (2) of this table), the gains form price leadership are outweighed by the cost of inflexibility. Even in the case that marginal cost is decreasing ((3) in this table), the net gain is likely to be negative. In the case of price-leadership, the follower usually gains much and the leader does little, although the leader's gain may be positive (see (4) in this table). Therefore, three is little incentive to be the price leader when demand fluctuates. Table 2 shows that the incentive is outweighed by the cost of inflexibility quite easily.

4.2. Mutual Commitment under Demand Fluctuation and Increasing Marginal Cost

I now show that the gain from pre-determined prices is larger than the cost of inflexibility in the mutual-commitment regime, even though demand fluctuation is substantial and marginal cost is increasing. This property of pre-determined prices is illustrated in Table 3. Here I consider a symmetric duopoly model with symmetric markets with linear demand and quadratic cost. In order to make results comparable, the value of parameters is set to be equal to that in Section 4.1. The demand for Firm 1's q-market products, $D_1{}^q$, its production cost, $C_1{}^q$, the demand for the firm's p-market products, $D_1{}^p$, and its production cost, $C_1{}^p$, are, respectively,

$$D_1{}^q = 10 - 0.5q_1 + 0.25q_2 + 0.12p_1 + 0.12p_2;\ C_1{}^q = (1/2)r(D_1{}^q)^2;$$

$$D_1{}^p = 10 - 0.5p_1 + 0.25p_2 + 0.12q_1 + 0.12q_2;\ C_1{}^q = (1/2)r(D_1{}^p)^2.$$

Here the cross-market substitutability of products is large: an increase in the p-market price of both firms has almost the same positive effect on the demand as an increase in the competitor's q-market price $(012 + 0.12 \approx 0.25)$. The demand and cost functions of Firm 2 are defined analogously.

The demand for p-market products fluctuates in Period 2, but it is not known in Period 1. As in the previous examples, I assume that the demand in the p-market is k % higher than the mean (10) with probability 1/2, and k % lower with probability 1/2. In the mutual-commitment regime, both firms determine their q-market prices in Period 1 without knowing the p-market demand condition in Period 2. Then, in Period 1, they set their p-market price after observing demand conditions and q-market prices. In the non-commitment regime, they determine prices in both markets in Period 2.

In Table 3, gain from pre-determined prices, cost of inflexibility, and net gain are defined analogously to those in Table 2. (Pure) gain from predetermined-prices is the difference between the *perfect-information* mutual-commitment profit (the profit level if firms could observe the demand of p-market products in determining pre-determined prices) and the non-commitment profit. Cost of inflexibility is the difference between the perfect-information mutual commitment profit and the imperfect-information mutual commitment profit (actual profit, i.e., profit when firms cannot observe p-market products in determining pre-determined prices). Net gain is the difference between the gain and the cost. Both the gain from predetermined-prices and cost of inflexibility are % of the non-commitment profit.

This table shows that the price-price-spiral effect of the mutual commitment is sufficiently strong to make mutual commitment still attractive even if the demand fluctuates substantially in the p-market. For example, in the case of constant returns ($r = 0$), mutual commitment gives firms positive 1.36% gain when demand fluctuates by 80%.

The effect of increasing marginal cost is of interest. Table 3 shows that the net gain from mutual commitment *changes little*, rather than decreases sharply, as the magnitude of increasing marginal cost (r) increases. This property of mutual commitment is in sharp contrast with the menu-cost argument and price-leadership explanation.

As marginal cost is more rapidly increasing, firms become more sensitive to prices of substitutes since a small increase in their price induces larger demand which results in larger cost. Consequently, firms increase their price more in the face of the substitutes' price increase. This implies that the magnitude of cross-market cross-strategic and own-strategic complementarity is increased, so that the pre-determined price in the mutual-commitment regime is raised further. The resulting increase in profit is larger than the cost of inflexibility, so that the net gain of mutual commitment increases as the magnitude of increasing marginal cost increases.

The major objection against the menu-cost and price-leadership argument for price rigidity is that they lose their ground rapidly as the degree of demand fluctuation is getting large and that marginal cost is increasing with production quantity. The example in Table 3 suggests this is not the case in the mutual-commitment argument for price rigidity, so long as the degree of substitutability of products between two markets is substantial.

5. EXTENSIONS

In the previous sections, I have shown that if firms in a dual-market duopoly with differentiated products can make a binding agreement, they are likely to agree on offering long-term contracts in one market and spot contracts in the other market. In this section, I examine two extensions of the model. First, I examine whether firms choose the mutual-commitment regime non-cooperatively if collusion on the regime choice is not possible. Second, I investigate the case where products of the two markets are *complements*, rather than substitutes as analyzed in the previous sections.

<u>*Mutual commitment as non-cooperative equilibrium*</u>

Suppose that for some reason it is not possible to enforce the agreement in which both firms pre-determine one market's prices before the other market's. In the following, I show that the mutual commitment is Nash equilibrium of non-cooperative game of regime choice, if the degree of substitutability of products between the two markets is not strong.

To illustrate the point, let us consider the symmetric linear-demand constant-marginal-cost duopoly model of Section 2, where marginal cost is equal to zero and demand functions are, respectively,

$$Dq = 10 - 0.5q_1 + 0.4q_2 + dp_1 + dp_2; \text{ and } Dp = 10 - 0.5p_1 + 0.4p_2 + dq_1 + dq_2.$$

I consider the Nash equilibrium of the following non-cooperative game of regime choice. For each firm, there are four possible strategies. First, the firm can determine both q-market price and p-market price in Period 1 (q-pre-determined/p-pre-determined). This strategy can be called perfect commitment. Second, it can pre-determine one market price, letting the other non-committed. This is strategy is partial commitment, which has two variations: (q-pre-determined/p-non-committed) and (q-non-committed/p-pre-determined). Finally, the firm can decide not to commit their prices (q-non-committed/p-non-commited), which I call non commitment. Both firms choose their strategy among them non-cooperatively.

If both choose perfect commitment, the resulting equilibrium is perfect commitment equilibrium. Similarly, if both choose non-commitment, then the non-commitment equilibrium is achieved. If both firms adopt partial commitment, we have mutual commitment equilibrium.

There are two types of mutual commitment equilibrium. In the "matching commitment" equilibrium, both firms choose the same market's price as pre-determined price. In this case, both firms pre-determine the p-market price and let the q-market price non-committed, or pre-determine the p-market price and let the q-market price non-committed. In the "cross commitment" equilibrium, firms choose different market's price as pre-determined price. In this case, Firm 1 pre-determines the p-market price, while Firm 2 pre-determines q-market price, or *vice versa*. Other than these symmetric equilibria, asymmetric equilibria are also possible in this framework.

Table 4 shows a typical example of the payoff matrix of this non-cooperative regime choice game, where $d = 0.03$. This table shows first that neither non-commitment nor perfect commitment are Nash equilibrium. If the competitor chooses non-commitment, the firm can gain by pre-determining its prices, as explained in the price-leadership literature. If the competitor pre-determines all his prices, then being a follower guarantees higher profit as explained in Section 3.

This table shows that the mutual commitment in the form of cross commitment is Nash equilibrium of the non-cooperative regime-choice game. Gain from making one market price pre-determined stems from the price-price spiral effect, which is realized only if both firms adopt the partial commitment strategy. Since the substitutability d of products between the markets is not large, firm cannot gain much from deviating from the partial commitment strategy and exploiting its competitor's price inflexibility in one market. Thus, this strategy is Nash equilibrium.

However, as the degree of substitutability d increases, non-commitment becomes increasingly more attractive than mutual commitment. Given that the competitor chooses partial commitment (q-pre-determined / p-non-committed), Figure 3 compares the gain from mutual commitment in the form of cross commitment (q-non-committed / p-pre-determined) with that from non-commitment (q-non-committed / p-non-committed), as the degree of substitutability d increases from zero to 0.05. The gain is measured by the ratio to the yardstick profit, which is the profit when both firms choose non-commitment. Both mutual commitment and non-commitment increase as the d increases, but non-commitment does more rapidly. Non-commitment which exploits price inflexibility of the competitor dominates mutual commitment, when the degree of substitutability is above $d = 0.035$.

The above argument has shown that the mutual commitment is Nash equilibrium of one-shot non-cooperative regime choice game, so long as the degree of substitutability of products between markets is not large. Moreover, the scope of mutual commitment may be further widened if we consider the regime choice as the repeated game, instead of one-shot game. Figure 3 shows that the gain from mutual commitment increases as the degree of substitutability d increases. Therefore, it may be possible to construct a sort of trigger strategy supporting mutual commitment even though mutual commitment is dominated by non-commitment in the one-shot framework.

Complementarity Of Products Between Markets And Cross-Market Cross-Strategic Substitutability

So far I have assumed that q-market products and p-market ones are substitutes (d, e, i, and j are positive). I now examine the case of complements (d, e, i, and j are negative). I show that whether mutual commitment yields higher profit than non-commitment is ambiguous, depending on cross-market demand parameters.

From (5), we have cross-market cross-strategic substitutability ($\partial p_1/\partial q_2 = \gamma < 0$) and cross-market own-strategic substitutability ($\partial p_2/\partial q_2 = \beta < 0$), instead of cross-market strategic complementarity. Suppose that the cross-market effect of complementarity is strong in such a way that $\partial V^1/\partial p_2 = eq_1 + hp_1 < 0$ and $\partial V^1/\partial q_2 = cq_1 + jp_1 > 0$, that is, the increase in the competitor's non-committed price is harmful while the increase in the competitor's pre-committed price is beneficial. This is the case in which (i) cross-market compelementarity between the two markets is strong, and at the same time (ii) substitutability between the firm's and its competitor's products within the market is also strong.

Note that an increase in the firm's pre-committed price induces a *decrease* in the competitor's non-committed price (cross-market cross-strategic substitutability). This decrease increases the firm's profit, since a *decrease* in the competitor's non-committed price is beneficial ($\partial V^1/\partial p_2 < 0$). Therefore, the firm's pre-determined price is higher than its counterpart in the non-commitment regime. Moreover, the profit is also higher in the mutual-commitment regime than in the non-commitment regime. Note that (11) still determines whether the mutual commitment increases the profit or not. In the assumed case, the term $(\partial V^1/\partial p_2)(\partial p_2/\partial q_2) = (eq_1 + hp_1)\beta$ is positive, and $\partial V^1/\partial q_2 = cq_1 + jp_1$ is also positive. Then, the profit is higher in the mutual-commitment regime than in the non-commitment regime.

In general, however, whether the increase in the competitor's price is beneficial or not, that is, the sign of $\partial V^1/\partial p_2 = eq_1 + hp_1$ and $\partial V^1/\partial q_2 = cq_1 + jp_1$, is ambiguous ($e$ and j are negative but h and c are positive). An increase in the competitor's p-market price increases the firm's p-market demand, which is beneficial. However, it reduces the firm's q-market demand through complementarity of products between the two markets. This is harmful for the firm. The same ambiguity holds for the q-market price. Therefore, there is no definitive result with respect to the case of complementarity of products between the two markets.

6. CONCLUDING REMARKS

In this paper, I have shown that two firms in differentiated-product multi-market duopoly can increase their profits by adopting the mutual-commitment regime in which some markets' prices are pre-determined before other markets' prices, even though prices themselves are determined non-cooperatively. Thus, if two firms can make a binding agreement making some markets as pre-determined-price markets, they have incentive to do so. I have also shown that the gain from mutual commitment is substantial, so that firms have positive net gain from price rigidity due to mutual commitment, even though demand fluctuation is substantial and marginal cost is increasing. Thus, the result of this paper suggests that price rigidity is likely to be observed in an imperfectly competitive economy with many differentiated products.

Although the model developed in this paper is based on product differentiation, the argument for mutual commitment seems valid even for markets of seemingly homogeneous products, if one takes account of the differentiation in service accompanying products. Here, service differentiation plays a role of product differentiation in the previous sections. For example, producers of commodities often offer long-term contracts having various accompanying long-term service such as lenient payment conditions, alongside with spot contracts of arm's length transaction. Long-term contracts and spot contracts constitute different sub markets, although products are the same. This fits the description of the mutual commitment. The distinctive feature of this service-differentiation interpretation of seemingly homogeneous product markets is that substitutability between markets is usually large. This implies the gain from mutual commitment is large, so that firms have strong incentive to agree on mutual commitment.[10]

The result of obtained in this paper also explains several business practices found in industrialized economies, other than price rigidity. Firstly, auto makers often sell both luxury models which are in effect made to order and whose prices are pre-determined well before its delivery time, and popular models which are mass-produced and whose prices are sensitive to

[10] At the same time, however, if it is not possible to make a binding agreement, the gain from deviating from the mutual commitment is also large and easily dominates the gain from mutual commitment in the market of seemingly homogeneous products. Thus, if one considers the non-cooperative setting, an explicit repeated game framework is needed to assess the plausibility of mutual commitment in this case.

current economic conditions. According to the principle of differentiation, auto makers have incentive to differentiate their products as much as possible. However, in reality, auto makers produce similar (though differentiated) products as the above example shows. This paper shows that there is a rationale for this practice, since the pre-determined prices of made-to-order luxury models reduce competition both in the luxury models and popular models.

Secondly, it is often observed that rival firms engage in price-cutting competition in one market, but their price is sticky in another market. For example, in the Japanese consumer electronics industry, firms typically introduce new products every half to one year,[11] and keep the price of these new products until another new products are introduced. However, they compete fiercely by cutting prices of existing products having been around in the market for more than one year. This practice can be considered as an attempt of making the market of new products a mutual-commitment market, although the market of existing products is left as a non-committed-price market. Since pre-determined prices make both pre-determined prices and non-committed prices higher than otherwise, firms have a common incentive to maintain this practice.

Thirdly, firms often pre-announce their future price change to their customers, and give customers opportunity to build a stock when price increases and to postpone purchase if price decreases (Okun 1981). Some firms establish a fixed pricing schedule, in which prices may be adjusted in fixed intervals. These practices are hard to be explained, since they seem to add extra costs to the firm without no efficiency improvement. However, the result of this paper presents one rationale: they are attempts to make prices "pre-determined". It is apparent for fixed pricing schedule and pre-announcement.

[11]This is a typical length of new-product cycle in the Japanese consumer electronics industry.

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APPENDIX:

PROOF OF THEOREMS

To simplify the notations, we hereafter use u for q_1 and z for q_2. I use

$$(u^*, z^*) \equiv (q_1{}^{mut}, q_2{}^{mut}) \text{ and } (u^+, z^+) \equiv (q_1{}^{non}, q_2{}^{non})$$

in the following analysis.

DEFINITION: $f(u,z) = V_3(x(u,z), y(u,z), u, z)$ and $g(u,z) = W_4(x(u,z), y(u,z), u, z)$.

Then, from (18) and (19), we have that $u(z)$ is defined by $f(u, z) = 0$; and that $z(u)$ is defined by $g(u, z) = 0$. Consequently, the non-commitment-regime equilibrium is determined by

(26) $f(u^+, z^+) = 0$; and $g(u^+, z^+) = 0$,

while the mutual-commitment-regime equilibrium is determined by

(27) $f(u^*, z^*) + V_2(x(u^*,z^*), y(u^*,z^*), u^*, z^*)y_1(u^*,z^*) = 0$; and

(28) $g(u^*, z^*) + W_1(x(u^*,z^*), y(u^*,z^*), u^*, z^*)x_2(u^*,z^*) = 0$.

LEMMA 1. $f_1 < 0$, $g_2 < 0$, $f_2 > 0$ and $g_1 > 0$.

Proof. Let $f(u, z; s) \equiv V_3(x(u, z), y(u, z), u, z; s)$ and let $u(z; s)$ be such that $f(u, z; s) = 0$, where s is a shift parameter such that $\partial f/\partial s > 0$. Then, Regularity implies $\partial u/\partial s > 0$. Since $f_1(u, z; s)(\partial u/\partial s) + \partial f/\partial s = 0$, we have $\partial u/\partial s = -(f_1)^{-1}(\partial f/\partial s)$. Therefore, we have $f_1 < 0$. Using the same procedure, we have $g_2 < 0$.

Next, since $u'(z)$ is determined by $f(u, z) = 0$ (see (18)), we have $u' = f_2/(-f_1)$. Strategic complementarity implies $u' > 0$, so that we have $f_2 > 0$ because $f_1 < 0$. Using the same procedure, we have $g_1 > 0$.□

LEMMA 2. $f_1 + f_2 < 0$; and $g_1 + g_2 < 0$.

Proof. Since $u' = f_2/(-f_1)$, stability ($u' < 1$) implies $f_1 + f_2 < 0$ because $f_1 < 0$. A symmetric argument establishes $g_1 + g_2 < 0$.□

LEMMA 3.

(29) $f(u^+, z^+) - f(u^*, z^*) > 0$;

(30) $g(u^+, z^+) - g(u^*, z^*) > 0$.

Prof. From (26), we have $f(u^+, z^+) = 0$ and $g(u^+, z^+) = 0$, while (27) and (28) mean that $f(u^*, z^*) + V_2y_1 = 0$ and $g(u^*, z^*) + W_1x_2 = 0$. Since $V_2 > 0$, $W_1 > 0$ (ASSUMPTION 1), $y_1 > 0$

and $x_2 > 0$ (ASSUMPTION 2), we have $f(u^+, z^+) = 0 > f(u^*, z^*)$ and $g(u^+, z^+) = 0 > g(u^*, z^*)$. □

DEFINITION. $\Delta u \equiv u^+ - u^*$; $\Delta z \equiv z^+ - z^*$;

$$\alpha_f(X) \equiv f(u^* + X, z^* + X) - f(u^*, z^*);$$

$$\alpha_g(X) \equiv g(u^* + X, z^* + X) - g(u^*, z^*);$$

$$\beta_f(Y) \equiv f(u^+, z^* + \Delta u + Y) - f(u^+, z^* + \Delta u);$$

$$\beta_g(Y) \equiv g(u^+, z^* + \Delta u + Y) - g(u^+, z^* + \Delta u);$$

$$\gamma_f(w) \equiv f(u^* + \Delta z + w, z^+) - f(u^* + \Delta z, z^+);$$

$$\gamma_g(w) \equiv g(u^* + \Delta z + w, z^+) - g(u^* + \Delta z, z^+).$$

LEMMA 4.

(31) $\alpha_f(0) = \alpha_g(0) = \beta_f(0) = \beta_g(0) = \gamma_f(0) = \gamma_g(0) = 0;$

(32) $\alpha_f' < 0; \alpha_g' < 0; \beta_g' < 0; \gamma_f' < 0; \beta_f' > 0;$ and $\gamma_g' > 0;$

(33) $f(u^+, z^+) - f(u^*, z^*) = \alpha_f(\Delta u) + \beta_f(\Delta z - \Delta u) = \alpha_f(\Delta z) + \gamma_f(\Delta u - \Delta z);$

(34) $g(u^+, z^+) - g(u^*, z^*) = \alpha_g(\Delta u) + \beta_g(\Delta z - \Delta u) = \alpha_g(\Delta z) + \gamma_g(\Delta u - \Delta z).$

Proof. By construction, we have (31). From LEMMA 2, we have $\alpha_f' = f_1(u^* + X, z^* + X) + f_2(u^* + X, z^* + X) < 0$ and $\alpha_g' = g_1(u^* + X, z^* + X) + g_2(u^* + X, z^* + X) < 0$. Similarly, the first part of LEMMA 1 implies $\beta_g' = g_2(u^+, z^* + \Delta u + Y) < 0$, and $\gamma_f' = f_1(u^* + \Delta z + w, z^+) < 0$. Since $\gamma_g' = g_1(u^* + \Delta z + w, z^+)$ and $\beta_f' = f_2(u^+, z^* + \Delta u + Y)$, the second part of LEMMA 1 implies $\gamma_g' > 0$ and $\beta_f' > 0$. Therefore, (32) holds.

Next, we have the following relations from the definition of α_f, β_f, and γ_f:

(35) $\alpha_f(\Delta u) = f(u^* + \Delta u, z^* + \Delta u) - f(u^*, z^*) = f(u^+, z^* + \Delta u) - f(u^*, z^*);$

(36) $\beta_f(\Delta z - \Delta u) = f(u^+, z^* + \Delta z) - f(u^+, z^* + \Delta u) = f(u^+, z^+) - f(u^+, z^* + \Delta u);$

(37) $\alpha_f(\Delta z) = f(u^* + \Delta z, z^* + \Delta z) - f(u^*, z^*) = f(u^* + \Delta z, z^+) - f(u^*, z^*);$

(38) $\gamma_f(\Delta u - \Delta z) = f(u^* + \Delta u, z^+) - f(u^* + \Delta z, z^+) = f(u^+, z^+) - f(u^* + \Delta z, z^+).$

Adding both sides of (35) and (36), and adding both sides of (37) and (38), we get (33). A symmetric argument establishes (34). □

LEMMA 5. Let $H(X)$ be a continuously differentiable function of X satisfying $H(0) = 0$ and $H'(X) < 0$. Then, $H(X) > 0$ implies $X < 0$, while $H(X) < 0$ implies $X > 0$.

Proof. Suppose, *ad absurdum*, that there exists $X > 0$ such that $H(X) > 0$. Since $H' < 0$, we have $H(0) > H(X) > 0$, which contradicts $H(0) = 0$. Therefore, $H(X) > 0$ implies $X < 0$. A symmetric argument establishes that $H(X) < 0$ implies $X > 0$.□

PROOF OF THEOREM 1. I first show that q-market prices are higher in the mutual-commitment regime than in the non-commitment regime ($\Delta u = u^+ - u^* < 0$ and $\Delta z = z^+ - z^* < 0$). We then prove that p-market prices are higher in the mutual-commitment regime than in the non-commitment regime ($x(u^*, z^*) > x(u^+, z^+)$ and $y(u^*, z^*) > y(u^+, z^+)$).

(1) To show $\Delta u < 0$ and $\Delta z < 0$.

We consider two mutually exclusive possibilities.

Possibility 1: $\Delta z \geq \Delta u$. Since $\gamma_g' = g_1(u^* + \Delta z + w, z^+) > 0$ and $\Delta u - \Delta z \leq 0$, we have

$$\gamma_g(\Delta u - \Delta z) \equiv \int_{w=0}^{w=\Delta u - \Delta z} \gamma_g'(w)dw \leq 0.$$

This, together with (30) and (34), implies $\alpha_g(\Delta z) > 0$. From LEMMA 4, we know $\alpha_g' < 0$ and $\alpha_g(0) = 0$. Then, we have $\Delta z < 0$ from LEMMA 5, so that we have $0 > \Delta z \geq \Delta u$.

Possibility 2: $\Delta z < \Delta u$. Since $\beta_f' > 0$ and $\Delta z - \Delta u < 0$, we have

$$\beta_f = (\Delta z - \Delta u) \equiv \int_{y=0}^{y=\Delta z - \Delta u} \beta_f'(y)dy < 0.$$

This, together with (29) and (33), implies $\alpha_f(\Delta u) > 0$. From LEMMA 4, we know $\alpha_f' < 0$ and $\alpha_f(0) = 0$. Then, we have $\Delta u < 0$ from LEMMA 5, so that we have $0 > \Delta u > \Delta z$.

Since both Δz and Δu are negative in both cases, we have $0 > \Delta z = z^+ - z^* = q_2^{non} - q_2^{mut}$ and $0 > \Delta u = u^+ - u^* = q_1^{non} - q_1^{mut}$ (so that $q_1^{mut} > q_1^{non}$ and $q_2^{mut} > q_2^{non}$).

(2) To show $x(u^*, z^*) > x(u^+, z^+)$ and $y(u^*, z^*) > y(u^+, z^+)$.

Since $u^* > u^+$, we have $x(u^*, z^*) > x(u^+, z^*)$ because $x_1 > 0$ (ASSUMPTION 2). Moreover, since $z^* > z^+$, we have $x(u^+, z^*) > x(u^+, z^+)$ because of $x_2 > 0$ (ASSUMPTION 2). Therefore, we have $x(u^*, z^*) > x(u^+, z^+)$ ($p_1^{mut} > p_1^{non.}$). A similar procedure establishes $y(u^*, z^*) > y(u^+, z^+)$ ($p_2^{mut} > p_2^{non}$).□

PROOF OF THEOREM 2

ASSUMPTION 3 implies $V(x(u^*,z^*), y(u^*,z^*), u^*, z^*) > V(x(u^+,z^*), y(u^+,z^*), u^+, z^*)$. Moreover, since we have $V_2 > 0$, $V_4 > 0$, $y_2 > 0$, and $z^* > z^+$, we get

$$V(x(u^+,z^*),\ y(u^+,z^*),\ u^+,\ z^*) - V(x(u^+,z^+),\ y(u^+,z^+),\ u^+,\ z^+)$$

$$= \int_{z=z^+}^{z=z^*} \{V_1 x_2 + V_2 y_2 + V_4\} dz > Min[V_2 y_2 + V_4](z^* - z^+) > 0$$

where the envelope relation $V_1 = 0$ (due to the definition of x) is utilized. Combining these two inequalities, we have (22). A parallel argument establishes (23). □

TABLE 1

MENU COST, MAGNITUDE OF FLUCTUATION, AND INCREASING MARGINAL COST

(1) EFFECT OF INCREASING FLUCTUATION				
Magnitude of Demand Fluctuation k =	1%	20%	40%	80%
Magnitude of Decreasing Marginal Cost: r =	0	0	0	0
Menu Cost Supporting Price Rigidity (% of Profits under Certainty)	0.01%	2.25%	9.00%	36.00%
(2) EFFECT OF INCREASING MARGINAL COST				
Magnitude of Demand Fluctuation k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	0.01	0.1	1	1.5
Menu Cost Supporting Price Rigidity (% of Profits under Certainty)	9.08%	9.76%	17.64%	22.78%
(3) EFFECT OF DECREASING MARGINAL COST				
Magnitude of Demand Fluctuation k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	-0.01	-0.1	-1	-1.5
Menu Cost Supporting Price Rigidity (% of Profits under Certainty)	0.01%	2.07%	2.78%	3.24%

TABLE 2

PRICE-LEADERSHIP UNDER DEMAND FLUCTUATION

(1) EFFECT OF INCREASING FLUCTUATION				
Magnitude of Demand Fluctuation: k =	1%	10%	20%	40%
Magnitude of Increasing Marginal Cost: r =	0	0	0	0
Gains from pre-determined prices	0.45%	0.45%	0.45%	0.45%
Cost of Inflexibility	0.01%	0.99%	3.86%	13.85%
Net Gain	0.44%	-0.55%	-3.42%	-13.41%
(% of Non-Commitment Profit)				
(2) EFFECT OF INCREASING MARGINAL COST				
Magnitude of Demand Fluctuation: k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	0.01	0.1	1	1.5
Gains from pre-determined prices	0.45%	0.45%	0.44%	0.42%
Cost of Inflexibility	13.98%	15.09%	28.13%	36.83%
Net Gain	-13.53%	-14.64%	-27.69%	-36.41%
(% of Non-Commitment Profit)				
(3) EFFECT OF DECREASING MARGINAL COST				
Magnitude of Demand Fluctuation: k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	-0.01	-0.1	-1	-1.5
Gains from pre-determined prices	0.45%	0.44%	0.33%	0.16%
Cost of Inflexibility	13.73%	12.66%	4.06%	1.14%
Net Gain	-13.29%	-12.22%	-3.73%	-0.98%
(% of Non-Commitment Profit)				
(4) GAIN FROM NON-COMMITMENT				
Magnitude of Demand Fluctuation: k =	1%	10%	20%	40%
Magnitude of Increasing Marginal Cost: r =	0	0	0	0
Net Gain from Non-Commitment	3.60%	3.13%	1.78%	-2.93%
(% of Non-Commitment Profit)				

TABLE 3

GAINS FROM MUTUAL COMMITMENT ON PRE-DETERMINED PRICES, COST OF INFLEXIBILITY, AND INCREASING MARGINAL COST:

(1) EFFECT OF INCREASING FLUCTUATION				
Magnitude of Demand Fluctuation: k =	1%	20%	40%	80%
Magnitude of Increasing Marginal Cost: r =	0	0	0	0
Gains from pre-determined prices	9.46%	9.44%	9.41%	9.29%
Cost of Inlfexibility	0.00%	0.58%	2.26%	7.93%
Net Gain (% of Non-Commitment Profit)	9.45%	8.86%	7.15%	1.36%

(2) EFFECT OF INCREASING MARGINAL COST				
Magnitude of Demand Fluctuation: k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	0.01	0.1	1	1.5
Gains from pre-determined prices	9.41%	9.44%	9.53%	9.50%
Cost of Inlfexibility	2.25%	2.20%	1.74%	1.59%
Net Gain (% of Non-Commitment Profit)	7.16%	7.24%	7.80%	7.91%

(3) EFFECT OF DECREASING MARGINAL COST				
Magnitude of Demand Fluctuation: k =	40%	40%	40%	40%
Magnitude of Increasing Marginal Cost: r =	-0.01	-0.1	-1	-1.5
Gains from pre-determined prices	9.45%	9.42%	9.01%	8.74%
Cost of Inlfexibility	0.00%	0.00%	0.00%	0.00%
Net Gain	9.45%	9.42%	9.01%	8.74%

TABLE 4
MUTUAL COMMITMENT
AS A NON-COOPERATIVE REGIME-CHOICE GAME

Payoff Matrix (V^A, V^B)

		B: q: non-committed p: non-committed	B: q: pre-committed p: non-committed	B: q: non-committed p: pre-committed	B: q: pre-committed p: pre-committed
A	q: non-committed p: non-committed	(361 , 361)	(427 , 375)	(427 , 375)	(490 , 389)
A	q: pre-committed p: non-committed	(375 , 427)	(370 , 370)	(428 , 428)	(415 , 376)
A	q: non-committed p: pre-committed	(375 , 427)	(428 , 428)	(370 , 370)	(415 , 376)
A	q: pre-committed p: pre-committed	(389 , 490)	(376 , 415)	(376 , 415)	(361 , 361)

Note: Demand functions are $D^q = 10 - 0.5q_1 + 0.4q_2 + 0.03p_1 + 0.03p_2$ and $D^p = 10 - 0.5p_1 + 0.4p_2 + 0.03q_1 + 0.03q_2$, respectively, and marginal cost is zero.

FIGURE 1: MUTUAL COMMITMENT VS. NON-COMMITMENT (Corel Draw Ver.4 rigid1.cdr) INSERTED HERE

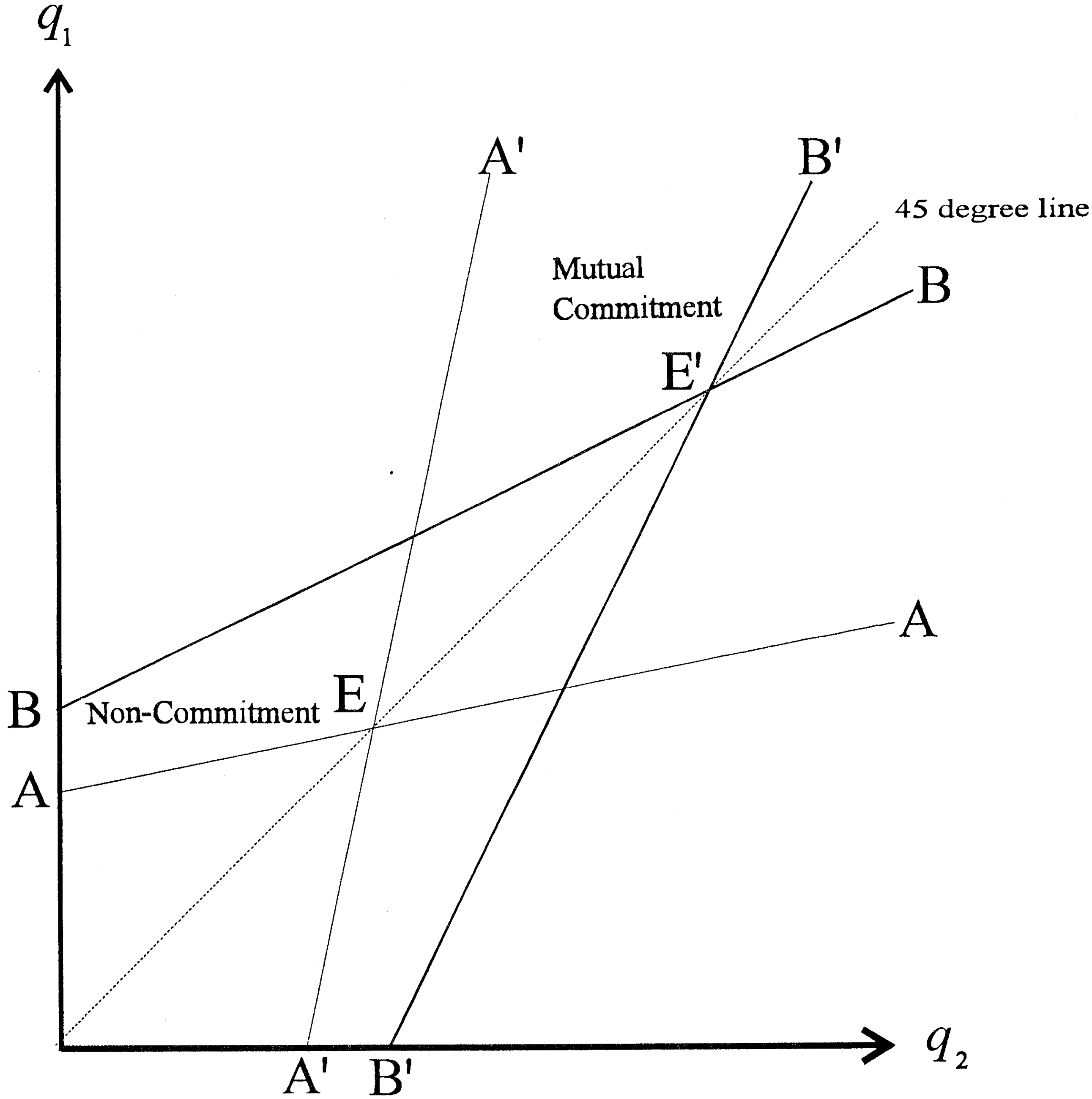

FIGURE 1
MUTUAL COMMITMENT VS. NON-COMMITMENT

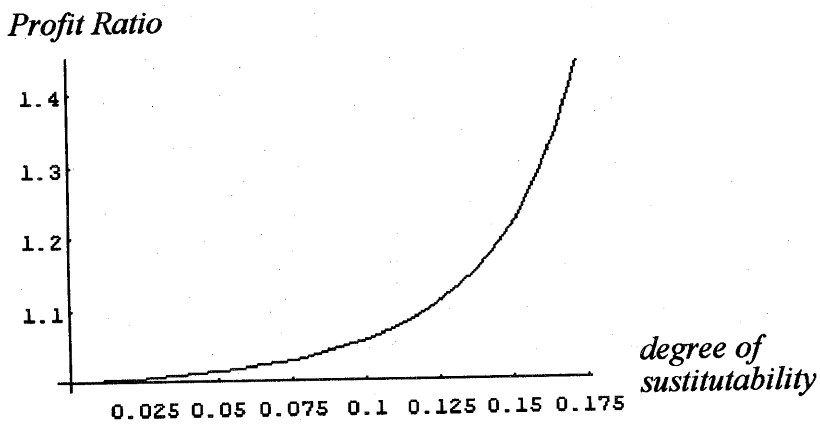

FIGURE 2
SUBSTITUTABILITY OF PRODUCTS AND GAINS FROM RIGIDITY

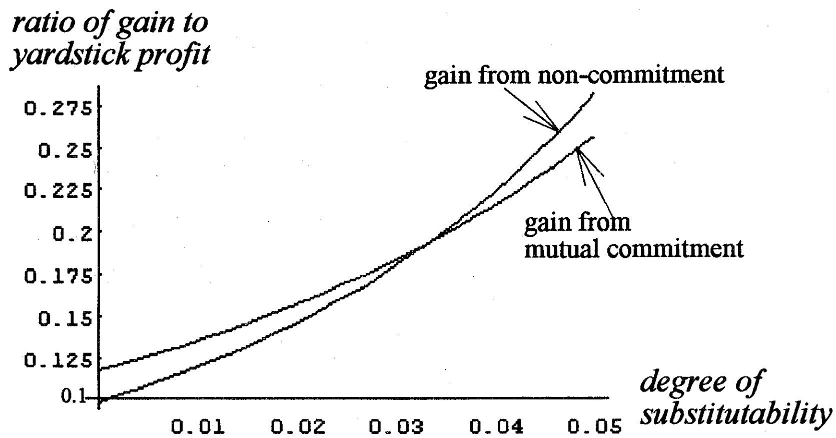

FIGURE 3
MUTUAL COMMITMENT
AS NON-COOPERATIVE EQUILIBRIUM