93-F-2

Bandwagon Effects and Long Run Technology Choice

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February 1993

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July 1992

This revision: February 1, 1993

Abstract

The present paper analyzes how a technological standard emerges in the long run, under the presence of bandwagon effects or network externalities. First, we present a set of axioms to capture what are popularly termed "bandwagon effects" in a general setting where there are more than two technological standards. The existing literature mainly assumes that the value of a technology is an increasing function of its own market share. However, the value is also affected by the composition of other technologies, because they bear various degrees of compatibility to the technology under consideration. Our formulation explicitly considers those cross effects. This enables us to examine the most likely way of dominating the market. A technology might dominate the market by directly increasing its share (direct domination), or it may let an intermediate technology dominate first and then overtake the market from it (indirect domination). We provide conditions to determine which way is more likely. Then we examine the long run technology choice according to the stochastic evolutionary game theory, which determines the unique outcome even when multiple equilibria exist. In particular, we show that pairwise risk dominance determines the long run standard, when bandwagon effects are strong enough.

1. Introduction

In this paper we examine the emergence of a technological standard when the value of a technology to any one user depends on how many others are adopting it, or in other words, when bandwagon effects are present. For instance, which video recording system or which disk operating system will emerge in the long-run? This subject has been much in vogue recently, and one major conclusion reached by several investigators is that the long-run standard is indeterminate. A host of papers (see Brian Arthur (1989) or Paul David (1985)) argue that the long run standard is determined by a series of historical events, which drift the population of users into a specific technology which, by virtue of its stand-alone value, may not have any intrinsic merit. Instead, its only reason for preeminence is extrinsic and ex-post, i.e., the fact that many people have already chosen it. Another branch of the literature stresses the role of multiple, self-fulfilling expectations (see Katz and Shapiro (1985, 1986, 1992) or Matsuyama (1991)). According to that approach once individuals believe that a critical mass will lock into a particular technology, it is in their best-interest to choose the same technology as well, leading to multiple consistent beliefs and, hence, to multiple equilibria. Therefore, the technological standard is indeterminate and, in particular, not related to the underlaying characteristics of the set of technologies that are ex-ante available.

The approach we take in this paper enables us to isolate a particular equilibrium, even when the static game of technology choices possesses multiple strict Nash equilibria, as in the formulations mentioned above. This is made possible by incorporating two features of the decentralized process of technology adoptions. First, we consider an explicitly dynamic formulation where technological choices are spread over time, and where individuals' decisions may depend on the pattern of technological adoptions present in the population at the time they choose a technology. For instance, in the case of the home videogame industry an individual's decision comes up for renewal only every once in a while (depending on the age of her existing base unit (if any), or on the age of her children), and at that time her choice (eg, Atari vs. Nintendo) may depend on how many other individuals are presently attached to each system, and therefore, on the size of the "library" associated with each system. A second feature we incorporate into the formulation is a small degree of randomness over the choice of technology, i.e., we allow different individuals facing the same situation to make different technological decisions. More explicitly, we hypothesize that most individuals will pick a (myopic) best-response given the present pattern of choices by others, but some will deviate and pick a non best - response. The basic reason for this is that some individuals are poorly informed about the present pattern of choices in the society (perhaps because this is the first time that they subscribe to a technology), or that it is very costly for them to collect such information, or that different individuals hold different expectations about the future evolution of technology choices, leading them to divergent decisions. Once these hypotheses are explicitly incorporated into the formulation we develop a user side theory of technological adoptions, using the stochastic evolutionary approach developed in our earlier paper (see Kandori, Mailath and Rob (1993) and Kandori and Rob (1991)). This approach examines the long run average behavior in a relatively small population, where its technological standard does fluctuate. For example, the dominant computer in a department of economics may change over time, and the present paper predicts which one the department uses most often in the long run.

The first step in applying this theory is to define the relevant class of games. All previous literatures consider special cases of such games, eg, games where individuals' choice is binary (Betamax vs. VHS) or games where individuals' willingness to pay is addictively separable in the "stand-alone" value and the size of the network. In practice, however, one often encounters cases where more than two technologies compete for market dominance, eg, MS/DOS, OS/2, and UNIX in the case of PC operating systems. In such situations, the pattern of compatibilities across the various pairs of technologies is much richer. For instance, in the case of PC operating systems that it is a well-known fact that only a fraction of the set of software packages designed for one operating system will be compatible with another, and that this fraction is different across different pairs of operating systems. Consequently, the value of a system to an individual will be affected by the whole structure of cross effects, i.e., it will depend on the entire configuration of market shares, and not just on how many other individuals are in his own network. Given these facts we introduce a more general definition of games with bandwagon effects which takes account of this global information.

A particularly interesting issue when there are more than two technologies is the comparison of direct and indirect domination of the market. Consider, for example, a market dominated by IBM computers, and examine how Next might capture the market. If IBM and Next are the only available technologies, the only way for Next to dominate the market is to capture directly a critical mass of IBM users. However, when there is another computer, Apple for example, there is an indirect way of dominating the market. That is, Next might let Apple dominate the market first, and then capture the market from Apple. One example of indirect domination can be found in the history of politics. When the Nazi party seized power in Germany, it first let the communists undermine Weimar. When the communist party became sufficiently strong and people started to worry, the Nazis won over and captured the power[1]. Another example is in the area of medical equipment where CT-scanners had taken a large market share from the traditional X-ray equipment, but were later replaced by the even more advanced MRI technology. We examine in this paper the basic elements determining which way, direct or indirect domination, is more likely to succeed. Those two points, the definitions of bandwagon effects in general and the comparison of direct and indirect domination, should be of independent interest apart from their relevance to the long run evolution of technological standards.

Once we delineate the class of games that are of interest, we generate several predictions about their long-run equilibrium, i.e., about how technological standards are expected to evolve over a long time horizon. First, we identify the measure of <u>risk dominance</u> as the relevant criterion in selecting among multiple equilibria. This measure was originally introduced (see Harsanyi and Selten (1988)) for binary choices, and was designed to reflect the tradeoff between the potential payoff to a strategy (here a technology) and its inherent risk (or compatibility in the present context). In the case of choice between two technologies, it is already known that the risk dominant technology emerges in the long run (Kandori, Mailath and Rob (1993)). In the present paper, we show that this result can be extended to the multiple-technology case in that the system of <u>pairwise risk dominance</u> measures (between all pairs of technologies) determines the long run equilibrium[2]. One particular instance of this is, when one technology risk-dominates all others. In this case we show that this technology is the unique long-run equilibrium.

Second, we contrast the long-run equilibrium with the Pareto-superior technology. For the class of games we analyze here the two need not coincide, although there are special circumstances when they do, and we give sufficient conditions to ensure this. Several models (see Meyer, Milgrom and Roberts (1991) or Harsanyi and Selten (1988)) analyze properties of the Pareto-efficient equilibrium, <u>on the</u>

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[1] We thank Matsuyama Kiminori for suggesting this example.

[2]When bandwagon effects are weak (or nonexistent) there are examples showing that pairwise risk-dominance is <u>not</u> the relevant criterion. See Young (1993), and section 4 below.

assumption that this equilibrium is somehow going to emerge. Our analysis complements these models by providing a way of verifying whether such assumption is justified.

More generally, we show that this set of circumstances is special and that the equilibrium which emerges in the long-run depends on the tradeoff between the technological advantage of a technology (which is manifested when everyone adopts it), and the degree to which it is compatible with other technologies (which is manifested when different individuals are adopting different technologies). Given such tradeoff it is quite possible (and we illustrate this) that a Pareto inferior technology is sustained in the long-run simply because the ease of 'mixing and matching' it with other technologies supersedes its technological disadvantage. This result (and the intuition which underlies it) are especially relevant given case-study data (see Cowan (1990)) and recent experimental evidence (see Van Huyack, et. al (1990) and Cooper et. al (1990)) which shows that the actual choices of people in coordination games corresponds (at least in some instances) to Pareto inefficient equilibria. Our framework provides a way of interpreting such data.

Third, we provide a complete characterization of the long-run equilibrium when there are exactly three technologies. In that case the maximin criterion determines which of the equilibria is sustained in the long-run. Fourth, we illustrate our results by constructing a parametric example of bandwagon games. The construction is based on a direct specification of the 'primitives', i.e., quality/risk characteristics of a set of available technologies. These characteristics determine the payoff structure of the game, and we show what its long-run equilibrium is, and how it relates to the specified characteristics (eg, we illustrate how the interplay between quality and compatibility determines the long-run equilibrium). Finally, we show that the long-run equilibrium is generically unique in band wagon games.

The rest of the paper is organized as follows. In the next section we present the model, introduce the class of bandwagon games in general and a particular parametric example illustrating their structure. Section 3 lays out the mechanics of the long run evolution of technological standard. In section 4 we compare direct and indirect dominance, and then provide various characterizations of the long run equilibrium, including the pairwise risk dominance and maxmin criteria. Section 5 concludes.

2. The Bandwagon Effects

In this section we develop a demand-side theory of technological adoptions in which users of the technology are the only strategic players. To capture the idea that identical choices by two users generate 'positive feedbacks', we consider a population of individuals who are randomly matched to play a symmetric, two person game in which the coordination of strategies as beneficial. There are scenarios in which positive feedbacks come about exactly in this fashion, eg, two scientists collaborating on a paper and using the same word processor package, or two households with compatible base units exchanging software for a home videogame. In other cases the positive feedbacks are more indirect, and are channeled through the manufacturer who can offer the product at a lower price, or offer a wider variety of peripherals if the number of subscribers is sufficiently large. In the first case our formulation applies directly, while in the second it can be viewed as an approximation. Generalizations of the basic model to encompass a broader class of payoffs is a clear target for future work.

We start here by defining and analyzing properties or the underlying game. The set of pure strategies of the game is denoted $N=\{1,2,...,n\}$. A player's payoff, when she and her match take strategies i and j respectively, is represented by u_{ij}. Let A be the set of mixed strategies (the n-1 dimensional simplex), and the carrier of a mixed strategy $a \in A$ is denoted $C(a)=\{i \in N \mid a_i > 0\}$. The set of pure strategy best responses against mixed strategy a is denoted BR(a). We consider the following three properties, which formalize mathematically what are popularly termed bandwagon effects:

(1) Total Bandwagon Property (TBP): For any $a \in A$, $BR(a) \subseteq C(a)$.

To interpret this condition, consider the situation where the distribution of the strategies in the society is given by a. The total bandwagon property says that the optimal strategy is one of the existing strategies in the society. For example, if each strategy represents a choice of technology, TBP says that the network externality is sufficiently strong that it always pays to adopt one of the existing technologies. Note that TBP in particular implies that the situation where everybody is using the same technology is always a Nash equilibrium: in terms of the component game, each strategy $i \in N$ constitutes a symmetric Nash equilibrium.

(2) Marginal Bandwagon Property (MBP): For any distinct strategies i, j, and k, $u_{ii} - u_{ji} > u_{ik} - u_{jk}$.

The marginal bandwagon property captures another aspect of the network externality. Namely, it says that the advantage of technology i over j is maximized when all other users are using technology i. In a random matching situation, this implies that the marginal gain of switching to technology i is an increasing function of the number of technology i-users in the society. As we will see in detail below, TBP and MBP are not nested assumptions.

The last property is somewhat subtler than the first two. To state it, we need a bit more notation. Under TBP, for any subset of strategies $S \subseteq N$, there is a unique mixed strategy equilibrium which is completely mixed in S. We denote it by m(S). We will sometimes denote m({i,...,j}) by m(i,..., j) for simplicity.

(3) Monotone Share Property (MSP): A game with TBP satisfies MSP when the following is true. Let S and S′ be subsets of strategies such that $S' \subsetneq S$. Then,

$$m_k(S) < m_k(S') \text{ for all } k \in S'.$$

The mixed strategy equilibrium m(S) can be regarded as the equilibrium market shares among technologies in the set S. When one takes this interpretation, the monotone share property captures the bandwagon effect in the following sense: when a technology exits from the market, each of the remaining technologies gets a larger market share. Although this condition may be the least intuitive, we show below that it is closely related to MBP. Moreover, it plays an important role in showing that direct domination is easier than indirect domination.

Proposition 1. Suppose TBP is satisfied. Then, MSP implies MBP. Furthermore, when $n=3$, MBP and MSP are equivalent.

Proof. Take any distinct strategies i, j, and k, and let $m=m(i, j)$ and $m'=m(i, j, k)$. Since the player is indifferent between i and j under m and m′, we have

$$u_{ii}m_i + u_{ij}m_j = u_{ji}m_i + u_{jj}m_j \tag{2.1}$$

$$u_{ii}m'_i + u_{ij}m'_j + u_{ik}m'_k = u_{ji}m'_i + u_{jj}m'_j + u_{jk}m'_k. \tag{2.2}$$

Subtracting (2.2) from (2.1) and rearranging terms yield

$$\begin{aligned} u_{ik}-u_{jk} &= (u_{ii}-u_{ji})\left(\frac{m_i-m_i'}{m_k'}\right) + (u_{ij}-u_{jj})\left(\frac{m_j-m_j'}{m_k'}\right) \\ &\equiv (u_{ii}-u_{ji})s + (u_{ij}-u_{jj})t. \end{aligned} \tag{2.3}$$

Note that $s+t=1$ because $m_i+m_j=m_i'+m_j'+m_k'=1$. Since MBP can be expressed as $u_{ii}-u_{ji} > u_{ik}-u_{jk} > u_{ij}-u_{jj}$, it is equivalent to $s,t>0$. (That is, $u_{ik}-u_{jk}$ being a convex combination of $u_{ii}-u_{ji}$ and $u_{ij}-u_{jj}$). But MSP implies $s,t>0$ by the definitions of s and t and, therefore, it implies MBP. Furthermore when $n=3$ the two are equivalent. ■

Let us illustrate these properties for 3x3 games, using the geometry of the 2-dimensional simplex. Figure 1 is an example where TBP is not satisfied. The numbers in parentheses indicate the best response for each region.

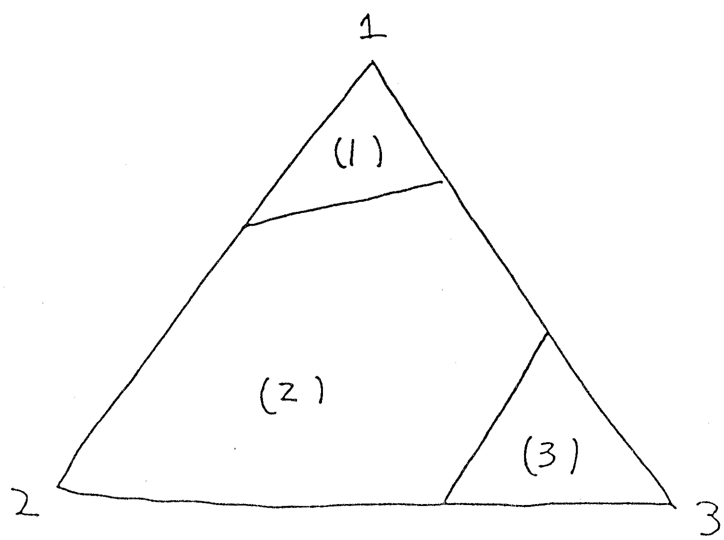

Figure 1.

TBP fails in this example because the best response region for strategy 2 contains a part of the edge between 1 and 3, where strategy 2 is assigned probability zero. The next example (Figure 2) satisfies TBP, but MBP (and hence MSP by Proposition 1) is violated.

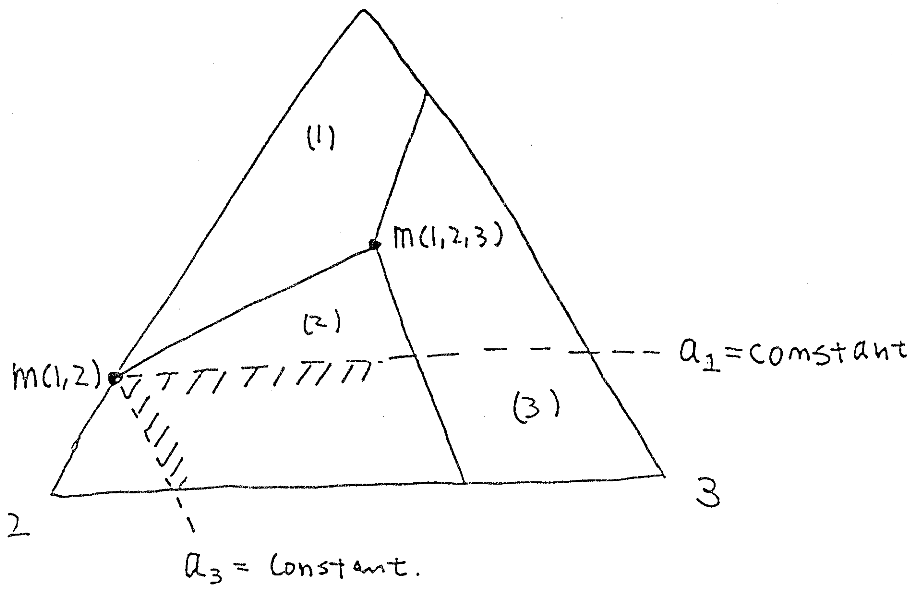

Figure 2.

MBP and MSP fail because the borderlines of best response regions do not have adequate angles. For example, one can see that $m_1(1, 2) < m_1(1, 2, 3)$, which violates MSP. For MSP to be satisfied, the borderline between the best response regions (1) and (2) must lie in the shaded region. Finally, Figure 3 shows an example satisfying all properties.

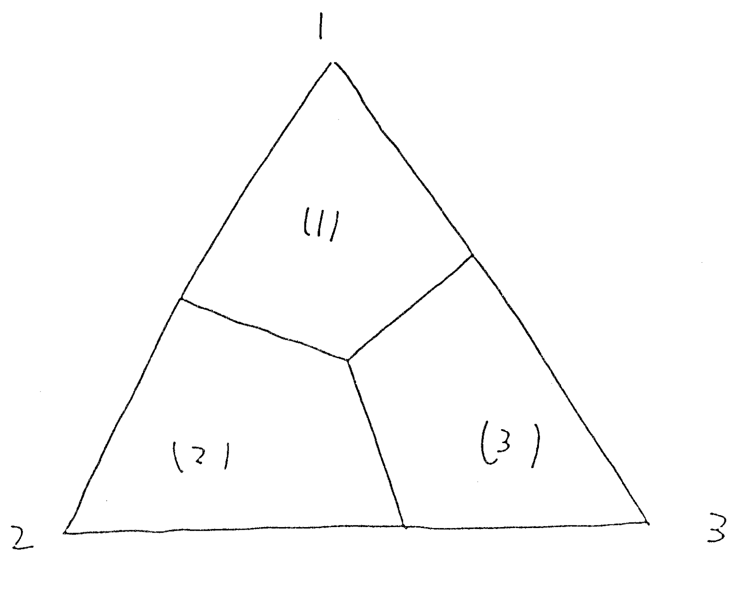

Figure 3.

We will now construct from primitives a game which satisfies all three "bandwagon" properties. Consider a scenario in which each individual can choose one among n different technologies available in the society. Let the inherent payoff (or quality) to technology i be q_i, and assume that this payoff is realized if two individuals with the same technology are matched. Otherwise, technology i user has to adapt to technology j user which costs her b_{ij} (technology - j user pays then b_{ji}). Assume for the moment that this cost depends only on the partner's technology, and let $c_j \equiv b_{ij}$[3]. Then the payoff is given by:

$$u_{ij} = \begin{cases} q_i & j=i \\ q_i - c_j & j \neq i. \end{cases} \tag{2.4}$$

We can readily check that for $i \neq k$

$$u_{ij} - u_{kj} = \begin{cases} q_i - q_k + c_i & j=i \\ q_i - q_k & j \neq i,k \\ q_i - q_k - c_k & j=k, \end{cases}$$

so that MBP is satisfied.

Let us now turn to TBP. Consider a mixed strategy $a \in A$. TBP is satisfied if for any $i \notin C(a)$, there exists a $j \in C(a)$ such that

$$\sum_{k \in C(a)} u_{ik} a_k < \sum_{k \in C(a)} u_{jk} a_k.$$

Since $i \notin C(a)$ and $j \in C(a)$, the left hand side of this inequality is $q_i - \Sigma_k c_k a_k$, and the right hand side is $q_j - \Sigma_{k \neq j} c_k a_k$. Hence TBP is equivalent to

$$\forall a \in A \ \forall i \notin C(a) \ \exists j \in C(a), \text{ so that } \ q_i - c_j a_j < q_j. \tag{2.5}$$

Condition (2.5) is vacuously satisfied if $a \in A$ is completely mixed. If $a \in A$ is not completely mixed, there

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[3]One instance of this is the ATM (Automatic teller machine) industry in which the customer of one bank (i) has to pay a 'switch fee' (c_j) to the bank which deployed the machine he is using. If the customer uses a machine deployed by his own bank this is an 'on us' transaction, and the fee is waived.

exists a j such that $a_j \geq 1/(n-1)$, where n is the number of strategies. So a sufficient condition for (2.5) is

$$\forall i,j: \ q_i - c_j/(n-1) < q_j. \tag{2.6}$$

Condition (2.6) is satisfied when the cost of adaptation is relatively large compared to the differences in the intrinsic qualities.

Lastly, we show that MSP is satisfied. Let S be a subset of strategies, and consider the equilibrium m(S), which is completely mixed in S. Then we have

$$\forall i,j \in S, \ \sum_{k \in S} u_{ik} m_k(S) = \sum_{k \in S} u_{jk} m_k(S).$$

By (2.4), this condition is equivalent to

$$\forall S \subset N \ \forall i,j \in S, \ q_i - c_j m_j(S) = q_j - c_i m_i(S). \tag{2.7}$$

Now consider $S^1 \supsetneq S^2$. Since $S^1 \supsetneq S^2$, there must be a $k \in S^2$ such that $m_k(S^2) > m_k(S^1)$. Then, applying (2.7) for $j = k$ and comparing across S^1 and S^2, we conclude that $m_i(S^2) > m_i(S^1)$ holds for all $i \in S^2$, which is nothing but MSP. We summarize the above results:

Proposition 2. Suppose the payoff function satisfies (2.4) and (2.6). Then, TBP and MSP (hence MBP) are satisfied.

In general, the cost of adaptation for a technology i-user to adopt technology j may depend on both i and j. In this case, the payoff function is given by

$$u_{ij} = q_i - b_{ij}, \ b_{ij} > 0 \text{ for } i \neq j \text{ and } b_{ii} = 0. \tag{2.8}$$

A special case which generalizes the above example is the case of additive adaptation cost: $b_{ij} = c_j + d_i$. By the same argument as above, one can verify MBP and MSP for this case. For TBP, we need a similar condition to (2.6),

$$\forall i,j \;\; q_i - d_i - q_j + (1-1/(n-1))d_j < c_j/(n-1), \tag{2.9}$$

which requires large cost of adaptation in terms of c_j. The proof that (2.9) is sufficient for TBP is similar and, therefore, omitted.

Lastly, we will provide a sufficient condition for TBP. This condition requires that: (1) The set of technologies be linearly ordered, closer technologies being more 'similar'. Denote the ordering by >. And, (2) compatibility increases at an increasing rate as products become more similar. Condition (2) is illustrated in figure 4, showing that it generates a 'one sided convex' payoff function.

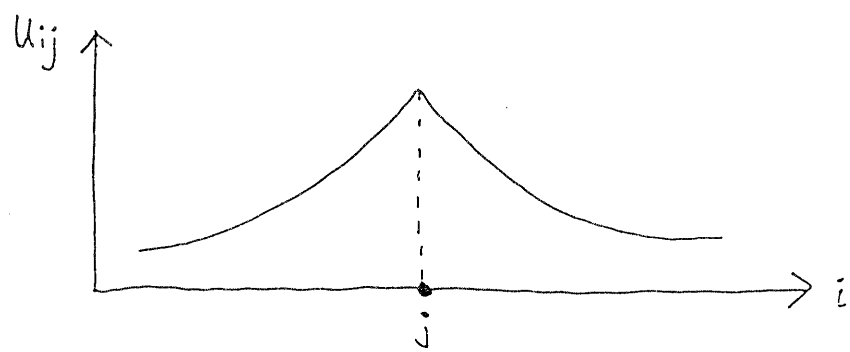

Figure 4.

Formally, we have the following.

Proposition 3. TBP is satisfied if

$$u_{i+1j} - u_{ij} > u_{ij} - u_{i-1j} > 0 \text{ for all } i < j, \text{ and}$$
$$0 < u_{ij} - u_{i+1j} < u_{i-1j} - u_{ij} \text{ for all } i > j. \tag{2.10}$$

Proof. Choose any mixed strategy a and consider its support C(a). We will show that for any $i \notin C(a)$ there is a mixture of strategies in C(a) which is a better response against a than strategy i. There are three possibilities to check.

(i) $i < \text{MinC}(a)$. Let $j = \text{MinC}(a)$. Since $u_{ik} < u_{jk}$ for all $k \in C(a)$ (see Figure 4), j is a better response than i against a.

(ii) $i > \text{MaxC}(a)$. Similar to case (i).

(iii) Otherwise. Let i′ and i″ be the closest strategies in C(a) to i ($i' < i < i''$). Consider the mixed strategy which assigns potability λ to i′ and $1-\lambda$ to i″, where $\lambda i' + (1-\lambda)i'' = i$. By the one-sided convexity of u_{ij} with respect to i (see Figure 4), we have $\lambda u_{i'j} + (1-\lambda)u_{i''j} > u_{ij}$ for all $j \leq i'$ and all $j \geq i''$. ■

3. The Long Run Evolution of the Technological Standard

In this section, we summarize the long run evolution model introduced by Kandori, Mailath and Rob (1993)[4], following the generalized formulation by Kandori and Rob (1992). We consider a population of M players, who are repeatedly and randomly matched to play the component game u_{ij} over an infinite time-horizon. For concreteness, the reader may imagine the faculty members in a department of economics, using different kinds of computers. The configuration of strategy-choices in the society is summarized by the <u>state-vector</u> z, whose i^{th} element, z_i, represents the number of players with strategy i. The <u>state space</u> is a finite set

$$Z \equiv \{(z_1,\ldots,z_n) \,|\, z_i \in \{0,1,\ldots,M\}, \sum_{i=1}^{n} z_i = M\}. \tag{3.1}$$

Under z, a player with strategy i receives the following expected payoff in random matchings:

$$\begin{aligned}\pi_i(z) &\equiv \frac{1}{M-1}[\sum_{j\neq i} z_j u_{ij} + (z_i - 1)u_{ii}] \\ &= \frac{1}{M-1}[\sum_{j=1}^{n} z_j u_{ij} - u_{ii}],\end{aligned}$$

where z_i-1 in the above expression comes from the fact that one cannot be matched with oneself.

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[4] A partial list of related literature includes the pioneering work of Foster and Young (1990) and important extensions by Ellison (1991), Fudenberg and Harris (1992), Nöldeke and Samuelson (1993) and Young (1993).

At each moment of time, the following events happen. First, each player exits from the society with probability ε. This event is independent across players and across time. If the player exits, a new comer (a newly recruited faculty member?) enters into the society. The new comer comes with her computer, which she has been using elsewhere. Or, she might buy a new computer, but we suppose that new comers are not always well informed about he configuration of computers in the population they enter into and, thus, may choose their computer on the basis of other considerations. Thus, we suppose that the probability that a new comer chooses computer i is strictly positive, denoted $m_i > 0$, for all i. This will introduce random flow of different computers into the department, and is called "mutation". If the player does not exit, she can potentially switch to a new computer. We suppose, however, that there is a switching cost and one may change computers only when the cost becomes sufficiently low. For example, one may switch to a new computer when her computer is broken or when she finishes writing a paper. We assume that the opportunity of adjustment arrives with probability η for each player, and that these events are independent across players and across time. Whenever this opportunity arrives we suppose that the player switches to the myopic best response. That is, the player assumes that the distribution of computers observed in the last period remains unchanged today and takes the best response against it. More precisely, let z be the strategy distribution in the last period and assume that the player took strategy i. The strategy distribution he expects to face, under the static expectation, is $\alpha(z,i)$ which is defined by

$$\alpha_j(z,i) = z_j/(M-1) \quad \text{for } j \neq i \text{ and } \alpha_i(z,i) = (z_i-1)/(M-1).$$

Then, the set of best response against this distribution is given by

$$\beta_i(z) \equiv \operatorname*{Argmax}_k \sum_{j=i}^{n} u_{kj}\alpha_j(z,i).$$

We assume that the player switches to an element of $\beta_i(z)$ if and only if $i \notin \beta_i(z)$[5]. This adaptive behavior, although naive, can be a completely rational one, if the adjustment speed is low (η being small) compared to the player's discount factor.

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[5] When there are multiple best responses, we assume that each one is chosen with a positive probability. It will turn out that our results do not depend on those probabilities.

Then we consider the behavior of the system when the mutation rate is much smaller than the rate of adjustment ($\epsilon < \eta$). To visualize the evolution in this case, let us intuitively examine what will happen when there are three computers: Apple, IBM and Next. If there are a large number of IBM users in the department, sooner or later everyone will end up using IBM by means of adaptive adjustments. After that, the department will be completely dominated by IBM for a while, but in the long run a significant number of new professors with Apple computers may enter into the department one after another. Once this happens, IBM will be driven out from the department by Apple, and the domination of Apple lasts for a while. This in turn may be broken by a large number of Next invasions, and so on. Therefore, the system fluctuates between different technological standards over a long time horizon. Then we can ask on average how much time the system will spend on each technology. As it will turn out, we can show that the system typically spends almost all the time on one technological standard, when the mutation rate is small. Figures 5 illustrate this in a stylized way. When the mutation rate ε is high, the system frequently fluctuates among three different regimes (Figure 5(a))[6]. As ε gets smaller, it becomes more difficult to upset each technological standard, so each regime expands (Figure 5(b)). However, one of the technologies, Apple for example, is typically more "stable" than others, and its regime expands much faster than others as ε becomes smaller (Figure 5(c)). We call such a technology, Apple in the above example, the long run equilibrium.

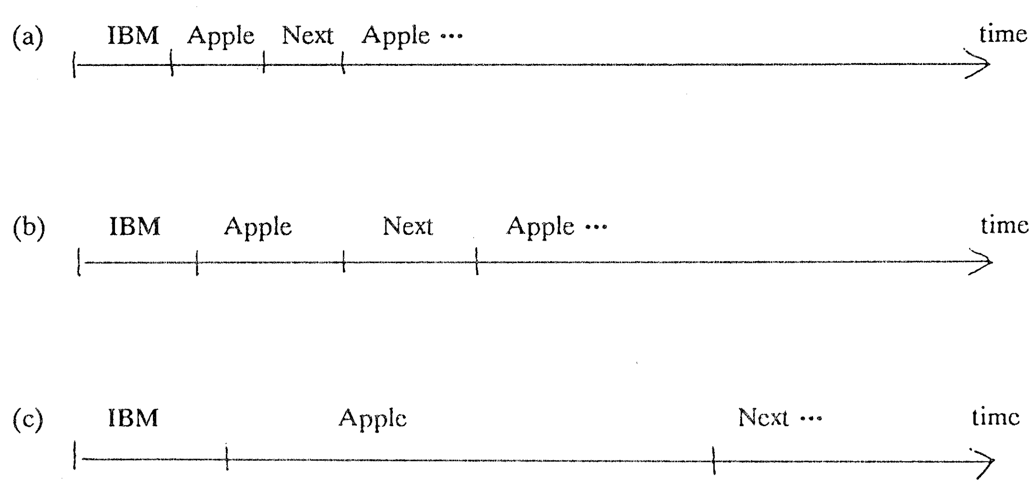

Figure 5.

[6] For simplicity, we do not denote transition periods, where different computers coexist.

As is clear from this description, the long run evolution model is most relevant for a society with a relatively small population. Otherwise, it takes a very long time to upset any given locally stable equilibrium. Nonetheless, our analysis applies to large populations in which each player is matched with a small number of close friends (or 'neighbors'). For more detail, see Ellison (1991).

The formal definition of long run equilibrium is as follows. The above setting defines a Markov chain on the finite state space, Z. That is, the distribution of tomorrow's state z' is completely determined by today's state. This is summarized by the transition matrix $P(\varepsilon) = (p_{zz'}(\varepsilon))$, where $p_{zz'}(\varepsilon)$ is the transition probability from state z to z' in one period. For $\varepsilon > 0$, the proportion of time spent on each state is represented by the stationary distribution $\mu(\varepsilon)$, which is uniquely determined by

$$\mu(\varepsilon)P(\varepsilon) = \mu(\varepsilon).$$

Then we consider the <u>limit distribution</u> $\mu^* = \lim_{\varepsilon \to 0}\mu(\varepsilon)$[7]. The set of <u>long run states</u> is the collection of states which receive positive probabilities under the limit distribution: $C(\mu^*) = \{z \mid \mu_z^* > 0\}$. Under the bandwagon assumptions, we will show that the set $C(\mu^*)$typically consists of one state were all players are using the same strategy (see Theorem 4). We call this state the <u>long run equilibrium</u>. In general, we use "equilibrium i" to denote a state where all players are using strategy i. (Mathematically, it is denoted by $e_i \equiv (0,...,0,M,0,...,0)$, where M is in the i^{th} element of the vector).

Having defined the basic solution concept, now we examine the basic nature of bandwagon games more closely. Let A_i be the <u>best response region</u> for strategy i: $A_i = \{a \in A \mid i \in BR(a)\}$. The <u>basin of attraction</u> for equilibrium i is defined as

$$BA_i = \{z \in Z \mid Pr(z(t) = e_i \mid z(0) = z, \varepsilon = 0) > 0 \text{ for some } t\}.$$

There are several differences between those two concepts. First, the best response region is a subset of the mixed strategy space (the n-1 dimensional simplex), while the basin of attraction is a subset of the state

[7] One of the striking features of the model is that the limit distribution does not depend such modeling details as the adjustment speed η, or the mutation distribution m. See Kandori and Rob (1991), Theorem 1.

space, which can be regarded as a set of grid points in the simplex. The second difference comes from the fact that players with different strategies face slightly different strategy distributions because of the finiteness of the population. The definition of the best response region does not capture this fact, but it is explicitly taken into account in the definition of the basin of attraction. Those two differences become inconsequential when the population size is large. There is, however, a third difference, which is more substantive. Consider Figure 6 below. The triangle can be regarded as the simplex of mixed strategies as well as the state space. Point x belongs to the best response region of e_2, but it is an element of both BA_2 and BA_1. This follows from the assumption of stochastic adjustment: the adaptive adjustment may lead to x′ or to x″, depending on who switches to strategy 2. This example shows one of the potential complications which do not arise in the 2x2 case, where the switched over players are uniquely defined. In higher dimension a cases, the basins of attraction may have substantial overlap, and they may have complicated shapes compared to the simple convex polyhedron structure of the best response region. Nonetheless, for the games we are analyzing here this complication does not occur because the MBP implies that the basin of attraction and the best response region coincide. This is shown in the next proposition.

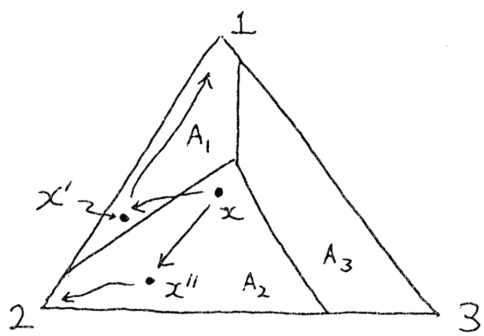

Figure 6.

Proposition 4. Under MBP, if $i \in \beta_i(z)$, then $\{i\} = \beta_j(z)$ for all $j \in C(z)$.

Remarks. (1) As is argued above, this implies that BA_i approximates A_i when the population size M is large. This is shown as follows. Take any state z in the interior of the best response region ($z/M \in \text{Int } A_i$). We argue that the adaptive dynamic starting from this point always converges to e_i. To this end, we show that any point along the adaptive adjustment, strategy i is the unique best response for all players. This is true for the initial point $z(0) = z$, when M is large enough. Take the next point z(1), and consider a hypothetical adjustment path leading to z(1), where one player adjusts at a time. In the first step, player with, say, strategy j adjusts to i. Since only one player is adjusting, strategy i is still a best response for this player in the next state. Then, Proposition 4 shows that i is the unique best response for all players. Repeating this argument proves that i is the best response for all players at z(1). Thus we conclude, by induction, that strategy i is the unique best response at z(2), z(3),....

(2) Proposition 4 also shows that the myopic adjustment always leads to a pure strategy Nash equilibrium. Take any state z(0). If all players are taking best responses, Proposition 4 shows that they have the same unique best response, so z(0) must be a pure strategy equilibrium. Otherwise, there is a positive probability that only one player adjusts to her best response. Then, as argued in remark (1) above, the process converges to a pure strategy equilibrium. Hence we have:

Corollary. Under the MBP, the system always converges to a pure strategy Nash equilibrium in the absence of mutations.

Proof of Proposition 4. Since the player with strategy i is optimizing, we have

$$\sum_{k\neq i} u_{ik}z_k + u_{ii}(z_i-1) \geq \sum_{k\neq i} u_{hk}z_k + u_{hi}(z_i-1), \tag{3.2}$$

for all h. For any other strategy $j \in C(z)$, MBP implies $u_{ii}-u_{hi} > u_{ij}-u_{hj}$. Rearranging this inequality as $u_{ii}-u_{ij} > u_{hi}-u_{hj}$ and adding this to inequality (3.2) yields

$$\sum_{k\neq j} u_{ik}z_k + u_{ij}(z_j-1) > \sum_{k\neq j} u_{hk}z_k + u_{hj}(z_j-1),$$

for all h. This shows that i is the best response for all players under z. ■

4. The Transition of Technological Standards

Next we consider how one technological standard might be upset. To this end, the following notion, called the <u>cost of transition</u> from equilibrium i to j (c_{ij}), plays a crucial role. The cost of transition c_{ij} represents the minimum number of mutations to change equilibrium i to j under the presence of adaptive adjustment. Formally it is determined as follows. First, we consider each path from equilibrium i to j in the state space Z, and count how many mutations occur on the path. Then we choose the path with the minimum number of mutations, and c_{ij} is the number of mutations associated with this most efficient path. When there are two technologies in the society, this cost represents the "critical mass" of individuals which is needed to convert equilibrium i to equilibrium j. That is, if c_{ij} individuals (or more) mutate from strategy i to strategy j, then the rest of the population will follow suit (following the adaptive adjustment), and the society will end up in equilibrium j. For example, if there are 100 individuals and if strategy 2 becomes the myopic best-response once more than 40% of the population is using it, then $c_{12} = 40$.

When there are more than two technologies, however, there are many ways of changing technological standards in the society. As is in the above example, a technology can dominate the market by directly steal the critical mass of customers from the prevailing technology. We call this <u>direct domination</u>. Or, it may let an intermediate technology invade first and then overtake the market form it. We call this <u>indirect domination</u>. An example of indirect domination is given by the payoff matrix

$$(u_{ij}) = \begin{pmatrix} 15 & 10 & 0 \\ 5 & 12 & 10 \\ 10 & 7 & 11 \end{pmatrix}.$$

In terms of the parametric model in section 2, this is given by inherent quality $q_1 = 15$, $q_2 = 12$, $q_3 = 11$, and the compatibility costs

$$(b_{ij}) = \begin{pmatrix} 0 & 5 & 15 \\ 7 & 0 & 2 \\ 1 & 4 & 0 \end{pmatrix}.$$

Figure 7 depicts the best response regions for this example.

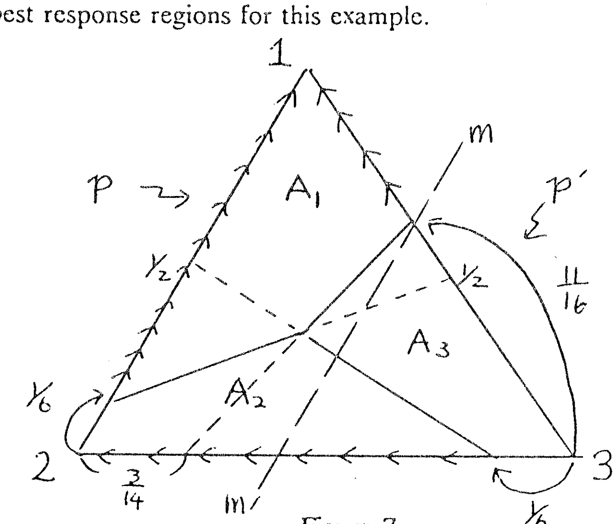

Figure 7.

Suppose all players are initially using technology 3 and examine how technology 1 can dominate the market. One way is 11/16 of the total population directly mutate from technology 3 to 1. This is indicated by "direct domination" path p in the figure. Note that it requires relatively large number of mutants, because of technology 1's poor compatibility to technology 3 (b_{13} being large). As is shown by the iso-mutation line mm', which is parallel to the side 12, this is the most efficient way to directly enter A_1 (the best response region of strategy 1). There are, however, another class of paths, which first enter A_2 and then reach A_1. Path p' is an example of such paths. On this path, technology 2 first dominate the market by directly converting 1/6 of the population towards technology 2. After technology 2 completely dominates the market, technology 1 steals 1/6 of its market share and finally capture the whole market. The total mutations on this "indirect domination" path is 1/3 of the population, which is smaller than the number of mutations associated with the direct domination path p (11/16). Hence the indirect domination is much more likely than the direct domination in this example. Furthermore, the indirect domination path p' turns out to be the most efficient way (in terms of the number of mutations) of converting equilibrium 1 to 3, and therefore c_{13} = M/3 (M is the total population). This is shown as follows. First, we have shown that p' is more efficient than any paths which directly enters A_1. So consider paths which enter A_1 through A_2, and call them indirect paths. It can be seen from the figure that entering into A_2 requires at least 1/6 mutants towards 2. By the same token, entering A_1 requires at least 1/6 mutants towards 1. Therefore, any indirect path must have at least 1/3 mutations, and this lower bound is achieved by path p'. This shows that p' has the least number of mutations among all paths.

As can be easily seen, this example satisfies the TBP property but not the MBP (so MSP is not satisfied either). The next proposition shows that the bandwagon conditions, TBP and MSP, ensure that the direct domination is the most efficient, and they enable us to use simple pairwise comparison of equilibria in much the same way as the risk dominance in 2x2 games.

Theorem 1. Under TBP and MSP, the minimum cost of transition c_{ij} is achieved by direct mutations from i to j, if the population size is sufficiently large. Namely, for large enough M,

$$\frac{c_{ij}}{M-1} \approx \frac{u_{ii}-u_{ji}}{(u_{ii}-u_{ji})+(u_{jj}-u_{ij})}.$$

Proof. We assume that the population size is large enough so that we can use the geometry of the mixed strategy space A as an approximation. In this proof, a state is regarded as a point in the simplex A. To determine c_{ij}, we need to count the minimum number of mutations when the state changes from z to z' in one step (namely, when $z(t)=z$ and $z(t+1)=z'$). We will call this the cost of immediate transition, and after normalization it is given by

$$c(z,z') = \sum_{k \notin BR(z)} (z_k' - z_k)_+,$$

where $(x)_+$ denotes max $\{x, 0\}$. Following Remark (1) of Proposition 4, we regard the best response region as the basin of attraction.

Consider any path $g=(z^0,...,z^T)$ where $z^0 = e_i$ and $z^T = e_j$. We consider two different cases. In case 1, the path goes through all the best response regions. Otherwise, we have case 2.

<u>Case 1</u>. Let c(g) be the cost associated with path g. Then we show the following:

$$c(g) \geq c(e_i, m(N)) \geq c(e_i, m(i, j)). \tag{4.1}$$

This says that any path which goes though all the best response regions is more costly than the direct jump to the completely mixed strategy equilibrium point, and the latter is in turn more costly than the direct jump from equilibrium i to j.

The first inequality in (4.1) is a consequence of TBP and MSP. Under TBP, best response region A_i is the convex hull of all the mixed strategy equilibria that assign positive probabilities to strategy i. Therefore, by MSP, we have

$$m_k(N) \leq a_k \text{ for all } k \text{ and all } a \in A_k. \tag{4.2}$$

Given this, the first inequality in (4.1) is shown as follows. Take any $k \neq i$ and consider the first point on g which lies in A_k. Inequality (4.2) shows that this point has at least $m_k(N)$ players with strategy k. Since strategy k has never been a best response on the previous part of the path, all those strategies must have been created by mutations. Since the path g goes through all the best response regions, this is true for all $k \neq i$, which shows the first inequality in (4.1).

The second inequality is also shown by TBP and MSP. Under TBP, each strategy is an equilibrium, and this implies that $c(e_i, m(N)) = 1-m_i(N)$ and $c(e_i, m(i, j)) = 1-m_i(i, j)$. Since $m_i(N) \leq m_i(i, j)$ by MSP, the second inequality in (4.1) is verified.

Case 2. Suppose path g, which starts with state e_i and ends with e_j, never passes the best response region of strategy h. In this case, we "project" the path $g=(z^0,...,z^T)$ by

$$x^t = z^t + \frac{z_h^t}{m_h}(m'-m),$$

where $m=m(N)$ and $m'=(N\backslash\{h\})$. Note that $x^t_h=0$ by construction and $x^t \in A$ (the nonnegativity of x^t is implied by MSP). Note also that $x^0=e_i$ and $x^T=e_j$. Furthermore, we assert $BR(z^t)=BR(x^t)$. The reason is twofold. First, $h \notin BR(z^t)$ by assumption and $h \notin BR(x^t)$ because of $x^t_h=0$ and TBP. Secondly, the choices between strategies $k,l \neq h$ do not change: that is, $(u_k-u_l)x^t=(u_k-u_l)z^t$, where $u_k=(u_{k1},...,u_{kn})$. This follows from $(u_k-u_l)(m'-m)=0$, which is implied by the definition of m′ and m.

Given that the projection preserves the best responses, we now show that the new path $(x^0,...,x^T)$ is less costly than the original path g. Namely, $c(x^t, x^{t+1}) \leq c(z^t, z^{t+1})$. This is shown as follows. First, observe that

$$c(x^t, x^{t+1}) = \sum_{k \notin BR(x^t)} (x_k^{t+1}-x_k^t)_+$$

$$= \sum_{\substack{k \notin BR(z^t) \\ k \neq h}} (x_k^{t+1}-x_k^t)_+ = \sum_{\substack{k \notin BR(z^t) \\ k \neq h}} ((z_k^{t+1}-z_k^t)+(\frac{z_h^{t+1}-z_h^t}{m_h})(m'_k-m_k))_+.$$

The second equality follows from $BR(x^t)=BR(z^t)$ and $x^t_h=0$ for all t, and the last one comes from the definition of x^t. Continuing the above equalities, we have

$$\leq \sum_{\substack{k \notin BR(z^t) \\ k \neq h}} (z_k^{t+1}-z_k^t)_+ + ((\frac{z_h^{t+1}-z_h^t}{m_h})(m'_k-m_k))_+$$

$$= \sum_{\substack{k \notin BR(z^t) \\ k \neq h}} (z_k^{t+1}-z_k^t)_+ + (\frac{z_h^{t+1}-z_h^t}{m_h})_+ \sum_{k \neq h} (m'_k-m_k).$$

The second inequality comes from $m'_k-m_k > 0$ by MSP. Since $\Sigma_{k \neq h}(m'_k-m_k) = 1-\Sigma_{k \neq h}m_k = m_h$, the last expression is equal to

$$\sum_{\substack{k \in BR(z^t) \\ k \neq h}} (z_k^{t+1} - z_k^t)_+ + \left(\frac{z_h^{t+1} - z_h^t}{m_h}\right)_+ m_h$$

$$= \sum_{k \in BR(z^t)} (z_k^{t+1} - z_k^t)_+ = c(z^t, z^{t+1}).$$

Hence we have shown that the new path is less costly.

If there is another strategy h′ which is never a best response on the new path, repeat the projection with respect to h′. Repeating this procedure, we get a path g′ with the following properties: (1) the path starts with e_i and ends with e_j, (2)it is less costly than the original path g, and (3) if we denote the set of strategies played on this path by J, the path g′ goes through all the best response regions A_k for $k \in J$. Then we can apply the same kind of reasoning as in Case 1 (replace m(N) with m(J)) to conclude that g′ (hence g) is more costly than the direct path (e_i, m(i, j), e_j). ■

Given Theorem 1, we are ready to determine the long run equilibrium. First, we define the notion of the pairwise risk dominance. Given two strategies i and j (each of them is an equilibrium under TBP), we say that i pairwise risk dominates j if

$$\frac{u_{ii} - u_{ji}}{(u_{ii} - u_{ji}) - (u_{ij} - u_{jj})} > \frac{1}{2},$$

which is equivalent to $u_{ii} - u_{ji} > u_{jj} - u_{ij}$. Note that the left hand side of the above inequality is the critical mass that technology j must steal from technology i, if there were no other potentially available technology in the society. This coincides with the definition of the risk dominance by Harsanyi and Selten (1988) for the hypothetical 2X2 game, where the strategy set is just {i, j}. Kandori, Mailath and Rob (1993) show that the long run equilibrium in a 2x2 game coincides with the risk dominant equilibrium. Young (1993), however, provides an example of a 3x3 game which suggests that the coincidence may not be generalized beyond 2x2 cases. The following is a slightly modified version of his example.

$$\begin{pmatrix} 6 & 0 & 0 \\ 5 & 7 & 5 \\ 4 & 5 & 8 \end{pmatrix}.$$

This example shows that an equilibrium which risk-dominates <u>every</u> other equilibrium need not be the long-run equilibrium. An important feature in this example, though, is that it does not satisfy TBP (see Figure 8), although it satisfies MBP.

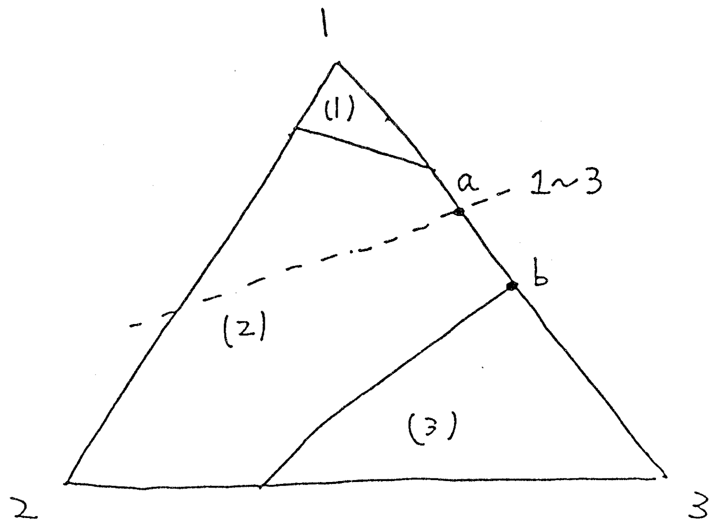

Figure 8

In this example, 3 pairwise risk dominates 1 and 2, but the long run equilibrium turns out to be 2. We can see why pairwise risk dominance fails to capture the stability of 3 against 1. The pairwise comparison of 1 and 3 indicates that a jump from 1 to point a (see the figure) is enough to make 3 a <u>better</u> response than 1. However, at point a, strategy 2, which is neglected in the pairwise comparison, becomes the best response, because the game does not satisfy TBP. Therefore, a much larger jump, from 1 to point b, is required, if we should convert equilibrium 1 to 3 by means of direct mutations. This example makes it clear that TBP is a necessary condition to make pairwise comparisons of equilibria the relevant criterion. It is, however, not a sufficient condition as we have seen in Figure 7. What goes wrong in Figure 7 is that it does not satisfy MSP. If TBP and MSP are both satisfied, we can utilize pairwise risk dominance.

Theorem 2. Suppose TBP and MSP hold. If strategy i pairwise risk dominates all other strategies, it is the unique long run equilibrium if the population size is large enough.

Proof. The long run equilibrium is computed as follows (See Kandori and Rob (1991) for the details). First, we must identify the candidates of long run behavior, called limit sets. Proposition 4 and TBP show that the collection of the limit sets consists of pure strategy equilibrium states, $\{\{e_1\},...,\{e_n\}\}$. Then we consider all directed trees defined over the limit sets (the n equilibria). Each branch of a tree is a directed pair of two equilibria (i,j), and the set of branches is directed into the root. The cost of a tree is defined to be the sum of c_{ij} over all its branches (i,j). Finally, the long run equilibrium is the root of the least cost tree.

Let h be the least cost tree, and assume k ($k \neq i$) is the root of the tree. This will lead to a contradiction. Construct a new tree h' by eliminating the outgoing branch from i and adding branch from k to i. Let l be the successor of i in the original tree h. Then, Theorem 1 and the assumption of pairwise risk dominance imply

$$c_{il}/(M-1) > 1/2 > c_{ki}/(M-1),$$

for large M. This shows that h' is less costly than h, which is a contradiction. ■

Theorem 2 provides only a sufficient condition. That is, there may be no strategy which pairwise risk dominates all other strategies. So we will examine when this sufficient condition is met. The first situation we consider is the case of symmetric adaptation costs.

Proposition 5. Suppose TBP and MSP are satisfied, and the payoff function is additive: $u_{ij} = q_i - b_{ij}$ ($b_{ii} = 0$). If the cost of adaptation is symmetric ($b_{ij} = b_{ji}$), the Pareto efficient technology is the unique long run equilibrium.

Proof. Under the symmetry, the pairwise risk dominance coincides with the Pareto dominance, because $u_{ii} - u_{ji} > u_{jj} - u_{ij}$ is equivalent to $q_i > q_j$. ■

A special case of this is the parametric example (2.4) where all adaptation costs are equal, $c_i = c_j$.

The second case where Theorem 2 applies is when technologies are linearly ordered according to inherent quality and compatibility. Namely, we assume the additive payoff function (2.4), and suppose that the quality and the cost curves have the following shapes.

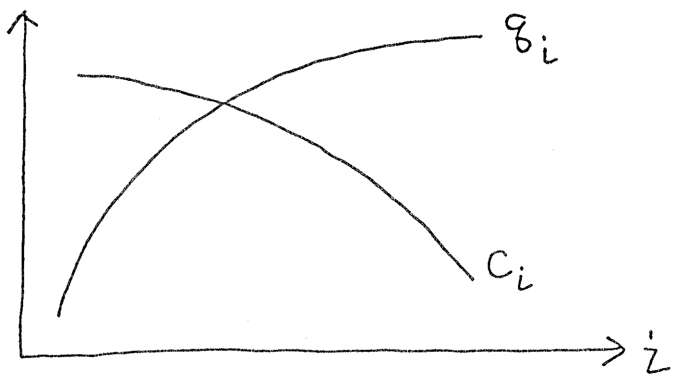

Figure 9.

This amounts to saying that a larger index i corresponds to higher quality but lower compatibility (that is, it costs more to be matched against other technologies on average), and that there are diminishing returns to scale in terms of quality and compatibility in appropriate senses. Formally, we assume

$$0 < q_{i+1} - q_i < q_{i-1} - q_i, \text{ and}$$
$$0 < c_{i-1} - c_i < c_i - c_{i+1} \quad \text{for all } 1 < i < n. \tag{4.3}$$

Then, we show that the technology that commands a balance between quality and compatibility emerges in the long run.

Proposition 6. Assume conditions (2.4), (2.6) and (4.3) are satisfied. Then, the unique long run equilibrium is the largest i such that $q_i - q_{i-1} > (c_{i-1} - c_i)/2$.

Proof. Let i be the strategy satisfying the above condition. By Proposition 2 and Theorem 2, we have only to show that i pairwise risk dominates all other strategies j, namely, $u_{ii} - u_{ji} > u_{jj} - u_{ij}$. A simple calculation shows that this is equivalent to $q_i - q_j > (c_j - c_i)/2$. For $j < i$, this is satisfied because

$$q_i - q_j = \sum_{j+1}^{i} (q_k - q_{k-1}) \geq \frac{1}{2}\sum_{j+1}^{i} (c_{k-1} - c_k) = (c_j - c_i)/2.$$

The case of $i < j$ is shown by a similar argument. ■

A special case of this is when $q_n - q_{n-1} > (c_{n-1} - c_n)/2$. In this case the Pareto-efficient technology is the long-run equilibrium.

Lastly, we provide the necessary and sufficient condition for the long run equilibrium, the maxmin criterion, when there are three technologies.

Theorem 3. If TBP and MSP are satisfied and $n = 3$, the long run equilibrium is given by $\text{ArgMax}_i(\text{Min}_{j\neq i} r_{ij})$ for sufficiently large M, where r_{ij} is given by

$$r_{ij} = \frac{u_{ii} - u_{ji}}{(u_{ii} - u_{ji}) + (u_{jj} - u_{ij})}.$$

Proof. Theorem 1 shows that $c_{ij} + c_{ji} \approx M-1$ for large M. Hence, if strategy i pairwise risk dominates all $k \neq i$, i achieves the maxmin criterion because $\text{Min}_{j\neq i} c_{ij} > (M-1)/2 > c_{ki} \geq \text{Min}_{j\neq k} c_{kj}$. By Theorem 2, the strategy i is indeed the long run equilibrium in this case. Next consider the case where no strategy pairwise risk dominates others. Define $r(i) \equiv \text{Min}_{j\neq i} r_{ij}$ and consider the case where $r(i)$, $i=1,2,3$, are all distinct. Without loss of generality, assume $r(1) < r(2) < r(3) \leq 1/2$, where the last inequality comes from the fact that no strategy pairwise risk dominates others. Next, let $\iota(i)$ be defined by $r_{i,\iota(i)} = r(i)$. Then, we claim $\iota(1) \neq \iota(2)$. Otherwise we would have $r(1) + r(2) = r_{12} + r_{21} = 1$ and $r(1) + r(2) < 1$ at the same time, which is a contradiction. Therefore, the combination of two branches, $(1, \iota(1))$ and $(2, \iota(2))$, forms a tree directed into root 1. Note that a tree is a collection of two branches whose roots are distinct. The tree we constructed utilizes the least cost branches whose roots are distinct, so it must be the optimal tree. Therefore, the long run equilibrium is 1, which satisfies the maxmin criterion. We can check that similar reasoning applies when there are ties in r_{ij}. ■

In all of the above cases the long-run equilibrium is uniquely determined. We now show that this situation is true in general.

Theorem 4. For a generic choice of payoff function u_{ij}, the long-run equilibrium is unique for large enough population M.

Proof. Assume that for some game there are two cost minimizing trees with two different roots, i and j. Let (j,k) be the branch which originates at j in the i-tree. Then we can lower the cost c_{jk} by increasing u_{kj}, leaving all other payoffs intact. This will decrease the cost of the i-tree, and will either leave the cost of the j-tree the same or will increase it (in case the branch (k,j) is part of the j-tree). It remains to be shown that the perturbed game satisfies TBP and MSP.

As to TBP, notice first that it is equivalent to $\operatorname{Max}_{r \neq s}(u_r - u_s)\, a > 0$, $a \in \Delta_s$, $s = 1, \ldots, n$, where Δ_s is face of the simplex on which $a_s = 0$. Since this condition is satisfied for the original game by assumption, since dot product is a continuous function and since each Δ_s is compact, there exists a $\delta > 0$ for which

$$\operatorname*{Max}_{r \neq s} (u_r - u_s)\, a \geq \delta > 0, \quad a \in \Delta_s, \quad s = 1, \ldots, n.$$

Now if $u'_{kj} - u_{kj} \leq \delta/2$ we have

$$\operatorname*{Max}_{r \neq s}(u'_r - u'_s)\, a \geq \operatorname*{Max}_{r \neq s}(u_r - u_s)a - \delta/2 \geq \delta/2 > 0,$$

where u'_{rh} denotes payoffs in the perturbed game. Therefore, the perturbed game satisfies TBP as well.

As to MSP, note that m(S) is a continuous function of the game's payoffs (being the solution to a system of linear equations). Let m'(S) denote the corresponding solution in the perturbed game, and let $\delta \equiv \operatorname{Min} \{m_k(S') - m_k(S) \mid S' \subset S, k \in S'\}$. Then there exists a $\delta' > 0$ so that if $u'_{kj} - u_{kj} < \delta'$, then

$$m_k(S) - m'_k(S) < \delta/3, \quad S \subseteq N.$$

Therefore, the MSP continues to hold for the perturbed game.

If there are more than one trees which tie with i, we can repeat the same argument for each such tree.■

5. Conclusion

This paper suggests a theory which pins down a unique ling-run standard in technology adoption games with multiple static (or even perfect-foresight) equilibria. The critical element which makes this selection feasible is that occurrence of abrupt transitions from one technological standard to another. The selection of ta technology is then determined by the relative ease of such transitions and this, in turn, depends on the inherent characteristics of the technologies. Therefore the theory here provides a link between the underlying properties of a technology and its chances for long-run survival.

By way of conclusion we suggest two avenues along which this line of attach can be extended. First we have confined ourselves here to the case where the benefit to each user is linear in the configuration of others' choices. In reality, however, there are cases with non-linearities -- either because of technological considerations (for instance, there might be large set-up costs to develop and design a new product but small cost of replicating it), or because of demand side considerations (the number of users with which any given user "communicates" is limited* or, the economic viability of applications and complementary product depends on the number of users, etc.) The treatment of such cases requires an analogous theory where each user's benefit is more generally (i.e. nonlinearly) specified.

A second interesting ramification is to consider firms' strategic behavior. Such behavior encompasses a wide spectrum: pricing over time, quality choice, degree of compatibility with others, vertical integration into (or from) complementary products, provision of product variety, date of new product introductions, licensing arrangements, and so on. We plan to analyze in the near future firms' strategic choice of direct vs indirect domination of the market by means of those rich menu of instruments. Ultimately it would be of interest to see to what extent the evolution of standards is driven by these factors as opposed to the demand side considerations on which our current approach here focuses.

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