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PRE-DETERMINED WAGES, DIFFERENTIAL PRICE FORECASTS, AND STRUCTURAL STABILITY OF AN ECONOMY

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ABSTRACT

One of perplexing observations is that some relations between key economic variables seem to be unstable over time. This paper explores the possibility of instability of wage equations under the predetermined-wage framework. If unions rely on their adviser's price forecast in determining their wages, there may be three possible configurations of equilibrium: unique flexible-price equilibrium; unique rigid-price equilibrium; and multiple equilibria. Moreover, the economy may be structurally unstable in the sense that a small change in the accuracy of the advisers' forecast may produces a jump in the sensitivity of the economy to demand and supply shocks.

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1. INTRODUCTION

One of most perplexing observations in the recent history of industrialized countries is that some of relations between key economic variables seem to be unstable over time. The well-known example is the Phillips curve. Seemingly stable relation between wage movements and the unemployment rate in 1960s became unstable in inflationary 1970s. An augmented version of the Phillips curve incorporating price expectations was relatively stable for some time, but it also became unstable in the midst of two oil crises. Then, a new variable such as oil prices was also augmented to the Phillips curve, and the curve seemed to be stable in 1980s.[1]

The Phillips curve is one of many examples of apparent instability in relations between economic variables. Equations in macroeconometric models are often altered because of "structural changes". Even if the structure of equations does not change, coefficients are very often "updated", keeping these equations in line with new data.

There may be several explanations of these phenomena. The first one, which many economists seem to entertain, is based on our insufficient knowledge about the economy. These equations are not stable over time, because we misspecify relations between economic variables. This line of argument implies that by accumulating knowledge about the economy, we will eventually come to equations which are stable and robust. On the other hand, the second explanation admits that the economy is actually subject to frequent structural changes. Because in most cases we cannot predict structural changes in advance, the best strategy to cope with the problem is to update equations whenever new information comes out.

However, there may be the third explanation, which emerged recently in the literature.[2] According to this explanation, the economy is inherently unstable. Even though underlying preferences and technology do not change very much, the behavior of economic variables may change drastically.

This paper explores the third explanation of apparent instability in economic relations. Specifically, I examine the possibility of instability in wage equations. I analyze the issue in a simple macroeconomic framework of monopolistically competitive firms and unions with predetermined wages, which has been extensively investigated in the last two decades. The new feature that I incorporate in this standard model is that unions rely on other economic agents' forecast about future prices in determining their wages. This small, but realistic change in the model may profoundly change the working of the economy. There may be three possible configurations of equilibrium: unique flexible-price equilibrium where unions rely heavily on other agents' forecast and that the economy is sensitive to economic shocks; unique rigid-price equilibrium where unions pay scant attention on the forecast and that prices are rigid to economic shocks; and multiple equilibria. Moreover, the economy may be structurally unstable in the sense that a small change in the accuracy of the forecast may produces a jump in the sensitivity of the economy to demand and supply shocks. These results stem from a particular kind of externality in expectation formation of unions.

In this economy, unions must determine their wages before they have perfect knowledge about the price level. It is well-known that this imperfect information makes prices rigid with respect to shocks in demand and supply.[3] However, I assume that each union has its own adviser, who supplies his forecast of the price level only to this union. Because union advisers' forecast contains information about the average price and ultimately information about the shocks, an increase in the confidence in these advisers' forecast increases flexibility of prices to the shocks.

Suppose that for some reason the other unions increase confidence in their advisers. This implies that prices become more flexible, so that the variance of the average price increases. Thus, for a given variance of forecast errors, the forecast of the average price supplied by the union's adviser becomes relatively more reliable, and thus the union increases its confidence in its adviser. In this case, the other unions' increase in their confidence in their adviser increases this union's confidence in its adviser, and the economy ends up with flexible-price equilibrium. Thus, externality in expectation formation plays a crucial role in this economy.

However, the same externality may work in the reverse way. A decrease in the confidence of the other unions may cause a decrease in the confidence of the union, and the economy may be "trapped" in a rigid-price equilibrium. In the in-between case, we have multiple equilibria having these two kinds of equilibrium at the same time. Whether the economy has flexible-price equilibrium, multiple equilibria, or rigid-price equilibrium depends on parameters of the model, especially the accuracy of union advisers' forecast. Thus, there are cases in which a small change in the accuracy moves the economy from flexible-price equilibrium to rigid-price equilibrium, and vice versa.

The plan of this paper is as follows. A reduced-form model with supply shocks is presented in Section 2, and the corresponding structural model is developed in APPENDIX A. In Section 3, I consider the simplest case in which unions try to keep their real wages constant no matter how they affect employment. If the technology of the economy is close to a constant-returns-to-scale one, we have unique flexible-price equilibrium, multiple equilibria, and unique rigid-price equilibrium, depending on the accuracy of forecasts. We also obtain structural instability of wage equations in this case.

In Section 4, the robustness of the result obtained in Section 3 is investigated. Even in the case that unions try to get the best combination of employment and real wages, I still have the same array of equilibria as in the case that unions try to fix real wages, so long as unions' marginal disutility of employment is not rapidly increasing. Moreover, the qualitatively the same result is obtained in the case of nominal demand shocks as in the supply shock cases. However, I show that if the economy exhibits rapidly decreasing returns to scale and/or marginal disutility of employment is rapidly increasing, then we have unique rigid-price equilibrium. Section 5 investigates the case where all unions have the same forecast of the price level. This is the case if all unions rely on the government's official forecast. Section 6 concludes the paper with remarks about future research.

2. THE MODEL

I consider the following simple two-equation reduced-form macroeconomic model, consisting of price and wage equations. All variables are in logarithm. (See the APPENDIX A for a microeconomic foundation of this model.)

The first equation is a price equation. There are n differentiated products in this economy, and each product is produced by one firm. Thus, firms are monopolistically competitive, and determines the price of their products. Here, I present the average price equation, which is the average of the individual price equation. In product-market equilibrium, the equilibrium average price is given by

$$\bar{p} = \frac{1}{1 + c_1}(\bar{w} + s) + \{1 - \frac{1}{1 + c_1}\}m. \tag{1}$$

Here $\bar{p}$ is the average price, $\bar{w}$ is the average wage, s is a productivity index representing the economy-wide productivity condition, and m is the money supply representing the economy-wide nominal demand condition. An increase in s implies an economy-wide decline in productivity, and thus puts upward pressure on the average price. An increase in m means an economy-wide increase in demand, which also raises the average price. The (equilibrium) average price is the weighted average of the cost condition (s and $\bar{w}$) and demand condition (m). The weight is determined by c_1, which is a parameter representing the degree of returns to scale. If firms' technology is close to constant returns to scale, then c_1 is close to zero. Then, (1) implies that the price is a mark-up over the cost condition, $\bar{w}$ + s, and that it is not influenced by the demand condition, m. In this paper, I assume that c_1 is positive (decreasing returns to scale).

The second equation is a wage equation. There are t differentiated labor inputs, and each labor input is controlled by one labor union. Thus, unions are monopolistically competitive, and determine the wage of their labor inputs. I assume that the number of unions (t) is so large that the effect of individual union's wage on the average wage is negligible. Thus, the union takes the average wage as given in the wage determination process.

Because I am concerned mostly with the wage decision of labor unions in this paper, I present here an <u>individual wage equation</u>, instead of the average wage equation. The j-th union's wage is determined by

$$(2)\qquad w_j - \bar{p} = z_1\{\frac{r}{1 + z_1 r}(\bar{w} - \bar{p}) + \frac{1 + c_1}{1 + z_1 r}(m - \bar{p}) - \frac{1}{1 + z_1 r}s\}.$$

Here z_1 represents the degree of increasing marginal disutility of employment, and determines the extent to which the union cares employment. I assume in this paper that $z_1 \geqq 0$.

In the literature of union preferences, it is often assumed that employment is indivisible and allocated randomly among members of the union.[4] Then, the j-th union's preferences are represented by expected utility of its members (expressed in level, not in log) such that $U_j = V(W_j/\bar{P})(L_j/L_j^*) + V'\{(L_j^* - L_j)/L_j^*\}$, where $V(W_j/\bar{P})$ is the utility of being employed in which $W_j/\bar{P}$ is the real wage, V' is that of being unemployed, L_j is the number of employed, and L_j^* is that of the j-th union's members (here all variables are in level, not in log.). In this case, the marginal utility of employment, $\partial U_j/\partial L_j$, is constant so that z_1 is zero.[5] As it is emphasized by McDonald and Solow (1981), if z_1 is zero, the optimal real wage of the union is constant so long as output is isoelastic with respect to employment (which I assume in the microfoundation model of APPENDIX A).

Thus, the union is indifferent with respect to employment fluctuations, and keeps the real wage constant regardless of resulting employment.

However, if there is work-sharing, z_1 is no longer constant. If work is shared equally by members of the union, then z_1 is equal to the degree of increasing marginal disutility of labor of its members. If z_1 is positive, then the union adjusts its real wage to the demand for its labor inputs. The demand for particular labor inputs depends on the real average wage, the real aggregate demand represented by the real balance, and the productivity condition.

If the real average wage ($\bar{w} - \bar{p}$) increases, the demand for the labor inputs that the union controls increases through substitution, so that the union increases its wage in order to trim the demand. This real-average-wage effect is represented by the first term in the wage equation. Here r represents the degree of substitutability among differentiated labor inputs, and it satisfies $r > 1$. Thus, the higher r is, the more the demand increases, so that the optimal real wage is higher.

Similarly, if the real aggregate demand represented by the real balance ($m - \bar{p}$) increases, the demand for labor increases so that the optimal wage increases. The more firms need labor inputs to produce their products (that is, the larger c_1 is), the higher the optimal real wage is. This real-aggregate-demand effect is the second term. By the same token, the more firms need labor inputs because of productivity decline (an increase in s), the higher the union's optimal real wage is. The third term in the wage equation represents this productivity effect.

To make analysis simple, I hereafter assume that the log of the money supply, m, is kept constant, and is equal to zero. All market participants know this, so that the only disturbance in this economy is a productivity shock, s.

Suppose for time being that labor unions have perfect information. Then, averaging (2) and incorporating it into (1), we obtain

$$\bar{p}_{PI} = \frac{1 - z_1}{c_1 + z_1 + c_1 z_1} s, \tag{3}$$

where $_{PI}$ denotes perfect information. Under the assumptions made so far ($c_1 > 0$ and $z_1 \geqq 0$), whether the perfect-information price increases or not when productivity declines depends on the value of z_1. A productivity decrease puts upward pressure on the price (see (1)), but lowers the wage because of resulting decline in employment (see (2)). If the former effect is larger than the latter effect (that is, if $1 > z_1$), then a productivity decline increases the equilibrium average price. However, if z_1 is sufficiently large ($1 < z_1$) that the latter wage-reducing effect dominates the former price-increasing effect, then a productivity decline decreases the equilibrium average price.

It is, however, unlikely that unions have perfect information about the economy (that is, $\bar{p}$, $\bar{w}$, and s) when they determine their wage. I am hereafter concerned with the case where unions do not have information about them. I retain in this framework the assumption that unions determine their wage and satisfy labor demand it creates. Thus, wages are predetermined, and firms are given the right to determine the level of employment.[6]

Although unions have no information of their own about the economy, there are various "advisers," who inform unions of their forecast of future economic variables. They may be consulting firms who have a contract with unions, or the research personnel within unions who supply their forecast to the union management. Some advisers supply their forecasts to all unions, and some are union-specific. I analyze the latter sort of advisers in this model. I will discuss later in Section 5 the effect of the former sort of advisers, those who supply their forecast to all unions.

Specifically, I assume that each union has one adviser who forecasts the average price $\bar{p}$. Let η_j is the forecast of the adviser attached to the j-th union. This forecast is private in the sense that only j-th union knows it. I assume that this forecast is unbiased, so that

$$(4)\qquad \eta_j = \bar{p} + x_j,$$

where x_j is the forecast error. For simplicity, I assume that the forecast error is symmetric and independent among advisers,[7] such that the distribution of x_j is normal with $E(x_j) = 0$ and $Var(x_j) = \sigma_x^2$. I assume that the number of unions, t, is so large that $\Sigma_{j=1}^{t} x_j = 0$ (by approximation).

In this paper, the forecast error of union advisers is assumed to be given to unions. This assumption implies that unions do not have information about the way union advisers forecast the average price, and that they only know the ex post performance of their advisers' forecast. This assumption is crucial in the following analysis.[8]

The union's optimal wage based on information η_j is from (2)

$$(5)\quad w_j = \frac{z_1 r}{1 + z_1 r} E(\bar{w}|\eta_j) + \frac{1 - z_1(1 + c_1)}{1 + z_1 r} E(\bar{p}|\eta_j) - \frac{z_1}{1 + z_1 r} E(s|\eta_j),$$

where $E(x|\eta_j)$ is the expectation of x (= $\bar{w}$, $\bar{p}$, s) conditional on information η_j.

Rational expectations equilibrium of this economy is defined in the following way.[9]

Rational Expectations Equilibrium

Rational expectations equilibrium of this economy is a triplet (θ_w, θ_p, θ_s) such that (i) $E(x|\eta_j) = \theta_x \eta_j$ for $x = \bar{w}$, $\bar{p}$, s, and (ii) the price equation (1), the individual wage equation (5), and the average wage relation

$$(6) \qquad \bar{w} = \Sigma_{j=1}^{t} w_j .$$

are simultaneously satisfied.

Computation of rational expectations equilibrium of this economy is explained in APPENDIX B. The APPENDIX also proves that, first, there exists at least one rational expectations equilibrium for any combination of (σ_x^2, σ_s^2, z_1, c_1, r) (PROPOSITION A.1), and that, second, we have $0 < \theta_p \leqq 1$ in equilibrium (PROPOSITION A.2). In the next section, I characterize rational expectations equilibrium.

3. MULTIPLE EQUILIBRIA AND STRUCTURAL INSTABILITY

Real-Wage-Keeping Unions and Near-Constant Returns

When unions' marginal disutility of employment is constant ($z_1 = 0$), then the model becomes simple and straightforward. If it is further assumed that firms' technology exhibits near-constant returns to scale (c_1 is small), we have an interesting array of equilibria: unique equilibrium with flexible prices, multiple equilibria, and unique equilibrium with rigid prices. In this section, I concentrate our attention in this simple case. In the next section, I investigate the case that c_1 is large and/or z_1 is positive, and I analyze how the basic results are modified in a different setting.

Let us first consider the case of perfect information as a frame of reference. If $z_1 = 0$, the individual wage equation is particularly simple. The union keeps the real wage constant (see (2)):

$$(7) \qquad w_j = \bar{p},$$

so that the average real wage is also constant: $\bar{w} = \bar{p}$. On the other hand, the average price is (see (1) and note that we have assumed that $m = 0$))

$$(8) \qquad \bar{p} = \frac{1}{1 + c_1}(\bar{w} + s).$$

Consequently, the equilibrium average price is from (3)

$$(9) \qquad \bar{p}_{PI} = \frac{1}{c_1}s.$$

Thus, the equilibrium price ($\bar{p}$) is sensitive to the productivity shock (s) when c_1 is close to zero (near-constant returns to scale).

The economy is characterized by wage-price spiral when $z_1 = 0$. Suppose that s is increased. In order to cope with a decrease in productivity, firms must increase their price in order to trim demand. This increases the average price. Then, since unions keep real wages constant, the average wage also increases by the same amount. This wage increase puts upward pressure on the average price, and the average price is increased. The process repeats itself until new equilibrium is reached.

How much a productivity decline increases the price and the wage depends on the degree of increasing marginal cost, c_1 (that is, decreasing returns to scale). Note that the optimal price for the firm is determined at the intersection of the marginal revenue curve and the marginal cost curve. An increase in the wage is the upward shift of the marginal cost curve. It is easily understood that the flatter the marginal cost curve is (that is, the smaller c_1 is), the larger the resulting price increase is. Consequently, the resulting increase in the price is larger when the marginal cost curve is flatter. Note that the more the price increases when the wage is increased, the higher the equilibrium price is. Thus, the equilibrium price is sensitive to a change in the productivity condition when c_1 is small.

In the case of imperfect information, the wage equation is

$$w_j = E(\bar{p}|\eta_j). \tag{10}$$

That is, the union tries to keep its real wage constant. Note that because $E(\bar{w}|\eta_j)$ and $E(s|\eta_j)$ do not appear in the individual wage equation, the rational expectation equilibrium is characterized only by θ_p. Rational expectations equilibrium is θ_p such that (1) $E(\bar{p}|\eta_j) = \theta_p \eta_j$, and (2) price equation (8), wage equation (10), and the average price formula (6) are simultaneously satisfied.

Let us now compute the rational expectations equilibrium. Consider the expectation formation process of the i-th union. The union solves the expectation formation problem using the undetermined-coefficient method.

Suppose that the other unions' expectations are $E(\bar{p}|\eta_j) = \theta_p \eta_j$, where θ_p is an undetermined coefficient. Then, by averaging the individual wage equation and using the law of large numbers, we have $\bar{w} = \theta_p \bar{p}$, so that from the price equation (8) we have

$$(11) \qquad \bar{p} = \{1 - \theta_p + c_1\}^{-1} s.$$

The above equation shows that, the larger the other unions' weight (θ_p) on their information (η_j) is, the more flexible the average price ($\bar{p}$) is. This dependence of the average price ($\bar{p}$) on the other unions' weight on the forecast (θ_p) is the primary source of the result obtained in this paper.

The i-th union's adviser provides the union with the average-price forecast such that $\eta_i = \bar{p} + x_i$ (see (4)). Consequently, by the linear least squares regression, we have

$$(12) \qquad E(\bar{p}|\eta_i) = [1 + \{1 - \theta_p + c_1\}^2 \{\sigma_x{}^2/\sigma_s{}^2\}]^{-1} \eta_i.$$

Let the union's best forecast of $\bar{p}$ be $\theta_p{}^* \eta_i$. Because $\theta_p{}^* \eta_i = E(\bar{p}|\eta_j)$, we obtain

(13) $$\theta_p^* = H(\theta_p; c_1, \sigma_x^2/\sigma_s^2) \equiv [1 + \{1 - \theta_p + c_1\}^2\{\sigma_x^2/\sigma_s^2\}]^{-1}.$$

Here we obtain important relationship between the union's weight on its adviser's average-price forecast and the other unions' weight on their adviser's forecast. If the other unions increase their weight, then the optimal weight for this union also increases. If the other firms' weight θ_p increases, the equilibrium price becomes more flexible ($\partial[\partial\bar{p}/\partial s]/\partial\theta_p > 0$). This implies that the variance of the average price increases relative to that of the forecast error, so that the adviser's average price forecast has more information about true $\bar{p}$. Consequently, the optimal weight on η_i increases ($\partial\theta_p^*/\partial\theta_p > 0$). This is the way externality in expectation formation operates when there is union advisers' forecast.

I hereafter call this property expectational complementarity. This is closely related with the concept of strategic complementarity.[10] Consider, for example, an oligopoly in which two firms compete with each other and where the price is a strategic variable (products are differentiated). If one firm's optimal price is positively related to the other firm's price, then the price is called a strategic complement. If we interpret θ_p as a "strategy" and take the monopolistically competitive setting into account, expectational complementarity is qualitatively the same as strategic complementarity.

In the oligopolistic setting just described, if the price is a strategic complement, then the reaction curve in the price-price plane is positively sloped. This often leads to multiple equilibria in the oligopolistic pricing game. By the same token, we may get multiple equilibria in the case of expectational complementarity.

Let us call $H(\theta_p; c_1, \sigma_x^2/\sigma_s^2)$ the <u>expectational reaction function</u>. Then, because of the symmetric assumption, the equilibrium value of θ_p is determined by $\theta_p = H(\theta_p; c_1, \sigma_x^2/\sigma_s^2)$. It is evident that $H(0; c_1, \sigma_x^2/\sigma_s^2) > 0$, $H(1; c_1, \sigma_x^2/\sigma_s^2) \leqq 1$, and H is continuous for $0 \leqq \theta_p \leqq 1$. Thus, there exists at least one equilibrium. In the remainder of this section, I examine equilibrium configuration of this economy.

<u>Equilibrium Configuration</u>

If c_1, the degree of decreasing returns to scale, is close to zero, we have unique rigid-price equilibrium for a large σ_x^2/σ_s^2, multiple equilibria for its medium value, and unique flexible-price equilibrium for its small value. Table 1 and Figures 1.1 through 1.4 depict numerical examples of these equilibria.

In this case, I set $c_1 = .05$.[11] If information is perfect, (3) shows that the average price is determined by $\bar{p} = 20s$. I use this perfect-information price as a frame of reference.

Let us take a look at Table 1. If σ_x^2/σ_s^2 is equal to 3, we have unique equilibrium, in which $\theta_p = 0.99$. In this case, the equilibrium price is determined by $\{1/(1 - 0.99 + 0.5)\}s$, which is approximately 16.6s. Thus, we have a very flexible equilibrium price. Consequently, the ratio of the standard deviation of the forecast error to that of the average price is 0.103, which implies relatively accurate forecast. This is the case of a "virtuous circle" of expectational inter-dependence. Because unions rely on advisers, advisers' forecast become accurate, so that unions rely more on advisers. Consequently, although advisers' forecast errors are non-negligible (in fact, σ_x^2 is three times as large as σ_s^2), the economy is close to perfect-information equilibrium. This case is shown in Figure 1.1.

A slight change in the variance ratio, however, drastically changes the picture. Table 1 shows that if σ_x^2/σ_s^2 is equal to 3.3, we have multiple equilibria, in which one equilibrium is close to the perfect-information equilibrium, but the other two are far from it. Figure 1.2 depicts these multiple equilibria.

Equation (13) shows that a slight increase in the variance ratio produces a small downward shift of $H(\theta_p)$. This does not change the characteristic of the far-right equilibrium. Thus, the near-perfect-information equilibrium is represented by $\theta_p = 0.99$. There is almost no change from the case of Figure 1.1, if we are concerned only with this equilibrium. However, the downward-shift of H produces two other equilibria, $\theta_p = 0.61$ and 0.51. In both cases, the sensitivity of the average price with respect to the productivity shock drops sharply from the perfect-information case. We here have a "vicious circle" of expectational inter-dependence. Because unions do not rely on advisers, prices become rigid, so that advisers' forecast error becomes large compared with the price variance. This means information produced by these advisers have small value, and thus justifies unions' initial distrust in their advisers.

A further decline of σ_x^2/σ_s^2 produces an interesting response of equilibria. Case 1.3 of Table 1 shows that when $\sigma_x^2/\sigma_s^2 = 5.2$, we still have near-perfect-information equilibrium, although the price sensitivity drops somewhat. However, the response of the two other equilibria to the change in the variance ratio is drastically different from each other. One equilibrium travels from the rigid-price zone to flexible-price zone. Thus, its value of $\theta_p = 0.93$, and the average price becomes more flexible. This is a perverse case because a decline of advisers' ability to forecast coincides with an increase in unions' "confidence" θ_p in their forecast. On the other hand, the remaining equilibrium shows a sharp decline in θ_p (= 0.22), and in the price sensitivity. Figure 1.3 shows how this decline in the variance ratio (a downward-shift of H) produces these results.

Case 1.4 of Table 1 shows that a still further decline of advisers' ability to forecast makes near-perfect-information equilibrium vanish, and we have only rigid-price equilibrium. When $\sigma_x^2/\sigma_s^2 = 7$, we have $\theta_p = 0.15$, and the insensitive average price (price sensitivity = 1.11). This case is depicted by Figure 1.4.

Hysteresis Criterion of Choosing Equilibrium and Structural Instability

I have shown that there exist multiple equilibria in the medium range of the variance ratio σ_x^2/σ_s^2. So far, I do not present any criterion to choose one among them. However, it is natural to assume that equilibrium changes continuously as far as possible, along with a change in a parameter. Thus, if there is a change in a parameter and that multiple equilibria exist in new equilibrium, I assume that the economy chooses one that is the nearest of the old equilibrium. This may be called as a hysteresis criterion.

Under this natural criterion, the economy moves in the following way as the variance ratio changes. Let us start from a very large ratio. Then, the economy is a rigid-price equilibrium as in Figure 1.4. As the ratio becomes smaller, the sensitivity of prices gradually increases. If the ratio is smaller than 5.2, there exist multiple equilibria. Then, under the hysteresis criterion, the economy chooses the least sensitive equilibrium $\theta_p = 0.22$, because it is the nearest of the old equilibrium. This continues until the variance ratio hits 3.3 and $\theta_p = .51$. Beyond that, there is a jump in θ_p, and a new θ_p is $\theta_p = .99$. Thus, a very small decrease in the variance ratio σ_x^2/σ_s^2 from 3.3 produces a quantum jump in the equilibrium value of θ_p (from .51 to .99). This implies that under the hysteresis criterion, the economy is structurally unstable in the sense that a very small change in a parameter (σ_x^2/σ_s^2 in this case) causes a large change in the behavior of the economy.

In addition to the structural instability, the equilibrium under the hysteresis criterion may exhibit asymmetric response to a change in the variance ratio. This can be easily understood if we increase, rather than decrease, the variance ratio from zero. The hysteresis criterion tells us the equilibrium is the near-perfect-information equilibrium until the ratio hits 5.2, and then the equilibrium jumps into the rigid-price region. Consequently, if the ratio is 3.3 and increasing, θ_p = .99 is the equilibrium. However, if the ratio is decreasing, θ_p = .51 is the equilibrium. Similarly, if the ratio is 5.2 and increasing, we have θ_p = .93, while if the ratio decreasing, we obtain θ_p = .22. This asymmetry is an important characteristic of equilibrium with union advisers under the hysteresis criterion.

It should be noted here that structural instability exhibited in this economy is the structural instability of the wage equation. Note that the wage equation is $w_j = \theta_p \eta_j$. I have shown that a small increase in the variance ratio from 3.3 causes a jump in θ_p from .51 to .99. This implies that the wage equation is generally unstable when σ_x^2/σ_s^2 fluctuates between, for example, 3 and 7.

4. EXTENSIONS

I have shown in the previous section that if unions try to keep their real wage constant ($z_1 = 0$) and that technology is close to constant returns to scale (c_1 is small), we have complex equilibrium configuration, and, under the hysteresis criterion of choosing unique equilibrium, structural instability. In this section, I investigate the robustness of the result with respect to variations in z_1 and c_1.

I first show that z_1 is positive but small (near-constant marginal disutility of employment), we still have qualitatively the same equilibrium configuration as in the case of the previous section. We have unique flexible-price equilibrium, multiple equilibria, and unique rigid-price equilibrium, depending on the magnitude of forecast errors. Thus, even though the union adjusts its real wage to employment conditions, the economy is still structurally unstable so long as the union puts more weight on maintaining its real wages than on maintaining employment. Then, I show that if c_1 is positive and large (rapidly decreasing returns to scale) and/or z_1 is positive and large (rapidly increasing marginal disutility of employment), then we have unique equilibrium. This unique equilibrium is characterized by rigid prices.

Finally, I investigate the case where we have nominal demand shocks. We show that basic result of this paper also holds for the case of nominal demand shocks.

<u>Unions Adjusting Its Real Wage to Employment Conditions, but Putting More Weight on Maintaining the Real Wage than on Maintaining Employment</u>

Consider the case of positive but small z_1. In this case, the union adjusts its real wage to employment conditions, but it puts more weight on maintaining the real wage than on maintaining employment (see (2)).

APPENDIX B shows that the expectational reaction function is in general dependent on z_1 (the degree of increasing marginal disutility of employment) and r (the degree of substitutability among labor inputs) in addition to c_1 and the variance ratio. We have (see (A36) in APPENDIX B)

$$(14)\ H(\theta_p;\ c_1,\ z_1,\ r,\ \sigma_x^2/\sigma_s^2) = \left[1 + \{\delta(\theta_p,\ z_1,\ c_1,\ r) + c_1\}^2\ \{\sigma_x^2/\sigma_s^2\}\right]^{-1},$$

where

$$\delta(\theta_p,\ z_1,\ c_1,\ r) = = 1 - \frac{1 - 2z_1(c_1 + 1)}{(1 + z_1 r) - (z_1 r + z_1)\theta_p}\cdot\theta_p.$$

The rational-expectations-equilibrium value of θ_p is determined by the equation $\theta_p = H(\theta_p;\ c_1,\ z_1,\ r,\ \sigma_x^2/\sigma_s^2)$. The rational-expectations-value of θ_w and θ_s is determined by θ_p (see APPENDIX B).

The formula of H (14) reveals that it is continuous with respect to z_1. Thus, we still have the same equilibrium configuration when z_1 is positive but small.

Case 1 of Table 2 and Figure 2 present a numerical example. In this example, the value of c_1 is the same as in Table 1 and Figure 1. We have multiple equilibria when z_1 = .011 and r = 5. From this result, it is easy to show that we obtain unique flexible-price equilibrium, multiple equilibria, and unique rigid-price equilibrium as σ_x^2/σ_s^2 increases from zero to infinity.

Rapidly Decreasing Returns to Scale and Unions Mostly Concerned with Maintaining Employment

Price equation (1) shows that if c_1 has a large positive value (rapidly decreasing returns to scale), then the average price is insensitive to the productivity shock. This is because the magnitude of wage-price spiral described in Section 3 is small when c_1 is large. Since the average price is insensitive to the shock, information contained in the adviser's forecast is not as important as in the flexible-price case. Thus, the economy ends up with rigid-price equilibrium, in which unions put a small weight on their adviser's forecast in their expectation formation. This case is depicted in Case 2 of Table 2 and Figure 3.

Qualitatively the same result is obtained in the case of large z_1 (see Case 3 of Table 2 and Figure 3). In this case, the union is concerned mostly with maintaining employment. If z_1 is large, a productivity decline (an increase in s) reduces the wage substantially (see (2)), which offsets an increase in the price directly stemmed from the productivity decline (see (1)). Thus, the price becomes rigid with respect to the productivity decline. Then, the same argument as in the case of large c_1 applies to this case, and the economy is in the rigid-price equilibrium.

Nominal Demand Shocks

Finally, let us consider the nominal demand shock. A closer look into the derivation of the expectational reaction function in Section 3 reveals that the existence of the nominal demand disturbance (the disturbance in the the money supply m in this economy) does not change at all the result of this paper.[12] Case 4 of Table 2 and Figure 5 illustrate an example. In this example, we have $z_1 = 0$, $c_1 = 0.01$, and $\sigma_x^2/\sigma_s^2 = 0.0004$, and we obtain multiple equilibria. The structural instability is then easily proved in this case.

Although we have qualitatively the same result in the nominal demand case, the range of the variance ratio that produces multiple equilibria is much narrower than in the case of productivity shocks and tilted toward zero. This is because the sensitivity of the price ($\bar{p}$) to the nominal demand shock (m) is at most equal to unity, while that to the supply shock (s) is $1/c_1$, which may be large if c_1 is small. Thus, in order that an information η_j is reliable information, the forecast error must be small.

5. THE PUBLIC FORECASTER

In the previous sections, I have investigated the effect of union advisers, the forecast of each of whom is private information known only to one union. In this section, I examine the case of a public forecaster whose forecast is known to all unions. Thus, this forecaster's forecast is common knowledge of all unions.

In order to examine the effect of the public forecaster, I return to the simplest model of Section 3. Price equation is (8), and individual wage equation is (10). Instead of assuming one union adviser for each union, I assume that there is one public forecaster who announces the following forecast of the average price:

$$\eta = \bar{p} + x, \tag{15}$$

where x is his forecast error, whose distribution is normal with $E(x) = 0$ and $Ex^2 = \sigma_x^2$.

Let the other unions' expectations be $E(\bar{p}|\eta) = \theta\eta$. Then, we obtain $w_j = \theta\eta$, so that $\bar{w} = \theta\eta$. Note that all unions demand the same wage, because information is homogeneous. Then, the average price is from (8)

$$\bar{p} = (1 + c_1)^{-1}(\theta\eta + s) = (1 + c_1)^{-1}\{\theta(\bar{p} + x) + s\},$$

which implies

$$\bar{p} = (1 + c_1 - \theta)^{-1}\{\theta x + s\}. \tag{16}$$

It should be noted here that in the case of the public forecaster, the average price depends on the public forecaster's forecast error as well as the productivity condition. (By contrast, in the union-adviser case of the previous sections, $\bar{p}$ depends only on s, because union advisers' forecast errors vanish in aggregation through the law of large numbers.) The dependence of the average price on the forecast error will be shown later to be the major factor differentiating the public-forecaster case from the union-adviser case. Specifically, an increase in θ increases the sensitivity of the average price to the forecast error x, as well as to the productivity shock s.

From (16) we obtain

$$\eta = \bar{p} + x = (1 + c_1 - \theta)^{-1}\{(1 + c_1)x + s\}$$

Consequently, the best estimate of $\bar{p}$ based on η is $\theta^*\eta$ where θ^* minimizes $E\{(\bar{p} - \theta\eta)^2\}$. Thus, we have

$$\text{(17)} \qquad \theta^* = \frac{E\bar{p}\eta}{E\eta^2} = \frac{E\bar{p}^2 + E\bar{p}x}{E\eta^2} = \frac{(1 + c_1)\theta\sigma_x^{\ 2} + \sigma_s^{\ 2}}{(1 + c_1)^2\sigma_x^{\ 2} + \sigma_s^{\ 2}}.$$

The equilibrium value of θ is obtained by setting $\theta^* = \theta$ in the above equation.

Equation (17) reveals that the same expectational complementarity is present in the case of the public forecaster. An increase in the other unions' weight θ on the public forecast increases this union's weight θ^* on the forecast. However, the magnitude of the complementarity is not strong enough to produce multiple equilibria in this case. It is evident from the above equation that equilibrium is unique in the case of public forecaster, in spite that there is expectational complementarity.

6. CONCLUDING REMARKS

I have shown in this paper that the existence of union advisers supplying their forecast to unions may profoundly change the working of the economy. In particular, it has been shown that if unions try to keep their real wage constant, the existence of union advisers lead the economy to an array of equilibria: unique flexible-price equilibrium, multiple equilibria, and unique rigid-price equilibrium. It is the magnitude of the forecast error that determines which equilibrium is realized. In addition, if we assume that the economy chooses from multiple equilibria unique equilibrium which is the closest to the old equilibrium before the change of conditions, we have structural instability: a small change in the forecast error variance causes a jump in the sensitivity of prices to the change in productivity.

This paper has revealed how externality in expectation formation lead us to the structural instability of wage equations. The key factor is expectational complementarity, in which one agent's weight on particular information is positively related to the other agents' weight on corresponding information. The existence of union advisers introduces expectational complementarity into the economy, and thus causes multiple equilibria and structural instability.

This paper, however, leaves several questions unanswered. First, I take union advisers' forecast-error-variance as given. Union advisers are also economic agents. Thus, they gather information and supply their forecast so long as such an activity is profitable. To gather information needs investment in physical capital (computer etc.) and human capital (training etc.). To analyze the behavior of union advisers and equilibrium of the market for information is an important subject of future research.

Second, I have assumed that union advisers' forecast is rational in the sense that their forecast is unbiased, and that unions form their expectations rationally. However, it is not certain that these assumptions are good approximation in the context of macroeconomics. It may be necessary to consider learning and expectational adjustment. In this line of work, multiplicity of equilibria may profoundly influence the process of learning. Moreover, the hysteresis criterion of choosing equilibrium may be more plausible in the context of learning.

NOTES

1. See Blanchard (1991) for an assessment of the history of the Phillips curve.

2. Gennotte and Leland (1990) is one example of this kind of approach in the field of finance.

3. See, for example, Nishimura (1989).

4. See Oswald (1985) for a survey of the literature.

5. This is the case so long as actual employment does not exceed the total number of the union's members.

6. This behavior is often found in industrialized economies. Thus, the predetermined-wage assumption is made in macroeconomics as a convenient short-cut assumption, although there has been a debate over whether this behavior can be explained as a rational behavior of labor unions and firms. See Nishimura (1992). Because the purpose of this paper is to investigate the working of the economy with private forecasters under realistic setting rather than to provide an explanation of the observed predetermined-wage behavior, we simply assume here that wages are predetermined and that employment is determined by firms.

7. The case of correlated forecast errors can be analyzed in the same framework as in the text. However, the result is qualitatively the same but notations become excessively messy.

8. Thus, I assume the "Nash behavior" of unions with respect to forecast errors of the union adviser.

9. It can be easily shown that rational expectations equilibrium defined below is the Bayesian Nash equilibrium of the wage determination game. See APPENDIX A, and also Nishimura (1992).

10. See, for example, Cooper and John (1988) for macroeconomic applications of this concept.

11. The choice of this particular value of c_1 is inconsequential. The term c_1 can take any value, so long as c_1 is positive but small.

12. In general cases reported in the APPENDIX, we must modify our analysis, taking account of the fact that the individual information η_j now has information about both the nominal shock (m) and the real shock (s). Although this makes analysis rather messy, the basic result does not change.

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APPENDIX A:

A MICROECONOMIC FOUNDATION

In this APPENDIX I prensent a microeconomic foundation of the two-equation reduced-form macroeconomic model in the text. The model I investigate in this APPENDIX is a version of monopolistically competitive macroeconomic models extensively investigated in the last decade (by, for example, Weitzman (1985), Blanchard and Kiyotaki (1987), and Nishimura (1989)). The economy consists of one representative consumer,[A1/] n firms, and t unions. Each firm produces a specific good that is an imperfect substitute for the other goods. It employs various labor inputs, each of which is an imperfect substitute for the other labor inputs. Each union controls the supply of one type of labor inputs. The household derives utility from the consumption of goods, liquidity services of real money balances, and leisure. The household gets initial money balances through transfer payments from the government. The household supplies labor to firms under unoins' control, and receives wages and dividends from firms.

The Sequence of Events

Before presenting the detail of the model, it is worthwhile to specify its sequence of events. There are three stages: the first is the wage-decison stage, the second is price-decision stage, and the third is the consumption-decision stage.

At the beginning of the first (wage-decision) stage, nature chooses a particular realization of a productivity disturbance common to all firms. The government then allocates money to the household through transfer payments.

At the first stage, unions determine their wage. At this stage, unions do not observe labor demand conditions (the pre-determined wage assumption).

Specifically, they do not know the money supply and the productivity disturbance.[A2/] However, unions have their own adiviser who supplies an estimate of the average price which will be determined in the second stage. Unions form rational expectations about their labor-demand conditions based on this information. Unions simultaneously choose their wages based on this imperfect information.

At the end of the first stage, all information is known to the public. At the second stage (price-decision stage), firms determine their prices simultaneously, taking wages as given. At the third (consumption-decision) stage, after all prices have been determined, the household decides how much to buy from each firm and places its orders. It determines consumption and the end-of-period real money holdings, taking prices, wages, dividends, and initial money holdings as given. Firms employ labor and produce the demanded quantities. Then, the household actually purchases goods from firms and consumes them, and firms pay wages and dividends to the household.

Unions and firms are assumed to be symmetric witin their own category. I am hereafter concerned with symmetric equilibrium.

In the model described below, I put normalization factors in the utility function and the production function so that equilibrium prices and wages are equal to unity when there is no disturbance in the economy. This no-disturbance case serves as a frame of reference in the following analysis.

A.1. CONSUMPTION DECISION

It convenient to analyze the economy backwards, from the third stage to the first.

<u>The Representative household</u>

The representative household gets utility from consumption of goods, consumption of liquidity services, and leisure. I assume

$$(A1) \qquad U = U(\bar{Y}, \frac{\tilde{M}}{\bar{P}}, L_1, \cdots, L_t) \equiv D(n\bar{Y})^{\zeta} (\frac{\tilde{M}}{\bar{P}})^{1-\zeta} - \gamma \cdot \Sigma_{j=1}^{t} L_j^{\mu},$$

where D is a normalization factor such that $D = \zeta^{-\zeta}(1 - \zeta)^{-(1-\zeta)}$; n is the number of goods (and the number of firms); $\bar{Y}$ is the average goods-consumption index defined below; $\tilde{M}$ represents the end-of-period nominal money holdings; and $\bar{P}$ is the price index associated with $\bar{Y}$, which is defined below. ζ is a parameter which satisfies $0 < \zeta < 1$. $\tilde{M}/\bar{P}$ are the end-of-period real balances. They are included in the total consumption index as a proxy of liquidity services that the real balances yield. L_j is the j-th labor input, and L_j^{μ} represents the disutility that comes from it. Thus, $\Sigma_{j=1}^{t} L_j^{\mu}$ is the total disutility of labor. γ is a normalization factor such that $\gamma = [(r - 1)/\mu r]\{\phi(k - 1)/k\}H]^{1-\mu}$, where r, ϕ, k, and H are parameters defined later in this section.

There are two possible interpretations of this model. Suppose that in this representative household, there are L_j^* members whose labor is of type j. In the first interpretation, employment is indivisible. In this case, L_j is the number of the representative household's members whose labor is of type j and who actually work. $L_j^* - L_j$ members do not work and enjoy leisure. In this case, $\mu = 1$ is a natural assumption. This case was analyzed in the implicit contract approach of 1970s, and in the labor-union literature of 1980s (see Blanchard and Fischer (1989; Chapter 9)).

In the second interpretation, employment is evenly ditributed among members. Then, L_j is work hours that each member of type j has to work. In this case, $1 < \mu$ is a natural assumption because it implies increasing marginal disutility of labor.

The average goods-consumption index $\bar{Y}$ is defined as follows:

$$\text{(A2)} \qquad \bar{Y} = \bar{Y}(\{Q_i\}:_{i = 1,\ldots,n}) \equiv \{(\Sigma_{i=1}^{n} Q_i^{(k-1)/k})/n\}^{k/(k-1)},$$

where Q_i is the consumption of the i-th product. The parameter k determines the degree of substitutability between goods, and satisfies $1 < k$.

$\bar{P}$ is the price index associated with the average goods-consumption index $\bar{Y}$:

$$\text{(A3)} \qquad \bar{P} = \bar{P}(\{P_i\}:_{i = 1,\ldots,n}) \equiv \{(\Sigma_{i=1}^{n} P_i^{1-k})/n\}^{1/(1-k)},$$

where P_i is the price of the i-th product.

The household's demand for each product and the demand for real balances are both derived from the maximization of Ψ with respect to Q_i and $\tilde{M}/\bar{P}$, subject to the following budget constraint:

$$\text{(A4)} \qquad \Sigma_{i=1}^{n} P_i Q_i + \tilde{M} = B,$$

where B is the beginning-of-period asset of the household.

Let us now consider B. The household obtains money from the government in the form of transfer payments, and wage payments and dividends from firms. Then,

$$\text{(A5)} \qquad B = \Sigma_{i=1}^{n} (\bar{P}\Lambda_i + \bar{P}\Pi_i) + M,$$

where Λ_i is the real wage payment, and Π_i the real dividend from the i-th firm. The beginning-of-period money holdings are equal to the money supply, M.

Demand Functions

Using the properties of the CES and Cobb-Douglas functions, we can derive the demand Q_i for the i-th product and the demand for real balances $\tilde{M}/\bar{P}$. They are

$$\text{(A6)} \qquad Q_i = \left(\frac{P_i}{\bar{P}}\right)^{-k} \bar{Y}, \text{ where } n\bar{Y} = \zeta\frac{B}{\bar{P}}, \text{ and } \frac{\tilde{M}}{\bar{P}} = (1 - \zeta)\frac{B}{\bar{P}}.$$

In order for the economy to be in monetary equilibrium at the third stage, the money demand must be equal to the money supply. Thus, the end-of-period money holdings must be equal to the beginning-of-period money holdings. That is,

$$\text{(A7)} \qquad \tilde{M} = M$$

must be satisfied. Because of (A6) and (A7), we obtain from the monetary equilibrium condition

$$\text{(A8)} \qquad \bar{Y} = H\,\frac{M}{\bar{P}}, \text{ where } H = \frac{\zeta}{1-\zeta}\,\frac{1}{n}.$$

Thus, in equilibrium, the average demand is proportional to the initial real money holdings.

The household's Utility in the Third-Stage Equilibrium

Substituting demand functions (A6) and (A8) into (A1), and using (A5), we obtain the household's utility in the third-stage equilibrium, such that

$$\text{(A9)} \qquad U \equiv \left(\frac{B}{\bar{P}}\right) - \Sigma_i L_i{}^{\mu} = \frac{1}{\bar{P}}\{\Sigma_{i=1}^{n} (\bar{P}\Lambda_i + \bar{P}\Pi_i) + M\} - \gamma\cdot(\Sigma_i L_i{}^{\mu}),$$

where L_i is determined by unions in the first stage.

A.2. PRICE DECISION

Let us now consider firms' decision in the second stage.

The Firm

I assume that the i-th firm needs t different labor inputs in order to produce its products. That is, the firm's production function is

$$\text{(A10)} \qquad Q_i = (\omega\cdot\frac{1}{S}\cdot t\bar{N}_i)^{\phi}$$

where ω is a normalization factor such that

$$\omega = \left[\frac{k}{\phi(k-1)}H^{(1/\phi)-1}\right].$$

I assume $\phi \leqq 1$ (non-increasing returns to scale).[A3/]

$\bar{N}_i$ is the average-labor-input index, which is defined as

$$\bar{N}_i = \bar{N}(\{N_{ij}\}:j=1, \cdots, t) \equiv \{(\Sigma_{j=1}^{t} N_{ij}{}^{(r-1)/r})/t\}^{r/(r-1)}.$$

Here N_{ij} is the j-th labor input of the i-th firm. The parameter satisfies $r > 1$. Thus, each labor input is an imperfect substitute of the other labor inputs, and r represents the degree of substitutability.

The firm's real profit is then

$$\text{(A11)} \qquad \Pi_i = \frac{1}{\bar{P}}\{P_i Q_i\} - \frac{1}{\bar{P}}\Sigma_{j=1}^{t} W_j N_{ij},$$

where the second term is the total real-wage payment. Here W_j is the wage of the j-th labor input.

Let us first consider the firm's cost minimization problem by taking Q_i as given. This yields

$$\text{(A12)} \qquad N_{ij} = \left(\frac{W_j}{\bar{W}}\right)^{-r}(\bar{N}_i) \quad \text{and} \quad \Sigma_{j=1}^{t} W_j N_{ij} = \bar{W} t \bar{N}_i = \bar{W}\omega' S(Q_i)^{1/\phi},$$

where $\omega' = \omega^{-1}$. Here $\bar{W}$ is the wage index corresponding to the labor-input index $\bar{N}_i$, such that

$$\text{(A13)} \qquad \bar{W} = \bar{W}(\{W_j\}:j=1,\cdots,t) = \{(\Sigma_{j=1}^{t} W_j^{(1-r)})/t\}^{1/(1-r)}.$$

Note that demand is still (A6). Consequently, the i-th firm's real profit is

$$\text{(A14)} \qquad \Pi_i = \Pi(P_i, \bar{P}, M, \bar{W}, S) \equiv H\left(\frac{P_i}{\bar{P}}\right)^{1-k}\left(\frac{M}{\bar{P}}\right) - \omega'\{S\frac{\bar{W}}{\bar{P}}\}\left(H\left(\frac{P_i}{\bar{P}}\right)^{-k}\left(\frac{M}{\bar{P}}\right)\right)^{1/\phi}.$$

Because the representative household is the sole stockholder of the firm, the firm maximizes the representative household's utility with respect to P_i. This turns out to be equal to maximizing the real profit (A14) with respect to P_i (see (A9)).[A4/]

Equilibrium Prices

I assume that the number of firms n is so large that the effect of the individual price on the average price is negligible. Thus, the firm takes the average price as given.

The symmetric equilibrum of this price game for given S, M and $\bar{W}$ is $\{P_i\}$ such that (i) P_i maximizes $\Pi(P_i, \bar{P}, M, \bar{W}, S)$ and (2) $P_i = P_j$ for $i \neq j$, and (3) the average-price formula (A3) is satisfied.

Because there is no uncertainty in the second stage, the optimal price is

$$\text{(A15)} \qquad P_i = \left[(\bar{P})^{1+c_1(k-1)} M^{c_1} (S\bar{W})\right]^{-(1+c_1 k)}, \text{ where } c_1 = (1/\phi) - 1.$$

Consequetly, the symmetric equilibrium average-price is

$$\text{(A16)} \qquad \bar{P} = \left[M^{c_1}(S\bar{W})\right]^{-(1+c_1)}.$$

A.3. WAGE DECISION

The Union

Let us look at labor demand. Let L_j be the demand for the j-th labor input, such that $L_j \equiv \Sigma_{i=1}^{t} N_{ij}$. Define the average-labor-demand index $\bar{L}$ such as

$$\text{(A17)} \qquad \bar{L} \equiv \frac{1}{t}\Sigma_{j=1}^{t}(\frac{W_j}{\bar{W}})L_j.$$

Then we obtain[A5/]

$$\text{(A18)} \qquad L_j = (\frac{W_j}{\bar{W}})^{-r}(\bar{L}) \text{ where } \bar{L} = \omega' S\{H(\frac{M}{\bar{P}})\}^{1/\phi}.$$

In a monopolistically competitive labor market, the supply of one type of labor is controlled by one union. The union controlling the j-th labor input sets the wage W_j in order to maximize the utility of the representative household. This turns out to be equal to maximize the following "union preference" function Φ_j (see (A9)):

$$\text{(A19)} \qquad \Phi_j \equiv \frac{1}{\bar{P}}W_jL_j - \gamma L_j^{\mu} = \Phi(W_j, \bar{W}, M, \bar{P}, S),$$

where

$$\Phi(W_j, \bar{W}, M, \bar{P}, S) \equiv \frac{W_j}{\bar{P}}(\frac{W_j}{\bar{W}})^{-r}\omega' S\{H(\frac{M}{\bar{P}})\}^{1/\phi} - \gamma\left[(\frac{W_j}{\bar{W}})^{-r}\omega' S\{H(\frac{M}{\bar{P}})\}^{1/\phi}\right]^{\mu}.$$

Equilibrium Wages

I assume that the number of unions t is so large that the effect of the individual wage on the average wage is negligible. Thus, the union takes the average wage as given.

Suppose for the time being that there is no productivity disturbance and the money supply is fixed so that M = S = 1, and that all unions know this. Then, unions can correctly infer $\bar{P}$ from (A16). Morever, unions can also infer $\bar{W}$ correctly because information unions possess is the same. Then, the symmetric equilibrum of this wage game for M = S = 1 is $\{W_j\}$ such that (i) W_j maximizes $\Phi(W_j, \bar{W}, M, \bar{P}, S)$ (2) $W_j = W_k$ for $j \neq k$, and (3) (A13) and (A16) are satisfied.

Then, the optimal wage is

$$\text{(A20)} \qquad W_j = \bar{P}\left[(\frac{\bar{W}}{\bar{P}})^{z_1 r}(\frac{M}{\bar{P}})^{z_1(1+c_1)}(S)^{-z_1}\right]^{-(1+z_1 r)}, \text{ where } z_1 = \mu - 1 \geqq 0.$$

Consequently, the equilibrium average wage is

$$\text{(A21)} \qquad \bar{W} = \bar{P}\left[(\frac{M}{\bar{P}})^{z_1(1+c_1)}(S)^{-z_1}\right].$$

It is evident from (A15), (A16), (A21) and A20), we have $P_i = \bar{P} = W_j = \bar{W} = 1$ in equilibrium when M = S = 1.

In this model, however, I assume that unions cannot observe M, $\bar{P}$, and S. Moreover, the union has its own adiviser who supplies the estimate of $\bar{P}$ such that

$$\text{(A22)} \qquad \exp(\eta_j) = \bar{P}\cdot\exp(x_j),$$

where x_j denotes this particular forecater's forecast errors, whose distribution is normal with mean zero and variance σ_x^2. The union forms rational expectations about $\bar{P}$, M, S and $\bar{W}$ based on this information η_j.

To make this rational expectation formation tractable, I use a log-quadratic approximation of the union's preference function (the second-order Tayler expansion of (A19) with respect to $\log\bar{P}$, $\log\bar{W}$, logM, and $\log W_j$) and a log-linear approximation of the average wage formula (the first-order Taylor expansion of (A13) with respect to $\log W_j$) around $\bar{P} = M = S = W_j = 1$. Let the lower-case variable be the log of the upper-case variable, that is, $\bar{p} = \log\bar{P}$, $w_j = \log W_j$, $\bar{w} = \log W_j$, and $m = \log M$. The latter approximation yields a very simple relation such that

(A23) $$\bar{w} = (1/t)\Sigma_{j=1}^{t} w_j.$$

In addition, we have from (A16) the following (exact) equilibrium price equaiton in the second-stage equilibrium

(A24) $$\bar{p} = \frac{1}{1 + c_1}\{\bar{w} + c_1 m + s\}.$$

Then, the symetric pure-strategy log-linear-policy Bayesian Nash Equilibrium of this wage determination game is a policy function $w_j^* = \xi_0 + \xi_1 \eta_j$ such that (1) w_j^* maximizes $E(\Phi^+(w_j, \bar{w}, m, \bar{p}, s)|\eta_j)$, where Φ^+ is the log-quadratic approximation of Φ, and (2) (A23) and (A24) are satisfied.

Note that the union's optimal wage formula based on the approximation is

(A25) $$w_j = \bar{p} + \frac{z_1 r}{1 + z_1 r}(\bar{w} - \bar{p}) + \frac{z_1(1 + c_1)}{1 + z_1 r}(m - \bar{p}) - \frac{z_1}{1 + z_1 r}s,$$

under perfect information in which $\bar{p}$, $\bar{w}$, m and s are known, and

(A26) $$w_j = E(\bar{p}|\eta_j) + \frac{z_1 r}{1 + z_1 r}\{E(\bar{w}|\eta_j) - E(\bar{p}|\eta_j)\}$$

$$+ \frac{z_1(1 + c_1)}{1 + z_1 r}\{E(m|\eta_j) - E(\bar{p}|\eta_j)\} - \frac{z_1}{1 + z_1 r}E(s|\eta_j),$$

under imperfect information where $\bar{p}$, $\bar{w}$, m and s are inferred from information η_j. Here $E(\bar{p}|\eta_j)$ is the expectation of $\bar{p}$ conditional on information η_j, and so on. Assuming that m and s are independently distributed normal random variables with $E(m) = E(s) = 0$; $Var(m) = \sigma_m^2$; and $Var(s) = \sigma_s^2$, we obtain the model described in the text.

APPENDIX B:

RATIONAL EXPECTATION FORMATION AND

EQUILIBRIUM DETEMINATION IN THE GENERAL CASE

Note that I assume $m \equiv 0$, so that there is no uncertainty about m. Let us consider rational expectation formation of one union.

Suppose that the other unions' expectations are: $E(\bar{p}|\eta_j) = \theta_p\eta_j$; $E(\bar{w}|\eta_j) = \theta_w\eta_j$; and $E(s|\eta_j) = \theta_s\eta_j$, for $j \neq i$, where θ_p, θ_w, and θ_s are undetermined coefficients. Because unions are symmetric except for η_j, the i-th union's rational expectations must be $E(\bar{p}|\eta_i) = \theta_p\eta_i$; $E(\bar{w}|\eta_i) = \theta_w\eta_i$; and $E(s|\eta_i) = \theta_s\eta_i$. Using this property, we determine rational expectations of the union.

Under the assumed expetations of the other unions' expectations, the other unions' individual wage equation is

$$w_j = \theta_p\eta_j + \frac{z_1 r}{1 + z_1 r}(\theta_w\eta_j - \theta_p\eta_j) + \frac{z_1(1 + c_1)}{1 + z_1 r}(- \theta_p\eta_j) - \frac{z_1}{1 + z_1 r}\theta_s\eta_j.$$

Because $\eta_j = \bar{p} + x_j$, the average wage $\bar{w} = (1/t)\Sigma_j w_j$ is

$$\text{(A27)} \qquad \bar{w} = \Theta\bar{p}, \text{ where } \Theta = \frac{z_1 r}{1 + z_1 r}\theta_w + \frac{1 - z_1(1 + c_1)}{1 + z_1 r}\theta_p - \frac{z_1}{1 + z_1 r}\theta_s.$$

where we utilize the fact that n is so large that we have $(1/n)\Sigma_j x_j = 0$ (by approximation) by the law of large numbers. Consequently, we have from the price equation (1), taking accout of the assumption that $m \equiv 0$,

$$\text{(A28)} \qquad \bar{p} = \{1 + c_1 - \Theta\}^{-1}s.$$

(A28) characterizes the equilibrium price in terms of θ_w, θ_p and θ_s (determinants of Θ). In the following, I first show that both θ_w and θ_s are a function of θ_p. By using this property, the firm's rational expectation formation is charaterized as a determination of θ_p.

Because of the price equation (1), the following relation must hold.

$$\text{(A29)} \qquad \theta_p \eta_j = E(\bar{p}|\eta_j) = \frac{1}{1 + c_1}\{E(\bar{w}|\eta_j) + E(s|\eta_j)\} = \frac{1}{1 + c_1}\{\theta_w \eta_j + \theta_s \eta_j\}.$$

From this we have

$$\text{(A30)} \qquad \theta_s = (1 + c_1)\theta_p - \theta_w.$$

Substituting this into Θ, we obtain

$$\Theta = (1 + z_1 r)^{-1}[\{z_1 r + z_1\}\theta_w + \{1 - 2z_1(c_1 + 1)\}\theta_p].$$

From (A27), $E(\bar{w}|\eta_j) = \Theta E(\bar{p}|\eta_j)$ must be satisfied. Thus, we get

$$\text{(A31)} \qquad \theta_w = \Theta\theta_p,$$

which implies

$$\text{(A32)} \qquad \theta_w = \frac{\{1 - 2z_1(c_1 + 1)\}\theta_p^2}{(1 + z_1 r) - (z_1 r + z_1)\theta_p}.$$

Let us define δ such that

(A33) $\delta(\theta_p, z_1, c_1, r) \equiv 1 - \Theta = 1 - (\theta_w/\theta_p)$.

Here the second equality is from (A31). Then, substituting this into (A28), we obtain

$$\text{(A34)} \qquad \bar{p} = \{\delta(\theta_p, z_1, c_1, r) + c_1\}^{-1}s.$$

Consequently, the union adiviser's average-price forecast η_i is

$$\eta_i = \bar{p} + x_i = \{\delta(\theta_p, z_1, c_1, r) + c_1\}^{-1}s + x_i.$$

Therefore, from the linear least squares regression, we have

$$E(\bar{p}|\eta_i) = \left[1 + \{\delta(\theta_p, z_1, c_1, r) + c_1\}^2 \{\sigma_x^2/\sigma_s^2\}\right]^{-1}\eta_i.$$

Because $E(\bar{p}|\eta_i) = \theta_p\eta_i$, we have the equation that determines the equilibrium value of θ_p such that

$$\text{(A35)} \qquad \theta_p = H(\theta_p; c_1, z_1, r, \sigma_x^2/\sigma_s^2),$$

where H is defined in the following way:

$$\text{(A36)} H(\theta_p; c_1, z_1, r, \sigma_x^2/\sigma_s^2) = \left[1 + \{\delta(\theta_p, z_1, c_1, r) + c_1\}^2 \{\sigma_x^2/\sigma_s^2\}\right]^{-1}.$$

(A35) determines the equilibrium value of θ_p as a function of σ_x^2/σ_s^2, z_1, c_1 and r: $\theta_p = f(\sigma_x^2/\sigma_s^2, z_1, c_1, r)$. Consequently, the equilibrium value of θ_w is determined by (A32) where θ_p is replaced by $f(\sigma_x^2/\sigma_s^2, z_1,$

c_1, r). Using these results, the equilibrium value of θ_s is determined by (A30). Finally, the equilibrium price is determined by (A28), which is

$$\bar{p} = \{1 - \frac{\{1 - 2z_1(c_1 + 1)\}f(\sigma_x^2/\sigma_s^2, z_1, c_1, r)}{(1 + z_1 r) - (z_1 r + z_1)f(\sigma_x^2/\sigma_s^2, z_1, c_1, r)} + c_1\}^{-1} s.$$

The next proposition establishes the existence of equilibrium.

PROPOSITION A.1

There exists at least one equilibrium for any combination of $(\sigma_x^2/\sigma_s^2, z_1, c_1, r)$ such that $\sigma_x^2/\sigma_s^2 \geqq 0$, $z_1 \geqq 0$, $c_1 \geqq 0$, and $r > 0$.

PROOF

If θ_p satisfying (A35) exists, then θ_w and θ_s are determined by (A32) and (A30). It is evident that by construction $(\theta_p, \theta_w, \theta_s)$ determined in this way are equilibrium values of expectations. Thus, it sufficient for the existence of equilibrium to show that there exists θ_p satisfying (A35) for any combination of $(\sigma_x^2/\sigma_s^2, z_1, c_1, r)$ such that $\sigma_x^2/\sigma_s^2 \geqq 0$, $z_1 \geqq 0$, $c_1 \geqq 0$, $r > 0$.

Note that the left-hand-side of (A35) is a function of θ_p. Let $H(\theta_p; z_1, c_1, r)$ denote this function. $H(\theta_p; z_1, c_1, r)$ is continuous with respect to θ_p for all θ_p. Moreover, we have $H(0; z_1, c_1, r) > 0$ and $H(1; z_1, c_1, r) \leqq 1$. Consequently, there exists at least one θ_p such that $\theta_p = H(\theta_p)$. Q.E.D.

NOTES TO APPENDIX

A1. This is assumption is made for notational simplicity. The model is the same as that of this APPENDIX if instead there are many identical households.

A2. This explains why we distinguish labor unions from households, and why we differ from other monopolistically competitive macroeconomic models (for example, Blanchard and Kiyotaki (1987)) in which households determine wages. If households determined wages, they whould know at least the money supply at the time of wage decision. We want to analyze the case where wage setters do not have perfect information about labor demand conditions including the money supply.

A3. Although I assume non-increasing returns to scale for simplicity, the model allows increasing returns to scale to a certain extent. That is, Φ may be greater than unity, but there should be an upper bound. I will specify the upper bound in the next footnote.

A4. Let me specify the upper bound for Φ in the case of increasing returns ($\Phi > 1$). Although Φ can be greater than unity, it must satisfy $1 + \{(1/\Phi) - 1\}k > 0$. This is necessary for profit maximization.

A5. Because by definition we have

$$t\bar{W}\bar{L} = \Sigma_{j=1}^{t} W_j L_j = \Sigma_{i=1}^{n} \Sigma_{j=1}^{t} W_j N_{ij},$$

the latter part of (A12) implies

$$\bar{L} = \omega' S' \frac{1}{t} \Sigma_{i=1}^{n} (Q_i)^{1/\Phi}.$$

Note that the union knows that in equilibrium $P_i = \bar{P}$ for all i. Thus, we have the formula in the text from the definition of L_j, (A17) and (A12).

TABLE 1

FORECAST ERROR VARIANCE AND RATIONAL EXPECTATIONS EQUILIBRIUM

σ_x^2	σ_s^2	c_1	θ_p	price sensitivity	$\sigma_x/\sigma_{\bar{p}}$
1.1. UNIQUE FLEXIBLE-PRICE EQUILIBRIUM					
3	1	0.05	0.99	16.666	0.103
			–		
			–		
1.2. MULTIPLE EQUILIBRIA: I					
3.3	1	0.05	0.99	16.666	0.108
			0.61	2.2727	0.799
			0.51	1.8518	0.980
1.3. MULTIPLE EQUILIBRIA: II					
5.2	1	0.05	0.96	11.111	0.205
			0.93	8.3333	0.273
			0.22	1.2048	1.892
1.4. UNIQUE RIGID-PRICE EQUILIBRIUM					
7	1	0.05	0.15	1.1111	2.381
			–		
			–		

Note: Price sensitivity to productivity shock is equal to 20 under perfect information.

TABLE 2

PARAMETER VALUES AND EQUILIBRIUM

σ_x^2	σ_s^2	c_1	z_1	r	θ_p	price sensitivity	$\sigma_x/\sigma_{\bar{p}}$	perfect-ifnfomation price sensitivity
1. Positive but small z_1								
3	1	0.05	0.011	5	0.98	12.0076	0.14434	16.0682
					0.64	2.31127	0.74939	16.0682
					0.55	1.91362	0.90511	16.0682
2. Large c_1								
3	1	3	0	5	0.03	0.25188	6.87624	0.33333
3. Large z_1								
3	1	0.05	3	5	0.21	0.87636	1.97640	-0.625

σ_x^2	σ_m^2	c_1	z_1	r	θ_p	price sensitivity	$\sigma_x/\sigma_{\bar{p}}$	perfect-ifnfomation price sensitivity
4. Nominal Demand Shock								
0.0004	1	0.01	0	5	1	100	0.02	100
					0.62	2.56410	0.78	100
					0.41	1.66666	1.2	100

Note: See text.

FIGURE 1.1 (σx2 = 3)
UNIQUE FLEXIBLE-PRICE EQUILIBRIUM

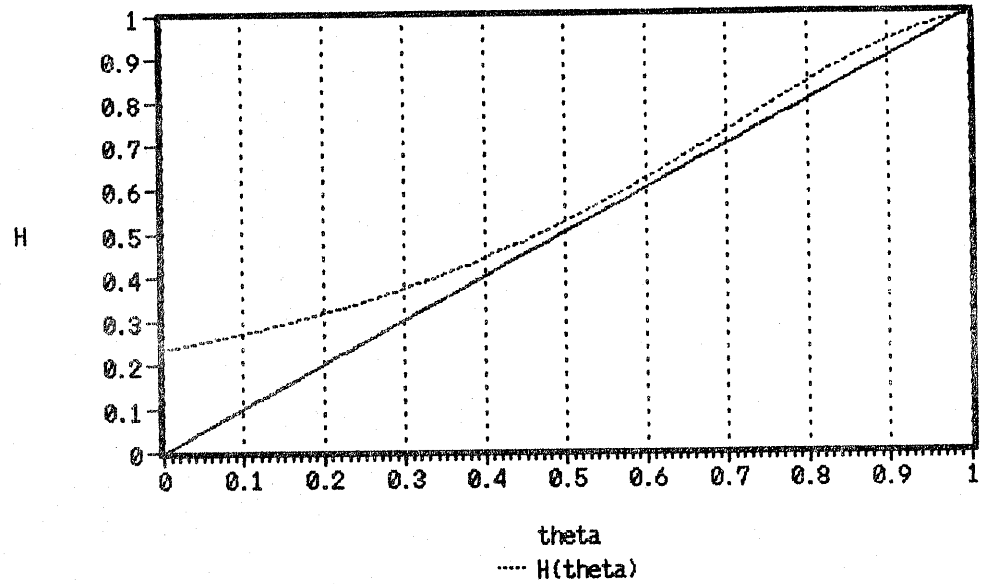

FIGURE 1.2 (σx2 = 3.3)
MULTIPLE EQUILIBRIA: I

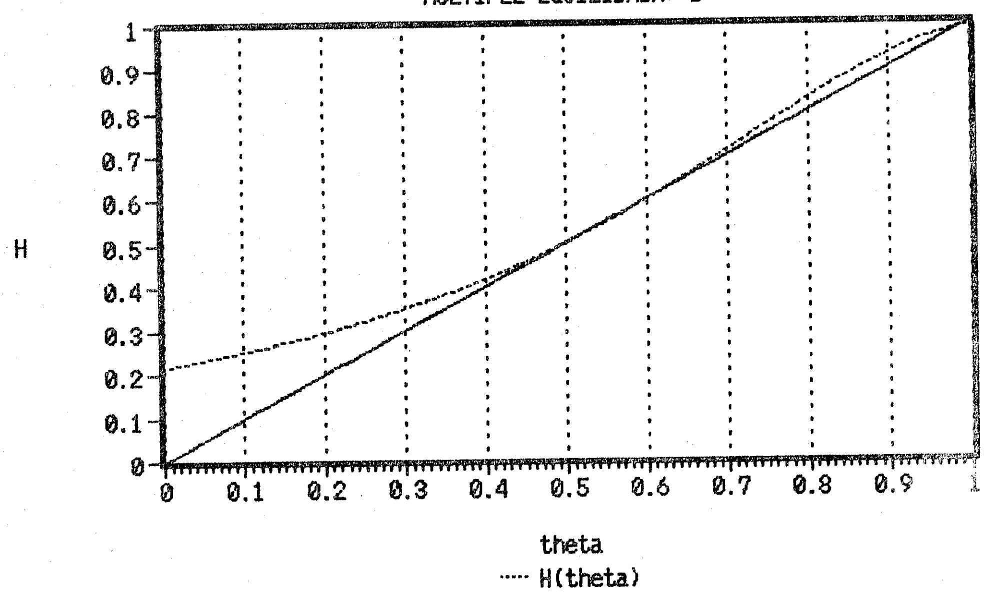

FIGURE 1.3 (σx2 = 5.2)
MULTIPLE EQUILIBRIA: II

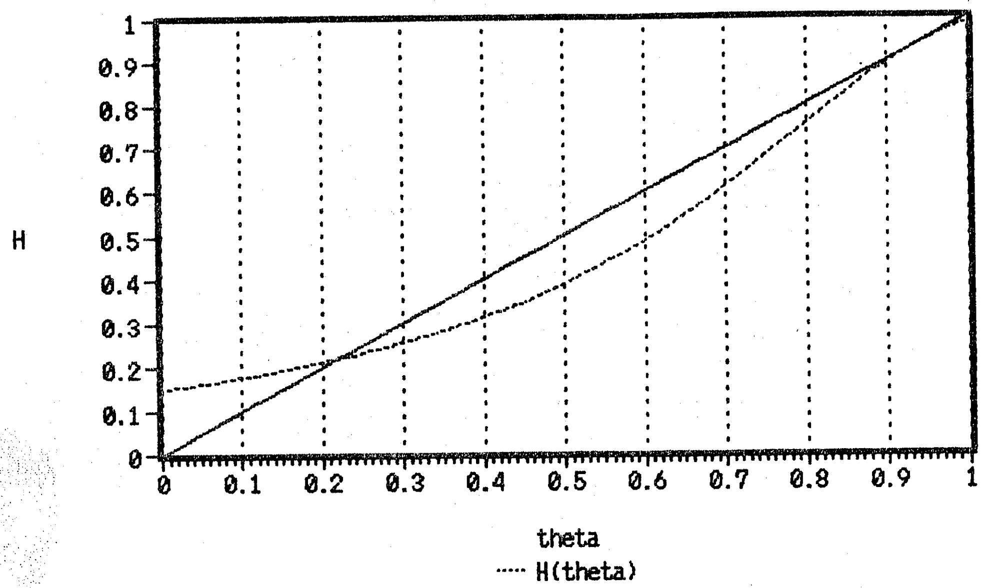

FIGURE 1.4 (σx2 = 7)
UNIQUE RIGID-PRICE EQUILIBRIUM

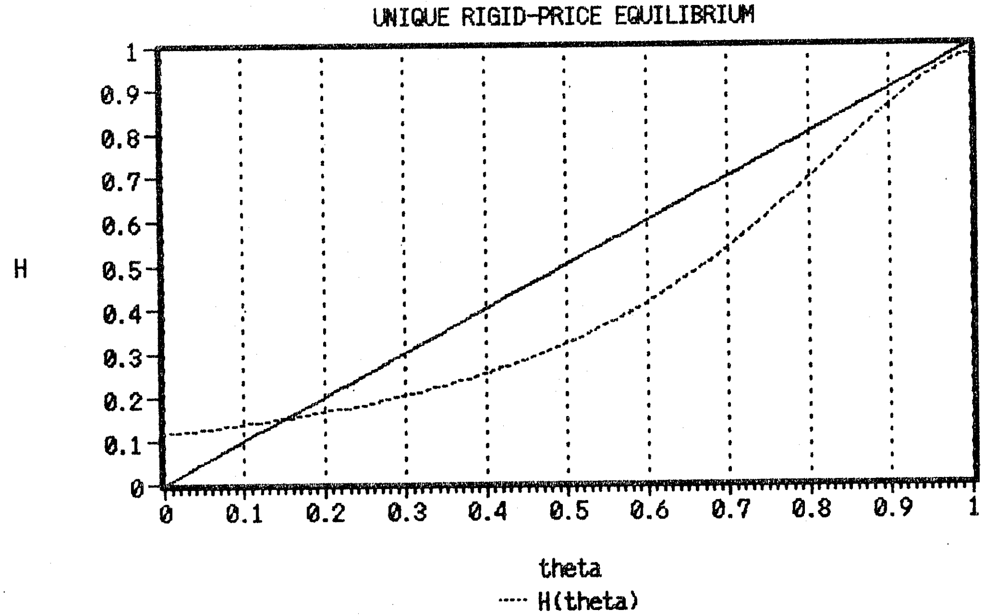

FIGURE 2: SMALL z1

c1=.05, r=5, z1=.011, and $\sigma x2/\sigma s2=3$

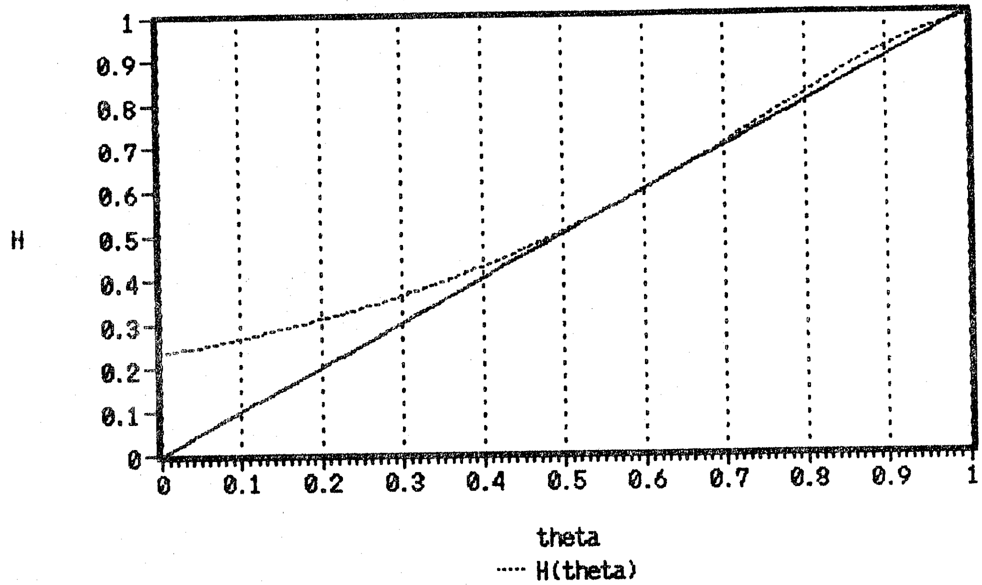

FIGURE 3: LARGE c1

c1=3, r=5, z1=0, and $\sigma x2/\sigma s2=3$

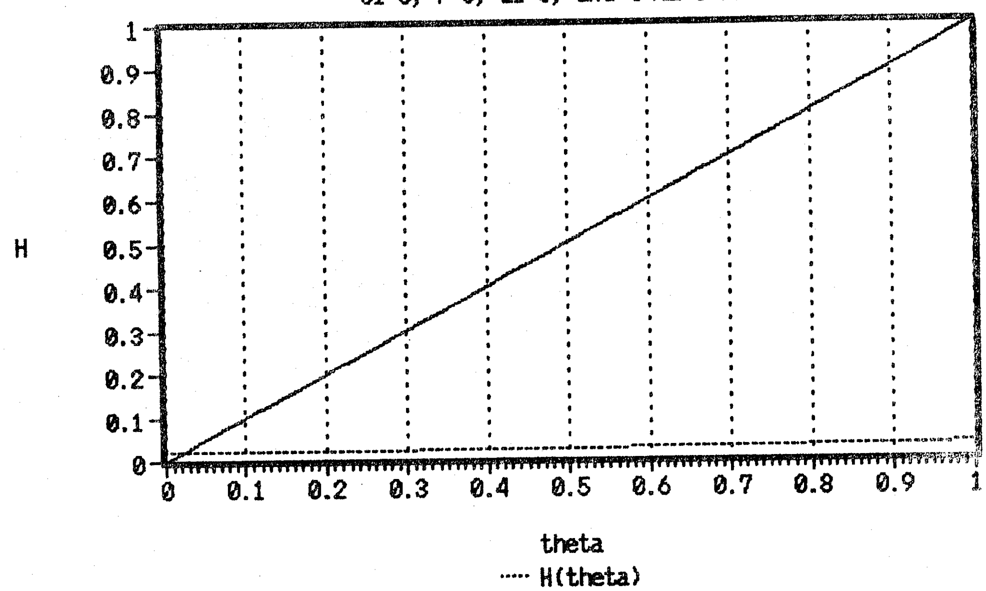

FIGURE 4: LARGE z1
c1=.05, r=5, z1=3, and σx2/σs2=3

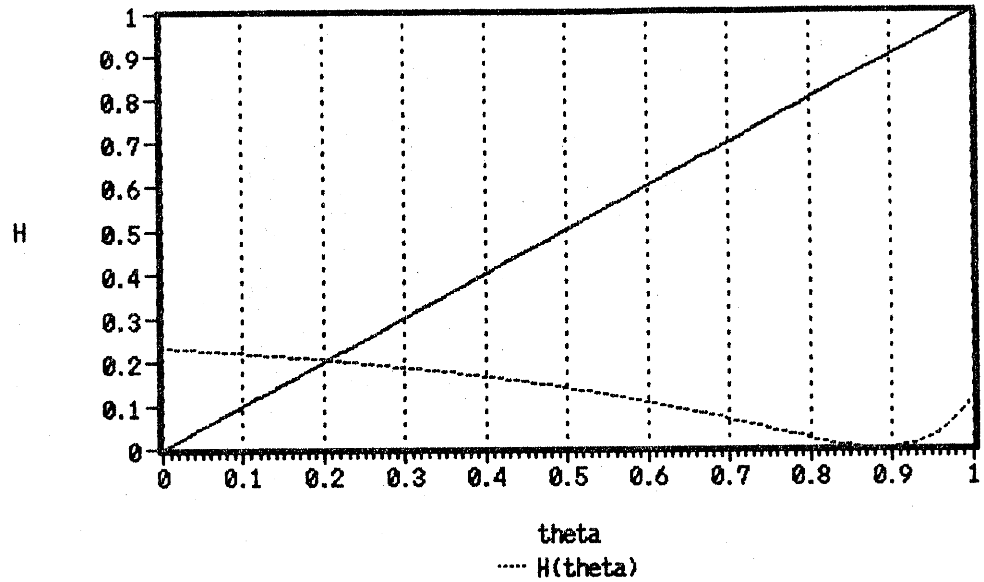

FIGURE 5: Nominal Demand Shock
c1=.01, r=5, z1=0, and σx2/σm2=.0004