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Competition with Two Part Pricing and Commodity Bundling: Applications to Clubs, Local Public Goods, and Networks

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Abstract

Introduction of two–part pricing modifies many of the results in the oligopoly theory. This paper constructs a model of competition with two–part pricing and analyzes its Bertrand–Nash and Cournot–Nash equilibria. Because our model includes club goods, local public goods, and shared facilities as its special cases, it helps to clarify relationships among these models. The model is also used to analyze commodity bundling (or tying) in the context of two–part pricing.

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1. Introduction

Two–part (or more generally, multi–part) pricing has been possible only in a limited number of industries such as telephone and electricity industries, since it requires prevention of resale activities. Although these industries had traditionally been considered as natural monopolies, competition emerged even in these network industries. For example, in many countries competition was allowed in long–distance telecommunication services. Monopoly has been maintained in the electricity industry at least at the distribution level, but competition with natural gas suppliers for energy demand is becoming more and more intense. Analysis of these industries requires an extention of the two–part pricing model to an oligopoly setting.

Calem and Spulber (1984) and Scotchmer (1985b), which analyzed two–part pricing in respectively differentiated–product and shared–facility models, are pioneering works in this direction. The two–part pricing models have close relationships with club good models of Buchanan (1965), McGuire (1974), Berglas (1976), and Scotchmer (1985a), and local public good models of Stiglitz (1977), Wooders (1978), Kanemoto (1980), Wildasin (1980), Brueckner (1983), and Scotchmer (1986). These papers can also be interpreted as attempts in the same direction. This paper constructs a general model of two–part pricing that includes models of clubs, local public goods, and shared facilities as its special cases and synthesizes results obtained in the literature. The model is also used to analyze commodity bundling in the context of two–part pricing.

As Oi (1971) has shown, two–part pricing avoids monopolistic price distortion when demand is homogeneous. That is, because a monopolist can use a lump–sum access fee which is non–distortionary to capture the consumer's surplus, he/she does not have to raise the unit price. This result carries over to oligopoly models as pointed out by Scotchmer (1985b), which distinguishes our model from the standard oligopoly model.

Bertrand and Cournot equilibria are the two equilibrium concepts that are most commonly used in the analysis of duopoly and oligopoly. We adapt these two concepts to two–part pricing and characterize the two types of equilibria. In both equilibria, no distortion exists for the unit price and capacity investment, but the access fee is in general higher than the marginal social cost of an additional subscriber. As the number of firms increases, the access fee approaches the marginal social cost. Even with a finite number of firms, however, there exists a case where the access fee is not distorted. This case occurs at a Bertrand equilibrium (a Nash equilibrium in prices) when no congestion exists on the consumption side.

In the context of multi–product firms, our result indicates that commodity bundling (or tie–in sale) is often efficiency enhancing if the commodities are complementary. The reason is that, if commodity bundling is prohibited, the unit price will become distorted to capture rents that would accrue to suppliers of the complementary good. This however assumes that scale economy in the complementary good is so large that monopolizing its production does not cause any efficiency losses. If scale *dis*economy exists in the production of the bundled commodity, commodity bundling may involve purchasing the bundled goods from competitive producers and selling them monopolistically to consumers. In such a case tying arrangements will cause distortion in the price of the bundled commodity. This does not however imply that tying must be prohibited. In the absence of tying the price of the coplementary commodity is not distorted, but distortion arises in the price and capacity investment of the other good. In a polar case where the marginal cost of the complementary good is constant, no distortion arises when tying is prohibited; and in the other polar case where the supply of the complementary good is fixed, tying does not cause any distortion.

The organization of this paper is as follows. Section 2 formulates the model, and a Nash equilibrium in prices (or a Bertrand equilibrium) is characterized in Section 3. Section 4 obtains a Nash equilibrium in quantities (or a Cournot equilibrium). Section 5 examines the effects of tying when producers of the tied good are competitive, and Section 6 concludes the paper.

2. Model

Consider firms that supply a vector of goods X charging a lump–sum access fee f and unit prices p. The goods X may be club goods such as golf courses and tennis courts, local public services such as parks and roads, and network services such as telecommunication and electricity.

Suppliers of X may differ in their cost structures. The cost function of the j–th firm is written as $C^j = C^j(X^j,n^j,k^j)$, where $C^j_X > 0$, $C^j_n \geq 0$, $C^j_k > 0$. The assumption of $C^j_n \geq 0$ reflects the possibility that an increase in the number of subscribers raises costs even if the total supply X^j is fixed. This is common in network industries where connecting a user to a network is costly. For clubs and local public goods, an increase in the membership may cause congestion even if the total consumption X^j is the same. The partial derivative C^j_n is strictly positive in such a case. The profits of firm j are $\Pi^j = f^j n^j + p^j X^j - C^j(X^j,n^j,k^j)$. We omit superscript j when this does not cause confusion.

We assume that all consumers have the same quasi–concave utility function, U(z,x,X,n,k), where z is the consumption of the composite consumer good which represents all goods other than goods X, and x is a consumption vector of goods X. Goods X may involve congestion so that utility derived from its consumption depends on the total supply by the supplier, X, the number of subscribers, n, and the capacity of the supplier, k. We assume that the utility function is twice differentiable and its first derivatives satisfy $U_z > 0$, $U_x > 0$, $U_X \leq 0$, $U_n \leq 0$, $U_k \geq 0$. The last three inequalities reflect our assumption on congestion.

A consumer may purchase X from more than one suppliers, but nobody will do that in equilibrium because he/she can save access fees by trading with only one firm. From this and the homogeneous consumer assumption, we have $X = nx$ in equilibrium.

The budget constraint of a consumer is $y = z + f + px$, where y is the income, f is the access fee, and p is a vector of unit prices of goods X. Define the expenditure function $E(p,X,n,k,u) = \min_{\{z,x\}}\{z + px\colon U(z,x,X,n,k) \geq u\}$. Note that this expenditure function does not include the access fee. In equilibrium the budget constraint satisfies $y = f + E(p,X,n,k,u)$. The partial derivatives of the expenditure function are $E_X(p,X,n,k,u) = -U_X/U_z \geq 0$, $E_k(p,X,n,k,u) = -U_k/U_z < 0$, $E_n(p,X,n,k,u) = -U_n/U_z \geq 0$, and $x(p,X,n,k,u) = E_p(p,X,n,k,u)$. The last equation yields a recursive relationship,

$$X(p,k,n,u) \equiv nx(p,X(p,k,n,u),n,k,u),$$

which defines a demand function that a firm is faced with, $X(p,k,n,u)$. This reduced–form demand function satisfies

$$X_p = nx_p/(1-nx_X)$$
$$X_n = (x+nx_n)/(1-nx_X)$$
$$X_u = nx_u/(1-nx_X).$$

We assume that $nx_X < 1$, $x+nx_n > 0$, and $x_u > 0$. The first two inequalities exclude perverse cases. If the first inequality does not hold, congestion externality is so strong that a rise in a unit price increases demand that the firm is faced with. The second inequality excludes the case where an increase in the number of subscribers reduces the total demand for X. The last inequality is equivalent to normality of goods X. Under these assumptions, we have $X_p < 0$, $X_n > 0$, and $X_u > 0$.

We assume that the total number of consumers is N and fixed. Because a consumer trades only with one firm, the population constraint, (2.1) $$\sum_{j=1}^{J} n^j = N,$$

must hold, where J is the number of firms. In equilibrium all consumers obtain the same utility level, which yields

(2.2) $$E^j(p^j,X^j(p^j,k^j,n^j,u),n^j,k^j,u) = y + f^j, \quad j = 1,2,\ldots,J.$$

Models of club goods, local public goods, and shared facilities are special cases of our model.

(a) Club Goods:

The club model of Buchanan (1965), Berglas (1976), and Scotchmer (1985a) assumes that consumption of X is fixed exogenously. This is equivalent to assuming a utility function U(z,n,k) and the cost function C(k). A firm then charges only the access fee (membership fee), and the budget constraint is $y = z + f$.

(b) Local Public Goods:

The local public good model of McGuire (1974), Wildasin (1980), Brueckner (1983), Scotchmer (1986) assumes that a consumer must purchase residential land to consume local public goods. If we interprete x and k in our model as land and local public goods respectively, the local public good model has a utility function U(z,x,k) and the cost function,

$$C(k,X) = \begin{cases} C(k) & \text{if } X \leq H \\ 0 & \text{if } X > H, \end{cases}$$

where H is the total available land in a jurisdiction. Except in Scotchmer (1986), the head tax (or the access fee) is assumed impossible and the costs of local public goods are financed by a (100%) tax on land rent. In such a case, the budget constraint for a consumer is $y = z + px$; and the 'profit' of a local government is the total land rent minus the cost of the public good, $pH-C(k)$. If the head tax is available, the budget constraint becomes $y = z + f + px$. The 'profit' of a local government in this case is $pH+fn-C(k)$.

(c) Shared Facilities:

The shared facility model of Scotchmer (1985b) assumes congestion on the consumption side for services provided by a shared facility such as a swimming pool. In particular, the utility function is U(z,x,X) and the cost of the facility is fixed. Suppliers adopt two–part pricing and the budget constraint for a consumer is $y = z + f + px$.

3. Nash Equilibrium in Prices

This section examines a Nash equilbrium in prices (or Bertrand equilibrium) where each firm maximizes its profits taking prices of other firms as given. We compare this equilibrium with a Nash equilibrium in quantities (or Cournot equilibrium) which is obtained in the next section.

If prices and capacities of all firms are given, equilibrium conditions (2.1) and (2.2) yield the number of subscribers and the utility level,

$$n^j = n^j(\mathbf{f},\mathbf{p},\mathbf{k}), \quad j = 1,\ldots,J, \tag{3.1}$$

$$u = u(\mathbf{f},\mathbf{p},\mathbf{k}), \tag{3.2}$$

where $\mathbf{f} = (f^1,\ldots,f^J)$, $\mathbf{p} = (p^1,\ldots,p^J)$, $\mathbf{k} = (k^1,\ldots,k^J)$. Let us first consider profit maximization of firm 1 which takes other firms' policies, $(f^2,\ldots,f^J)$, $(p^2,\ldots,p^J)$, $(k^2,\ldots,k^J)$, as given. Suppressing other firms' policies and abusing notation, we can write

$$n^1 = n^1(f^1,p^1,k^1) \tag{3.3}$$

$$u = u(f^1,p^1,k^1). \tag{3.4}$$

Firm 1 maximizes

$$\Pi = f^1n^1 + p^1X(p^1,k^1,n^1,u) - C^1(n^1,X(p^1,k^1,n^1,u),k^1) \tag{3.5}$$

with respect to (f^1, p^1, k^1) subject to (3.3) and (3.4). We suppress superscript 1 when obvious.

The first order conditions for profit maximization yield

$$f - C_n = -\frac{n}{n_f} - (p-C_X)(X_n n_f + X_u u_f) \tag{3.6}$$

(3.7) $$p - C_X = -\frac{X+(f-C_n)n_p}{X_p+X_u u_p+X_n n_p}$$

(3.8) $$(f-C_n)n_k + (p-C_X)(X_n n_k+X_k+X_u u_k) = C_k\,.$$

If the unit price equaled the marginal cost of production (i.e., $p = C_X$), then the usual monopoly pricing formula would hold for the access fee f: $MC_n = MR_n = f(1+\frac{n}{f n_f}) \leq f$. If the access fee equaled the marginal cost of a subscriber (i.e., $f = C_n$), then the same would be true for the unit price p: $MC_X = MR_X = p(1+\frac{\frac{X}{p}}{X_p+X_u u_p+X_n n_p}) \leq p$. In a monopoly model, Oi (1971) showed that the access fee is more efficient than the unit price in capturing the monopoly rent because it is a non–distortionary lump–sum charge. As noted by Scotchmer (1985b), this result extends to an oligopoly model, and distortion in unit price does not occur also in our model.

Let us first evaluate the derivatives of $n^1(f^1,p^1,k^1)$ and $u(f^1,p^1,k^1)$.

Lemma 1. Partial derivatives of $n^1(f^1,p^1,k^1)$ and $u(f^1,p^1,k^1)$ satisfy

$$n_f^1 = -\frac{1}{e_n^1}\frac{\sum_{j=2}^{J}(e_u^j/e_n^j)}{\sum_{j=1}^{J}(e_u^j/e_n^j)}$$

$$u_f = -\frac{1}{e_n^1}\frac{1}{\sum_{j=1}^{J}(e_u^j/e_n^j)}$$

$$n_p^1 = (x^1+E_X^1X_p^1)n_f^1$$

$$u_p = (x^1+E_X^1X_p^1)u_f$$

$$n_k^1 = (E_k^1+E_X^1X_k^1)n_f^1$$

$$u_k = (E_k^1+E_X^1X_k^1)u_f,$$

where

$$e_n^j = E_X^jX_n^j+E_n^j \geq 0,\ j = 1,\ldots,J,$$

and

$$e_u^j = E_X^j X_u^j + E_u^j > 0,\ j = 1,\ldots,J.$$

Proof:

The derivatives of $n^1(f^1,p^1,k^1)$ and $u(f^1,p^1,k^1)$ follow from

$$(3.9)\qquad \begin{bmatrix} & & \\ & \Omega & \\ & & \end{bmatrix}\begin{bmatrix} dn^1 \\ \vdots \\ dn^J \\ du \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} df^1 - \begin{bmatrix} x^1 + E_X^1 X_p^1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dp^1 - \begin{bmatrix} E_k^1 + E_X^1 X_k^1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dk^1,$$

where

$$\Omega = \begin{bmatrix} e_n^1 & \ddots & 0 & e_u^1 \\ 0 & \ddots & e_n^J & e_u^J \\ 1 & \cdots & 1 & 0 \end{bmatrix}.$$

Define

$$\omega = |\Omega| = -\left[\prod_{j=1}^{J} e_n^j\right] \sum_{j=1}^{J} \frac{e_u^j}{e_n^j},$$

$$\phi = \begin{vmatrix} 1 & 0 & \cdots & 0 & e_u^1 \\ 0 & e_n^2 & & 0 & e_u^2 \\ & 0 & \ddots & & \vdots \\ & & & e_n^J & e_u^J \\ 0 & 1 & \cdots & 1 & 0 \end{vmatrix} = -\left[\prod_{j=2}^{J} e_n^j\right] \sum_{j=2}^{J} \frac{e_u^j}{e_n^j},$$

and

$$\psi = \begin{vmatrix} e_n^1 & \ddots & 0 & 1 \\ 0 & \ddots & e_n^J & 0 \\ & & & \vdots \\ 1 & \cdots & 1 & 0 \end{vmatrix} = -\left[\prod_{j=2}^{J} e_n^j\right].$$

Then

$$n_f^1 = -\frac{\phi}{\omega}$$

$$u_f = -\frac{\psi}{\omega},$$

which yield the first two equalities in the lemma. The rest of the lemma is obvious from (3.9). Q.E.D.

The following proposition shows that no distortion occurs for the unit price and capacity investment. The monopoly rent is captured by the access fee which is in general higher than the marginal social cost of an additional subscriber.

Proposition 1. In a Nash equilibrium in prices, unit prices equal the marginal costs of producing goods X plus the marginal congestion costs,

$$p^j = C^j_X - n^j \frac{U^j_X}{U^j_z},$$

and capacity investment is carried out until its marginal benefit equals the marginal cost,

$$-n^j \frac{U^j_k}{U^j_z} = C^j_k .$$

The access fee diverges from the marginal social cost of a new subscriber, where the divergence satisfies

$$f^j - C^j_n + n^j U^j_n / U^j_z = n^j \frac{1}{\sum_{i \neq j} (1/e^i_n)(e^i_u/E^j_u)} .$$

Proof:

Let us restrict our attention to j=1 and suppress the superscript. The same result is obtained for other firms. From Lemma 1, we have $n_p = (x+E_X X_p) n_f$, $u_p = (x+E_X X_p) u_f$, $n_k = (E_k + E_X X_k) n_f$, and $u_k = (E_k + E_X X_k) u_f$, which yield

$$\Pi_p = (x+E_X X_p)\Pi_f + (p-C_X-nE_X)X_p,$$

and

$$\Pi_k = (E_k+E_X X_k)\Pi_f + (p-C_X-nE_X)X_k - (nE_k+C_k).$$

Combining these relationships with the first order conditions for profit maximization shows that, if $X_p \neq 0$, then $p = C_X + nE_X$ and $-nE_k = C_k$.

Next, from Lemma 1 and

$$\Pi_f = n + \{(f-C_n)+(p-C_X)X_n\}n_f + (p-C_X)X_u u_f = 0,$$

we obtain

$$f = C_n - nE_X X_n - n/n_f - nE_X X_u u_f/n_f$$
$$= C_n + nE_n + n^1 \frac{1}{\sum_{j=2}^{J} (1/e_n^j)(e_u^j/E_u^1)} .$$

Q.E.D.

Athough the access fee is in general distorted, the total number of subscribers is fixed at N and will not be distorted. Distortions in real resource allocation are therefore limited to those in the distribution of subscribers among firms.

The formula for the access fee becomes simpler in a symmetric equilibrium where all firms have the same cost structure and charge the same price.

Corollary 1. In a symmetric equilibrium, we have

$$f - C_n + nU_n/U_z = \frac{1}{J-1} n(E_X X_n + E_n) \frac{E_u}{E_X X_u + E_u} .$$

This corollary implies that, as the number of firms becomes larger, the access fee approaches the social marginal cost of a subscriber. The distortion in access fee is proportional to 1/(J–1). For example, the distortion becomes a half as the number of firms increases from 2 to 3. In our model the first best allocation is attained in the symmetric case. Because, as mentioned above, the total number of consumers of X is fixed at N, the access fees are equivalent to non–distortionary lump–sum taxes so long as all firms charge equal fees.

A surprising result is obtained in a special case where no congestion exists on the consumer side. In such a case, $E_X^j = E_n^j = e_n^j = 0$, which implies that no distortion occurs for the access fee even when the number of firms is finite.

Corollary 2. If no congestion exists on the consumer side, then the access fee equals the social marginal cost of an additional subscriber, i.e., $f^j = C_n^j$ for any j.

This corresponds to the well–known result that in a Bertrand model the price equals the marginal cost. Because we assumed that a firm takes other firms' access fees as given, it is faced with a horizontal demand curve if no congestion exists. With congestion on the consumer side, however, a reduction in the number of customers raises their utility level, which yields a downward sloping demand curve.

Without congestion on the consumption side, the access fee is zero if and only if $C_n = 0$. The local public good model satisfies this condition, which implies that the head tax is not necessary to attain the efficient supply of local public goods. This however depends crucially on our Bertrand assumption. We shall see below that in Nash equilibrium in quantities the access fee is positive even in this case.

Equilibrium prices in the examples are as follows.

(a) Club Goods: $p = 0$ and $f = \frac{J}{J-1}(-nU_n/U_z)$.

When the number of clubs is two, the access fee is twice as high as the congestion cost of an additional club member. The ratio between the access fee and the marginal congestion cost quickly approaches one as the number of clubs increases: it is 3/2 with 3 clubs, 4/3 with 4 clubs, 5/4 with 5 clubs, and so on.

(b) Local Public Goods: $p = U_x/U_z$ and $f = 0$.

In this case a local government owns all the land in its jurisdiction and sets the head tax and land rent to maximize its fiscal surplus, i.e., the sum of the head tax and land rent revenues minus the cost of local public goods. In equilibrium land rent is equal to the marginal rate of substitution between land and the consumer good. The supply of local public goods is optimal even if the number of local governments is finite. It turns out that the head tax is zero in equilbrium. As mentioned above, this implies that, even if the head tax is restricted to zero, the same equilibrium is obtained, i.e., equilibrium levels of land rent and local public goods are efficient.

(c) Shared Facilities: $p = -n\frac{U_X}{U_z}$ and $f = \frac{1}{J-1}\frac{x}{1-x_x}p\frac{E_u}{E_X X_u + E_u}$.

The unit price equals the marginal congestion cost of X. Because an additional user of a facility does not cause any increase in social costs so long as the total consumption X is the same, the efficient level of the access fee is zero. With a finite number of firms, the access fee is positive, but it approaches zero as the number increases. If the utility function is quasi–linear as in Scotchmer (1985b), then $X_u = 0$ and $x_X = 0$, which yields $f = \frac{1}{J-1}xp$. This coicides with Scotchmer's result. In this case the access fee happens to equal the quantity dependent part of the price, f = px, when the number of firms is two.

4. Nash Equilibrium in Quantities

In Nash equilibrium in quantities (Cournot equilibrium), a firm takes other firms' outputs as given. Our model has two prices, i.e., the access fee and the unit price however and one of them must be taken as given in addition to quantities. We assume that the access fees are taken as given as well as outputs.

Consider profit maximization of firm 1 which takes other firms' policies, $(f^2,\ldots,f^J)$, $(X^2,\ldots,X^J)$, $(k^2,\ldots,k^J)$, as given. From $X^j=X^j(p^j,k^j,n^j,u)$, we obtain $p^j=p^j(X^j,k^j,n^j,u)$. Using this relationship, we can rewrite the market clearing condition (2.2) as

(4.1) $$E^j(p^j(X^j,k^j,n^j,u),X^j,n^j,k^j,u) = w + f^j, \quad j = 1,2,\ldots,J$$

Then, in the same way as in the preceding section, we can write the number of subscribers and their utility level as functions of firm 1's choice variables:

(4.2) $$n^1 = n^1(f^1,p^1,k^1)$$

(4.3) $$u = u(f^1,p^1,k^1).$$

Firm 1 maximizes

$$\Pi^1 = f^1n^1 + p^1(X^1,k^1,n^1,u)X^1 - C^1(n^1,X^1,k^1) \tag{4.4}$$

with respect to $(f^1,\ X^1,\ k^1)$ subject to (4.2) and (4.3). We suppress superscript 1 when obvious.

The next lemma obtains the derivatives of (4.2) and (4.3).

Lemma 2. Partial derivatives of $n^1(f^1,p^1,k^1)$ and $u(f^1,p^1,k^1)$ satisfy

$$n^1_f = -\frac{1}{e^1_n}\frac{\sum_{j=2}^{J}(e^j_u/e^j_n)}{\sum_{j=1}^{J}(e^j_u/e^j_n)} \le 0$$

$$u_f = -\frac{1}{e^1_n}\frac{1}{\sum_{j=1}^{J}(e^j_u/e^j_n)} \le 0$$

$$n_X = (xp_X+E_X)n_f$$

$$u_X = (xp_X+E_X)u_f$$

$$n_k = (E_k+xp_k)n_f$$

$$u_k = (E_k+xp_k)u_f.$$

Proof:

If we replace elements of Ω in the preceding section by

$$e^j_n = x^jp^j_n + E^j_n = -x^j\frac{X^j_n}{X^j_p} + E^j_n \ge 0,$$

$$e^j_u = x^jp^j_u + E^j_u = -x^j\frac{X^j_u}{X^j_p} + E^j_u \ge 0,$$

then we obtain

$$\begin{bmatrix} & & \\ & \Omega & \\ & & \end{bmatrix}\begin{bmatrix} dn^1 \\ \vdots \\ dn^J \\ du \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} df^1 - \begin{bmatrix} x^1p^1_X+E^1_X \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dX^1 - \begin{bmatrix} E^1_k+x^1p^1_k \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} dk^1.$$

The lemma then immediately follows by applying the same argument as in Lemma

1. Q.E.D.

Equilibrium prices can be characterized in the same way as in the preceding section.

Proposition 2. In a Nash equilibrium in quantities, price prices and capacity investment satisfy the same conditions as in a Nash equilibrium in prices. The condition for the access fee is also the same as in Proposition 1 if e_n^j and e_u^j are modified as

$$e_n^j \equiv -\frac{U_n^j}{U_z^j} - x^j\frac{X_n^j}{X_p^j},$$

and

$$e_u^j = E_u^j - x^j\frac{X_u^j}{X_p^j}.$$

Proof:

From Lemma 2,

$$\Pi_X = (xp_X + E_X)\Pi_f + (p - C_X - nE_X)$$
$$\Pi_k = (E_k + xp_k)\Pi_f - (nE_k + C_k).$$

Hence, the first order conditions for profit maximization, $\Pi_f = \Pi_X = \Pi_k = 0$, yield

$$p = C_X + nE_X$$

and

$$-nE_k = C_k.$$

Now, from

$$\Pi_f = n + (f - C_n + Xp_n)n_f + Xp_u u_f = 0$$

we get

$$f = C_n + nE_n + n^1\frac{1}{\sum_{j=2}^{J}(1/e_n^j)(e_u^j/E_u^1)}.$$

Q.E.D.

Thus, no distortion arises in unit prices and capacity investment also in a Nash equilibrium in quantities. The access fee is distorted as in the Bertrand equilibrium but the precise formulas are different. In a symmetric equilibrium we obain the following corollary.

COROLLARY 1. In a symmetric equilibrium we have

$$f = C_n + nE_n + \frac{1}{J-1}[-X(X_n/X_p)+nE_n]\frac{E_u}{-x(X_u/X_p)+E_u}.$$

As in the Bertrand case, the distortion in the access fee will vanish as the number of firms approaches infinity. Even if the access fee is distorted, however, a Cournot symmetric equilibrium is first best efficient as in the Bertrand case, since the total number of consumers is fixed. An important difference from the previous case is that distortion in the access fee does not vanish even when no congestion exists on the consumer side. This can be seen from the following result for the symmetric case.

COROLLARY 2. If no congestion exists on the consumer side, then the access fee exceeds the social marginal cost of an additional subscriber by

$$f - [C_n + nE_n] = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p)+E_u}.$$

In a Nash equilibrium in prices, the head tax (the access fee) is zero in a local public good model where congestion does not exist on the consumer side and C_n is zero. This corollary shows that the head tax is positive in a Nash equilibrium in quantities. An implication of this result is that if the head tax is restricted to zero, the unit price and/or capacity choice will be distorted in a Nash equilibrim in quantities.[2]

Equilibrium prices in our three examples are as follows.

(a) Club Goods: $p = 0$ and $f = \frac{J}{J-1}(-nU_n/U_z)$.

Hence, equilibrium prices are the same as in the Bertrand equilibrium.

(b) Local Public Goods: $p = U_X/U_z$ and $f = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p)+E_u}$.

The head tax (access fee) is different from that in the Bertrand equilibrium where it is zero. In the limit as the number of firms approaches infinity, the access fee becomes zero. Except in this limit, a restriction of the head tax to zero will cause distortions in the supply of local public goods. If the utility function is quasi–linear, then $X_u = 0$ and we obtain the same result as in Scotchmer (1986): $f = -\frac{1}{J-1}\frac{(x)^2}{x_p}$.

Which equilibrium concept is relevant in the local public good model depends on the ownership structure of land. If land is owned by private individuals and land rent is determined by the market, land rent, p, is not a direct choice variable for a local government. In such a case a Nash equilibrium in quantities is more appropriate. If the local government owns the land and sets the land rent, then a Nash equilibrium in prices may be applicable although there is no a priori reason to exclude the Cournot equilibrium in this case.

(c) Shared Facilities: $p = -nU_X/U_z$ and $f = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p)+E_u}$.

As in the local public good case (b), the access fee is different from that in the Bertrand equilibrium.

5. Tying and Rent Extraction

[2] Scotchmer (1986) obtained this result in a local public good model.

Because X in our model can be a vector of goods, the analysis in the preceding sections is relevant to commodity bundling. For example, it can be applied to a firm bundling intra–city and inter–city telecommunication network services as well as telephone equipments. The local public good model is another example where they are bundled with residential land. Our results indicate that commodity bundling with two–part pricing can often be efficiency enhancing. In a competitive case with many suppliers, commodity bundling with two part pricing yields efficient decisions on pricing and capacity investment. If the number of firms is small, distortion arises in the access fee but unit prices and capacity investment are not distorted.

Our model does not explicitly treat scale economy in production, however. For example, scale economy for production of telephone equipment is usually much smaller than that for telecommunication network services. In such a case it is not efficient for a telecommunication network to produce telephone equipments as our model implicitly assumes. If commodity bundling (or tying) is allowed, a network company may purchase telephone equipments from other firms and sell or lease them to users of its network. This adds to our model monopsony power of the network in the purchase of telephone equipments. We next analyze the effects of commodity bundling in this setting.

Suppose that there exists a complement H that must be used with good X. Although they must be used together, proportions with which they are used are flexible. In order to simplify the analysis, we assume that consumption of good H does not cause any congestion on the consumer side. The utility function of a consumer is then U(z,x,X,h,n,k), where h is the consumption of good H. Production of H is carried out competitively by many firms and the cost function is $R(H^j)$, where H^j is demand for good H by customers of the j–th supplier of good X. This cost function implicitly assumes that good H is differentiated according to the supplier of good X. This is satisfied if good X is a local public good and good H is land. In telecommunications, if each network uses a particular technology or frequency, different equipments must be used to access different networks. If exactly the same equipments can be used by all networks, the cost function must be written as $R(\sum_{j=1}^{J} H^j)$.

If tying is allowed, a supplier of X buys good H and sells them monopolistically to its users. If tying is prohibited, then producers of good H sell them directly to users. We next compare these two cases.

First, it is rather obious that, if tying is allowed, the supplier of X will always bundle good H with good X. Because there is no additional costs for bundling, the firm can always bundle the two goods, and buy and sell H at a competitive price that would prevail if they were not bundled. This will produce the same profit level as in the no–bundling case and the firm can always increase profits by changing the price of good H.

With tying, the monopsony power in the market for good H causes divergence of its consumer price from its producer price. Denote the consumer price by r and the producer price by $\tilde{r}$. Because producers of good H are competitive, the producer price at which a supplier of X buys good H equals its marginal cost: $\tilde{r} = R_H(H)$. The cost of good H for the firm is then $\hat{R}(H) = HR_H(H)$, and the cost function in the preceding sections must be replaced by $C(n,X,H,k) = C(n,X,k) + \hat{R}(H)$. With this change we get the same results as in Propositions 1 and 2. The prices of goods X and H are then

$$(5.1) \qquad p = C_X - n\frac{U_X}{U_z},$$

$$(5.2) \qquad r = \hat{R}_H = R_H + HR_{HH} \geq R_H .$$

Thus, the monopsony power in the good H market makes its consumer price higher than the marginal cost. Pricing and investment in good X are not however distorted. These results are summarized in the following proposition.

Proposition 3. If tying is possible, the firm will choose to bundle the two goods together. There is no distortion in the price of good X and investment decision for good X, but the price of good H is higher than its marginal cost.

This result can be interpreted as an example of the "leverage theory" of tying: a firm with monopoly power in one market uses the leverage provided by this power to monopolize a second market. The leverage theory was criticised by Posner (1976) and others, but recently Whinston (1990) offered its theoretical foundations based on scale economies and strategic interaction. Our model does not have strategic interaction because producers of tied goods are competitive. If the industry–wide scale diseconomy (due for example to fixed factors) produces rents in the tied–good industry, however, a monopolist adopts tying to capture part of the rents.

Next, consider the case where tying is prohibited. Competition among producers of good H yields $r = R_H(H)$ ($\equiv dR(H)/dH$). Demand for H by an individual consumer is $h = h(p,r,X,n,k,u) = \partial E(p,r,X,n,k,u)/\partial r$ and summing this over all subscribers yields $H = nh(p,r,X,n,k,u)$. From $r = R_H(nh(p,r,X,n,k,u))$ and $X = X(p,r,k,n,u)$, we obtain

$$r = r^*(p,k,n,u). \tag{5.3}$$

The price of H therefore depends on p and k. From this relationship, a change in p and k induces a change in r, which in turn changes the number of subscribers and demand for X. Distortions in p and k are introduced because the firm takes into account the effect of this induced change on profits.

From (5.3) the expenditure function and demand for X can be rewritten as

$$E(p,r^*(p,k,n,u),X,n,k,u) \equiv E^*(p,X,k,n,u), \tag{5.4}$$

$$X = X(p,r^*(p,k,n,u),k,n,u) \equiv X^*(p,k,n,u). \tag{5.5}$$

If we replace the expenditure function and the demand function in the preceding sections with (5.4) and (5.5), the same analysis as before can be carried out. In a Nash equilibrium in prices, for example, the unit price and capacity satisfy

$$p = C_X + nU_X/U_z + hr_p^*/X_p^*, \tag{5.6}$$

and

$$nU_k/U_z - nhr_k^* = C_k, \tag{5.7}$$

if $X_p^* \neq 0$. Thus, in general the unit price and capacity choices are distorted.

Proposition 4. If tying is prohibited, the price of good H equals its marginal cost, but in general pricing and investment decisions for good X are distorted.

Equations (5.6) and (5.7) show that, if $\partial r^*/\partial p = \partial r^*/\partial k = 0$, no distortion arises. From

$$\partial r^*/\partial p = \frac{R_{HH}(nh_p + nh_X X_p)}{1 - R_{HH}(nh_r + nh_X X_r)}$$

and

$$\partial r^*/\partial k = \frac{R_{HH}(nh_k + nh_X X_k)}{1 - R_{HH}(nh_r + nh_X X_r)},$$

this occurs when

$$R_{HH}(H) = \text{constant}$$

or

$$h_p + h_X X_p = h_k + h_X X_k = 0.$$

Hence, if the marginal cost of good H is constant or if a change in p and k does not change demand for H by an individual consumer, the prohibition of tying does not introduce distortion. An important example of such a case occurs when good H is common for all suppliers of X. As noted before, the cost function in such a case is $R(\sum_{j=1}^{J} H^j)$, and if J is large enough, $\partial^2 R/\partial(H^j)^2$ becomes close to zero.

If tying is allowed, no distortion arises for p and k, but the price of good H is distorted. This distortion does not affect the 'real' resource allocation if the supply of H is fixed. The local public good case is an example of this. These observations are summarized in the following corollary.

Corollary. If the marginal cost of good H is constant, then no distortion arises when tying is prohibited; and if the supply of good H is fixed, then there is no distortion when tying is allowed.

The reason why prohibition of tying results in distortion of p and k is that an induced change in the price of good H has the effect of transfering the consumer surplus to producers of good H. Because a supplier of good X must guarantee the same utility level as other suppliers, he/she must compensate for this change in consumer's surplus by a change in p or k. One way of avoiding distortion of p and k is for the supplier of good X to capture rents that accrue to producers of H. For example, if producers of H use a certain input (e.g., land) which is owned by the supplier of good X, then the supplier can extract rents in industry H by pricing the input. Let us now examine what happens if this is possible.

Competition among producers yields $r = R_H$, and the price of H satisfies $r = r^*(p,k,n,u)$. The amount of rents that the X supplier captures is $r^*(p,k,n,u)H - R(H)$ and the profits for the firm are

$$\Pi = fn + pX - C(n,X,k) + r^*(p,k,n,u)H - R(H).$$

Then, the profit maximization problem for the firm is formally the same as those in preceding sections with an additional constraint that the price of H equals its marginal cost. We have seen however that marginal cost pricing is obtained as solutions to the problems without the constraint. This constraint is not therefore effective and the solution to this problem is exactly the same as those in preceding sections.

Proposition 5. If tying is prohibited but if the firm can extract rents earned in the supply of good H, then pricing and investment decisions for both goods X and H are non–distortionary.

6. Concluding Remarks

With two–part pricing, many of the results in the oligopoly theory must be modified. The most important modification is that, with homogeneous consumers, unit prices and capacity investment are not distorted even with a small number of competing firms. The access fee is distorted, but the distortion disappears as the number of firms increases. The speed of convergence is of the order of 1/(J–1) where J is the number of firms.

Scotchmer (1985b) obtained these results in a Bertrand equilibrium of a simple shared facility model. This paper extends the results to Bertrand and Cournot equilibria of a fairly general model that includes club goods, local public goods, and shared facilities as special cases. This extension clarifies relationships among these models and between the two types of equilibria. Furthermore, we obtained an interesting new result that, if there is no congestion on the consumption side, there is no distortion in the access fee in a Bertrand equilibrium. An example of this case is the local public good model. In a Cournot equilibrium, however, the access fee is not zero even in this case.

Commodity bundling is efficiency enhancing if the bundled commodities are complementary and if there is no loss in production efficiency by monopolizing production. If the latter condition does not hold, then commodity bundling may involve separation of production from sales. That is, a firm purchases the complementary good from competitive producers and sells them to its customers.

The firm then has monopsony power as well as monopoly power in the market for the bundled commodity, and tying arrangements will cause distortion in its price. Prohibition of tying also causes distortion because, even though the price of the complementary good is not distorted, distortion arises in the price and capacity investment of the other good.

We assumed that the profits of the firms are given to absentee share holders. If the consumers own the shares of the firms, then repercussions through distribution of profit income are introduced. The analysis of such a case is somewhat more complicated, but similar (though more complicated) formulas are obtained. All the qualitative results remain the same.

The assumption of homogeneous consumers is crucial to our results. With heterogeneous consumers, the analysis becomes much more complicated because the price structure serves an additional role of a self–selection device. Much of the literature on nonlinear pricing in a monopoly model focused on this aspect, and extending their results to oligopolistic competition is a fruitful direction of future research.[3]

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[3] See Brown and Sibley (1986) for an excellent textbook treatment of nonlinear pricing with heterogeneous demand.

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