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Malthus seen from a Smithian Growth Model

根岸 隆  
(東京大学)

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Takashi Negishi

(1)

Adam Smith is typically a growth economist. It can be clearly seen if we compare his definition of natural prices with that of Ricardo. While the latter defines the natural price of labor only in the context of a stationary population ( Ricardo (1951a), p. 93 ), Smith argues that the natural rate of wage is higher in a growing economy than in a stationary economy and that a high natural rate of wage and a high natural rate of profit coexist in an economy with a high rate of growth ( Smith (1976), pp. 91, 109 ). Elsewhere we have constructed a Smithian growth model, a von-Neumann model of production of commodities by means of commodities where commodities include the labor power commodity ( Negishi (1985), pp. 1 - 16, (1989), pp. 83 - 89 ). It is shown that both the rate of profit and the rate of wage are higher if the rate of growth is made higher by the higher productivity of labor, owing, for example, to the existence of rich lands. The model is further extended to consider the effects of capitalists' consumption and saving on the rate of profit ( Negishi (1988) ). It can be shown that the rate of profit falls as the rate of growth made higher by the higher rate of saving, when the labor productivity is kept unchanged. This confirms Smith's theory of the falling rate of profit ( Smith (1976), p. 105 ).

As is well known, Smith's theory of the falling rate of profit was criticized by Ricardo, since the former "uniformly ascribes the fall of

profits to accumulation of capital, and to the competition which will result from it, without ever adverting to the increasing difficulty of providing food for the additional number of labourers which the additional capital will employ" ( Ricardo (1951a), p. 289 ). Smith was defended, however, by Malthus who argued against Ricardo that "in the actual state of things in most countries of the world, and within limited periods of moderate extent, the rate of profits will practically depend more upon the causes which affect the relative abundance or scarcity of capital, and the demand for produce compared with the supply, than on the fertility of the last land taken into cultivation. And consequently, to dwell on this latter point as the sole, or even the main cause which determines profits, must lead to the most erroneous conclusions. Adam Smith, in stating the cause of the fall of profits, has omitted this point, and in so doing has omitted a most important consideration; but in dwelling solely upon the abundance and competition of capital, he is practically much nearer the truth than those who dwell almost exclusively on the quality of the last land taken into cultivation" ( Malthus (1836), p. 289 ).

Our aim in this note is, therefore, to consider firstly Malthus's theory of profit, particularly the so-called regulating principle, by the use of our Smithian growth model. As will be seen in the below, higher rate of saving, which raises the real wage and reduces the rate of profit, raises the rate of growth of the economy in our model. This contradicts, however, with the conclusion of Costabile and Rowthorn (1985) that an increase in the saving propensities leads to slower growth in Malthus's theory of wages and growth. The second aim in this note is, then, to defend our Smithian interpretation of Malthus against

interpretation of Malthus adopted by Costabile and Rowthorn, by the use of Malthus's notion of optimal propensity to save ( Malthus (1836), p. 7 ). This will, of course, clarify also the point at which Malthus distinguishes himself from Smith.

(2)

Let us sketch our Smithian growth model. Suppose that the period necessary for the reproduction of labor ( power ) in laborers' households is identical to the period of production of labor products and that a units of labor must be expended one period before to produce one unit of product and b units of labor product must be consumed in households one period before to produce one units of labor. Capitalists are assumed to use e of their stock of products to employ labor and to use  $(1-c)$  of their employees in the households as unproductive labor, where  $0 < e < 1$ ,  $0 < c < 1$ . In other words, they consume  $(1-e)$  of the stock of products and capitalists' rate of (gross) saving is  $ec$  while laborers are assumed not to save.

Denote the capitalists' aggregate stock of the product at time  $t$  by  $X(t)$  and the labor population at time  $t$  by  $L(t)$ . Then, from definitions of coefficients  $a$ ,  $b$ ,  $e$  and  $c$ , we have

$$(1) \quad eX(t) = bL(t+1)$$

and

$$(2) \quad cL(t) = aX(t+1).$$

To consider a balanced growth solution of our model (1) and (2) in which both  $X(t)$  and  $L(t)$  grow at the common rate of  $g$ , substitute  $X(t+1) = (1+g)X(t)$  and  $L(t+1) = (1+g)L(t)$  into (1) and (2). It can be easily seen that the rate of growth  $g$  and the given coefficients must satisfy the

condition

$$(3) \quad ab(1+g)^2 = ec.$$

Particularly, the given coefficients must satisfy the condition  $ec > ab$  to assure the positive rate of growth. The rate of growth  $g$  is higher, if the rate of saving  $ec$  and the labor productivity  $1/a$  are higher, or the real rate of subsistence wage  $b$  is lower.

Since equilibrium relative prices remain unchanged through time on such a balanced growth path of the economy, let us denote the natural price of the product by  $p$  and the natural rate of wage by  $w$ . According to Smith, then,

$$(4) \quad p = (1+r)aw$$

and

$$(5) \quad w = (1+s)bp$$

where  $r$  is the natural rate of profit, and  $r$  and  $s$  are assumed to be positive if  $g$  is positive. In other words, the natural price of the product is the sum of the wage  $\underline{aw}$  and profit  $\underline{raw}$  at their natural rates, since we assumed away the land rent, and in a growing economy the natural rate of wage is higher than the subsistence wage  $\underline{bp}$ , which is the wage at the natural rate in a stationary economy. From (4) and (5), it can be easily seen that

$$(6) \quad 1 = (1+s)(1+r)ab$$

must be satisfied by  $s$  and  $r$ .

From the definition of  $e$ ,

$$(7) \quad wL(t) = epX(t).$$

By substituting  $L(t+1) = (1+g)L(t)$  and (7) into (1), we have

$$(8) \quad w = (1+g)bp.$$

Smith's assumption that  $s$  is positive when  $g$  is positive is justified,

therefore, in the balance growth path of our model, since  $s = g$  from (5) and (8). Similarly, by substituting  $X(t+1) = (1+g)X(t)$  and (7) into (2), we have

$$(9) \quad ecp = (1+g)aw.$$

As Smith assumed, therefore,  $r$  is higher if  $g$  is higher, since (4) and (9) imply that  $ec(1+r) = (1+g)$ . In view of the fact that  $s = g$ , then, both  $s$  and  $r$  can be higher, if  $g$  is higher, provided that the rate of saving  $ec$  remains constant. In other words, the coexistence of high profit and high real wage is possible, if the labor productivity  $1/a$  is high so that the rate of growth is high.

If the rate of saving is increased either through an increase in  $e$  or through an increase in  $c$ , there is an increase in the rate of growth  $g$ , as is seen in (3). This increases  $s$  and the real wage  $w/p$  in (5), since  $s = g$  as we saw in the above. The increased  $s$  implies that the rate of profit  $r$  decreases from (6). As far as the balanced growth path is concerned, therefore, the analysis of our Smithian growth model confirms Smith's theory of the falling rate of profit caused by the capital accumulation, which, unlike Ricardo's, does not require the falling productivity of labor.

(3)

According to Pasinetti (1960)'s mathematical model of Ricardian economics, the rate of profit  $r$  in the natural equilibrium is determined in the production of the wage good, corn, by

$$(10) \quad r = f'(N_1) / \bar{x} - 1$$

where  $N_1$  is the number of workers employed in the corn production,  $f(N_1)$  is the production function of corn such that  $f''(N_1) < 0$ , and  $\bar{x}$  is the

constant natural wage rate in terms of corn. As capital is accumulated so that the labor population is increased, the rate of profit falls only through the reduction of the marginal productivity of labor ( $f'(N_1)$ ) in the production of corn.

To this Ricardian theory of the profit, Malthus added the second theory of profit. In the first edition of his Principles of Political Economy (1820), he argues that "The two main causes which influence the means of supporting labour, are 1st. The difficulty or facility of production on the land, by which a greater or less proportion of the value of the whole produce is capable of supporting the labourers employed. And 2dly, the varying relation of the quantity of capital to the quantity of labour employed by it, by which more or less of the necessaries of life may go to each individual labourer. Each of these causes is alone sufficient to occasion all the variations of which profits are susceptible" ( Ricardo (1951b), pp. 253 - 254 ).

In the second edition of Principles of Political Economy (1836), Malthus gave the names to these two theories of the rate of profit. The first of the two main causes which affect the rate of profit is "the productiveness, or unproductiveness of the last capitals employed upon the land, by which a smaller, or a greater proportion of the value of the produce is capable of supporting the labourers employed. This may be called the limiting principle of profits" ( Malthus (1836), p. 271 ). The second of causes which affect the rate of profit is "the varying value of the produce of the same quantity of labour occasioned by the accidental or ordinary state of the demand and supply, by which a greater or smaller proportion of that produce falls to the share of the labourers employed. This may be called the regulating principle of profits" ( Malthus

(1836), p. 271 ). "The second cause which affects profits, is the varying value of the produce of the same quantity of labour on the same value of capital, determined by the state of the demand and supply. This may be called the regulating principle of profits, as within the extreme limits prescribed by the state of land, all the variations of profits, whether temporary or durable, are regulated by it. Such variations in the value of produce are occasioned principally by the abundance or scantiness of capital, including the funds for the maintenance of labour, as compared with the labour which it employs" ( Malthus (1836), p. 276 ).

To understand Malthus's arguments fully, we have to remember that Malthus followed Smith to measure the value of commodities by their commanding value of labor. "When we consider labour as a measure of value in the sense in which it is most frequently applied by Adam Smith, that is, when the value of an object is estimated by the quantity of labour of a given description which it can command, it will appear to be a measure essentially distinct from all others, and to approach as near to a standard measure, both of relative and of intrinsic value in exchange, as the nature of the subject will admit" ( Malthus (1836), pp. 93 - 94 ). As was pointed out by Schumpeter ( (1954), p. 188 ), the commanding labor theory of value to measure the value of a commodity by the quantity of labor which it will exchange for is simply an attempt to use labor as numéraire.

The variation of the value of the produce is, therefore, nothing but the variation of the real wage in the definition of the regulating principle of profits. This variation is, of course, caused by the relation between demand and supply of capital to employ labor. By the definition of value, the value of capital to employ the same quantity of



labor remains unchanged in the definition of the regulating principle. The quantity of the produce of the same quantity of labor, i.e., the labor productivity, need not be changed in the regulating principle of profits, while changes in the marginal productivity of labor is the essence of the limiting principle of profits. The real ( corn ) wage, however, need not be changed and therefore there may be no variation in the value of the produce in the definition of the limiting principle of profits. Only the proportion of the value of the produce necessary to support the laborers is changed as the productivity of labor changes. In this respect, Ricardo's comment on Malthus is confusing, though not incorrect from his point of view. "Whenever the difficulty of production on the land is such that a greater proportion of the value of the whole produce is employed in supporting labour, I call wages high" ( Ricardo (1951b), p. 252 ). The value of wage is high only from the point of view of Ricardo's embodied labor value theory, when the labor productivity is reduced.

(4)

Malthus emphasized the role of the regulating principle against the limiting principle and argued "that profits never fall but when the value of the produce of the same quantity of labour falls, and never rise but when the value of the produce of the same quantity of labor rises" ( Malthus (1836), p.291 ). To support the regulating principle, he compared Poland and America. "In Poland, and some other parts of Europe, where capital is scarce, profits are said to be higher than in America; yet it is probable that the last land taken into cultivation in America is much richer than the last land taken into cultivation in Poland. But in America

the labourer earns perhaps the value of eighteen or twenty quarters of wheat in the year; in Poland only the value of eight or nine quarters of rye. This difference in the division of the produce, must make a great difference in the rate of profits; yet the causes which determine this division, far from being of so temporary a nature that they may be safely overlooked, might operate most powerfully for a great length of time. Such is the extent of America, that the corn wage of its labour may not essentially fall for a long term of years; and the effects of a scanty but stationary capital on an overflowing but stationary population might last for ever" ( Malthus (1836), pp. 280 - 281 ).

Malthus's conclusion from the comparison of Poland and America suggests that the regulating principle is considered to work even in the long run. In other words, the principle is concerned with the determination of the rate of profit by the ordinary and average relation of the supply and the demand of capital to employ labor, rather than the extraordinary and accidental relation. The rate of profit considered is, therefore, the natural rate of profit rather than the market rate, since "Natural and necessary prices appear to be regulated by this principle [ of demand and supply ], as well as market prices; and the only difference is, that the former are regulated by the ordinary and average relation of the supply to the demand; and the latter, when they differ from the former, are determined by the extraordinary and accidental relations of the supply to the demand" ( Malthus (1863), p. 78 ).

Malthus's definition of natural and necessary prices follows the definition of Smith's. "It [ the definition of necessary price ] will be, the price necessary, in the actual circumstances of the society, to bring the commodity regularly to the market. This is only a shorter

description of what Adam Smith means by natural price, as distinguished from market price or the price at which commodities actually sell in the market, which --- are sometimes sold higher and sometimes lower than the price which is necessary to fulfill the conditions of a regular supply" ( Ricardo (1951), pp. 53 - 54 ).

Since Malthus's regulating principle of profits is thus concerned with Smith's system of natural prices and the land as a limiting factor of production is assumed away in our Smithian growth model, it is interesting to use the model to make the implications of the regulating principle clear. The ratio of the aggregate stock of product  $X(t)$  and the labor population  $L(t)$  remains unchanged through time on the balanced growth path of our Smithian model (1) and (2), which was sketched in the section (2). This ratio changes, of course, as coefficients  $a$ ,  $b$ ,  $c$  and  $e$  change in (1) and (2). Changes in these coefficients, furthermore, cause variations in the rate of growth  $g$  and the natural rate of profit  $r$ . From our point of view, what is particularly interesting is a change in the rate of saving  $ec$ . As was shown in the section (2), an increase in  $ec$  increases  $g$  and  $w/p$  and reduces  $r$ . Since we have

$$(11) \quad eX(t)/L(t) = b(1+g)$$

from the substitution of  $L(t+1) = L(t)(1+g)$  into (1), we can see the working of the regulating principle that the natural rate of profit is reduced if the ratio of the funds employed to demand labor and the supply of labor is increased by an increase in the rate of saving. As was noted by Malthus ( Ricardo (1951b), p. 258 ), the funds employed to demand labor is not  $ecX(t)$  but  $eX(t)$  in (11), since "it does not depend upon capital alone, but upon capital and revenue together."<sup>1)</sup>

In our Smithian model, an increase in the rate of saving, which reduces the rate of profit through Malthusian regulating principle, always increases the rate of growth. This contradicts with the conclusion from the Malthusian growth model of Costabile and Rowthorn (1985) that an increase in the saving propensities always leads to slower growth. The difference arises from the fact that we are concerned with the comparative dynamics of different balanced growth paths generated by different values of the rate of saving, while Costabile and Rowthorn considers comparative dynamics of growth paths starting from a given combination of population and aggregate capital. To defend our own approach, let us consider the famous problem of the optimal rate of saving proposed by Malthus.

"No considerable and continued increase of wealth could possibly take place without that degree of frugality which occasions, annually, the conversion of some revenue into capital, and creates a balance of produce above consumption; but it is quite obvious that they are not true to an indefinite extent, and that the principle of saving, pushed to excess, would destroy the motive to production. --- If consumption exceed production, the capital of the country must be diminished, and its wealth must be gradually destroyed from its want of power to produce; if production be in a great excess above consumption, the motive to accumulate and produce must cease from the want of an effectual demand in those who have the principal means of purchasing. The two extremes are obvious; and it follows that there must be some intermediate point, though the resources of political economy may not be able to ascertain it, where, taking into consideration both the power to produce and the

will to consume, the encouragement to increase of wealth is the greatest" ( Malthus (1863), pp. 6 - 7 ).

Obviously Malthus admits that the principle of saving, unless it is pushed to excess, implies the encouragement to increase of wealth. If an increase in the rate of saving always leads to slower growth as is insisted by Costabile and Rowthorn, the encouragement to increase of wealth must be the greatest when the rate of saving is extremely low and there cannot be some intermediate point at which it is the greatest. In spite of Costabile and Rowthorn, then, the problem of the optimal rate of saving which Malthus posed but was unable to solve remains unsolved.<sup>2)</sup> This problem is, however, a double-edged sword. If an increase in the rate of saving always increases the rate of growth as is concluded from the analysis of our Smithian growth model, the encouragement to increase of wealth must be the greatest when the rate of saving is extremely high and there cannot be some intermediate point at which it is the greatest. The problem can be answered neither by Malthus's theory of wages and growth reconstructed by Costabile and Rowthorn nor by the application of our Smithian growth model.

To deal with this Marthusian problem of the optimal rate of saving, we have to modify our Smithian growth model by taking into consideration Malthus theory of the motive to production, by which Malthus distinguished himself even from Smith, let alone from Ricardo. When the first edition of Principles was published, Malthus wrote to Ricardo. "You constantly say that it is not a question about the motives to produce. Now I have certainly intended to make it almost entirely a question about motives. We see in almost every part of the world vast powers of production which are not put into action, and I explain this phenomenon by saying that

from the want of proper distribution of the actual produce adequate motives are not furnished to continued production" ( Ricardo (1952), p. 10 ).

When the rate of saving is raised excessively, the rate of profit falls and the real wage rises, as a result of an increase in supply against demand of the produce, to such an extent that the capitalists' motive to production is destroyed. A rise in the real wage implies that commodities fall in value compared with labor. "The deficiency in the value of what they produced would necessarily make them either consume more, or produce less; and when the mere pleasure of present expenditure, without the accompaniments of an improved local situation and an advance in rank, is put in opposition to the continued labour of attending to business during the greatest part of the day, the probability is that a considerable body of them will be induced to prefer the latter alternative, and produce less" ( Malthus (1836), pp. 400 - 401 ). "By employing ten families he might perhaps, owing to the richness of the soil, obtain food for fifty; but he would find no proportionate market for this additional food, and would be soon sensible that he had wasted his time and attention in superintending the labour of so many persons. He would be disposed therefore to employ a smaller number" ( Malthus (1836), p.332 ).

(6)

Let us consider the situation of a capitalist who owns one unit of the stock of the product and saves  $ec$  of his stock. To save is to refrain from consumption and his loss of utility from consumption is  $u_1 ec$ , where  $u_1$  is the constant marginal utility of the current consumption. By saving  $ec$  to employ labor in the production, however, he can have

$(1+r)ec$  of the product in the next period, the utility of which is  $u_2(1+r)ec$  where  $u_2$  is the constant marginal utility of the future consumption and the holding of the stock ( Steedman (1989), pp. 225 - 229 ), while the utility is merely  $u_2ec$  if he does not employ labor to produce. In addition to the comparison of the current and future consumption and stock holding, furthermore, he has to take into consideration his time and attention in superintending the laborers in the production. If real wage is  $w/p$ , he employs  $ecp/w$  employees, which is equal to  $(1+r)aec$  in view of the definition of  $r$ , i.e., (4) in section (2). Let us assume that the disutility of superintending  $(1+r)aec$  laborers in the production is  $u_3(1+r)aec$ , where  $u_3$  is the constant marginal disutility of superintending.

For the adequate motives to be furnished to production, then,

$$(12) \quad u_2(1+r) - u_3(1+r)a > u_2$$

must be satisfied. To assure the inequality in (12), the rate of profit must be higher than

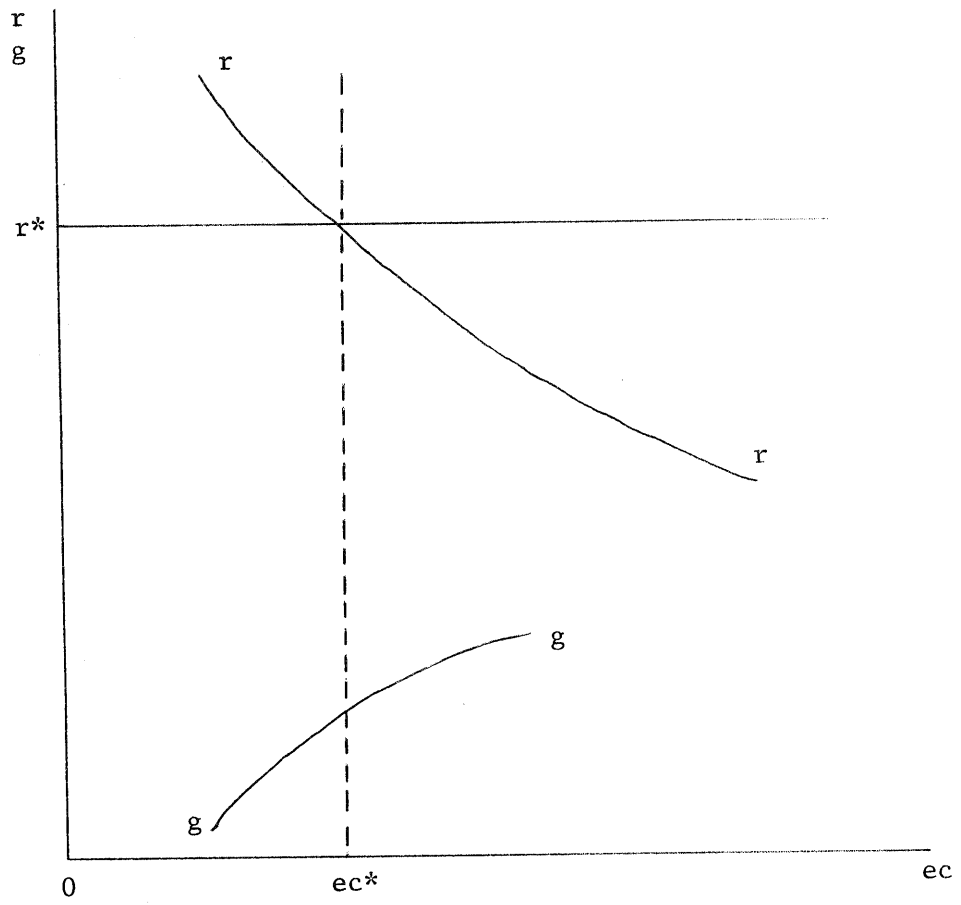
$$(13) \quad r^* = u_3 / ( u_2 - au_3 ).$$

We may assume that  $u_2 - au_3 > 0$ , otherwise no motives can be furnished to production, however high the rate of profit is. Then, the critical value of the rate of profit  $r^*$  is positive. If the rate of profit  $r$  is lower than  $r^*$ , the motive to production would be destroyed and our capitalist would cease to produce.<sup>3)</sup> The higher the rate of saving and the lower the rate of profit, the more capitalists lose the motive to production. As Malthus expected, then, capitalists are induced to produce less, which " is precisely what is meant by the term glut " ( Malthus (1863), p. 316 ).

In the figure, the rate of saving  $ec$  is measured horizontally, and the rate of profit  $r$  and the rate of growth  $g$ , vertically. Curves  $rr$  and  $gg$  show, respectively, the relation between  $ec$  and  $r$  and the relation between  $ec$  and  $g$ , which are generated from the Smithian growth model in section (2). If the rate of saving is very low, the encouragement

to increase of wealth is very small. An increase in the rate of saving increases the encouragement to increase of wealth, as is shown by the curve  $gg$ . If the rate of saving is very high so that  $r$  is lower than  $r^*$ , however, the motives to production is destroyed and there is no encouragement to increase of wealth. If the rate of saving is slightly less than  $ec^*$ , then, the encouragement to increase of wealth is the greatest. This is our answer to Malthus's problem of the optimal rate of saving.<sup>4)</sup>





## Footnotes

- 1) To speak of the proportion which capital bears to labor in the regulating principle of profits, we must assume that there is no unproductive labor so that  $c = 1$  and  $ecX(t) = eX(t)$ .
- 2) Another difficulty with Costabile and Rowthorn is their insistence that the labor market may not be cleared since the rate of profit and therefore the real wage are determined in the product market so as to equate investment and saving. If so, in spite of Costabile and Rowthorn, there is nothing to do with Malthusian regulating principle in which the rate of profit is, as we saw, determined " by the abundance or scarcity of capital --- as compared with the labor which it employs " ( Malthus (1863), p.276 ).
- 3) For Malthus, there is no possibility of the reaction of capitalist to increase the current consumption. As was mentioned in the previous section, this is because " pleasure of present expenditure " (  $u_1$  ) can not be expected to work.
- 4) If the rate of saving of capitalists  $ec$  is higher than  $ec^*$ , there must be landlords, government, etc. with the lower rate of saving so as to make the rate of aggregate saving optimal. This is the reason why Malthus emphasized the importance of the consumption of the unproductive classes.

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