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DIFFERENTIAL INFORMATION, MONOPOLISTIC COMPETITION, AND INVESTMENT

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ABSTRACT

The investment behavior of monopolistically competitive firms is investigated under the assumption of incomplete information about the average investment. Firms' profits are affected by productivity and demand shocks. These shocks consist of two types of disturbances, industry-wide and idiosyncratic ones. The main results are: (1) Investment under incomplete information is more volatile than that under complete information if firms can observe their own productivity and demand conditions (but not their components), while the result is ambiguous if firms can have only noisy signals of them. (2) Increased competition always destabilizes investment and raises its volatility.

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1. INTRODUCTION

In this paper, I investigate the investment behavior of monopolistically competitive firms under the assumption of incomplete information about the average investment. A fundamental assumption in this paper is that a typical firm does not have complete information about its competitors' investment; hence the average investment, which affects the firm's equilibrium profit, is not known with certainty when the firm determines its investment. The firm forms rational expectations about the unknown average investment on the basis of available information. Two questions are addressed in this environment. First, does incomplete information destabilize investment? Second, under incomplete information, is increased competition stabilizing?

The answer to the first question depends on whether or not there exists additional incomplete information about individual demand and productivity conditions. If the firm can observe its productivity and demand conditions in determining its investment, incomplete information about the average investment increases the sensitivity of investment to aggregate shocks, and thus destabilizes investment. This result is related to the one in Lucas (1975). However, if the firm has only incomplete information about its own productivity and demand conditions, the result is dependent on the accuracy of the firm's forecast about those conditions. If they can be accurately forecasted, incomplete information increases the sensitivity of investment and destabilizes it, while if the forecast contains a large error, incomplete information reduces the sensitivity and stabilizes it.

By contrast, the answer to the second question is unambiguous. Increased competition always <u>increases</u> the sensitivity of investment to aggregate shocks and thus destabilizes investment, regardless of whether the firm's own demand and cost conditions are observed or not.

The plan of this paper is as follows. In Section 2, the model is laid out. In order to concentrate our analysis on the effect of the incomplete average investment information, I assume that the product market is under complete information. The case of productivity shocks is investigated in this section. The firm does not have complete information about the industry-wide productivity shock in determining its investment. I consider two cases. In the first case, the firm observes its own productivity condition in determining its investment, while in the second case, the firm has only incomplete information about the condition. In Section 3 the case of demand shocks is analyzed. The future demand condition is not observable when the firm determines its investment, but the firm has a noisy information about it. The main findings of this paper are presented in these two sections. In Section 4, I extend the analysis to the cases of increasing unit variable costs, risk aversion, and free entry. There I show that the main results in Sections 2 and 3 still hold true in a general setting. In addition, I show the sensitivity of investment may be different between the peak and the trough of a business cycle if we allow free entry. Section 5 concludes the paper.

2. PRODUCTIVITY SHOCKS AND INDUSTRY INVESTMENT

In this paper, I consider the investment decision of firms in a monopolistically competitive industry. The model is the same as in Nishimura (1986), except that capital stocks are explicitly analyzed.

In order to make analysis clear, I assume the simplest investment technology. That is, capital goods in this industry are assumed to be depreciated fully in one period. They are used up in one period, like seeds in agriculture. The characteristic of investment analyzed in this paper is that the investment decision takes place before actual market conditions are known to firms. (An alternative interpretation of the model is also possible, in which the demand for variable inputs of production is analyzed, so long as decision about such variable inputs has to be made before actual market conditions are known to firms. In this interpretation, "investment" in the following analysis is replaced by the "demand for variable inputs.")

In this setting, the firm's problem is reduced to the following two-period problem. In the first period, the firm determines its capital stock in the next period. In the second period, the firm determines the price of its products, taking account of its own capital stock and the average price in the market. At the end of the second period, the firm produces the products and sells them to consumers. The product market in the second period is assumed to be monopolistically competitive, while for simplicity (1) the labor market in the second period, (2) the financial market in the first period, and (3) the capital goods market in the first period are all assumed to be perfectly competitive.

In Sections 2 and 3, I assume (1) constant unit variable costs,[1] (2) the risk neutrality of firms, and (3) the exogenously determined number of firms.[2] The effects of increasing unit variable costs, risk aversion, and free entry will be discussed in Section 4. In Section 2, I analyze the case of productivity shocks. There is no shock in demand. I investigate demand shocks in Section 3.

Let us consider information available to the firm in each period. In the first period (investment period), productivity shocks are realized. There are two kinds of productivity shocks: one is industry-wide, and the other is firm-specific. The individual productivity shock at the firm level consists of these two shocks. The firm is assumed to observe its own productivity shock, but does not observe the two components separately.[3] The firm is also assumed to have no information about other firms' productivity levels and their investment levels. The firm has to determine its investment before knowing the average investment and the industry-wide productivity condition.

In the second period (production period), all information is assumed to be known to the firm. It knows the average capital stock, the average productivity and the average price in the market. Thus, there is no uncertainty in the second period. This assumption is made in order to concentrate the effect of competition in the investment process.[4,5]

The following analysis depends crucially on the assumption of this first-period (investment) incomplete information and induced strategic uncertainty. This seems a realistic assumption about industries in which many relatively small firms compete with one another. In such industries, information sharing is not practical because of large administrative costs as well as anti-trust consideration.[6] The model of this paper is intended to describe these industries.[7]

It is helpful to consider the firm's decision backward from the second period to the first.[8] Throughout this paper, all variables are in logarithm, if not noted otherwise. Investment goods are taken as a numeraire, so that all prices in this paper are relative ones.

The Second Period

The demand for the firm's products in the second period is

(1) $$q^d = -m(p - \bar{p}) + \bar{q}^d,$$

where

(2) $\bar{q}^d = - b\bar{p}$,

in which p is the firm's price, $\bar{p}$ is the average price, q^d is the individual demand, and $\bar{q}^d$ is the average demand. In the following, I use the geometric average. The parameter m is the price elasticity of the individual demand, while b is that of the average demand. I assume m > b, that is, the slope of the individual demand curve is flatter than that of the market demand curve. I also assume b > 1, which implies that the optimal monopoly price would be finite if the market were monopolized by one firm.[9]

In the case of productivity shocks, the production function is

(3) $q = \ell + \gamma k + \beta$, where $\beta = g + x$.

Here q is output, ℓ is labor, and k is capital. I assume that the parameter γ in (3) satisfies $1/(m - 1) > \gamma > 0$. This condition is necessary for profit maximization. The term β represents the productivity shock, which consists of the industry-wide component g and the firm-specific one x. The random variables g and x satisfy $Eg = Ex = Egx = 0$, $Eg^2 = \sigma_g^2$, and $Ex^2 = \sigma_x^2$.

Note that the capital stock, k, is determined in the first period, and thus it is given in the second period. The capital stock k differs from firm to firm. The firm is identified by a pair (k, x) in the second period. Because of complete information, the firm (k, x)'s problem is to maximize $\exp[\pi] = \exp[p]\exp[q^d] - \exp[w]\exp[\ell]$ with respect to p. Then the individual price is

$$(4)\qquad p = \log\{m/(m - 1)\} + w - \gamma k - \beta.$$

The average price is $\bar{p} = \log\{m/(m - 1)\} + w - \gamma\bar{k} - g$, where $\bar{k}$ is the average of k.

Using (4) we have the second-period equilibrium profit function

$$(5)\quad \pi(k, \bar{k}, \beta, g) = \phi_0 + \phi_v(k - \bar{k}) + \phi_{\bar{k}}\bar{k} + (m - 1)\beta - (m - b)g, \text{ where}$$

$$(6)\quad \phi_0 = - b\log m + (b-1)\log(m-1) - (b-1)w,\ \phi_v = (m-1)\gamma, \text{ and } \phi_{\bar{k}} = (b-1)\gamma.$$

Because $m > b > 1$ and $1/(m - 1) > \gamma > 0$, we have $1 > \phi_v > \phi_{\bar{k}} > 0$.

The second-period profit function has the following properties. First, if <u>ceteris paribus</u> the firm's productivity (β) increases, the firm's profit increases ($m - 1 > 0$), while if other firms' productivity (g) increases, the firm's profit decreases ($-(m - b) < 0$). Second, if <u>ceteris paribus</u> the firm's capital stock (k) increases, the firm's profit increases ($\phi_v > 0$), while if other firms' capital stock ($\bar{k}$) increases, its profit decreases ($-\phi_v + \phi_{\bar{k}} < 0$). If the firm's investment is not accompanied by other firms', the firm has cost advantage relative to other firms in the second period. Then the firm can lower its price relative to other firms', attract more customers, and thus make more profits. On the other hand, if other firms' investment increases, the firm suffers from cost disadvantage, so that the firm's profit decreases. This is the way competition among firms works in the investment process.

<u>The First Period</u>

In the first period, the firm knows the functional form of the profit function, and the cost of investment:

(7) $c = c_0 + k.$

Here c_0 is a constant. Thus, I assume the constant marginal cost of investment.[10]

Information available to the firm is incomplete. First, the average investment $\bar{k}$ is not observable in determining its own investment. Second, although β is observable, g and x in β are not independently observable. All other parameters in (3) through (7) are known to the firm. The discount rate, r, is also known to the firm.

The firm forms expectations about $\bar{k}$ and g rationally, based on available information including β. Specifically, $\bar{k}$ and g are assumed to be jointly normally distributed with mean $(e(\bar{k}|\Omega), e(\bar{g}|\Omega))$ and variance-covariance matrix $V(\bar{k}, g|\Omega)$, where $e(\bar{k}|\Omega)$ and $e(g|\Omega)$ are, respectively, the linear least squares regressions of $\bar{k}$ and g on Ω, and $V(\bar{k}, g|\Omega)$ is their error variance-covariance matrix. The firm's expectation formation problem is a signal extraction problem, which has extensively been analyzed in the literature.

Note that the firm is identified in the first period by the firm-specific productivity disturbance x. Then, the firm x's problem in the first period is to maximize $\hat{E}\left[(1 + r)^{-1}\exp[\pi(k, \bar{k}, \beta, g)] - \exp[c] \right]$ with respect to k, subject to (5) and (7). Here $\hat{E}$ is the expectation operator with respect to the firm's subjective distribution of $\bar{k}$ and g.

The following optimal investment formula is derived from the first order condition of optimality.

(8) $k = (1 - \phi_V)^{-1}[z_g + (m - 1)\beta - (m - b)e(g|\Omega) - (\phi_V - \phi_{\bar{k}})e(\bar{k}|\Omega)]$, where

$$z_g = z^* + \frac{1}{2}f_g{}^t V(\bar{k}, g|\Omega)f_g \text{ and } z^* = \log \phi_v + \phi_0 - c_0 - \log(1 + r),$$

in which $f_g{}^t = (-(\phi_v - \phi_{\bar{k}}), -(m - b))$. Here t denotes the transpose.[11]

Note that $e(\bar{k}|\Omega)$ and $e(g|\Omega)$ are based on information β. Consequently, (8) implies that the average investment $\bar{k}$ is solely dependent on the average of β, that is, g. Thus, $\bar{k}$ and g are perfectly correlated. Uncertainty in (8) is reduced to uncertainty about only one variable, $\bar{k}$ (or g). Using this fact and the undetermined coefficient method,[12] we obtain

$$e(g|\Omega) = \lambda\beta \text{ and } e(\bar{k}|\Omega) = \frac{1}{1-\phi_{\bar{k}}} z_g + \frac{m-1-(m-b)\lambda}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\lambda}\lambda\beta, \text{ where}$$

(9)

$$\lambda = \frac{\sigma_g{}^2}{\sigma_g{}^2+\sigma_x{}^2}, \text{ and } z_g = z^* + \frac{1}{2}\{(\phi_v - \phi_{\bar{k}})\frac{m-1-(m-b)\lambda}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\lambda} + (m-b)\}^2\lambda\sigma_x{}^2.$$

Consequently, we get, taking (6) into account,

(10) $$\bar{k} = \frac{1}{1-\gamma(b-1)}z_g + \frac{m-1-(m-b)\lambda}{1-\gamma\{m-1-(m-b)\lambda\}}g.$$

The formula (10) completely characterizes the average investment under incomplete information.

Let us now compare the incomplete information case with the complete information case. Complete information about $\bar{k}$ implies $e(\bar{k}|\Omega) = \bar{k}$ and $V(\bar{k}|\Omega) = 0$. From (8) it is straightforward to show that the average investment under complete information is

(11) $$\bar{k} = \frac{1}{1-\gamma(b-1)}z^* + \frac{b-1}{1-\gamma(b-1)}g.$$

From the above two average investment equations, we immediately obtain two characteristics about the sensitivity of the average investment to the aggregate productivity shock g. First, incomplete information makes the average investment more sensitive to the aggregate productivity shock than complete information ($(\{m-1-(m-b)\lambda\}/\{1-\gamma\{m-1-(m-b)\lambda\} > (b-1)/\{1-\gamma(b-1)\})$}). Second, competition makes the average investment more sensitive to the aggregate productivity shock under incomplete information ($\partial[\partial\bar{k}/\partial g$ in $(10)]/\partial m > 0$), though it has no effect on the sensitivity under complete information ($\partial[\partial\bar{k}/\partial g$ in $(11)]/\partial m = 0$).

<u>The Reaction Curve</u>

The above results can be explained in the "reaction function" framework. Suppose that $g = 1$. Let $\Delta\bar{k}$ denote the response of the the average investment such that $\Delta\bar{k} = \bar{k} - \bar{\bar{k}}$, where $\bar{\bar{k}}$ is the unconditional mean of $\bar{k}$. Under incomplete information, we have $\bar{\bar{k}} = \{1 - \gamma(b - 1)\}^{-1}z_g$, while under complete information, $\bar{\bar{k}} = \{1 - \gamma(b - 1)\}^{-1}z^*$. Next, let the average expectation about the average investment, $\bar{e}(\bar{k}|\Omega)$, be the average of $e(\bar{k}|\Omega)$ over all firms. Using this definition, I define the response of the average expectation, $\Delta\bar{e}(\bar{k}|\Omega)$, such that $\Delta\bar{e}(\bar{k}|\Omega) = \bar{e}(\bar{k}|\Omega) - \bar{\bar{k}}$. Similarly, the average expectation about g is defined as the average of $e(g|\Omega)$ over all firms. The response of the average expectation about g, $\Delta\bar{e}(g|\Omega)$ is defined analogously, but we have $\Delta\bar{e}(g|\Omega) = \bar{e}(g|\Omega)$ because the unconditional mean of g is zero.

Using these definitions, (6), and the assumption that $g = 1$, we can transform the individual investment formula (8) into the following "reaction function."

(12) $\Delta\bar{k} = \frac{1}{1-\gamma(m-1)}\{(m - 1) + (m - b)\Delta\bar{e}(g|\Omega)\} - \frac{(m-b)\gamma}{1-\gamma(m-1)}\Delta\bar{e}(\bar{k}|\Omega).$

This equation characterizes the firm's response in investment as a reaction to the expected response of the average investment. The average is taken here in order to cancel out the effect of the firm-specific disturbance x.

On the one hand, under complete information, we have $\Delta\bar{e}(g|\Omega) = g = 1$ by definition, so that the complete-information reaction function is

$$(13) \qquad \Delta\bar{k} = \frac{b-1}{1-\gamma(m-1)} - \frac{(m-b)\gamma}{1-\gamma(m-1)}\Delta\bar{e}(\bar{k}|\Omega).$$

This reaction function is represented by AA in Figure 1. In this figure, the vertical axis is $\Delta\bar{k}$, and the horizontal axis is $\Delta\bar{e}(\bar{k}|\Omega)$. The equilibrium is the intersection of this reaction curve and the "expectation curve," which relates the average expectation to the average investment. The expectation curve under complete information is OC, the forty-five degree line, because by definition we have $\Delta\bar{k} = \Delta\bar{e}(\bar{k}|\Omega)$. Thus, the complete information equilibrium is E.

On the other hand, under incomplete information, we have $e(g|\Omega) = \lambda g = \lambda$. Consequently, the incomplete-information reaction function is

$$(14) \qquad \Delta\bar{k} = \frac{m-1-(m-b)\lambda}{1-\gamma(m-1)} - \frac{(m-b)\gamma}{1-\gamma(m-1)}\Delta\bar{e}(\bar{k}|\Omega).$$

This is represented by BB in Figure 1. BB is above AA by $\{(m - b)(1 - \lambda)\}/\{1 - \gamma(m - 1)\} > 0$. Next, the expectation curve under incomplete information is no longer a forty-five degree line. We obtain $\Delta\bar{k} = (1/\lambda)\Delta\bar{e}(\bar{k}|\Omega)$ under incomplete information (see (9) and (10)), which is represented by OD. Because λ is less than unity (the actual average investment is more sensitive than the average expectations), OD is steeper than OC. The incomplete information equilibrium is the intersection F of BB and OD. It is evident from this figure that so long as $\lambda < 1$, incomplete information makes investment more responsive to g, and thus more volatile.

The effect of incomplete information can conveniently be decomposed into two parts: the impact effect and the repercussion effect. First, the impact effect is the effect of incomplete information taking expectations about $\bar{k}$ held constant. This is the effect of uncertainty about the average productivity g. Note that, for given β, an increase in g reduces the demand for the firm's products through an increase in the average price, and thus reduces the incentive to investment. However, because of the signal extraction problem, the estimate of g is less sensitive to the signal β. This is because part of the signal could be local. Since the firm underestimates the average productivity increase (and the average price in the next period) when g = 1, it believes that it has room to expand its own investment, even if its expectations about the average investment do not change. Thus, the impact effect is positive. The vertical distance between BB and AA measures this effect.

Second, the repercussion effect is the effect of incomplete information through expectations about $\bar{k}$, taking the impact effect as given. Because of the signal extraction problem with respect to $\bar{k}$, λ is less than unity, implying that the estimate of the average investment is less sensitive to the signal β. Since the firm underestimates the average investment when g = 1, it again believes that it has room to expand own investment. Thus, the repercussion effect is also positive. This effect is measured by $(1/\lambda)$, the degree of rotation of OD from OC.[13]

The two effects of incomplete information affect investment in the same direction and thus investment is unambiguously more sensitive to the aggregate productivity shock under incomplete information.

Next consider the effect of increased competition. An increase in m rotates AA clockwise around E. Thus, the complete information equilibrium does not change. However, BB shifts to B'B' and the new incomplete information equilibrium is F', which is above F so long as $\lambda < 1$. Increased competition unambiguously increases the sensitivity of investment.

This result can also be explained by local-global confusion in the signal extraction problem. Take again the case of g = 1. Increased competition implies that the firm expects larger returns from an increase in the capital stock (an increase in Φ_V in the equilibrium profit function), if such an increase is not accompanied by an increase in other firms' capital stock. If the firm's investment is matched by other firms' investment, the profit opportunity due to increased competition dissipates ($\Phi_{\bar{k}}$ in the profit function is independent of m). Under incomplete information the average investment has to be estimated relying on β. Because the estimate of the average investment is less sensitive to the signal than the actual average investment, the initial incentive to increase investment does not dissipate even after estimating the average investment rationally. Thus, the average investment becomes more sensitive to g, and its volatility increases.

Additional Uncertainty about the Individual Productivity Condition

So far I have assumed that the individual productivity shock is observed in the first (investment) period. Thus, there has been no uncertainty about the firm's own productivity condition. This is the case, for example, if the firm hires workers and observes their productivity in the first period.[14] In this setting, I have investigated incomplete information about the average investment (incomplete investment information). Uncertainty caused by this incomplete information can be called the strategic uncertainty, because it is uncertainty about other firms' strategies (levels of investment).

In some cases, however, the firm's own productivity condition may not be observable in the first period. For example, the firm may have to determine its investment well before knowing the productivity of its workers. In such cases, the firm has to face another kind of incomplete information, namely, incomplete information about the firm's own productivity condition (incomplete own productivity information). Uncertainty caused by incomplete own productivity information can be called the non-strategic uncertainty, because it does not involve uncertainty about strategic interaction among firms.

The effect of the non-strategic uncertainty is opposite to the effect of the strategic uncertainty. The non-strategic uncertainty <u>reduces</u> the sensitivity of investment.

Suppose that β is now not observable in the first period, and that a productivity signal:[15]

(15) $$\hat{\beta} = \beta + y$$

is assumed to be observed, where y is an observational error. The random variable y is independent of g and x, and satisfies $Ey = 0$ and $Ey^2 = \sigma_y^2$. Using the same procedure employed earlier, we obtain

(16) $$\bar{k} = \frac{1}{1-\gamma(b-1)}\hat{z}_g + \frac{(m-1)\hat{\nu} - (m-b)\hat{\lambda}}{1 - \gamma\{m-1-(m-b)\hat{\lambda}\}}g. \text{ where}$$

$$\hat{\lambda} = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_x^2 + \sigma_y^2}, \quad \hat{\nu} = \frac{\sigma_g^2 + \sigma_x^2}{\sigma_g^2 + \sigma_x^2 + \sigma_y^2} \text{ and } \hat{z}_g = z^* + \frac{1}{2}\hat{f}_g^{\,t} V(\bar{k}, g, \hat{\beta}|\Omega)\hat{f}_g,$$

in which $\hat{f}_g^{\,t} = (-(\phi_v - \phi_{\bar{k}}), -(m - b), m - 1)$. From this relation, it can be shown that competition increases the sensitivity of the average investment even in this framework. However, because $\hat{\lambda} < \lambda$ and $\hat{\nu} < 1$, (16) and (10) show that incomplete information about the firm's own productivity condition reduces the sensitivity of the average investment to the industry-wide productivity shock.

The intuition behind this result is simple. Suppose that the firm observes an increase in the signal $\hat{\beta}$. Every firm knows that there is a chance that the change in the signal is only an observational error, so that it increases investment by less than it does if there is no uncertainty about the productivity condition. Thus, the average investment is less responsive to the aggregate productivity shock if the firm's own productivity condition is not observable.

3. DEMAND SHOCKS AND INDUSTRY INVESTMENT

Let us turn to the case of demand shocks. In the following, I show qualitatively the same results are obtained in the demand shock case as in the productivity shock case.

The Second period

The average demand and the individual demand are, respectively,

(17) $\bar{q}^d = - b\bar{p} + d$, and

(18) $q^d = - m(p - \bar{p}) + \bar{q}^d + u = - m(p - \bar{p}) - \bar{p} + \alpha$, where $\alpha = d + u$,

in which the random variables d and u are normally distributed, satisfying $Ed = Eu = Edu = 0$, $Ed^2 = \sigma_d^2$, and $Eu^2 = \sigma_u^2$. The term α is the individual demand disturbance, which consists of d, the disturbance common to all firms, and u, the idiosyncratic one. Because information is complete in the second period, the firm's optimal price is $p = \log\{m/(m - 1)\} + w - \gamma k$. Consequently, the second-period profit function is

$$(19) \qquad \pi(k, \bar{k}, \alpha) = \phi_0 + \phi_v(k - \bar{k}) + \phi_{\bar{k}}\bar{k} + \alpha.$$

The First Period

Note that α is the next period's demand condition, so that it is not in general observable. However, in order to make analysis clear, I first assume α is observable. This case corresponds to the case of the observable own productivity shock in the previous section. A more realistic case, in which α is not observable, is discussed later in this section.

When α is observable, the firm is identified by u in the second period. The firm u maximizes $\hat{E}\ (1 + r)^{-1}\exp[\pi(k, \bar{k}, \alpha)] - \exp[c_0 + k]$.

The second-period profit function (19) is simpler in the demand shock case than the one (5) in the productivity shock case, because the average demand d does not independently enter the profit function. Otherwise, the two profit functions are quite similar. The constant term, and the coefficients of $k - \bar{k}$ and $\bar{k}$ are the same. The coefficient of α in (19) is positive as the coefficient of β in (5). Thus, one can expect that the demand shock case is qualitatively similar to the productivity shock case.

Using the same procedure to the one employed in the previous section, we have

(20) $$\bar{k} = \frac{1}{1-\gamma(b-1)} z_d + \frac{1}{1-\gamma\{(m-1)-(m-b)\xi\}} d.$$

where $z_d = z^* + \frac{1}{2}(\phi_v - \phi_{\bar{k}})^2 V(\bar{k}|\Omega)$ and $\xi = \sigma_d^2/(\sigma_d^2 + \sigma_u^2)$.[16] By contrast, the average investment under complete information is

(21) $$\bar{k} = \frac{1}{1-\gamma(b-1)} z^* + \frac{1}{1-\gamma(b-1)} d.$$

Comparing these two expressions, we know that the same conclusion is obtained in the demand shock case as in the productivity case. Incomplete investment information increases the sensitivity of the average investment to the aggregate demand shock. An increase in competition also increases its sensitivity.

The Effect of Incomplete Own Demand Information

Next, consider a more realistic case in which the firm has only incomplete information about α. Suppose that the firm can obtain a noisy signal $\hat{\alpha}$:[17]

(22) $$\hat{\alpha} = \alpha + v,$$

where v is a normally distributed forecast error, which is independent of d and u, and satisfies $Ev = 0$ and $Ev^2 = \sigma_v^2$. This case corresponds to the case in which the firm's own productivity is not observable, in the previous section.

The determination of the average investment in this case can be explained in Figure 2, which is similar to Figure 1. In this figure, I assume $d = 1$, and $\Delta\bar{k}$ and $\Delta\bar{e}(\bar{k}|\Omega)$ are defined in the same way as in Figure 1.

Note that the optimal investment formula in the case of incomplete information about α is

(23) $$k = (1 - \phi_v)^{-1}[\hat{z}_d + e(\alpha|\Omega) - (\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)], \text{ where}$$

$$\hat{z}_d = z^* + \frac{1}{2}f_d{}^tV(\bar{k}, \alpha|\Omega)f_d, \text{ in which } f_d{}^t = (-(\phi_v - \phi_{\bar{k}}), 1).$$

Thus, the reaction function is from (23)

(24) $$\Delta\bar{k} = \frac{1}{1-\gamma(m-1)}\Delta\bar{e}(\alpha|\Omega) - \frac{(m-b)\gamma}{1-\gamma(m-1)}\Delta\bar{e}(\bar{k}|\Omega).$$

Under complete information we have $\Delta\bar{e}(\alpha|\Omega) = d = 1$. AA represents this complete information reaction curve. OC is the forty-five degree line, representing the expectation curve. Thus the complete information equilibrium is the intersection E of AA and OC.

Under incomplete information, $\Delta\bar{e}(\alpha|\Omega) = \hat{\tau}d = \hat{\tau}$, where $\hat{\tau} = \sigma_\alpha^2/(\sigma_\alpha^2 + \sigma_v^2) = (\sigma_d^2 + \sigma_u^2)/(\sigma_d^2 + \sigma_u^2 + \sigma_v^2)$. The reaction curve in this case is BB. BB is below AA by $(1 - \hat{\tau})/\{1 - \gamma(m - 1)\}$. OD is the expectation curve such as $\Delta\bar{k} = (1/\hat{\xi})\Delta\bar{e}(\bar{k}|\Omega)$, where $\hat{\xi} = \sigma_d^2/(\sigma_d^2 + \sigma_u^2 + \sigma_v^2)$. The incomplete information equilibrium is the intersection F of BB and OD. From this figure, it is evident that the overall effect of incomplete information on the sensitivity of investment is ambiguous.

We obtain the ambiguous result in this case because we have two conflicting effects of incomplete information. As explained earlier, if α is observable, incomplete investment information increases the sensitivity of $\bar{k}$ to d. This effect can be represented in Figure 2 as the effect of switching from OC to OD, where AA is held unchanged. However, as it can be inferred from the case of incomplete information about own productivity, incomplete information about own demand reduces the sensitivity. The possibility of confusion makes the firm underestimate α, so that it will not increase investment as much as in the case of complete own demand information. Thus, BB in Figure 2 lies below AA. This effect reduces the sensitivity of the average investment to the aggregate demand shock.

By contrast, increased competition always increases the sensitivity. The reason is the same as in the case of productivity shocks.

Summary

The results in the previous and this sections can be summarized in the following way. Regardless of the nature of shocks, increased competition always increases the sensitivity of the average investment to aggregate shocks under incomplete information, while it has no effect on the sensitivity under complete information. Thus, competition is destabilizing.

The effect of incomplete investment information is also destabilizing, because it increases the sensitivity of the average investment. However, incomplete information about the individual productivity shock and the individual demand shock decreases the sensitivity. Consequently, the overall effect of incomplete information depends on the relative magnitudes of these two effects.

If the productivity condition in the production stage can be relatively predictable in the investment stage compared with the demand condition, the effect of incomplete information is determined by the nature of shocks. If productivity is subject to shocks, incomplete information increases the volatility of investment. However, if demand is subject to shocks, the result is ambiguous.

4. EXTENSION

In this section, I show that the results obtained in the previous sections still hold even if three changes are made in the basic assumptions of the model. The first is increasing unit variable costs, the second is risk aversion, and the third is free entry. The model considered in this section is the simplest one in the previous sections, namely, the model of demand shocks in which α is observable ($v = 0$).

Increasing Unit Variable Costs

I sketch the main result here. The detailed discussion is relegated to the unpublished APPENDIX.[18] Suppose that the production function is now

$$(25) \qquad q = \delta\Omega + \gamma k,$$

where δ and γ satisfy (i) $1 > \delta > 0$, (ii) $1 > \gamma > 0$, and (iii) $\{m/(m - 1)\} > \delta + \gamma$.[19] Then, the second-period profit function has the form

$$(26) \qquad \pi(k, \bar{k}, d, u) = \phi_0^* + \phi_v^*(k - \bar{k}) + \phi_{\bar{k}}^*\bar{k} + \phi_d^* d + \phi_u^* u,$$

where ϕ_0^*, ϕ_v^*, $\phi_{\bar{k}}^*$, ϕ_d^*, and ϕ_u^* are constants depending on the parameters of the model. Specifically, we obtain $\phi_d^* \neq \phi_u^*$ so long as $m > b$. This implies that even if α is observable, this is not sufficient to infer $\phi_d^* d + \phi_u^* u$ correctly. Thus, increasing unit variable costs introduces additional uncertainty into the profit function. The random variables d and u have to be estimated from α. Because $\phi_d^* > 0$ and $\phi_u^* > 0$, the effect of local-global confusion in estimating d and u is similar to the one in estimating α in the case of the unobservable α. Increasing unit variable costs reduce the sensitivity of investment under incomplete information.

Whether this sensitivity-reducing effect of increasing unit variable costs dominates the sensitivity-increasing effect of incomplete investment information depends on the parameters of the model. In the unpublished APPENDIX I show that in this particular example the overall effect of incomplete information is to increase the volatility if and only if $\gamma + \delta > 1$. Thus, the results obtained in the previous sections still hold even though unit variable costs of production are increasing, so long as the technology exhibits increasing returns to scale.

Risk Aversion

Next, consider the effect of risk aversion. Suppose that a firm is owned by one investor having constant relative risk aversion. The investor receives income Y in the current period but no income in the next period. Then, the firm maximizes $\hat{E}\,[(1 + r)^{-1}u(V') + u(V)]$, where $u(V) = V^{\zeta}/\zeta$, $V' = \exp[\phi_0 + \phi_v k - (\phi_v - \phi_{\bar{k}})\bar{k} + \alpha]$, and $V = Y - \exp[c_0 + k]$. Here ζ satisfies $\zeta \neq 0$ and $\zeta < 1$, and r is the (utility) discount rate.

The optimal investment formula in this case is

$$k = \{1 - \zeta\phi_v + (1 - \zeta)h\}^{-1}[z_r + \zeta\alpha - \zeta(\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)], \text{ where}$$

(27)

$$z_r = z_r^* + \tfrac{1}{2}\zeta^2(\phi_v - \phi_{\bar{k}})^2\, V(\bar{k}|\Omega), \text{ and } z_r^* = z^* + (1 - \zeta)(g - \phi_0).$$

Here $h = \exp(c_0 + \bar{\bar{k}})/\{Y - \exp(c_0 + \bar{\bar{k}})\}$ and $g = \log\{Y - \exp(c_0 + \bar{\bar{k}})\} + h\bar{\bar{k}}$.[20] Using a similar procedure to the one in the previous section,[21] we obtain the average investment equation such as

$$\text{(28)} \qquad \bar{k} = \frac{z_r}{1 - \zeta\gamma(b-1) + (1-\zeta)h} + \frac{\zeta}{1 - \zeta\gamma(m-1) + \zeta\gamma(m-b)\xi + (1-\zeta)h}\, d.$$

The qualitative results of the previous sections still hold under risk aversion. It is easy to see from (28) that incomplete information increases the sensitivity of the average investment, and that increased competition increases the sensitivity under incomplete information. However, the magnitude of these effects are now dependent on the degree of risk aversion ς.

Free Entry

In the preceding analysis, the number of firms is fixed. However, in many industries new firms enter the market if there is profit opportunity, and old firms exit if their operation is unprofitable. Thus, the number of firms is endogenously determined rather than exogenous. In the remaining of this section, I investigate the effect of free entry on the sensitivity of investment to aggregate shocks.

The effect of free entry can be decomposed into two parts. First, free entry affects the average demand, for the given own-price elasticity m of the individual demand. Second, free entry may change m, which is the degree of competitiveness in the industry.[22]

Let us first consider the first effect. The average demand is

$$(29) \qquad \bar{q}^d = (-b\bar{p} + d) - t, \text{ where } t = \log T,$$

in which T is the number of firms. If the number of firms increases then the average demand (total demand divided by the number of firms) decreases. The individual demand is now $q^d = -m(p - \bar{p}) - b\bar{p} + \alpha - t$, which also decreases when t increases. However, under the constant unit variable cost, the second-period price is independent of t. Using this fact, we can show that the second-period profit function is

(30) $$\pi(k, \bar{k}, \alpha, t) = (\Phi_0 - t) + \Phi_V(k - \bar{k}) + \Phi_{\bar{k}}\bar{k} + \alpha.$$

Thus, if we replace Φ_0 with $\Phi_0 - t$, the analysis in the previous two sections holds true without any change, so long as t is known with certainty.

Consider the determination of the number of firms. There is a fixed cost of production. (Up until now I have ignored the fixed cost, because it does not affect the short-run behavior of firms.) A new firm enters the market (an old firm stays in the market) if the unconditional mean of its profit is non-negative. The number of firms is determined by the zero expected profit condition. Using the perfect information about the structure of the economy including the distributional characteristics of the disturbances, the firm can calculate this unconditional mean of the profit, and then correctly predicts the number of firms in the market. Thus, there is no uncertainty about t. Consequently, the sensitivity of the average investment to aggregate productivity and demand shocks does not depend on the number of firms, t, as it is independent of Φ_0. Free entry does not at all affect the result obtained in the fixed-number model.

Next, let us turn to the case in which the number of firms affects the own-price elasticity m. If an increase in the number of firms (t) increases the competitiveness of the industry (m), free entry produces a asymmetric response of investment to aggregate shocks between peaks and troughs in business cycles.

In order to incorporate the effect of business cycles, let us now assume $\bar{q}^d = g - b\bar{p} + d$, where g represents the observable general economic condition. The number of firms is larger under the strong average demand (a large g) than under the weak average demand (a small g), because the number of firms is determined by the zero expected profit condition. The results of Sections 2 and 3 imply that an increase in m increases the sensitivity of investment to aggregate shocks. Consequently, the sensitivity is larger under large g, than under small g. Thus, the average investment is more sensitive to aggregate shocks, and thus more volatile, in the peak of a business cycle than in the trough.

5. CONCLUDING REMARKS

The major finding of this paper is that increased competition unambiguously increases the volatility of investment under incomplete information about the average investment, regardless of whether the shocks are on the demand side or on the supply side, and of whether they are observed with a noise or without it. This is in a sharp contrast with the effect of competition on the volatility of prices under incomplete information about the average price. I show in Nishimura (1986) that increased competition unambiguously reduces the volatility of prices, or equivalently, increases the rigidity of prices.

The model developed here has many shortcomings as the model of investment. The most serious one may be the characterization of capital as an input used up in one period. This assumption reduces the firm's investment problem to a static two-period problem, and thus simplifies expectation formation of firms. To generalize the monopolistically competitive investment model into an explicit dynamic rational expectations framework is a challenging task, and an important topic of future research.

NOTES

1. Weitzman (1985) among others argues that this is a reasonable assumption for the short-run analysis.

2. In fact, I assume a continuum of firms.

3. A possible story is: workers are hired in the first period already, and found to be good, but the firm does not know whether it is its luck or a general phenomenon. The case in which the productivity condition is not observable will be discussed later in this section.

4. I show in Nishimura (1986) that competition increases the elasticity of output to aggregate shocks under incomplete second-period price information, although competition does not influence the elasticity under complete information. More volatile output may induce more volatile investment. Thus, competition is likely to destabilize investment under incomplete price information. However, by assuming complete information in the second period, I rule out this possibility. Consequently, the effect of competition found in this paper is mainly the one in the investment process.

5. What we are concerned with in the following analysis is the <u>sensitivity</u> (or more precisely, elasticity) of investment to aggregate shocks. It should be noted here that competition affects the <u>average level</u> of investment for an obvious reason. An increase in competition increases the average level of output. In order to increase output, investment should be increased. Thus, competition increases the average level of investment.

6. Partial information sharing involving only a subset of firms is also possible, but it does not perfectly eliminate incomplete information about $\bar{k}$.

7. Information sharing may not be practical even if the number of firms is relatively small. Explicit information sharing through trade associations and the like may not be feasible because of anti-trust law considerations. The authority may be suspicious of such activities because they may lead to tacit collusion. Moreover, even if we ignore such consideration, non-cooperative firms may not find it profitable to agree on a binding information-sharing agreement (see the literature of information sharing in oligopolistic markets such as Clarke (1983) and Gal-Or (1986)). Although collusion coupled with information sharing always yields the maximum profit (monopoly profit), a binding information sharing agreement under non-cooperative behavior is not always profit-increasing.

In addition, there are some doubts about the practicality of information sharing if a binding agreement is not possible. In order to have true information sharing it is necessary for firms to announce their information truthfully. However, if the agreement is not binding, there may be the possibility of strategic misrepresentation (see Okuno-Fujiwara, Postlewaite, and Suzumura (1987)). Information sharing and the accompanying possibility of strategic misrepresentation are confounding issues, which are beyond the scope of this paper.

8. In this paper, I analyze the subgame-perfect Bayesian-Nash investment equilibrium in this industry.

9. It is possible to derive the demand function from a log-linear log-normal model of the consumer having CES utility (see Nishimura (1988)).

10. Increasing costs of investment does not change the results obtained in this paper. See the next footnote.

11. In the case of increasing marginal costs of investment (that is, $c = c_0 + (1 + c_1)k$, where $c_1 > 0$), the optimal investment formula is

$$k = (1 + c_1 - \phi_v)^{-1}[\tilde{z}_g + (m - 1)\beta - (m - b)e(g|\Omega) - (\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)],$$

where $\tilde{z}_g$ is appropriately defined. Thus, the investment formula under increasing marginal costs differs from the one in the text in that $1 - \phi_{\bar{k}}$ and z_g are replaced by, respectively, $1 + c_1 - \phi_{\bar{k}}$ and $\tilde{z}_g$. Because such changes do not significantly affect the following analysis, increasing marginal costs of investment do not change the results of this paper.

12. The detailed derivation of rational expectations in this paper is presented in the unpublished supplement, which is available from the author upon request.

13. The smaller λ is, the larger $(1/\lambda)$ is, and thus the more volatile investment is. Thus an increase in the volatility of the firm-specific productivity shock (an increase in σ_x) increases the volatility of investment.

14. Another possible story is: the productivity of capital goods is a random variable. Machines are bought in the first period, and found to be good, but the firm does not know whether it is its luck or a general phenomenon.

15. For example, consider an automobile industry. There is a continuum of product types such as passenger cars, within which there is a continuum of firms producing differentiated brands, such as Pontiac and Celica. The

disturbance x is "type-specific," and is uniform over all brands of the same type. However, firms producing the brands of the same type have different information about their productivity conditions and thus have different estimates of them. Thus the firm is characterized by its own error in the forecast, y, and the type of products it produces, x.

16. The optimal investment formula is $k = (1-\phi_v)^{-1}[z_d + \alpha - (\phi_v-\phi_{\bar{k}})e(\bar{k}|\Omega)]$. Using the undetermined coefficient method similar to the one employed in the case of productivity shock, we obtain $e(d|\Omega) = \xi\alpha$ and $e(\bar{k}|\Omega) = (1-\phi_{\bar{k}})^{-1}z_d + \{1-\phi_v+(\phi_v-\phi_{\bar{k}})\xi\}^{-1}\xi\alpha$. Consequently, taking (6) into account, we obtain the average investment in the text.

17. A more desirable procedure is to assume explicit dynamic stochastic processes about d, u and v, and to analyze the optimal forecast. The model in the text can be considered as an approximation to such dynamic models.

18. The APPENDIX is available from the author upon request.

19. The last assumption is necessary for profit maximization.

20. g and h are the coefficients of the linear approximation to $\log[Y - \exp(c_0 + k)]$ around $\bar{\bar{k}}$, where $\bar{\bar{k}}$ is the unconditional mean of $\bar{k}$. That is, $\log\{Y - \exp(c_0 + k)\} \approx g - hk$.

21. The derivation of rational expectations is presented in the unpublished supplement, which is available from the author upon request.

22. Sattinger (1984) presents a market model in which m is equal to the number of firms in the market.

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SUPPLEMENT

DERIVATION OF RATIONAL EXPECTATIONS

Productivity Shocks

I first consider the case in which β is observable. Recall the firm's optimal investment formula (8). Note that because $\beta = g + x$, we have $e(g|\Omega) = \lambda\beta$ and $V(g|\Omega) = \lambda\sigma_x^2$, where $\lambda = \sigma_g^2/(\sigma_g^2 + \sigma_x^2)$. Let us assume that

$$\text{(S1)} \qquad \bar{k} = L + Mg.$$

Then, we obtain $g = (1/M)(\bar{k} - L)$, so that the second period profit function is

$$\pi = \phi_0 + \phi_v k - \{\phi_v - \phi_{\bar{k}} + \frac{m - b}{M}\}\bar{k} + (m - 1)\beta + (m - b)\frac{L}{M}.$$

(S1) also implies

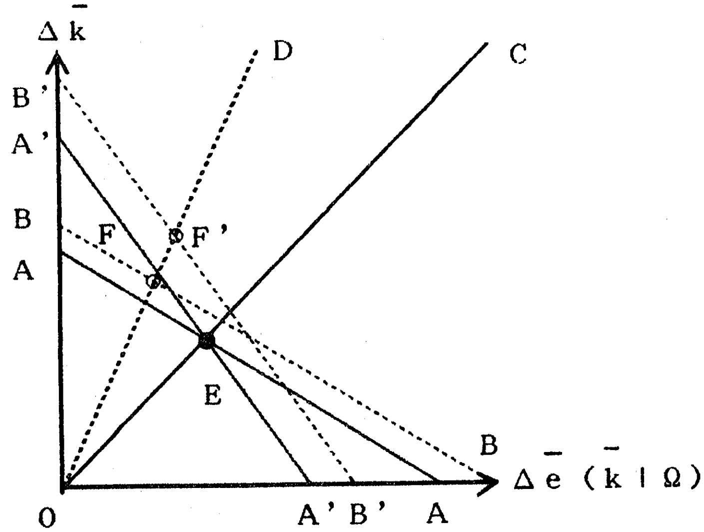
$$e(\bar{k}|\Omega) = L + M\lambda\beta \text{ and } V(\bar{k}|\Omega) = M^2\lambda\sigma_x^2.$$

Using these expectations, we can transform the first-order condition (8) into

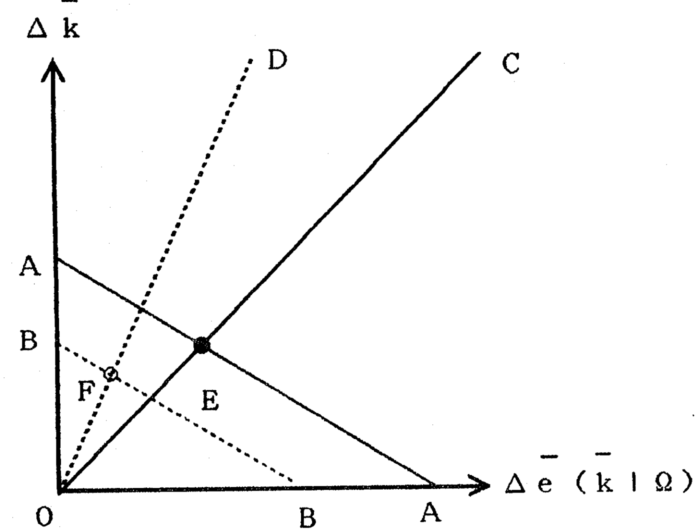
$$\text{(S2)} \quad (1 - \phi_v)k = z_g + (m - 1)\beta + (m - b)\frac{L}{M} - \{\phi_v - \phi_{\bar{k}} + \frac{m - b}{M}\}e(\bar{k}|\Omega)$$

$$= z_g + (m - 1)\beta + (m - b)\frac{L}{M} - \{\phi_v - \phi_{\bar{k}} + \frac{m - b}{M}\}(L + M\lambda\beta),$$

where $\qquad z_g = z^* + \frac{1}{2}\{\phi_v - \phi_{\bar{k}} + \frac{m - b}{M}\}^2 V(\bar{k}|\Omega)$



F I G U R E 1
P r o d u c t i v i t y S h o c k s



F I G U R E 2
D e m a n d S h o c k s

$$= z^* + \frac{1}{2}\{(\phi_v - \phi_{\bar{k}})M + (m - b)\}^2 \lambda\sigma_x^2.$$

From the above transformed condition, rational expectations can be obtained in the following steps. First, average (S2) over all firms to get the expression of $\bar{k}$ in terms of g. Then, apply $e(\cdot|\Omega)$ on both sides of the resulting expression. Collecting terms in the resulting expression, we obtain

$$L = (1 - \phi_{\bar{k}})^{-1} z_g \text{ and } M = \frac{m-1-(m-b)\lambda}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\lambda}.$$

Consequently we get

$$e(\bar{k}|\Omega) = (1 - \phi_{\bar{k}})^{-1} z_g + \{1 - \phi_v + (\phi_v - \phi_{\bar{k}})\lambda\}^{-1}\{m - 1 - (m - b)\lambda\}\lambda\beta.$$

Second, let us now consider the case in which only $\hat{\beta} = \beta + y$ is observable. The first order condition is

$$\text{(S3)}\quad (1 - \phi_v)k = [\hat{z}_g + (m - 1)e(\beta|\Omega) - (m - b)e(g|\Omega) - (\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)].$$

Note that because $\hat{\beta} = \beta + y = g + x + y$, we have $e(g|\Omega) = \hat{\lambda}\hat{\beta}$ and $V(\beta|\Omega) = \hat{\nu}\hat{\beta}$, where $\hat{\lambda} = \sigma_g^2/(\sigma_g^2 + \sigma_x^2 + \sigma_y^2)$ and $\hat{\nu} = (\sigma_g^2 + \sigma_x^2)/(\sigma_g^2 + \sigma_x^2 + \sigma_y^2)$. Let us assume that

$$\text{(S4)}\qquad \bar{k} = \hat{L} + \hat{M}g.$$

From now on, follow the same procedure as in the case of observable β. Then we have

$$\hat{L} = \frac{\hat{z}_g}{1 - \phi_{\bar{k}}}, \quad \hat{M} = \frac{(m-1)\hat{\nu}-(m-b)\hat{\lambda}}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\hat{\lambda}}, \text{ and } e(\bar{k}|\Omega) = \frac{\hat{z}_g}{1 - \phi_{\bar{k}}} + \frac{(m-1)\hat{\nu}-(m-b)\hat{\lambda}}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\hat{\lambda}}\hat{\lambda}\hat{\beta}.$$

Demand Shocks

I consider the case in which only $\hat{\alpha} = \alpha + v$ is observable. The case in which α is observable can be obtained by setting $v = 0$. The firm's optimal investment formula is

$$(1\ ) \qquad k = (1 - \phi_v)^{-1}[z_d + e(\alpha|\Omega) - (\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)].$$

Note first that from $\hat{\alpha} = \alpha + v = d + u + v$, we have $e(\alpha|\Omega) = \hat{\tau}\hat{\alpha}$ and $e(d|\Omega) = \hat{\xi}\hat{\alpha}$. Let $\bar{k} = L + Kd$. Insert these expressions into the optimal investment formula (1), and average over all firms. Then we obtain $\bar{k}$ as a function of d. Apply $e(\cdot|\Omega)$ to the both sides of the resulting expression, and collect terms. Then we have

$$L = \frac{\hat{z}_d}{1 - \phi_{\bar{k}}}, \quad K = \frac{\hat{\tau}}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\hat{\xi}} \text{ and } e(\bar{k}|\Omega) = \frac{\hat{z}_d}{1 - \phi_{\bar{k}}} + \frac{\hat{\tau}}{1-\phi_v+(\phi_v-\phi_{\bar{k}})\hat{\xi}}\hat{\xi}\hat{\alpha}.$$

Risk Aversion

The firm's optimal investment formula is

$$k = \{1 - \zeta\phi_v + (1 - \zeta)h\}^{-1}[z_r + \zeta\alpha - \zeta(\phi_v - \phi_{\bar{k}})e(\bar{k}|\Omega)].$$

Note that $e(d|\Omega) = \xi\alpha$, where $\xi = \sigma_d^2/(\sigma_d^2 + \sigma_u^2)$. Let $e(\bar{k}|\Omega) = J + K\alpha$. Then using the same procedure in the demand shock case, we obtain from the above investment formula

$$e(\bar{k}|\Omega) = \{1 - \zeta\phi_{\bar{k}} + (1 - \zeta)h\}^{-1}z_r + \{1 - \zeta\phi_v + \zeta(\phi_v - \phi_{\bar{k}})\xi + (1 - \zeta)h\}^{-1}\zeta\xi\alpha$$

Using this we obtain the average investment equation in the text.

APPENDIX

THE CASE OF INCREASING UNIT VARIABLE COSTS

The Second Period

The firm's problem is

$$\underset{p}{\text{Max}}\ [\exp[p]\exp[q^d] - \exp[w]\exp[\ell]] ,$$

subject to the demand function $q^d = - m(p - \bar{p}) - b\bar{p} + \alpha$, and the production function $q = \delta\ell + \gamma k$. The optimal price is

(A1) $$p = (1 + c_1 m)^{-1}\{a + h(k) + c_1\alpha + c_1(m - b)\bar{p}\},\ \text{where}$$

(A2) $$a^* = \log\{m/(m - 1)\},\ h(k) = w - \delta^{-1}\gamma k - \log\delta,\ \text{ and } c_1 = \delta^{-1} - 1.$$

Then the average price is

(A3) $$\bar{p} = (1 + c_1 b)^{-1}\{a^* + h(\bar{k}) + c_1 d\}.$$

Using the above results, we have the following second-period profit function

$$\pi(k, \bar{k}, d, u) = \phi_0^* + \phi_v^*(k - \bar{k}) + \phi_{\bar{k}}^*\bar{k} + \phi_d^* d + \phi_u^* u,\ \text{where}$$

(A4)

$$\phi_0^* = \log\{1 - \frac{\delta(m - 1)}{m}\} - \frac{b-1}{1+c_1 b}a^* - \frac{b-1}{1+c_1 b}(w-\log\delta),$$

$$1 > \phi_v^* = \frac{(m - 1)\gamma}{(1 + c_1 m)\delta} > 0, \qquad 1 > \phi_{\bar{k}}^* = \frac{(b - 1)\gamma}{(1 + c_1 b)\delta} > 0,$$

$$\phi_d^* = \frac{1 + c_1}{1 + c_1 b} > 0, \qquad \phi_u^* = \frac{1 + c_1}{1 + c_1 m} > 0.$$

The First Period

The firm's problem in the first period is now

$$\underset{k}{\text{Max}}\ \hat{E}\ [\ (1 + r)^{-1}\exp[\pi(k, \bar{k}, u, d)] - \exp[c]\ ],$$

where $\hat{E}$ is taken with respect to the subjective distribution of $\bar{k}$, d, and u. The optimal investment formula is

(A5) $k = (1 - \phi_v)^{-1}\{z_i - (\phi_v^* - \phi_{\bar{k}}^*)e(\bar{k}|\Omega) + \phi_d^* e(d|\Omega) + \phi_u^* e(u|\Omega)\}$. where

(A6) $z_i = z_i^* + \frac{1}{2}f_i^t V(\bar{k}, u, d|\Omega)f_i$, and $z_i^* = \log\phi_v^* + \phi_0^* - c_0 - \log(1+r)$.

in which $f_i = (-(\phi_v^* - \phi_{\bar{k}}^*), \phi_u^*, \phi_d^*)^t$. By the method of undetermined coefficients, the following forecasts in (A5) are obtained.

(A7) $e(u|\Omega) = (1 - \xi)\alpha$, $e(d|\Omega) = \xi\alpha$, and $e(\bar{k}|\Omega) = \psi_0 + \psi_d \xi\alpha$, where

(A8) $\psi_0 = (1 - \phi_{\bar{k}}^*)^{-1} z_i$, and $\psi_d = \{1 - \xi\phi_{\bar{k}}^* - (1-\xi)\phi_v^*\}^{-1}\{\phi_u^*(1-\xi) + \phi_d^*\xi\}$.

Here $\xi = \sigma_d^2/(\sigma_d^2 + \sigma_u^2)$. Using the above results we obtain the following average investment equation under incomplete information

(A9) $\bar{k} = \psi_0 + \psi_d d.$

Next consider the case of complete information. It is straightforward to show that the corresponding average investment equation under complete information is

(A10) $k^* = \psi_0^* + \psi_d^* d$, where $\psi_0^* = (1 - \phi_{\bar{k}})^{-1} z_i^*$, and $\psi_d^* = (1 - \phi_{\bar{k}})^{-1} \phi_d^*$.

The effect of incomplete information is summarized in the ratio ψ_d/ψ_d^*. From (A8) through (A10) we obtain

(A11) $$\psi_d/\psi_d^* = \left[\frac{1 - \phi_{\bar{k}}^*}{1 - \xi\phi_{\bar{k}}^* - (1 - \xi)\phi_v^*}\right]\left[\frac{\phi_u^*(1 - \xi) + \phi_d^*\xi}{\phi_d^*}\right].$$

The term in the first bracket is the effect of incomplete information about $\bar{k}$ (repercussion effect). This is larger than unity. The term in the second bracket is the effect incomplete information about d and u (impact effect). This term is less than unity. The overall effect depends on whether technology exhibits <u>long-run</u> increasing returns to scale ($\gamma + \delta > 1$) or decreasing ones ($\gamma + \delta < 1$), as shown below.

Note that

(A12) $$(\psi_d/\psi_d^*) - 1 = \frac{\Delta(1 - \xi)(\gamma + \delta - 1)}{\phi_d^*\{\delta(1 - \phi_{\bar{k}}) - \Delta\gamma(1 - \xi)\}}, \text{ where}$$

(A13) $$\Delta = (\phi_v^* - \phi_{\bar{k}}^*)(\frac{\delta}{\gamma}) = (\phi_d^* - \phi_u^*)\frac{1}{c_1} = \frac{(1 + c_1)(m - b)}{(1 + c_1 m)(1 + c_1 b)} > 0.$$

It is straightforward to show that the denominator in (A12) is positive under our assumptions. Consequently, if $\delta + \gamma > 1$, then we obtain $(\Psi_d/\Psi_d^*) > 1$.

Next consider the effect of competition. As in the effect of incomplete information <u>per se</u>, the sign of $\partial(\Psi_d/\Psi_d^*)/\partial m$ is dependent on the degree of long-run returns to scale. Recall (A11). As explained in the previous subsection, the first term in the right hand side of (A11) is the repercussion effect, while the second term is the impact effect. The derivative of the first term with respect to m is shown to be positive. However, because $\partial\Phi_u/\partial m < 0$, the derivative of the second term is negative. Since we have $\partial\Delta/\partial m > 0$, it can be shown from (A12) that we have $\partial(\Psi_d/\Psi_d^*)/\partial m > 0$ if and only if $\gamma + \delta > 1$.