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Bertrand Competition with Potential Entry and the Theory of Contestable Markets*

by

Takao Kobayashi
and
Akihiko Matsui

The University of Tokyo

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1. INTRODUCTION

Given a common U-shaped average cost curve for an industry the classical model of perfect competition and the recent contestable market theory by Baumol, Panzar and Willig [1982] both predict that the industry equilibriates in the long run at a point where price equals the minimum average cost and each incumbent firm operates at the minimum-average-cost level of output, which is known to be an efficient outcome. The former obtains the result by the assumption of price-taking firms and free entry, and the latter by the assumption of a contestable market.

Consider an industry producing a single product. Let $D(\cdot)$ be the industry demand function, and let $C(\cdot)$ be the cost function which applies to every firm. A contestable market is defined to be one in which a potential entrant faces no disadvantage *vis à vis* incumbents with respect to either the available production techniques (i.e., the cost function) or the perceptions of consumers as to the desirability of his product. Assuming that the market is contestable in this sense, a tuple $(p; m; y_1, \cdots, y_m)$, in which p denotes a price, m denotes the number of incumbent firms, and $y_1, \cdots, y_m$ denote positive output quantities of the m firms, is said to be a *sustainable equilibrium* if

(a) $\sum_{i=1}^{m} y_i = D(p)$;

(b) $p\, y_i - C(y_i) \geq 0$ for $i = 1, \cdots, m$;

(c) $p^e\, y^e - C(y^e) < 0$ for all $p^e < p$ and $y^e \leq D(p^e)$.

That is, the contestable market theory depicts an equilibrium industry configuration in which total demand equals total supply, each incumbent firm obtains nonnegative profit, and no further entry is possible. They then show that these three conditions imply the efficient industry outcome with the socially "right" amount of entry and with each firm operating at the minimum average cost. The classical assumption of price-taking firms plays no role in this new theory, and thus they claim that the threat of entry can serve to discipline the market even in the absence of the intra-industry competitive forces generated by the presence of many rival firms.

One can easily see that the key assumption which drives their result is condition (c), the no-profitable-entry condition. This condition reflects the contestability assumption: If an entrant offers a price below the price quoted by incumbents, he can sell any quantities he wishes, constrained only by the market demand at his price. And if an entrant cannot find an entry plan (p^e, y^e) which yields sufficient revenue to cover the production cost $C(y^e)$, then the industry is thought to be immune against new entry. However, there is more to it. The essential ingredient of condition (c) is that possibility of a new entry is evaluated relative to the incumbents' pre-entry price p. Taken literally, it implies that the entrant supposes the incumbents to be constrained from adjusting prices in response to entry, or more precisely, the incumbents set their prices with the very "timid" conjecture that a potential entrant would judge profitability of his entry taking no account of possible counteractions from the incumbents.[1/]

Theoretically there could be an infinite sequence of strategic actions and counteractions between incumbents and entrants, and so one must go into the arena of repeated game formulations to investigate the validity of their assumption. Maskin and Tirole [1986] is such an example. They show an interesting result that the (Markov) perfect equilibrium of their alternating-move dynamic game coincides with the prediction of the contestable market theory if the time discount factor is near one; that is, if firms are sufficiently far-sighted or one period (commitment time) is very short.[2/] But, this is not the direction we pursue in this paper.

The prediction of the contestable market theory that optimality of industry performance is attained even with a small number of firms is similar to that of Bertrand [1883] who modelled a market with price-setting oligopolists. But Baumol, Panzar and Willig claim that this resemblance is only superficial:

> "However, though the result corresponds precisely with that of our small group case, the underlying forces in the Bertrand model are fundamentally quite distinct from ours. The basic distinction once again resides in the crucial role of potential entrants in our analysis, as contrasted with that of incumbents in Bertrand's.[3/]

In fact the implicit competition between incumbents and potential entrants plays a crucial role in their result, but the assumption that the potential entrants evaluate the profitability of entry at the incumbent firms' pre-entry prices is the key element of condition (c), in spite of their assertion that the mode of competition is irrelevant to the outcome.[4/] It at least on the surface implies that potential entrants act in accordance with the Bertrand-Nash price-setting game. And this observation suggests that the efficiency result of the contestable market theory and the outcome of the Bertrand model of oligopolistic price competition may be related much more closely than they think. Yet, the contestable market theorists further discriminate between the two by saying:

> As is well known, the Bertrand's result is highly dependent upon the assumption that marginal costs are constant and equal to average cost throughout the relevant range. If marginal cost begins to rise, or lies below average cost at (what might otherwise be) an equilibrium output, serious problem arise for the existence and uniqueness of equilibrium price and output. Furthermore, the zero profit property tends to

evaporate, and with it the optimality of the quantity of resource allocated to the industry."[5/]

In this paper we extend the Bertrand's game of an oligopoly to include potential entrants and show that there exists a unique pure-strategy equilibrium which attains the efficient outcome under general conditions on the industry cost function. We construct a two-stage game in which firms, including potential entrants, announce their prices simultaneously in the first stage and then choose quantities of output, only constrained by the demand allocated to them depending on the price configuration.[6/]

Apparently, the sustainable equilibrium notion of the contestable market theory does not correspond to any game-theoretic equilibrium: The uniform price should be a result of the equilibrium outcome rather than an assumption, and the mechanism by which the market clears is ambiguous. Furthermore, as Elizabeth Bailey writes, the theory has an unfortunate feature that the sustainable equilibria may exist only rarely unless the average cost curves are "flat-bottomed".[7/] The latter problem, usually called the integer problem, is easily resolved in our formulation which accommodates unfulfilled demand in equilibrium. Thus, the present paper provides a general game-theoretic scenario for the contestable market theory.

In section 2 we illustrate the main result assuming that the average cost curve is distinctly U-shaped. In section 3 we show that the analysis is easily extended to the case of natural monopoly, to the case of flat-bottomed average cost functions, and to the case of large "residual demands" whose meaning will be clarified in the main text.

2. THE CASE OF A U-SHAPED AVERAGE COST CURVE

Consider a market for a single product. The market demand is given by a downward-sloping demand function $D(p)$. There are N firms who can access to an identical technology to manufacture the product. If y is the quantity of output each firm incurs production cost $C(y)$. The number N is assumed to be sufficiently large, whose meaning will be clarified in a moment. In this section we deal with the standard textbook case as depicted in Fig. 1, where the average cost curve is U-shaped with a unique minimum at (p_c, y_c) and the marginal cost curve is strictly monotone increasing in the range $y > y_c$.

<u>Fig. 1</u>

We start our discussion with the following basic observation. Suppose that a certain number of firms set their prices at p_c. Clearly the optimal (positive) quantity of output at this price is y_c. The total market demand at this price is $D(p_c)$. Therefore, the maximum number of firms who can operate at this price is an integer m_c which satisfies

$$m_c\, y_c \leq D(p_c) < (m_c+1)\, y_c. \tag{1}$$

Assume that the residual market demand is not large enough to cover the production cost for any level of output, as shown in the same figure. Then, given that m_c firms operate at price p_c there is no incentive for another to operate in the industry. And, if there is at least one outsider among the N firms who sets his price at p_c, no one among the m_c firms can gain by increasing his price, since it would only shift his current market share to the outsider and by assumption no profit opportunity would be left in the residual demand. This essentially corresponds to the equilibrium outcome of the contestable market theory.

To formalize this observation and to show that there would be no other equilibria we construct a two-stage noncooperative game as follows. All N

firms first announce their prices, which they cannot alter throughout the game. In the second stage, each firm chooses his quantity of output. The objective of each firm is to maximize his profit. Demand is allocated purely on the basis of price: Consumers first visit the lowest-price firm(s), and if goods are sold out the unfulfilled consumers visit the second-lowest-price firm(s), and so on. Several scenarios are possible to determine the amounts of demand which firms with higher prices can get, but our result does not depend on them.[8/] It is also irrelevant how demand is allocated among firms who set the same price.[9/]

We impose the following four assumptions.

Assumption 1. $N \geq m_c + 1$, where m_c is a positive integer defined by (1).

Assumption 2. If a firm is to choose between serving the market with zero profit and staying out of the market, he chooses the former.

Assumption 3. Given that m_c firms each produce y_c at price p_c the residual market demand curve, denote by R(p), lies below the average cost curve.

Assumption 4. If at least $m_c y_c$ units of demand are fulfilled at prices no less than p_c, the residual market demand is smaller than or equal to R(p) for all p.

Assumption 1 embodies the threat of potential entry. Assumption 2 is a technical assumption to ensure the uniqueness of equilibrium. Assumption 3 is necessary to sustain the competitive outcome as an equilibrium of our game. We will discuss the implication of deleting this assumption in the next section. Assumption 4 will be satisfied by any plausible scenario regarding the generation of the residual demand.

The output decision at the second stage is rather trivial. Since each firm is comitted to the price which he announced in the first stage, he acts as a price-taker with respect to this price. For the relevant price range $p \geq p_c$ define $Y(p)$ by

$$Y(p) = \underset{y > 0}{\text{argmax}} \ (p\, y - C(y)). \qquad (2)$$

We call $Y(p)$ the optimal supply schedule. This is obviously the inverse marginal cost function, and Assumption 2 is implicit in the definition of $Y(p)$ for $p=p_c$. Each firm is subject to a demand restriction which is determined by the market demand and the output decisions of other firms who set lower prices. Since his profit is monotone increasing in output below $Y(p)$, he will choose as his actual output the smaller of $Y(p)$ and the amount of demand allocated to him.[10/]

We have already shown that one (subgame-perfect) equilibrium of this game is the competitive outcome. In this equilibrium, every firm follows the output decision described above and at least $m_c + 1$ firms set prices at p_c, while the remaining firms set prices above p_c. This will result in m_c firms operating at (p_c, y_c) and others staying outside the industry with at least one of them announcing the price p_c. So the remaining task is to show that there is no other (pure-strategy) equilibria.

PROPOSITION 1. <u>No firm earns positive profit in any pure-strategy equilibrium of our game.</u>

PROOF. Suppose that a firm, say i, obtains positive equilibrium profit with a price-output pair (p_i, y_i). Then any other firm potentially can earn positive profit by setting his price at less than but sufficiently close to

p_i (which would give him at least y_i units of demand) and producing y_i. Therefore, positive equilibrium profit for at least one firm implies that every other firm must also earn positive profit in that equilibrium.

Since demand is allocated in the order of prices, all firms except the highest-price setter(s) must operate on the optimal supply schedule. We show that this in turn implies that there must exist at least two highest-price firms whose demands are less than the optimal supply level. Suppose the contrary and let firm i be a highest-price setter with price p_i, and other highest-price setters, if exist, operate on the optimal supply schedule. It implies that all firms except i produce on the optimal supply schedule; namely, if firm j's ($j \neq i$) price is p_j he is producing $Y(p_j)$. Then the sum of demands fulfilled by these N-1 firms is $\Sigma_{j \neq i} Y(p_j)$. Since $Y(p_j) \geq y_c$, it follows that $\Sigma_{j \neq i} Y(p_j) \geq (N-1)y_c \geq m_c y_c$, where the last inequality follows from Assumption 1. This means that the N-1 firms set their prices in the interval $(p_c, p_i]$ and obtain total demand of at least $m_c y_c$. Then Assumption 4 implies that the residual demand to firm i is not more than $R(p_i)$, and this coupled with Assumption 3 implies that firm i cannot get sufficient demand to cover the production cost. This obviously contradicts the assumption that firm i earns positive profit.

Thus, if at least one firm earns positive profit in equilibrium, there must exist at least two highest-price setters who hold positive shares of the market but not sufficient to operate on the optimal supply schedule. Then either of them can gain by undercutting his price and capturing an additional share from the other highest-price setter(s). This is incompatible with an equilibrium. ■

PROPOSITION 2. _Any firm who sets his price above_ p_c _has no market share in any pure-strategy equilibrium._

We use the following lemma to prove this proposition.

LEMMA. _At most one firm can charge price above_ p_c _and obtain a positive market share in any pure-strategy equilibrium._

PROOF. Suppose that a firm i charges a price $p_i > p_c$ and gets a positive market share in equilibrium. From Proposition 1 his profit must be zero, so his price-output pair (p_i, y_i) must lie on the downward-sloping portion of the average cost curve. Let A denote the point (see Fig. 2). Then he is operating below his optimal supply $Y(p_i)$, and this implies that his price must be highest among all active firms. Now note that no firm can set his price in the open interval (p_c, p_i); otherwise it would mean that he is operating below his optimal supply although his price is not the highest (Proposition 1 again implies that he must be producing on the average cost curve). Thus all other active firms must be operating either at A or at the bottom of the average cost curve, B. However, no one except i can operate at A, since if two firms are at A then either of the two would gain by undercutting his price and capturing an additional share from the other. Therefore, if a firm is at A in an equilibrium, all other active firms must be on B. ■

Fig. 2

PROOF OF PROPOSITION 2. Suppose a firm sets his price above p_c and gets a positive market share. From the previous lemma, all other active firms set their prices at p_c. Assumption 2 implies that there are m_c firms in the second group of firms. Then the residual demand to the first firm must be insufficient to cover his production cost, owing to Assumption 3. This means that the first firm's profit is negative, a contradiction. Therefore, all active firms must charge price p_c in any equilibrium. ■

Proposition 2 immediately implies that there is no pure-strategy equilibrium to our game except the competitive outcome with at least one outsider firm announcing price p_c.

3. EXTENSIONS OF THE RESULT

The result of the previous section can be extended to other situations. Here, we examine three of them. The first and economically most important is the case in which the industry turns out to be monopolized by a single firm at equilibrium . This is usually called the case of natural monopoly. The second is the case in which the average cost curve is flat-bottomed. The third is the case in which residual demands are large enough to intersect the average cost curve.

(1) Natural monopoly

The exposition in the last section already included a case of natural monopoly: namely, if m_c defined by (1) is 1 then a single firm operates in the industry with the price-output pair (p_c, y_c). Here we assume that either m_c is zero or the average cost curve is monotone decreasing (i.e., y_c is $+\infty$). The relation between the average cost curve and the industry demand curve typically looks like Fig. 3.[11] In this case we replace Assumption 1 by $N \geqq 2$ and delete Assumptions 3 and 4.

Fig. 3

Let p_ℓ denote the minimum price at which the average cost curve and the industry demand curve intersect. And let $y_\ell = D(p_\ell)$. An obvious equilibrium for this case is one in which a monopolist firm operates at (p_ℓ, y_ℓ) and at least one outsider firm announces price p_ℓ. This outcome is usually called the Ramsey second-best solution, and it again coincides with the prediction of the contestable market theory. Note that the presence of a potential entrant quoting the incumbent's price is critical to sustain this outcome as an equilibrium of our game. It is straightforward to prove that no other (pure-strategy) equilibrium exists.

PROPOSITION 3. The unique pure-strategy equilibrium for the game is one in which one firm operates at (p_ℓ, y_ℓ) and other N-1 firms stay outside the market setting prices higher than or equal to p_ℓ with at least one of them at p_ℓ.

PROOF. Since the average cost is declining in the relevant output range, the amount of optimal supply for every firm is larger than the total industry demand. Hence, the lowest-price firm(s) will produce as many goods as are demanded and other firms cannot capture any market share. Now, if a firm gains positive profit in an equilibrium, all others' equilibrium profits must also be positive, as eatablished in the proof of Proposition 1. This implies that every firm must set the same price, but then each has an incentive to undercut his price; contradicting to the nature of equilibrium. Therefore, no firm obtains positive profit in any equilibrium.

If the industry demand curve intersects the average cost curve only at p_ℓ, this completes the proof. If the two curves intersect at another price

$p' > p_\ell$, then there is another candidate for an equilibrium in which a firm operates at $(p', D(p'))$ and other firms stay outside the industry with prices higher than or equal to p'. However, Assumption 2 implies that any of the outsiders is better off by setting his price at p_ℓ and capturing the entire market with zero profit. Hence the latter is not an equilibrium. ■

(2) Flat-bottomed Average Cost Curve

The equilibrium notion of the contestable market theory requires market-clearing. Thus, if the total industry demand at the equilibrium price (= minimum average cost) is not a multiple of the minimum-average-cost level of output, there is no sustainable equilibrium. Baumol et. al. resolve this nonexistence problem, also called the integer problem, by resorting to the casual observation that the average cost curve is nearly flat around the bottom. Fig. 4 exhibits a typical flat-bottomed average cost curve.

Fig. 4

This assumption is unnecessary for our construction, since it permits unfulfilled demands in equilibrium. But, the discussion in the previous section is fully compatible with a flat-bottomed average cost curve.[12/] That is, all the statements remain valid by changing the definition of y_c to

$$y_c = \max \{\operatorname{argmin} C(y)/y\}.$$

(3) Large Residual Demand

The final situation we consider is one in which, given that m_c firms each supply y_c units of the good at price p_c, the residual market demand is large enough to cover the production cost for some output level. So we replace Assumption 3 by:

$$pR(p) - C(R(p)) > 0 \qquad \text{for some } p > 0.$$

Fig. 5 exhibits this case. We maintain Assumptions 2 and 4. The assumption securing a sufficient number of potential entrants, Assumption 1, is replaced by $N \geq m_c + 2$.

Fig. 5

In this case, a sort of natural monopoly occurs in the market for the residual demand. Let p_ℓ denote the minimum price at which the residual demand curve intersects the average cost curve. Let y_ℓ denote the corresponding quantity. Given that m_c firms operate at (p_c, y_c) there is some room for another firm to operate at (p_ℓ, y_ℓ). And if there is at least one outsider who announces price p_c, none of the $m_c + 1$ active firms can raise their price. Thus, this is clearly an equilibrium. Note that the outsider firm who announces price p_c rejects his orders since he cannot cover the production cost at this price level.

It is again straightforward to show that this is the only pure-strategy equilibrium of the game. The zero-equilibrium-profit property is proved by repeating the argument made in Proposition 1 and using the assumption that $N \geq m_c + 2$. The lemma of the previous section establishes that at most one firm can get a positive market share with price higher than p_c. And Assumption 2 implies that the minimum of the zero-profit prices, p_ℓ, is the only price that he can charge.

FOOTNOTES

1/ Baumol, Panzar and Willig claim that this assumption is irrelevant to motivate condition (c). They argue that condition (c) is a natural consequence of the assumption of costless exit, which is also a requirement for a contestable market. Namely, a firm will enter the market (without any concern about the incumbents' possible countermoves) if he expects a profit opportunity under the current price, since if the incumbents should react by undercutting prices and preclude all further profit to the entrant he could readily exit from the market without loss of investment. We think that this argument does not endorse condition (c), because such hit-and-run incursions into the market would only cause temporal loss of incumbents' profits and hence would not affect the long-run equilibrium configuration of the industry.

2/ The analysis of Maskin and Tirole is restricted to the case of declining average cost; hence only one monopolist firm operates in equilibrium. They also assume that firms choose quantities (capacities) and prices are set by an auctioneer to clear the market.

3/ Baumol, Panzar and Willig, op. cit. p.44.

4/ See footnote 1.

5/ Baumol, Panzar and Willig, op. cit. p.44.

6/ In spirit our work has a very similar perspective to Mirman, Tauman and Zang [1984]. But they only dealt with the case of natural monopoly and their result holds only in special situations where either there is a continuum of potential entrants or the demand curve is tangent to the average cost curve at a point and uniformly below elsewhere. Grossman [1981] obtained a similar result to ours by modifying the Bertrand's game in such a way that firms submit supply schedules and price is announced by an auctioneer to clear the market.

7/ Foreword to Baumol, Panzar and Willig, op. cit. p.xxi.

8/ For example, one can assume that each consumer demands only one unit of the product and purchasing orders are fulfilled according to the consumers' reservation prices (i.e., consumers with the lowest reservation prices are served first). In such a case, the residual demand to firms with price p will be D(p) - (the sum of orders fulfilled by firms charging prices less than p).

9/ The above discussion which sustains the competitive outcome as an equilibrium implies that demands are allocated unequally between "the incumbents" and "the outsiders", since outsiders who set price at p_c do not get the demand y_c.

10/ Rigorously speaking our game in the current format has a flaw in that the set of admissible quantities of output for each firm depends on other firms' output decisions. But it is easily remedied by assuming that in the second stage firms simply announce the maximum and the minimum quantities of output that they wish to supply (firms prefer the largest quantity in this range) and the actual outputs are determined by the demand allocation rule.

11/ In the case of decreasing average cost we assume that the two curves have an intersection. In the case of $m_c = 0$ they obviously intersect at an output level below y_c.

12/ This case is not covered in Grossman's formulation [1981]. We can extend our results to cases with more oddly-shaped average cost curves. The critical assumption in our proof is that the average cost is non-increasing in the range where output is smaller than y_c.

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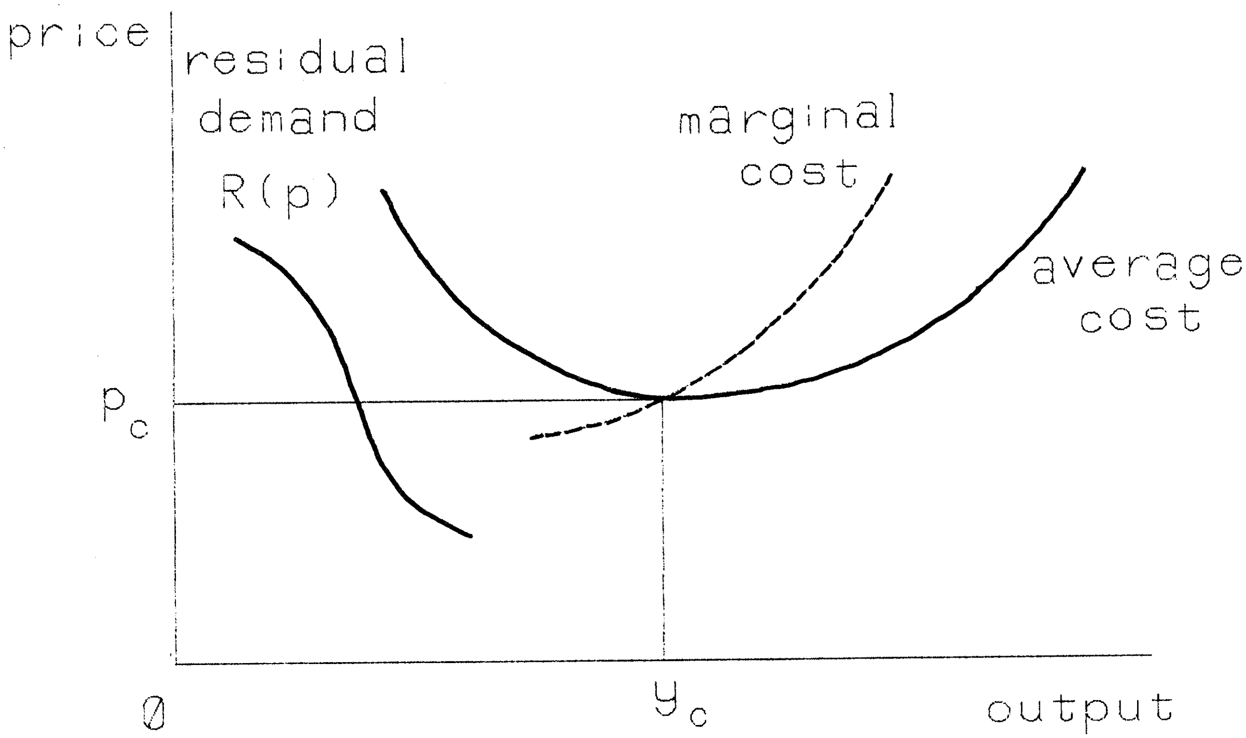
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Fig. 1

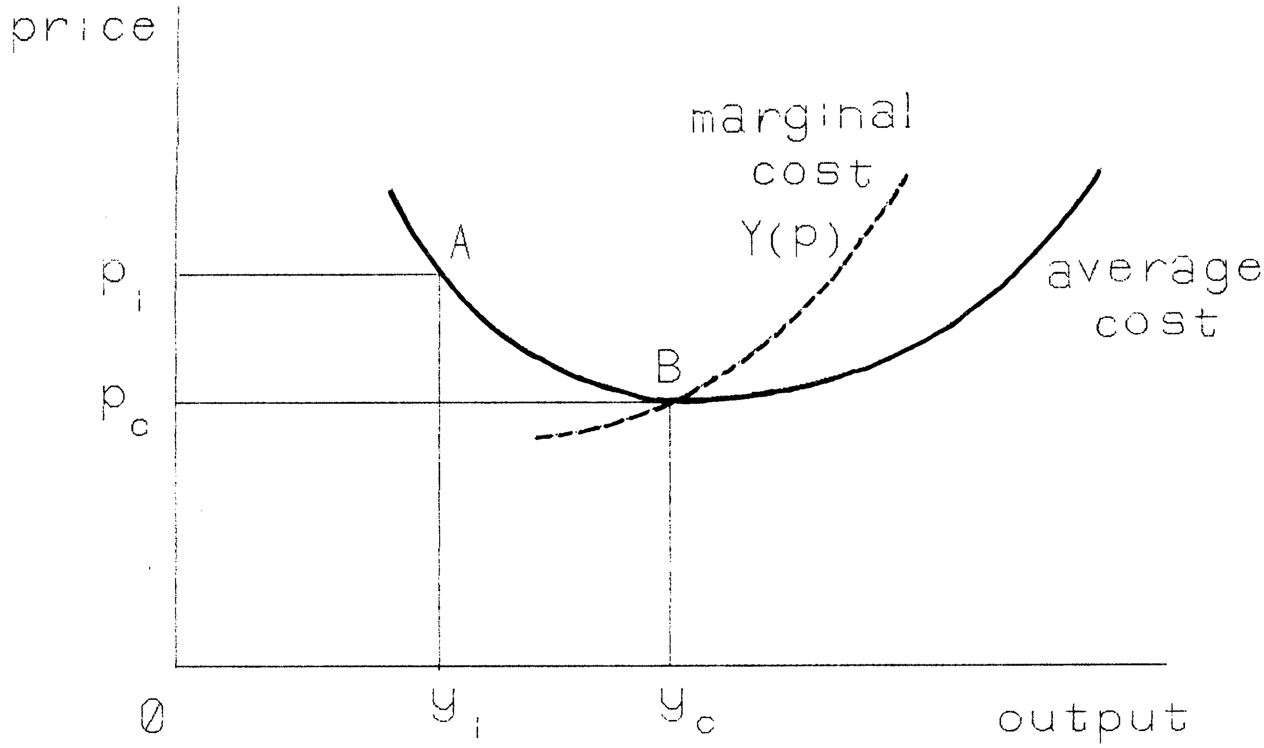


Fig. 2

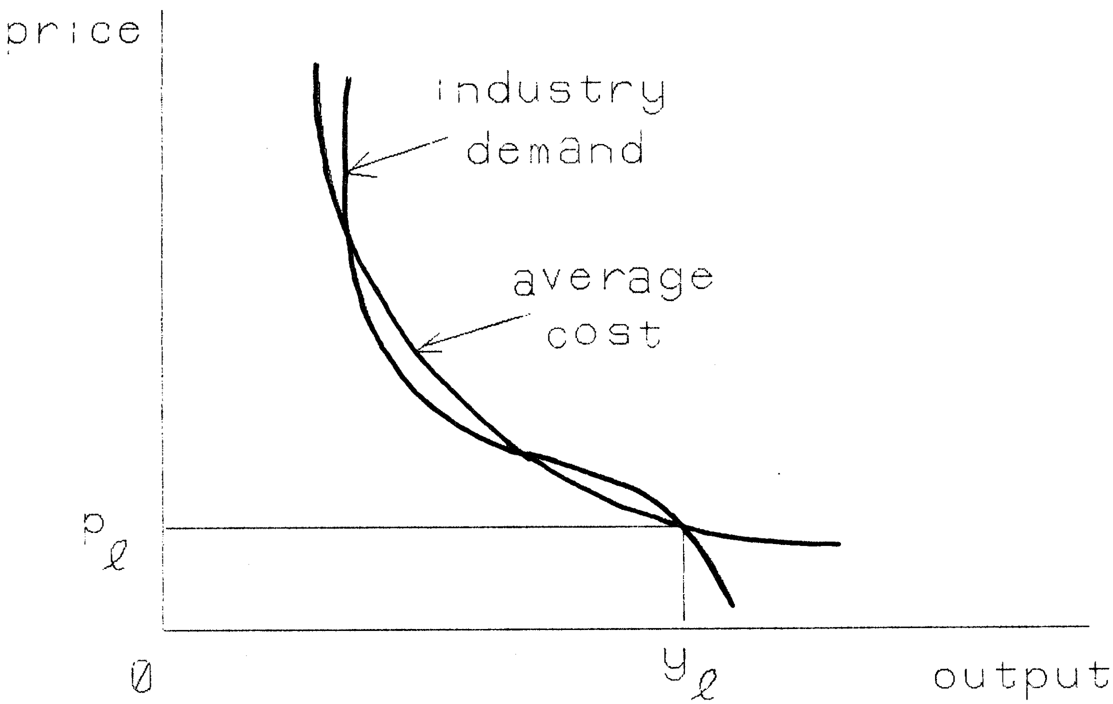


Fig. 3

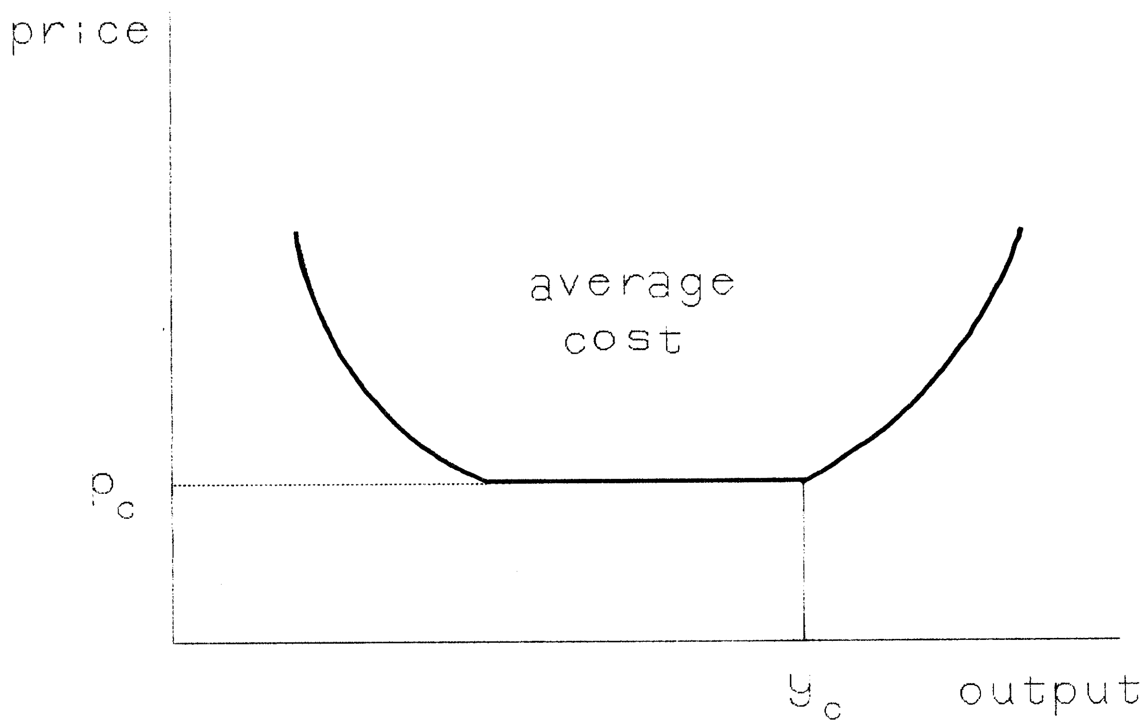
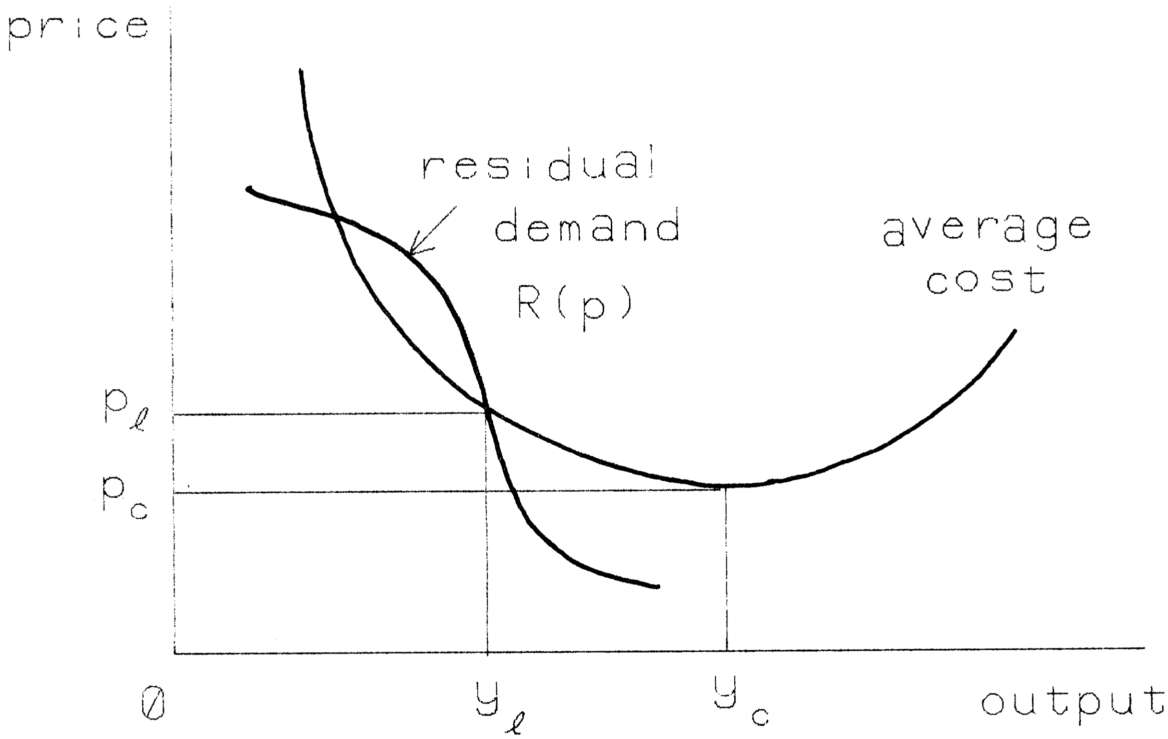


Fig. 4


price
residual
demand
R(p)
average
cost
p_ℓ
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0
y_ℓ
y_c
output

Fig. 5