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Tax Competition and Efficient Fiscal Transfers under Capital and Labor Income Taxes

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Abstract

This paper considers efficient fiscal transfer policies in a tax competition setting with ad valorem taxation (i.e., income taxation) on mobile capital and immobile labor. We show that fiscal equalization of regions' capital income tax bases eliminates the inefficiency of horizontal tax competition if these tax bases are evaluated by the average taxable return on capital in all regions, rather than the taxable return in each region. This equalization system, together with revenue matching grants that correct vertical externalities, achieves efficiency. By investigating the nature of horizontal and vertical externalities arising from non-cooperative regional tax policies, we derive formulas for efficient fiscal transfer policies and explain their workings.

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Keywords: horizontal and vertical tax competition; horizontal and vertical externalities; ad valorem taxes; fiscal transfers.

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1. Introduction

Much of the tax competition literature has argued that non-cooperative regional tax policies cause inefficiencies because they involve horizontal and vertical externalities. Although each region's tax policies affect other regions' welfare and federal/central government's tax revenue, the resulting external impacts are ignored under decentralized decision making. Wilson (1999), Wilson and Wildasin (2004), Fuest et al (2005), and Keen and Konrad (2013) provide comprehensive reviews of related studies.

In the literature, there has been continued interest in the role of fiscal transfer policies in achieving efficiency. Wildasin (1989) and Dahlby (1996) derive efficient revenue matching grants under which regions are compensated for the externalities caused by their tax policies.[1] Köthenbürger (2002) and Bucovetsky and Smart (2006) argue that tax base equalization, which primarily aims to reduce disparities in regional fiscal capacity, has the advantage of removing the inefficiency of horizontal capital tax competition. Under this equalization system, a region receives a transfer equal to the difference between its tax base and the average tax base of all regions, multiplied by the average tax rate of all regions.[2] This system is helpful in internalizing horizontal externalities because regions are compensated for the loss of tax bases when they raise tax rates. Kotsogiannis (2010) extends the analysis of tax base equalization to examine vertical externalities. However, Matsumoto (2021) notes that correcting vertical externalities through tax base equalization entails a trade-off between efficiency and regional equality. Whereas tax base equalization serves as a means of internalizing horizontal externalities, vertical externalities should be corrected by other policy tools such as revenue matching grants.

Sas (2017) and Liesegang and Runkel (2018) question the efficiency effect of tax base equalization. Whereas this effect has been discussed for per unit taxes, it may not be applicable to ad valorem taxes. Liesegang and Runkel (2018) show that when regions impose a tax on corporate income, inefficient horizontal tax competition persists even if tax base equalization is implemented.[3]

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[1] For example, under horizontal capital tax competition, a region's tax increase causes capital outflows and creates a positive externality on other regions. The efficient revenue matching grant at the margin is equal to the resulting increase in other regions' tax revenues.

[2] Tax base equalization, which is called the representative tax system, is the basis of fiscal equalization programs in Canadian, Denmark, Sweden, and Switzerland; see Smart (2007a, b).

[3] Sas (2017) considers both horizontal and vertical externalities. Although Sas's main attention is focused on

In general, when factor taxes are ad valorem, the external impacts of regional tax policies arise not only from tax base mobility but also from changes in factor returns or prices. Liesegang and Runkel (2018) argue that a transfer system composed of tax revenue and private income equalization can correct these externalities.[4]

This paper stands in sharp contrast to Sas (2017) and Liesegang and Runkel (2018). We show that the efficiency effect of tax base equalization is preserved even under factor income taxation if the equalization formula is modified such that each region's capital income tax base is evaluated by the average taxable return on capital in all regions, rather than the regional taxable return. Our framework is built on Matsumoto (2021) with multiple tax bases, which is more general than the single tax models of Sas (2017) and Liesegang and Runkel (2018). Per unit taxes on mobile capital and immobile labor in Matsumoto (2021) are replaced with taxes on capital and labor income. The supply of these factors is variable because of endogenous savings and labor-leisure decisions. To remedy inefficient horizontal tax competition, our equalization system should be applied only to capital income tax base. By investigating the nature of horizontal externalities due to non-cooperative regional tax policies, we provide detailed arguments on how our modified tax base equalization system works. Moreover, we compare our equalization system with the standard tax base equalization system considered by Sas (2017) and Liesegang and Runkel (2018), and discuss the reason for why the latter system fails to internalize horizontal externalities under ad valorem taxes.

Our model with multiple tax bases also yields interesting results regarding tax base overlap between federal and regional governments. We allow for both net and gross fiscal transfers to distinguish between tax base equalization and revenue matching grants. The latter grants correct vertical externalities. We analyze whether the efficient matching rates for regional income taxes are positive or negative, which reflects the sign of the vertical externalities that should be corrected by fiscal transfer policies. The standard argument on negative vertical externalities in the tax

the distortion caused by a residence-based income tax on mobile labor, the case of a source-based income tax on mobile capital is also examined. Sas's (2017) critical argument on tax base equalization holds even if vertical externalities are omitted.

[4] Under tax revenue equalization, fiscal capacity is measured by regional tax revenues, rather than regional tax bases. This system is the basis of fiscal transfer programs at, for example, the German interstate level. (At the German municipal level, fiscal equalization is based on the representative tax system.)

competition literature assumes overlapping per unit taxes.[5] Dahlby and Wilson (2003) show that vertical externalities can be positive under overlapping taxes on labor income. In our model with capital and labor income taxes, the possibility of positive vertical externalities occurs because of overlapping taxes on capital income. The efficient matching rate for capital income taxation may be positive, whereas that for labor income taxation is always negative. These matching rates can be expressed in terms of tax rates, tax bases, and the wage elasticity of labor supply.

The analysis of ad valorem taxes in the context of tax competition deserves more attention. As stated above, the difference between per unit and ad valorem taxes has important implications for vertical tax competition. The same can be said about horizontal tax competition. The seminal works of Zodrow and Mieszkowski (1986) and Wilson (1986) consider a per unit tax on mobile capital when there are numerous perfectly competitive regions. If regions have no market powers in the capital market, per unit and ad valorem capital taxes are equivalent. However, this equivalence does not hold in the presence of a finite number of regions. Lockwood (2004) and Hoffmann and Runkel (2016) show that ad valorem capital taxes do not lead to the same non-cooperative equilibrium as per unit capital taxes.[6] Their arguments suggest that corrective fiscal transfer policies under factor income taxes is worth considering. Following Sas (2017) and Liesegang and Runkel (2018), we tackle this research subject by using a more general framework than theirs.

The remainder of this paper is organized as follows. Section 2 describes our model in which capital and labor income taxes are imposed at both federal and regional levels. Section 3 considers the horizontal and vertical externalities caused by non-cooperative regional tax policies. Although our model includes various externalities, fiscal transfer policies need not correct all these externalities because some of them are offset. Examining the nature of non-offset horizontal and vertical externalities enables us to draw similarities and differences with related studies on tax competition. Based on this examination, our formulas for tax base equalization and revenue matching

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[5] See Boadway and Keen (1996), Keen (1998), and Keen and Kotsogiannis (2002, 2004). Tsakiris et al. (2019, Section 5) argue that in a small open federation, a positive vertical externality can occur even under overlapping per unit taxes on labor and capital if these factors are substitutes in production. As in Matsumoto (2021), this paper assumes factor complementarity.

[6] Akai et. al (2011) and Ogawa (2016) consider the endogenous choice of a per unit or ad valorem capital tax in two-stage frameworks. See also Aiura and Ogawa (2013) for a similar analysis in the context of consumption tax competition.

grants are derived and discussed in Section 4. Finally, concluding remarks are given in Section 5.

2. The model

We consider a federation composed of N identical regions. The assumption of identical regions may not seem appropriate when fiscal equalization is analyzed. However, as in related studies, we make this assumption to highlight the efficiency implications of fiscal transfer policies. In our analysis, production and consumption activities are formalized in a two-period framework. The number of each region's residents is normalized to equal one.

2.1. Households

In the first period, a representative resident in each region divides his or her endowment (E) into current consumption and savings (S). In the second period, a labor-leisure decision is made.[7] Labor income and the return from savings are spent on the second-period consumption. Then, the lifetime utility function is given by

$$U(E-S)+(1+r)S+\omega L-\phi(L)+\Gamma(g,G), \tag{1}$$

where U is the first-period utility function, r is the net return on capital, ω is the net wage rate, L is labor supply, and ϕ is the disutility from labor. $\Gamma(g,G)$ is the utility of public expenditures, where g is the level of regional public expenditure and G is the level of federal public expenditure per region. We assume that $U(E-S)$ and $\Gamma(g,G)$ are increasing and concave, whereas $\phi(L)$ is increasing and convex. Given the factor prices and public expenditures, maximizing (1) yields the savings and labor supply functions, denoted by $S(r)$ and $L(\omega)$, respectively. The derivatives of these functions are positive: $S'(r)>0$ and $L'(\omega)>0$. Then, the indirect utility function is

$$V(\omega,r)+\Gamma(g,G), \text{ where } \partial V/\partial\omega=V_\omega=L \text{ and } \partial V/\partial r=V_r=S. \tag{2}$$

Throughout this paper, the subscripts attached to functions represent partial derivatives. The term $V(\omega,r)$ is referred to as private utility.

2.2. Firms

[7] This paper assumes full employment. See Kikuchi and Tamai (2019) for an analysis of tax competition and fiscal equalization under unemployment.

Production occurs in the second period. In each region, competitive firms produce a numeraire output from immobile labor and mobile capital. One unit of output can be transformed into one unit of g, G, or a private good. The well-behaved constant returns to scale (CRS) production function is given by $F(K, L)$, where K is the regional capital stock. Note that the CRS assumption implies that K and L are complements in production. Each regional government imposes taxes on labor income and source-based capital income. The federal government also imposes labor and capital income taxes. Following Liesegang and Runkel's (2018) analysis of corporate income taxation, we assume that a fixed share of capital income, $e \in (0,1)$, is exempted from taxation because of debt financing.[8] Then, the objective function of private firms' profit maximization is given by

$$(1-t)(F - reK - WL) - r(1-e)K, \tag{3}$$

where t is the aggregate tax rate on capital income and W is the gross wage rate. The regional and federal capital income tax rates are denoted as t^R and t^F, respectively. Thus, we have $t = t^R + t^F$. Similarly, we denote T as the aggregate tax rate on labor income: $T = T^R + T^F$, where T^R and T^F are the regional and federal labor income tax rates, respectively. Thus, we have $\omega = (1-T)W$.

The first-order conditions for profit maximization are

$$F_K - re = r\frac{1-e}{1-t} = q(r,t) \text{ and } F_L = W, \tag{4}$$

where q represents the taxable return on capital. The capital and labor income tax bases in each region are equal to qK and WL, respectively. The first equation in (4) can be rewritten as

$$F_K = q(r,t) + re = Q(r,t), \tag{5}$$

where Q represents the gross return on capital. This return is higher than the taxable return on capital as capital income deduction is available to private firms. The zero-profit condition yields W as a function of Q, denoted as $W(Q)$. The derivative of this function satisfies the following conditions:

$$W'(Q) = -K/L = -\delta(Q) \text{ and } \delta'(Q) < 0, \tag{6}$$

where δ is the capital-labor ratio.[9] Note that the regional capital stock K is equal to δL.

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[8] Under CRS production technology, source-base taxation on capital income is equivalent to corporate income taxation. In the case where $e = 0$, private firms depend on equity financing only.

[9] The zero-profit condition can be expressed in terms of the unit cost function: $1 = C(W, Q)$. Differentiating this equation gives (6).

2.3. Public budget constraints

Taxation and public good provision occur in the second period. This paper analyzes a symmetric equilibrium among identical regions, in which all regions choose the same tax rates and the same level of public good provision. As described later in this section, each regional government non-cooperatively sets its public policies, taking all other governments' tax rates as given. Each regional government's budget constraint is as follows:

$$g = [t^R q\delta(Q) + T^R W(Q)]L((1-T)W(Q)) + \Lambda + \Omega, \tag{7}$$

where Λ and Ω represents fiscal transfers. Λ is the net interregional fiscal transfer, and Ω is the gross fiscal transfer between the federal and regional governments.

The federal tax revenue, net of gross transfers, is equally shared by all regions through the provision of G. In our analysis of symmetric allocations, the level of G in each region can be expressed as

$$G = \frac{1}{N}\{[t^F q\delta(Q) + T^F W(Q)]L\big((1-T)W(Q)\big) + (N-1)[t^F q^*\delta(Q^*) + T^F W(Q^*)]L\big((1-T^*)W(Q^*)\big)\} - \frac{1}{N}[\Omega + (N-1)\Omega^*], \tag{8}$$

where the asterisk indicates the variables of the other $N-1$ regions. Denoting the aggregate tax rates in these regions as $t^* = t^{R*} + t^F$ and $T^* = T^{R*} + T^F$, we have $q^* = q(r, t^*)$ and $Q^* = Q(r, t^*)$.[10] Net transfers do not appear in (8) because $\Lambda + (N-1)\Lambda^* = 0$.

2.4. Capital market

The capital market equilibrium determines the net return on capital r. Let b be the net capital export of a single region.

$$b = S(r) - \delta(Q)L((1-T)W(Q)) = b(r, t, T). \tag{9}$$

Equations (4), (5), (6), and (9) imply that

$$b_r = S' + [\delta^2 L'(1-T) - \delta' L]Q_r > 0,\ b_t = [\delta^2 L'(1-T) - \delta' L]Q_t > 0, \text{ and } b_T = W\delta L' > 0, \tag{10}$$

where

[10] In (8), we need not distinguish between other regions' tax rates because each region takes all other governments' tax rates as given and because all derivatives and functions are eventually evaluated in a symmetric equilibrium, where $t = t^*$ and $T = T^*$. The same remark applies to (12), too.

$$Q_r = q_r + e = (1 - et)/(1 - t) \text{ and } Q_t = q_t = q/(1 - t). \quad (11)$$

For each of the other $N - 1$ regions, we have $b(r, t^*, T^*) = S(r) - \delta(Q^*)L((1 - T^*)W(Q^*))$. The equilibrium condition for the capital market is

$$b(r, t, T) + (N - 1)b(r, t^*, T^*) = 0, \quad (12)$$

which yields the net return on capital as a function of the tax rates: $r(t, T, t^*, T^*)$. In a symmetric equilibrium, where $t = t^*$ and $T = T^*$, the partial derivatives of this function with respect to t^R and T^R are as follows:

$$r_{t^R} = -b_t/(Nb_r) \text{ and } r_{T^R} = -b_T/(Nb_r). \quad (13)$$

Using (10), (11) and (13), we can see how a region's tax policies affect Q and Q^*. In the equilibrium, these impacts are given by

$$dQ/dt^R = Q_r r_{t^R} + Q_t,\ dQ/dT^R = Q_r r_{T^R},\ dQ^*/dt^R = Q_r r_{t^R}, \text{ and } dQ^*/dT^R = dQ/dT^R. \quad (14)$$

From (5), the impacts on q and q^* are given by

$$dq/d\tau = dQ/d\tau - er_\tau \text{ and } dq^*/d\tau = dQ^*/d\tau - er_\tau, \quad (15)$$

where τ is either t^R or T^R.

2.5. Nash game in taxes

Non-cooperative regions engage in a Nash game in taxes. Taking t^{R*}, T^{R*}, t^F, and T^F as given, each region chooses t^R and T^R to maximize (2) subject to (4)-(8) and (12). In this maximization, the external impacts of t^R and T^R on other regions' welfare are ignored. Horizontal and vertical externalities arise from the impacts on other regions' tax revenues and private incomes, and from the impacts on federal tax revenue, respectively. Note that although each regional government considers the impacts of its tax policies on G through (8), the resulting welfare impact on other regions is ignored under non-cooperative decision making. This ignorance results in the distortions caused by vertical tax competition.[11] To consider corrective fiscal transfers for horizontal and vertical externalities, we assume that both Λ and Ω are conditioned on the regional capital and labor income tax rates.

The federal government also plays a Nash game in taxes. Taking the regional income tax rates

[11] See Hoyt (2001) and Keen and Kotsogiannis (2002, 2004) for (bottom-up) vertical externalities when regions play a Nash game in taxes.

as given, t^F and T^F are set to maximize the sum of all regions' welfare. In this maximization, the impacts of t^F and T^F on each region's g through (7) are fully considered. Therefore, federal tax policies do not cause distortions owning to (top down) vertical externalities. However, the Nash assumption implies that federal tax policies cannot correct inefficient regional tax policies. In this paper, we focus on the marginal subsidies to regional taxes that corrects horizontal and vertical externalities, not on corrective federal tax policies.[12] The federal and regional income tax rates are assumed to be positive in both non-cooperative and efficient allocations.[13]

3. Analysis

3.1. Regional and federal tax policies

Our analysis begins with regional tax policies. Differentiating $V((1-T)W(Q),r)+\Gamma(g,G)$ with respect to t^R and T^R yields the first-order conditions for non-cooperative regional tax policies. Under the assumption that all regions choose the same tax rates in equilibrium, these conditions are given by

$$(t^R)\quad \Theta\delta L r_{t^R} - \delta L(1-T)Q_t + \Gamma_g\left[q\delta L + Z^R\left(Q_r r_{t^R} + Q_t\right) - t^R e\delta L r_{t^R}\right]$$
$$+\Gamma_G\left[Z^F\left(Q_r r_{t^R} + \frac{1}{N}Q_t\right) - t^F e\delta L r_{t^R}\right] + \Gamma_g \partial\Lambda/\partial t^R + (\Gamma_g - \frac{1}{N}\Gamma_G)(\partial\Omega/\partial t^R) = 0, \quad (16)$$

$$(T^R)\quad \Theta\delta L r_{T^R} - WL + \Gamma_g\left[WL + Z^R Q_r r_{T^R} - (t^R q\delta + T^R W)WL' - t^R e\delta L r_{T^R}\right]$$
$$+\Gamma_G\left[Z^F Q_r r_{T^R} - \frac{1}{N}(t^R q\delta + T^R W)WL' - t^F e\delta L r_{T^R}\right] + \Gamma_g \partial\Lambda/\partial T^R + (\Gamma_g - \frac{1}{N}\Gamma_G)(\partial\Omega/\partial T^R) = 0, \quad (17)$$

where

$$\Theta = 1 - \frac{(1-T)(1-et)}{1-t} \text{ and } Z^i = [t^i(\delta + q\delta') - T^i\delta]L - (t^i q\delta + T^i W)\delta L'(1-T) \ \ (i = R \text{ or } F). \quad (18)$$

The derivation procedures for (16) and (17) are described in the Appendix.

In (16) and (17), the first two terms represent the impacts of regional tax policies on private

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[12] This follows studies on tax competition and fiscal equalization, referred to in Section 1. When the federal government commits itself to its policies before regional decision making, federal tax policies can remedy or mitigate inefficient regional tax policies: related studies include Boadway and Keen (1996), Hoyt and Jensen (1996), Köthenbürger (2008, Section 5), Kotsogiannis and Martinez (2008), and Tsakiris et al. (2019, Section 6). For example, in Boadway and Keen's model of overlapping per unit labor taxes, vertical externalities are nullified through a negative federal tax, while the federal budget is maintained through a negative lump-sum transfer.

[13] Although the gross transfer Ω can be negative in our model, we assume that this revenue of the federal government is not enough to finance federal public good provision. We also assume that even if Ω is positive, the regional tax rates must be positive to finance regional public good provision. The net transfer Λ is equal to zero in a symmetric equilibrium.

utility. The third and fourth bracketed terms are the impacts on regional and federal tax revenues, respectively. The terms with e arise because changes in r affect the amount of capital income that is deducted from the tax base. The last two terms capture the impacts on the entitlement to fiscal transfers.

For federal tax policies, whereas the sum of all regions' welfare is maximized, the symmetry assumption implies that considering the welfare impact on a representative region is sufficient because the federal tax rates are uniform across all regions.[14] Equation (12) implies that the impacts of t^F and T^F on r are equal to $r_{t^F} = Nr_{t^R}$ and $r_{T^F} = Nr_{T^R}$, respectively. Therefore, the first-order conditions for t^F and T^F can be expressed as follows:

$$(t^F) \qquad \Theta\delta LNr_{t^R} - \delta L(1-T)Q_t + \Gamma_g\left[Z^R\left(Q_rNr_{t^R} + Q_t\right) - t^Re\delta LNr_{t^R}\right] + \Gamma_G\left[q\delta L + Z^F\left(Q_rNr_{t^R} + Q_t\right) - t^Fe\delta LNr_{t^R}\right] = 0, \tag{19}$$

$$(T^F) \qquad \Theta\delta LNr_{T^R} - WL + \Gamma_g\left[Z^RQ_rNr_{T^R} - (t^Rq\delta + T^RW)WL' - t^Re\delta LNr_{T^R}\right] + \Gamma_G\left[WL + Z^FQ_rNr_{T^R} - (t^Rq\delta + T^RW)WL' - t^Fe\delta NLr_{t^R}\right] = 0. \tag{20}$$

The interpretations of these equations are similar to those of (16) and (17).[15] In our analysis of a Nash game in taxes, the federal government takes Λ and Ω in all regions as given because they are conditioned on the regional tax rates. As argued at the end of Section 2, given regional tax policies, federal tax policies are efficient under the constraint of available taxes.

3.2. Efficient tax and transfer policies

If the federal government directly chose both the regional and federal tax rates, t^R and T^R would also be second-best efficient even in the absence of fiscal transfer policies. From (12), increasing t^R and T^R in all regions alters r by Nr_{t^R} and Nr_{T^R}, respectively. Therefore, the first-order conditions for efficient regional tax policies are given by

$$(t^R) \qquad \Theta\delta LNr_{t^R} - \delta L(1-T)Q_t + \Gamma_g\left[q\delta L + Z^R\left(Q_rNr_{t^R} + Q_t\right) - t^Re\delta LNr_{t^R}\right] + \Gamma_G\left[Z^F\left(Q_rNr_{t^R} + Q_t\right) - t^Fe\delta LNr_{t^R}\right] = 0, \tag{21}$$

$$(T^R) \qquad \Theta\delta LNr_{T^R} - WL + \Gamma_g\left[WL + Z^RQ_rNr_{T^R} - (t^Rq\delta + T^RW)WL' - t^Re\delta LNr_{T^R}\right]$$

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[14] The aggregate marginal welfare impacts of federal tax policies are equal to the left-hand sides of (19) and (20) multiplied by the number of regions N.

[15] The derivation procedures for (19) and (20) are very similar to those for (16) and (17), which are described in the Appendix. The same remark applies to (21) and (22) in the next subsection.

$$+\Gamma_G\left[Z^F Q_r N r_{T^R} - (t^F q\delta + T^F W)WL' - t^F e\delta L r_{T^R}\right] = 0. \tag{22}$$

These equations, together with (19) and (20), imply that the following condition holds in the efficient allocation in our model:

$$\Gamma_g = \Gamma_G. \tag{23}$$

When the federal and regional governments use the same set of taxes (taxes on labor and capital income) to finance different public goods (federal and regional public goods), efficiency requires that the marginal benefits of these goods be equalized.

The efficient fiscal transfer policies in our model are such that the first-order conditions for non-cooperative regional tax policies, (16) and (17), coincide with (21) and (22). Moreover, (23) also holds when the federal tax rates are set efficiently according to (19) and (20). Subtracting (16) and (17) from (21) and (22), respectively, and applying (23) to the outcomes yields the following conditions for efficient marginal transfers:

Proposition 1.

To achieve second-best efficiency, fiscal transfer policies should be set to satisfy the following conditions:

$$\Gamma_g(\partial\Lambda/\partial t^R + \tfrac{N-1}{N}\ \partial\Omega/\partial t^R) =$$
$$\Gamma_g\left[(Z^R + Z^F)Q_r(N-1)r_{t^R} + Z^F \tfrac{N-1}{N} Q_t - te\delta L(N-1)r_{t^R}\right] + \Theta\delta L(N-1)r_{t^R}, \tag{24}$$

$$\Gamma_g(\partial\Lambda/\partial T^R + \tfrac{N-1}{N}\ \partial\Omega/\partial T^R) =$$
$$\Gamma_g\left[(Z^R + Z^F)Q_r(N-1)r_{T^R} - \tfrac{N-1}{N}(t^F q\delta + T^F W)WL' - te\delta L(N-1)r_{T^R}\right] + \Theta\delta L(N-1)r_{T^R}. \tag{25}$$

3.3. Horizontal and vertical externalities

The right-hand sides of (24) and (25) capture the externalities caused by non-cooperative regional tax policies. The terms with Z^R represent horizontal fiscal externalities, whereas the terms with Z^F and WL' capture vertical fiscal externalities. A region's tax increase affects the tax revenues of other regions and the federal government through interregional capital mobility, endogenous savings and labor-leisure decisions, and vertical tax base overlap. As argued regarding (16) and (17), the terms with e are the horizontal and vertical fiscal externalities that occur because of capital income deduction. A regions' tax increase reduces the deducted capital income by decreasing the net return on capital. This raises the capital income tax revenues of all governments.

The terms with Θ capture the horizontal externalities that arise from the impacts of policy-induced changes in r on private utility. These externalities are "pecuniary" in the sense that they do not arise through public budgets (see Liesegang and Runkel 2018 for a detailed discussion). In our model with capital and labor income taxes, (2) and (6) imply that an increase in r changes V by $S - \frac{(1-T)(1-et)}{1-t}\delta L = \Theta\delta L$ when the interregional allocation of capital is symmetric: $S = K$. Thus, the impacts of t^R and T^R on each rival region's private utility are given by $\Theta\delta L r_{t^R}$ and $\Theta\delta L r_{T^R}$, respectively.

Equations (24) and (25) show that non-cooperative regional tax settings with capital and labor income taxes cause various horizontal and vertical externalities. However, the conditions for efficient fiscal transfers can be considerably simplified because some externalities are offset.

Corollary 1.

In the second-best optimum where (21), (22) and (23) hold, the marginal conditions for efficient fiscal transfers are given by

$$\partial\Lambda/\partial t^R + \frac{N-1}{N}\,\partial\Omega/\partial t^R = \frac{N-1}{N}\{t^R q b_t + t^F\delta L Q_t - T^F[WL'(1-T) + L]\delta Q_t\}, \tag{26}$$

$$\partial\Lambda/\partial T^R + \frac{N-1}{N}\,\partial\Omega/\partial T^R = \frac{N-1}{N}(t^R q b_T - T^F W^2 L'). \tag{27}$$

Proof. See Appendix. □

The right-hand sides of (26) and (27) represent the "non-offset" externalities that should be corrected by fiscal transfer policies. The terms with the regional capital income tax rate capture the non-offset horizontal fiscal externalities that consist of the impacts of t^R and T^R on other regions' capital income tax revenues when the total supply of capital is hypothetically fixed. The terms with the federal tax rates capture the non-offset vertical fiscal externalities that consist of the impacts of t^R and T^R on federal tax revenue when the net capital return is hypothetically fixed. Other horizontal and vertical externalities that does not appear in (26) and (27), including those related to e and Θ, are mutually offset.

The non-offset horizontal externalities correspond to the conventional fiscal externalities caused by tax base mobility, which have been discussed in the literature on capital tax competition (see Wilson 1986 and Wildasin 1989 for the basic arguments). Holding the total supply of capital fixed hypothetically, a region's increase in t^R and T^R reduces the capital supply in that region by $b_t(N-1)/N$ and $b_T(N-1)/N$, respectively, thereby increasing other regions' capital by the same amount.[16] These horizontal externalities are positive whenever t^R is positive. One would expect that the impacts of t^R and T^R on $q^* = q(r, t^*)$ affect other regions' capital income tax revenues and distort regional tax policies. However, these externalities through the taxable return on capital in other regions are included in the offset externalities.

One would also expect that the endogeneity of savings creates a negative horizontal fiscal externality because increasing t^R and T^R reduces the total supply of capital. In addition, a region's tax increase raises other regions' labor income tax revenues by increasing W^*L^*. These horizontal fiscal externalities, with opposite signs, are included in the offset externalities. Thus, in our model, the impact on other regions' capital income tax revenues due to interregional mobility is the sole source of horizontal externalities that should be corrected by fiscal transfer policies.

Although vertical externalities can occur through the impact of regional tax policies on r, these external impacts are also offset. Therefore, the impacts of t^R and T^R on federal tax revenue through $S(r)$, $q^* = q(r, t^*)$, $Q^* = Q(r, t^*)$, and $W^* = W(Q^*)$ can be ignored when considering the influence of inefficient vertical tax competition.[17] This in turn implies that the impacts on federal tax revenue through δL can be ignored because of the irrelevance of changes in $S(r)$ and $\delta^* L^*$.[18] Moreover, the impacts of T^R on federal tax revenue through $q(r,t)$, $Q(r,t)$, and $W(Q)$ are irrelevant because these impacts are due to policy-induced changes in r.

In (26) and (27), the term $t^F \delta L Q_t$ only remains as the positive external impact on federal capital income tax revenue.[19] For changes in federal revenue from labor income, the term

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[16] Using (10), the changes in the regional capital stock δL can be expressed as $-(b_r r_{t^R} + b_t)$ and $-(b_r r_{T^R} + b_T)$ when $S' = 0$. Equation (13) implies that these changes are equal to $-b_t(N-1)/N$ and $-b_T(N-1)/N$, respectively.

[17] This implies that the non-offset vertical externalities caused by a region's tax increase consist of the changes in the federal revenue raised in the region with higher taxes. The impacts on the federal revenue raised in all other regions are included in the offset vertical externalities.

[18] This argument for the "offset vertical" externalities should not be confused with that for the "non-offset horizontal" externalities in which the changes in δL are crucial.

[19] Noting from (11) that $Q_t = q_t$, this term effectively represents the impact through the taxable return on capital.

$-T^F[WL'(1-T)+L]\delta Q_t$ captures the impact of t^R on $Q(r,t)$ at a given r, whereas $-T^F W^2 L'$ is caused by the impact of T^R on ω at a given $W(Q)$. These changes multiplied by $(N-1)/N$ are equal to the sum of the changes in G in other regions that represents the non-offset vertical fiscal externalities. When the federal tax rates are positive, the sign of the vertical externality term is negative in (27), while it can be positive in (26).

To summarize, the offset externalities are: (i) pecuniary externalities that directly affect other regions' private utility; (ii) horizontal and vertical fiscal externalities through capital income deduction; (iii) horizontal and vertical fiscal externalities that occur because of the impacts on the taxable return on capital in other regions; (iv) horizontal and vertical fiscal externalities arising from changes in other regions' labor income; (v) horizontal and vertical fiscal externalities due to the endogeneity of savings; and, (vi) vertical fiscal externalities caused by the impacts of regional labor income taxation on the taxable return on capital and the gross wage rate in all regions. The sign and magnitude of these externalities do not affect the fiscal transfer policies that correct inefficient regional tax policies.

3.4. Discussion

This paper is closely related to Liesegang and Runkel's (2018) analysis of horizontal corporate income tax competition. Following their analysis, we have considered the pecuniary externalities arising from the impacts on private utility and the fiscal externalities caused by capital income deduction and by policy-induced changes in the taxable return on capital in other regions. We have shown that these externalities are included in the offset externalities; see (i), (ii), and (iii) in the previous subsection. Liesegang and Runkel (2018, equation 16) effectively reach the same result by demonstrating that the sum of all (horizontal) externalities in their model is equal to the external impact caused by interregional capital mobility, which corresponds to the first term on the right-hand side of our (26). Our findings are distinctive as our model includes a variety of other externalities due to the endogeneity of savings and labor supply, the availability of labor income taxation, and the presence of vertical tax overlap, which Liesegang and Runkel (2018) do not consider.

For this extension of Liesegang and Runkel (2018), the availability of labor income taxation is crucial. Indeed, Sas's (2017, Proposition 3) analysis of source-based ad valorem taxation implies that when only capital income tax is available, endogenous savings cause both horizontal and vertical externalities that should be corrected by fiscal transfer policies.[20] In our model where labor income is also taxed, such externalities are offset by those arising from changes in other regions' labor income; see (iv) and (v) in the previous subsection. Thus, the set of available taxes has a significant impact on the nature of externalities. Interestingly, increasing tax instruments enhances the potential for offsetting externalities and thus simplifies the analysis.

This implication of multiple tax bases is similar to that discussed by Matsumoto (2021) regarding per unit factor taxes. Bucovetsky and Smart (2006) and Kotsogiannis (2010) show that if a per unit capital tax is the only available tax instrument, the elasticity of savings is relevant to efficient fiscal transfer policies. However, this relevance does not hold when both labor and capital are taxed. As Matsumoto (2021, Section 3.2) argues, the offsetting nature of externalities under multiple tax bases implies that horizontal externalities need to be corrected only for the distortion caused by tax base mobility. This argument of inefficient horizontal tax competition is similar between per unit and ad valorem taxes.[21]

Our analysis is also comparable to Dahlby and Wilson's (2003) analysis of vertical fiscal externalities under ad valorem taxes. In their model with overlapping taxes on labor income, a region's tax increase can raise labor income. This positive correlation between the tax rate and tax base is possible because while the supply of labor declines, the gross wage rate rises. The sign of the resulting vertical externality depends on the relative magnitude of these impacts. However, this argument does not apply to our model with multiple tax bases. As (vi) in the previous section shows, we can ignore changes in the gross wage rate when considering the vertical external impacts of regional labor income taxation. Consequently, the vertical externality term in (27) is negative. This result for labor taxation is akin to the case of per unit taxes in which vertical externalities are negative.

In our model, the possibility of positive vertical externalities occurs because of overlapping taxes on capital income. This is due to the impact of regional capital income taxation on the taxable return on capital (see footnote 19), which is specific to the case of ad valorem taxes. Moreover, the

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[20] Sas (2017, equation 30) implies that under source-based taxation on capital income, efficient fiscal transfers depend on the elasticity of savings.

[21] As Lockwood (2004) and Hoffmann and Runkel (2016) show, the magnitude of the distortion due to tax base mobility differs between these tax regimes.

irrelevance of endogenous savings to inefficient regional tax policies contrasts with the case of per unit capital taxes in Keen and Kotsogiannis (2002, 2004) and Kotsogiannis (2010), where the sign of vertical externalities is negative because of the negative impact of regional tax policies on the total supply of capital.

4. Implementing efficient fiscal transfers

4.1. Tax base equalization and revenue matching transfers

In our model, setting the net and gross fiscal transfers Λ and Ω to satisfy (26) and (27) achieves second-best efficiency. Among the various possible combinations of these transfers, we investigate the case in which horizontal and vertical externalities are internalized by Λ and Ω, respectively. Our net fiscal transfer is built on the concept of tax base equalization, whereas the gross transfer is formalized as a system of revenue matching grants.

The standard system of tax base equalization (the representative tax system) assumes that each region's entitlement is equal to the difference between its tax base and the average tax base of all regions, multiplied by the average tax rate of all regions. When this system is applied to the regional capital income tax in our model, each region's entitlement, denoted as Λ^s, is equal to

$$\Lambda^s = \bar{t}^R\,(\overline{q\delta L} - q\delta L), \tag{28}$$

where the bars indicate the average values of the variables in all regions: $\bar{t}^R = [(N-1)t^{R*} + t^R]/N$ and $\overline{q\delta L} = [(N-1)q^*\delta^* L^* + q\delta L]/N$. In contrast, our net fiscal transfer assumes that each region's tax base is evaluated by the "average" taxable return on capital in all regions.

$$\Lambda = \bar{t}^R\,\bar{q}(\overline{\delta L} - \delta L), \tag{29}$$

where $\bar{q} = [(N-1)q^* + q]/N$ and $\overline{\delta L} = [(N-1)\delta^* L^* + \delta L]/N$. This formula is called the return-averaged tax base equalization. In the next subsection, we compare these different equalization systems.

We demonstrate how our return-averaged tax base equalization works. It should be emphasized that for efficiency, there is no need to apply fiscal equalization to regional labor income taxation. This is due to the offsetting nature of the external impacts on other regions' labor income tax revenues; see (iv) in Section 3.3. To derive the marginal impacts of regional taxes on Λ, we consider the impacts on the regional capital stock:

$$d(\delta^* L^*)/dt^R = BdQ^*/dt^R,\ d(\delta L)/dt^R = BdQ/dt^R,$$

$$d(\delta^* L^*)/dT^R = BdQ^*/dT^R,\ d(\delta L)/dT^R = BdQ/dT^R - b_T, \tag{30}$$

where $B = \delta' L - \delta^2 L'(1-T)$. Using (10), (14), and (30), it can be shown that under the symmetry assumption,

$$\partial \Lambda / \partial t^R = \frac{N-1}{N} t^R q[d(\delta^* L^*)/dt^R - d(\delta L)/dt^R] = \frac{N-1}{N} t^R q b_t, \tag{31}$$

$$\partial \Lambda / \partial T^R = \frac{N-1}{N} t^R q[d(\delta^* L^*)/dT^R - d(\delta L)/dT^R] = \frac{N-1}{N} t^R q b_T. \tag{32}$$

These equations implies that the non-offset horizontal fiscal externalities in (26) and (27) are corrected by applying the return-averaged tax base equalization to the regional capital income tax.

Our gross transfer system is defined as follows:

$$\Omega = mt^R + MT^R, \tag{33}$$

where m and M are the matching rates for t^R and T^R, respectively. To correct the non-offset vertical fiscal externalities in (26) and (27), these matching rates are set such that

$$m = t^F \delta L Q_t - T^F[WL'(1-T) + L]\delta Q_t = \frac{t^F - (1+\varepsilon)T^F}{1-t} q\delta L, \tag{34}$$

$$M = -T^F W^2 L' = \frac{-\varepsilon T^F}{1-T} WL, \tag{35}$$

where $\varepsilon = L'W(1-T)/L$ is the net wage elasticity of labor supply. The efficient matching rates can be expressed in terms of the tax rates, the income tax bases, and the elasticity of labor supply. The sign of the efficient matching rates indicates the sign of the non-offset vertical fiscal externalities caused by regional tax policies. In our model, the amount of gross fiscal transfers can be either positive or negative because of the ambiguous sign of m. This result can be related to Kotsogiannis and Martinez's (2008) argument regarding vertical fiscal imbalance, where the efficient direction of gross transfers between federal and regional governments is generally ambiguous under overlapping ad valorem taxes.[22]

These arguments are summarized in the following proposition:

Proposition 2.

An efficient combination of net and gross fiscal transfers is given by (29) and (33)-(35). The return-

[22] This contrasts with Boadway and Keen's (1996) argument regarding negative transfers to lower-level government (see footnote 12). Although Boadway and Keen (1996) and Kotsogiannis and Martinez (2008) consider lump-sum fiscal transfers under federal leadership, the efficient direction of gross transfers depends on the sign of vertical externalities.

averaged capital income tax base equalization corrects inefficient horizontal tax competition, while the revenue matching grant system corrects inefficient vertical tax competition.

4.2. Discussion

The standard tax base equalization system in (28) fails to eliminate the inefficiency of horizontal tax competition. Indeed, differentiating (28) with respect to the regional tax rates and evaluating the outcomes in a symmetric equilibrium gives the following equations:

$$\partial \Lambda^s/\partial t^R = \frac{N-1}{N} t^R \{q[d(\delta^* L^*)/dt^R - d(\delta L)/dt^R] + \delta L(dq^*/dt^R - dq/dt^R)\}$$
$$= \frac{N-1}{N} t^R (qb_t - \delta L Q_t), \quad (36)$$
$$\partial \Lambda^s/\partial T^R = \frac{N-1}{N} t^R \{q[d(\delta^* L^*)/dT^R - d(\delta L)/dT^R] + \delta L(dq^*/dT^R - dq/dT^R)\}$$
$$= \frac{N-1}{N} t^R q b_T, \quad (37)$$

where (14) and (15) are used to derive the second equality. By comparing (31) and (32) with (36) and (37), we can see that although the standard tax base equalization system corrects the horizontal externalities due to regional labor income taxation, it cannot fully internalize the externalities caused by regional capital income taxation. Equation (36) implies that under-taxation of capital income occurs because $\partial \Lambda^s/\partial t^R$ is smaller than $\partial \Lambda/\partial t^R$.

Under the standard equalization system, each region's entitlement is influenced by the changes in relative fiscal capacity due to the impacts of regional tax policies on the taxable return on capital. For the impact of T^R, these changes are irrelevant because $dq^*/dT^R = dq/dT^R$. However, for the impact of t^R, dq/dt^R is higher than dq^*/dt^R, implying that given interregional capital allocation, fiscal capacity is relatively improved in the region with higher t^R. Thus, the marginal impact of t^R on the equalization entitlement is small in comparison with our return-averaged tax base equalization. This reduction in the entitlement is harmful to removing the inefficiency of horizontal tax competition.

With per unit factor taxes, this defect of the standard equalization system is not revealed. Indeed, Matsumoto's (2021) analysis of per unit capital and labor taxes implies that applying this system to regions' capital tax bases corrects horizontal fiscal externalities.[23] As emphasized in

[23] In Matsumoto's (2021) analysis where tax base equalization is the only policy instrument to deal with both horizontal and vertical externalities, efficiency requires that equalization is implemented for regional capital

Section 3.4, the nature of non-offset horizontal externalities is similar between per unit and ad valorem taxes in the sense that only the distortion due to interregional tax base mobility is relevant. However, the functioning of the standard tax base equalization system differs considerably depending on whether taxes are per unit or ad valorem. Our return-averaged system resolves this mismatch.

Our argument in this subsection is closely related to the critical argument of Liesegang and Runkel (2018, Proposition 1) on tax base equalization. In their model, the standard tax base equalization results in under-taxation of corporate income, which is consistent with our result derived from (36). To internalize horizontal externalities, Liesegang and Runkel (2018, Proposition 3) propose a combination of tax revenue and private income equalization. Unlike our return-averaged equalization, this policy combination is effectively oriented towards interregional welfare equalization. As Myers (1990) and Wellisch (2000) argue, welfare equalization through interregional population mobility can eliminate inefficiencies associated with horizontal tax competition. The essence of Liesegang and Runkel's (2018) remedy is that fiscal transfer policies can have the same effect as population mobility.[24]

Our equalization system is applicable to Liesegang and Runkel's (2018) model with corporate income taxation in which the tax base is defined as revenue minus the cost of labor employment and capital income deduction is also considered. In this setting, for fiscal equalization, the cost of labor employment should be evaluated by the average wage rate of all regions, not by the regional wage rates.[25] With this modified system of tax base equalization, which is analogous to ours, the inefficiency of horizontal tax competition is completely removed from their model.

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tax base while the rate of equalization is negative for regional labor tax base. That is, fiscal transfers are given to regions with an above average labor tax base. This negative rate of equalization is needed to internalize vertical externalities. If revenue matching grants are also used to correct vertical externalities, the capital tax base only should be subject to equalization in order to internalize horizontal externalities.

[24] Silva (2017) provides a similar argument in the context of ex post fiscal transfers.

[25] In Liesegang and Runkel (2018), the corporate income tax base includes pure profits under decreasing returns to scale. Still, the essence of our arguments holds. In terms of our notation, ignoring capital income deduction for simplicity, the return-averaged equalization implies that region i's entitlement is given by

$$\Lambda_i = \bar{t}^R \left[\frac{\sum_{j=1}^{N}\left(F_j - \bar{W}L_j\right)}{N} - F_i - \bar{W}L_i \right],$$

where $\bar{W} = \sum_{j=1}^{N} W_j / N$.

5. Concluding remarks

Under fiscal decentralization, inefficiencies arise from horizontal and vertical interactions among governments. Fiscal transfer policies can be used to correct such inefficiencies. In the context of tax competition, we have considered a combination of tax base equalization and revenue matching grants that internalize horizontal and vertical externalities. Undoubtedly, revenue matching grants serve as a means of correcting inefficient regional tax policies. However, starting with the work of Köthenbürger (2002), opinions vary regarding the efficiency effect of tax base equalization. This equalization system is not suitable for dealing with vertical externalities (c.f., Matsumoto 2021). Furthermore, it cannot correct horizontal externalities if mobile capital is subject to an ad valorem tax (c.f., Sas 2017; Liesegang and Runkel 2018). This is particularly important as factor taxes are usually implemented as income or asset value taxes, not as specific taxes.

Our return-averaged tax base equalization demonstrates that fiscal equalization can internalize horizontal externalities even under ad valorem taxes. When this system is used together with revenue matching grants that correct vertical externalities, non-cooperative regional tax policies become efficient. We have considered how our fiscal transfer policies work in a framework with multiple tax bases and endogenous factor supply. Although this framework is very complex, the offsetting nature of externalities enables us to makes the analysis transparent.

We do not assert that our fiscal transfer policies are superior to other possible policy options with the same efficiency effect. Liesegang and Runkel's (2018) combination of tax revenue and private income equalization seems to be an interesting option for correcting inefficient horizontal tax competition, despite their concern about its practicality. Alternatively, following Wildasin (1989) and Dahlby (1996), revenue matching grants can be used to internalize both horizontal and vertical externalities. Whereas the focus of this paper is on the distortions of tax competition, we think that more sophisticated models with other distortions are needed to compare alternative fiscal transfer policies. Obviously, different set of federal interventions would be required to achieve efficiency under different economic and political environments.

Appendix

Derivation of equations (16) and (17).

The marginal impacts of t^R and T^R on $V((1-T)W(Q),r)$ are derived by using (2), (6), (11), (14), and (18):

$$dV/dt^R = r_{t^R}S - (1-T)\delta L\big(Q_r r_{t^R} + Q_t\big) = \delta L\left[1 - \frac{(1-T)(1-et)}{1-t}\right] r_{t^R} - (1-T)\delta L Q_t$$

$$= \Theta\delta L r_{t^R} - (1-T)\delta L Q_t, \tag{A.1}$$

$$dV/dT^R = r_{T^R}S - (1-T)\delta L Q_r r_{T^R} - WL = \delta L\left[1 - \frac{(1-T)(1-et)}{1-t}\right] r_{T^R} - WL$$

$$= \Theta\delta L r_{T^R} - WL. \tag{A.2}$$

The second equality of these equations holds because $S = K$ in a symmetric equilibrium.

Differentiating (7) with respect to t^R yields

$$dg/dt^R = L\{q\delta + t^R[\delta(Q_r r_{t^R} + Q_t - e r_{t^R}) + q\delta'(Q_r r_{t^R} + Q_t)] - T^R\delta(Q_r r_{t^R} + Q_t)\}$$

$$-(t^R q\delta + T^R W)\delta L'(1-T)\big(Q_r r_{t^R} + Q_t\big) + \partial\Lambda/\partial t^R + \partial\Omega/\partial t^R, \tag{A.3}$$

where (6), (14), and (15) were applied. Using (18), this equation is rewritten as follows:

$$dg/dt^R = q\delta L + Z^R(Q_r r_{t^R} + Q_t) - t^R e\delta L r_{t^R} + \partial\Lambda/\partial t^R + \partial\Omega/\partial t^R. \tag{A.4}$$

Similarly, we have

$$dg/dT^R = WL + Z^R Q_r r_{t^R} - (t^R q\delta + T^R W)WL' - t^R e\delta L r_{T^R} + \partial\Lambda/\partial T^R + \partial\Omega/\partial T^R. \tag{A.5}$$

Differentiating (8) with respect to t^R and using (6), (14), and (15) yields

$$N(dG/dt^R) = L\{t^F[\delta(Q_r r_{t^R} + Q_t - e r_{t^R}) + q\delta'(Q_r r_{t^R} + Q_t)] - T^F\delta(Q_r r_{t^R} + Q_t)\}$$

$$-(t^F q\delta + T^F W)\delta L'(1-T)\big(Q_r r_{t^R} + Q_t\big) - \partial\Omega/\partial t^R$$

$$+(N-1)L\{t^F\big[\delta\big(Q_r r_{t^R} - e r_{t^R}\big) + q\delta' Q_r r_{t^R}\big] - T^F\delta Q_r r_{t^R}\}$$

$$-(N-1)(t^F q\delta + T^F W)\delta L'(1-T)Q_r r_{t^R}, \tag{A.6}$$

where this equation is evaluated at $t = t^*$ and $T = T^*$. The terms with $N-1$ represent the change in the federal tax revenue raised in the other $N-1$ regions. Using (18), (A.6) can be rewritten as

$$dG/dt^R = \tfrac{1}{N}\{[t^F(\delta + q\delta') - T^F\delta]L - (t^F q\delta + T^F W)\delta L'(1-T)\}[NQ_r r_{t^R} + Q_t]$$

$$-t^F e\delta L r_{t^R} - \tfrac{1}{N}\partial\Omega/\partial t^R$$

$$= Z^F(Q_r r_{t^R} + \tfrac{1}{N}Q_t) - t^F e\delta L r_{t^R} - \tfrac{1}{N}\partial\Omega/\partial t^R. \tag{A.7}$$

Similarly, differentiating (8) with respect to T^R yields

$$dG/dT^R = Z^F Q_r r_{T^R} - \frac{1}{N}(t^F q\delta + T^F W)WL' - t^F e\delta L r_{T^R} - \frac{1}{N}\partial\Omega/\partial T^R. \tag{A.8}$$

The first-order conditions for t^R and T^R are given by $dV/dt^R + \Gamma_g dg/dt^R + \Gamma_G dG/dt^R = 0$ and $dV/dT^R + \Gamma_g dg/dT^R + \Gamma_G dG/dT^R = 0$, respectively. By substituting (A.1), (A.2), (A.4), (A.5), (A.7), and (A.8) into these equations, (16) and (17) are derived.

Derivation of equations (26) and (27).

For (26) and (27), the following lemma is crucial.

Lemma.

The efficient regional capital and labor income taxes satisfy the following condition:

$$\Theta\delta L + \Gamma_g[(Z^R + Z^F)Q_r - te\delta L] = -\Gamma_g tqb_r. \tag{A.9}$$

Proof. Subtracting (22) multiplied by $\delta(1-T)Q_t/W$ from (21) and applying (23) to the outcome yields

$$\begin{aligned} &\{\Theta\delta L + \Gamma_g[(Z^R + Z^F)Q_r - te\delta L]\}N\left(r_{t^R} - \mu r_{T^R}\right) \\ &\quad + \Gamma_g[q\delta L - \mu WL + (Z^R + Z^F)Q_t + \mu(tq\delta + TW)WL'] = 0, \end{aligned} \tag{A.10}$$

where $\mu = \delta(1-T)Q_t/W$. Using (10), (11), (13), and (18), we obtain the following equations:

$$N\left(r_{t^R} - \mu r_{T^R}\right) = \delta' L Q_t / b_r,$$

$$q\delta L - \mu WL = \delta L Q_t (T - t),$$

$$(Z^R + Z^F)Q_t + \mu(tq\delta + TW)WL' = [t(\delta + q\delta') - T\delta]LQ_t.$$

Substituting these equations into (A.10) and manipulating the terms yields (A.9). □

We now transform (24) and (25) into (26) and (27), respectively. Substituting (A.9) into (24) yields

$$\partial\Lambda/\partial t^R + \frac{N-1}{N}\,\partial\Omega/\partial t^R = \frac{N-1}{N}(tqb_t + Z^F Q_t), \tag{A.11}$$

where $b_r r_{t^R} = -b_t/N$ is applied using (13). Equations (10) and (18) imply

$$t^F qb_t + Z^F Q_t = t^F \delta L Q_t - T^F[WL'(1-T) + L]\delta Q_t. \tag{A.12}$$

Equation (26) is derived from (A.11) and (A.12).

Similarly, substituting (A.9) into (25) yields

$$\partial \Lambda / \partial T^R + \frac{N-1}{N}\ \partial \Omega / \partial T^R = \frac{N-1}{N}[tqb_T - (t^F q\delta + T^F W)WL'], \tag{A.13}$$

where $b_r r_{T^R} = -b_T/N$ is applied. As (10) implies that $t^F qb_T - (t^F q\delta + T^F W)WL' = -T^F W^2 L'$, (A.13) reduces to (27).

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